13. Complex Numbers

Exercise 13.1

1. Question

Evaluate the following:

(i) i^{457}

(ii) i^{528}

(iii) $\frac{1}{i^{58}}$

(iv) $i^{37}+\frac{1}{i^{67}}$

(v) $\left(i^{41}+\frac{1}{i^{257}}\right)^9$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(vii) (vii) $i^{30} + i^{40} + i^{60}$

(viii) $i^{49} + i^{68} + i^{89} + i^{118}$

Answer

i. $i^{457} = i^{(456 + 1)}$

$= i^{4(114)} \times i$

$= (1)^{114} \times i = i$ since $i^4 = 1$

ii. $i^{528} = i^{4(132)}$

$= (1)^{132} = 1$ since $i^4 = 1$

iii. $\frac{1}{i^{58}} = \frac{1}{i^{56+2}}$

$= \frac{1}{i^{56}\times i^2} = \frac{1}{(i^4)^{14}\times i^2}$ since $i^4 = 1$

$= \frac{1}{i^2} = \frac{1}{-1} = -1$ since $i^2 = -1$

iv. $i^{37} + \frac{1}{i^{67}} = i^{36+1}+\frac{1}{i^{64+3}} = i+\frac{1}{i^3}$

[since $i^4 = 1$]

$i^{37} + \frac{1}{i^{67}} = i+\frac{i}{i^4}$

$i^{37} + \frac{1}{i^{67}} = i+i = 2i$

v. $\left(i^{41} + \frac{1}{i^{257}}\right)^9 = \left(i^{40+1} + \frac{1}{i^{256+1}}\right)^9$

$= \left(i + \frac{1}{i}\right)^9 = (i - i) = 0$

[since $\frac{1}{i} = -i$]

vi. $(i^{77} + i^{70} + i^{87} + i^{414})^3 = (i^{(76 + 1)} + i^{(68 + 2)} + i^{(84 + 3)} + i^{(412 + 2)})^3$

$(i^{77} + i^{70} + i^{87} + i^{414})^3 = (i + i^2 + i^3 + i^2)^3$

[since $i^3 = -i$, $i^2 = -1$]

$= (i + (-1) + (-i) + (-1))^3 = (-2)^3$

$(i^{77} + i^{70} + i^{87} + i^{414})^3 = -8$

vii. $i^{30} + i^{40} + i^{60} = i^{(28 + 2)} + i^{40} + i^{60}$

$= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{15}$

$= i^2 + 1^{10} + 1^{15} = -1 + 1 + 1 = 1$

viii. $i^{49} + i^{68} + i^{89} + i^{118} = i^{(48 + 1)} + i^{68} + i^{(88 + 1)} + i^{(116 + 2)}$

$= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{11} \times i + (i^4)^{29} \times i^2$

$= i + 1 + i - 1 = 2i$

2. Question

Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number ?

Answer

$1 + i^{10} + i^{20} + i^{30} = 1 + i^{(8 + 2)} + i^{20} + i^{(28 + 2)}$

$= 1 + (i^4)^2 \times i^2 + (i^4)^5 + (i^4)^7 \times i^2$

$= 1 - 1 + 1 - 1 = 0$

[since $i^4 = 1$, $i^2 = -1$]

Hence , $1 + i^{10} + i^{20} + i^{30}$ is a real number.

3 A. Question

Find the value of following expression:

$i^{49} + i^{68} + i^{89} + i^{110}$

Answer

$i^{49} + i^{68} + i^{89} + i^{110} = i^{(48 + 1)} + i^{68} + i^{(88 + 1)} + i^{(108 + 2)}$

$= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{11} \times i + (i^4)^{27} \times i^2$

$= i + 1 + i - 1 = 2i$

[since $i^4 = 1$, $i^2 = -1$]

$i^{49} + i^{68} + i^{89} + i^{110} = 2i$

3 B. Question

Find the value of following expression:

$i^{30} + i^{80} + i^{120}$

Answer

$i^{30} + i^{80} + i^{120} = i^{(28 + 2)} + i^{80} + i^{120}$

$= (i^4)^7 \times i^2 + (i^4)^{20} + (i^4)^{30}$

$= -1 + 1 + 1 = 1$

[since $i^4 = 1, i^2 = -1$]

$i^{30} + i^{80} + i^{120} = 1$

3 C. Question

Find the value of following expression:

$i + i^2 + i^3 + i^4$

Answer

$i + i^2 + i^3 + i^4 = i + i^2 + i^2 \times i + i^4$

$= i - 1 + (-1) \times i + 1$

since $i^4 = 1, i^2 = -1$

$= i - 1 - i + 1 = 0$

3 D. Question

Find the value of following expression:

$i^5 + i^{10} + i^{15}$

Answer

$i^5 + i^{10} + i^{15} = i^{(4+1)} + i^{(8+2)} + i^{(12+3)}$

$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^3$

$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^2 \times i$

$= 1 \times i + 1 \times (-1) + 1 \times (-1) \times i$

$= i - 1 - i = -1$

3 E. Question

Find the value of following expression:

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$$

Answer

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = \frac{i^{10}(i^{582} + i^{580} + i^{578} + i^{576} + i^{574})}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$$

$= i^{10}$

$= i^8 i^2$

$= (i^4)^2 i^2$ Since $i^4 = 1, i^2 = -1$

$= (1)^2 (-1)$

$= -1$

3 F. Question

Find the value of following expression:

$1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$

Answer

$1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} = 1 + (-1) + 1 + (-1) + 1 + \ldots + 1$

$= 1$

3 G. Question

Find the value of following expression:

$(1 + i)^6 + (1 - i)^3$

Answer

$(1 + i)^6 + (1 – i)^3 = \{(1 + i)^2\}^3 + (1 – i)^2 (1 – i)$

$= \{1 + i^2 + 2i\}^3 + (1 + i^2 - 2i)(1 – i)$

$= \{1 – 1 + 2i\}^3 + (1 – 1 – 2i)(1 – i)$

$= (2i)^3 + (– 2i)(1 – i)$

$= 8i^3 + (– 2i) + 2i^2$

[since $i^3 = – i$, $i^2 = – 1$]

$= – 8i – 2i – 2$

$= – 10 i – 2$

$= – 2(1 + 5i)$

Exercise 13.2

1 A. Question

Express the following complex numbers in the standard form a + i b :

(1 + i) (1 + 2i)

Answer

Given:

⇒ a+ib = (1+i)(1+2i)

⇒ a+ib = 1(1+2i)+i(1+2i)

⇒ $a+ib = 1+2i+i+2i^2$

We know that $i^2=-1$

⇒ a+ib = 1+3i+2(-1)

⇒ a+ib = 1+3i-2

⇒ a+ib=-1+3i

∴ The values of a, b are -1, 3.

1 B. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{3+2i}{-2+i}$$

Answer

Given:

$\Rightarrow a+ib = \frac{3+2i}{-2+i}$

Multiplying and dividing with -2-i

$\Rightarrow a+ib = \frac{3+2i}{-2+i} \times \frac{-2-i}{-2-i}$

$\Rightarrow a+ib = \frac{3(-2-i)+2i(-2-i)}{(-2)^2-(i)^2}$

We know that $i^2=-1$

$\Rightarrow a + ib = \frac{-6-3i-4i-2i^2}{4-i^2}$

$\Rightarrow a + ib = \frac{-6-7i-2(-1)}{4-(-1)}$

$\Rightarrow a + ib = \frac{-4-7i}{5}$

∴ The values of a, b are $-\frac{4}{5}, -\frac{7}{5}$.

1 C. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{1}{(2+i)^2}$$

Answer

Given:

$\Rightarrow a + ib = \frac{1}{(2+i)^2}$

$\Rightarrow a + ib = \frac{1}{2^2+i^2+2(2)(i)}$

We know that i^2=-1

$\Rightarrow a + ib = \frac{1}{4-1+4i}$

$\Rightarrow a + ib = \frac{1}{3+4i}$

Multiplying and diving with 3-4i

$\Rightarrow a + ib = \frac{1}{3+4i} \times \frac{3-4i}{3-4i}$

$\Rightarrow a + ib = \frac{3-4i}{3^2-(4i)^2}$

$\Rightarrow a + ib = \frac{3-4i}{9-16i^2}$

$\Rightarrow a + ib = \frac{3-4i}{9-16(-1)}$

$\Rightarrow a + ib = \frac{3-4i}{25}$

∴ The values of a, b is $\frac{3}{25}, \frac{-4}{25}$.

1 D. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{1-i}{1+i}$$

Answer

Given:

$\Rightarrow a + ib = \frac{1-i}{1+i}$

Multiplying and dividing by 1-i

$\Rightarrow a + ib = \frac{1-i}{1+i} \times \frac{1-i}{1-i}$

$\Rightarrow a + ib = \frac{1^2+i^2-2(1)(i)}{1^2-(i)^2}$

We know that i^2=-1

$\Rightarrow a + ib = \frac{1+(-1)-2i}{1-(-1)}$

$\Rightarrow a + ib = \frac{-2i}{2}$

$\Rightarrow a + ib = -i$

$\Rightarrow a + ib = 0 - i$

∴ The values of a, b is 0, -1.

1 E. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{(2+i)^3}{2+3i}$$

Answer

Given:

$\Rightarrow a + ib = \frac{(2+i)^3}{2+3i}$

$\Rightarrow a + ib = \frac{2^3+i^3+3(2)^2(i)+3(i)^2(2)}{2+3i}$

$\Rightarrow a + ib = \frac{8+(i^2.i)+3(4)(i)+6i^2}{2+3i}$

We know that i^2=-1

$\Rightarrow a + ib = \frac{8+(-1)i+12i+6(-1)}{2+3i}$

$\Rightarrow a + ib = \frac{2+11i}{2+3i}$

Multiplying and dividing with 2-3i

$\Rightarrow a + ib = \frac{2+11i}{2+3i} \times \frac{2-3i}{2-3i}$

$\Rightarrow a + ib = \frac{2(2-3i)+11i(2-3i)}{2^2-(3i)^2}$

$\Rightarrow a + ib = \frac{4-6i+22i-33i^2}{4-9i^2}$

We know that i^2=-1

$\Rightarrow a + ib = \frac{4+16i-33(-1)}{4-9(-1)}$

$\Rightarrow a + ib = \frac{37+16i}{13}$

∴ The values of a, b are $\frac{37}{13}, \frac{16}{13}$.

1 F. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{(1+i)(1+\sqrt{3}i)}{1-i}$$

Answer

Given:

$\Rightarrow a+ib=\frac{(1+i)(1+\sqrt{3}i)}{1-i}$

$\Rightarrow a+ib=\frac{1(1+\sqrt{3}i)+i(1+\sqrt{3}i)}{1-i}$

$\Rightarrow a+ib=\frac{1+\sqrt{3}i+i+\sqrt{3}i^2}{1-i}$

We know that i^2=-1

$\Rightarrow a+ib=\frac{1+(\sqrt{3}+1)i+\sqrt{3}(-1)}{1-i}$

$\Rightarrow a+ib=\frac{(1-\sqrt{3})+(1+\sqrt{3})i}{1-i}$

Multiplying and dividing with 1+i

$\Rightarrow a+ib=\frac{(1-\sqrt{3})+(1+\sqrt{3})i}{1-i}\times\frac{1+i}{1+i}$

$\Rightarrow a+ib=\frac{(1-\sqrt{3})(1+i)+(1+\sqrt{3})i(1+i)}{1^2-i^2}$

$\Rightarrow a+ib=\frac{1-\sqrt{3}+(1-\sqrt{3})i+(1+\sqrt{3})i+(1+\sqrt{3})i^2}{1-(-1)}$

$\Rightarrow a+ib=\frac{(1-\sqrt{3})+(1-\sqrt{3}+1+\sqrt{3})i+(1+\sqrt{3})(-1)}{2}$

$\Rightarrow a+ib=\frac{-2\sqrt{3}+2i}{2}$

$\Rightarrow a+ib=-\sqrt{3}+i$

∴ The values of a, b are $-\sqrt{3}$,1.

1 G. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{2+3i}{4+5i}$$

Answer

Given:

$\Rightarrow a+ib=\frac{2+3i}{4+5i}$

Multiplying and dividing with 4-5i

$\Rightarrow a+ib=\frac{2+3i}{4+5i}\times\frac{4-5i}{4-5i}$

$\Rightarrow a+ib=\frac{2(4-5i)+3i(4-5i)}{4^2-(5i)^2}$

$\Rightarrow a+ib=\frac{8-10i+12i-15i^2}{16-25i^2}$

We know that i^2=-1

$\Rightarrow a+ib=\frac{8+2i-15(-1)}{16-25(-1)}$

$\Rightarrow a+ib=\frac{23+2i}{41}$

∴ The values of a, b are $\frac{23}{41},\frac{2}{41}$.

1 H. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{(1-i)^3}{1-i^3}$$

Answer

Given:

$\Rightarrow a+ib=\frac{(1-i)^3}{1-i^3}$

$\Rightarrow a+ib=\frac{1^3-3(1)^2(i)+3(1)(i)^2-i^3}{1-i^2.i}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{1-3i+3(-1)-i^2.i}{1-(-1)i}$

$\Rightarrow a+ib=\frac{-2-3i-(-1)i}{1+i}$

$\Rightarrow a+ib=\frac{-2-4i}{1+i}$

Multiplying and diving with 1-i

$\Rightarrow a+ib=\frac{-2-4i}{1+i}\times\frac{1-i}{1-i}$

$\Rightarrow a+ib=\frac{-2(1-i)-4i(1-i)}{1^2-i^2}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{-2+2i-4i+4i^2}{1-(-1)}$

$\Rightarrow a+ib=\frac{-2-2i+4(-1)}{2}$

$\Rightarrow a+ib=\frac{-6-2i}{2}$

⇒ a+ib=-3-i

∴ The values of a, b are -3, -1.

1 I. Question

Express the following complex numbers in the standard form a + i b :

$(1 + 2i)^{-3}$

Answer

Given:

$\Rightarrow a+ib=(1+2i)^{-3}$

$\Rightarrow a+ib=\frac{1}{(1+2i)^3}$

$\Rightarrow a+ib=\frac{1}{1^3+3(1)^2(2i)+2(1)(2i)^2+(2i)^3}$

$\Rightarrow a+ib=\frac{1}{1+6i+4i^2+8i^3}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{1}{1+6i+4(-1)+8(i^2)(i)}$

$\Rightarrow a+ib=\frac{1}{-3+6i+8(-1)i}$

$\Rightarrow a+ib=\frac{1}{-3-2i}$

$\Rightarrow a+ib=\frac{-1}{3+2i}$

Multiplying and dividing with 3-2i

$\Rightarrow a+ib=\frac{-1}{3+2i}\times\frac{3-2i}{3-2i}$

$\Rightarrow a+ib=\frac{-3+2i}{3^2-(2i)^2}$

$\Rightarrow a+ib=\frac{-3+2i}{9-4i^2}$

$\Rightarrow a+ib=\frac{-3+2i}{9-4(-1)}$

$\Rightarrow a+ib=\frac{-3+2i}{13}$

∴ the values of a, b are $\frac{-3}{13},\frac{2}{13}$.

1 J. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{3-4i}{(4-2i)(1+i)}$$

Answer

Given:

$\Rightarrow a+ib=\frac{3-4i}{(4-2i)(1+i)}$

$\Rightarrow a+ib=\frac{3-4i}{4(1+i)-2i(1+i)}$

$\Rightarrow a+ib=\frac{3-4i}{4+4i-2i-2i^2}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{3-4i}{4+2i-2(-1)}$

$\Rightarrow a+ib=\frac{3-4i}{6+2i}$

Multiplying and dividing with 6-2i

$\Rightarrow a+ib=\frac{3-4i}{6+2i}\times\frac{6-2i}{6-2i}$

$\Rightarrow a+ib=\frac{3(6-2i)-4i(6-2i)}{6^2-(2i)^2}$

$\Rightarrow a + ib = \frac{18-6i-24i+8i^2}{36-4i^2}$

$\Rightarrow a + ib = \frac{18-30i+8(-1)}{36-4(-1)}$

$\Rightarrow a + ib = \frac{10-30i}{40}$

$\Rightarrow a + ib = \frac{1-3i}{4}$

∴ The values of a, b are $\frac{1}{4}, \frac{-3}{4}$.

1 K. Question

Express the following complex numbers in the standard form a + i b :

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

Answer

Given:

$\Rightarrow a + ib = \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

$\Rightarrow a + ib = \left(\frac{1+i-2(1-4i)}{(1-4i)(1+i)}\right)\left(\frac{3-4i}{5+i}\right)$

$\Rightarrow a + ib = \left(\frac{1+i-2+8i}{1(1+i)-4i(1+i)}\right)\left(\frac{3-4i}{5+i}\right)$

$\Rightarrow a + ib = \left(\frac{-1+9i}{1+i-4i-4i^2}\right)\left(\frac{3-4i}{5+i}\right)$

We know that i^2=-1

$\Rightarrow a + ib = \left(\frac{-1+9i}{1-3i-4(-1)}\right)\left(\frac{3-4i}{5+i}\right)$

$\Rightarrow a + ib = \frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}$

$\Rightarrow a + ib = \frac{-1(3-4i)+9i(3-i)}{5(5+i)-3i(5+i)}$

$\Rightarrow a + ib = \frac{-3+4i+27i-9i^2}{25+5i-15i-3i^2}$

$\Rightarrow a + ib = \frac{-3+31i-9(-1)}{25-10i-3(-1)}$

$\Rightarrow a + ib = \frac{6+31i}{28-10i}$

Multiplying and dividing with 28+10i

$\Rightarrow a + ib = \frac{6+31i}{28-10i} \times \frac{28+10i}{28+10i}$

$\Rightarrow a + ib = \frac{6(28+10i)+31i(28+10i)}{28^2-(10i)^2}$

$\Rightarrow a + ib = \frac{168+60i+868i+310i^2}{784-100i^2}$

$\Rightarrow a + ib = \frac{168+928i+310(-1)}{784-100(-1)}$

$\Rightarrow a + ib = \frac{478+928i}{884}$

∴ The values of a, b is $\frac{478}{884}, \frac{928}{884}$.

1 L. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$$

Answer

Given:

⇒ $a + ib = \frac{5+\sqrt{2}i}{1-\sqrt{2}i}$

Multiplying and dividing with 1+$\sqrt{2}$i

⇒ $a + ib = \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i}$

⇒ $a + ib = \frac{5(1+\sqrt{2}i)+\sqrt{2}i(1+\sqrt{2}i)}{1^2-(\sqrt{2}i)^2}$

⇒ $a + ib = \frac{5+5\sqrt{2}i+\sqrt{2}i+2i^2}{1-2i^2}$

We know that i^2=-1

⇒ $a + ib = \frac{5+6\sqrt{2}i+2(-1)}{1-2(-1)}$

⇒ $a + ib = \frac{3+6\sqrt{2}i}{3}$

⇒ $a + ib = 1 + 2\sqrt{2}i$

∴ The values of a, b are 1, 2$\sqrt{2}$.

