

# 11. Trigonometric Equations

## Exercise 11.1

### 1 A. Question

Find the general solutions of the following equations :

i.  $\sin x = \frac{1}{2}$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

we have,

$$\sin x = \frac{1}{2}$$

We know that  $\sin 30^\circ = \sin \pi/6 = 0.5$

$$\therefore \sin x = \sin \frac{\pi}{6}$$

$\therefore$  it matches with the form  $\sin x = \sin y$

Hence,

$$x = n\pi + (-1)^n \frac{\pi}{6}, \text{ where } n \in \mathbb{Z}$$

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We know that  $\sin 30^\circ = \sin \pi/6 = 0.5$

$$\therefore \sin x = \sin \frac{\pi}{6}$$

$\therefore$  it matches with the form  $\sin x = \sin y$

Hence,

$$x = n\pi + (-1)^n \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

### 1 B. Question

Find the general solutions of the following equations :

$$\cos x = -\frac{\sqrt{3}}{2}$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\text{We know that, } \cos 150^\circ = \left(-\frac{\sqrt{3}}{2}\right) = \cos \frac{5\pi}{6}$$

$$\therefore \cos x = \cos \frac{5\pi}{6}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = 5\pi / 6$

$$\therefore x = 2n\pi \pm 5\pi / 6, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

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$$\therefore x = 2n\pi \pm 5\pi / 6, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

### 1 C. Question

Find the general solutions of the following equations :

$$\operatorname{cosec} x = -\sqrt{2}$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\operatorname{cosec} x = -\sqrt{2}$$

We know that  $\sin x$ , and  $\operatorname{cosec} x$  have negative values in the 3<sup>rd</sup> and 4<sup>th</sup> quadrant.

While giving a solution, we always try to take the least value of  $y$

The fourth quadrant will give the least magnitude of  $y$  as we are taking an angle in a clockwise sense (i.e., negative angle)

$$-\sqrt{2} = -\operatorname{cosec}(\pi/4) = \operatorname{cosec}(-\pi/4) \{ \because \sin -\theta = -\sin \theta \}$$

$$\therefore \operatorname{cosec} x = \operatorname{cosec} \left( -\frac{\pi}{4} \right)$$

$$\Rightarrow \sin x = \sin \left( \frac{-\pi}{4} \right)$$

If  $\sin x = \sin y$ , then  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$$\text{For above equation } y = -\frac{\pi}{4}$$

$$\therefore x = n\pi + (-1)^n \left( -\frac{\pi}{4} \right), \text{ where } n \in \mathbb{Z}$$

$$\text{Or } x = n\pi + (-1)^{n+1} \left( \frac{\pi}{4} \right), \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for given trigonometric equation.

### 1 C. Question

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We know that  $\sin x$ , and  $\operatorname{cosec} x$  have negative values in the 3<sup>rd</sup> and 4<sup>th</sup> quadrant.

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The fourth quadrant will give the least magnitude of  $y$  as we are taking an angle in a clockwise sense (i.e., negative angle)

$$-\sqrt{2} = -\operatorname{cosec}(\pi/4) = \operatorname{cosec}(-\pi/4) \{ \because \sin -\theta = -\sin \theta \}$$

$$\therefore \operatorname{cosec} x = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$$

$$\Rightarrow \sin x = \sin\left(\frac{-\pi}{4}\right)$$

If  $\sin x = \sin y$ , then  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$$\text{For above equation } y = -\frac{\pi}{4}$$

$$\therefore x = n\pi + (-1)^n\left(-\frac{\pi}{4}\right), \text{ where } n \in \mathbb{Z}$$

$$\text{Or } x = n\pi + (-1)^{n+1}\left(\frac{\pi}{4}\right), \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for given trigonometric equation.

#### 1 D. Question

Find the general solutions of the following equations :

$$\sec x = \sqrt{2}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sec x = \sqrt{2}$$

We know that  $\sec x$  and  $\cos x$  have positive values in the 1<sup>st</sup> and 4<sup>th</sup> quadrant.

While giving a solution, we always try to take the least value of  $y$

both quadrants will give the least magnitude of  $y$ .

We can choose any one, in this solution we are assuming a positive value.

$$\sec x = \sec \frac{\pi}{4}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{4}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = \pi / 4$

$$\therefore x = 2n\pi \pm \frac{\pi}{4}, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

### 1 D. Question

Find the general solutions of the following equations :

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#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sec x = \sqrt{2}$$

We know that  $\sec x$  and  $\cos x$  have positive values in the 1<sup>st</sup> and 4<sup>th</sup> quadrant.

While giving a solution, we always try to take the least value of  $y$  both quadrants will give the least magnitude of  $y$ .

We can choose any one, in this solution we are assuming a positive value.

$$\sec x = \sec \frac{\pi}{4}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{4}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = \pi / 4$

$$\therefore x = 2n\pi \pm \frac{\pi}{4}, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

### 1 E. Question

Find the general solutions of the following equations :

$$\tan x = -\frac{1}{\sqrt{3}}$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan x = -\frac{1}{\sqrt{3}}$$

We know that  $\tan x$  and  $\cot x$  have negative values in the 2<sup>nd</sup> and 4<sup>th</sup> quadrant.

While giving solution, we always try to take the least value of  $y$ .

The fourth quadrant will give the least magnitude of  $y$  as we are taking an angle in a clockwise sense (i.e. negative angle)

$$\tan x = \tan\left(-\frac{\pi}{6}\right)$$

If  $\tan x = \tan y$  then  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = -\frac{\pi}{6}$

$$\therefore x = n\pi + \left(-\frac{\pi}{6}\right), \text{ where } n \in \mathbb{Z}$$

$$\text{Or } x = n\pi - \frac{\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

### 1 E. Question

Find the general solutions of the following equations :

$$\tan x = -\frac{1}{\sqrt{3}}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan x = -\frac{1}{\sqrt{3}}$$

We know that  $\tan x$  and  $\cot x$  have negative values in the 2<sup>nd</sup> and 4<sup>th</sup> quadrant.

While giving solution, we always try to take the least value of  $y$ .

The fourth quadrant will give the least magnitude of  $y$  as we are taking an angle in a clockwise sense (i.e. negative angle)

$$\tan x = \tan\left(-\frac{\pi}{6}\right)$$

If  $\tan x = \tan y$  then  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = -\frac{\pi}{6}$

$$\therefore x = n\pi + \left(-\frac{\pi}{6}\right), \text{ where } n \in \mathbb{Z}$$

$$\text{Or } x = n\pi - \frac{\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

### 1 F. Question

Find the general solutions of the following equations :

$$\sqrt{3} \sec x = 2$$

#### Answer

#### Ideas required to solve the problem:

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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sqrt{3} \sec x = 2$$

$$\Rightarrow \sec x = \frac{2}{\sqrt{3}}$$

We know that  $\sec x$  and  $\cos x$  have positive values in the 1<sup>st</sup> and 4<sup>th</sup> quadrant.

While giving solution, we always try to take the least value of  $y$

both quadrants will give the least magnitude of  $y$ .

We can choose any one, in this solution we are assuming a positive value.

$$\sec x = \sec \frac{\pi}{6}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{6}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = \pi / 6$

$$\therefore x = 2n\pi \pm \frac{\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

### 1 F. Question

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$$\sqrt{3} \sec x = 2$$

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#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
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Given,

$$\sqrt{3} \sec x = 2$$

$$\Rightarrow \sec x = \frac{2}{\sqrt{3}}$$

We know that  $\sec x$  and  $\cos x$  have positive values in the 1<sup>st</sup> and 4<sup>th</sup> quadrant.

While giving solution, we always try to take the least value of  $y$  both quadrants will give the least magnitude of  $y$ .

We can choose any one, in this solution we are assuming a positive value.

$$\sec x = \sec \frac{\pi}{6}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{6}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

For above equation  $y = \pi / 6$

$$\therefore x = 2n\pi \pm \frac{\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Thus,  $x$  gives the required general solution for the given trigonometric equation.

## 2 A. Question

Find the general solutions of the following equations :

$$\sin 2x = \frac{\sqrt{3}}{2}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x = \frac{\sqrt{3}}{2}$$

We know that  $\sin x$ , and  $\cos x$  have positive values in the 1<sup>st</sup> and 2<sup>nd</sup> quadrant.

While giving solution, we always try to take the least value of  $y$

The first quadrant will give the least magnitude of  $y$ .

$$\therefore \sin 2x = \sin \frac{\pi}{3}$$

If  $\sin x = \sin y$  then  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

Clearly on comparing we have  $y = \pi/3$

$$\therefore 2x = n\pi + (-1)^n \frac{\pi}{3}$$

$$\Rightarrow x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 A. Question

Find the general solutions of the following equations :



$$\sin 2x = \frac{\sqrt{3}}{2}$$

**Answer**

**Ideas required to solve the problem:**

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Given,

$$\sin 2x = \frac{\sqrt{3}}{2}$$

We know that  $\sin x$ , and  $\cos x$  have positive values in the 1<sup>st</sup> and 2<sup>nd</sup> quadrant.

While giving solution, we always try to take the least value of  $y$

The first quadrant will give the least magnitude of  $y$ .

$$\therefore \sin 2x = \sin \frac{\pi}{3}$$

If  $\sin x = \sin y$  then  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

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$$\therefore 2x = n\pi + (-1)^n \frac{\pi}{3}$$

$$\Rightarrow x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 B. Question

Find the general solutions of the following equations :

$$\cos 3x = \frac{1}{2}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos 3x = \frac{1}{2}$$

We know that  $\cos x$  and  $\sec x$  have positive values in the 1<sup>st</sup> and 4<sup>th</sup> quadrant.

While giving solution, we always try to take the least value of  $y$

both quadrant will give the least magnitude of  $y$ . We prefer the first quadrant.

$$\therefore \cos 3x = \cos \frac{\pi}{3}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

Clearly on comparing we have  $y = \pi/3$

$$\therefore 3x = 2n\pi \pm \frac{\pi}{3}$$

$$\Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 B. Question

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$$\cos 3x = \frac{1}{2}$$

### Answer

#### Ideas required to solve the problem:

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$$\Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 C. Question

Find the general solutions of the following equations :

$$\sin 9x = \sin x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 9x = \sin x$$

$$\Rightarrow \sin 9x - \sin x = 0$$

Using transformation formula:  $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$$\therefore 2 \cos \frac{9x+x}{2} \sin \frac{9x-x}{2} = 0$$

$$\Rightarrow \cos 5x \sin 4x = 0$$

$$\therefore \cos 5x = 0 \text{ or } \sin 4x = 0$$

If either of the equation is satisfied, the result will be 0

So we will find the solution individually and then finally combined the solution.

$$\therefore \cos 5x = 0$$

$$\Rightarrow \cos 5x = \cos \pi/2$$

$$\therefore 5x = (2n + 1) \frac{\pi}{2}$$

$$x = (2n + 1) \frac{\pi}{10}, \text{ where } n \in \mathbb{Z} \dots\dots\dots \text{eqn 1}$$

Also,

$$\sin 4x = \sin 0$$

$$\therefore 4x = n\pi$$

$$\text{Or } x = \frac{n\pi}{4}, \text{ where } n \in \mathbb{Z} \dots\dots\dots \text{eqn 2}$$

From equation 1 and eqn 2,

$$x = (2n + 1) \frac{\pi}{10} \text{ or } x = \frac{n\pi}{4}, \text{ where } n \in \mathbb{Z} \dots\dots \text{ans}$$

### 2 C. Question

Find the general solutions of the following equations :

$$\sin 9x = \sin x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

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Using transformation formula:  $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

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$$\therefore \cos 5x = 0 \text{ or } \sin 4x = 0$$

If either of the equation is satisfied, the result will be 0

So we will find the solution individually and then finally combined the solution.

$$\therefore \cos 5x = 0$$

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From equation 1 and eqn 2,

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### 2 D. Question

Find the general solutions of the following equations :

$$\sin 2x = \cos 3x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x = \cos 3x$$

$$\Rightarrow \cos\left(\frac{\pi}{2} - 2x\right) = \cos 3x \quad \{\because \sin \theta = \cos(\pi/2 - \theta)\}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

Clearly on comparing we have  $y = 3x$

$$\therefore \frac{\pi}{2} - 2x = 2n\pi \pm 3x$$

$$\frac{\pi}{2} - 2x = 2n\pi + 3x \text{ or } \frac{\pi}{2} - 2x = 2n\pi - 3x$$

$$\therefore 5x = \frac{\pi}{2} - 2n\pi = \frac{\pi}{2}(1 - 4n) \text{ or } x = 2n\pi - \frac{\pi}{2} = \frac{\pi}{2}(4n - 1)$$

$$x = \frac{\pi}{10}(1 - 4n), \text{ where } n \in \mathbb{Z}$$

Hence,

$$x = \frac{\pi}{10}(1 - 4n) \text{ or } \frac{\pi}{2}(4n - 1), \text{ where } n \in \mathbb{Z} \dots\dots \text{ans}$$

### 2 D. Question

Find the general solutions of the following equations :

$$\sin 2x = \cos 3x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x = \cos 3x$$

$$\Rightarrow \cos\left(\frac{\pi}{2} - 2x\right) = \cos 3x \quad \{\because \sin \theta = \cos(\pi/2 - \theta)\}$$

If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

Clearly on comparing we have  $y = 3x$

$$\therefore \frac{\pi}{2} - 2x = 2n\pi \pm 3x$$

$$\therefore \frac{\pi}{2} - 2x = 2n\pi + 3x \text{ or } \frac{\pi}{2} - 2x = 2n\pi - 3x$$

$$\therefore 5x = \frac{\pi}{2} - 2n\pi = \frac{\pi}{2}(1 - 4n) \text{ or } x = 2n\pi - \frac{\pi}{2} = \frac{\pi}{2}(4n - 1)$$

$$x = \frac{\pi}{10}(1 - 4n), \text{ where } n \in \mathbb{Z}$$

Hence,

$$x = \frac{\pi}{10}(1 - 4n) \text{ or } \frac{\pi}{2}(4n - 1), \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 E. Question

Find the general solutions of the following equations :

$$\tan x + \cot 2x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan x + \cot 2x = 0$$

$$\Rightarrow \tan x = -\cot 2x$$

We know that:  $\cot \theta = \tan(\pi/2 - \theta)$

$$\therefore \tan x = -\tan\left(\frac{\pi}{2} - 2x\right)$$

$$\Rightarrow \tan x = \tan\left(2x - \frac{\pi}{2}\right) \quad \{\because -\tan \theta = \tan -\theta\}$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = (2x - \frac{\pi}{2})$$

$$\therefore x = n\pi + 2x - \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2} - n\pi = \frac{\pi}{2} (1 - 2n), \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 E. Question

Find the general solutions of the following equations :

$$\tan x + \cot 2x = 0$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan x + \cot 2x = 0$$

$$\Rightarrow \tan x = -\cot 2x$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan x = -\tan (\frac{\pi}{2} - 2x)$$

$$\Rightarrow \tan x = \tan(2x - \frac{\pi}{2}) \{ \because -\tan \theta = \tan -\theta \}$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = (2x - \frac{\pi}{2})$$

$$\therefore x = n\pi + 2x - \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2} - n\pi = \frac{\pi}{2} (1 - 2n), \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 F. Question

Find the general solutions of the following equations :

$$\tan 3x = \cot x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan 3x = \cot x$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan 3x = \tan \left(\frac{\pi}{2} - x\right)$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(\frac{\pi}{2} - x\right)$$

$$\therefore 3x = n\pi + \frac{\pi}{2} - x$$

$$\Rightarrow 4x = n\pi + \frac{\pi}{2}$$

$$\therefore x = \frac{n\pi}{4} + \frac{\pi}{8}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 F. Question

Find the general solutions of the following equations :

$$\tan 3x = \cot x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan 3x = \cot x$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan 3x = \tan \left(\frac{\pi}{2} - x\right)$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(\frac{\pi}{2} - x\right)$$

$$\therefore 3x = n\pi + \frac{\pi}{2} - x$$

$$\Rightarrow 4x = n\pi + \frac{\pi}{2}$$

$$\therefore x = \frac{n\pi}{4} + \frac{\pi}{8}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 G. Question

Find the general solutions of the following equations :

$$\tan 2x \tan x = 1$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan 2x \tan x = 1$$

$$\Rightarrow \tan 2x = \frac{1}{\tan x}$$

$$\Rightarrow \tan 2x = \cot x$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan 2x = \tan\left(\frac{\pi}{2} - x\right)$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(\frac{\pi}{2} - x\right)$$

$$\therefore 2x = n\pi + \frac{\pi}{2} - x$$

$$\Rightarrow 3x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{6}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 G. Question

Find the general solutions of the following equations :

$$\tan 2x \tan x = 1$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan 2x \tan x = 1$$

$$\Rightarrow \tan 2x = \frac{1}{\tan x}$$

$$\Rightarrow \tan 2x = \cot x$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan 2x = \tan\left(\frac{\pi}{2} - x\right)$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(\frac{\pi}{2} - x\right)$$



$$\therefore 2x = n\pi + \frac{\pi}{2} - x$$

$$\Rightarrow 3x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{6}, \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

## 2 H. Question

Find the general solutions of the following equations :

$$\tan mx + \cot nx = 0$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan mx + \cot nx = 0$$

$$\Rightarrow \tan mx = -\cot nx$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan mx = -\tan \left(\frac{\pi}{2} - nx\right)$$

$$\Rightarrow \tan mx = \tan \left(nx - \frac{\pi}{2}\right) \{ \because -\tan \theta = \tan -\theta \}$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = k\pi + y$ , where  $k \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(nx - \frac{\pi}{2}\right)$$

$$\therefore mx = k\pi + nx - \frac{\pi}{2}$$

$$\Rightarrow (m - n)x = k\pi - \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2} \left(\frac{2k-1}{m-n}\right), \text{ where } k \in \mathbb{Z} \dots \text{ans}$$

## 2 H. Question

Find the general solutions of the following equations :

$$\tan mx + \cot nx = 0$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan mx + \cot nx = 0$$

$$\Rightarrow \tan mx = -\cot nx$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan mx = -\tan \left(\frac{\pi}{2} - nx\right)$$

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If  $\tan x = \tan y$ , then  $x$  is given by  $x = k\pi + y$ , where  $k \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(nx - \frac{\pi}{2}\right)$$

$$\therefore mx = k\pi + nx - \frac{\pi}{2}$$

$$\Rightarrow (m - n)x = k\pi - \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2} \left(\frac{2k-1}{m-n}\right), \text{ where } k \in \mathbb{Z} \dots \text{ans}$$

