11. Constructions

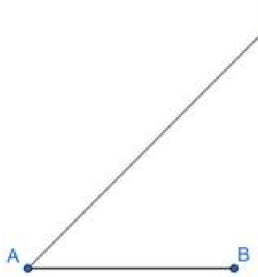
Exercise 11.1

1. Question

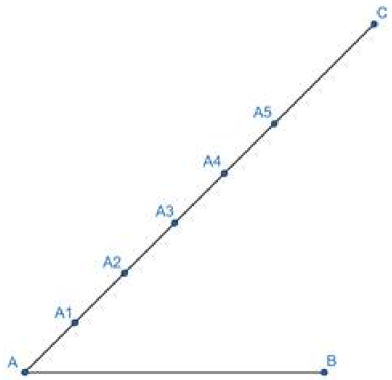
Determine a point which divides a line segment of length 12 cm internally in the ratio 2 : 3. Also, justify your construction.

Answer

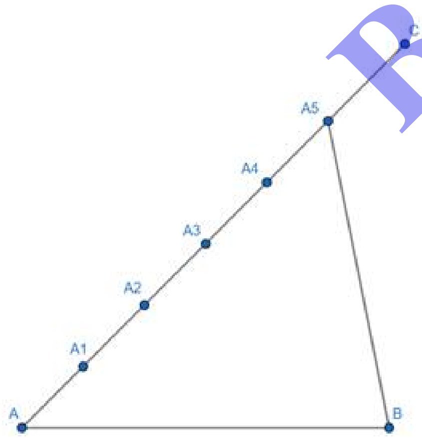
We need to divide this line segment AB of length 12 cm internally in the ratio 2 : 3.



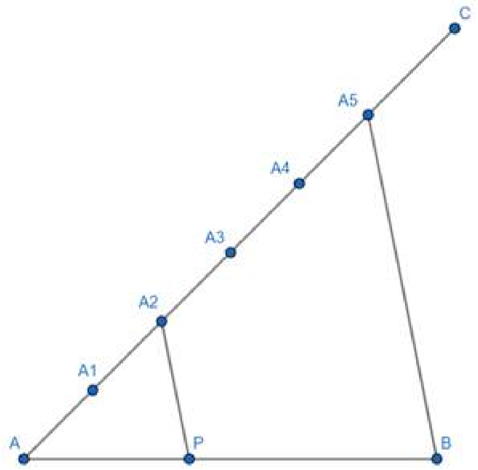
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



Step 2: We plot (2 + 3 =) 5 points A_1, A_2, A_3, A_4, and A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.



Step 3: We join points A_5 and B.



Step 4: We draw line segment A_2P such that $A_2P \parallel A_5B$ and P is the point of intersection of this line segment with AB.

Point P divides AB in the ratio 2 : 3.

Justification –

In ΔAA_2P and ΔAA_5B,

i. $\angle A$ is common.

ii. $\angle AA_2P = \angle AA_5B$ (corresponding angles $\because A_2P \parallel A_5B$)

Hence, $\Delta AA_2P \sim \Delta AA_5B$

So, ratio of lengths of corresponding sides must be equal.

$\Rightarrow \frac{AA_2}{AP} = \frac{AA_5}{AB}$

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = x$

So, the previous relation can be re – written as –

$$\frac{2x}{AP} = \frac{5x}{AP + PB}$$

$\Rightarrow 2(AP + PB) = 5AP$

$\Rightarrow 2PB = 3AP$

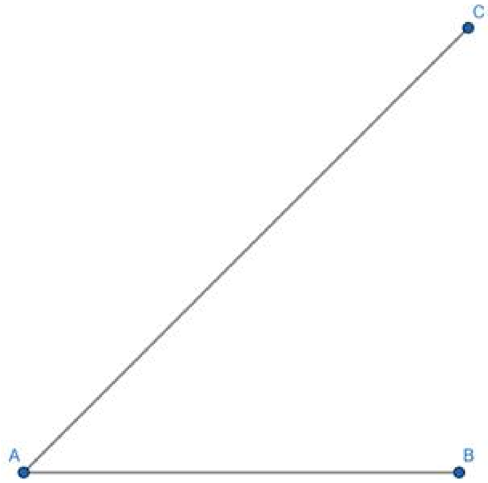
$\Rightarrow AP/PB = 2/3$, or, $AP : PB = 2 : 3$

2. Question

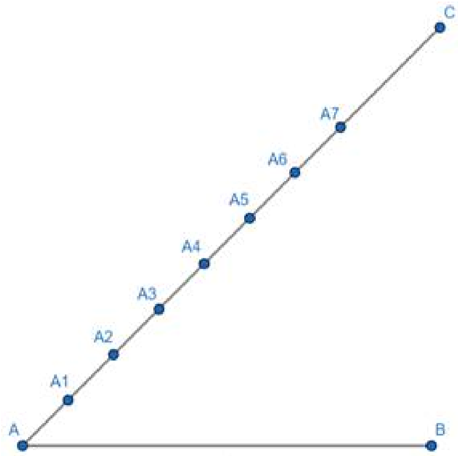
Divide a line segment of length 9 cm internally in the ratio 4 : 3. Also, give justification of the construction.

Answer

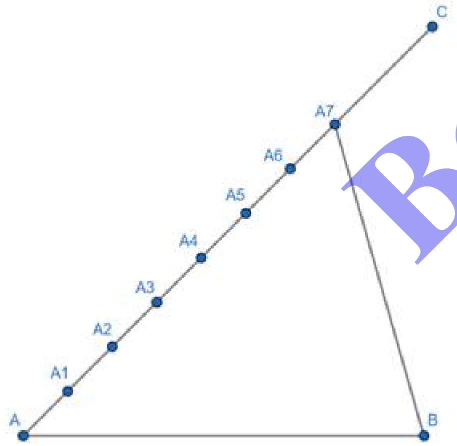
We need to divide this line segment AB of length 9 cm internally in the ratio 4 : 3.



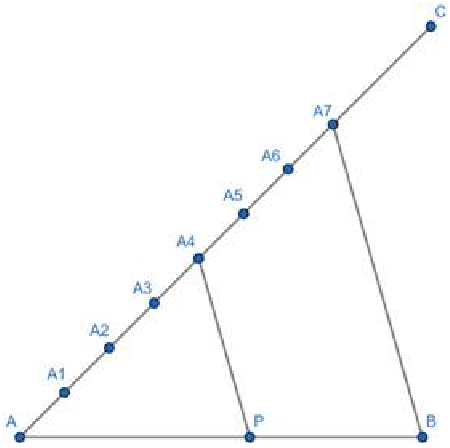
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



Step 2: We plot (4 + 3 =) 7 points A_1, A_2, A_3, A_4, A_5, A_6, and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$



Step 3: We join points A_7 and B.



Step 4: We draw line segment A_4P such that $A_4P \parallel A_7B$ and P is the point of intersection of this line segment with AB.

Point P divides AB in the ratio 4 : 3.

Justification –

In ΔAA_4P and ΔAA_7B,

iii. $\angle A$ is common.

iv. $\angle AA_4P = \angle AA_7B$ (corresponding angles $\because A_4P \parallel A_7B$)

Hence, $\Delta AA_4P \sim \Delta AA_7B$

So, ratio of lengths of corresponding sides must be equal.

$\Rightarrow \frac{AA_4}{AP} = \frac{AA_7}{AB}$

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = x$

So, the previous relation can be re – written as –

$$\frac{4x}{AP} = \frac{7x}{AP + PB}$$

$\Rightarrow 4(AP + PB) = 7AP$

$\Rightarrow 4PB = 3AP$

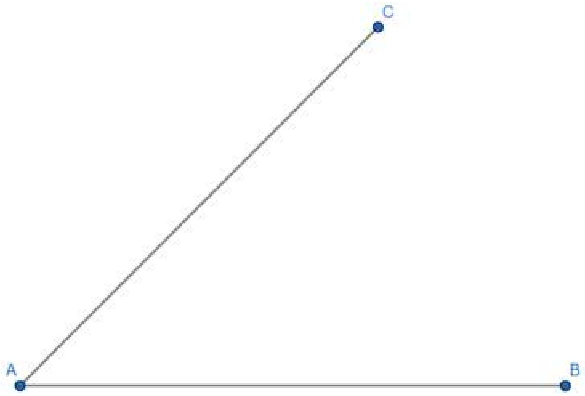
$\Rightarrow AP/PB = 4/3$, or, $AP : PB = 4 : 3$

3. Question

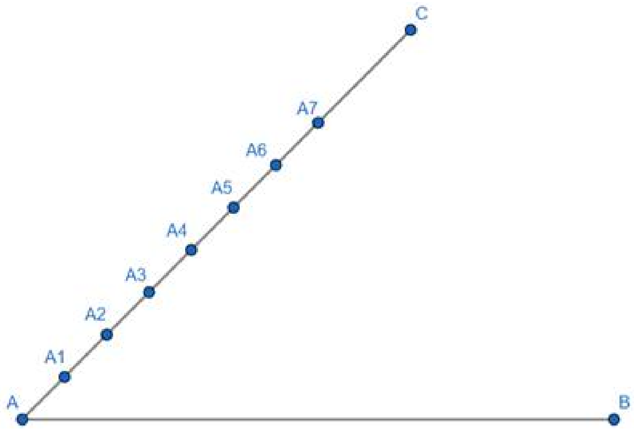
Divide a line segment of length 14 cm internally in the ratio 2 : 5. Also, justify your construction.