2 A. Question

Find the real values of x and y, if

(x + i y) (2 – 3i) = 4 + i

Answer

Given:

⇒ (x+iy)(2-3i)=4+i

⇒ x(2-3i)+iy(2-3i)=4+i

⇒ 2x-3xi+2yi-3yi^2=4+i

We know that i^2=-1

⇒ 2x+(-3x+2y)i-3y(-1)=4+i

⇒ (2x+3y)+(-3x+2y)=4+i

Equating Real and Imaginary parts on both sides, we get

⇒ 2x+3y=4 and -3x+2y=1

On solving we get,

⇒ $x = \frac{5}{13}, y = \frac{14}{13}$

∴ The real values of x and y are $\frac{5}{13}, \frac{14}{13}$.

2 B. Question

Find the real values of x and y, if

$(3x - 2i\,y)\,(2 + i)^2 = 10\,(1 + i)$

Answer

Given:

⇒ $(3x-2iy)(2+i)^2=10(1+i)$

⇒ $(3x-2yi)(2^2+i^2+2(2)(i))=10+10i$

We know that $i^2=-1$

⇒ $(3x-2yi)(4+(-1)+4i)=10+10i$

⇒ $(3x-2yi)(3+4i)=10+10i$

Dividing with 3+4i on both sides

$$\Rightarrow 3x - 2yi = \frac{10+10i}{3+4i}$$

Multiplying and dividing with 3-4i

$$\Rightarrow 3x - 2yi = \frac{10+10i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$\Rightarrow 3x - 2yi = \frac{10(3-4i)+10i(3-4i)}{3^2-(4i)^2}$$

$$\Rightarrow 3x - 2yi = \frac{30-40i+30i-40i^2}{9-16i^2}$$

$$\Rightarrow 3x - 2yi = \frac{30-10i-40(-1)}{9-16(-1)}$$

$$\Rightarrow 3x - 2yi = \frac{70-10i}{25}$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow 3x = \frac{70}{25} \; and \; -2y = -\frac{10}{25}$$

$$\Rightarrow x = \frac{70}{75} \; and \; y = \frac{1}{5}$$

∴ The values of x and y are $\frac{70}{75} \; and \; \frac{1}{5}$.

2 C. Question

Find the real values of x and y, if

$$\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$$

Answer

Given:

$$\Rightarrow \frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$$

$$\Rightarrow \frac{\left(\left((1+i)x-2i\right)(3-i)\right)+\left(\left((2-3i)y+i\right)(3+i)\right)}{(3+i)(3-i)} = i$$

$\Rightarrow \frac{((1+i)(3-i)x)-(2i)(3-i)+((2-3i)(3+i)y)+(i)(3+i)}{3^2-i^2} = i$

We know that $i^2=-1$

$\Rightarrow \frac{(3-i+3i-i^2)x-6i+2i^2+(6+2i-9i-3i^2)y+3i+i^2}{9-(-1)} = i$

$\Rightarrow \frac{(3+2i-(-1))x-6i+2(-1)+(6-7i-3(-1))y+3i+(-1)}{10} = i$

⇒ (4+2i)x-3i-3+(9-7i)y=10i

⇒ (4x+9y-3)+i(2x-7y-3)=10i

Equating Real and Imaginary parts on both sides we get

⇒ 4x+9y-3=0 and 2x-7y-3=10

⇒ 4x+9y=3 and 2x-7y=13

On solving these equations we get

⇒ x=3 and y=-1

∴ Thee real values of x and y are 3 and -1

2 D. Question

Find the real values of x and y, if

(1 + i) (x + i y) = 2 – 5*i*

Answer

Given:

⇒ (1+i)(x+iy)=2-5i

Dividing with 1+i on both sides we get

$\Rightarrow x + iy = \frac{2-5i}{1+i}$

Multiplying and dividing with 1-i

$\Rightarrow x + iy = \frac{2-5i}{1+i} \times \frac{1-i}{1-i}$

$\Rightarrow x + iy = \frac{2(1-i)-5i(1-i)}{1^2-i^2}$

We know that $i^2=-1$

$\Rightarrow x + iy = \frac{2-2i-5i+5i^2}{1-(-1)}$

$\Rightarrow x + iy = \frac{2-7i+5(-1)}{2}$

$\Rightarrow x + iy = \frac{-3-7i}{2}$

Equating Real and Imaginary parts on both sides we get

$\Rightarrow x = \frac{-3}{2}\ and\ y = \frac{-7}{2}$

∴ The real values of x and y are $\frac{-3}{2}, \frac{-7}{2}$.

3 A. Question

Find the conjugates of the following complex numbers:

4 – 5 i

Answer

Given complex number is 4-5i

We know that conjugate of a complex number a+ib is a-ib

∴ The conjugate of 4-5i is 4+5i.

3 B. Question

Find the conjugates of the following complex numbers:

$$\frac{1}{3+5i}$$

Answer

Given

complex number is $\frac{1}{3+5i}$

Let us convert this to standard form a+ib,

Multiplying and dividing with 3-5i

$\Rightarrow a+ib=\frac{1}{3+5i}\times\frac{3-5i}{3-5i}$

$\Rightarrow a+ib=\frac{3-5i}{3^2-(5i)^2}$

$\Rightarrow a+ib=\frac{3-5i}{9-25i^2}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{3-5i}{9-25(-1)}$

$\Rightarrow a+ib=\frac{3-5i}{34}$

We know that complex conjugate of a complex number a+ib is a-ib.

$\Rightarrow a-ib=\frac{3+5i}{34}$

∴ The conjugate of $\frac{1}{3+5i}$ is $\frac{3+5i}{34}$.

3 C. Question

Find the conjugates of the following complex numbers:

$$\frac{1}{1+i}$$

Answer

Given complex number is $\frac{1}{1+i}$

Let us convert this to the standard form a+ib

Multiplying and dividing with 1-i

$\Rightarrow a+ib=\frac{1}{1+i}\times\frac{1-i}{1-i}$

$\Rightarrow a+ib=\frac{1-i}{1^2-i^2}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{1-i}{1-(-1)}$

$\Rightarrow a+ib=\frac{1-i}{2}$

We know that complex conjugate of a complex number a+ib is a-ib.

$\Rightarrow a-ib=\frac{1+i}{2}$

∴ The conjugate of $\frac{1}{1+i}$ is $\frac{1+i}{2}$

3 D. Question

Find the conjugates of the following complex numbers:

$$\frac{(3-i)^2}{2+i}$$

Answer

Given complex number is $\frac{(3-i)^2}{2+i}$

Let us convert this to the standard form a+ib

$\Rightarrow a+ib=\frac{(3-i)^2}{2+i}$

$\Rightarrow a+ib=\frac{3^2+i^2-2(3)(i)}{2+i}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{9+(-1)-6i}{2+i}$

$\Rightarrow a+ib=\frac{8-6i}{2+i}$

Multiplying and dividing with 2-i

$\Rightarrow a+ib=\frac{8-6i}{2+i}\times\frac{2-i}{2-i}$

$\Rightarrow a+ib=\frac{8(2-i)-6i(2-i)}{2^2-i^2}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{16-8i-12i+6i^2}{4-(-1)}$

$\Rightarrow a+ib=\frac{16-20i+6(-1)}{5}$

$\Rightarrow a+ib=\frac{10-20i}{5}$

⇒ a+ib=2-4i

We know that the complex conjugate of a complex number a+ib is a-ib

⇒ a-ib=2+4i

∴ the conjugate of $\frac{(3-i)^2}{2+i}$ is 2+4i.

3 E. Question

Find the conjugates of the following complex numbers:

$$\frac{(1+i)(2+i)}{3+i}$$

Answer

Given complex number is $\frac{(1+i)(2+i)}{3+i}$

Let us convert this to the standard form a+ib

$\Rightarrow a+ib=\frac{(1+i)(2+i)}{3+i}$

$\Rightarrow a+ib=\frac{1(2+i)+i(2+i)}{3+i}$

$\Rightarrow a+ib=\frac{2+i+2i+i^2}{3+i}$

We know that $i^2=-1$

$\Rightarrow a+ib=\frac{2+3i+(-1)}{3+i}$

$\Rightarrow a+ib=\frac{1+3i}{3+i}$

Multiplying and dividing with 3-i

$\Rightarrow a+ib=\frac{1+3i}{3+i}\times\frac{3-i}{3-i}$

$\Rightarrow a+ib=\frac{1(3-i)+3i(3-i)}{3^2-i^2}$

$\Rightarrow a+ib=\frac{3-i+9i-3i^2}{9-(-1)}$

$\Rightarrow a+ib=\frac{3+8i-3(-1)}{10}$

$\Rightarrow a+ib=\frac{6+8i}{10}$

We know that complex conjugate of a complex number a+ib is a-ib

$\Rightarrow a-ib=\frac{6-8i}{10}$

∴ The conjugate of $\frac{(1+i)(2+i)}{3+i}$ is $\frac{6-8i}{10}$.

3 F. Question

Find the conjugates of the following complex numbers:

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

Answer

Given complex number is $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Let us convert this into the standard form a+ib

$\Rightarrow a + ib = \frac{3(2+3i)-2i(2+3i)}{1(2-i)+2i(2-i)}$

$\Rightarrow a + ib = \frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$

We know that $i^2=-1$

$\Rightarrow a + ib = \frac{6+5i-6(-1)}{2+3i-2(-1)}$

$\Rightarrow a + ib = \frac{12+5i}{4+3i}$

Multiplying and dividing with 4-3i

$\Rightarrow a + ib = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$

$\Rightarrow a + ib = \frac{12(4-3i)+5i(4-3i)}{4^2-(3i)^2}$

$\Rightarrow a + ib = \frac{48-36i+20i-15i^2}{16-9i^2}$

$\Rightarrow a + ib = \frac{48-16i-15(-1)}{16-9(-1)}$

$\Rightarrow a + ib = \frac{63-46i}{25}$

We know that the complex conjugate of a complex number a+ib is a-ib

$\Rightarrow a - ib = \frac{63+46i}{25}$

∴ The conjugate of complex number $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is $\frac{63+46i}{25}$.

4 A. Question

Find the multiplicative inverse of the following complex numbers :

1 – i

Answer

Given complex number is **Z**=1-i

We know that the multiplicative inverse of a complex number **Z** is Z^{-1} (or) $\frac{1}{Z}$.

$\Rightarrow Z^{-1} = \frac{1}{1-i}$

Multiplying and dividing with 1+i

$\Rightarrow Z^{-1} = \frac{1}{1-i} \times \frac{1+i}{1+i}$

$\Rightarrow Z^{-1} = \frac{1+i}{1^2-(i)^2}$

We know that $i^2=-1$

$\Rightarrow Z^{-1} = \frac{1+i}{1-(-1)}$

$\Rightarrow Z^{-1} = \frac{1+i}{2}$

∴ The Multiplicative inverse of 1-i is $\frac{1+i}{2}$

4 B. Question

Find the multiplicative inverse of the following complex numbers :

$(1 + i\sqrt{3})^2$

Answer

Given complex number is $\mathbf{Z}=(1+\sqrt{3}i)^2$

$\Rightarrow \mathbf{Z}=1^2+(\sqrt{3}i)^2+2(1)(\sqrt{3}i)$

$\Rightarrow \mathbf{Z}=1+3i^2+2\sqrt{3}i$

We know that $i^2=-1$

$\Rightarrow \mathbf{Z}=1+3(-1)+2\sqrt{3}i$

$\Rightarrow \mathbf{Z}=-2+2\sqrt{3}i$

We know that the multiplicative inverse of a complex number **Z** is Z^{-1} (or) $\frac{1}{Z}$.

$\Rightarrow Z^{-1}=\frac{1}{-2+2\sqrt{3}i}$

Multiplying and dividing with $-2-2\sqrt{3}i$

$\Rightarrow Z^{-1}=\frac{1}{-2+2\sqrt{3}i}\times\frac{-2-2\sqrt{3}i}{-2-2\sqrt{3}i}$

$\Rightarrow Z^{-1}=\frac{-2-2\sqrt{3}i}{(-2)^2-(2\sqrt{3}i)^2}$

$\Rightarrow Z^{-1}=\frac{-2-2\sqrt{3}i}{4-12i^2}$

$\Rightarrow Z^{-1}=\frac{-2-2\sqrt{3}i}{4-12(-1)}$

$\Rightarrow Z^{-1}=\frac{-2-2\sqrt{3}i}{16}$

∴ The Multiplicative inverse of $(1+\sqrt{3}i)^2$ is $\frac{-2-2\sqrt{3}i}{16}$.

4 C. Question

Find the multiplicative inverse of the following complex numbers :

4 – 3 i

Answer

Given complex number is $\mathbf{Z}=4-3i$

We know that the multiplicative inverse of a complex number **Z** is Z^{-1} (or) $\frac{1}{Z}$.

$\Rightarrow Z^{-1}=\frac{1}{4-3i}$

Multiplying and dividing with 4+3i

$\Rightarrow Z^{-1}=\frac{1}{4-3i}\times\frac{4+3i}{4+3i}$

$\Rightarrow Z^{-1}=\frac{4+3i}{4^2-(3i)^2}$

$\Rightarrow Z^{-1}=\frac{4+3i}{16-9i^2}$

We know that $i^2=-1$

$\Rightarrow Z^{-1} = \frac{4+3i}{16-9(-1)}$

$\Rightarrow Z^{-1} = \frac{4+3i}{25}$

∴ The Multiplicative inverse of 4-3i is $\frac{4+3i}{25}$.

4 D. Question

Find the multiplicative inverse of the following complex numbers :

√5 + 3i

Answer

Given complex number is **Z**=$\sqrt{5}$+3i

We know that the multiplicative inverse of a complex number **Z** is Z^{-1} (or) $\frac{1}{Z}$.

$\Rightarrow Z^{-1} = \frac{1}{\sqrt{5}+3i}$

Multiplying and dividing with $\sqrt{5}$-3i

$\Rightarrow Z^{-1} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$

$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{5-9i^2}$

We know that i^2=-1

$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{5-9(-1)}$

$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{14}$

∴ The Multiplicative inverse of $\sqrt{5}$+3i is $\frac{\sqrt{5}-3i}{14}$.

5. Question

If z_1 = 2 – i, z_2 = 1 + i, find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$.

Answer

Given:

$\Rightarrow z_1$=2-i and z_2=1+i

We have to find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

We know that $\left|\frac{a}{b}\right|$ is $\frac{|a|}{|b|}$

$\Rightarrow \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{|z_1+z_2+1|}{|z_1-z_2+i|}$

$\Rightarrow \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{|2-i+1+i+1|}{|2-i-(1+i)+i|}$

$\Rightarrow \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{|4|}{|1-i|}$

We know that |a+ib| is $\sqrt{a^2+b^2}$.

$\Rightarrow \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{\sqrt{4^2+0^2}}{\sqrt{1^2+(-1)^2}}$

$\Rightarrow \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{4}{\sqrt{2}}$

$\Rightarrow \left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = 2\sqrt{2}$

∴ The value of $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ is $2\sqrt{2}$.

6. Question

If $z_1 = 2 - i$, $z_2 = -2 + i$, find

i. $\mathrm{Re}\left(\frac{z_1 z_2}{z_1}\right)$

ii. $\mathrm{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$

Answer

Given:

⇒ $z_1=2-i$ and $z_2=-2+i$

i. We need to find $Re\left(\frac{z_1 z_2}{z_1}\right)$

$\Rightarrow Re\left(\frac{z_1 z_2}{z_1}\right) = Re(z_2)$

$\Rightarrow Re\left(\frac{z_1 z_2}{z_1}\right) = Re(-2+i)$

$\Rightarrow Re\left(\frac{z_1 z_2}{z_1}\right) = -2$

∴ The Real part of $\frac{z_1 z_2}{z_1}$ is -2.

ii. We need to find $Im\left(\frac{1}{z_1 \bar{z}_1}\right)$

We know that $z\bar{z} = |z|^2$

$\Rightarrow z_1\bar{z}_1 = |2-i|^2$

We know that for a complex number **Z**=a+ib it's magnitude is given by $|z| = \sqrt{a^2+b^2}$

$\Rightarrow z_1\bar{z}_1 = \left(\sqrt{2^2+(-1)^2}\right)^2$

$\Rightarrow z_1\bar{z}_1 = 5$

$\Rightarrow Im\left(\frac{1}{z_1\bar{z}_1}\right) = Im\left(\frac{1}{5}\right)$

$\Rightarrow Im\left(\frac{1}{z_1\bar{z}_1}\right) = \frac{1}{5}$

∴ The Imaginary part of the $\frac{1}{z_1\bar{z}_1}$ is $\frac{1}{5}$.

7. Question

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$

Answer

Given complex number is $\boldsymbol{Z}=\frac{1+i}{1-i}-\frac{1-i}{1+i}$

$\Rightarrow \boldsymbol{Z}=\frac{(1+i)(1+i)-(1-i)(1-i)}{1^2-i^2}$

We know that $i^2=-1$

$\Rightarrow \boldsymbol{Z}=\frac{1^2+i^2+2(1)(i)-\left(1^2+i^2-2(1)(i)\right)}{1-(-1)}$

$\Rightarrow \boldsymbol{Z}=\frac{4i}{2}$

⇒ **Z**=2i

We know that for a complex number **Z**=a+ib it's magnitude is given by $|z|=\sqrt{a^2+b^2}$

$\Rightarrow |\boldsymbol{Z}|=\sqrt{0^2+2^2}$

⇒ |**Z**|=2

∴ The modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$ is 2

8. Question

If $x+iy=\frac{a+ib}{a-ib}$, prove that $x^2+y^2=1$

Answer

Given:

$\Rightarrow x+iy=\frac{a+ib}{a-ib}$

We know that for a complex number **Z**=a+ib it's magnitude is given by $|z|=\sqrt{a^2+b^2}$

We know that $\left|\frac{a}{b}\right|$ is $\frac{|a|}{|b|}$

Applying Modulus on both sides we get,

$\Rightarrow |x+iy|=\left|\frac{a+ib}{a-ib}\right|$

$\Rightarrow \sqrt{x^2+y^2}=\frac{|a+ib|}{|a-ib|}$

$\Rightarrow \sqrt{x^2+y^2}=\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+(-b)^2}}$

$\Rightarrow \sqrt{x^2+y^2}=\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$

$\Rightarrow \sqrt{x^2+y^2}=1$

Squaring on both sides

$\Rightarrow \left(\sqrt{x^2+y^2}\right)^2=1^2$

$\Rightarrow x^2+y^2=1$

∴ Thus Proved.