## 2 I. Question

Find the general solutions of the following equations :

$$\tan px = \cot qx$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan px = \cot qx$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan px = \tan \left(\frac{\pi}{2} - qx\right)$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(\frac{\pi}{2} - qx\right)$$

$$\therefore px = n\pi + \frac{\pi}{2} - qx$$

$$\Rightarrow (p + q)x = n\pi + \frac{\pi}{2}$$

$$\therefore x = \frac{n\pi}{(p+q)} + \frac{\pi}{2(p+q)}, \text{ where } n \in \mathbb{Z}$$

## 2 I. Question

Find the general solutions of the following equations :

$$\tan px = \cot qx$$

## Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan px = \cot qx$$

We know that:  $\cot \theta = \tan (\pi/2 - \theta)$

$$\therefore \tan px = \tan \left(\frac{\pi}{2} - qx\right)$$

If  $\tan x = \tan y$ , then  $x$  is given by  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

From above expression, on comparison with standard equation we have

$$y = \left(\frac{\pi}{2} - qx\right)$$

$$\therefore px = n\pi + \frac{\pi}{2} - qx$$

$$\Rightarrow (p + q)x = n\pi + \frac{\pi}{2}$$

$$\therefore x = \frac{n\pi}{(p+q)} + \frac{\pi}{2(p+q)}, \text{ where } n \in \mathbb{Z}$$

## 2 J. Question

Find the general solutions of the following equations :

$$\sin 2x + \cos x = 0$$

## Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x + \cos x = 0$$

We know that:  $\sin \theta = \cos (\pi/2 - \theta)$

$$\therefore \cos x = -\sin 2x$$

$$\Rightarrow \cos x = -\cos\left(\frac{\pi}{2} - 2x\right)$$

We know that:  $-\cos \theta = \cos (\pi - \theta)$

$$\therefore \cos x = \cos\left(\pi - \left(\frac{\pi}{2} - 2x\right)\right)$$

$$\Rightarrow \cos x = \cos \left(\frac{\pi}{2} + 2x\right)$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \left(\frac{\pi}{2} + 2x\right)$$

$$\therefore x = 2n\pi \pm \left(\frac{\pi}{2} + 2x\right)$$

Hence,

$$x = 2n\pi + \frac{\pi}{2} + 2x \text{ or } x = 2n\pi - \frac{\pi}{2} - 2x$$

$$\therefore x = -\frac{\pi}{2} - 2n\pi \text{ or } 3x = 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow x = -\frac{\pi}{2} (1 + 4n) \text{ or } x = \frac{\pi}{6}(4n - 1)$$

$$\therefore x = -\frac{\pi}{2} (4n + 1) \text{ or } \frac{\pi}{6} (4n - 1), \text{ where } n \in \mathbb{Z}$$

## 2 J. Question

Find the general solutions of the following equations :

$$\sin 2x + \cos x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x + \cos x = 0$$

We know that:  $\sin \theta = \cos (\pi/2 - \theta)$

$$\therefore \cos x = -\sin 2x$$

$$\Rightarrow \cos x = -\cos\left(\frac{\pi}{2} - 2x\right)$$

We know that:  $-\cos \theta = \cos (\pi - \theta)$

$$\therefore \cos x = \cos\left(\pi - \left(\frac{\pi}{2} - 2x\right)\right)$$

$$\Rightarrow \cos x = \cos\left(\frac{\pi}{2} + 2x\right)$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \left(\frac{\pi}{2} + 2x\right)$$

$$\therefore x = 2n\pi \pm \left(\frac{\pi}{2} + 2x\right)$$

Hence,

$$x = 2n\pi + \frac{\pi}{2} + 2x \text{ or } x = 2n\pi - \frac{\pi}{2} - 2x$$

$$\therefore x = -\frac{\pi}{2} - 2n\pi \text{ or } 3x = 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow x = -\frac{\pi}{2} (1 + 4n) \text{ or } x = \frac{\pi}{6}(4n - 1)$$

$$\therefore x = -\frac{\pi}{2} (4n + 1) \text{ or } \frac{\pi}{6} (4n - 1), \text{ where } n \in \mathbb{Z}$$

## 2 K. Question

Find the general solutions of the following equations :

$$\sin x = \tan x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x = \tan x$$

$$\Rightarrow \sin x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \sin x \cos x = \sin x$$

$$\Rightarrow \sin x (\cos x - 1) = 0$$

either,

$$\sin x = 0 \text{ or } \cos x = 1$$

$$\Rightarrow \sin x = \sin 0 \text{ or } \cos x = \cos 0$$

We know that,

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

$$\therefore \sin x = \sin 0$$

$$\therefore y = 0$$

And hence,

$$\mathbf{x = n\pi \text{ where } n \in \mathbb{Z}}$$

Also,

If  $\cos x = \cos y$ , implies  $x = 2m\pi \pm y$ , where  $m \in \mathbb{Z}$

$$\therefore \cos x = \cos 0$$

$$\therefore y = 0$$

Hence, x is given by

$$\mathbf{x = 2m\pi \text{ where } m \in \mathbb{Z}}$$

$$\therefore \mathbf{x = n\pi \text{ or } 2m\pi, \text{ where } m, n \in \mathbb{Z} \dots \text{ans}}$$

## 2 K. Question

Find the general solutions of the following equations :

$$\sin x = \tan x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x = \tan x$$

$$\Rightarrow \sin x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \sin x \cos x = \sin x$$

$$\Rightarrow \sin x (\cos x - 1) = 0$$

either,

$$\sin x = 0 \text{ or } \cos x = 1$$

$$\Rightarrow \sin x = \sin 0 \text{ or } \cos x = \cos 0$$

We know that,

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

$$\therefore \sin x = \sin 0$$

$$\therefore y = 0$$

And hence,

$$\mathbf{x = n\pi \text{ where } n \in \mathbb{Z}}$$

Also,

If  $\cos x = \cos y$ , implies  $x = 2m\pi \pm y$ , where  $m \in \mathbb{Z}$

$$\therefore \cos x = \cos 0$$

$$\therefore y = 0$$

Hence,  $x$  is given by

$$\mathbf{x = 2m\pi \text{ where } m \in \mathbb{Z}}$$

$$\therefore \mathbf{x = n\pi \text{ or } 2m\pi, \text{ where } m, n \in \mathbb{Z} \dots \text{ans}}$$

## 2 L. Question

Find the general solutions of the following equations :

$$\sin 3x + \cos 2x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 3x + \cos 2x = 0$$

We know that:  $\sin \theta = \cos (\pi/2 - \theta)$

$$\therefore \cos 2x = -\sin 3x$$

$$\Rightarrow \cos 2x = -\cos\left(\frac{\pi}{2} - 3x\right)$$

We know that:  $-\cos \theta = \cos (\pi - \theta)$

$$\therefore \cos 2x = \cos\left(\pi - \left(\frac{\pi}{2} - 3x\right)\right)$$

$$\Rightarrow \cos 2x = \cos \left(\frac{\pi}{2} + 3x\right)$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \left(\frac{\pi}{2} + 3x\right)$$

$$\therefore 2x = 2n\pi \pm \left(\frac{\pi}{2} + 3x\right)$$

Hence,

$$2x = 2n\pi + \frac{\pi}{2} + 3x \text{ or } 2x = 2n\pi - \frac{\pi}{2} - 3x$$

$$\therefore x = -\frac{\pi}{2} - 2n\pi \text{ or } 5x = 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow x = -\frac{\pi}{2} (1 + 4n) \text{ or } x = \frac{\pi}{10}(4n - 1)$$

$$\therefore x = -\frac{\pi}{2} (4n + 1) \text{ or } \frac{\pi}{10} (4n - 1), \text{ where } n \in \mathbb{Z}$$

## 2 L. Question

Find the general solutions of the following equations :

$$\sin 3x + \cos 2x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 3x + \cos 2x = 0$$

We know that:  $\sin \theta = \cos (\pi/2 - \theta)$

$$\therefore \cos 2x = -\sin 3x$$

$$\Rightarrow \cos 2x = -\cos\left(\frac{\pi}{2} - 3x\right)$$

We know that:  $-\cos \theta = \cos (\pi - \theta)$

$$\therefore \cos 2x = \cos\left(\pi - \left(\frac{\pi}{2} - 3x\right)\right)$$

$$\Rightarrow \cos 2x = \cos \left(\frac{\pi}{2} + 3x\right)$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \left(\frac{\pi}{2} + 3x\right)$$

$$\therefore 2x = 2n\pi \pm \left(\frac{\pi}{2} + 3x\right)$$

Hence,

$$2x = 2n\pi + \frac{\pi}{2} + 3x \text{ or } 2x = 2n\pi - \frac{\pi}{2} - 3x$$

$$\therefore x = -\frac{\pi}{2} - 2n\pi \text{ or } 5x = 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow x = -\frac{\pi}{2} (1 + 4n) \text{ or } x = \frac{\pi}{10}(4n - 1)$$

$$\therefore x = -\frac{\pi}{2} (4n + 1) \text{ or } \frac{\pi}{10} (4n - 1), \text{ where } n \in \mathbb{Z}$$

### 3 A. Question

Solve the following equations :

$$\sin^2 x - \cos x = \frac{1}{4}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin^2 x - \cos x = \frac{1}{4}$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable  $k$  and we will solve for  $k$

$$\text{As, } \sin^2 x = 1 - \cos^2 x$$

$\therefore$  we have,

$$1 - \cos^2 x - \cos x = \frac{1}{4}$$

$$\Rightarrow 4 - 4\cos^2 x - 4\cos x = 1$$

$$\Rightarrow 4\cos^2 x + 4\cos x - 3 = 0$$

Let,  $\cos x = k$

$$\therefore 4k^2 + 4k - 3 = 0$$

$$\Rightarrow 4k^2 - 2k + 6k - 3$$

$$\Rightarrow 2k(2k - 1) + 3(2k - 1) = 0$$

$$\Rightarrow (2k - 1)(2k + 3) = 0$$

$$\therefore k = 1/2 \text{ or } k = -3/2$$



$$\Rightarrow \cos x = \frac{3}{2} \text{ or } \cos x = -3/2$$

As  $\cos x$  lies between -1 and 1

$\therefore \cos x$  can't be  $-3/2$

So we ignore that value.

$$\therefore \cos x = \frac{3}{2}$$

$$\Rightarrow \cos x = \cos 60^\circ = \cos \pi/3$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = \pi/3$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 A. Question

Solve the following equations :

$$\sin^2 x - \cos x = \frac{1}{4}$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin^2 x - \cos x = \frac{1}{4}$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable  $k$  and we will solve for  $k$

$$\text{As, } \sin^2 x = 1 - \cos^2 x$$

$\therefore$  we have,

$$1 - \cos^2 x - \cos x = \frac{1}{4}$$

$$\Rightarrow 4 - 4\cos^2 x - 4\cos x = 1$$

$$\Rightarrow 4\cos^2 x + 4\cos x - 3 = 0$$

Let,  $\cos x = k$

$$\therefore 4k^2 + 4k - 3 = 0$$

$$\Rightarrow 4k^2 - 2k + 6k - 3$$

$$\Rightarrow 2k(2k - 1) + 3(2k - 1) = 0$$

$$\Rightarrow (2k - 1)(2k + 3) = 0$$

$$\therefore k = 1/2 \text{ or } k = -3/2$$

$$\Rightarrow \cos x = \frac{3}{2} \text{ or } \cos x = -3/2$$

As  $\cos x$  lies between -1 and 1

$\therefore \cos x$  can't be  $-3/2$

So we ignore that value.

$$\therefore \cos x = \frac{3}{2}$$

$$\Rightarrow \cos x = \cos 60^\circ = \cos \pi/3$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = \pi/3$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 B. Question

Solve the following equations :

$$2 \cos^2 x - 5 \cos x + 2 = 0$$

#### Answer

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$2 \cos^2 x - 5 \cos x + 2 = 0$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable  $k$  and we will solve for  $k$

$$\text{Let, } \cos x = k$$

$$\therefore 2k^2 - 5k + 2 = 0$$

$$\Rightarrow 2k^2 - 4k - k + 2 = 0$$

$$\Rightarrow 2k(k - 2) - 1(k - 2) = 0$$

$$\Rightarrow (k - 2)(2k - 1) = 0$$

$$\therefore k = 2 \text{ or } k = \frac{1}{2}$$

$$\Rightarrow \cos x = 2 \text{ \{which is not possible\} or } \cos x = \frac{1}{2} \text{ (acceptable)}$$

$$\therefore \cos x = \frac{1}{2}$$

$$\Rightarrow \cos x = \cos 60^\circ = \cos \pi/3$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = \pi/3$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 B. Question

Solve the following equations :

$$2 \cos^2 x - 5 \cos x + 2 = 0$$

#### Answer

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The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$2 \cos^2 x - 5 \cos x + 2 = 0$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable  $k$  and we will solve for  $k$

Let,  $\cos x = k$

$$\therefore 2k^2 - 5k + 2 = 0$$

$$\Rightarrow 2k^2 - 4k - k + 2 = 0$$

$$\Rightarrow 2k(k - 2) - 1(k - 2) = 0$$

$$\Rightarrow (k - 2)(2k - 1) = 0$$

$$\therefore k = 2 \text{ or } k = \frac{1}{2}$$

$$\Rightarrow \cos x = 2 \text{ \{which is not possible\} or } \cos x = \frac{1}{2} \text{ (acceptable)}$$

$$\therefore \cos x = \frac{1}{2}$$

$$\Rightarrow \cos x = \cos 60^\circ = \cos \frac{\pi}{3}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = \frac{\pi}{3}$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 C. Question

Solve the following equations :

$$2 \sin^2 x + \sqrt{3} \cos x + 1 = 0$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$2\sin^2 x + \sqrt{3} \cos x + 1 = 0$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

$$\text{As, } \sin^2 x = 1 - \cos^2 x$$

∴ we have,

$$2(1 - \cos^2 x) + \sqrt{3} \cos x + 1 = 0$$

$$\Rightarrow 2 - 2\cos^2 x + \sqrt{3} \cos x + 1 = 0$$

$$\Rightarrow 2\cos^2 x - \sqrt{3} \cos x - 3 = 0$$

Let,  $\cos x = k$

$$\therefore 2k^2 - \sqrt{3}k - 3 = 0$$

$$\Rightarrow 2k^2 - 2\sqrt{3}k + \sqrt{3}k - 3 = 0$$

$$\Rightarrow 2k(k - \sqrt{3}) + \sqrt{3}(k - \sqrt{3}) = 0$$

$$\Rightarrow (2k + \sqrt{3})(k - \sqrt{3}) = 0$$

$$\therefore k = \sqrt{3} \text{ or } k = -\sqrt{3}/2$$

$$\Rightarrow \cos x = \sqrt{3} \text{ or } \cos x = -\sqrt{3}/2$$

As  $\cos x$  lies between -1 and 1

∴  $\cos x$  can't be  $\sqrt{3}$

So we ignore that value.

$$\therefore \cos x = -\sqrt{3}/2$$

$$\Rightarrow \cos x = \cos 150^\circ = \cos 5\pi/6$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = 5\pi/6$$

$$\therefore x = 2n\pi \pm \frac{5\pi}{6} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 C. Question

Solve the following equations :

$$2\sin^2 x + \sqrt{3} \cos x + 1 = 0$$

#### Answer

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$2\sin^2 x + \sqrt{3} \cos x + 1 = 0$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

$$\text{As, } \sin^2 x = 1 - \cos^2 x$$

∴ we have,

$$2(1 - \cos^2 x) + \sqrt{3} \cos x + 1 = 0$$

$$\Rightarrow 2 - 2\cos^2 x + \sqrt{3} \cos x + 1 = 0$$

$$\Rightarrow 2\cos^2 x - \sqrt{3} \cos x - 3 = 0$$

Let,  $\cos x = k$

$$\therefore 2k^2 - \sqrt{3}k - 3 = 0$$

$$\Rightarrow 2k^2 - 2\sqrt{3}k + \sqrt{3}k - 3 = 0$$

$$\Rightarrow 2k(k - \sqrt{3}) + \sqrt{3}(k - \sqrt{3}) = 0$$

$$\Rightarrow (2k + \sqrt{3})(k - \sqrt{3}) = 0$$

$$\therefore k = \sqrt{3} \text{ or } k = -\sqrt{3}/2$$

$$\Rightarrow \cos x = \sqrt{3} \text{ or } \cos x = -\sqrt{3}/2$$

As  $\cos x$  lies between -1 and 1

∴  $\cos x$  can't be  $\sqrt{3}$

So we ignore that value.

$$\therefore \cos x = -\sqrt{3}/2$$

$$\Rightarrow \cos x = \cos 150^\circ = \cos 5\pi/6$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = 5\pi/6$$

$$\therefore x = 2n\pi \pm \frac{5\pi}{6} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 D. Question

Solve the following equations :

$$4 \sin^2 x - 8 \cos x + 1 = 0$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$4\sin^2 x - 8 \cos x + 1 = 0$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

$$\text{As, } \sin^2 x = 1 - \cos^2 x$$

∴ we have,

$$4(1 - \cos^2 x) - 8 \cos x + 1 = 0$$

$$\Rightarrow 4 - 4 \cos^2 x - 8 \cos x + 1 = 0$$

$$\Rightarrow 4 \cos^2 x + 8 \cos x - 5 = 0$$

Let,  $\cos x = k$

$$\therefore 4k^2 + 8k - 5 = 0$$

$$\Rightarrow 4k^2 - 2k + 10k - 5 = 0$$

$$\Rightarrow 2k(2k - 1) + 5(2k - 1) = 0$$

$$\Rightarrow (2k + 5)(2k - 1) = 0$$

$$\therefore k = -5/2 = -2.5 \text{ or } k = 1/2$$

$$\Rightarrow \cos x = -2.5 \text{ or } \cos x = 1/2$$

As  $\cos x$  lies between -1 and 1

∴  $\cos x$  can't be -2.5

So we ignore that value.

$$\therefore \cos x = 1/2$$

$$\Rightarrow \cos x = \cos 60^\circ = \cos \pi/3$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = \pi/3$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 D. Question

Solve the following equations :

$$4 \sin^2 x - 8 \cos x + 1 = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$4 \sin^2 x - 8 \cos x + 1 = 0$$