Answer

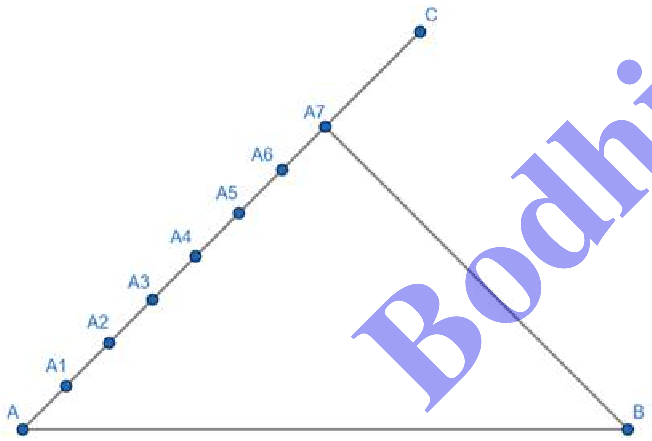
We need to divide this line segment AB of length 14 cm internally in the ratio 2 : 5.



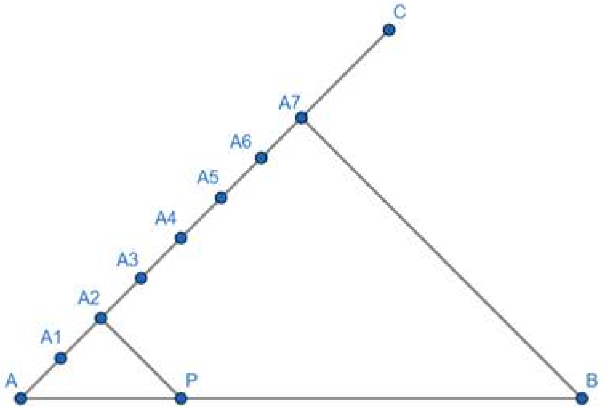
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



Step 2: We plot (2 + 5 =) 7 points A_1, A_2, A_3, A_4, A_5, A_6, and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$



Step 3: We join points A_7 and B.



Step 4: We draw line segment A_2P such that $A_2P \parallel A_7B$ and P is the point of intersection of this line segment with AB.

Point P divides AB in the ratio 2 : 5.

Justification –

In ΔAA_2P and ΔAA_7B,

v. $\angle A$ is common.

vi. $\angle AA_2P = \angle AA_7B$ (corresponding angles ∵ $A_2P \parallel A_7B$)

Hence, $\Delta AA_2P \sim \Delta AA_7B$

So, ratio of lengths of corresponding sides must be equal.

$\Rightarrow \frac{AA_2}{AP} = \frac{AA_7}{AB}$

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = x$

So, the previous relation can be re – written as –

$$\frac{2x}{AP} = \frac{7x}{AP + PB}$$

$\Rightarrow 2(AP + PB) = 7AP$

$\Rightarrow 2PB = 5AP$

$\Rightarrow AP/PB = 2/5$, or, $AP : PB = 2 : 5$

Exercise 11.2

1. Question

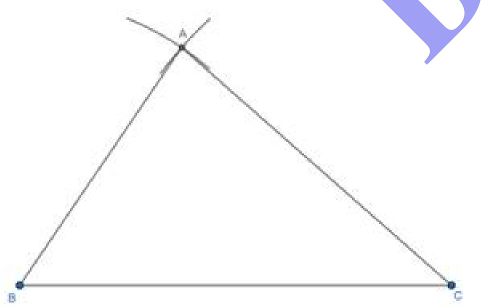
Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are (2/3) of the corresponding sides of it.

Answer

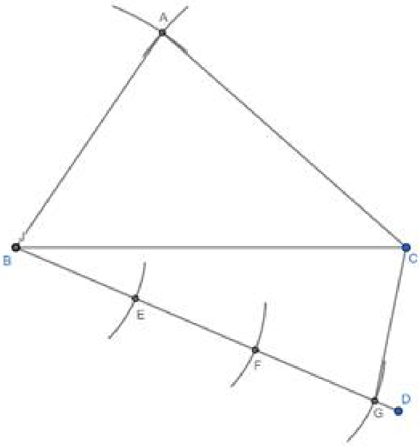
The steps involved in the required construction are:

1) Draw a line segment BC=6 cm.

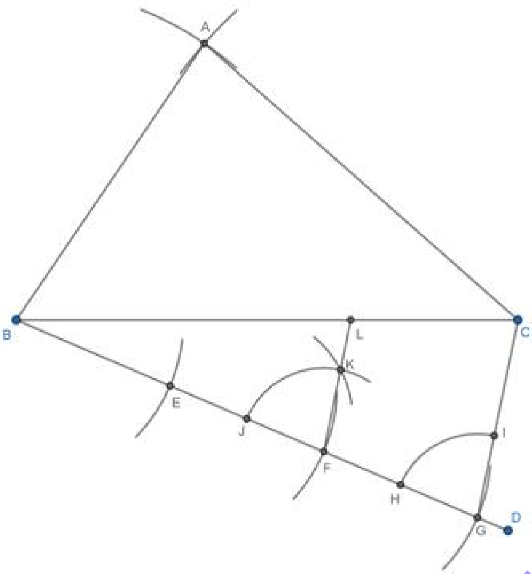
2) Taking B as the center and radius 4 cm, draw an arc. Now taking C as the center and radius 5 cm draw another arc, intersecting the previous arc at A. Join AB and AC.



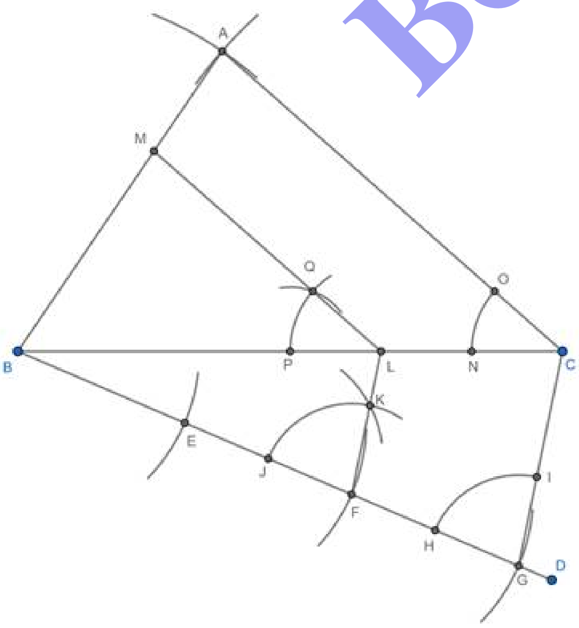
3) Draw any line segment BD, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BD at E. Taking E as the center and radius BE, draw an arc, intersecting BD at F. Taking F as the center and radius BE, draw an arc, intersecting BD at G. Join CG.



4) Taking G as the center and any radius, draw an arc., intersecting BD and CG at H and I respectively. Taking F as the center and radius GH, draw an arc., intersecting BD at J. Taking J as the center and radius HI, draw an arc, intersecting previous arc at K. Join and extend FK, intersecting BC at L.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at N and O respectively. Taking L as the center and radius CN, draw an arc., intersecting BC at P. Taking P as the center and radius NO, draw an arc, intersecting previous arc at Q. Join and extend LQ, intersecting AB at M.



6) ΔBLM is the required triangle.