9. Question

Find the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is real.

Answer

Let us assume the given complex number be $Z = \left(\frac{1+i}{1-i}\right)$

Multiplying and dividing with 1+i

$\Rightarrow Z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$\Rightarrow Z = \frac{(1+i)^2}{1^2-i^2}$

We know that i^2=-1

$\Rightarrow Z = \frac{1^2+i^2+2(1)(i)}{1-(-1)}$

$\Rightarrow Z = \frac{1-1+2i}{2}$

$\Rightarrow Z = \frac{2i}{2}$

$\Rightarrow Z = i$

We know that i^{2k} is real for k>0.

So, the smallest positive integral 'n' that can make $\left(\frac{1+i}{1-i}\right)^n$ real is 2.

∴ The smallest positive integral value of 'n' is 2.

10. Question

Find the real values of θ for which the complex number $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

Answer

Let us assume the given complex number as $Z = \frac{1+icos\theta}{1-2icos\theta}$

Multiplying and dividing with 1+2icosθ

$\Rightarrow Z = \frac{1+icos\theta}{1-2icos\theta} \times \frac{1+2icos\theta}{1+2icos\theta}$

$\Rightarrow Z = \frac{1(1+2icos\theta)+icos\theta(1+2icos\theta)}{1^2-(2icos\theta)^2}$

$\Rightarrow Z = \frac{1+2icos\theta+icos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$

We know that i^2=-1

$\Rightarrow Z = \frac{1+3icos\theta+2(-1)\cos^2\theta}{1-4(-1)\cos^2\theta}$

$\Rightarrow Z = \frac{1-2\cos^2\theta+3icos\theta}{1+4\cos^2\theta}$

For a complex number to be purely real, the imaginary part equals to zero.

$\Rightarrow \frac{3cos\theta}{1+4\cos^2\theta} = 0$

⇒ 3cosθ=0 (∵ 1+4cos^{2}θ≥1)

⇒ cos θ=0

⇒ $\theta = \frac{(2n+1)\pi}{2}$, for n$\epsilon$I

∴ The values of θ to get the complex number to be purely real is $\frac{(2n+1)\pi}{2}$ for nϵI.

11. Question

Find the smallest positive integer value of *n* for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is a real number.

Answer

Let us write the given complex number as $\boldsymbol{Z} = \frac{(1+i)^n}{(1-i)^{n-2}}$

Multiplying and dividing with (1-i)^2

$\Rightarrow \boldsymbol{Z} = \frac{(1+i)^n}{(1-i)^{n-2}} \times \frac{(1-i)^2}{(1-i)^2}$

$\Rightarrow \boldsymbol{Z} = \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2$

$\Rightarrow \boldsymbol{Z} = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n \times \left(1^2 + i^2 - 2(1)(i)\right)$

$\Rightarrow \boldsymbol{Z} = \left(\frac{(1+i)^2}{1^2-i^2}\right)^n \times (1 + i^2 - 2i)$

We know that i^2=-1

$\Rightarrow \boldsymbol{Z} = \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^n \times (1 + (-1) - 2i)$

$\Rightarrow \boldsymbol{Z} = \left(\frac{1-1+2i}{2}\right)^n \times (-2i)$

$\Rightarrow \boldsymbol{Z} = \left(\frac{2i}{2}\right)^n \times (-2i)$

$\Rightarrow \boldsymbol{Z} = i^n \times (-2i)$

$\Rightarrow \boldsymbol{Z} = -2i^{n+1}$

We know that i^{2k} is real for k≥0.

∴ The least positive integral of n is 1.

12. Question

If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, find (x, y)

Answer

Given:

$\Rightarrow \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$

Rationalising denominator

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^3 - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^3 = x + iy$

$\Rightarrow \left(\frac{(1+i)^2}{1^2-i^2}\right)^3 - \left(\frac{(1-i)^2}{1^2-i^2}\right)^3 = x + iy$

We know that $i^2=-1$

$\Rightarrow \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^3 - \left(\frac{1^2+i^2-2(1)(i)}{1-(-1)}\right)^3 = x + iy$

$\Rightarrow \left(\frac{1-1+2i}{2}\right)^3 - \left(\frac{1-1-2i}{2}\right)^3 = x + iy$

$\Rightarrow \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 = x + iy$

$\Rightarrow i^3-(-i)^3=x+iy$

$\Rightarrow 2i^3=x+iy$

$\Rightarrow 2i^2.i=x+iy$

$\Rightarrow 2(-1)i=x+iy$

$\Rightarrow -2i=x+iy$

Equating Real and Imaginary parts on both sides we get

$\Rightarrow x=0$ and $y=-2$

∴ The values of x and y are 0 and -2.

13. Question

If $\frac{(1+i)^2}{2-i} = x + iy,$ find x + y.

Answer

Given:

$\Rightarrow \frac{(1+i)^2}{2-i} = x + iy$

$\Rightarrow \frac{1^2+i^2+2(1)(i)}{2-i} = x + iy$

We know that $i^2=-1$

$\Rightarrow \frac{1+(-1)+2i}{2-i} = x + iy$

$\Rightarrow \frac{2i}{2-i} = x + iy$

Multiplying and dividing with 2+i

$\Rightarrow \frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$

$\Rightarrow \frac{4i+2i^2}{2^2-i^2} = x + iy$

$\Rightarrow \frac{2(-1)+4i}{4-(-1)} = x + iy$

$\Rightarrow \frac{-2+4i}{5} = x + iy$

Equating Real and Imaginary parts on both sides we get

$\Rightarrow x = \frac{-2}{5}\ and\ y = \frac{4}{5}$

$\Rightarrow x+y=\frac{-2}{5}+\frac{4}{5}$

$\Rightarrow x+y=\frac{2}{5}$

∴ The value of x+y is $\frac{2}{5}$.

14. Question

If $\left(\frac{1-i}{1+i}\right)^{100}=a+ib$, find (a, b).

Answer

Given:

$\Rightarrow \left(\frac{1-i}{1+i}\right)^{100}=a+ib$

$\Rightarrow \left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right)^{100}=a+ib$

$\Rightarrow \left(\frac{(1-i)^2}{1^2-i^2}\right)^{100}=a+ib$

We know that $i^2=-1$

$\Rightarrow \left(\frac{1^2+i^2-2(1)(i)}{1-(-1)}\right)^{100}=a+ib$

$\Rightarrow \left(\frac{1-1-2i}{2}\right)^{100}=a+ib$

$\Rightarrow \left(\frac{-2i}{2}\right)^{100}=a+ib$

$\Rightarrow (-i)^{100}=a+ib$

$\Rightarrow i^{100}=a+ib$

$\Rightarrow (i^2)^{50}=a+ib$

$\Rightarrow (-1)^{50}=a+ib$

$\Rightarrow 1=a+ib$

Equating Real and Imaginary parts on both sides we get

$\Rightarrow a=1$ and $b=0$

∴ The values of a and b are 1 and 0.

15. Question

If a = cos θ + i sin θ, find the value of $\frac{1+a}{1-a}$.

Answer

Given:

$\Rightarrow a=\cos\theta+i\sin\theta$

$\Rightarrow \frac{1+a}{1-a}=\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}$

We know that $1+\cos2\theta=2\cos^2\theta$, $1-\cos2\theta=2\sin^2\theta$ and $\sin2\theta=2\sin\theta\cos\theta$

$\Rightarrow \frac{1+a}{1-a} = \frac{2\cos^2\left(\frac{\theta}{2}\right)+i\left(2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)}{2\sin^2\left(\frac{\theta}{2}\right)-i\left(2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)}$

$\Rightarrow \frac{1+a}{1-a} = \frac{2\cos\left(\frac{\theta}{2}\right)\left(\cos\left(\frac{\theta}{2}\right)+i\sin\left(\frac{\theta}{2}\right)\right)}{2\sin\left(\frac{\theta}{2}\right)\left(\sin\left(\frac{\theta}{2}\right)-i\cos\left(\frac{\theta}{2}\right)\right)}$

$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right)+i\sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)-i\cos\left(\frac{\theta}{2}\right)} \times \frac{\sin\left(\frac{\theta}{2}\right)+i\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)+i\cos\left(\frac{\theta}{2}\right)}$

$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right)\left(\sin\left(\frac{\theta}{2}\right)+i\cos\left(\frac{\theta}{2}\right)\right)+i\sin\left(\frac{\theta}{2}\right)\left(\sin\left(\frac{\theta}{2}\right)+i\cos\left(\frac{\theta}{2}\right)\right)}{\sin^2\left(\frac{\theta}{2}\right)-i^2\cos^2\left(\frac{\theta}{2}\right)}$

We know that $i^2=-1$

$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)+i\cos^2\left(\frac{\theta}{2}\right)+i\sin^2\left(\frac{\theta}{2}\right)+i^2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}{\sin^2\left(\frac{\theta}{2}\right)-\left(-\cos^2\left(\frac{\theta}{2}\right)\right)}$

$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)+i+\left(-\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\right)}{1}$

$\Rightarrow \frac{1+a}{1-a} = i\tan\left(\frac{\theta}{2}\right)$

∴ The value of $\frac{1+a}{1-a}$ is $i\tan\left(\frac{\theta}{2}\right)$

16 A. Question

Evaluate the following :

$2x^3 + 2x^2 - 7x + 72$, when $x = \frac{3-5i}{2}$

Answer

Given:

$\Rightarrow x = \frac{3-5i}{2}$

$\Rightarrow 2x^3+2x^2-7x+72$

$\Rightarrow 2\left(\frac{3-5i}{2}\right)^3 + 2\left(\frac{3-5i}{2}\right)^2 - 7\left(\frac{3-5i}{2}\right) + 72$

$\Rightarrow 2\left(\frac{3^3-3(3)^2(5i)+3(3)(5i)^2-(5i)^3}{8}\right) + 2\left(\frac{3^2-2(3)(5i)+(5i)^2}{4}\right) - \left(\frac{21-35i}{2}\right) + 72$

We know that $i^2=-1$

$\Rightarrow \left(\frac{27-135i+225i^2-125i^3}{4}\right) + \left(\frac{9-30i+25i^2}{2}\right) - \left(\frac{21-35i}{2}\right) + 72$

$\Rightarrow \left(\frac{27-135i+225(-1)-125(-1)(i)}{4}\right)+$
$\left(\frac{9-30i+25(-1)}{2}\right)-\left(\frac{21-35i}{2}\right)+72$

$\Rightarrow \left(\frac{-198-10i}{4}\right)+\left(\frac{-16-30i}{2}\right)-\left(\frac{21-35i}{2}\right)+72$

$\Rightarrow \left(\frac{-99-5i}{2}\right)+\left(\frac{-37+5i}{2}\right)+72$

$\Rightarrow \left(\frac{-136}{2}\right)+72$

⇒ -68+72

⇒ 4

∴ $2x^3+2x^2-7x+72=4$

16 B. Question

Evaluate the following :

$x^4 - 4x^3 + 4x^2 + 8x + 44$, when x = 3 + 2i

Answer

Given:

⇒ x=3+2i

⇒ $x^4-4x^3+4x^2+8x+44$

⇒ $(3+2i)^4-4(3+2i)^3+4(3+2i)^2+8(3+2i)+44$

⇒ $(3^4+4(3)^3(2i)+6(3)^2(2i)^2+4(3)(2i)^3+(2i)^4)-4(3^3+3(3)^2(2i)+3(3)(2i)^2+(2i)^3)+4(3^2+(2i)^2+2(3)(2i))+24+16i+44$

⇒ $81+216i+216i^2+96i^3+16i^4-108-216i-144i^2-32i^3+36+16i^2+48i+24+16i+44$

We know that $i^2=-1$

⇒ $77+64i+88i^2+64i^3+16i^4$

⇒ $77+64i+88(-1)+64(-1)(i)+16(-1)^2$

⇒ 5

∴ $x^4-4x^3+4x^2+8x+44=5$

16 C. Question

Evaluate the following :

$x^4 + 4x^3 + 6x^2 + 4x + 9$, when x = - 1 + i√2

Answer

Given:

⇒ $x=-1+i\sqrt{2}$

⇒ $x+1=i\sqrt{2}$

⇒ $(x+1)^4=(i\sqrt{2})^4$

⇒ $x^4+4x^3+6x^2+4x+1=2i^4$

We know that $i^2=-1$

$\Rightarrow x^4+4x^3+6x^2+4x+1=2(-1)^2$

$\Rightarrow x^4+4x^3+6x^2+4x+1=2$

$\Rightarrow x^4+4x^3+6x^2+4x+1+8=2+8$

$\Rightarrow x^4+4x^3+6x^2+4x+9=10$

$\therefore x^4+4x^3+6x^2+4x+9=10$

16 D. Question

Evaluate the following :

x6 + x4 + x2 + 1, when $x=\frac{1+i}{\sqrt{2}}$.

Answer

Given:

$\Rightarrow x=\frac{1+i}{\sqrt{2}}$

$\Rightarrow x^6+x^4+x^2+1$

$\Rightarrow x^4(x^2+1)+1(x^2+1)$

$\Rightarrow (x^4+1)(x^2+1)$

$\Rightarrow \left(\left(\frac{1+i}{\sqrt{2}}\right)^4+1\right)\left(\left(\frac{1+i}{\sqrt{2}}\right)^2+1\right)$

$\Rightarrow \left(\left(\frac{1^4+4(1)^3(i)+6(1)^2(i)^2+4(1)(i)^3+i^4}{4}\right)+1\right)\left(\left(\frac{1^2+i^2+2(i)(1)}{2}\right)+1\right)$

$\Rightarrow \left(\left(\frac{1+4i+6i^2+4i^3+i^4}{4}\right)+1\right)\left(\left(\frac{1+2i+i^2}{2}\right)+1\right)$

We know that $i^2=-1$

$\Rightarrow \left(\left(\frac{1+4i+6(-1)+4(-1)i+(-1)^2}{4}\right)+1\right)\left(\left(\frac{1+2i+(-1)}{2}\right)+1\right)$

$\Rightarrow \left(\left(\frac{-4}{4}\right)+1\right)\left(\frac{2i}{2}+1\right)$

$\Rightarrow (-1+1)(i+1)$

$\Rightarrow (0)(i+1)$

$\Rightarrow 0$

$\therefore x^6+x^4+x^2+1=0$

16 E. Question

Evaluate the following :

2x4 + 5x3 + 7x2 – x + 41, when x = - 2 - √3i

Answer

Given:

⇒ $x=-2-\sqrt{3}i$

⇒ $2x^4+5x^3+7x^2-x+41$

⇒ $2(-2-\sqrt{3})^4+5(-2-\sqrt{3}i)^3+7(-2-\sqrt{3}i)^2-(-2-\sqrt{3}i)+41$

⇒ $2(2^4+4(2)^3(\sqrt{3}i)+6(2)^2(\sqrt{3}i)^2+4(2)(\sqrt{3}i)^3+(\sqrt{3}i)^4)-5(2^3+3(2)^2(\sqrt{3}i)+3(2)(\sqrt{3}i)^2+(\sqrt{3}i)^3)+7(2^2+2(2)(\sqrt{3}i)+(\sqrt{3}i)^2)+2+\sqrt{3}i+41$

⇒ $16+64\sqrt{3}i+144i^2+48\sqrt{3}i^3+18i^4-40-60\sqrt{3}i-90i^2-15\sqrt{3}i^3+28+28\sqrt{3}i+21i^2+\sqrt{3}i+43$

We know that $i^2=-1$

⇒ $127+33\sqrt{3}i+75i^2+33\sqrt{3}i^3+18i^4$

⇒ $127+33\sqrt{3}i+75(-1)+33\sqrt{3}(-1)(i)+18(-1)^2$

⇒ 70

∴ $2x^4+5x^3+7x^2-x+41=70$

17. Question

For a positive integer n, find the value of $(1-i)^n\left(1-\frac{1}{i}\right)^n$.

Answer

Given:

⇒ $(1-i)^n\left(1-\frac{1}{i}\right)^n=\left((1-i)\left(1-\frac{1}{i}\right)\right)^n$

⇒ $(1-i)^n\left(1-\frac{1}{i}\right)^n=\left(\frac{(1-i)(i-1)}{i}\right)^n$

⇒ $(1-i)^n\left(1-\frac{1}{i}\right)^n=\left(\frac{i-1-i^2+i}{i}\right)^n$

We know that $i^2=-1$

⇒ $(1-i)^n\left(1-\frac{1}{i}\right)^n=\left(\frac{2i-1-(-1)}{i}\right)^n$

⇒ $(1-i)^n\left(1-\frac{1}{i}\right)^n=\left(\frac{2i}{i}\right)^n$

⇒ $(1-i)^n\left(1-\frac{1}{i}\right)^n=2^n$

∴ The values of $(1-i)^n\left(1-\frac{1}{i}\right)^n=2^n$.

18. Question

If $(1+i)z=(1-i)\bar{z}$, then show that $z=-i\bar{z}$.

Answer

Given:

⇒ $(1+i)z=(1-i)\bar{z}$

Dividing with (1+i) on both sides we get,

$\Rightarrow \frac{(1+i)z}{1+i} = \frac{(1-i)\bar{z}}{1+i}$

$\Rightarrow z = \frac{1-i}{1+i} \times \frac{1-i}{1-i}\bar{z}$

$\Rightarrow z = \frac{(1-i)^2}{1^2-i^2}\bar{z}$

We know that i^2=-1

$\Rightarrow z = \frac{1^2+i^2-2(1)(i)}{1-(-1)}\bar{z}$

$\Rightarrow z = \frac{1-(-1)-2i}{2}\bar{z}$

$\Rightarrow z = \frac{-2i}{2}\bar{z}$

$\Rightarrow z=-i\bar{z}$

∴ Thus proved

19. Question

Solve the system of equations $Re(z^2) = 0$, $|z| = 2$.

Answer

Given:

$\Rightarrow Re(z^2)=0$ and $|z|=2$

Let us assume Z=x+iy

$\Rightarrow Re(z^2)=0$

$\Rightarrow Re((x+iy)^2)=0$

$\Rightarrow Re(x^2+(iy)^2+2(x)(iy))=0$

$\Rightarrow Re(x^2+i^2y^2+i(2xy))=0$

We know that i^2=-1

$\Rightarrow Re(x^2-y^2+i(2xy))=0$

$\Rightarrow x^2-y^2=0$----------------------(1)

$\Rightarrow |z|=2$

$\Rightarrow \sqrt{(x^2+y^2)} = 2$

$\Rightarrow (x^2+y^2)=2^2$

$\Rightarrow (x^2+y^2)=4$-------------------(2)

Solving (1) and (2) we get

$\Rightarrow x=\sqrt{2}$ and $y=\sqrt{2}$.