As the equation is of 2<sup>nd</sup> degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable  $k$  and we will solve for  $k$

$$\text{As, } \sin^2 x = 1 - \cos^2 x$$

∴ we have,

$$4(1 - \cos^2 x) - 8 \cos x + 1 = 0$$

$$\Rightarrow 4 - 4 \cos^2 x - 8 \cos x + 1 = 0$$

$$\Rightarrow 4 \cos^2 x + 8 \cos x - 5 = 0$$

Let,  $\cos x = k$

$$\therefore 4k^2 + 8k - 5 = 0$$

$$\Rightarrow 4k^2 - 2k + 10k - 5 = 0$$

$$\Rightarrow 2k(2k - 1) + 5(2k - 1) = 0$$

$$\Rightarrow (2k + 5)(2k - 1) = 0$$

$$\therefore k = -5/2 = -2.5 \text{ or } k = 1/2$$

$$\Rightarrow \cos x = -2.5 \text{ or } \cos x = 1/2$$

As  $\cos x$  lies between -1 and 1

$\therefore \cos x$  can't be -2.5

So we ignore that value.

$$\therefore \cos x = 1/2$$

$$\Rightarrow \cos x = \cos 60^\circ = \cos \pi/3$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

On comparing our equation with standard form, we have

$$y = \pi/3$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \text{ ..ans}$$

### 3 E. Question

Solve the following equations :

$$\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$$

$$\Rightarrow \tan^2 x + \tan x - \sqrt{3} \tan x - \sqrt{3} = 0$$

$$\Rightarrow \tan x (\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$$

$$\Rightarrow (\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\therefore \tan x = -1 \text{ or } \tan x = \sqrt{3}$$

As,  $\tan x \in (-\infty, \infty)$  so both values are valid and acceptable.

$$\Rightarrow \tan x = \tan (-\pi/4) \text{ or } \tan x = \tan (\pi/3)$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Clearly by comparing standard form with obtained equation we have

$$y = -\pi/4 \text{ or } y = \pi/3$$

$$\therefore x = m\pi - \frac{\pi}{4} \text{ or } x = n\pi + \frac{\pi}{3}$$

**Hence,**

$$x = m\pi - \frac{\pi}{4} \text{ or } n\pi + \frac{\pi}{3}, \text{ where } m, n \in \mathbb{Z}$$

### 3 E. Question

Solve the following equations :

$$\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$$

$$\Rightarrow \tan^2 x + \tan x - \sqrt{3} \tan x - \sqrt{3} = 0$$

$$\Rightarrow \tan x (\tan x + 1) - \sqrt{3} (\tan x + 1) = 0$$

$$\Rightarrow (\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\therefore \tan x = -1 \text{ or } \tan x = \sqrt{3}$$

As,  $\tan x \in (-\infty, \infty)$  so both values are valid and acceptable.

$$\Rightarrow \tan x = \tan (-\pi/4) \text{ or } \tan x = \tan (\pi/3)$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Clearly by comparing standard form with obtained equation we have

$$y = -\pi/4 \text{ or } y = \pi/3$$

$$\therefore x = m\pi - \frac{\pi}{4} \text{ or } x = n\pi + \frac{\pi}{3}$$

**Hence,**

$$x = m\pi - \frac{\pi}{4} \text{ or } n\pi + \frac{\pi}{3}, \text{ where } m, n \in \mathbb{Z}$$

### 3 F. Question

Solve the following equations :

$$3 \cos^2 x - 2\sqrt{3} \sin x \cos x - 3 \sin^2 x = 0$$

**Answer**



**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$3 \cos^2 x - 2\sqrt{3} \sin x \cos x - 3 \sin^2 x = 0$$

$$\Rightarrow 3 \cos^2 x - 3\sqrt{3} \sin x \cos x + \sqrt{3} \sin x \cos x - 3 \sin^2 x = 0$$

$$\Rightarrow 3 \cos x (\cos x - \sqrt{3} \sin x) + \sqrt{3} \sin x (\cos x - \sqrt{3} \sin x) = 0$$

$$\Rightarrow \sqrt{3} (\cos x - \sqrt{3} \sin x)(\sqrt{3} \cos x + \sin x) = 0$$

$$\therefore \text{either, } \cos x - \sqrt{3} \sin x = 0 \text{ or } \sin x + \sqrt{3} \cos x = 0$$

$$\Rightarrow \cos x = \sqrt{3} \sin x \text{ or } \sin x = -\sqrt{3} \cos x$$

$$\Rightarrow \tan x = \frac{1}{\sqrt{3}} \text{ or } \tan x = -\sqrt{3}$$

$$\Rightarrow \tan x = \tan \frac{\pi}{6} \text{ or } \tan x = \tan\left(-\frac{\pi}{3}\right)$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Clearly by comparing standard form with obtained equation we have:

$$y = \pi/6 \text{ or } y = -\pi/3$$

$$\therefore x = m\pi + \frac{\pi}{6} \text{ or } x = n\pi - \frac{\pi}{3}$$

**Hence,**

$$x = m\pi + \frac{\pi}{6} \text{ or } n\pi - \frac{\pi}{3}, \text{ where } m, n \in \mathbb{Z}$$

**3 F. Question**

Solve the following equations :

$$3 \cos^2 x - 2\sqrt{3} \sin x \cos x - 3 \sin^2 x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$3 \cos^2 x - 2\sqrt{3} \sin x \cos x - 3 \sin^2 x = 0$$

$$\Rightarrow 3 \cos^2 x - 3\sqrt{3} \sin x \cos x + \sqrt{3} \sin x \cos x - 3 \sin^2 x = 0$$

$$\Rightarrow 3 \cos x (\cos x - \sqrt{3} \sin x) + \sqrt{3} \sin x (\cos x - \sqrt{3} \sin x) = 0$$

$$\Rightarrow \sqrt{3} (\cos x - \sqrt{3} \sin x)(\sqrt{3} \cos x + \sin x) = 0$$

$$\therefore \text{either, } \cos x - \sqrt{3} \sin x = 0 \text{ or } \sin x + \sqrt{3} \cos x = 0$$

$$\Rightarrow \cos x = \sqrt{3} \sin x \text{ or } \sin x = -\sqrt{3} \cos x$$

$$\Rightarrow \tan x = \frac{1}{\sqrt{3}} \text{ or } \tan x = -\sqrt{3}$$

$$\Rightarrow \tan x = \tan \frac{\pi}{6} \text{ or } \tan x = \tan\left(-\frac{\pi}{3}\right)$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Clearly by comparing standard form with obtained equation we have:

$$y = \pi/6 \text{ or } y = -\pi/3$$

$$\therefore x = m\pi + \frac{\pi}{6} \text{ or } x = n\pi - \frac{\pi}{3}$$

**Hence,**

$$x = m\pi + \frac{\pi}{6} \text{ or } n\pi - \frac{\pi}{3}, \text{ where } m, n \in \mathbb{Z}$$

### 3 G. Question

Solve the following equations :

$$\cos 4x = \cos 2x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos 4x = \cos 2x$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = 2x$$

$$\therefore 4x = 2n\pi \pm 2x$$

Hence,

$$4x = 2n\pi + 2x \text{ or } 4x = 2m\pi - 2x$$

$$\therefore 2x = 2n\pi \text{ or } 6x = 2m\pi$$

$$\Rightarrow x = n\pi \text{ or } x = \frac{2m\pi}{6} = \frac{m\pi}{3}$$

$$\therefore x = n\pi \text{ or } \frac{m\pi}{3} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

### 3 G. Question

Solve the following equations :

$$\cos 4x = \cos 2x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos 4x = \cos 2x$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = 2x$$

$$\therefore 4x = 2n\pi \pm 2x$$

Hence,

$$4x = 2n\pi + 2x \text{ or } 4x = 2m\pi - 2x$$

$$\therefore 2x = 2n\pi \text{ or } 6x = 2m\pi$$

$$\Rightarrow x = n\pi \text{ or } x = \frac{2m\pi}{6} = \frac{m\pi}{3}$$

$$\therefore x = n\pi \text{ or } \frac{m\pi}{3} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

**4 A. Question**

Solve the following equations :

$$\cos x + \cos 2x + \cos 3x = 0$$

**Answer****Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x + \cos 2x + \cos 3x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \cos x + \cos 2x + \cos 3x = 0$$

$\therefore$  we will use  $\cos x$  and  $\cos 2x$  for transformation as after transformation it will give  $\cos 2x$  term which can be taken common.

$$\therefore \cos x + \cos 2x + \cos 3x = 0$$

$$\Rightarrow \cos 2x + (\cos x + \cos 3x) = 0$$

$$\left\{ \because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right\}$$

$$\Rightarrow \cos 2x + 2\cos\left(\frac{3x+x}{2}\right)\cos\frac{3x-x}{2} = 0$$

$$\Rightarrow \cos 2x + 2\cos 2x \cos x = 0$$

$$\Rightarrow \cos 2x (1 + 2\cos x) = 0$$

$$\therefore \cos 2x = 0 \text{ or } 1 + 2\cos x = 0$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = -1/2$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = \cos (\pi - \pi/3) = \cos (2\pi/3)$$

If  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \pi/2 \text{ or } y = 2\pi/3$$

$$\therefore 2x = 2n\pi \pm \pi/2 \text{ or } x = 2m\pi \pm 2\pi/3$$

$$\therefore x = n\pi \pm \frac{\pi}{4} \text{ or } x = 2m\pi \pm \frac{2\pi}{3} \text{ where } m, n \in \mathbb{Z}$$

#### 4 A. Question

Solve the following equations :

$$\cos x + \cos 2x + \cos 3x = 0$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x + \cos 2x + \cos 3x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \cos x + \cos 2x + \cos 3x = 0$$

$\therefore$  we will use  $\cos x$  and  $\cos 2x$  for transformation as after transformation it will give  $\cos 2x$  term which can be taken common.

$$\therefore \cos x + \cos 2x + \cos 3x = 0$$

$$\Rightarrow \cos 2x + (\cos x + \cos 3x) = 0$$

$$\left\{ \because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right\}$$

$$\Rightarrow \cos 2x + 2\cos\left(\frac{3x+x}{2}\right)\cos\frac{3x-x}{2} = 0$$

$$\Rightarrow \cos 2x + 2\cos 2x \cos x = 0$$

$$\Rightarrow \cos 2x (1 + 2\cos x) = 0$$

$$\therefore \cos 2x = 0 \text{ or } 1 + 2\cos x = 0$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = -1/2$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = \cos (\pi - \pi/3) = \cos (2\pi/3)$$

If  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \pi/2 \text{ or } y = 2\pi/3$$

$$\therefore 2x = 2n\pi \pm \pi/2 \text{ or } x = 2m\pi \pm 2\pi/3$$

$$\therefore x = n\pi \pm \frac{\pi}{4} \text{ or } x = 2m\pi \pm \frac{2\pi}{3} \text{ where } m, n \in \mathbb{Z}$$

#### 4 B. Question

Solve the following equations :

$$\cos x + \cos 3x - \cos 2x = 0$$

#### Answer

##### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x - \cos 2x + \cos 3x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \cos x - \cos 2x + \cos 3x = 0$$

$\therefore$  we will use  $\cos x$  and  $\cos 2x$  for transformation as after transformation it will give  $\cos 2x$  term which can be taken common.

$$\therefore \cos x - \cos 2x + \cos 3x = 0$$

$$\Rightarrow -\cos 2x + (\cos x + \cos 3x) = 0$$

$$\left\{ \because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right\}$$

$$\Rightarrow -\cos 2x + 2\cos\left(\frac{3x+x}{2}\right)\cos\frac{3x-x}{2} = 0$$

$$\Rightarrow -\cos 2x + 2\cos 2x \cos x = 0$$

$$\Rightarrow \cos 2x (-1 + 2\cos x) = 0$$

$$\therefore \cos 2x = 0 \text{ or } 1 + 2\cos x = 0$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = 1/2$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = \cos (\pi/3) = \cos (\pi/3)$$

If  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \pi/2 \text{ or } y = \pi/3$$

$$\therefore 2x = 2n\pi \pm \pi/2 \text{ or } x = 2m\pi \pm \pi/3$$

$$\therefore x = n\pi \pm \frac{\pi}{4} \text{ or } x = 2m\pi \pm \frac{\pi}{3} \text{ where } m, n \in \mathbb{Z}$$

#### 4 B. Question

Solve the following equations :

$$\cos x + \cos 3x - \cos 2x = 0$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x - \cos 2x + \cos 3x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \cos x - \cos 2x + \cos 3x = 0$$

$\therefore$  we will use  $\cos x$  and  $\cos 2x$  for transformation as after transformation it will give  $\cos 2x$  term which can be taken common.

$$\therefore \cos x - \cos 2x + \cos 3x = 0$$

$$\Rightarrow -\cos 2x + (\cos x + \cos 3x) = 0$$

$$\left\{ \because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right\}$$

$$\Rightarrow -\cos 2x + 2\cos\left(\frac{3x+x}{2}\right)\cos\frac{3x-x}{2} = 0$$

$$\Rightarrow -\cos 2x + 2\cos 2x \cos x = 0$$

$$\Rightarrow \cos 2x (-1 + 2\cos x) = 0$$

$$\therefore \cos 2x = 0 \text{ or } 1 + 2\cos x = 0$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = -1/2$$

$$\Rightarrow \cos 2x = \cos \pi/2 \text{ or } \cos x = \cos(\pi/3) = \cos(\pi/3)$$

If  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

From above expression and on comparison with standard equation we have:

$$y = \pi/2 \text{ or } y = \pi/3$$

$$\therefore 2x = 2n\pi \pm \pi/2 \text{ or } x = 2m\pi \pm \pi/3$$

$$\therefore x = n\pi \pm \frac{\pi}{4} \text{ or } x = 2m\pi \pm \frac{\pi}{3} \text{ where } m, n \in \mathbb{Z}$$

#### 4 C. Question

Solve the following equations :

$$\sin x + \sin 5x = \sin 3x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x + \sin 5x = \sin 3x$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want to transform.

$$\text{As, } \sin x + \sin 5x = \sin 3x$$

$$\therefore \sin x + \sin 5x - \sin 3x = 0$$

$\therefore$  we will use  $\sin x$  and  $\sin 5x$  for transformation as after transformation it will give  $\sin 3x$  term which can be taken common.

$$\left\{ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right\}$$

$$\Rightarrow -\sin 3x + 2 \sin \left( \frac{5x+x}{2} \right) \cos \frac{5x-x}{2} = 0$$

$$\Rightarrow 2\sin 3x \cos 2x - \sin 3x = 0$$

$$\Rightarrow \sin 3x (2\cos 2x - 1) = 0$$

$$\therefore \text{either, } \sin 3x = 0 \text{ or } 2\cos 2x - 1 = 0$$

$$\Rightarrow \sin 3x = \sin 0 \text{ or } \cos 2x = \frac{1}{2} = \cos \pi/3$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

Comparing obtained equation with standard equation, we have:

$$3x = n\pi \text{ or } 2x = 2m\pi \pm \pi/3$$

$$\therefore x = \frac{n\pi}{3} \text{ or } x = m\pi \pm \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

#### 4 C. Question

Solve the following equations :

$$\sin x + \sin 5x = \sin 3x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x + \sin 5x = \sin 3x$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want to transform.

$$\text{As, } \sin x + \sin 5x = \sin 3x$$

$$\therefore \sin x + \sin 5x - \sin 3x = 0$$

$\therefore$  we will use  $\sin x$  and  $\sin 5x$  for transformation as after transformation it will give  $\sin 3x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \}$$

$$\Rightarrow -\sin 3x + 2 \sin \left( \frac{5x+x}{2} \right) \cos \frac{5x-x}{2} = 0$$

$$\Rightarrow 2 \sin 3x \cos 2x - \sin 3x = 0$$

$$\Rightarrow \sin 3x (2 \cos 2x - 1) = 0$$

$$\therefore \text{either, } \sin 3x = 0 \text{ or } 2 \cos 2x - 1 = 0$$

$$\Rightarrow \sin 3x = \sin 0 \text{ or } \cos 2x = \frac{1}{2} = \cos \frac{\pi}{3}$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

Comparing obtained equation with standard equation, we have:

$$3x = n\pi \text{ or } 2x = 2m\pi \pm \frac{\pi}{3}$$

$$\therefore x = \frac{n\pi}{3} \text{ or } x = m\pi \pm \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

#### 4 D. Question

Solve the following equations :

$$\cos x \cos 2x \cos 3x = \frac{1}{4}$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x \cos 2x \cos 3x = \frac{1}{4}$$

$$\Rightarrow 4 \cos x \cos 2x \cos 3x - 1 = 0$$

$$\{ \because 2 \cos A \cos B = \cos (A + B) + \cos (A - B) \}$$

$$\therefore 2(2 \cos x \cos 3x) \cos 2x - 1 = 0$$



$$\Rightarrow 2(\cos 4x + \cos 2x)\cos 2x - 1 = 0$$

$$\Rightarrow 2(2\cos^2 2x - 1 + \cos 2x)\cos 2x - 1 = 0 \text{ \{using } \cos 2\theta = 2\cos^2\theta - 1 \text{ \}}$$

$$\Rightarrow 4\cos^3 2x - 2\cos 2x + 2\cos^2 2x - 1 = 0$$

$$\Rightarrow 2\cos^2 2x (2\cos 2x + 1) - 1(2\cos 2x + 1) = 0$$

$$\Rightarrow (2\cos^2 2x - 1)(2\cos 2x + 1) = 0$$

$$\therefore \text{either, } 2\cos 2x + 1 = 0 \text{ or } (2\cos^2 2x - 1) = 0$$

$$\Rightarrow \cos 2x = -1/2 \text{ or } \cos 4x = 0 \text{ \{using } \cos 2\theta = 2\cos^2\theta - 1 \text{ \}}$$

$$\Rightarrow \cos 2x = \cos(\pi - \pi/3) = \cos 2\pi/3 \text{ or } \cos 4x = \cos \pi/2$$

If  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

In case of  $\cos x = 0$  we can give solution directly as  $\cos x = 0$  is true for  $x = \text{odd multiple of } \pi/2$

Comparing obtained equation with standard equation, we have:

$$y = 2\pi/3 \text{ or } y = \pi/2$$

$$\therefore 2x = 2m\pi \pm 2\pi/3 \text{ or } 4x = (2n+1)\pi/2$$

$$\therefore x = m\pi \pm \frac{\pi}{3} \text{ or } x = (2n+1)\frac{\pi}{8} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