2. Question

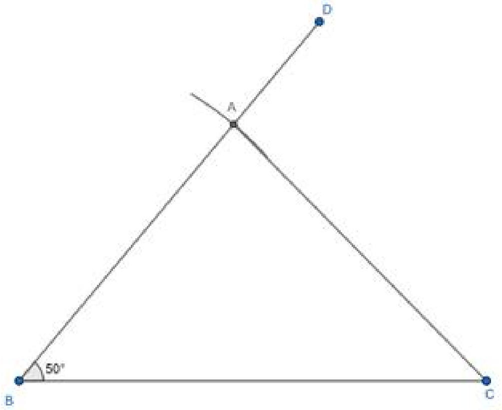
Construct a triangle similar to a given ΔABC such that each of its sides is $(5/7)^{th}$ of the corresponding sides of ΔABC. It is given that AB = 5 cm, BC = 7 cm and $\angle ABC = 50^{0}$.

Answer

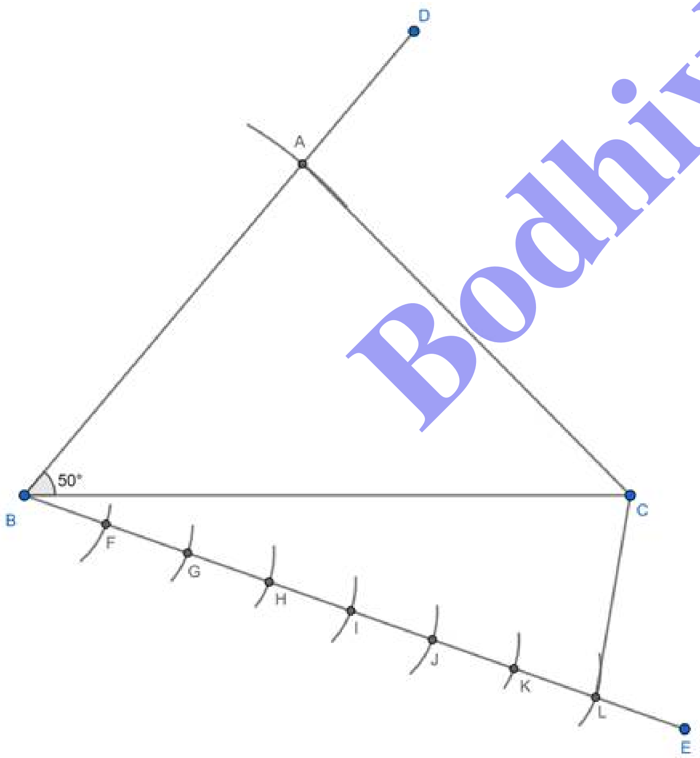
The steps involved in the required construction are:

1) Draw a line segment BC=6 cm.

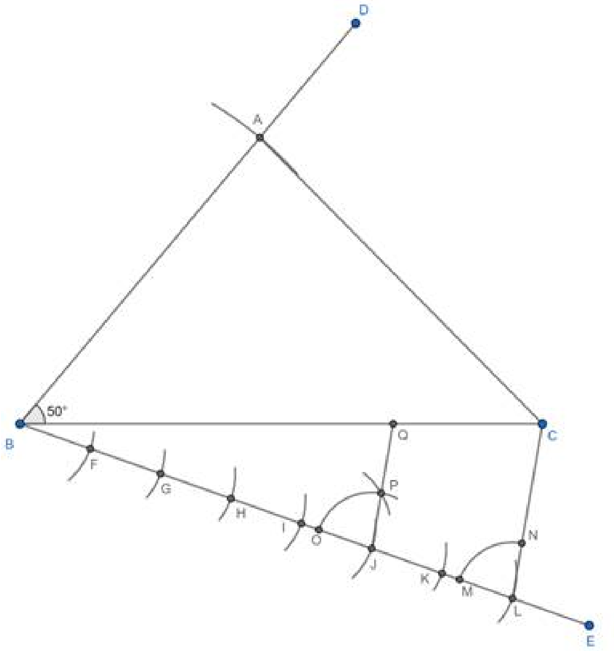
2) Using a protractor, draw ∠CBD=50°. Taking B as the center and radius 5 cm draw an arc, intersecting BD at A. Join AC.



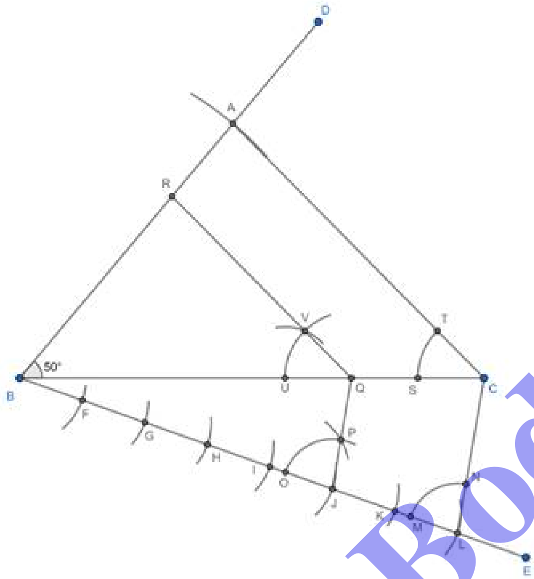
3) Draw any line segment BE, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BE at F. Taking F as the center and radius BF, draw an arc, intersecting BE at G. Similarly, repeat the process 5 more times to get points H, I, J, K and L. Join CL.



4) Taking L as the center and any radius, draw an arc., intersecting BE and CL at M and N respectively. Taking J as the center and radius LM, draw an arc., intersecting BE at O. Taking O as the center and radius MN, draw an arc, intersecting previous arc at P. Join and extend JP, intersecting BC at Q.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at S and T respectively. Taking Q as the center and radius CS, draw an arc., intersecting BC at U. Taking U as the center and radius ST, draw an arc, intersecting previous arc at V. Join and extend QV, intersecting AB at R.



6) ΔBQR is the required triangle.

3. Question

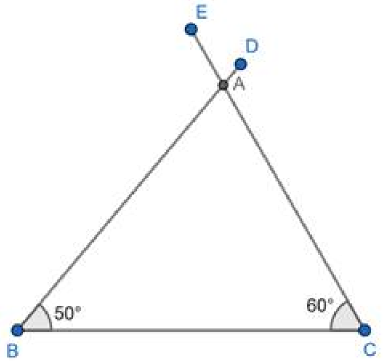
Construct a triangle similar to a given ΔABC such that each of its sides is $(2/3)^{rd}$ of the corresponding sides of ΔABC. It is given that BC = 6 cm, $\angle B = 50^0$ and $\angle C = 60^0$.

Answer

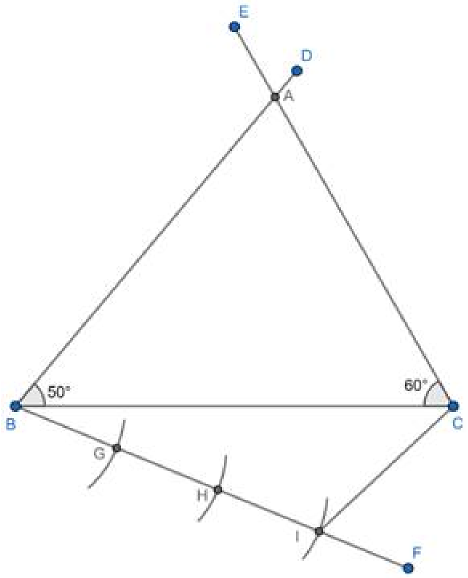
The steps involved in the required construction are:

1) Draw a line segment BC=6 cm.