$\therefore Z = \sqrt{2} + i\sqrt{2}$.

20. Question

If $\frac{z-1}{z+1}$ is purely imaginary number ($z \neq -1$), find the value of $|z|$.

Answer

Given:

⇒ $\frac{z-1}{z+1}$ is purely imaginary

⇒ Let us assume $\frac{z-1}{z+1} = ki$, where K is any real number

Let us assume z=x+iy

⇒ $\frac{x+iy-1}{x+iy+1} = ki$

Multiplying and dividing with (x+1)-iy

⇒ $\frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy} = ki$

⇒ $\frac{((x-1)(x+1))-((x-1)(iy))+((x+1)(iy))-((iy)(iy))}{(x+1)^2-(iy)^2} = ki$

⇒ $\frac{x^2-1-i(xy-y)+i(xy+y)-i^2y^2}{x^2+2x+1-i^2y^2} = ki$

We know that $i^2=-1$

⇒ $\frac{x^2-(-y^2)-1+i(2y)}{x^2+2x+1-(-y^2)} = ki$

⇒ $\frac{x^2+y^2+1+i(2y)}{x^2+y^2+2x+1} = ki$

Equating Real and Imaginary parts on both sides we get

⇒ $\frac{x^2+y^2-1}{x^2+y^2+2x+1} = 0$

⇒ $x^2+y^2-1=0$

⇒ $x^2+y^2=1$

⇒ $\sqrt{x^2+y^2} = \sqrt{1}$

⇒ |z|=1

∴ |z|=1

21. Question

If z_1 is a complex number other than -1 such that $|z_1|$ = 1 and $z_2 = \frac{z_1-1}{z_1+1}$, then show that the real parts of z_2 is zero.

Answer

Given:

⇒ $|Z_1| = 1 \text{ and } z_2 = \frac{z_1-1}{z_1+1}$

Let us assume z_1=x+iy

⇒ $|Z_1|=1$

⇒ $\sqrt{x^2+y^2} = 1$

⇒ $x^2+y^2=1$-------------------(1)

⇒ $z_2 = \frac{z_1-1}{z_1+1}$

$\Rightarrow z_2 = \frac{x+iy-1}{x+iy+1}$

$\Rightarrow z_2 = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$\Rightarrow z_2 = \frac{((x-1)(x+1))-((iy)(x-1))+((iy)(x+1))-((iy)(iy))}{(x+1)^2-(iy)^2}$

$\Rightarrow z_2 = \frac{x^2-1+i(-xy+y+xy+y)-i^2y^2}{x^2+2x+1-i^2y^2}$

We know that i^2=-1

$\Rightarrow z_2 = \frac{x^2-(-y^2)-1+i(2y)}{x^2-(-y^2)+2x+1}$

$\Rightarrow z_2 = \frac{x^2+y^2-1+i(2y)}{x^2+y^2+2x+1}$

$\Rightarrow z_2 = \frac{1-1+i(2y)}{1+1+2x}$

$\Rightarrow z_2 = \frac{i(2y)}{2+2x}$

$\Rightarrow z_2 = \frac{iy}{1+x}$

∴ z_2 is an imaginary one.

22. Question

If |z + 1| = z + 2(1 + i), find z.

Answer

Given:

⇒ |z+1|=z+2(1+i)

Let us assume z=x+iy

⇒ |x+iy+1|=x+iy+2+2i

$\Rightarrow \sqrt{(x+1)^2+y^2} = (x+2)+i(y+2)$

Equating Real and Imaginary parts on both sides

⇒ y+2=0

⇒ y=-2----------------(1)

$\Rightarrow \sqrt{(x+1)^2+y^2} = x+2$

⇒ $(x+1)^2+y^2=(x+2)^2$

⇒ $x^2+2x+1+(-2)^2=x^2+4x+4$

⇒ 2x=1+4-4

⇒ 2x=1

$\Rightarrow x = \frac{1}{2}.$

$\therefore z = \frac{1}{2} - 2i.$

23. Question

Solve the equation |z| = z + 1 + 2i.

Answer

Given:

⇒ $|z|=z+1+2i$

Let us assume $z=x+iy$

⇒ $|x+iy|=x+iy+1+2i$

⇒ $\sqrt{x^2+y^2}=(x+1)+i(y+2)$

Equating Real and Imaginary parts on both sides we get

⇒ $y+2=0$

⇒ $y=-2$----------------------(1)

⇒ $\sqrt{x^2+y^2}=(x+1)$

⇒ $x^2+(-2)^2=(x+1)^2$

⇒ $x^2+4=x^2+2x+1$

⇒ $2x=3$

⇒ $x=\frac{3}{2}$

∴ $z=\frac{3}{2}-2i.$

24. Question

What is the smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$?

Answer

Given:

⇒ $(1+i)^{2n}=(1-i)^{2n}$

⇒ $((1+i)^2)^n=((1-i)^2)^n$

⇒ $(1^2+i^2+2(1)(i))^n=(1^2+i^2-2(1)(i))^n$

We know that $i^2=-1$

⇒ $(1-1+2i)^n=(1-1-2i)^n$

⇒ $(2i)^n=(-2i)^n$

We can see that the Relation holds only when n is an even integer.

∴ The smallest positive integer n is 2.

25. Question

If z_1, z_2, z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then find the value of $|z_1 + z_2 + z_3|$.

Answer

Given:

⇒ $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$

$\Rightarrow |z_1+z_2+z_3| = \left|\frac{z_1\overline{z_1}}{\overline{z_1}}+\frac{z_2\overline{z_2}}{\overline{z_2}}+\frac{z_3\overline{z_3}}{\overline{z_3}}\right|$

We know that $z\bar{z}=|z|^2$

$\Rightarrow |z_1+z_2+z_3| = \left|\frac{|z_1|^2}{\overline{z_1}}+\frac{|z_2|^2}{\overline{z_2}}+\frac{|z_3|^2}{\overline{z_3}}\right|$

$\Rightarrow |z_1+z_2+z_3| = \left|\frac{1}{\overline{z_1}}+\frac{1}{\overline{z_2}}+\frac{1}{\overline{z_3}}\right|$

We know that $|z|=|\bar{z}|$

$\Rightarrow |z_1+z_2+z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$

$\Rightarrow |z_1+z_2+z_3| = 1$

$\therefore |z_1+z_2+z_3|=1.$

26. Question

Find the number of solutions of $z^2 + |z|^2 = 0$.

Answer

Given:

$\Rightarrow z^2+|z|^2=0$

Let us assume z=x+iy

$\Rightarrow (x+iy)^2+\left(\sqrt{x^2+y^2}\right)^2=0$

$\Rightarrow x^2+(iy)^2+2(x)(iy)+x^2+y^2=0$

$\Rightarrow 2x^2+y^2+i^2y^2+i2xy=0$

We know that $i^2=-1$

$\Rightarrow 2x^2+y^2-y^2+i2xy=0$

$\Rightarrow 2x^2+i2xy=0$

Equating Real and Imaginary parts on both sides we get,

$\Rightarrow 2x^2=0$ and $2xy=0$

$\Rightarrow x=0$ and $y\epsilon R$

$\therefore z=0+iy$ where $y\epsilon R$. i.e, Infinite solutions.

Exercise 13.3

1 A. Question

Find the square root of the following complex numbers :

$-5 + 12i$

Answer

Given:

$\Rightarrow x+iy=-5+12i$

Here y>0

We know that for a complex number z=a+ib

$$\sqrt{a+ib} = \Rightarrow \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b > 0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b < 0 \end{cases}$$

$$\Rightarrow \sqrt{-5+12i} = \pm\left[\left(\frac{-5+\sqrt{(-5)^2+12^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{(-5)^2+12^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm\left[\left(\frac{-5+\sqrt{25+144}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{25+144}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm\left[\left(\frac{-5+\sqrt{169}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{169}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm\left[\left(\frac{-5+13}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+13}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm\left[\left(\frac{8}{2}\right)^{\frac{1}{2}} + i\left(\frac{18}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm\left[4^{\frac{1}{2}} + i9^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm[2+3i]$$

$$\therefore \sqrt{-5+12i} = \pm[2+3i].$$

1 B. Question

Find the square root of the following complex numbers :

-7 – 24_i

Answer

Given:

⇒ x+iy=-7+24i

Here y<0

We know that for a complex number z=a+ib

$$\sqrt{a+ib} = \Rightarrow \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b > 0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b < 0 \end{cases}$$

$$\Rightarrow \sqrt{-7-24i} = \pm\left[\left(\frac{-7+\sqrt{(-7)^2+(-24)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{(-7)^2+(-24)^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm\left[\left(\frac{-7+\sqrt{49+576}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{49+576}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm\left[\left(\frac{-7+\sqrt{625}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{625}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm\left[\left(\frac{-7+25}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+25}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm\left[\left(\frac{18}{2}\right)^{\frac{1}{2}} - i\left(\frac{32}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm\left[9^{\frac{1}{2}} - i16^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm[3-4i]$$

$$\therefore \sqrt{-7-24i} = \pm[3-4i].$$

1 C. Question

Find the square root of the following complex numbers :

1 – *i*

Answer

Given:

⇒ x+iy=1-i

Here y<0

We know that for a complex number z=a+ib

$$\Rightarrow \sqrt{a+ib} = \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b>0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b<0 \end{cases}$$

$$\Rightarrow \sqrt{1-i} = \pm\left[\left(\frac{1+\sqrt{(1)^2+(-1)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{(1)^2+(-1)^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1-i} = \pm\left[\left(\frac{1+\sqrt{1+1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{1+1}}{2}\right)^{\frac{1}{2}}\right]$$

$\Rightarrow \sqrt{1-i} = \pm\left[\left(\frac{1+\sqrt{2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{2}}{2}\right)^{\frac{1}{2}}\right]$

$\therefore \sqrt{-5+12i} = \pm\left[\left(\frac{1+\sqrt{2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{2}}{2}\right)^{\frac{1}{2}}\right].$

1 D. Question

Find the square root of the following complex numbers :

-8 – 6*i*

Answer

Given:

⇒ x+iy=-8-6i

Here y<0

We know that for a complex number z=a+ib

$$\Rightarrow \sqrt{a+ib} = \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b>0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b<0 \end{cases}$$

$$\Rightarrow \sqrt{-8-6i} = \pm\left[\left(\frac{-8+\sqrt{(-8)^2+(-6)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{(-8)^2+(-6)^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-8-6i} = \pm\left[\left(\frac{-8+\sqrt{64+36}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{64+36}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-8-6i} = \pm\left[\left(\frac{-8+\sqrt{100}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{100}}{2}\right)^{\frac{1}{2}}\right]$$

$\Rightarrow \sqrt{-8-6i} = \pm\left[\left(\frac{-8+10}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+10}{2}\right)^{\frac{1}{2}}\right]$

$\Rightarrow \sqrt{-8-6i} = \pm\left[\left(\frac{2}{2}\right)^{\frac{1}{2}} - i\left(\frac{18}{2}\right)^{\frac{1}{2}}\right]$

$\Rightarrow \sqrt{-8-6i} = \pm\left[1^{\frac{1}{2}} - i9^{\frac{1}{2}}\right]$

$\Rightarrow \sqrt{-8-6i} = \pm[1-3i]$

$\therefore \sqrt{-8-6i} = \pm[1-3i].$

1 E. Question

Find the square root of the following complex numbers :

8 – 15*i*

Answer

Given:

⇒ x+iy=8-15i

Here y<0

We know that for a complex number z=a+ib

$$\sqrt{a+ib} = \Rightarrow \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b>0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b<0 \end{cases}$$

$$\Rightarrow \sqrt{8-15i} = \pm\left[\left(\frac{8+\sqrt{8^2+(-15)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{8^2+(-15)^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{8-15i} = \pm\left[\left(\frac{8+\sqrt{64+225}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{64+225}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{8-15i} = \pm\left[\left(\frac{8+\sqrt{289}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{289}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{8-15i} = \pm\left[\left(\frac{8+17}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+17}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{8-15i} = \pm\left[\left(\frac{25}{2}\right)^{\frac{1}{2}} - i\left(\frac{9}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{8-15i} = \pm\left[\frac{5}{\sqrt{2}} - \frac{i3}{\sqrt{2}}\right]$$

$$\therefore \sqrt{8-15i} = \pm\left[\frac{5}{\sqrt{2}} - \frac{i3}{\sqrt{2}}\right].$$

1 F. Question

Find the square root of the following complex numbers :

-11 – 60 √-1

Answer

Given:

$\Rightarrow x + iy = -11 - 60\sqrt{-1}$

⇒ x+iy=-11-60i

Here y<0

We know that for a complex number z=a+ib

$$\sqrt{a+ib} = \Rightarrow \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b>0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b<0 \end{cases}$$

$$\Rightarrow \sqrt{-11-60i} = \pm\left[\left(\frac{-11+\sqrt{(-11)^2+(-60)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{(-11)^2+(60)^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-11-60i} = \pm\left[\left(\frac{-11+\sqrt{121+3600}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{121+3600}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-11-60i} = \pm\left[\left(\frac{-11+\sqrt{3721}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{3721}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-11-60i} = \pm\left[\left(\frac{-11+61}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+61}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-11-60i} = \pm\left[\left(\frac{50}{2}\right)^{\frac{1}{2}} - i\left(\frac{72}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-11-60i} = \pm\left[25^{\frac{1}{2}} - i36^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-11-60i} = \pm[5-6i]$$

$$\therefore \sqrt{-11-60i} = \pm[5-6i].$$

1 G. Question

Find the square root of the following complex numbers :

1 + 4 √-3

Answer

Given:

$$\Rightarrow x+iy = 1+4\sqrt{-3}$$

$$\Rightarrow x+iy = 1+4(\sqrt{3})(\sqrt{-1})$$

$$\Rightarrow x+iy = 1+4\sqrt{3}i$$

Here y>0

We know that for a complex number z=a+ib

$$\sqrt{a+ib} = \Rightarrow \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b > 0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b < 0 \end{cases}$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{1+\sqrt{1^2+(4\sqrt{3})^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{1^2+(4\sqrt{3})^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{1+\sqrt{1+48}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{1+48}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{1+\sqrt{49}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{49}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{1+7}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+7}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{8}{2}\right)^{\frac{1}{2}} + i\left(\frac{6}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm\left[4^{\frac{1}{2}} + i3^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm[2+\sqrt{3}i]$$

$$\therefore \sqrt{1+4\sqrt{3}i} = \pm[2+\sqrt{3}i]$$

1 H. Question

Find the square root of the following complex numbers :

$4i$

Answer

Given:

⇒ x+iy=4i

Here y>0

We know that for a complex number z=a+ib

$$\Rightarrow \sqrt{a+ib} = \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b>0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b<0 \end{cases}$$

$$\Rightarrow \sqrt{4i} = \pm\left[\left(\frac{0+\sqrt{0^2+4^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{0^2+4^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{4i} = \pm\left[\left(\frac{0+\sqrt{0+16}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{0+16}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{4i} = \pm\left[\left(\frac{0+\sqrt{16}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{16}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{4i} = \pm\left[\left(\frac{0+4}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+4}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{4i} = \pm\left[\left(\frac{4}{2}\right)^{\frac{1}{2}} + i\left(\frac{4}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{4i} = \pm\left[2^{\frac{1}{2}} + i2^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{4i} = \pm[\sqrt{2} + \sqrt{2}i]$$

$$\therefore \sqrt{4i} = \pm[\sqrt{2} + \sqrt{2}i].$$

1 I. Question

Find the square root of the following complex numbers :

-*i*

Answer

Given:

⇒ x+iy=-i

Here y<0

We know that for a complex number z=a+ib

$$\Rightarrow \sqrt{a+ib} = \begin{cases} \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b>0 \\ \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right], if\ b<0 \end{cases}$$

$$\Rightarrow \sqrt{-i} = \pm\left[\left(\frac{0+\sqrt{0^2+(-1)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{0^2+(-1)^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-i} = \pm\left[\left(\frac{0+\sqrt{0+1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{0+1}}{2}\right)^{\frac{1}{2}}\right]$$

$$\Rightarrow \sqrt{-i} = \pm\left[\left(\frac{0+\sqrt{1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{1}}{2}\right)^{\frac{1}{2}}\right]$$

$\Rightarrow \sqrt{-i} = \pm\left[\left(\frac{0+1}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+1}{2}\right)^{\frac{1}{2}}\right]$

$\Rightarrow \sqrt{-i} = \pm\left[\left(\frac{1}{2}\right)^{\frac{1}{2}} - i\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$

$\Rightarrow \sqrt{-i} = \pm\left[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right]$

$\therefore \sqrt{-i} = \pm[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}]$.

Exercise 13.4

1 A. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

1 + i

Answer

Given complex number is Z=1+i

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{1^2+1^2}$

$\Rightarrow |z| = \sqrt{1+1}$

$\Rightarrow |z| = \sqrt{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{1}\right)$

Since x>0,y>0 complex number lies in 1^{st} quadrant and the value of θ will be as follows $0^0 \le \theta \le 90^0$.

$\Rightarrow \theta = \tan^{-1}(1)$

$\Rightarrow \theta = \frac{\pi}{4}$.

$\Rightarrow Z = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

∴ The Polar form of Z=1+i is $z = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$.

1 B. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

√3 + i

Answer

Given Complex number is Z=$\sqrt{3}$+i

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{(\sqrt{3})^2 + 1^2}$

$\Rightarrow |z| = \sqrt{3+1}$

$\Rightarrow |z| = \sqrt{4}$

$\Rightarrow |Z| = 2$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Since x>0,y>0 complex number lies in 1st quadrant and the value of θ will be as follows $0^0 \leq \theta \leq 90^0$.

$\Rightarrow \theta = \frac{\pi}{6}$.

$\Rightarrow Z = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

∴ The Polar form of Z=$\sqrt{3}$+i is $z = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$.

1 C. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

1 – i

Answer

Given complex number is z=1-i

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{1^2 + (-1)^2}$

$\Rightarrow |z| = \sqrt{1+1}$

$\Rightarrow |z| = \sqrt{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{1}\right)$

Since x>0,y<0 complex number lies in 4th quadrant and the value of θ will be as follows $-90^0 \leq \theta \leq 0^0$.

$\Rightarrow \theta = \tan^{-1}(1)$

$\Rightarrow \theta = \frac{-\pi}{4}$.

$\Rightarrow Z=\sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)$

$\Rightarrow z=\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)-i\sin\left(\frac{\pi}{4}\right)\right)$

∴ The Polar form of Z=1+i is $z=\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)-i\sin\left(\frac{\pi}{4}\right)\right)$.