#### 4 D. Question

Solve the following equations :

$$\cos x \cos 2x \cos 3x = \frac{1}{8}$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x \cos 2x \cos 3x = \frac{1}{8}$$

$$\Rightarrow 4\cos x \cos 2x \cos 3x - 1 = 0$$

$$\{\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)\}$$

$$\therefore 2(2\cos x \cos 3x)\cos 2x - 1 = 0$$

$$\Rightarrow 2(\cos 4x + \cos 2x)\cos 2x - 1 = 0$$

$$\Rightarrow 2(2\cos^2 2x - 1 + \cos 2x)\cos 2x - 1 = 0 \text{ \{using } \cos 2\theta = 2\cos^2\theta - 1 \text{ \}}$$

$$\Rightarrow 4\cos^3 2x - 2\cos 2x + 2\cos^2 2x - 1 = 0$$

$$\Rightarrow 2\cos^2 2x (2\cos 2x + 1) - 1(2\cos 2x + 1) = 0$$

$$\Rightarrow (2\cos^2 2x - 1)(2\cos 2x + 1) = 0$$

$$\therefore \text{either, } 2\cos 2x + 1 = 0 \text{ or } (2\cos^2 2x - 1) = 0$$

$$\Rightarrow \cos 2x = -1/2 \text{ or } \cos 4x = 0 \text{ \{using } \cos 2\theta = 2\cos^2\theta - 1 \text{ \}}$$

$$\Rightarrow \cos 2x = \cos(\pi - \pi/3) = \cos 2\pi/3 \text{ or } \cos 4x = \cos \pi/2$$

If  $\cos x = \cos y$  implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

In case of  $\cos x = 0$  we can give solution directly as  $\cos x = 0$  is true for  $x = \text{odd multiple of } \pi/2$

Comparing obtained equation with standard equation, we have:

$$y = 2\pi/3 \text{ or } y = \pi/2$$

$$\therefore 2x = 2m\pi \pm 2\pi/3 \text{ or } 4x = (2n+1)\pi/2$$

$$\therefore x = m\pi \pm \frac{\pi}{3} \text{ or } x = (2n+1)\frac{\pi}{8} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

#### 4 E. Question

Solve the following equations :

$$\cos x + \sin x = \cos 2x + \sin 2x$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x + \sin x = \cos 2x + \sin 2x$$

$$\cos x - \cos 2x = \sin 2x - \sin x$$

$$\{ \because \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \text{ \& } \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \}$$

$$\therefore -2 \sin\left(\frac{x+2x}{2}\right) \sin\left(\frac{x-2x}{2}\right) = 2 \cos\left(\frac{2x+x}{2}\right) \sin\left(\frac{2x-x}{2}\right)$$

$$\Rightarrow 2 \sin\frac{3x}{2} \sin\frac{x}{2} = 2 \cos\frac{3x}{2} \sin\frac{x}{2}$$

$$\therefore \sin\frac{x}{2} (\sin\frac{3x}{2} - \cos\frac{3x}{2}) = 0$$

Hence,

$$\text{Either, } \sin\frac{x}{2} = 0 \text{ or } \sin\frac{3x}{2} = \cos\frac{3x}{2}$$

$$\Rightarrow \sin\frac{x}{2} = \sin m\pi \text{ or } \tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore \frac{x}{2} = m\pi \text{ or } \frac{3x}{2} = n\pi + \frac{\pi}{4}$$

$$\Rightarrow x = 2m\pi \text{ or } x = \frac{2n\pi}{3} + \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

#### 4 E. Question

Solve the following equations :

$$\cos x + \sin x = \cos 2x + \sin 2x$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x + \sin x = \cos 2x + \sin 2x$$

$$\cos x - \cos 2x = \sin 2x - \sin x$$

$$\left\{ \because \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \text{ \& } \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \right\}$$

$$\therefore -2 \sin\left(\frac{x+2x}{2}\right) \sin\left(\frac{x-2x}{2}\right) = 2 \cos\left(\frac{2x+x}{2}\right) \sin\left(\frac{2x-x}{2}\right)$$

$$\Rightarrow 2 \sin \frac{3x}{2} \sin \frac{x}{2} = 2 \cos \frac{3x}{2} \sin \frac{x}{2}$$

$$\therefore \sin \frac{x}{2} (\sin \frac{3x}{2} - \cos \frac{3x}{2}) = 0$$

Hence,

$$\text{Either, } \sin \frac{x}{2} = 0 \text{ or } \sin \frac{3x}{2} = \cos \frac{3x}{2}$$

$$\Rightarrow \sin \frac{x}{2} = \sin m\pi \text{ or } \tan \frac{3x}{2} = 1 = \tan \frac{\pi}{4}$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore \frac{x}{2} = m\pi \text{ or } \frac{3x}{2} = n\pi + \frac{\pi}{4}$$

$$\Rightarrow x = 2m\pi \text{ or } x = \frac{2n\pi}{3} + \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

#### 4 F. Question

Solve the following equations :

$$\sin x + \sin 2x + \sin 3x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x + \sin 2x + \sin 3x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \sin x + \sin 2x + \sin 3x = 0$$

$\therefore$  we will use  $\sin x$  and  $\sin 3x$  for transformation as after transformation it will give  $\sin 2x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \}$$

$$\Rightarrow \sin 2x + 2 \sin\left(\frac{3x+x}{2}\right) \cos\frac{3x-x}{2} = 0$$

$$\Rightarrow 2\sin 2x \cos x + \sin 2x = 0$$

$$\Rightarrow \sin 2x (2\cos x + 1) = 0$$

$\therefore$  either,  $\sin 2x = 0$  or  $2\cos x + 1 = 0$

$$\Rightarrow \sin 2x = \sin 0 \text{ or } \cos x = -\frac{1}{2} = \cos(\pi - \pi/3) = \cos 2\pi/3$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

Comparing obtained equation with standard equation, we have:

$$2x = n\pi \text{ or } x = 2m\pi \pm 2\pi/3$$

$$\therefore x = \frac{n\pi}{2} \text{ or } x = 2m\pi \pm \frac{2\pi}{3} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

#### 4 F. Question

Solve the following equations :

$$\sin x + \sin 2x + \sin 3x = 0$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x + \sin 2x + \sin 3x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \sin x + \sin 2x + \sin 3x = 0$$

$\therefore$  we will use  $\sin x$  and  $\sin 3x$  for transformation as after transformation it will give  $\sin 2x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \}$$

$$\Rightarrow \sin 2x + 2 \sin\left(\frac{3x+x}{2}\right) \cos\frac{3x-x}{2} = 0$$

$$\Rightarrow 2\sin 2x \cos x + \sin 2x = 0$$

$$\Rightarrow \sin 2x (2\cos x + 1) = 0$$

$\therefore$  either,  $\sin 2x = 0$  or  $2\cos x + 1 = 0$

$$\Rightarrow \sin 2x = \sin 0 \text{ or } \cos x = -\frac{1}{2} = \cos(\pi - \pi/3) = \cos 2\pi/3$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

Comparing obtained equation with standard equation, we have:

$$2x = n\pi \text{ or } x = 2m\pi \pm 2\pi/3$$

$$\therefore x = \frac{n\pi}{2} \text{ or } x = 2m\pi \pm \frac{2\pi}{3} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

#### 4 G. Question

Solve the following equations :

$$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

#### Answer

##### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{As, } \sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

$\therefore$  we will use  $\sin x$  and  $\sin 3x$  together in 1 group for transformation and  $\sin 4x$  and  $\sin 2x$  common in other group as after transformation both will give  $\cos x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \}$$

$$(\sin x + \sin 3x) + (\sin 2x + \sin 4x) = 0$$

$$\Rightarrow 2 \sin \left( \frac{4x+2x}{2} \right) \cos \frac{4x-2x}{2} + 2 \sin \left( \frac{3x+x}{2} \right) \cos \frac{3x-x}{2} = 0$$

$$\Rightarrow 2 \sin 2x \cos x + 2 \sin 3x \cos x = 0$$

$$\Rightarrow 2 \cos x (\sin 2x + \sin 3x) = 0$$

Again using transformation formula, we have:

$$\Rightarrow 2 \cos x \cdot 2 \sin \frac{3x+2x}{2} \cos \frac{3x-2x}{2} = 0$$

$$\Rightarrow 4 \cos x \sin \frac{5x}{2} \cos \frac{x}{2} = 0$$

$$\therefore \text{either, } \cos x = 0 \text{ or } \sin \frac{5x}{2} = 0 \text{ or } \cos \frac{x}{2} = 0$$

In case of  $\cos x = 0$  we can give solution directly as  $\cos x = 0$  is true for  $x =$  odd multiple of  $\pi/2$

In case of  $\sin x = 0$  we can give solution directly as  $\sin x = 0$  is true for  $x =$  integral multiple of  $\pi$

$$\therefore x = (2n+1) \frac{\pi}{2} \text{ or } \frac{5x}{2} = k\pi \text{ or } \frac{x}{2} = (2p+1) \frac{\pi}{2}$$

$$\Rightarrow x = (2n+1) \frac{\pi}{2} \text{ or } x = \frac{2k\pi}{5} \text{ or } x = (2p+1)\pi \text{ where } n, p, m \in \mathbb{Z}$$

#### 4 G. Question

Solve the following equations :

$$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want to transform.

$$\text{As, } \sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

$\therefore$  we will use  $\sin x$  and  $\sin 3x$  together in 1 group for transformation and  $\sin 4x$  and  $\sin 2x$  common in other group as after transformation both will give  $\cos x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \}$$

$$(\sin x + \sin 3x) + (\sin 2x + \sin 4x) = 0$$

$$\Rightarrow 2 \sin\left(\frac{4x+2x}{2}\right) \cos\frac{4x-2x}{2} + 2 \sin\left(\frac{3x+x}{2}\right) \cos\frac{3x-x}{2} = 0$$

$$\Rightarrow 2\sin 2x \cos x + 2\sin 3x \cos x = 0$$

$$\Rightarrow 2\cos x (\sin 2x + \sin 3x) = 0$$

Again using transformation formula, we have:

$$\Rightarrow 2 \cos x \cdot 2\sin\frac{3x+2x}{2} \cos\frac{3x-2x}{2} = 0$$

$$\Rightarrow 4 \cos x \sin\frac{5x}{2} \cos\frac{x}{2} = 0$$

$$\therefore \text{either, } \cos x = 0 \text{ or } \sin\frac{5x}{2} = 0 \text{ or } \cos\frac{x}{2} = 0$$

In case of  $\cos x = 0$  we can give solution directly as  $\cos x = 0$  is true for  $x =$  odd multiple of  $\pi/2$

In case of  $\sin x = 0$  we can give solution directly as  $\sin x = 0$  is true for  $x =$  integral multiple of  $\pi$

$$\therefore x = (2n+1)\frac{\pi}{2} \text{ or } \frac{5x}{2} = k\pi \text{ or } \frac{x}{2} = (2p+1)\frac{\pi}{2}$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2} \text{ or } x = \frac{2k\pi}{5} \text{ or } x = (2p+1)\pi \text{ where } n, p, m \in \mathbb{Z}$$

#### 4 H. Question

Solve the following equations :

$$\sin 3x - \sin x = 4 \cos^2 x - 2$$

**Answer**

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 3x - \sin x = 4 \cos^2 x - 2$$

$$\Rightarrow \sin 3x - \sin x = 2(2 \cos^2 x - 1)$$

$$\Rightarrow \sin 3x - \sin x = 2 \cos 2x \quad \{\because \cos 2\theta = 2\cos^2 \theta - 1\}$$

$$\{\because \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)\}$$

$$\Rightarrow 2 \cos\left(\frac{3x+x}{2}\right) \sin\left(\frac{3x-x}{2}\right) = 2 \cos 2x$$

$$\Rightarrow 2 \cos 2x \sin x - 2 \cos 2x = 0$$

$$\Rightarrow 2 \cos 2x (\sin x - 1) = 0$$

$\therefore$  either,  $\cos 2x = 0$  or  $\sin x = 1 = \sin \pi/2$

In case of  $\cos x = 0$  we can give solution directly as  $\cos x = 0$  is true for  $x =$  odd multiple of  $\pi/2$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$$\therefore 2x = (2n+1)\frac{\pi}{2} \text{ or } x = m\pi + (-1)^m \frac{\pi}{2}$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \text{ or } x = m\pi + (-1)^m \frac{\pi}{2} \text{ where } m, n \in \mathbb{Z}$$

### 4 H. Question

Solve the following equations :

$$\sin 3x - \sin x = 4 \cos^2 x - 2$$

### Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 3x - \sin x = 4 \cos^2 x - 2$$

$$\Rightarrow \sin 3x - \sin x = 2(2 \cos^2 x - 1)$$

$$\Rightarrow \sin 3x - \sin x = 2 \cos 2x \quad \{\because \cos 2\theta = 2\cos^2 \theta - 1\}$$

$$\{\because \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)\}$$

$$\Rightarrow 2 \cos\left(\frac{3x+x}{2}\right) \sin\left(\frac{3x-x}{2}\right) = 2 \cos 2x$$

$$\Rightarrow 2 \cos 2x \sin x - 2 \cos 2x = 0$$

$$\Rightarrow 2 \cos 2x (\sin x - 1) = 0$$

$\therefore$  either,  $\cos 2x = 0$  or  $\sin x = 1 = \sin \pi/2$

In case of  $\cos x = 0$  we can give solution directly as  $\cos x = 0$  is true for  $x =$  odd multiple of  $\pi/2$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$$\therefore 2x = (2n+1)\frac{\pi}{2} \text{ or } x = m\pi + (-1)^m \frac{\pi}{2}$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \text{ or } x = m\pi + (-1)^m \frac{\pi}{2} \text{ where } m, n \in \mathbb{Z}$$

#### 4 I. Question

Solve the following equations :

$$\sin 2x - \sin 4x + \sin 6x = 0$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x - \sin 4x + \sin 6x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

$$\text{we have, } \sin 2x - \sin 4x + \sin 6x = 0$$

$\therefore$  we will use  $\sin 6x$  and  $\sin 2x$  for transformation as after transformation it will give  $\sin 4x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \}$$

$$\Rightarrow -\sin 4x + 2 \sin \left( \frac{2x+6x}{2} \right) \cos \frac{6x-2x}{2} = 0$$

$$\Rightarrow 2\sin 4x \cos 2x - \sin 4x = 0$$

$$\Rightarrow \sin 4x (2\cos 2x - 1) = 0$$

$\therefore$  either,  $\sin 4x = 0$  or  $2\cos 2x - 1 = 0$

$$\Rightarrow \sin 4x = \sin 0 \text{ or } \cos 2x = \frac{1}{2} = \cos \pi/3$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

Comparing obtained equation with standard equation, we have:

$$4x = n\pi \text{ or } 2x = 2m\pi \pm \pi/3$$

$$\therefore x = \frac{n\pi}{4} \text{ or } x = m\pi \pm \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

#### 4 I. Question



Solve the following equations :

$$\sin 2x - \sin 4x + \sin 6x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\sin 2x - \sin 4x + \sin 6x = 0$$

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want to transform.

$$\text{we have, } \sin 2x - \sin 4x + \sin 6x = 0$$

$\therefore$  we will use  $\sin 6x$  and  $\sin 2x$  for transformation as after transformation it will give  $\sin 4x$  term which can be taken common.

$$\{ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \}$$

$$\Rightarrow -\sin 4x + 2 \sin \left( \frac{2x+6x}{2} \right) \cos \frac{6x-2x}{2} = 0$$

$$\Rightarrow 2\sin 4x \cos 2x - \sin 4x = 0$$

$$\Rightarrow \sin 4x (2\cos 2x - 1) = 0$$

$$\therefore \text{either, } \sin 4x = 0 \text{ or } 2\cos 2x - 1 = 0$$

$$\Rightarrow \sin 4x = \sin 0 \text{ or } \cos 2x = \frac{1}{2} = \cos \frac{\pi}{3}$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

Comparing obtained equation with standard equation, we have:

$$4x = n\pi \text{ or } 2x = 2m\pi \pm \frac{\pi}{3}$$

$$\therefore x = \frac{n\pi}{4} \text{ or } x = m\pi \pm \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \text{ ..ans}$$

### 5 A. Question

Solve the following equations :

$$\tan x + \tan 2x + \tan 3x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

•  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\tan x + \tan 2x + \tan 3x = 0$$

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand  $\tan 3x = \tan (x + 2x)$  we will get  $\tan x + \tan 2x$  common.

$$\therefore \tan x + \tan 2x + \tan 3x = 0$$

$$\Rightarrow \tan x + \tan 2x + \tan (x + 2x) = 0$$

$$\text{As, } \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore \tan x + \tan 2x + \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( 1 + \frac{1}{1 - \tan x \tan 2x} \right) = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( \frac{2 - \tan x \tan 2x}{1 - \tan x \tan 2x} \right) = 0$$

$$\therefore \tan x + \tan 2x = 0 \text{ or } 2 - \tan x \tan 2x = 0$$

$$\text{Using, } \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \text{ we have,}$$

$$\Rightarrow \tan x = \tan (-2x) \text{ or } 2 - \frac{2 \tan^2 x}{1 - \tan^2 x} = 0$$

$$\Rightarrow \tan x = \tan(-2x) \text{ or } 2 - 4 \tan^2 x = 0 \Rightarrow \tan x = 1/\sqrt{2}$$

Let  $1/\sqrt{2} = \tan \alpha$  and if  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi + (-2x) \text{ or } \tan x = \tan \alpha \Rightarrow x = m\pi + \alpha$$

$$\Rightarrow 3x = n\pi \text{ or } x = m\pi + \alpha$$

$$\therefore x = \frac{n\pi}{3} \text{ or } x = m\pi + \alpha \text{ where } \tan \alpha = \frac{1}{\sqrt{2}} \text{ and } m, n \in \mathbb{Z}$$

### 5 A. Question

Solve the following equations :

$$\tan x + \tan 2x + \tan 3x = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

•  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

•  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

•  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\tan x + \tan 2x + \tan 3x = 0$$

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand  $\tan 3x = \tan (x + 2x)$  we will get  $\tan x + \tan 2x$  common.