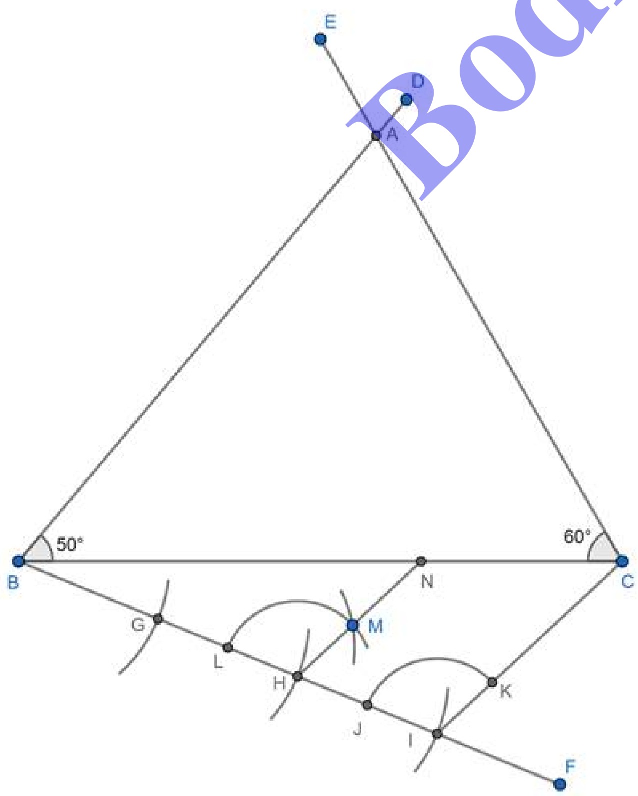
2) Using a protractor, draw ∠CBD=50° and ∠BCE=60°. BD and CE intersect at point A.



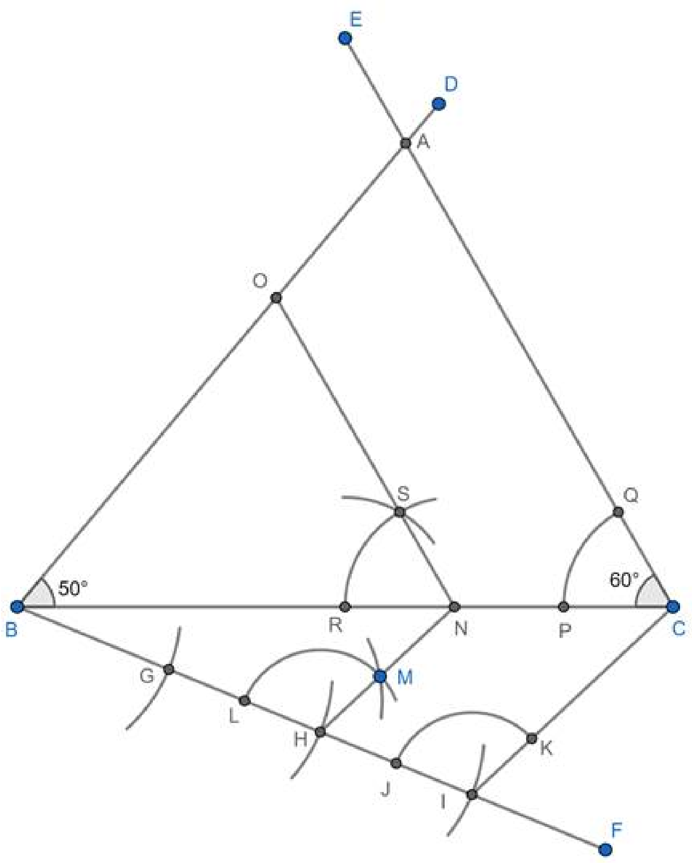
3) Draw any line segment BF, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BF at G. Taking G as the center and radius BG, draw an arc, intersecting BF at H. Taking H as the center and radius BG, draw an arc, intersecting BF at I. Join CI.



4) Taking I as the center and any radius, draw an arc., intersecting BF and CI at J and K respectively. Taking H as the center and radius IJ, draw an arc., intersecting BF at L. Taking L as the center and radius JK, draw an arc, intersecting previous arc at M. Join and extend HM, intersecting BC at N.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at P and Q respectively. Taking N as the center and radius CP, draw an arc., intersecting BC at R. Taking R as the center and radius PQ, draw an arc, intersecting previous arc at S. Join and extend NS, intersecting AB at O.



6) ΔBNO is the required triangle.

4. Question

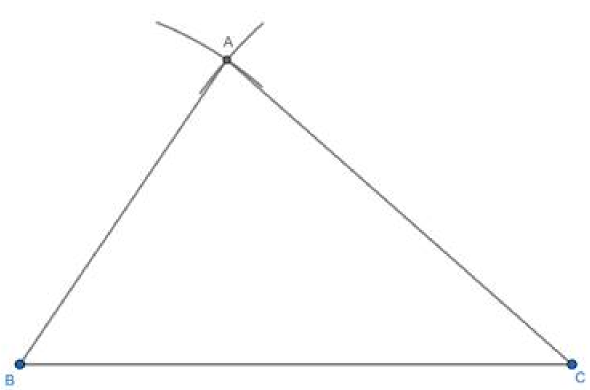
Draw a $\triangle ABC$ in which BC = 6 cm, AB = 4 cm and AC = 5 cm. Draw a triangle similar to $\triangle ABC$ with its sides equal to $(3/4)^{th}$ of the corresponding sides of $\triangle ABC$.

Answer

The steps involved in the required construction are:

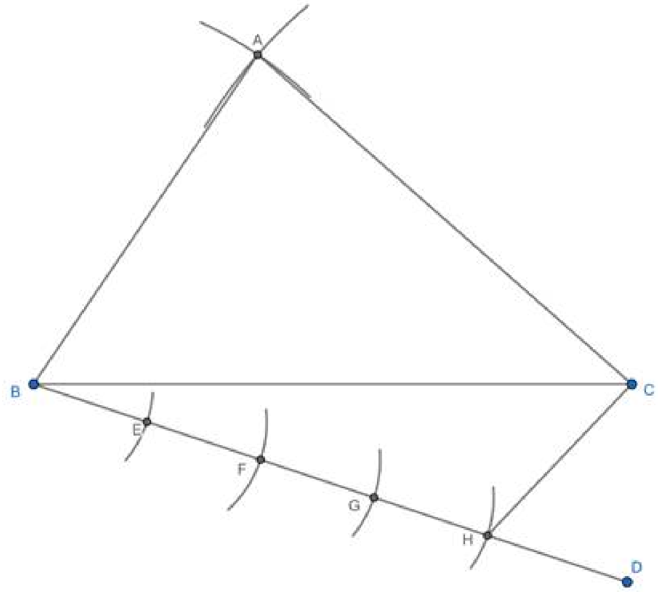
1) Draw a line segment BC=6 cm.

2) Taking B as the center and radius 4 cm, draw an arc. Now taking C as the center and radius 5 cm draw another arc, intersecting the previous arc at A. Join AB and AC.

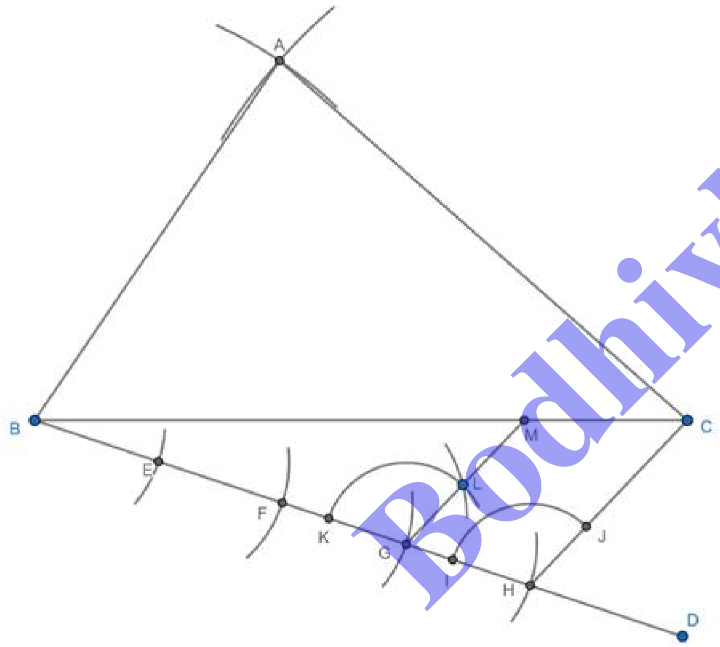


3) Draw any line segment BD, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BD at E. Taking E as the center and radius BE, draw an arc,

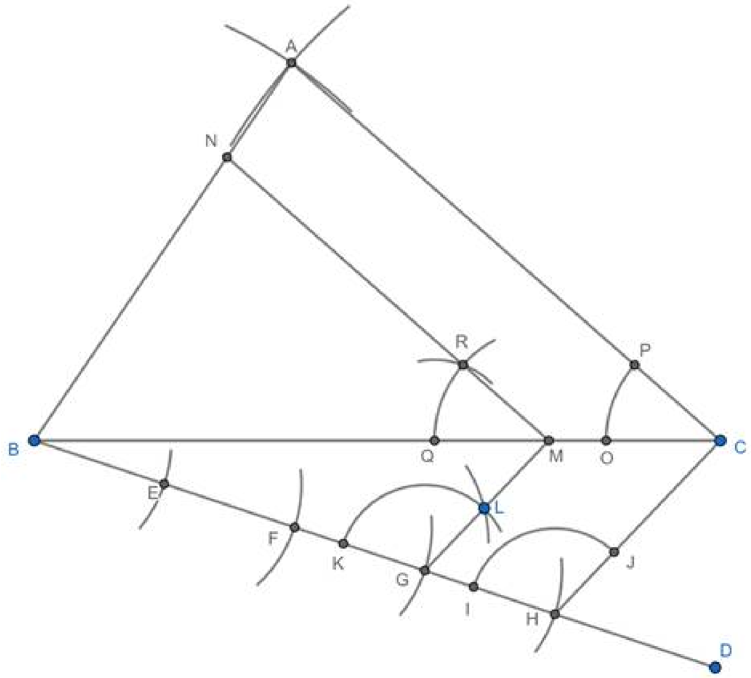
intersecting BD at F. Similarly, repeat the process 2 more times to get points G and H. Join CH.