1 D. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$\frac{1-i}{1+i}$

Answer

Given complex number is $z=\frac{1-i}{1+i}$

$\Rightarrow z=\frac{1-i}{1+i}\times\frac{1-i}{1-i}$

$\Rightarrow z=\frac{(1-i)^2}{1^2-i^2}$

We know that $i^2=-1$

$\Rightarrow z=\frac{1^2+i^2-2(1)(i)}{1-(-1)}$

$\Rightarrow z=\frac{1+(-1)-2i}{2}$

$\Rightarrow z=\frac{-2i}{2}$

⇒ z=0-i

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z|=\sqrt{0^2+(-1)^2}$

$\Rightarrow |z|=\sqrt{0+1}$

$\Rightarrow |z|=\sqrt{1}$

⇒ |z|=1

$\Rightarrow \theta=\tan^{-1}\left(\frac{1}{0}\right)$

Since x≥0,y<0 complex number lies in 4^{th} quadrant and the value of θ will be as follows $-90^0\leq\theta\leq0^0$.

$\Rightarrow \theta=\tan^{-1}(\infty)$

$\Rightarrow \theta=\frac{-\pi}{2}$.

$\Rightarrow Z = 1\left(\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right)$

$\Rightarrow z = 1\left(\cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right)\right)$

∴ The Polar form of $Z = \frac{1-i}{1+i}$ is $z = 1\left(\cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right)\right)$.

1 E. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\frac{1}{1+i}$$

Answer

Given complex number is $z = \frac{1}{1+i}$.

$\Rightarrow z = \frac{1}{1+i} \times \frac{1-i}{1-i}$

$\Rightarrow z = \frac{1-i}{1^2-i^2}$

We know that $i^2=-1$

$\Rightarrow z = \frac{1-i}{1-(-1)}$

$\Rightarrow z = \frac{1-i}{2}$

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$

$\Rightarrow |z| = \sqrt{\frac{1}{4} + \frac{1}{4}}$

$\Rightarrow |z| = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right)$

Since x>0,y<0 complex number lies in 4^{th} quadrant and the value of θ will be as follows $-90^0 \leq \theta \leq 0^0$.

$\Rightarrow \theta = \tan^{-1}(1)$

$\Rightarrow \theta = \frac{-\pi}{4}$.

$\Rightarrow Z = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right)$

$\Rightarrow Z = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right)$

∴ The Polar form of $Z = \frac{1}{1+i}$ is $z = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{4}\right) - isin\left(\frac{\pi}{4}\right)\right)$.

1 F. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\frac{1+2i}{1-3i}$$

Answer

Given complex number is $z = \frac{1+2i}{1-3i}$.

$\Rightarrow z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$

$\Rightarrow z = \frac{1(1+3i)+2i(1+3i)}{1^2-(3i)^2}$

$\Rightarrow z = \frac{1+3i+2i+6i^2}{1-9i^2}$

We know that $i^2=-1$

$\Rightarrow z = \frac{1+5i+6(-1)}{1-9(-1)}$

$\Rightarrow z = \frac{-5+5i}{10}$

$\Rightarrow z = \frac{-1+i}{2}$

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$\Rightarrow |z| = \sqrt{\frac{1}{4} + \frac{1}{4}}$

$\Rightarrow |z| = \sqrt{\frac{1}{2}}$

$\Rightarrow |z| = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right)$

Since x<0,y>0 complex number lies in 2nd quadrant and the value of θ will be as follows $90^0 \leq \theta \leq 180^0$.

$\Rightarrow \theta = \tan^{-1}(1)$

$\Rightarrow \theta = \frac{3\pi}{4}$.

$\Rightarrow Z = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{3\pi}{4}\right) + isin\left(\frac{3\pi}{4}\right)\right)$

∴ The Polar form of $Z = \frac{1+2i}{1-2i}$ is $z = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{3\pi}{4}\right) + isin\left(\frac{3\pi}{4}\right)\right)$.

1 G. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

sin 120° – i cos 120°

Answer

Given complex number is $z=\sin 120^0 - i\cos 120^0$

$\Rightarrow z = \frac{\sqrt{3}}{2} - i\left(\frac{-1}{2}\right)$

$\Rightarrow z = \frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)$

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{(\frac{\sqrt{3}}{2})^2 + \left(\frac{1}{2}\right)^2}$

$\Rightarrow |z| = \sqrt{\frac{3}{4} + \frac{1}{4}}$

$\Rightarrow |z| = \sqrt{1}$

⇒ |z|=1

$\Rightarrow \theta = \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$

Since x>0,y>0 complex number lies in 1st quadrant and the value of θ will be as follows $0^0 \le \theta \le 90^0$.

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta = \frac{\pi}{6}$.

$\Rightarrow Z = 1\left(\cos\left(\frac{\pi}{6}\right) + isin\left(\frac{\pi}{6}\right)\right)$

∴ The Polar form of $Z=\sin 120^0 - i\cos 120^0$ is $z = 1\left(\cos\left(\frac{\pi}{6}\right) + isin\left(\frac{\pi}{6}\right)\right)$.

1 H. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$\frac{-16}{1+i\sqrt{3}}$

Answer

Given complex number is $z = \frac{-16}{1+i\sqrt{3}}$

$\Rightarrow z = \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$\Rightarrow z = \frac{-16+i16\sqrt{3}}{1^2-(i\sqrt{3})^2}$

$\Rightarrow z = \frac{-16+i16\sqrt{3}}{1-3i^2}$

We know that $i^2=-1$

$\Rightarrow z = \frac{-16+i16\sqrt{3}}{1-3(-1)}$

$\Rightarrow z = \frac{-16+i16\sqrt{3}}{4}$

$\Rightarrow z=-4+i4\sqrt{3}$

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{(-4)^2+(4\sqrt{3})^2}$

$\Rightarrow |z| = \sqrt{16+48}$

$\Rightarrow |z| = \sqrt{64}$

$\Rightarrow |z|=8$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$

Since x<0,y>0 complex number lies in 2nd quadrant and the value of θ will be as follows $90^0 \le \theta \le 180^0$.

$\Rightarrow \theta = \tan^{-1}(\sqrt{3})$

$\Rightarrow \theta = \frac{2\pi}{3}$.

$\Rightarrow Z = 8\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)$

∴ The Polar form of $Z = \frac{-16}{1+i\sqrt{3}}$ is $z = 8\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)$.

2. Question

Write $(i^{25})^3$ in polar form.

Answer

Given Complex number is $Z=(i^{25})^3$

$\Rightarrow Z=i^{75}$

⇒ $Z=i^{74}.i$

⇒ $Z=(i^2)^{37}.i$

We know that $i^2=-1$

⇒ $Z=(-1)^{37}.i$

⇒ Z=(-1).i

⇒ Z=-i

⇒ Z=0-i

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

⇒ $|z|=\sqrt{0^2+(-1)^2}$

⇒ $|z|=\sqrt{0+1}$

⇒ $|z|=\sqrt{1}$

⇒ |z|=1

⇒ $\theta=\tan^{-1}\left(\frac{1}{0}\right)$

Since x>0,y<0 complex number lies in 4th quadrant and the value of θ will be as follows $-90^0 \le \theta \le 0^0$.

⇒ $\theta=\tan^{-1}(\infty)$

⇒ $\theta=\frac{-\pi}{2}$.

⇒ $Z=1\left(\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)\right)$

⇒ $Z=1\left(\cos\left(\frac{\pi}{2}\right)-i\sin\left(\frac{\pi}{2}\right)\right)$

∴ The Polar form of $Z=(i^{25})^3$ is $z=1\left(\cos\left(\frac{\pi}{2}\right)-i\sin\left(\frac{\pi}{2}\right)\right)$.

3 A. Question

Express the following complex numbers in the form $r(\cos\theta+i\sin\theta)$:

1 + i tan α

Answer

Given Complex number is Z=1+itanα

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

We know that tanα is a periodic function with period π.

We have α lying in the interval $\left[0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\pi\right]$

Case1:

$\Rightarrow \alpha\epsilon\left[0,\frac{\pi}{2}\right)$

$\Rightarrow |z| = r = \sqrt{1^2 + \tan^2\alpha}$

$\Rightarrow |z| = r = \sqrt{\sec^2\alpha}$

$\Rightarrow |z| = r = |sec\alpha|$

Since secα is positive in the interval $\left[0,\frac{\pi}{2}\right)$

$\Rightarrow |z| = r = sec\alpha$

$\Rightarrow \theta = \arg(z) = \tan^{-1}\left(\frac{tan\alpha}{1}\right)$

$\Rightarrow \theta = \tan^{-1}(tan\alpha)$

Since tanα is positive in the interval $\left[0,\frac{\pi}{2}\right)$

⇒ θ=α

∴ The polar form is z=secα(cosα+isinα).

Case2:

$\Rightarrow \alpha\epsilon\left(\frac{\pi}{2},\pi\right]$

$\Rightarrow |z| = r = \sqrt{1^2 + \tan^2\alpha}$

$\Rightarrow |z| = r = \sqrt{\sec^2\alpha}$

$\Rightarrow |z| = r = |sec\alpha|$

Since secα is negative in the interval $\left(\frac{\pi}{2},\pi\right]$.

$\Rightarrow |z| = r = -sec\alpha$

$\Rightarrow \theta = \arg(z) = \tan^{-1}\left(\frac{tan\alpha}{1}\right)$

$\Rightarrow \theta = \tan^{-1}(tan\alpha)$

Since tanα is negative in the interval $\left(\frac{\pi}{2},\pi\right]$.

$\Rightarrow \theta = -\pi + \alpha$.(∵ θ lies in 4^{th} quadrant)

⇒ z=-secα(cos($\alpha - \pi$)+isin($\alpha - \pi$))

⇒ z=-secα(-cosα-isinα)

⇒ z=secα(cosα+isinα)

∴ The polar form is z=secα(cosα+isinα)

3 B. Question

Express the following complex numbers in the form $r(\cos\theta + i\sin\theta)$:

tan α – i

Answer

Given Complex number is tanα-i

We know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

We know that tanα is a periodic function with period π.

We have α lying in the interval $\left[0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\pi\right]$

Case1:

$\Rightarrow \alpha\epsilon\left[0,\frac{\pi}{2}\right)$

$\Rightarrow |z| = r = \sqrt{\tan^2\alpha + 1^2}$

$\Rightarrow |z| = r = \sqrt{\sec^2\alpha}$

$\Rightarrow |z| = r = |sec\alpha|$

Since secα is positive in the interval $\left[0,\frac{\pi}{2}\right)$

$\Rightarrow |z| = r = sec\alpha$

$\Rightarrow \theta = \arg(z) = \tan^{-1}\left(\frac{1}{tan\alpha}\right)$

$\Rightarrow \theta = \tan^{-1}(cot\alpha)$

Since cotα is positive in the interval $\left[0,\frac{\pi}{2}\right)$

$\Rightarrow \theta = \alpha - \frac{\pi}{2}$ (∵ θ lies in 4th quadrant)

$\Rightarrow z = sec\alpha\left(\cos\left(\frac{-\pi}{2}+\alpha\right) + isin\left(\frac{-\pi}{2}+\alpha\right)\right)$

⇒ z=secα(sinα-icosα)

∴ The polar form is z=secα(sinα-icosα)

Case2:

$\Rightarrow \alpha\epsilon\left(\frac{\pi}{2},\pi\right]$

$\Rightarrow |z| = r = \sqrt{\tan^2\alpha + 1^2}$

$\Rightarrow |z| = r = \sqrt{\sec^2\alpha}$

$\Rightarrow |z| = r = |sec\alpha|$

Since secα is negative in the interval $\left(\frac{\pi}{2},\pi\right]$.

$\Rightarrow |z| = r = -sec\alpha$

$\Rightarrow \theta = \arg(z) = \tan^{-1}\left(\frac{1}{tan\alpha}\right)$

$\Rightarrow \theta = \tan^{-1}(cot\alpha)$

Since cotα is negative in the interval $\left(\frac{\pi}{2}, \pi\right]$.

$\Rightarrow \theta = \frac{\pi}{2} + \alpha$. ($\because$ θ lies in3[rd] quadrant)

$\Rightarrow z = -sec\alpha\left(cos\left(\frac{\pi}{2}+\alpha\right) + isin\left(\frac{\pi}{2}+\alpha\right)\right)$

⇒ z=-secα(-sinα+icosα)

⇒ z=secα(sinα-icosα)

∴ The polar form is z=secα(sinα-icosα).

3 C. Question

Express the following complex numbers in the form $r(\cos\theta + i\sin\theta)$:

1 – sin α + i cos α

Answer

Given Complex number is z=1-sinα+icosα

We know that $\sin^2\theta+\cos^2\theta=1$, $\sin 2\theta=2\sin\theta\cos\theta$, $\cos 2\theta=\cos^2\theta-\sin^2\theta$.

$\Rightarrow z = \left(\sin^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right) - 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right) + i\left(\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)$

$\Rightarrow z = \left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)^2 + i\left(\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)$

e know that the polar form of a complex number Z=x+iy is given by Z=|Z|(cosθ+isinθ)

Where,

|Z|=modulus of complex number=$\sqrt{x^2+y^2}$

θ =arg(z)=argument of complex number=$\tan^{-1}\left(\frac{|y|}{|x|}\right)$

$\Rightarrow |z| = \sqrt{(1-sin\alpha)^2 + \cos^2\alpha}$

$\Rightarrow |z| = \sqrt{1+\sin^2\alpha - 2sin\alpha + \cos^2\alpha}$

$\Rightarrow |z| = \sqrt{1+1-2sin\alpha}$

$\Rightarrow |z| = \sqrt{(2)(1-sin\alpha)}$

$\Rightarrow |z| = \sqrt{(2)\left(\sin^2\left(\frac{\alpha}{2}\right) + \cos^2\left(\frac{\alpha}{2}\right) - 2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right)}$

$\Rightarrow |z| = \sqrt{(2)\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)^2}$

$\Rightarrow |z| = \left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)\right|$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\cos^2\left(\frac{\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)}{\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)^2}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)\left(\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)\right)}{\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)^2}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\cos\left(\frac{\alpha}{2}\right)\left(1+\tan\left(\frac{\alpha}{2}\right)\right)}{\cos\left(\frac{\alpha}{2}\right)\left(1-\tan\left(\frac{\alpha}{2}\right)\right)}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\alpha}{2}\right)}{1-\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\alpha}{2}\right)}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)$

We know that sine and cosine functions are periodic with period 2π

Here We have 3 intervals as follows:

(i) $0 \le \alpha \le \frac{\pi}{2}$

(ii) $\frac{\pi}{2} \le \alpha \le \frac{3\pi}{2}$

(iii) $\frac{3\pi}{2} \le \alpha < 2\pi$

Case(i):

In the interval $0 \le \alpha < \frac{\pi}{2}$, $cos\left(\frac{\alpha}{2}\right) > \sin\left(\frac{\alpha}{2}\right)$ and also $0 < \frac{\pi}{4} + \frac{\alpha}{2} < \frac{\pi}{2}$

so,

$\Rightarrow |z| = \left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)\right|$

$\Rightarrow |z| = \sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)$

$\Rightarrow \theta = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)$

$\Rightarrow \theta = \frac{\pi}{4} + \frac{\alpha}{2}$.($\because$ θ lies in 1st quadrant)

$\therefore$ The polar form is $\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)\left(\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right) + isin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)$.

Case(ii):

In the interval $\frac{\pi}{2} \le \alpha < \frac{3\pi}{4}$, $cos\left(\frac{\alpha}{2}\right) < \sin\left(\frac{\alpha}{2}\right)$ and also $\frac{\pi}{2} < \frac{\pi}{4} + \frac{\alpha}{2} < \pi$

so,

$\Rightarrow |z| = \left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)\right|$

$\Rightarrow |z| = -\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)$

$\Rightarrow |z| = \sqrt{2}\left(\sin\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\alpha}{2}\right)\right)$

$\Rightarrow \theta = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)$

$\Rightarrow \theta = \frac{\pi}{4} + \frac{\alpha}{2} - \pi$. ($\because$ θ lies in 4th quadrant)

$\Rightarrow \theta = \frac{\alpha}{2} - \frac{3\pi}{4}$

∴ The polar form is $\sqrt{2}\left(\sin\left(\frac{\alpha}{2}\right)-\cos\left(\frac{\alpha}{2}\right)\right)\left(\cos\left(\frac{\alpha}{2}-\frac{3\pi}{4}\right)+isin\left(\frac{\alpha}{2}-\frac{3\pi}{4}\right)\right)$.

Case(iii):

In the interval $\frac{3\pi}{2}\leq\alpha<2\pi$, $cos\left(\frac{\alpha}{2}\right)<\sin\left(\frac{\alpha}{2}\right)$ and also $\pi<\frac{\pi}{4}+\frac{\alpha}{2}<\frac{5\pi}{4}$

so,

$\Rightarrow |z|=\left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)\right|$

$\Rightarrow |z|=-\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)$

$\Rightarrow |z|=\sqrt{2}\left(\sin\left(\frac{\alpha}{2}\right)-\cos\left(\frac{\alpha}{2}\right)\right)$

$\Rightarrow \theta=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)$

$\Rightarrow \theta=\pi-\frac{\pi}{4}-\frac{\alpha}{2}$.(since θ presents in first quadrant and tan's period is π)

$\Rightarrow \theta=\frac{3\pi}{4}-\frac{\alpha}{2}$.

∴ The polar form is $\sqrt{2}\left(\sin\left(\frac{\alpha}{2}\right)-\cos\left(\frac{\alpha}{2}\right)\right)\left(\cos\left(\frac{3\pi}{4}-\frac{\alpha}{2}\right)+isin\left(\frac{3\pi}{4}-\frac{\alpha}{2}\right)\right)$.