$$\therefore \tan x + \tan 2x + \tan 3x = 0$$

$$\Rightarrow \tan x + \tan 2x + \tan (x + 2x) = 0$$

$$\text{As, } \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore \tan x + \tan 2x + \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( 1 + \frac{1}{1 - \tan x \tan 2x} \right) = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( \frac{2 - \tan x \tan 2x}{1 - \tan x \tan 2x} \right) = 0$$

$$\therefore \tan x + \tan 2x = 0 \text{ or } 2 - \tan x \tan 2x = 0$$

$$\text{Using, } \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \text{ we have,}$$

$$\Rightarrow \tan x = \tan (-2x) \text{ or } 2 - \frac{2 \tan^2 x}{1 - \tan^2 x} = 0$$

$$\Rightarrow \tan x = \tan(-2x) \text{ or } 2 - 4 \tan^2 x = 0 \Rightarrow \tan x = 1/\sqrt{2}$$

Let  $1/\sqrt{2} = \tan \alpha$  and if  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi + (-2x) \text{ or } \tan x = \tan \alpha \Rightarrow x = m\pi + \alpha$$

$$\Rightarrow 3x = n\pi \text{ or } x = m\pi + \alpha$$

$$\therefore x = \frac{n\pi}{3} \text{ or } x = m\pi + \alpha \text{ where } \tan \alpha = \frac{1}{\sqrt{2}} \text{ and } m, n \in \mathbb{Z}$$

## 5 B. Question

Solve the following equations :

$$\tan x + \tan 2x = \tan 3x$$

### Answer

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\tan x + \tan 2x - \tan 3x = 0$$

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand  $\tan 3x = \tan (x + 2x)$  we will get  $\tan x + \tan 2x$  common.

$$\therefore \tan x + \tan 2x - \tan 3x = 0$$

$$\Rightarrow \tan x + \tan 2x - \tan (x + 2x) = 0$$

$$\text{As, } \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore \tan x + \tan 2x - \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( 1 - \frac{1}{1 - \tan x \tan 2x} \right) = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( \frac{-\tan x \tan 2x}{1 - \tan x \tan 2x} \right) = 0$$

$$\therefore \tan x + \tan 2x = 0 \text{ or } -\tan x \tan 2x = 0$$

Using,  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$  we have,

$$\Rightarrow \tan x = \tan(-2x) \text{ or } \frac{2 \tan^2 x}{1 - \tan^2 x} = 0$$

$$\Rightarrow \tan x = \tan(-2x) \text{ or } \tan x = 0 = \tan 0$$

if  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi + (-2x) \text{ or } x = m\pi + 0$$

$$\Rightarrow 3x = n\pi \text{ or } x = m\pi$$

$$\therefore x = \frac{n\pi}{3} \text{ or } x = m\pi \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

## 5 B. Question

Solve the following equations :

$$\tan x + \tan 2x = \tan 3x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\tan x + \tan 2x - \tan 3x = 0$$

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand  $\tan 3x = \tan(x + 2x)$  we will get  $\tan x + \tan 2x$  common.

$$\therefore \tan x + \tan 2x - \tan 3x = 0$$

$$\Rightarrow \tan x + \tan 2x - \tan(x + 2x) = 0$$

$$\text{As, } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore \tan x + \tan 2x - \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( 1 - \frac{1}{1 - \tan x \tan 2x} \right) = 0$$

$$\Rightarrow (\tan x + \tan 2x) \left( \frac{-\tan x \tan 2x}{1 - \tan x \tan 2x} \right) = 0$$

$$\therefore \tan x + \tan 2x = 0 \text{ or } -\tan x \tan 2x = 0$$

Using,  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$  we have,

$$\Rightarrow \tan x = \tan(-2x) \text{ or } \frac{2 \tan^2 x}{1 - \tan^2 x} = 0$$

$$\Rightarrow \tan x = \tan(-2x) \text{ or } \tan x = 0 = \tan 0$$

if  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi + (-2x) \text{ or } x = m\pi + 0$$

$$\Rightarrow 3x = n\pi \text{ or } x = m\pi$$

$$\therefore x = \frac{n\pi}{3} \text{ or } x = m\pi \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 5 C. Question

Solve the following equations :

$$\tan 3x + \tan x = 2 \tan 2x$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\tan x + \tan 3x = 2 \tan 2x$$

$$\Rightarrow \tan x + \tan 3x = \tan 2x + \tan 2x$$

$$\Rightarrow \tan 3x - \tan 2x = \tan 2x - \tan x$$

$$\Rightarrow \frac{(\tan 3x - \tan 2x)(1 + \tan 3x \tan 2x)}{1 + \tan 3x \tan 2x} = \frac{(\tan 2x - \tan x)(1 + \tan x \tan 2x)}{1 + \tan 2x \tan x}$$

$$\text{As, } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore \tan(3x - 2x)(1 + \tan 3x \tan 2x) = \tan(2x - x)(1 + \tan x \tan 2x)$$

$$\Rightarrow \tan x \{1 + \tan 3x \tan 2x - 1 - \tan 2x \tan x\} = 0$$

$$\Rightarrow \tan x \tan 2x (\tan 3x - \tan x) = 0$$

$$\therefore \tan x = 0 \text{ or } \tan 2x = 0 \text{ or } \tan 3x = \tan x$$

if  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi \text{ or } 2x = m\pi \text{ or } 3x = k\pi + x$$

$$\therefore x = n\pi \text{ or } x = \frac{m\pi}{2} \text{ or } x = \frac{k\pi}{2}$$

$$\therefore x = n\pi \text{ or } x = \frac{m\pi}{2} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 5 C. Question

Solve the following equations :

$$\tan 3x + \tan x = 2 \tan 2x$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\tan x + \tan 3x = 2 \tan 2x$$

$$\Rightarrow \tan x + \tan 3x = \tan 2x + \tan 2x$$

$$\Rightarrow \tan 3x - \tan 2x = \tan 2x - \tan x$$

$$\Rightarrow \frac{(\tan 3x - \tan 2x)(1 + \tan 3x \tan 2x)}{1 + \tan 3x \tan 2x} = \frac{(\tan 2x - \tan x)(1 + \tan x \tan 2x)}{1 + \tan 2x \tan x}$$

$$\text{As, } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore \tan(3x - 2x)(1 + \tan 3x \tan 2x) = \tan(2x - x)(1 + \tan x \tan 2x)$$

$$\Rightarrow \tan x \{1 + \tan 3x \tan 2x - 1 - \tan 2x \tan x\} = 0$$

$$\Rightarrow \tan x \tan 2x (\tan 3x - \tan x) = 0$$

$$\therefore \tan x = 0 \text{ or } \tan 2x = 0 \text{ or } \tan 3x = \tan x$$

if  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi \text{ or } 2x = m\pi \text{ or } 3x = k\pi + x$$

$$\therefore x = n\pi \text{ or } x = \frac{m\pi}{2} \text{ or } x = \frac{k\pi}{2}$$

$$\therefore x = n\pi \text{ or } x = \frac{m\pi}{2} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 6 A. Question

Solve the following equations :

$$\sin x + \cos x = \sqrt{2}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x + \cos x = \sqrt{2}$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = 1$$

$$\Rightarrow \sin x \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos x = 1 \left\{ \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right\}$$

$$\Rightarrow \sin \left( \frac{\pi}{4} + x \right) = 1 \left\{ \because \sin A \cos B + \cos A \sin B = \sin(A + B) \right\}$$

$$\Rightarrow \sin \left( \frac{\pi}{4} + x \right) = \sin \frac{\pi}{2}$$

**NOTE:** We can also make the ratio of cos instead of sin, the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

$$\therefore \frac{\pi}{4} + x = n\pi + (-1)^n \frac{\pi}{2}$$

$$\therefore x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4} \text{ where } n \in Z \dots \text{ans}$$

### 6 A. Question

Solve the following equations :

$$\sin x + \cos x = \sqrt{2}$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in Z$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in Z$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in Z$ .

given,

$$\sin x + \cos x = \sqrt{2}$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = 1$$

$$\Rightarrow \sin x \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos x = 1 \{ \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \}$$

$$\Rightarrow \sin \left( \frac{\pi}{4} + x \right) = 1 \{ \because \sin A \cos B + \cos A \sin B = \sin (A + B) \}$$

$$\Rightarrow \sin \left( \frac{\pi}{4} + x \right) = \sin \frac{\pi}{2}$$

**NOTE:** We can also make the ratio of cos instead of sin, the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in Z$

$$\therefore \frac{\pi}{4} + x = n\pi + (-1)^n \frac{\pi}{2}$$

$$\therefore x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4} \text{ where } n \in Z \dots \text{ans}$$

### 6 B. Question

Solve the following equations :

$$\sqrt{3} \cos x + \sin x = 1$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in Z$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in Z$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in Z$ .

given,

$$\sqrt{3} \cos x + \sin x = 1$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2}$$

$$\Rightarrow \sin x \sin \frac{\pi}{6} + \cos \frac{\pi}{6} \cos x = \frac{1}{2} \quad \{ \because \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{ and } \sin \frac{\pi}{6} = \frac{1}{2} \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{6} \right) = \frac{1}{2} \quad \{ \because \cos A \cos B + \sin A \sin B = \cos (A - B) \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{6} \right) = \cos \frac{\pi}{3}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in Z$

$$\therefore x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6} \text{ where } n \in Z$$

$$\therefore x = 2n\pi + \frac{\pi}{2} \text{ or } 2n\pi - \frac{\pi}{6} \text{ where } n \in Z \dots \text{ans}$$

### 6 B. Question

Solve the following equations :

$$\sqrt{3} \cos x + \sin x = 1$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in Z$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in Z$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in Z$ .

given,

$$\sqrt{3} \cos x + \sin x = 1$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2}$$

$$\Rightarrow \sin x \sin \frac{\pi}{6} + \cos \frac{\pi}{6} \cos x = \frac{1}{2} \quad \{ \because \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{ and } \sin \frac{\pi}{6} = \frac{1}{2} \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{6} \right) = \frac{1}{2} \quad \{ \because \cos A \cos B + \sin A \sin B = \cos (A - B) \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{6} \right) = \cos \frac{\pi}{3}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in Z$

$$\therefore x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6} \text{ where } n \in Z$$

$$\therefore x = 2n\pi + \frac{\pi}{2} \text{ or } 2n\pi - \frac{\pi}{6} \text{ where } n \in Z \dots \text{ans}$$

### 6 C. Question

Solve the following equations :

$$\sin x + \cos x = 1$$



## Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x + \cos x = 1.$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \quad \{ \text{dividing by } \sqrt{2} \text{ both sides} \}$$

$$\Rightarrow \sin x \sin \frac{\pi}{4} + \cos x \cos \frac{\pi}{4} = \cos \frac{\pi}{4} \quad \{ \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \quad \{ \because \cos A \cos B + \sin A \sin B = \cos (A - B) \}$$

**NOTE:** We can also make the ratio of sin instead of cos, the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x - \frac{\pi}{4} = \left( 2n\pi \pm \frac{\pi}{4} \right)$$

$$\therefore x = \left( 2n\pi \pm \frac{\pi}{4} \right) + \frac{\pi}{4} \quad \text{where } n \in \mathbb{Z}.$$

$$x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} \quad \text{where } n \in \mathbb{Z} \dots \text{ans.}$$

## 6 C. Question

Solve the following equations :

$$\sin x + \cos x = 1$$

## Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x + \cos x = 1.$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \quad \{ \text{dividing by } \sqrt{2} \text{ both sides} \}$$

$$\Rightarrow \sin x \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \cos x = \cos \frac{\pi}{4} \quad \{ \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \quad \{ \because \cos A \cos B + \sin A \sin B = \cos (A - B) \}$$

**NOTE:** We can also make the ratio of sin instead of cos, the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x - \frac{\pi}{4} = \left( 2n\pi \pm \frac{\pi}{4} \right)$$

$$\therefore x = \left( 2n\pi \pm \frac{\pi}{4} \right) + \frac{\pi}{4} \text{ where } n \in \mathbb{Z}.$$

$$x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} \text{ where } n \in \mathbb{Z} \dots \text{ans.}$$

#### 6 D. Question

Solve the following equations :

$$\operatorname{cosec} x = 1 + \cot x$$

**Answer**

**deas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\operatorname{cosec} x = 1 + \cot x$$

$$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$$

$$\Rightarrow \sin x + \cos x = 1.$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \quad \{ \text{dividing by } \sqrt{2} \text{ both sides} \}$$

$$\Rightarrow \sin x \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \cos x = \cos \frac{\pi}{4} \quad \{ \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \quad \{ \because \cos A \cos B + \sin A \sin B = \cos (A - B) \}$$

**NE:** We can also make the ratio of sin instead of cos, the answer remains same but the form of answer may

look different, when you put values of n you will get same values with both forms

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x - \frac{\pi}{4} = \left( 2n\pi \pm \frac{\pi}{4} \right).$$

$$\therefore x = \left( 2n\pi \pm \frac{\pi}{4} \right) + \frac{\pi}{4} \text{ where } n \in \mathbb{Z}$$

$$x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

#### 6 D. Question

Solve the following equations :

$$\operatorname{cosec} x = 1 + \cot x$$

#### Answer

**deas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\operatorname{cosec} x = 1 + \cot x$$

$$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$$

$$\Rightarrow \sin x + \cos x = 1.$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \text{ { dividing by } \sqrt{2} \text{ both sides}}$$

$$\Rightarrow \sin x \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \cos x = \cos \frac{\pi}{4}. \text{ { } \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}. \text{ } }$$

$$\Rightarrow \cos \left( x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4}. \text{ { } \because \cos A \cos B + \sin A \sin B = \cos (A - B) \text{ } }$$

**NE:** We can also make the ratio of sin instead of cos , the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x - \frac{\pi}{4} = \left( 2n\pi \pm \frac{\pi}{4} \right).$$

$$\therefore x = \left( 2n\pi \pm \frac{\pi}{4} \right) + \frac{\pi}{4} \text{ where } n \in \mathbb{Z}$$

$$x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 6 E. Question

Solve the following equations :

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 2$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

•  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

•  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 2$$

Dividing both sides by  $2\sqrt{2}$  :

We have,

$$\Rightarrow \cos \alpha \cos x + \sin \alpha \sin x = \cos \frac{\pi}{4} \text{ where } \cos \alpha = \frac{\pi}{4}$$

$$\Rightarrow \cos(x - \alpha) = \cos \frac{\pi}{4} \{ \cos \frac{\pi}{4} = 1/\sqrt{2} \}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x - \alpha = 2n\pi \pm \frac{\pi}{4} \frac{(\sqrt{3}-1)}{2\sqrt{2}} \cos x + \frac{(\sqrt{3}+1)}{2\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$$

$$x = 2n\pi \pm \frac{\pi}{4} + \alpha \text{ where } \cos \alpha = \frac{(\sqrt{3}-1)}{2\sqrt{2}} \text{ and } n \in \mathbb{Z}$$

### 6 E. Question

Solve the following equations :

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 2$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

•  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

•  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 2$$

Dividing both sides by  $2\sqrt{2}$  :

We have,

$$\Rightarrow \cos \alpha \cos x + \sin \alpha \sin x = \cos \frac{\pi}{4} \text{ where } \cos \alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos(x - \alpha) = \cos \frac{\pi}{4} \text{ { } \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ } }$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x - \alpha = 2n\pi \pm \frac{\pi}{4} \frac{(\sqrt{3}-1)}{2\sqrt{2}} \cos x + \frac{(\sqrt{3}+1)}{2\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$$

$$x = 2n\pi \pm \frac{\pi}{4} + \alpha \text{ where } \cos \alpha = \frac{(\sqrt{3}-1)}{2\sqrt{2}} \text{ and } n \in \mathbb{Z}$$

### 7 . Question

Solve the following equations :

$$\cot x + \tan x = 2$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\cot x + \tan x = 2$$

$$\frac{1}{\tan x} + \tan x = 2$$

$$\Rightarrow \tan^2 x - 2 \tan x + 1 = 0$$

$$\Rightarrow (\tan x - 1)^2 = 0$$

$$\therefore \tan x = 1 \Rightarrow \tan x = \tan \frac{\pi}{4}$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + \frac{\pi}{4} \text{ where } n \in \mathbb{Z} \text{ ....ans}$$

### 7 . Question

Solve the following equations :

$$\cot x + \tan x = 2$$

## Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\cot x + \tan x = 2$$

$$\frac{1}{\tan x} + \tan x = 2$$

$$\Rightarrow \tan^2 x - 2 \tan x + 1 = 0$$

$$\Rightarrow (\tan x - 1)^2 = 0$$

$$\therefore \tan x = 1 \Rightarrow \tan x = \tan \pi/4$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + \pi/4 \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 7 B. Question

Solve the following equations :

$$2 \sin^2 x = 3 \cos x, 0 \leq x \leq 2\pi$$

## Answer

### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$2 \sin^2 x = 3 \cos x, 0 \leq x \leq 2\pi$$

$$\Rightarrow 2(1 - \cos^2 x) = 3 \cos x$$

$$\Rightarrow 2 \cos^2 x + 3 \cos x - 2 = 0$$

$$\Rightarrow 2 \cos^2 x + 4 \cos x - \cos x - 2 = 0$$

$$\Rightarrow 2 \cos x(\cos x + 2) - 1(\cos x + 2) = 0$$

$$\Rightarrow (2 \cos x - 1)(\cos x + 2) = 0$$

$$\therefore \cos x = -2$$

or  $\cos x = -2$  { as  $\cos x$  lies between  $-1$  and  $1$  so this value is rejected }

$$\therefore \cos x = \frac{1}{2} = \cos \pi/3$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$

$$\therefore x = 2n\pi \pm \pi/3$$

But,  $0 \leq x \leq 2\pi$

$\therefore x = \pi/3$  and  $x = 2\pi - \pi/3 = 5\pi/3$  ....ans

### 7 B. Question

Solve the following equations :

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$$\Rightarrow (2 \cos x - 1)(\cos x + 2) = 0$$

$$\therefore \cos x = \frac{1}{2}$$

or  $\cos x = -2$  { as  $\cos x$  lies between  $-1$  and  $1$  so this value is rejected }

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$$\therefore x = 2n\pi \pm \pi/3$$

But,  $0 \leq x \leq 2\pi$

$\therefore x = \pi/3$  and  $x = 2\pi - \pi/3 = 5\pi/3$  ....ans

### 7 C. Question

Solve the following equations :

$$\sec x \cos 5x + 1 = 0, 0 < x < \pi/2$$

#### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sec x \cos 5x + 1 = 0, 0 < x < \pi/2$$

$$\Rightarrow \sec x \cos 5x = -1$$

$$\Rightarrow \cos 5x = -\cos x$$

$$\because -\cos x = \cos(\pi - x)$$

$$\therefore \cos 5x = \cos(\pi - x)$$

If  $\cos x = \cos y$ , implies  $2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\therefore 5x = 2n\pi \pm (\pi - x)$$

$$\Rightarrow 5x = 2n\pi + (\pi - x) \text{ or } 5x = 2n\pi - (\pi - x)$$

$$\Rightarrow 6x = (2n+1)\pi \text{ or } 4x = (2n-1)\pi$$

$$\therefore x = (2n+1)\frac{\pi}{6} \text{ or } x = (2n-1)\frac{\pi}{4} \text{ where } n \in \mathbb{Z}$$