4) Taking H as the center and any radius, draw an arc., intersecting BD and CH at I and J respectively. Taking G as the center and radius HI, draw an arc., intersecting BD at K. Taking K as the center and radius IJ, draw an arc, intersecting previous arc at L. Join and extend HL, intersecting BC at M.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at O and P respectively. Taking M as the center and radius CO, draw an arc., intersecting BC at Q. Taking Q as the center and radius OP, draw an arc, intersecting previous arc at R. Join and extend MR, intersecting AB at N.



6) ΔBMN is the required triangle.

5. Question

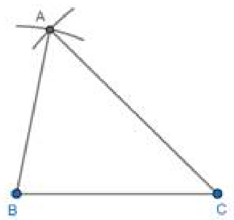
Construct a triangle with sides 5cm, 6 cm and 7 cm and then another triangle whose sides are 7/5 of the corresponding sides of the first triangle.

Answer

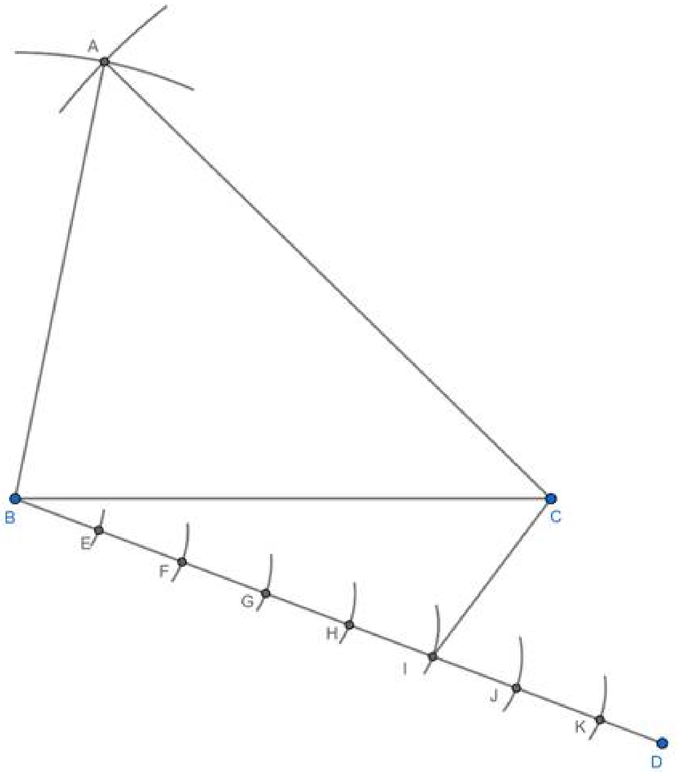
The steps involved in the required construction are:

1) Draw a line segment BC=6 cm.

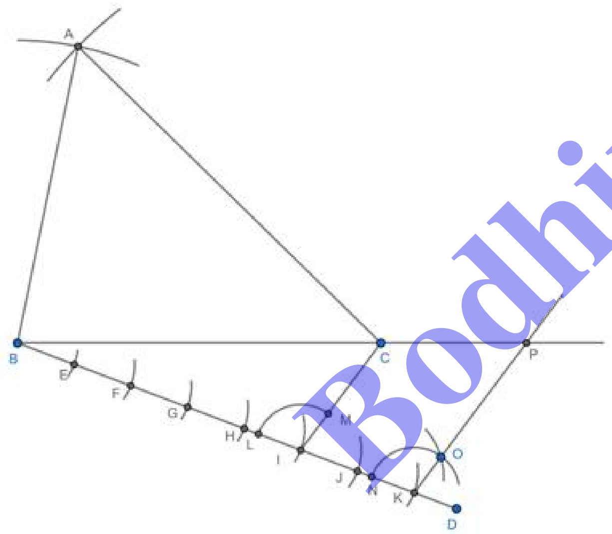
2) Taking B as the center and radius 5 cm, draw an arc. Now taking C as the center and radius 7 cm draw another arc, intersecting the previous arc at A. Join AB and AC.



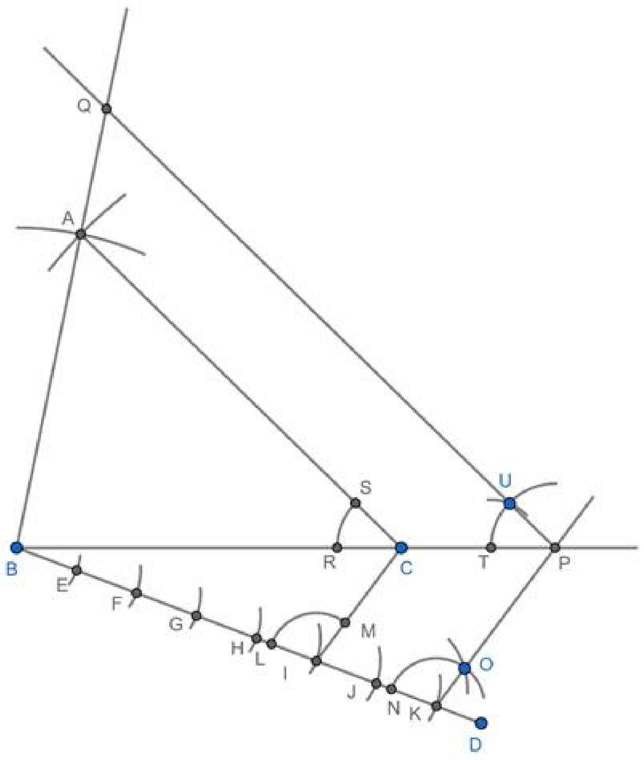
3) Draw any line segment BD, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BD at E. Taking E as the center and radius BE, draw an arc, intersecting BD at F. Similarly, repeat the process 5 more times to get points G, H, I, J and K. Join CI.



4) Taking I as the center and any radius, draw an arc., intersecting BD and CI at L and M respectively. Taking K as the center and radius IL, draw an arc., intersecting BD at N. Taking N as the center and radius LM, draw an arc, intersecting previous arc at O. Join and extend KO, intersecting extended BC at P.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at R and S respectively. Taking P as the center and radius CR, draw an arc., intersecting BP at T. Taking T as the center and radius RS, draw an arc, intersecting previous arc at U. Join and extend PU, intersecting extended AB at Q.



6) ΔBPQ is the required triangle.

6. Question

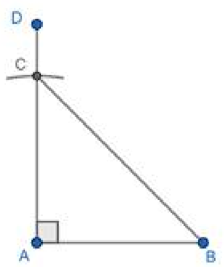
Draw a right triangle ABC in which AC = AB = 4.5 cm and $\angle A = 90^0$. Draw a triangle similar to $\triangle ABC$ with its sides equal to $(5/4)$th of the corresponding sides of $\triangle ABC$.

Answer

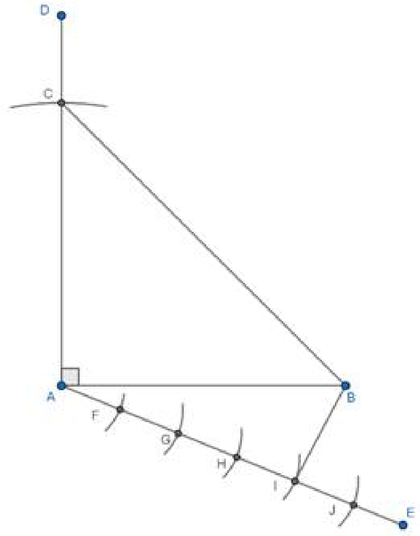
The steps involved in the required construction are:

1) Draw a line segment AB=4.5 cm.