3 D. Question

Express the following complex numbers in the form $r(\cos\theta+i\sin\theta)$:

$$\frac{1-i}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$$

Answer

Given complex number is $z=\frac{1-i}{\cos\left(\frac{\pi}{3}\right)+isin\left(\frac{\pi}{3}\right)}$

$\Rightarrow z=\frac{1-i}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}$

$\Rightarrow z=2\times\frac{1-i}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$\Rightarrow z=2\times\frac{1(1-i\sqrt{3})-i(1-i\sqrt{3})}{1^2-(i\sqrt{3})^2}$

$\Rightarrow z=2\times\frac{1+i^2\sqrt{3}-i(1+\sqrt{3})}{1-i^2 3}$

We know that i^2=-1

$\Rightarrow z=2\times\frac{(1+(-\sqrt{3})-i(1+\sqrt{3}))}{1-(-3)}$

$\Rightarrow z=2\times\frac{(1-\sqrt{3})-i(1+\sqrt{3})}{4}$

$\Rightarrow z=\frac{(1-\sqrt{3})-i(1+\sqrt{3})}{2}$

We know that the polar form of a complex number Z=x + iy is given by Z=|Z|(cos θ+ i sin θ)

Where,

|Z|=modulus of complex number= $\sqrt{x^2+y^2}$

θ = arg(z)=argument of complex number= $tan^{-1}\left(\frac{|y|}{|x|}\right)$

Now for the given problem,

$\Rightarrow |z| = \sqrt{\left(\frac{1-\sqrt{3}}{2}\right)^2 + \left(\frac{-1-\sqrt{3}}{2}\right)^2}$

$\Rightarrow |z| = \sqrt{\frac{1+3-2\sqrt{3}+1+2+2\sqrt{3}}{4}}$

$\Rightarrow |z| = \sqrt{\frac{8}{4}}$

$\Rightarrow |z| = \sqrt{2}$

$\Rightarrow \theta = \tan^{-1}\left(\left|\frac{\frac{1+\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}}\right|\right)$

$\Rightarrow \theta = \tan^{-1}\left(\left|\frac{1+\sqrt{3}}{1-\sqrt{3}}\right|\right)$

$\Rightarrow \theta = \tan^{-1}\left(\left|\frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}\right|\right)$

$\Rightarrow \theta = \tan^{-1}\left(\left|\frac{1+3+2\sqrt{3}}{1-3}\right|\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4+2\sqrt{3}}{2}\right)$

Since x<0,y<0 complex number lies in 3rd quadrant and the value of θ will be as follows $-180^0 \le \theta \le -90^0$.

$\Rightarrow \theta = \tan^{-1}(2+\sqrt{3})$

$\Rightarrow \theta = \frac{-7\pi}{12}$.

$\Rightarrow Z = \sqrt{2}\left(\cos\left(\frac{-7\pi}{12}\right) + isin\left(\frac{-7\pi}{12}\right)\right)$

$\Rightarrow z = \sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) - isin\left(\frac{7\pi}{12}\right)\right)$

∴ The Polar form of $Z = \frac{1-i}{\cos\left(\frac{\pi}{3}\right)+isin\left(\frac{\pi}{3}\right)}$ is $z = \sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) - isin\left(\frac{7\pi}{12}\right)\right)$.

4. Question

If z_1 and z_2 are two complex number such that $|z_1| = |z_2|$ and arg (z_1) + arg $(z_2) = \pi$, then show that $z_1 = -\bar{z}_2$.

Answer

Given:

⇒ $|z_1|=|z_2|$ and $\arg(z_1)+\arg(z_2)=\pi$

Let us assume $\arg(z_1)=\theta$

⇒ $\arg(z_2)=\pi-\theta$

We know that z=|z|(cosθ+isinθ)

⇒ $z_1=|z_1|(\cos\theta+i\sin\theta)$-----------------(1)

$\Rightarrow z_2=|z_2|(\cos(\pi-\theta)+i\sin(\pi-\theta))$

$\Rightarrow z_2=|z_2|(-\cos\theta+i\sin\theta)$

$\Rightarrow z_2=-|z_2|(\cos\theta-i\sin\theta)$

Now we find the conjugate of z_2

$\Rightarrow \bar{z}_2=-|z_2|(\cos\theta+i\sin\theta)$ ($\because \overline{|z_2|} = |z_2|$)

Now,

$\Rightarrow \frac{z_1}{z_2} = \frac{|z_1|(\cos\theta+i\sin\theta)}{-|z_2|(\cos\theta+i\sin\theta)}$

$\Rightarrow \frac{z_1}{z_2} = -1$ ($\because |z_1|=|z_2|$)

$\Rightarrow z_1=-\bar{z}_2$

∴ Thus proved.

5. Question

If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, prove that $\arg\left(\frac{z_1}{z_4}\right)+\arg\left(\frac{z_2}{z_3}\right)=0.$

Answer

Given:

$\Rightarrow z_1 = \bar{z}_2$

$\Rightarrow z_3 = \bar{z}_4$

We know that $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$

$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg(\bar{z}_2) - \arg(z_4) + \arg(z_2) - \arg(\bar{z}_4)$

$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = (\arg(z_2) + \arg(\bar{z}_2)) - (\arg(z_4) + \arg(\bar{z}_4))$

We know that $\arg(z)+\arg(\bar{z})=0$

$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = 0 - 0$

$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = 0$

∴ Thus proved.

6. Question

Express $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$ in polar form.

Answer

Given Complex number is $z = \sin\frac{\pi}{5} + i\left(1-\cos\frac{\pi}{5}\right)$

We know that $\sin2\theta=2\sin\theta\cos\theta$ and $1-\cos2\theta=2\sin^2\theta$

$\Rightarrow z = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i\left(2\sin^2\frac{\pi}{10}\right)$

$\Rightarrow z = 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$

∴ The Polar form of $z = \sin\left(\frac{\pi}{5}\right) + i\left(1 - \cos\frac{\pi}{5}\right)$ is $z = 2\sin\frac{\pi}{10}\left(\cos\left(\frac{\pi}{10}\right) + isin\left(\frac{\pi}{10}\right)\right)$.

Very Short Answer

1. Question

Write the value of the square root of i.

Answer

Let $\sqrt{i} = \sqrt{a+ib}$..................1

Squaring both sides, we get

$i^2 = (a^2-b^2) + 2aib$

By comparing real and imaginary term, we get

$2ab = 1$ and $a^2-b^2 = 0$

By solving these we get

$a = b = \frac{1}{\sqrt{2}}$

By putting value of a and b in 1, we get

$\sqrt{i} = \pm\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$\sqrt{i} = \pm\frac{1}{\sqrt{2}}(1+i)$

2. Question

Write the values of the square root of –i.

Answer

$\sqrt{-i} = x + iy$

Squaring both side

$-i = (x + iy)^2$

$= (x^2-y^2)+2ixy$

$x^2-y^2 = 0$

$2xy = -1$

As we know all that,

$(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$(x^2+y^2)^2 = 0+1$

$(x^2+y^2)^2 = 1$

$x^2+y^2 = 1$

$X^2-y^2 = 0$ .... (1)

$x^2+y^2 = 1$ ....(2)

From (1)

$x^2 = y^2$...............(3)

$2x^2 = 1$ (because $x^2 = y^2$)

$2xy = -1$

it means $xy<0$

Either $x<0$, $y > 0$

Or $x>0$, $y<0$

X and y have different sign

$$\therefore x + iy = \pm\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) \text{ or } -\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$$

3. Question

If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then write the value of $(x^2 + y^2)^2$.

Answer

$$x + iy = \sqrt{\frac{a+ib}{c+id}}$$

$$x + iy = \sqrt{\left(\frac{a+ib}{c+id}\right) \times \left(\frac{c-id}{c-id}\right)}$$

$$= \sqrt{\left(\frac{(a+ib)(c-id)}{c^2+d^2}\right)}$$

$$= \sqrt{\left(\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}\right)}$$

By squaring both sides, we get

$$\Rightarrow (x+iy)^2 = \left(\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}\right)$$

$$\Rightarrow x^2 - y^2 + 2ixy = \left(\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}\right)$$

On comparing real and imaginary parts, we get

$$x^2 - y^2 = \left(\frac{ac+bd}{c^2+d^2}\right), 2xy = \left(\frac{bc-ad}{c^2+d^2}\right)$$

We know that,

$$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$$

$$\Rightarrow \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{bc-ad}{c^2+d^2}\right)^2$$

$$\Rightarrow \left(\frac{a^2c^2 + b^2d^2 + 2abcd + b^2c^2 + a^2d^2 - 2abcd}{(c^2 + d^2)^2}\right)$$

$$\Rightarrow \left(\frac{a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2}{(c^2 + d^2)^2}\right)$$

$$\Rightarrow \left(\frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}\right)$$

$$\Rightarrow \left(\frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}\right)$$

$$\Rightarrow \left(\frac{(a^2 + b^2)}{c^2 + d^2}\right)$$

4. Question

If π< θ < 2π and z = 1 + cos θ + i sin θ, then write the value of |z|.

Answer

As we all know that,

z = 1 + cos θ + i sin θ

a = (1+cosθ) and b = sin θ

$$|z| = \sqrt{a^2 + b^2}$$

$$\Rightarrow \sqrt{(1 + \cos\theta)^2 + \sin\theta} \Rightarrow \sqrt{1 + \cos^2\theta + 2\cos\theta + \sin^2\theta} \Rightarrow \sqrt{2 + 2\cos\theta}$$

$$\Rightarrow \sqrt{2(1 + \cos\theta)} \Rightarrow \sqrt{4\cos^2\frac{\theta}{2}} = 2\cos^2\frac{\theta}{2}$$

π< θ < 2π it means z lies in second quadrant

z = -θ

$$\Rightarrow -2\cos^2\frac{\theta}{2}$$

5. Question

If n is any positive integer, write the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$.

Answer

Explanation

As we know that $i = \sqrt{-1}$, i^2 = -1, i^3 = -i , i^4 = 1

$$z = \frac{i^{4n+1} - i^{4n-1}}{2}$$

$$= \frac{((i^4)^n)(i^1 - i^{-1})}{2}$$

$$= \frac{1^n\left(i - \left(\frac{1}{i}\right)\right)}{2}$$

$$= \frac{i^2 - 1}{2i}$$

$= \frac{-1-1}{2i}$

$= -\frac{1}{i} \times \frac{i}{i}$

$= -\frac{i}{i^2}$

$= -\frac{i}{-1}$

= i

6. Question

Write the value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$.

Answer

Explanation

$z = \frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$

$= \frac{(i^2)^{296}+(i^2)^{295}+(i^2)^{294}+(i^2)^{293}+(i^2)^{292}}{(i^2)^{291}+(i^2)^{290}+(i^2)^{289}+(i^2)^{288}+(i^2)^{287}}$

$= \frac{1-1+1-1+1}{-1+1-1+1-1}$

= -2

7. Question

Write 1 – i in polar form.

Answer

Z = 1-i = a+ib

So, a = 1 , b = -1

$|z| = \sqrt{a^2+b^2} \Rightarrow \sqrt{1^2+(-1)^2} = \sqrt{2} = r$

$\tan\alpha = \left|\frac{b}{a}\right| \Rightarrow \left|\frac{-1}{1}\right| \Rightarrow 1$

$\alpha = \frac{\pi}{4}$ or $45°$

tan α a>0 , b<1

∴z lies in forth quadrant

arg (z) = θ

$\Rightarrow -\alpha = -\frac{\pi}{4}$

Required polar form

$= z(\cos\theta + i\sin\theta) \Rightarrow \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) \Rightarrow \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right)$

8. Question

Write -1 + i√3 in polar form.

Answer

$$z = -1 + \sqrt{3}i = a + ib$$

So, a = 1 , b = -1

$$|z| = \sqrt{a^2 + b^2} \Rightarrow \sqrt{-1^2 + \left(\sqrt{3}\right)^2} = 2 = r$$

$$\tan\alpha = \left|\frac{b}{a}\right| \Rightarrow \left|\frac{\sqrt{3}}{-1}\right| \Rightarrow \sqrt{3} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{4} \text{ or } 45°$$

tan α a<0 , b>1

∴z lies in second quadrant

$$\arg(z) = \theta \Rightarrow \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Required polar form

$$= z(\cos\theta + i\sin\theta) \Rightarrow 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

9. Question

Write the argument of –i.

Answer

$$z = 0 - i = a + ib$$

So, a = 0 , b = -1

$$\tan\alpha = \left|\frac{b}{a}\right| \Rightarrow \left|\frac{-1}{0}\right| \Rightarrow \text{not defined} = \infty = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4} \text{ or } 45°$$

a = 0 , b = -1

∴z lies in forth quadrant

arg(z) = θ

$$\Rightarrow -\alpha = -\frac{\pi}{2}$$

10. Question

Write the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is real.

Answer

$$\left(\frac{1+i}{1-i}\right)^n = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n$$

$$= \left(\frac{(1+i)^2}{(1)^2 - (i)^2}\right)^n$$

$= \left(\frac{1+i^2+2i}{2}\right)^n$

$= \left(\frac{1-1+2i}{2}\right)^n$

$= \left(\frac{2i}{2}\right)^n$

$= i^n$

As we know that $i^2 = -1$

And value of n is real number so,

n = 2

11. Question

Find the principal argument of $\left(1+i\sqrt{3}\right)^2$.

Answer

As we know that, z = a+ib

$z = \left(1+\sqrt{3}i\right)^2$

$= 1^2 + \left(\sqrt{3}i\right)^2 + 2\times 1\times\sqrt{3}i$

$= 1+i^2+2i\sqrt{3}$

= 1-3+2i√3

= -2+2i√3

a = -2 b = 2√3

$\tan\alpha = \left|\frac{b}{a}\right|$

$= \left|\frac{2\sqrt{3}}{-2}\right|$

= |√3|

$\alpha = \frac{\pi}{3}$ or $60°$

α<0 , b>1

∴z lies in second quadrant

arg(z) = θ

$= \pi - a$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

12. Question

Find z, if |z| = 4 and $\arg(z) = \frac{5\pi}{6}$.

Answer

r = |z| = 4 ,

$$\arg(z) = \frac{5\pi}{6} = \theta$$

z = r(cos θ + i sin θ)

$$= 4(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$$

$$= 4\left(\cos\left(\pi - \frac{\pi}{6}\right) + i\sin\left(\pi - \frac{\pi}{6}\right)\right)$$

$$= 4(-\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$$

$$= 4\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$= \frac{4}{2}(-\sqrt{3} + i)$$

z = -2√3+2i

13. Question

If |z – 5i| = |z + 5i|, then find the locus of z.

Answer

z = a + ib

|a+ib-5i| = |a+ib+5i|

$|a+ib-5i|^2 = |a+ib+5i|^2$

$|a + i(b-5)|^2 = |a + i(b+5)|^2$

$a^2+(b-5)^2 = a^2+(b+5)^2$

$a^2+b^2+25-10b = a^2+b^2+25+10b$

20b = 0

b = 0

b is a imaginary part of z

$$\operatorname{Im}(z) = \frac{z - \bar{z}}{2}$$

= y

= 0

Im (z) = 0

So, the locus point is real axis

14. Question

If $\frac{\left(a^2+1\right)^2}{2a-i} = x + iy,$ find the value of $x^2 + y^2$.

Answer

$$\frac{(a^2+1)^2}{2a-i} = x+iy$$

$$= \frac{a^4+1+2a^2}{2a-i} \times \frac{2a+i}{2a+i}$$

$$= \frac{\left(2a(a^4+1+2a^2)\right)+\left(i(a^4+1+2a^2)\right)}{4a^2+1}$$

$$x+iy = \frac{\left(2a(a^4+1+2a^2)\right)+\left(i(a^4+1+2a^2)\right)}{4a^2+1}$$

Comparing real and imaginary part, we get

$$x = \frac{\left(2a(a^4+1+2a^2)\right)}{4a^2+1}, y = \frac{\left(i(a^4+1+2a^2)\right)}{4a^2+1}$$

So, X^2+Y^2

$$x^2+y^2 = \frac{\left(2a(a^4+1+2a^2)\right)^2}{(4a^2+1)^2} + \frac{\left(i(a^4+1+2a^2)\right)^2}{(4a^2+1)^2}$$

$$= \frac{4a^2\left((a^4+1+2a^2)\right)^2}{(4a^2+1)^2} + \frac{i^2\left((a^4+1+2a^2)\right)^2}{(4a^2+1)^2}$$

$$= \frac{4a^2\left((a^4+1+2a^2)\right)^2}{(4a^2+1)^2} - \frac{1\left((a^4+1+2a^2)\right)^2}{(4a^2+1)^2}$$

$$= \frac{(4a^2-1)(a^4+1+2a^2)^2}{(4a^2+1)^2}$$

$$= \frac{(4a^2-1)(a^2+1)^4}{(4a^2+1)^2}$$

$$= \frac{(a^2+1)^4}{(4a^2+1)}$$

15. Question

Write the value of $\sqrt{-25} \times \sqrt{-9}$.

Answer

$$\sqrt{-25} \times \sqrt{-9} = \sqrt{25}\,\sqrt{-1} \times \sqrt{9}\,\sqrt{-1}$$

= 5i×3i

= $15i^2$

= -15

16. Question

Write the sum of the series i + i^2 + i^3 + ... Upto 1000 terms.

Answer

0

Explanation

Here, $a = i$

$$r = \frac{i^2}{i}$$

= i

n = 1000 terms

$$s_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow s_{1000} = \frac{i(i^{1000} - 1)}{1000 - 1}$$

$$= \frac{i(1 - 1)}{999}$$

= 0

17. Question

Write the value of $\arg(z) + \arg(\bar{z})$.

Answer

As we all know that,

z = r(cos θ + i sin θ) , θ = arg(z)

$\bar{z} = r(\cos\theta - \sin\theta)$

= r(cos(-θ)+sin(-θ)

$, -\theta = \arg(\bar{z})$

So, $\arg(z) + \arg(\bar{z}) = \theta - \theta$

= 0

18. Question

If |z + 4| ≤ 3, then find the greatest and least values of |z + 1|.

Answer

6 and 0

Explanation

As we all know that,

$|z_1 + z_2| \le |z_1| + |z_2|$ and $|z_1 + z_2| \ge |z_1| - |z_2|$

Suppose,

Z_1 = z+4

Z_2 = -3

$|z_1| - |z_2| \le |z_1 + z_2| \le |z_1| + |z_2|$

|z+4|-|-3|≤|z+4-3|≤ |z+4|+|-3|

|z+4|-3≤|z+1|≤ |z+4|+3

3-3≤|z+1|≤ 3+3 (Given-|z + 4| ≤ 3)

0≤|z+1|≤ 6

19. Question

for any two complex numbers z_1 and z_2 and any two real numbers a, b find the value of $|az_1 - bz_2|^2 + |az_2 + bz_1|^2$.

Answer

$|az_1-bz_2|^2 + |az_2+bz_1|^2$

$= (az_1 - bz_2)\overline{(az_1 - bz_2)} + (az_2 + bz_1)\overline{(az_2 + bz_1)}$ ($\therefore$ by using $z\,\bar{z} = |z|^2$)

$= (az_1 - bz_2)(a\overline{z_1} - b\overline{z_2}) + (az_2 + bz_1)(a\overline{z_2} - b\overline{z_1})$

$= a^2 z_1\overline{z_1} - abz_1\overline{z_2} - abz_2\overline{z_1} + b^2 z_2\overline{z_2} + a^2 z_2\overline{z_2} + abz_2\overline{z_1} + abz_1\overline{z_2} + b^2 z_1\overline{z_1}$

$= z_1\overline{z_1}(a^2 + b^2) + z_2\overline{z_2}(a^2 + b^2)$

$= |z_1|^2 (a^2+b^2) + |z_2|^2 (a^2+b^2)$

$= (a^2+b^2)(|z_1|^2+|z_2|^2)$

20. Question

Write the conjugate of $\frac{2-i}{(1-2i)^2}$.