But,  $0 < x < \pi/2$

$$\therefore x = \frac{\pi}{6} \text{ and } x = \frac{\pi}{4} \dots \text{ans}$$

### 7 C. Question

Solve the following equations :

$$\sec x \cos 5x + 1 = 0, 0 < x < \pi/2$$

#### Answer

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
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given,

$$\sec x \cos 5x + 1 = 0, 0 < x < \pi/2$$

$$\Rightarrow \sec x \cos 5x = -1$$

$$\Rightarrow \cos 5x = -\cos x$$

$$\because -\cos x = \cos(\pi - x)$$

$$\therefore \cos 5x = \cos(\pi - x)$$

If  $\cos x = \cos y$ , implies  $2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\therefore 5x = 2n\pi \pm (\pi - x)$$

$$\Rightarrow 5x = 2n\pi + (\pi - x) \text{ or } 5x = 2n\pi - (\pi - x)$$

$$\Rightarrow 6x = (2n+1)\pi \text{ or } 4x = (2n-1)\pi$$

$$\therefore x = (2n+1)\frac{\pi}{6} \text{ or } x = (2n-1)\frac{\pi}{4} \text{ where } n \in \mathbb{Z}$$

But,  $0 < x < \pi/2$

$$\therefore x = \frac{\pi}{6} \text{ and } x = \frac{\pi}{4} \dots \text{ans}$$

### 7 D. Question



Solve the following equations :

$$5 \cos^2 x + 7 \sin^2 x - 6 = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$5 \cos^2 x + 7 \sin^2 x - 6 = 0$$

$$\Rightarrow 5 \cos^2 x + 5 \sin^2 x + 2 \sin^2 x - 6 = 0$$

$$\Rightarrow 2 \sin^2 x - 6 + 5 = 0 \quad \{\because \sin^2 x + \cos^2 x = 1\}$$

$$\Rightarrow 2 \sin^2 x - 1 = 0$$

$$\Rightarrow \sin^2 x = (1/2)$$

$$\therefore \sin x = \pm(1/\sqrt{2})$$

$$\Rightarrow \sin x = \pm \sin \pi/4$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + (-1)^n (\pm(\pi/4)) \text{ where } n \in \mathbb{Z}$$

$$\therefore x = n\pi \pm \frac{\pi}{4} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

**7 D. Question**

Solve the following equations :

$$5 \cos^2 x + 7 \sin^2 x - 6 = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$5 \cos^2 x + 7 \sin^2 x - 6 = 0$$

$$\Rightarrow 5 \cos^2 x + 5 \sin^2 x + 2 \sin^2 x - 6 = 0$$

$$\Rightarrow 2 \sin^2 x - 6 + 5 = 0 \quad \{\because \sin^2 x + \cos^2 x = 1\}$$

$$\Rightarrow 2 \sin^2 x - 1 = 0$$

$$\Rightarrow \sin^2 x = (1/2)$$

$$\therefore \sin x = \pm(1/\sqrt{2})$$

$$\Rightarrow \sin x = \pm \sin \pi/4$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + (-1)^n (\pm(\pi/4)) \text{ where } n \in \mathbb{Z}$$

$$\therefore x = n\pi \pm \frac{\pi}{4} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 7 E. Question

Solve the following equations :

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$$

$$\Rightarrow (\sin x + \sin 3x) - 3 \sin 2x - (\cos x + \cos 3x) + 3 \cos 2x = 0$$

$$\therefore \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \text{ and } \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$$

$$\therefore 2 \sin \left( \frac{x+3x}{2} \right) \cos \left( \frac{3x-x}{2} \right) - 3 \sin 2x - 2 \cos \left( \frac{3x+x}{2} \right) \cos \left( \frac{3x-x}{2} \right) + 3 \cos 2x = 0$$

$$\Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos x - 3) - \cos 2x (2 \cos x - 3) = 0$$

$$\Rightarrow (2 \cos x - 3)(\sin 2x - \cos 2x) = 0$$

$$\therefore \cos x = 3/2 = 1.5 \text{ (not accepted as } \cos x \text{ lies between } -1 \text{ and } 1)$$

$$\text{Or } \sin 2x = \cos 2x$$

$$\therefore \tan 2x = 1 = \tan \pi/4$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore 2x = n\pi + \pi/4$$

$$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 7 E. Question

Solve the following equations :

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$$

$$\Rightarrow (\sin x + \sin 3x) - 3 \sin 2x - (\cos x + \cos 3x) + 3 \cos 2x = 0$$

$$\because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \text{ and } \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore 2 \sin\left(\frac{x+3x}{2}\right) \cos\left(\frac{3x-x}{2}\right) - 3 \sin 2x - 2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) + 3 \cos 2x = 0$$

$$\Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos x - 3) - \cos 2x (2 \cos x - 3) = 0$$

$$\Rightarrow (2 \cos x - 3)(\sin 2x - \cos 2x) = 0$$

$$\therefore \cos x = 3/2 = 1.5 \text{ (not accepted as } \cos x \text{ lies between } -1 \text{ and } 1)$$

$$\text{Or } \sin 2x = \cos 2x$$

$$\therefore \tan 2x = 1 = \tan \pi/4$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore 2x = n\pi + \pi/4$$

$$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 7 F. Question

Solve the following equations :

$$4 \sin x \cos x + 2 \sin x + 2 \cos x + 1 = 0$$

### Answer

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$4 \sin x \cos x + 2 \sin x + 2 \cos x + 1 = 0$$

$$\Rightarrow 2 \sin x (2 \cos x + 1) + 1(2 \cos x + 1) = 0$$

$$\Rightarrow (2 \cos x + 1)(2 \sin x + 1) = 0$$

$$\therefore \cos x = -1/2 \text{ or } \sin x = -1/2$$

$$\Rightarrow \cos x = \cos (\pi - \pi/3) \text{ or } \sin x = \sin (-\pi/6)$$

$$\Rightarrow \cos x = \cos 2\pi/3 \text{ or } \sin x = \sin (-\pi/6)$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

And  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = 2n\pi \pm 2\pi/3 \text{ or } x = m\pi + (-1)^m (-\pi/6)$$

Hence,

$$x = 2n\pi \pm \frac{2\pi}{3} \text{ or } x = m\pi + (-1)^m \left(-\frac{\pi}{6}\right) \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 7 F. Question

Solve the following equations :

$$4 \sin x \cos x + 2 \sin x + 2 \cos x + 1 = 0$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$4 \sin x \cos x + 2 \sin x + 2 \cos x + 1 = 0$$

$$\Rightarrow 2\sin x (2\cos x + 1) + 1(2\cos x + 1) = 0$$

$$\Rightarrow (2\cos x + 1)(2\sin x + 1) = 0$$

$$\therefore \cos x = -1/2 \text{ or } \sin x = -1/2$$

$$\Rightarrow \cos x = \cos (\pi - \pi/3) \text{ or } \sin x = \sin (-\pi/6)$$

$$\Rightarrow \cos x = \cos 2\pi/3 \text{ or } \sin x = \sin (-\pi/6)$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

And  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = 2n\pi \pm 2\pi/3 \text{ or } x = m\pi + (-1)^m (-\pi/6)$$

Hence,

$$x = 2n\pi \pm \frac{2\pi}{3} \text{ or } x = m\pi + (-1)^m \left(-\frac{\pi}{6}\right) \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 7 G. Question

Solve the following equations :

$$\cos x + \sin x = \cos 2x + \sin 2x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x + \sin x = \cos 2x + \sin 2x$$

$$\cos x - \cos 2x = \sin 2x - \sin x$$

$$\left\{ \because \sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \text{ \& } \cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \right\}$$

$$\therefore -2 \sin \left( \frac{x+2x}{2} \right) \sin \left( \frac{x-2x}{2} \right) = 2 \cos \left( \frac{2x+x}{2} \right) \sin \left( \frac{2x-x}{2} \right)$$

$$\Rightarrow 2 \sin \frac{3x}{2} \sin \frac{x}{2} = 2 \cos \frac{3x}{2} \sin \frac{x}{2}$$

$$\therefore \sin \frac{x}{2} \left( \sin \frac{3x}{2} - \cos \frac{3x}{2} \right) = 0.$$

Hence,

$$\text{Either, } \sin \frac{x}{2} = 0 \text{ or } \sin \frac{3x}{2} = \cos \frac{3x}{2}$$

$$\Rightarrow \sin \frac{x}{2} = \sin m\pi \text{ or } \tan \frac{3x}{2} = 1 = \tan \frac{\pi}{4}$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore \frac{x}{2} = m\pi \text{ or } \frac{3x}{2} = n\pi + \frac{\pi}{4}$$

$$\Rightarrow x = 2m\pi \text{ or } x = \frac{2n\pi}{3} + \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 7 G. Question

Solve the following equations :

$$\cos x + \sin x = \cos 2x + \sin 2x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$\cos x + \sin x = \cos 2x + \sin 2x$$

$$\cos x - \cos 2x = \sin 2x - \sin x$$

$$\left\{ \because \sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \text{ \& } \cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \right\}$$

$$\therefore -2 \sin \left( \frac{x+2x}{2} \right) \sin \left( \frac{x-2x}{2} \right) = 2 \cos \left( \frac{2x+x}{2} \right) \sin \left( \frac{2x-x}{2} \right)$$

$$\Rightarrow 2 \sin \frac{3x}{2} \sin \frac{x}{2} = 2 \cos \frac{3x}{2} \sin \frac{x}{2}$$

$$\therefore \sin \frac{x}{2} \left( \sin \frac{3x}{2} - \cos \frac{3x}{2} \right) = 0$$

Hence,

$$\text{Either, } \sin \frac{x}{2} = 0 \text{ or } \sin \frac{3x}{2} = \cos \frac{3x}{2}$$

$$\Rightarrow \sin \frac{x}{2} = \sin m\pi \text{ or } \tan \frac{3x}{2} = 1 = \tan \frac{\pi}{4}$$

If  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore \frac{x}{2} = m\pi \text{ or } \frac{3x}{2} = n\pi + \frac{\pi}{4}$$

$$\Rightarrow x = 2m\pi \text{ or } x = \frac{2n\pi}{3} + \frac{\pi}{6} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 7 H. Question

Solve the following equations :

$$\sin x \tan x - 1 = \tan x - \sin x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x \tan x - 1 = \tan x - \sin x$$

$$\Rightarrow \sin x \tan x - \tan x + \sin x - 1 = 0$$

$$\Rightarrow \tan x(\sin x - 1) + (\sin x - 1) = 0$$

$$\Rightarrow (\sin x - 1)(\tan x + 1) = 0$$

$$\therefore \sin x = 1 \text{ or } \tan x = -1$$

$$\Rightarrow \sin x = \sin \pi/2 \text{ or } \tan x = \tan (-\pi/4)$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

and  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + (-1)^n (\pi/2) \text{ or } x = m\pi + (-\pi/4)$$

$$\therefore x = n\pi + (-1)^n \left( \frac{\pi}{2} \right) \text{ or } x = m\pi - \frac{\pi}{4} \text{ where } n, m \in \mathbb{Z} \dots \text{ans}$$

### 7 H. Question

Solve the following equations :

$$\sin x \tan x - 1 = \tan x - \sin x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

given,

$$\sin x \tan x - 1 = \tan x - \sin x$$

$$\Rightarrow \sin x \tan x - \tan x + \sin x - 1 = 0$$

$$\Rightarrow \tan x(\sin x - 1) + (\sin x - 1) = 0$$

$$\Rightarrow (\sin x - 1)(\tan x + 1) = 0$$

$$\therefore \sin x = 1 \text{ or } \tan x = -1$$

$$\Rightarrow \sin x = \sin \pi/2 \text{ or } \tan x = \tan (-\pi/4)$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .

and  $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + (-1)^n (\pi/2) \text{ or } x = m\pi + (-\pi/4)$$

$$\therefore x = n\pi + (-1)^n \left(\frac{\pi}{2}\right) \text{ or } x = m\pi - \frac{\pi}{4} \text{ where } n, m \in \mathbb{Z} \dots \text{ans}$$

**7 I. Question**

Solve the following equations :

$$3 \tan x + \cot x = 5 \operatorname{cosec} x$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$3 \tan x + \cot x = 5 \operatorname{cosec} x$$

$$\Rightarrow 3 \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\sin x}$$

$$\Rightarrow 3 \frac{\sin x}{\cos x} = \frac{5}{\sin x} - \frac{\cos x}{\sin x}$$

$$\Rightarrow 3 \sin^2 x = (5 - \cos x) \cos x$$

$$\Rightarrow 3 \sin^2 x + \cos^2 x = 5 \cos x$$

$$\Rightarrow 2\sin^2 x - 5\cos x + 1 = 0 \quad \{\because \sin^2 x + \cos^2 x = 1\}$$

$$\therefore 2(1 - \cos^2 x) - 5\cos x + 1 = 0$$

$$\Rightarrow 2\cos^2 x + 5\cos x - 3 = 0$$

$$\Rightarrow 2\cos^2 x + 6\cos x - \cos x - 3 = 0$$

$$\Rightarrow 2\cos x(\cos x + 3) - 1(\cos x + 3) = 0$$

$$\Rightarrow (\cos x + 3)(2\cos x - 1) = 0$$

$\therefore \cos x = -3$  (neglected as  $\cos x$  lies between -1 and 1)

or  $\cos x = \frac{1}{2}$  (accepted value)

$$\therefore \cos x = \cos \frac{\pi}{3}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 7 I. Question

Solve the following equations :

$$3 \tan x + \cot x = 5 \operatorname{cosec} x$$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$3 \tan x + \cot x = 5 \operatorname{cosec} x$$

$$\Rightarrow 3 \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\sin x}$$

$$\Rightarrow 3 \frac{\sin x}{\cos x} = \frac{5}{\sin x} - \frac{\cos x}{\sin x}$$

$$\Rightarrow 3\sin^2 x = (5 - \cos x) \cos x$$

$$\Rightarrow 3\sin^2 x + \cos^2 x = 5\cos x$$

$$\Rightarrow 2\sin^2 x - 5\cos x + 1 = 0 \quad \{\because \sin^2 x + \cos^2 x = 1\}$$

$$\therefore 2(1 - \cos^2 x) - 5\cos x + 1 = 0$$

$$\Rightarrow 2\cos^2 x + 5\cos x - 3 = 0$$

$$\Rightarrow 2\cos^2 x + 6\cos x - \cos x - 3 = 0$$



$$\Rightarrow 2 \cos x (\cos x + 3) - 1(\cos x + 3) = 0$$

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$\therefore \cos x = -3$  (neglected as  $\cos x$  lies between -1 and 1)

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If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = 2n\pi \pm \frac{\pi}{3} \text{ where } n \in \mathbb{Z} \dots \text{ans}$$

### 8. Question

Solve :  $3 - 2 \cos x - 4 \sin x - \cos 2x + \sin 2x = 0$

### Answer

#### Ideas required to solve the problem:

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$3 - 2 \cos x - 4 \sin x - \cos 2x + \sin 2x = 0$$

As,  $\cos 2x = 1 - 2\sin^2 x$  and  $\sin 2x = 2\sin x \cos x$

$$\therefore 3 - 2\cos x - 4\sin x - (1 - 2\sin^2 x) + 2\sin x \cos x = 0$$

$$\Rightarrow 2\sin^2 x - 4\sin x + 2 - 2\cos x + 2\sin x \cos x = 0$$

$$\Rightarrow 2(\sin^2 x - 2\sin x + 1) + 2\cos x(\sin x - 1) = 0$$

$$\Rightarrow 2(\sin x - 1)^2 + 2\cos x(\sin x - 1) = 0$$

$$\Rightarrow (\sin x - 1)(2\cos x + 2\sin x - 2) = 0$$

$$\therefore \sin x = 1 \text{ or } \sin x + \cos x = 1$$

When,  $\sin x = 1$

We have,

$$\sin x = \sin \frac{\pi}{2}$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi + (-1)^n \left( \frac{\pi}{2} \right) \text{ where } n \in \mathbb{Z}$$

When,  $\sin x + \cos x = 1$

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \text{ { dividing by } \sqrt{2} \text{ both sides}}$$

$$\Rightarrow \sin x \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \cos x = \cos \frac{\pi}{4} \left\{ \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right\}$$

$$\Rightarrow \cos \left( x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \left\{ \because \cos A \cos B + \sin A \sin B = \cos (A - B) \right\}$$

If  $\cos x = \cos y$ , implies  $x = 2m\pi \pm y$ , where  $m \in \mathbb{Z}$

$$\therefore x - \frac{\pi}{4} = \left( 2m\pi \pm \frac{\pi}{4} \right)$$

$$\therefore x = \left( 2m\pi \pm \frac{\pi}{4} \right) + \frac{\pi}{4} \text{ where } m \in \mathbb{Z}$$

$$\Rightarrow x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2} \text{ where } m \in \mathbb{Z}$$

Hence,

$$x = n\pi + (-1)^n \left( \frac{\pi}{2} \right) \text{ or } x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2} \text{ where } m, n \in \mathbb{Z}$$

### 8. Question

Solve :  $3 - 2 \cos x - 4 \sin x - \cos 2x + \sin 2x = 0$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$3 - 2 \cos x - 4 \sin x - \cos 2x + \sin 2x = 0$$

$$\text{As, } \cos 2x = 1 - 2\sin^2 x \text{ and } \sin 2x = 2\sin x \cos x$$

$$\therefore 3 - 2\cos x - 4\sin x - (1 - 2\sin^2 x) + 2\sin x \cos x = 0$$

$$\Rightarrow 2\sin^2 x - 4\sin x + 2 - 2\cos x + 2\sin x \cos x = 0$$

$$\Rightarrow 2(\sin^2 x - 2\sin x + 1) + 2\cos x(\sin x - 1) = 0$$

$$\Rightarrow 2(\sin x - 1)^2 + 2\cos x(\sin x - 1) = 0$$

$$\Rightarrow (\sin x - 1)(2\cos x + 2\sin x - 2) = 0$$

$$\therefore \sin x = 1 \text{ or } \sin x + \cos x = 1$$

When,  $\sin x = 1$

We have,

$$\sin x = \sin \frac{\pi}{2}$$

If  $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$

$$\therefore x = n\pi + (-1)^n \left( \frac{\pi}{2} \right) \text{ where } n \in \mathbb{Z}$$