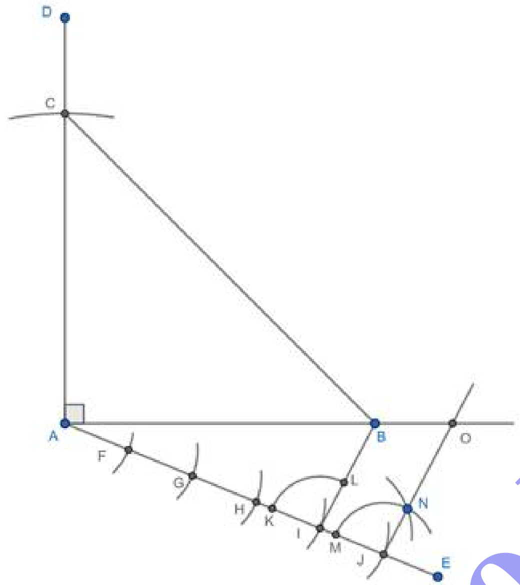
2) Using a protractor, draw ∠BAD=90°. Taking A as the center and radius 4.5 cm, draw an arc, intersecting AD at C. Join BC.



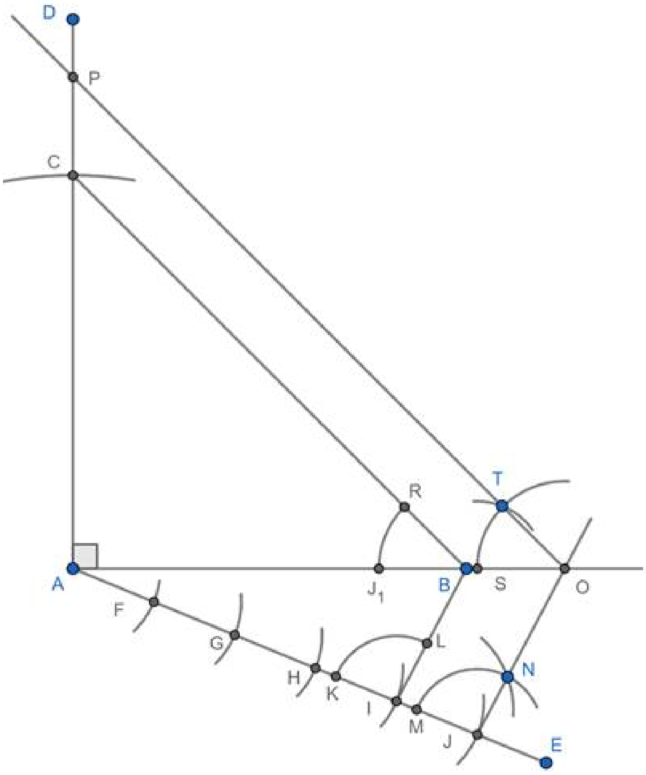
3) Draw any line segment AE, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AE at F. Taking F as the center and radius AF, draw an arc, intersecting AE at G. Similarly, repeat the process 3 more times to get points H, I and J. Join BI.



4) Taking I as the center and any radius, draw an arc., intersecting AE and BI at K and L respectively. Taking J as the center and radius IK, draw an arc., intersecting AE at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend JN, intersecting extended AB at O.



5) Taking B as the center and any radius, draw an arc., intersecting BA and BC at Q and R respectively. Taking O as the center and radius BQ, draw an arc., intersecting AO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting AD at P.



6) ΔAOP is the required triangle.

7. Question

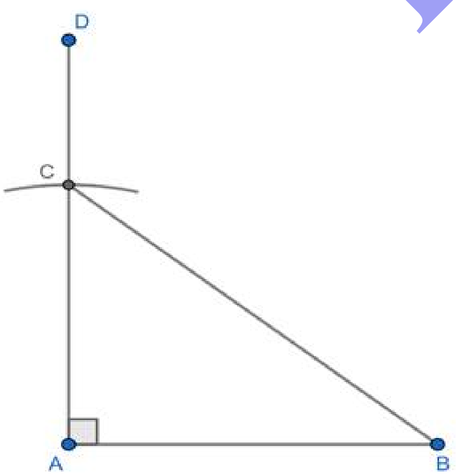
Draw a right triangle in which the sides (other than hypotenuse) are of lengths 5 cm and 4 cm. Then construct another triangle whose sides are 5/3 times the corresponding sides of the given triangle.

Answer

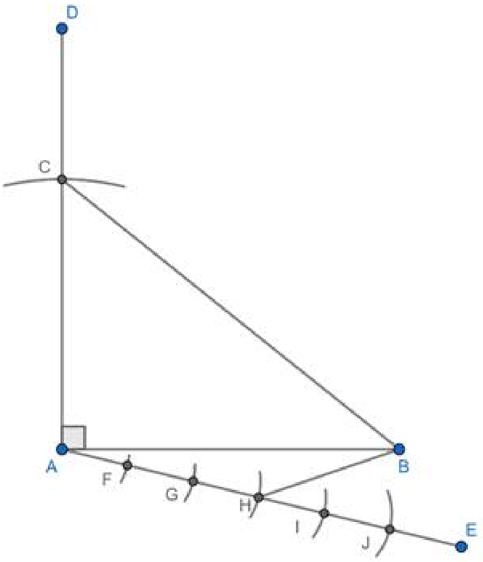
The steps involved in the required construction are:

1) Draw a line segment AB=5 cm.

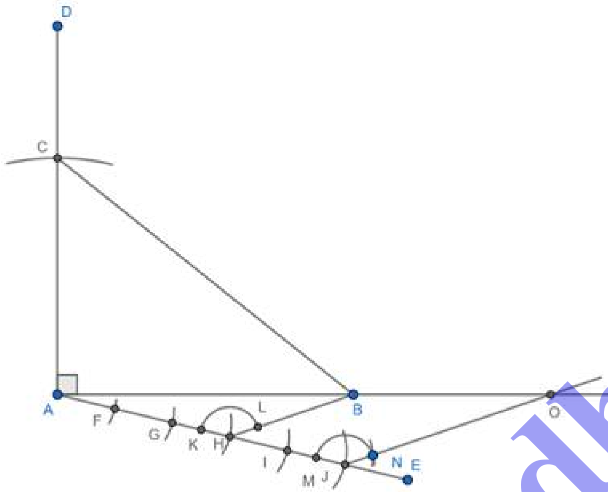
2) Using a protractor, draw ∠BAD=90°. Taking A as the center and radius 4 cm, draw an arc, intersecting AD at C. Join BC.



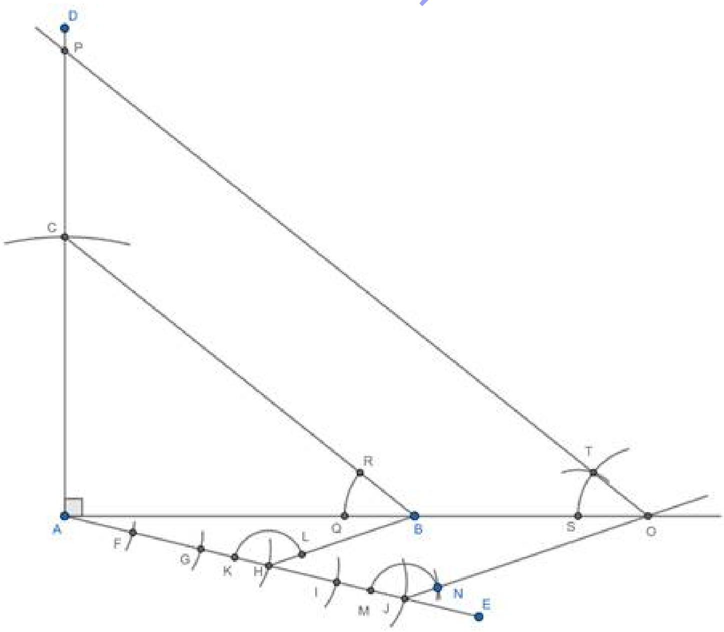
3) Draw any line segment AE, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AE at F. Taking F as the center and radius AF, draw an arc, intersecting AE at G. Similarly, repeat the process 3 more times to get points H, I and J. Join BH.