Answer

$z = \frac{2-i}{(1-2i)^2}$

$= \frac{2-i}{1+4i^2-4i}$

$= \frac{2-i}{1-4-4i}$

$= \frac{2-i}{-3-4i}$

$= \frac{2-i}{-3-4i} \times \frac{-3+4i}{-3+4i}$

$= \left(\frac{(2-i)(-3+4i)}{(-3)^2-(4i)^2}\right)$

$= \frac{-6+8i+3i+4}{9+16}$

$= \frac{-2+11i}{25}$

$= -\frac{2}{25} + i\frac{11}{25}$

21. Question

If $n \in N$, then find the value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$.

Answer

As we know that,

$i = \sqrt{(-1)}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$z = i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$= i^n (1+i^1+i^2+i^3)$

$= i^n (1+i-1-i)$

$= i^n(0)$

= 0

22. Question

Find the real value of a for which $3i^3 - 3ai^2 + (1 - a)i + 5$ is real.

Answer

a = 2

Explanation

Z is a purely real, it means Im (z) = 0

$Z = 3i^3 - 3ai^2 + (1-a)i + 5$

= -3i+3a+ (1-a) i+5

= (3a+5)+i(-3+1-a)

= (3a+5)+i(-2-a)

Re(z) = 3a+5 , Im(z) = (-2-a)

Z is a real so, Im (z) = 0

-2-a = 0

a = -2

23. Question

If |z| = 2 and $\arg(z) = \frac{\pi}{4}$, find z.

Answer

r = |z| = 2 , $\arg(z) = \frac{\pi}{4} = \theta$

z = r(cos θ + i sin θ)

$= 2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

$= 2\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}}(1+i)$

z = √2 (1+i)

24. Question

Write the argument of $\left(1+\sqrt{3}\right)(1+i)\left(\cos\theta + i\sin\theta\right)$.

Answer

As we know that,

$\arg(z_1z_2) = \arg(z_1)+\arg(z_2)$ so,

$\arg(z_1z_2z_3) = \arg(z_1)+\arg(z_2)+\arg(z_3)$

arg((1+√3 i)(1+i)(cos θ+ i sin θ)

= arg(1+√3 i)+arg(1+i)+arg(cos θ + i sin θ).......... (1)

$$z_1 = \arg\left(1+\sqrt{3}\,i\right) = \tan\alpha = \left|\frac{\mathrm{Im}(z_1)}{\mathrm{Re}(z_1)}\right| \Rightarrow \left|\frac{\sqrt{3}}{1}\right| = \frac{\pi}{6}$$

∴arg(z_1) = θ

$$\alpha = \frac{\pi}{6}$$

z_2 = arg(1+i)

$$\tan\alpha = \left|\frac{\mathrm{Im}(z_1)}{\mathrm{Re}(z_1)}\right|$$

$$\Rightarrow \left|\frac{1}{1}\right| = \frac{\pi}{4}$$

$$\therefore \arg(z_2) = \theta$$

$$\alpha = \frac{\pi}{4}$$

z_3 = arg(cos θ + i sin θ)

⇒1(cos θ + i sin θ)

arg(z_3) = θ

∵ In r(cos θ + i sin θ) , arg(z) = θ

By putting the value of all arg in 1, we get

$$\frac{\pi}{6}+\frac{\pi}{4}+\theta = \frac{5\pi}{12}+\theta$$

MCQ

1. Question

Mark the Correct alternative in the following:

The value of (1 + i) (1 + i^2) (1 + i^3)(1 + i^4) is

A. 2

B. 0

C. 1

D. i

Answer

We know that

i= √(-1)

i^2= i × i

$$= \sqrt{-1} \times \sqrt{-1}$$

= -1

(1+i)(1+i^2)(1+i^3)(1+i^4) = (1+i)(1+(-1))(1+i^3)(1+i^4)

= (1+i)(0)(1+i^3)(1+i^4)

= 0

2. Question

Mark the Correct alternative in the following:

If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and $0 < \theta < 2\pi$, then θ =

A. π

B. π/2

C. π/3

D. π/6

Answer

$$\frac{3+2i\sin\theta}{1-2i\sin\theta} = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$= \frac{3-4\sin^2\theta + 8i\sin\theta}{1+4\sin^2\theta}$$

$$= \frac{3-4\sin^2\theta}{1+4\sin^2\theta} + i\frac{8\sin\theta}{1+4\sin^2\theta}$$

For real number, imaginary part should be 0

$$\Rightarrow \frac{8\sin\theta}{1+4\sin^2\theta} = 0$$

$\Rightarrow 8\sin\theta = 0$

$\Rightarrow \theta = n\pi$

As θ belongs to (0,2π) so θ = π

3. Question

Mark the Correct alternative in the following:

If (1 + i) (1 + 2i) (1 + 3i) …. (1 + n i) = a + i b, then 2 × 5 × 10 × … × $(1 + n^2)$ is equal to

A. $\sqrt{a^2+b^2}$

B. $\sqrt{a^2-b^2}$

C. $a^2 + b^2$

D. $a^2 - b^2$

E. a + b

Answer

Given that (1 + i) (1 + 2i) (1 + 3i) …. (1 + n i) = a + i b …(1)

We can also say that

(1 - i) (1 - 2i) (1 - 3i) …. (1 - n i) = a - i b …(2)

Multiply and divide the eq no. 2 with eq no. 1

$$\frac{(1+i)(1-i)(1+2i)(1-2i)\ \dots(1+ni)(1-ni)}{(1-i)\ (1-2i)\ \dots(1-ni)} = \frac{(a+ib)(a-ib)}{a-ib}$$

$((1)^2 - (i)^2)((1)^2 - (2i)^2)\dots\dots((1)^2 - (ni)^2) = ((a)^2 - (ib)^2)$

$2 \times 5 \times 10 \times \dots\dots \times (1 + n^2) = a^2 + b^2$

4. Question

Mark the Correct alternative in the following:

If $\sqrt{a+ib}=x+iy$, then possible value of $\sqrt{a-ib}$ is

A. x^2+y^2 B. $\sqrt{x^2+y^2}$

C. x + i y D. x – i y

E. $\sqrt{x^2-y^2}$

Answer

$\sqrt{a+ib}=x+iy$

Square both sides

a + ib = $(x + iy)^2 = x^2 + i2xy - y^2$

So, we can say that a = $x^2 - y^2$ and b = 2xy

a – ib = $(x^2 - y^2) - i(2xy)$

= $(x)^2 + 2(x)(-iy) + (-iy)^2$

= $(x + (-iy))^2$

= $(x - iy)^2$

$\sqrt{a-ib}= x-iy$

5. Question

Mark the Correct alternative in the following:

If $z=\cos\frac{\pi}{4}+i\sin\frac{\pi}{6}$, then

A. $|z|=1, \arg(z)=\frac{\pi}{4}$

B. $|z|=1, \arg(z)=\frac{\pi}{6}$

C. $|z|=\frac{\sqrt{3}}{2}, \arg(z)=\frac{5\pi}{24}$

D. $|z|=\frac{\sqrt{3}}{2}, \arg(z)=\tan^{-1}\frac{1}{\sqrt{2}}$

Answer

$z= \cos\frac{\pi}{4}+i\sin\frac{\pi}{6}$

$|z| = \sqrt{\cos^2\frac{\pi}{4}+ \sin^2\frac{\pi}{6}}$

$=\sqrt{\frac{1}{2}+\frac{1}{4}}$

$=\frac{\sqrt{3}}{2}$

$$\arg(z) = \tan^{-1}\left(\frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{4}}\right)$$

$$= \tan^{-1}\frac{1}{\sqrt{2}}$$

6. Question

Mark the Correct alternative in the following:

The polar form of $(i^{25})^3$ is

A. $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

B. $\cos\pi + i\sin\pi$

C. $\cos\pi - i\sin\pi$

D. $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

Answer

$z = (i^{25})^3 = i^{75} = i^{4\times18+3}$

We know that $i^4 = 1$ and $i^3 = -i$

$z = i^{4\times18}.i^3 = 0 - i$

$$|z| = \sqrt{0^2 + (-1)^2} = 1$$

$$\theta = \tan^{-1}\left(\frac{-1}{0}\right) = \frac{-\pi}{2}$$

$z = |z|(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right)$$

$$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$$

7. Question

Mark the Correct alternative in the following:

If $i^2 = -1$, then the sum $i + i^2 + i^3 + \ldots$ upto 1000 terms is equal to

A. 1

B. -1

C. i

D. 0

Answer

We know that

$i^{4n+1} = i$

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3 = -i$

$i^{4n+4} = i^4 = 1$

$i^{4n+1} + i^{4n+2} + i^{4n+3} + i^{4n+4} = i + (-1) + (-i) + 1$

$= 0$

$S = i + i^2 + i^3 + ...$ upto 1000 terms

We can make the pair of 4 terms because we know that value is repeat after every 4th terms. So, there are total 250 pairs are made and each pair have value equal to 0.

$S = 0$

8. Question

Mark the Correct alternative in the following:

If $z = \frac{-2}{1+i\sqrt{3}}$, then the value of arg(z) is

A. π

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{\pi}{4}$

Answer

$z = \frac{-2}{(1+i\sqrt{3})}\frac{(1-i\sqrt{3})}{(1-i\sqrt{3})} = \frac{-2(1-i\sqrt{3})}{4}$

$z = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$

$Arg(z) = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}\right)$

$= \tan^{-1}(-\sqrt{3})$

$= \frac{2\pi}{3}$

9. Question

Mark the Correct alternative in the following:

If $a = \cos\theta + i\sin\theta$, then $\frac{1+a}{1-a} =$

A. $\cot\frac{\theta}{2}$

B. $\cot\theta$

C. $i\cot\frac{\theta}{2}$

D. $i\tan\frac{\theta}{2}$

Answer

$\frac{1+a}{1-a} = \frac{1+(\cos\theta+i\sin\theta)}{1-(\cos\theta+i\sin\theta)} = \frac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta} \times \frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)+i\sin\theta}$

$= \frac{2i\sin\theta}{2-2\cos\theta}$

$= 0 + i\frac{\sin\theta}{1-\cos\theta}$

$= i\cot\frac{\theta}{2}$

10. Question

Mark the Correct alternative in the following:

If (1 + i) (1 + 2i) (1 + 3i) ... (1 + ni) = a + i b, then 2 . 5. 10 . 17 ....$(1 + n^2)$ =

A. a – ib

B. $a^2 - b^2$

C. $a^2 + b^2$

D. None of these

Answer

Given that (1 + i) (1 + 2i) (1 + 3i) .... (1 + n i) = a + i b ...(1)

We can also say that

(1 - i) (1 - 2i) (1 - 3i) .... (1 - n i) = a - i b ...(2)

Multiply and divide the eq no. 2 with eq no. 1

$\frac{(1+i)(1-i)(1+2i)(1-2i)\ \ldots.(1+ni)(1-ni)}{(1-i)\ (1-2i)\ \ldots.(1-ni)} = \frac{(a+ib)(a-ib)}{a-ib}$

$((1)^2 - (i)^2)((1)^2 - (2i)^2)\ldots\ldots((1)^2 - (ni)^2) = ((a)^2 - (ib)^2)$

$2 \times 5 \times 10 \times \ldots\ldots \times (1 + n^2) = a^2 + b^2$

11. Question

Mark the Correct alternative in the following:

If $\frac{(a^2+1)^2}{2a-i} = x + iy,$ then $x^2 + y^2$ is equal to

A. $\frac{(a^2+1)^4}{4a^2+1}$

B. $\frac{(a+1)^2}{4a^2+1}$

C. $\frac{(a^2-1)^2}{(4a^2-1)^2}$

D. None of these

Answer

$\frac{(a^2+1)^2}{2a-i} = x + iy$

$x + iy = \frac{(a^2+1)^2}{2a-i} \times \frac{2a+i}{2a+i}$

$= \frac{(a^2+1)^2(2a+i)}{4a^2+1}$

$= \frac{2a(a^2+1)^2}{4a^2+1} + i\frac{(a^2+1)^2}{4a^2+1}$

$x = \frac{2a(a^2+1)^2}{4a^2+1}$ and $y = \frac{(a^2+1)^2}{4a^2+1}$

$x^2 + y^2 = \left(\frac{2a(a^2+1)^2}{4a^2+1}\right)^2 + \left(\frac{(a^2+1)^2}{4a^2+1}\right)^2$

$= (4a^2+1)\frac{(a^2+1)^4}{(4a^2+1)^2}$

$= \frac{(a^2+1)^4}{4a^2+1}$

12. Question

Mark the Correct alternative in the following:

The principal value of the amplitude of(1 + i) is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{12}$

C. $\frac{3\pi}{4}$

D. π

Answer

We know that the principal value of amplitude is value of argument lie between (-π,π]

arg(z) = $\tan^{-1}(1) = \frac{\pi}{4}$

So, $\frac{\pi}{4}$ is called the principal value of the amplitude of (1 + i) because it lies between (-π,π]

13. Question

Mark the Correct alternative in the following:

The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer, is

A. 16

B. 8

C. 4

D. 2

Answer

$\frac{2i}{1+i} = \frac{2i}{(1+i)} \times \frac{(1-i)}{(1-i)} = 1 + i$

$\left(\frac{2i}{1+i}\right)^n = (1+i)^n$

Let check the value of $(1 + i)^n$ for different value of n

at n =1 , 1+ i (no)

at n =2 , $(1 + i)^2 = 1 + i^2 + 2i = 2i$ (no)

at n =3 , $(1 + i)^2(1 + i) = (1 + i)(2i) = 2i – 2$ (no)

at n =4 , $(1 + i)^2(1 + i)^2 = (2i)^2 = -4$ (no)

at n =5 , $(1 + i)^4(1 + i) = -4(1 + i)$ (no)

at n =6 , $(1 + i)^4(1 + i)^2 = -4(2i)$ (no)

at n =7 , $(1 + i)^6(1 + i) = -8i(1 + i) = -8i + 8$ (no)

at n =8 , $(1 + i)^4(1 + i)^4 = (-4)(-4) = 8$ (yes)

So, we can say that n = 8 is the least positive integer for which $\left(\frac{2i}{1+i}\right)^n$ is positive integer.

14. Question

Mark the Correct alternative in the following:

If z is a non-zero complex number, then $\left|\frac{|\bar{z}|^2}{z\bar{z}}\right|$ is equal to

A. $\left|\frac{|\bar{z}|}{z}\right|$

B. |z|

C. $|\bar{z}|$

D. None of these

Answer

Let, $z = re^{i\theta}$

$\bar{z} = re^{-i\theta}$

$z\bar{z} = re^{i\theta}.re^{-i\theta} = r^2$

$|\bar{z}| = r$

$|\bar{z}|^2 = r^2$

$\left|\frac{|\bar{z}|^2}{z\bar{z}}\right| = \left|\frac{r^2}{r^2}\right|$

=1

Solve option A

$$\left|\frac{|\bar{z}|}{z}\right| = \left|\frac{r}{re^{i\theta}}\right|$$

$$= \left|\frac{1}{e^{i\theta}}\right|$$

$=|e^{-i\theta}|$

=1

15. Question

Mark the Correct alternative in the following:

If a = 1 + i, then a^2 equals

A. 1 – i

B. 2i

C. (1 + i) (1 – i)

D. i – 1.

Answer

$a^2 = (1 + i)(1 + i)$

$= 1^2 + 2i + i^2$

= 1 – 1 + 2i

= 2i

16. Question

Mark the Correct alternative in the following:

If $(x + iy)^{1/3} = a + ib$, then $\frac{x}{a} + \frac{y}{b} =$

A. 0

B. 1

C. -1

D. None of these

Answer

$(x + iy)^{1/3} = a + ib$

$x + iy = (a + ib)^3$

$= a^3 + (ib)^3 + 3a^2(ib) + 3a(ib)^2$

$= a^3 - ib^3 + i3a^2b - 3ab^2$

$= (a^3 - 3ab^2) + i(3a^2b - b^3)$

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

$$\frac{x}{a} + \frac{y}{b} = \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b}$$

$= a^2 - 3b^2 + 3a^2 - b^2$

$= 4(a^2 - b^2)$

17. Question

Mark the Correct alternative in the following:

$\left(\sqrt{-2}\right)\left(\sqrt{-3}\right)$ is equal to

A. $\sqrt{6}$

B. $-\sqrt{6}$

C. $i\sqrt{6}$

D. None of these

Answer

$\sqrt{-2} = \sqrt{2}\,i$ and $\sqrt{-3} = \sqrt{3}\,i$

$\sqrt{-2}\sqrt{-3} = \sqrt{2}i \times \sqrt{3}i$

$= i^2 \sqrt{6}$

$= -\sqrt{6}$

18. Question

Mark the Correct alternative in the following:

The argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is

A. 60^o

B. 120^o

C. 210^o

D. 240^o

Answer

$\frac{1-i\sqrt{3}}{1+i\sqrt{3}} = \frac{(1-i\sqrt{3})(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}$

$= \frac{-2-2i\sqrt{3}}{4}$

$= \frac{-1}{2} + i\frac{-\sqrt{3}}{2}$

$\text{Arg } z = \tan^{-1}\left(\frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}}\right)$

$= \frac{\pi}{3}$

$= 60^o$

But answer is going in 3^{rd} quadrant because tan θ is positive but sin θ and cos θ both are negative and it is possible only in 3^{rd} quadrant.