When,  $\sin x + \cos x = 1$

$$\therefore \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \text{ { dividing by } \sqrt{2} \text{ both sides}}$$

$$\Rightarrow \sin x \sin \frac{\pi}{4} + \cos x \cos \frac{\pi}{4} = \cos \frac{\pi}{4} \text{ { } \because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ } }$$

$$\Rightarrow \cos \left( x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4} \text{ { } \because \cos A \cos B + \sin A \sin B = \cos (A - B) \text{ } }$$

If  $\cos x = \cos y$ , implies  $x = 2m\pi \pm y$ , where  $m \in \mathbb{Z}$

$$\therefore x - \frac{\pi}{4} = \left( 2m\pi \pm \frac{\pi}{4} \right)$$

$$\therefore x = \left( 2m\pi \pm \frac{\pi}{4} \right) + \frac{\pi}{4} \text{ where } m \in \mathbb{Z}$$

$$\Rightarrow x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2} \text{ where } m \in \mathbb{Z}$$

Hence,

$$x = n\pi + (-1)^n \left( \frac{\pi}{2} \right) \text{ or } x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2} \text{ where } m, n \in \mathbb{Z}$$

### 9. Question

$$3\sin^2 x - 5 \sin x \cos x + 8 \cos^2 x = 2$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
- $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .
- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$3\sin^2 x - 5 \sin x \cos x + 8 \cos^2 x = 2$$

$$\Rightarrow 3\sin^2 x + 3 \cos^2 x - 5 \sin x \cos x + 5 \cos^2 x = 2$$

$$\Rightarrow 3 - 5 \sin x \cos x + 5 \cos^2 x = 2 \text{ { } \because \sin^2 x + \cos^2 x = 1 \text{ } }$$

$$\Rightarrow 5 \cos^2 x + 1 = 5 \sin x \cos x$$

Squaring both sides:

$$\Rightarrow (5 \cos^2 x + 1)^2 = (5 \sin x \cos x)^2$$

$$\Rightarrow 25 \cos^4 x + 10 \cos^2 x + 1 = 25 \sin^2 x \cos^2 x$$

$$\Rightarrow 25 \cos^4 x + 10 \cos^2 x + 1 = 25 (1 - \cos^2 x) \cos^2 x$$

$$\Rightarrow 50\cos^4 x - 15\cos^2 x + 1 = 0$$

$$\Rightarrow 50\cos^4 x - 10\cos^2 x - 5\cos^2 x + 1 = 0$$

$$\Rightarrow 10\cos^2 x (5\cos^2 x - 1) - (5\cos^2 x - 1) = 0$$

$$\Rightarrow (10\cos^2 x - 1)(5\cos^2 x - 1) = 0$$

$$\therefore \cos^2 x = 1/10 \text{ or } \cos^2 x = 1/5$$

Hence, when  $\cos^2 x = 1/10$

$$\text{We have, } \cos x = \pm \frac{1}{\sqrt{10}}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\text{let } \cos \alpha = 1/\sqrt{10}$$

$$\therefore \cos(\pi - \alpha) = -1/\sqrt{10}$$

$$\therefore x = 2n\pi \pm \alpha \text{ or } x = 2n\pi \pm (\pi - \alpha)$$

$$\therefore \text{when, } \cos x = \pm \frac{1}{\sqrt{10}}$$

$$x = 2n\pi \pm \alpha \text{ or } x = 2n\pi \pm (\pi - \alpha) \text{ where } n \in \mathbb{Z} \text{ and } \cos \alpha = \frac{1}{\sqrt{10}}$$

When  $\cos^2 x = 1/5$

$$\text{We have, } \cos x = \pm \frac{1}{\sqrt{5}}$$

If  $\cos x = \cos y$ , implies  $x = 2m\pi \pm y$ , where  $m \in \mathbb{Z}$ .

$$\text{let } \cos \beta = 1/\sqrt{5}$$

$$\therefore \cos(\pi - \beta) = -1/\sqrt{5}$$

$$\therefore x = 2m\pi \pm \beta \text{ or } x = 2m\pi \pm (\pi - \beta)$$

$$\therefore \text{when, } \cos x = \pm \frac{1}{\sqrt{5}}$$

$$x = 2m\pi \pm \beta \text{ or } x = 2m\pi \pm (\pi - \beta) \text{ where } m \in \mathbb{Z} \text{ and } \cos \beta = \frac{1}{\sqrt{5}}$$

...ans

### 9. Question

$$3\sin^2 x - 5\sin x \cos x + 8\cos^2 x = 2$$

**Answer**

**Ideas required to solve the problem:**

The general solution of any trigonometric equation is given as -

- $\sin x = \sin y$ , implies  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .
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- $\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

Given,

$$3\sin^2 x - 5 \sin x \cos x + 8 \cos^2 x = 2$$

$$\Rightarrow 3\sin^2 x + 3 \cos^2 x - 5 \sin x \cos x + 5 \cos^2 x = 2$$

$$\Rightarrow 3 - 5 \sin x \cos x + 5 \cos^2 x = 2 \quad \{\because \sin^2 x + \cos^2 x = 1\}$$

$$\Rightarrow 5 \cos^2 x + 1 = 5 \sin x \cos x$$

Squaring both sides:

$$\Rightarrow (5 \cos^2 x + 1)^2 = (5 \sin x \cos x)^2$$

$$\Rightarrow 25 \cos^4 x + 10 \cos^2 x + 1 = 25 \sin^2 x \cos^2 x$$

$$\Rightarrow 25 \cos^4 x + 10 \cos^2 x + 1 = 25 (1 - \cos^2 x) \cos^2 x$$

$$\Rightarrow 50 \cos^4 x - 15 \cos^2 x + 1 = 0$$

$$\Rightarrow 50 \cos^4 x - 10 \cos^2 x - 5 \cos^2 x + 1 = 0$$

$$\Rightarrow 10 \cos^2 x (5 \cos^2 x - 1) - (5 \cos^2 x - 1) = 0$$

$$\Rightarrow (10 \cos^2 x - 1)(5 \cos^2 x - 1) = 0$$

$$\therefore \cos^2 x = 1/10 \text{ or } \cos^2 x = 1/5$$

Hence, when  $\cos^2 x = 1/10$

$$\text{We have, } \cos x = \pm \frac{1}{\sqrt{10}}$$

If  $\cos x = \cos y$ , implies  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .

$$\text{let } \cos \alpha = 1/\sqrt{10}$$

$$\therefore \cos(\pi - \alpha) = -1/\sqrt{10}$$

$$\therefore x = 2n\pi \pm \alpha \text{ or } x = 2n\pi \pm (\pi - \alpha)$$

$$\therefore \text{when, } \cos x = \pm \frac{1}{\sqrt{10}}$$

$$x = 2n\pi \pm \alpha \text{ or } x = 2n\pi \pm (\pi - \alpha) \text{ where } n \in \mathbb{Z} \text{ and } \cos \alpha = \frac{1}{\sqrt{10}}$$

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$$\text{let } \cos \beta = 1/\sqrt{5}$$

$$\therefore \cos(\pi - \beta) = -1/\sqrt{5}$$

$$\therefore x = 2m\pi \pm \beta \text{ or } x = 2m\pi \pm (\pi - \beta)$$

$$\therefore \text{when, } \cos x = \pm \frac{1}{\sqrt{5}}$$

$$x = 2m\pi \pm \beta \text{ or } x = 2m\pi \pm (\pi - \beta) \text{ where } m \in \mathbb{Z} \text{ and } \cos \beta = \frac{1}{\sqrt{5}}$$

...ans

### 10. Question

$$\text{Solve : } 2^{\sin^2 x} + 2^{\cos^2 x} = 2\sqrt{2}$$

### Answer

Given,

$$\Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} = 2^{\frac{1}{2}} + 2^{\frac{1}{2}}$$

On comparing both sides, we have

$$\sin^2 x = \cos^2 x = \diamond$$

**Note:** If we want to give solution using above two equations then task will become tedious as sin x can be positive at that time cos will be negative and similar 4-5 cases will arise. So inspite of combining all solutions at the end, we proceed as follows

combining both we can say that,

all the solutions of first 2 equations combined will satisfy this single equation

$$\tan^2 x = 1$$

$$\tan x = \pm 1 = \tan\left(\pm \frac{\pi}{4}\right)$$

$\tan x = \tan y$ , implies  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .

$$\therefore x = n\pi + \frac{\pi}{4} \text{ or } x = m\pi - \frac{\pi}{4} \text{ where } m, n \in \mathbb{Z} \dots \text{ans}$$

### 10. Question

$$\text{Solve : } 2^{\sin^2 x} + 2^{\cos^2 x} = 2\sqrt{2}$$

### Answer

Given,

$$\Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} = 2^{\frac{1}{2}} + 2^{\frac{1}{2}}$$

On comparing both sides, we have

$$\sin^2 x = \cos^2 x = \diamond$$

**Note:** If we want to give solution using above two equations then task will become tedious as sin x can be positive at that time cos will be negative and similar 4-5 cases will arise. So inspite of combining all solutions at the end, we proceed as follows

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## Very Short Answer

### 1. Question

Write the number of solutions of the equation  $\tan x + \sec x = 2 \cos x$  in the interval  $[0, 2\pi]$ .

### Answer

$$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \times \cos x$$

$$\sin x + 1 = 2 \times (\cos x)^2$$

$$\sin x + 1 = 2 \times (1 - (\sin x)^2)$$

$$\sin x + 1 = 2 - 2(\sin x)^2$$

$$2(\sin x)^2 + \sin x - 1 = 0$$

Consider  $a = \sin x$

So, the equation will be

$$2a^2 + a - 1 = 0$$

From the equation  $a = 0.5$  or  $-1$

Which implies

$$\sin x = 0.5 \text{ or } \sin x = (-1)$$

Therefore  $x = 30^\circ$  or  $270^\circ$

But for  $x = 270^\circ$  our equation will not be defined as  $\cos(270^\circ) = 0$

So, the solution for  $x = 30^\circ$

According to trigonometric equations

If  $\sin x = \sin a$

Then  $x = n\pi - na$

Here  $\sin x = \sin 30$

$$\text{So, } x = n\pi + (-1)^n \times 30$$

For  $n=0$ ,  $x=30$  and  $n=1$ ,  $x=150^\circ$  and for  $n=2$ ,  $x=390$

Hence between 0 to  $2\pi$  there are only 2 possible solutions.

### 2. Question

Write the number of solutions of the equation  $4 \sin x - 3 \cos x = 7$ .

### Answer

$$4 \sin x - 3 \cos x = 7$$

$$4 \sin x - 7 = 3 \cos x$$

Squaring both sides

$$16(\sin x)^2 + 49 - 56 \sin x = 9(\cos x)^2$$

$$16(\sin x)^2 + 49 - 56 \sin x = 9((\sin x)^2 - 1)$$

$$16(\sin x)^2 - 9(\sin x)^2 - 56 \sin x + 49 + 9 = 0$$

$$7(\sin x)^2 - 56\sin x + 58 = 0$$

Solving the quadratic equation

$$\sin x = 6.7774 \text{ or } 1.2225$$

But we know that  $\sin\theta$  lies between  $[-1, 1]$

So there are no solutions for this given equation

### 3. Question

Write the general solution of  $\tan^2 2x = 1$ .

**Answer**

$$\frac{\sin^2 2x}{\cos^2 2x} = 1$$

$$\sin^2 2x = \cos^2 2x$$

$$\sin^2 2x = 1 - \sin^2 2x$$

$$2 \sin^2 2x = 1$$

$$\sin 2x = \frac{1}{\sqrt{2}}$$

$$\sin 2x = \sin 45^\circ$$

So

$$2x = n\pi + \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$

### 4. Question

Write the set of values of  $a$  for which the equation  $\sqrt{3} \sin x - \cos x = a$  has no solution.

**Answer**

$$\frac{\sqrt{3}}{2} \sin x - \frac{\cos x}{2} = a$$

$$\cos 30^\circ \sin x - \sin 30^\circ \cos x = a$$

$$\sin(x - 30^\circ) = a$$

As the range of  $\sin$  function is from  $[-1, 1]$

So the value of  $a$  can be  $R - [-1, 1]$

$$\text{i.e. } a \in (-\infty, -2) \cup (2, \infty)$$

### 5. Question

If  $\cos x = k$  has exactly one solution in  $[0, 2\pi]$ , then write the value(s) of  $k$ .

**Answer**

$$\text{As } \cos x = \cos \theta$$

$$\text{Then } x = 2n\pi \pm \theta$$

And it is said that it has exactly one solution.

So  $\theta = 0$  and



$$x = \frac{2n\pi}{2}$$

$$= n\pi$$

In the given interval taking  $n=1, x=\pi$  { $n=0$  is not possible as  $\cos 0 = 1$  not  $-1$  but  $\cos \pi$  is  $-1$ }

### 6. Question

Write the number of points of intersection of the curves  $2y = 1$  and  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ .

### Answer

$$2y=1$$

$$\text{i.e. } y = \frac{1}{2}$$

$$\text{and } y = \cos x$$

so, to get the intersection points we must equate both the equations

$$\text{i.e. } \cos x = \frac{1}{2}$$

$$\text{so, } \cos x = \cos 60^\circ$$

$$\text{and we know if } \cos x = \cos a$$

$$\text{then } x=2n\pi \pm a \text{ where } a \in [0, \pi]$$

so here

$$x = 2n\pi \pm \frac{\pi}{3}$$

So the possible values which belong  $[0, 2\pi]$  are  $\frac{\pi}{3}$  and  $\frac{5\pi}{3}$ .

There are a total of 2 points of intersection.

### 7. Question

Write the values of  $x$  in  $[0, \pi]$  for which  $\sin 2x, \frac{1}{2}$  and  $\cos 2x$  are in A.P.

### Answer

$$a, a+r, a+2r$$

$$\text{so } A_1 + A_3 = 2A_2$$

$$\text{here } \sin 2x + \cos 2x = 1$$

$$2\sin x \cos x + 1 - 2\sin^2 x = 1$$

$$\sin x \cos x - \sin^2 x = 0$$

$$\sin x (\cos x - \sin x) = 0$$

$$\text{if } \sin x = 0$$

$$\text{then } x = 0, \pi$$

$$\text{if } \sin x = \cos x$$

$$\text{then } x = \pi/4$$

So, all possible values are  $0, \frac{\pi}{4}, \pi$

### 8. Question

Write the number of points of intersection of the curves  $2y = -1$  and  $y = \operatorname{cosec} x$ .

**Answer**

$$Y = \operatorname{cosec} x \text{ and } y = -\frac{1}{2}$$

So

$$\frac{1}{\sin x} = \frac{-1}{2}$$

$$\sin x = -2$$

Which is not possible

So

There are 0 points of intersection.

**9. Question**

Write the solution set of the equation  $(2 \cos x + 1)(4 \cos x + 5) = 0$  in the interval  $(0, 2\pi]$ .

**Answer**

$$8 \cos^2 x + 10 \cos x + 4 \cos x + 5 = 0$$

$$8 \cos^2 x + 14 \cos x + 5 = 0$$

Solving the quadratic equation, we get,

$$\cos x = -0.5$$

$$\cos x = \cos 120^\circ$$

$$x = 2n\pi \pm \frac{2\pi}{3}$$

$$\text{So } x = \frac{2\pi}{3} \text{ when } n = 0,$$

$$\text{And when } n=1 \text{ } x = \frac{4\pi}{3}$$

**10. Question**

Write the number of values of  $x$  in  $[0, 2\pi]$  that satisfy the equation  $\sin^2 x - \cos x = \frac{1}{4}$ .

**Answer**

$$1 - \cos^2 x - \cos x = 0.25$$

$$\cos^2 x + \cos x - 0.75 = 0$$

Solving the quadratic equation we get

$$\cos x = 0.5$$

$$\cos x = \cos 60^\circ$$

$$x = 2n\pi \pm \frac{\pi}{3}$$

$$x = 60^\circ \text{ when } n=0$$

$$\text{And } x = 300^\circ \text{ when } n=1$$

**11. Question**

If  $3 \tan(x - 15^\circ) = \tan(x + 15^\circ)$ ,  $0 \leq x \leq 90^\circ$ , find  $x$ .

**Answer**

Let  $\tan(15^\circ) = \tan(45^\circ - 30^\circ)$

We know that

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$\tan(45 - 30) = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\tan 15 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

We now

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \times \tan b}$$

And

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \times \tan b}$$

So,  $3 \tan(x - 15^\circ) = \tan(x + 15^\circ)$  can be written as follows

$$3 \times \frac{\tan x - \tan 15}{1 + \tan x \times \tan 15} = \frac{\tan x + \tan 15}{1 - \tan x \times \tan 15}$$

$$(3 \tan x - 3 \tan 15)(1 - \tan x \times \tan 15) = (1 + \tan x \times \tan 15)(\tan x + \tan 15)$$

$$3 \tan x - 3 \tan 15 - 3 \tan^2 x \tan 15 - 3 \tan x \tan^2 15 = \tan x + \tan 15 + \tan^2 x \tan 15 + \tan x \tan^2 15$$

Solving the equation,

And putting

$$\tan 15 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

We get  $\tan x - 1 = 0$

Therefore,  $\tan x = 1$

So,  $x = 45^\circ$

Or

$$x = \frac{\pi}{4}$$

## 12. Question

If  $2 \sin^2 x = 3 \cos x$ , where  $0 \leq x \leq 2\pi$ , then find the value of  $x$ .

### Answer

$$2 \sin^2 x = 3 \cos x$$

$$2 - 2 \cos^2 x = 3 \cos x$$

Solving the quadratic equation, we get

$$\cos x = 1/2$$

Therefore  $x = 60^\circ$  and  $300^\circ$

i.e.

$$\theta = \frac{\pi}{3} \text{ and } \frac{5\pi}{3}$$

### 13. Question

If  $\sec x \cos 5x + 1 = 0$ , where  $0 < x \leq \frac{\pi}{2}$ , find the value of  $x$ .