4) Taking H as the center and any radius, draw an arc., intersecting AE and BH at K and L respectively. Taking J as the center and radius HK, draw an arc, intersecting AE at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend JN, intersecting extended AB at O.



5) Taking B as the center and any radius, draw an arc., intersecting BA and BC at Q and R respectively. Taking O as the center and radius BQ, draw an arc., intersecting AO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting AD at P.



6) ΔAOP is the required triangle.

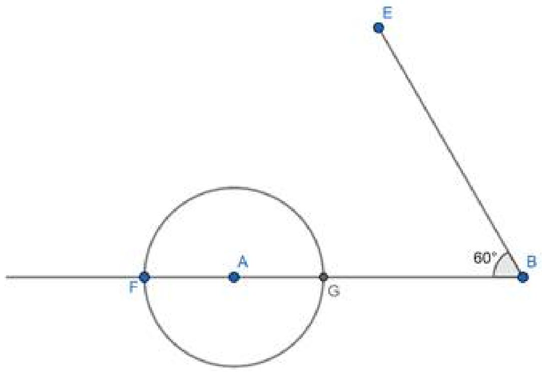
8. Question

Construct a $\triangle ABC$ in which AB = 5 cm. $\angle B = 60^{o}$ altitude CD = 3 cm. Construct a $\triangle AQR$ similar to $\triangle ABC$ such that side of $\triangle AQR$ is 1.5 times that of the corresponding sides of $\triangle ACB$.

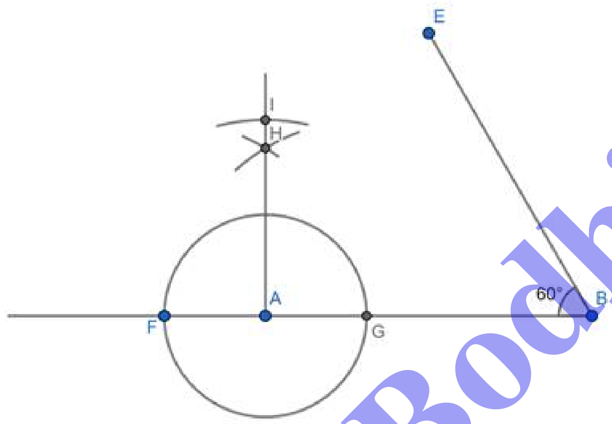
Answer

The steps involved in the required construction are:

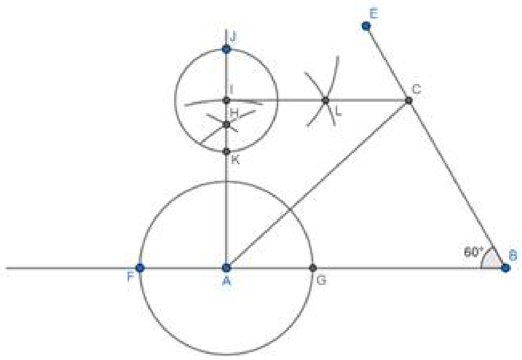
1) Draw a line segment AB= 5 cm. Using a protractor, draw ∠ABE=60°. Keeping A as the center, draw a circle of any radius, intersecting extended AB at F and G.



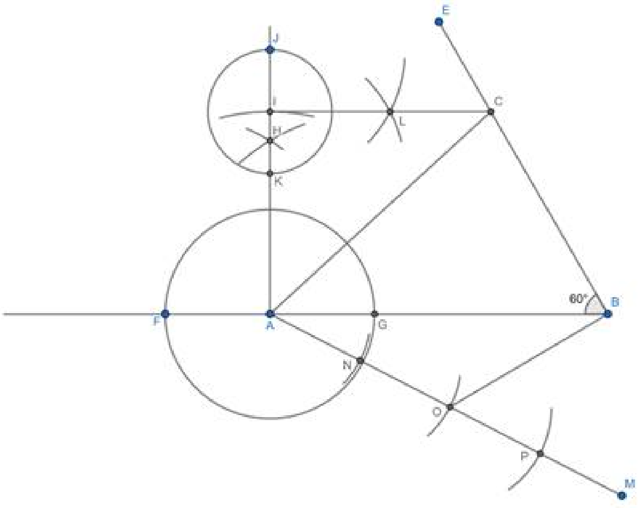
2) Keeping F as the center and any radius draw a circular arc. Now, Keeping G as the center and same radius as before, draw another arc, intersecting previous arc at H. Join and extend AH. Keeping A as the center and radius 3 cm, draw an arc, intersecting extended AH at I.



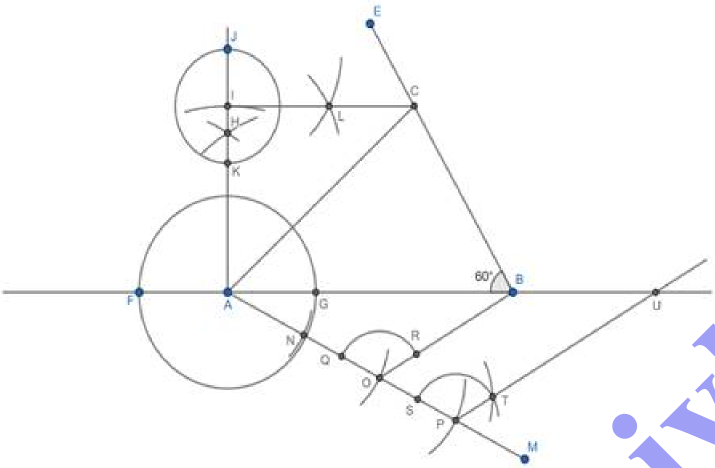
3) Keeping I as the center, draw a circle of any radius, intersecting extended AH at J and K. Keeping J as the center and any radius draw a circular arc. Now, Keeping K as the center and same radius as before, draw another arc, intersecting previous arc at L. Join and extend IL, intersecting BE at C. Join AC.



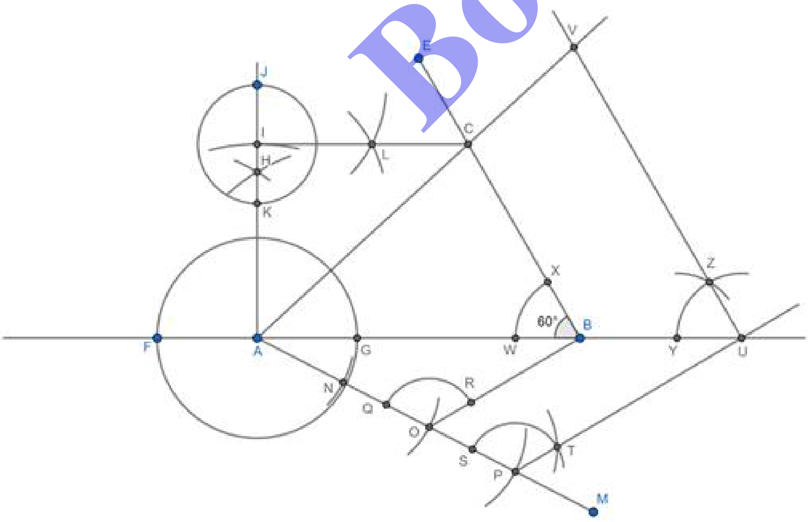
4) Draw any line segment AM, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AM at N. Taking N as the center and radius AN, draw an arc, intersecting AM at O. Taking O as the center and radius AN, draw an arc, intersecting AM at P. Join BO.



5) Taking O as the center and any radius, draw an arc., intersecting AM and BO at Q and R respectively. Taking P as the center and radius OQ, draw an arc, intersecting AM at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend PT, intersecting extended AB at U.



6) Taking B as the center and any radius, draw an arc., intersecting BA and BC at W and X respectively. Taking U as the center and radius BW, draw an arc., intersecting AU at Y. Taking Y as the center and radius WX, draw an arc, intersecting previous arc at Z. Join and extend UZ, intersecting extended AC at V.



7) ΔAUV is the required triangle.