So, answer is $\pi + 60^o = 180^o$

19. Question

Mark the Correct alternative in the following:

If $z = \left(\frac{1+i}{1-i}\right)$, then z^4 equals

A. 1

B. -1

C. 0

D. None of these

Answer

$z = \frac{1+i}{1-i}$

$= \frac{(1+i)}{(1-i)} \times \frac{(1+i)}{(1+i)}$

$= \frac{(1+i)^2}{1^2 - i^2}$

$= \frac{1+i^2+2i}{1+1}$

$= \frac{1-1+2i}{2}$

=i

$z^4 = i^4$

=1

20. Question

Mark the Correct alternative in the following:

If $z = \frac{1+2i}{1-(1-i)^2}$, then arg(z) equals

A. 0

B. $\frac{\pi}{2}$

C. π

D. None of these

Answer

$z = \frac{1+2i}{1-(1-1-2i)}$

$= \frac{1+2i}{1+2i}$

=1+i0

$\text{Arg } z = \tan^{-1}\frac{0}{1}$

=0

21. Question

Mark the Correct alternative in the following:

If s$z = \frac{1}{(2+3i)^2}$, then |z| =

A. $\frac{1}{13}$

B. $\frac{1}{5}$

C. $\frac{1}{12}$

D. None of these

Answer

$z = \frac{1}{(2+3i)^2}$

$= \frac{1}{(2+3i)^2} \times \frac{(2-3i)^2}{(2-3i)^2}$

$= \frac{-5-12i}{169}$

$= \frac{-5}{169} + i\frac{-12}{169}$

$|z| = \sqrt{\left(\frac{-5}{169}\right)^2 + \left(\frac{-12}{169}\right)^2}$

$= \sqrt{\frac{25+144}{(169)^2}}$

$= \sqrt{\frac{169}{(169)^2}}$

$= \frac{1}{13}$

22. Question

Mark the Correct alternative in the following:

If $z = \frac{1}{(1+i)(2+3i)}$, then |z| =

A. 1

B. $1/\sqrt{26}$

C. $5/\sqrt{26}$

D. None of these

Answer

$z = \frac{1}{(1+i)(2+3i)} = \frac{1}{-1+5i}$

$= \frac{1}{(-1+5i)} \times \frac{(-1-5i)}{(-1-5i)}$

$= \frac{-1}{26} + i\frac{-5}{26}$

$|z| = \sqrt{\left(\frac{-1}{26}\right)^2 + \left(\frac{-5}{26}\right)^2}$

$= \sqrt{\frac{1+25}{(26)^2}}$

$= \frac{1}{\sqrt{26}}$

23. Question

Mark the Correct alternative in the following:

If z = 1 – cos θ + i sin θ, then |z| =

A. $2\sin\frac{\theta}{2}$

B. $2\cos\frac{\theta}{2}$

C. $2\left|\sin\frac{\theta}{2}\right|$

D. $2\left|\cos\frac{\theta}{2}\right|$

Answer

$|z| = \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}$

$= \sqrt{2 - 2\cos\theta}$

$= 2\left|\sin\frac{\theta}{2}\right|$

24. Question

Mark the Correct alternative in the following:

If x + i y = (1 + i) (1 + 2 i) (1 + 3i), then $x^2 + y^2$ =

A. 0

B. 1

C. 100

D. None of these

Answer

Given that (1 + i) (1 + 2i) (1 + 3i) = x + i y ...(1)

We can also say that

(1 - i) (1 - 2i) (1 - 3i) = x - i y ...(2)

Multiply and divide the eq no. 2 with eq no. 1

$$\frac{(1+i)(1-i)(1+2i)(1-2i)(1+3i)(1-3i)}{(1-i)(1-2i)(1-3i)} = \frac{(x+iy)(x-iy)}{x-iy}$$

$((1)^2 - (i)^2)((1)^2 - (2i)^2)((1)^2 - (3i)^2) = ((x)^2 - (iy)^2)$

$x^2 + y^2 = 2 \times 5 \times 10 = 100$

25. Question

Mark the Correct alternative in the following:

If $z = \frac{1}{1-\cos\theta - i\sin\theta}$, then Re (z) =

A. 0

B. $\frac{1}{2}$

C. $\cot\frac{\theta}{2}$

D. $\frac{1}{2}\cot\frac{\theta}{2}$

Answer

$$z = \frac{1}{1-\cos\theta - i\sin\theta} = \frac{1}{(1-\cos\theta - i\sin\theta)} \times \frac{(1-\cos\theta + i\sin\theta)}{(1-\cos\theta + i\sin\theta)}$$

$$= \frac{(1-\cos\theta) + i\sin\theta}{2 - 2\cos\theta}$$

$$= \frac{1}{2} + i\frac{\cot\frac{\theta}{2}}{2}$$

$$\text{Re}(z) = \frac{1}{2}$$

26. Question

Mark the Correct alternative in the following:

If $x + iy = \frac{3+5i}{7-6i}$, then y =

A. 9/85

B. -9/85

C. 53/85

D. None of these

Answer

$$x + iy = \frac{3+5i}{7-6i}$$

$$= \frac{(3+5i)}{(7-6i)} \times \frac{(7+6i)}{(7+6i)}$$

$= \frac{-9+53i}{85}$

$= \frac{-9}{85} + i\frac{53}{85}$

$y = \frac{53}{85}$

27. Question

Mark the Correct alternative in the following:

If $\frac{1-ix}{1+ix} = a + ib$, then $a^2 + b^2 =$

A. 1

B. -1

C. 0

D. None of these

Answer

$a + ib = \frac{(1-ix)}{(1+ix)} \times \frac{(1-ix)}{(1-ix)}$

$= \frac{1-x^2-2ix}{1+x^2}$

$= \frac{1-x^2}{1+x^2} + i\frac{-2x}{1+x^2}$

$a = \frac{1-x^2}{1+x^2}$ and $b = \frac{-2x}{1+x^2}$

$a^2 + b^2 = \left(\frac{1-x^2}{1+x^2}\right)^2 + \left(\frac{-2x}{1+x^2}\right)^2$

$= \frac{(1-x^2)^2 + (-2x)^2}{(1+x^2)^2}$

$= \frac{1+x^4-2x^2+4x^2}{(1+x^2)^2}$

$= \frac{(1+x^2)^2}{(1+x^2)^2}$

$=1$

28. Question

Mark the Correct alternative in the following:

If θ is the amplitude of $\frac{a+ib}{a-ib}$, then tan θ =

A. $\frac{2a}{a^2+b^2}$

B. $\frac{2ab}{a^2-b^2}$

C. $\frac{a^2-b^2}{a^2+b^2}$

D. None Of these

Answer

$\frac{a+ib}{a-ib} = \frac{(a+ib)}{(a-ib)} \times \frac{(a+ib)}{(a+ib)}$

$= \frac{a^2-b^2+2iab}{a^2+b^2}$

$= \frac{a^2-b^2}{a^2+b^2} + i\frac{2ab}{a^2+b^2}$

$\mathrm{Tan}\,\theta = \frac{\left(\frac{2ab}{a^2+b^2}\right)}{\left(\frac{a^2-b^2}{a^2+b^2}\right)}$

$= \frac{2ab}{a^2-b^2}$

29. Question

Mark the Correct alternative in the following:

If $z = \frac{1+7i}{(2-i)^2}$, then

A. |z| = 2

B. $|z| = \frac{1}{2}$

C. $\mathrm{amp}(z) = \frac{\pi}{4}$

D. $\mathrm{amp}(z) = \frac{3\pi}{4}$

Answer

$z = \frac{1+7i}{3-4i} = \frac{(1+7i)}{(3-4i)} \times \frac{(3+4i)}{(3+4i)}$

$= \frac{-25+25i}{25}$

$= -1+i$

$|z| = \sqrt{(-1)^2+1^2}$

$= \sqrt{2}$

$\mathrm{amp}(z) = \tan^{-1}\frac{1}{-1}$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

30. Question

Mark the Correct alternative in the following:

The amplitude of $\frac{1}{i}$ is equal to

A. 0

B. $\frac{\pi}{2}$

C. $-\frac{\pi}{2}$

D. π

Answer

$\frac{1}{i} = \frac{1}{i} \times \frac{i}{i}$

$= \frac{i}{-1}$

=0+i(-1)

$\text{amp} = \tan^{-1}\frac{-1}{0}$

$= \frac{-\pi}{2}$

31. Question

Mark the Correct alternative in the following:

The argument of $\frac{1-i}{1+i}$ is

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. $\frac{5\pi}{2}$

Answer

$\frac{1-i}{1+i} = \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)}$

=0+i(-1)

$\arg = \tan^{-1}\frac{-1}{0}$

$= \frac{-\pi}{2}$

32. Question

Mark the Correct alternative in the following:

The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $-\frac{\pi}{6}$

Answer

$\frac{1+i\sqrt{3}}{\sqrt{3}+i} = \frac{(1+i\sqrt{3})}{(\sqrt{3}+i)} \times \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}$

$= \frac{2\sqrt{3}+2i}{4}$

$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$

$\text{amp} = \tan^{-1}\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= \tan^{-1}\frac{1}{\sqrt{3}}$

$= \frac{\pi}{6}$

33. Question

Mark the Correct alternative in the following:

The value of $(i^5 + i^6 + i^7 + i^8 + i^9)/(1 + i)$ is

A. $\frac{1}{2}(1+i)$

B. $\frac{1}{2}(1-i)$

C. 1

D. $\frac{1}{2}$

Answer

We know that

$i^{4n+1} = i$

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3 = -i$

$i^{4n+4} = i^4 = 1$

$i^5 + i^6 + i^7 + i^8 + i^9 = i + (-1) + (-i) + 1 + i$

$= i$

$$\frac{(i^5 + i^6 + i^7 + i^8 + i^9)}{(1+i)} = \frac{i}{1+i}$$

$$= \frac{i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{1}{2}(1+i)$$

34. Question

Mark the Correct alternative in the following:

$\frac{1+2i+3i^2}{1-2i+3i^2}$ equals

A. i

B. -1

C. -i

D. 4

Answer

$$\frac{1+2i+3i^2}{1-2i+3i^2} = \frac{1+i(2+3i)}{1+i(-2+3i)}$$

$$= \frac{i\left(\frac{1}{i} + (2+3i)\right)}{i\left(\frac{1}{i} + (-2+3i)\right)}$$

$$= \frac{-i + (2+3i)}{-i + (-2+3i)}$$

$$= \frac{2+2i}{-2+2i}$$

$$= \frac{1+i}{-1+i} \times \frac{-1-i}{-1-i}$$

$= -i$

35. Question

Mark the Correct alternative in the following:

The value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$ is

A. -1

B. -2

C. -3

D. -4

Answer

We know that

$i^{4n+1} = i$

$i^{4n+2} = i^2$

$= -1$

$i^{4n+3} = i^3$

$= -i$

$i^{4n+4} = i^4$

$= 1$

$i^{592} = i^{4(147)+4}$

$= 1$

$i^{582} = i^{4(145)+2}$

$= -1$

$i^{590} = i^{4(147)+2}$

$= -1$

$i^{580} = i^{4(144)+4}$

$= 1$

$i^{588} = i^{4(146)+4}$

$= 1$

$i^{578} = i^{4(144)+2}$

$= -1$

$i^{586} = i^{4(146)+2}$

$= -1$

$i^{576} = i^{4(143)+4}$

$= 1$

$i^{584} = i^{4(145)+4}$

$= 1$

$i^{574} = i^{4(143)+2}$

$= -1$

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} - 1 = \frac{1-1+1-1+1}{-1+1-1+1-1} - 1$$

$= -2$

36. Question

Mark the Correct alternative in the following:

The value of $(1 + i)^4 + (1 - i)^4$ is

A. 8

B. 4

C. -8

D. -4

Answer

$(1 + i)^4 + (1 - i)^4 = ((1 + i)^2)^2 + ((1 - i)^2)^2$

$= (2i)^2 + (-2i)^2$

$= -4 + -4$

$= -8$

37. Question

Mark the Correct alternative in the following:

If z = a + ib lies in third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant if

A. a > b > 0

B. a < b < 0

C. b < a < 0

D. b > a > 0

Answer

If z = a + ib lies in third quadrant then a and b both are less than zero

$\bar{z} = a - ib$

$\frac{\bar{z}}{z} = \frac{a - ib}{a + ib}$

$= \frac{a - ib}{a + ib} \times \frac{a - ib}{a - ib}$

$= \frac{a^2 - b^2 - 2iab}{a^2 + b^2}$

$= \frac{a^2 - b^2}{a^2 + b^2} + i\frac{-2ab}{a^2 + b^2}$

$\frac{a^2 - b^2}{a^2 + b^2} < 0 \text{ and } \frac{-2ab}{a^2 + b^2} < 0$

$a^2 - b^2 < 0$ and ab > 0 because $a^2 + b^2$ is always greater than zero

(a – b)(a + b) < 0

Here a and b both are less than zero that means (a + b) is always less than zero

So, a – b > 0 ⇒ a > b

Then, final answer is b < a < 0

38. Question

Mark the Correct alternative in the following:

If $f(z) = \frac{7 - z}{1 - z^2}$, where z = 1 + 2i, then |f(z)| is

A. $\frac{|z|}{2}$

B. |z|

C. 2|z|

D. None of these

Answer

$|z| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$

$f(z) = \frac{7 - z}{1 - z^2}$

$= \frac{7 - (1 + 2i)}{1 - (1 + 2i)^2}$

$= \frac{6 - 2i}{4 - 4i}$

$= \frac{3 - i}{2 - 2i} \times \frac{2 + 2i}{2 + 2i}$

$= \frac{8 + 4i}{8}$

$= 1 + i\frac{1}{2}$

$|f(z)| = \sqrt{(1)^2 + \left(\frac{1}{2}\right)^2}$

$= \frac{\sqrt{5}}{2}$

$= \frac{|z|}{2}$

39. Question

Mark the Correct alternative in the following:

A real value of x satisfies the equation $\frac{3 - 4ix}{3 + 4ix} = a - ib(a, b \in R)$, if $a^2 + b^2 =$

A. 1

B. -1

C. 2

D. -2

Answer

$a - ib = \frac{(3-4ix)}{(3+4ix)} \times \frac{(3-4ix)}{(3-4ix)}$

$= \frac{9 - 16x^2 - 24ix}{9 + 16x^2}$

$= \frac{9 - 16x^2}{9 + 16x^2} - i\frac{24x}{9 + 16x^2}$

$a = \frac{9 - 16x^2}{9 + 16x^2}$ and $b = \frac{24x}{9 + 16x^2}$

$$a^2 + b^2 = \left(\frac{9 - 16x^2}{9 + 16x^2}\right)^2 + \left(\frac{24x}{9 + 16x^2}\right)^2$$

$$= \frac{81 + 256x^4 - 288x^2 + 576x^2}{(9 + 16x^2)^2}$$

$$= \frac{(9 + 16x^2)^2}{(9 + 16x^2)^2}$$

=1

40. Question

Mark the Correct alternative in the following:

The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on

A. circle $x^2 + y^2 = 1$

B. the x-axis

C. the y-axis

D. the line x + y = 1

Answer

Let, z = x + iy

$$\frac{i+z}{i-z} = \frac{x + i(y+1)}{-x + i(-y+1)}$$

$$= \frac{x + i(y+1)}{-x + i(-y+1)} \times \frac{-x - i(-y+1)}{-x - i(-y+1)}$$

$$= \frac{-x^2 - y^2 + 1 - 2ix}{x^2 + y^2 - 2y + 1}$$

$$= \frac{-x^2 - y^2 + 1}{x^2 + y^2 - 2y + 1} + i\frac{-2x}{x^2 + y^2 - 2y + 1}$$

$$\left|\frac{i+z}{i-z}\right| = \sqrt{\left(\frac{-x^2 - y^2 + 1}{x^2 + y^2 - 2y + 1}\right)^2 + \left(\frac{-2x}{x^2 + y^2 - 2y + 1}\right)^2}$$

$$= \sqrt{\frac{(x^4 + y^4 + 1 + 2x^2y^2 - 2x^2 - 2y^2) + 4x^2}{(x^2 + y^2 - 2y + 1)^2}}$$

$$= \sqrt{\frac{(x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 - 2y^2)}{(x^2 + y^2 - 2y + 1)^2}}$$

$$\left|\frac{i+z}{i-z}\right| = 1$$

$$\sqrt{\frac{(x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 - 2y^2)}{(x^2 + y^2 - 2y + 1)^2}} = 1$$

$x^4+y^4+1+2x^2 y^2+2x^2-2y^2= x^4+y^4+1+2x^2 y^2+2x^2+$

$6y^2-4y^3-2xy(x+y)-4y$

$8y^2 - 4y^3 - 2xy(x + y) - 4y = 0$

$y(8y - 4y^2 - 2x(x + y) - 4) = 0$

$y = 0$ and $8y - 4y^2 - 2x(x + y) - 4 = 0$

So, by y = 0 we can say that it lies on x axis

41. Question

Mark the Correct alternative in the following:

If z is a complex number, then

A. $|z|^2 > |\bar{z}|^2$

B. $|z|^2 = |\bar{z}|^2$

C. $|z|^2 < |\bar{z}|^2$

D. $|z|^2 \geq |\bar{z}|^2$

Answer

Let, z = a + ib

$\bar{z} = a - ib = a + i(-b)$

$|z| = \sqrt{a^2 + b^2}$

$|z|^2 = a^2 + b^2$

$|\bar{z}| = \sqrt{a^2 + (-b)^2}$

$= \sqrt{a^2 + b^2}$

$|\bar{z}|^2 = a^2 + b^2$

$|z|^2 = |\bar{z}|^2$

42. Question

Mark the Correct alternative in the following:

Which of the following is correct for any two complex numbers z_1 and z_2?

A. $|z_1 z_2| = |z_1| |z_2|$

B. $\arg(z_1 z_2) = \arg(z_1) \arg(z_2)$

C. $|z_1 + z_2| = |z_1| + |z_2|$

D. $|z_1 + z_2| \geq |z_1| + |z_2|$

Answer

Let, $z_1 = r_1 e^{i\alpha}$ and $z_2 = r_2 e^{i\beta}$

$|z_1| = r_1$ and $|z_2| = r_2$

<u>Option A</u>

$z_1 z_2 = r_1 r_2 e^{i(\alpha+\beta)}$

$|z_1 z_2| = r_1 r_2 = |z_1| |z_2|$

Option A correct

Option B

$\arg(z_1 z_2) = \alpha + \beta$

$= \arg(z_1) + \arg(z_2)$

Option B not correct

Let, z_1 = a+ ib and z_2 = c + id

Option C

$z_1 + z_2 = (a+c) + i(b+d)$

$|z_1 + z_2| = \sqrt{(a+c)^2 + (b+d)^2}$

$|z_1| = \sqrt{a^2 + b^2}$ and $|z_2| = \sqrt{c^2 + d^2}$

We cannot say anything about option c and option d

43. Question

Mark the Correct alternative in the following:

If the complex number z = x + iy satisfies the condition |z + 1| = 1, then z lies on

A. x-axis

B. circle with centre (-1, 0) and radius 1

C. y-axis

D. None of these

Answer

$|z + 1| = 1$

$|x + iy + 1| = 1$

$|(1 + x) + iy| = 1$

$\sqrt{(1+x)^2 + y^2} = 1$

$(x + 1)^2 + y^2 = 1$

$(x - (-1))^2 + (y - 0)^2 = (1)^2$

So, we can say that it is a circle with centre (-1,0) and radius 1