### Answer

$$\frac{\cos 5x}{\cos x} = -1$$

$$\cos 5x = -\cos x$$

$$\cos 5x + \cos x = 0$$

We know

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

Here

$$\cos 5x + \cos x = 2 \cos\left(\frac{5x+x}{2}\right) \cos\left(\frac{5x-x}{2}\right)$$

Now from the above equation it would be,

$$2 \cos 3x \cos 2x = 0$$

$$\cos 3x \cos 2x = 0$$

$$\cos 3x = 0 \text{ or } \cos 2x = 0$$

for  $\cos 3x = 0$

$$3x = (2n+1)\left(\frac{\pi}{2}\right)$$

$$x = (2n+1)\left(\frac{\pi}{6}\right)$$

for  $\cos 2x = 0$

$$2x = (2n+1)\left(\frac{\pi}{2}\right)$$

$$x = (2n+1)\left(\frac{\pi}{4}\right)$$

so the values of the  $x$  less than equal to  $90^\circ$  are  $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}$

### MCQ

#### 1. Question

Mark the Correct alternative in the following:

The smallest value of  $x$  satisfying the equation  $\sqrt{3}(\cot x + \tan x) = 4$  is

A.  $2\pi/3$

B.  $\pi/3$

C.  $\pi/6$

D.  $\pi/12$

**Answer**

$$\sqrt{3} \left( \frac{1}{\tan x} + \tan x \right) = 4$$

$$\sqrt{3} \left( \frac{1 + \tan^2 x}{\tan x} \right) = 4$$

$$\sqrt{3} + \sqrt{3} \tan^2 x = 4 \tan x$$

$$\sqrt{3} \tan^2 x - 4 \tan x + \sqrt{3} = 0$$

Therefore

$$\tan x = \sqrt{3} \text{ or } \tan x = \frac{1}{\sqrt{3}}$$

$$\text{Therefore } x = \frac{\pi}{3} \text{ or } \frac{\pi}{6}$$

But here the smallest angle is  $\pi/6$

Option C

**2. Question**

Mark the Correct alternative in the following:

If  $\cos x + \sqrt{3} \sin x = 2$ , then  $x =$

- A.  $\pi/3$
- B.  $2\pi/3$
- C.  $4\pi/3$
- D.  $5\pi/3$

**Answer**

$$\cos^2 x = (2 - \sqrt{3} \sin x)^2$$

$$1 - \sin^2 x = 4 + 3 \sin^2 x - 4\sqrt{3} \sin x$$

$$4 \sin^2 x - 4\sqrt{3} \sin x + 3 = 0$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}$$

Option A

**3. Question**

Mark the Correct alternative in the following:

If  $\tan px - \tan qx = 0$ , then the values of  $\theta$  form a series in

- A. AP
- B. GP
- C. HP
- D. None of these

**Answer**

Tan x = tana

$$X = n\pi + a$$

$$\text{So } \tan px - \tan qx = 0$$

$$\tan px = \tan qx$$

$$px = n\pi + qx$$

$$(p-q)x = n\pi$$

$$x = \frac{n\pi}{p-q}$$

$$x = \frac{\pi}{p-q}, \frac{2\pi}{p-q}, \frac{3\pi}{p-q}$$

$$\text{Here in this series } a = r = \frac{\pi}{p-q}$$

So, this is in AP.

Option A

#### 4. Question

Mark the Correct alternative in the following:

If  $a$  is any real number, the number of roots of  $\cot x - \tan x = a$  in the first quadrant is (are).

- A. 2
- B. 0
- C. 1
- D. None of these

#### Answer

$$\frac{1}{\tan x} - \tan x = a$$

$$\frac{1 - \tan^2 x}{\tan x} = a$$

$$1 - \tan^2 x = a \tan x$$

$$\tan^2 x + a \tan x - 1 = 0$$

$$\tan x = \frac{-a \pm \sqrt{a^2 - 4(-1)}}{2}$$

$$\tan x = \frac{-a \pm \sqrt{a^2 + 4}}{2}$$

As it is given  $a$  be any real number take  $a=0$ ,

For  $a=0$

$$\tan x = \frac{\pm\sqrt{0+4}}{2}$$

$$\tan x = +1 \text{ or } -1$$

In first quadrant only  $\tan(\pi/4)=1$

So, there is only one root that lies in the first quadrant.

Option C

#### 5. Question

Mark the Correct alternative in the following:

The general solution of the equation  $7 \cos^2 x + 3 \sin^2 x = 4$  is

A.  $x = 2n\pi \pm \frac{\pi}{6}, n \in Z$

B.  $x = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

C.  $x = 2n\pi \pm \frac{\pi}{3}, n \in Z$

D. none of these

**Answer**

$$7\cos^2x+3(1-\cos^2x)=4$$

$$7\cos^2x+3-3\cos^2x=4$$

$$4\cos^2x-1=0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = \cos 60^\circ$$

Then

$$x = 2n\pi \pm \frac{\pi}{3}, n \in Z$$

**6. Question**

Mark the Correct alternative in the following:

A solution of the equation  $\cos^2 x + \sin x + 1 = 0$ , lies in the interval

A.  $(-\pi/4, \pi/4)$

B.  $(\pi/4, 3\pi/4)$

C.  $(3\pi/4, 5\pi/4)$

D.  $(5\pi/4, 7\pi/4)$

**Answer**

$$1-\sin^2x+\sin x+1=0$$

$$\sin^2x-\sin x-2=0$$

$$\sin x=-1$$

$$x = \frac{3\pi}{2}$$

Option D

**7. Question**

Mark the Correct alternative in the following:

The number of solution in  $[0, \pi/2]$  of the equation  $\cos 3x \tan 5x = \sin 7x$  is

A. 5

B. 7

C. 6

D. None of these

**Answer**

$$\cos 3x \tan 5x = \sin 7x$$

$$\cos 3x \left( \frac{\sin 5x}{\cos 5x} \right) = \sin 7x$$

$$2 \cos 3x \sin 5x = 2 \cos 5x \sin 7x$$

$$\sin 8x + \sin 2x = \sin 12x + \sin 2x$$

$$\sin 8x = \sin 12x$$

$$\sin 12x - \sin 8x = 0$$

$$2 \sin 2x \cos 10x = 0$$

$$\text{If } \sin 2x = 0$$

$$\text{Then, } x = 0$$

$$\text{If } \cos 10x = 0$$

$$\text{Then } 10x = \frac{\pi}{2}$$

$$x = \frac{\pi}{20}, \frac{3\pi}{20}, \frac{5\pi}{20}, \frac{7\pi}{20}, \frac{9\pi}{20}$$

and  $x = 0$  (from above equation  $\sin 2x = 0$ )

So, there are 6 possible solutions.

Option C

**8. Question**

Mark the Correct alternative in the following:

The general value of  $x$  satisfying the equation  $\sqrt{3} \sin x + \cos x = \sqrt{3}$  is given by

A.  $x = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{3}, n \in Z$

B.  $x = n\pi + (-1)^n \frac{\pi}{3} - \frac{\pi}{6}, n \in Z$

C.  $x = n\pi \pm \frac{\pi}{6}, n \in Z$

D.  $x = n\pi \pm \frac{\pi}{3}, n \in Z$

**Answer**

$$\cos^2 x = (\sqrt{3} - \sqrt{3} \sin x)^2$$

$$1 - \sin^2 x = 3 + 3 \sin^2 x - 6 \sin x$$

$$4 \sin^2 x - 6 \sin x + 2 = 0$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$\sin x = 1 \text{ or } 0.5$$



We know,

$$x = n\pi + (-1)^n\theta$$

$$x = n\pi + (-1)^n\left(\frac{\pi}{2}\right) \text{ or } x = n\pi + (-1)^n\left(\frac{\pi}{6}\right)$$

Therefore, the values of  $x$  are

$$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{13\pi}{6}$$

So, these values are obtained for different value of  $n$  from the equation

$$x = n\pi + (-1)^n\frac{\pi}{3} - \frac{\pi}{6}, n \in Z$$

So, Option B

### 9. Question

Mark the Correct alternative in the following:

The smallest positive angle which satisfies the equation  $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$  is

A.  $\frac{5\pi}{6}$

B.  $\frac{2\pi}{3}$

C.  $\frac{\pi}{3}$

D.  $\frac{\pi}{6}$

### Answer

$$2(1 - \cos^2 x) + \sqrt{3}\cos x + 1 = 0$$

$$2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$$

$$2\cos^2 x - \sqrt{3}\cos x - 3 = 0$$

$$\cos x = \sqrt{3} \text{ or } \frac{-\sqrt{3}}{2}$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

Option A

### 10. Question

Mark the Correct alternative in the following:

If  $4\sin^2 x = 1$ , then the values of  $x$  are

A.  $2n\pi \pm \frac{\pi}{3}, n \in Z$

$$B. n\pi \pm \frac{\pi}{3}, n \in Z$$

$$C. n\pi \pm \frac{\pi}{6}, n \in Z$$

$$D. 2n\pi \pm \frac{\pi}{6}, n \in Z$$

**Answer**

$$\sin x = \frac{1}{2} \text{ or } \frac{-1}{2}$$

$$\sin x = \sin a$$

Here  $a = 30^\circ$  or  $-30^\circ$

$$X = n\pi + (-1)^n a$$

So, the values of  $x$  are

$$n\pi \pm \frac{\pi}{6}, n \in Z$$

Option C

### 11. Question

Mark the Correct alternative in the following:

If  $\cot x - \tan x = \sec x$ , then  $x$  is equal to

$$A. 2n\pi + \frac{3\pi}{2}, n \in Z$$

$$B. n\pi + (-1)^n \frac{\pi}{6}, n \in Z$$

$$C. n\pi + \frac{\pi}{2}, n \in Z$$

D. None of these

**Answer**

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{1}{\cos x}$$

$$\frac{1 - \sin^2 x - \sin^2 x}{\sin x} = 1$$

$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$\sin x = 0.5 \text{ or } -1$$

But the equation is invalid for  $\sin x = -1$

$$\text{So, } \sin x = 0.5 = \sin(\pi/6)$$

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$$\text{Hence } x = n\pi + (-1)^n \frac{\pi}{6}, n \in Z$$

Option B

### 12. Question

Mark the Correct alternative in the following:

A value of  $x$  satisfying is

A.  $\frac{5\pi}{3}$

B.  $\frac{4\pi}{3}$

C.  $\frac{2\pi}{3}$

D.  $\frac{\pi}{3}$

**Answer**

$$\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = 1$$

$$\cos 60 \cos x + \sin 60 \sin x = 1$$

$$\cos (60-x) = 1$$

$$\cos (60-x) = \cos 0^\circ$$

$$x = 60^\circ$$

Option D

### 13. Question

Mark the Correct alternative in the following:

In  $(0, \pi)$ , the number of solutions of the equation  $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$  is

A. 7

B. 5

C. 4

D. 2

**Answer**

$$\tan x + \tan 2x + \tan 3x - \tan x \tan 2x \tan 3x = 0$$

$$\tan x + \tan 2x + \tan 3x (1 - \tan x \tan 2x) = 0$$

$$\tan x + \tan 2x = -\tan 3x (1 - \tan x \tan 2x)$$

$$\frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = -\tan 3x$$

$$\tan 3x = -\tan 3x$$

$$2 \tan 3x = 0$$

$$\tan 3x = 0$$

$$3x = 2n\pi$$

$$x = \frac{2n\pi}{3}$$

For

$$n=0, x=0$$

$$n=1,$$

$$x = \frac{2\pi}{3}$$

$$n = 2,$$

$$x = \frac{4\pi}{3} > \pi$$

so, there are only two possible solutions

Option D

#### 14. Question

Mark the Correct alternative in the following:

The number of values of  $x$  in  $[0, 2\pi]$  that satisfy the equation  $\sin^2 x - \cos x = \frac{1}{4}$

- A. 1
- B. 2
- C. 3
- D. 4

#### Answer

$$1 - \cos^2 x - \cos x - 0.25 = 0$$

$$\cos^2 x + \cos x - \frac{3}{4} = 0$$

Solving the quadratic equation, we get

$$\cos x = 0.5$$

$$\text{So } x = 60^\circ \text{ or } 300^\circ$$

Hence there are 2 values

Option B

#### 15. Question

Mark the Correct alternative in the following:

If  $e^{\sin x} - e^{-\sin x} - 4 = 0$ , then  $x =$

- A. 0
- B.  $\sin^{-1} \{\log_e(2 - \sqrt{5})\}$
- C. 1
- D. None of these

#### Answer

$$\log_e (e^{\sin x} \cdot e^{-\sin x}) = \log_e(4)$$

$$\frac{\log_e e^{\sin x}}{\log_e e^{-\sin x}} = \log_e 4$$

$$\frac{\sin x}{-\sin x} = \log_e 4$$

$$-1 = \log_e 4$$

But the above equation is not true so there are no possible values of  $x$  for this given equation

Option D

### 16. Question

Mark the Correct alternative in the following:

The equation  $3 \cos x + 4 \sin x = 6$  has .... Solution

A. finite

B. infinite

C. one

D. no

### Answer

$$4 \sin x = 6 - 3 \cos x$$

$$16 \sin^2 x = 36 + 9 \cos^2 x - 36 \cos x$$

$$16 - 16 \cos^2 x = 36 + 9 \cos^2 x - 36 \cos x$$

$$25 \cos^2 x - 36 \cos x + 20 = 0$$

As both the roots are imaginary there exists no value of  $x$  satisfying this given equation.

No Solution

Option D

### 17. Question

Mark the Correct alternative in the following:

If  $\sqrt{3} \cos x + \sin x = \sqrt{2}$ , then general value of  $\theta$  is

A.  $n\pi + (-1)^n \frac{\pi}{4}, n \in Z$

B.  $(-1)^n \frac{\pi}{4} - \frac{\pi}{3}, n \in Z$

C.  $n\pi + \frac{\pi}{4} - \frac{\pi}{3}, n \in Z$

D.  $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}, n \in Z$

### Answer

$$3 \cos^2 x = (\sqrt{2} - \sin x)^2$$

$$3 - 3 \sin^2 x = 2 + \sin^2 x - 2\sqrt{2} \sin x$$

$$4 \sin^2 x - 2\sqrt{2} \sin x - 1 = 0$$

$$\sin x = \frac{\sqrt{3} + 1}{2\sqrt{2}} \text{ or } \frac{-\sqrt{3} + 1}{2\sqrt{2}}$$

So,  $x = 15^\circ$  or  $345^\circ$

And these values are obtained by the following equation

$$n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}, n \in \mathbb{Z}$$

Option D

### 18. Question

Mark the Correct alternative in the following:

General solution of  $\tan 5x = \cot 2x$  is

A.  $\frac{11\pi}{7} + \frac{\pi}{2}, n \in \mathbb{Z}$

B.  $x = \frac{11\pi}{7} + \frac{\pi}{3}, n \in \mathbb{Z}$

C.  $x = \frac{11\pi}{7} + \frac{\pi}{14}, n \in \mathbb{Z}$

D.  $x = \frac{11\pi}{7} - \frac{\pi}{14}, n \in \mathbb{Z}$

**Answer**

$$\tan 5x = \tan\left(\frac{\pi}{2} - 2x\right)$$

$$\tan 5x - \tan\left(\frac{\pi}{2} - 2x\right) = 0$$

$$\frac{\sin 5x}{\cos 5x} - \frac{\sin\left(\frac{\pi}{2} - 2x\right)}{\cos\left(\frac{\pi}{2} - 2x\right)} = 0$$

$$\frac{\sin 5x \cos\left(\frac{\pi}{2} - 2x\right) - \sin\left(\frac{\pi}{2} - 2x\right) \cos 5x}{\cos 5x \cos\left(\frac{\pi}{2} - 2x\right)} = 0$$

$$\frac{\sin\left(5x - \frac{\pi}{2} + 2x\right)}{\cos 5x \cos\left(\frac{\pi}{2} - 2x\right)} = 0$$

$$\text{This implies } \sin\left(7x - \frac{\pi}{2}\right) = 0$$

$$\text{But } \cos 5x \cos\left(\frac{\pi}{2} - 2x\right) \neq 0$$

$$\text{So } \sin\left(7x - \frac{\pi}{2}\right) = 0$$

$$\sin\left(7x - \frac{\pi}{2}\right) = \sin 0$$

$$7x - \frac{\pi}{2} = n\pi$$

$$x = \frac{n\pi}{7} + \frac{\pi}{14}$$

Option C

### 19. Question

Mark the Correct alternative in the following:

The solution of the equation  $\cos^2 x + \sin x + 1 = 0$  lies in the interval

- A.  $(-\pi/4, \pi/4)$
- B.  $(-\pi/3, \pi/4)$
- C.  $(3\pi/4, 5\pi/4)$
- D.  $(5\pi/4, 7\pi/4)$

### Answer

$$1 - \sin^2 x + \sin x + 1 = 0$$

$$\sin^2 x - \sin x - 2 = 0$$

$$\sin x = -1$$

$$\text{so, } x = 3\pi/2$$

and it lies between  $(5\pi/4, 7\pi/4)$

Option D

### 20. Question

Mark the Correct alternative in the following:

If and  $0 < x < 2\pi$ , then the solution are

- A.  $x = \frac{\pi}{3}, \frac{4\pi}{3}$
- B.  $x = \frac{2\pi}{3}, \frac{4\pi}{3}$
- C.  $x = \frac{2\pi}{3}, \frac{7\pi}{6}$
- D.  $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

### Answer

We know if  $\cos x = \cos a$

Then

$$x = 2n\pi \pm a$$

$$\text{here } \cos x = \cos\left(\frac{2\pi}{3}\right)$$

when  $n=0$ ,

$$x = \frac{2\pi}{3}$$

when  $n=1$ ,

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$$x = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Option B

### 21. Question

Mark the Correct alternative in the following:

The number of values of  $x$  in the interval  $[0, 5\pi]$  satisfying the equation  $3\sin^2 x - 7\sin x + 2 = 0$  is

- A. 0
- B. 5
- C. 6
- D. 10

### Answer

$$3\sin^2 x - 7\sin x + 2 = 0$$

Solving the equation, we get

$$\sin x = \frac{1}{3}$$

$$a = \sin^{-1}\left(\frac{1}{3}\right)$$

$$= 19.47122$$

$$x = n\pi + (-1)^n a$$

For

$$n=0, x=a$$

$$n=1, x = \pi - a$$

$$n=2, x = 2\pi + a$$

$$n=3, x = 3\pi - a$$

$$n=4, x = 4\pi + a$$

$$n=5, x = 5\pi + a$$

So, there are 6 values less than  $5\pi$ .

Option C.

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