9. Question

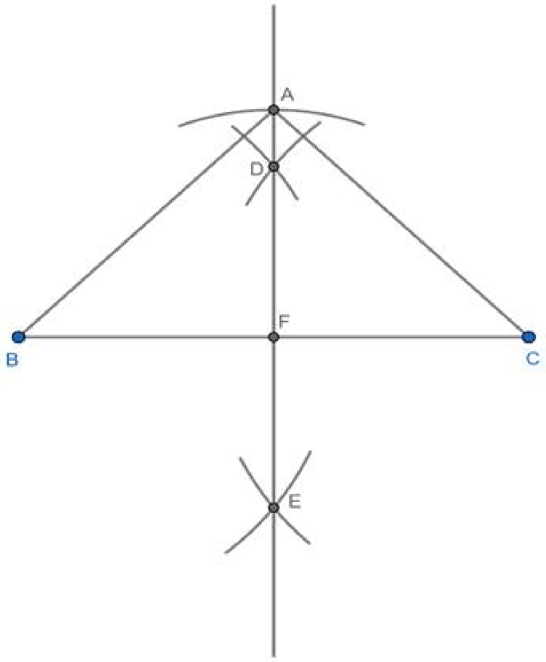
Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are 3/2 times the corresponding sides of the isosceles triangle.

Answer

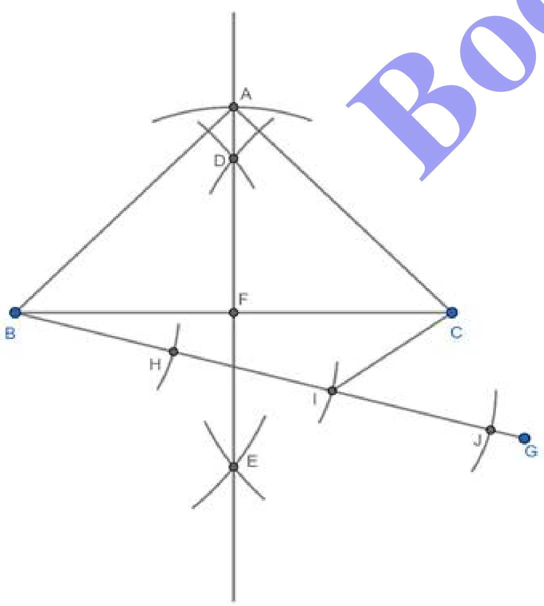
The steps involved in the required construction are:

1) Draw a line segment BC= 8 cm.

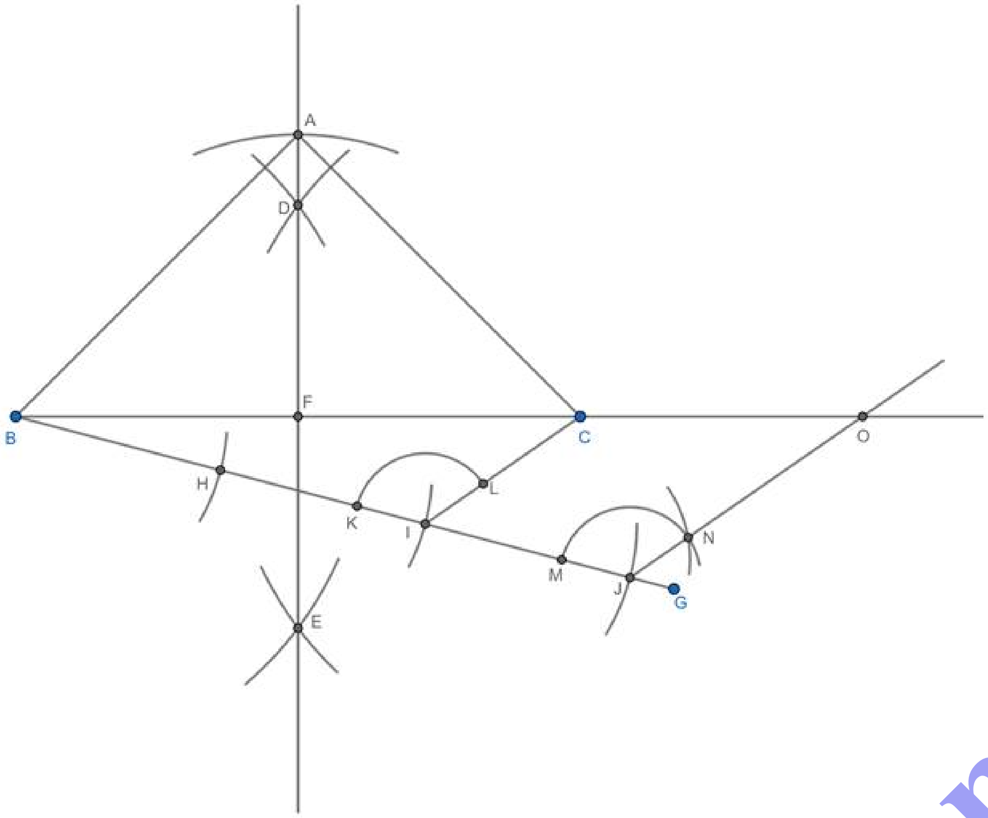
2) Taking B as the center and any radius greater than $\frac{BC}{2}$, draw two arcs on each side of BC. Taking C as the center and same radius, draw 2 more arcs, intersecting previous arcs at D and E. DE, intersecting BC at F. Taking F as the center and radius 4 cm, draw an arc, intersecting FD at A. Join AB and AC.



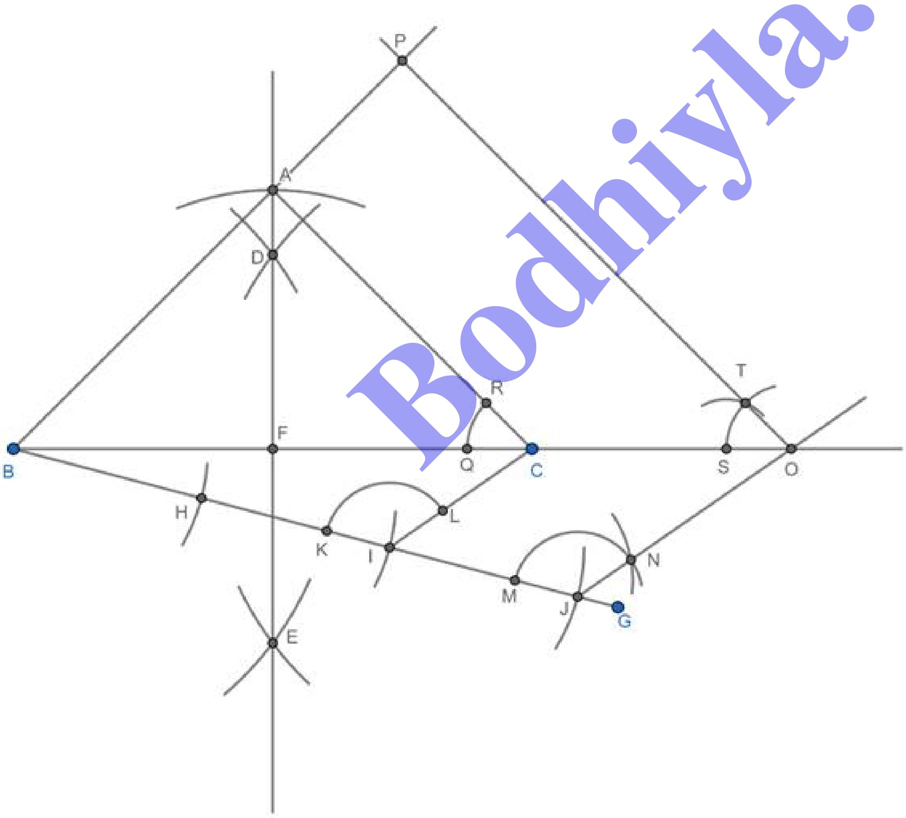
3) Draw any line segment BG, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BG at H. Taking H as the center and radius BH, draw an arc, intersecting BG at I. Taking I as the center and radius BH, draw an arc, intersecting BG at J. Join CI.



4) Taking I as the center and any radius, draw an arc., intersecting BG and CI at K and L respectively. Taking J as the center and radius IK, draw an arc, intersecting BG at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend JN, intersecting extended BC at O.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at Q and R respectively. Taking O as the center and radius CQ, draw an arc., intersecting BO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting extended AB at P.



6) ΔBOP is the required triangle.

10. Question

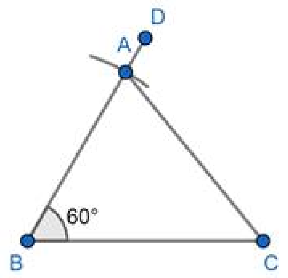
Draw a ΔABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^0$. Then, construct a triangle whose sides are $(3 / 4)^{th}$ of the corresponding sides of the ΔABC.

Answer

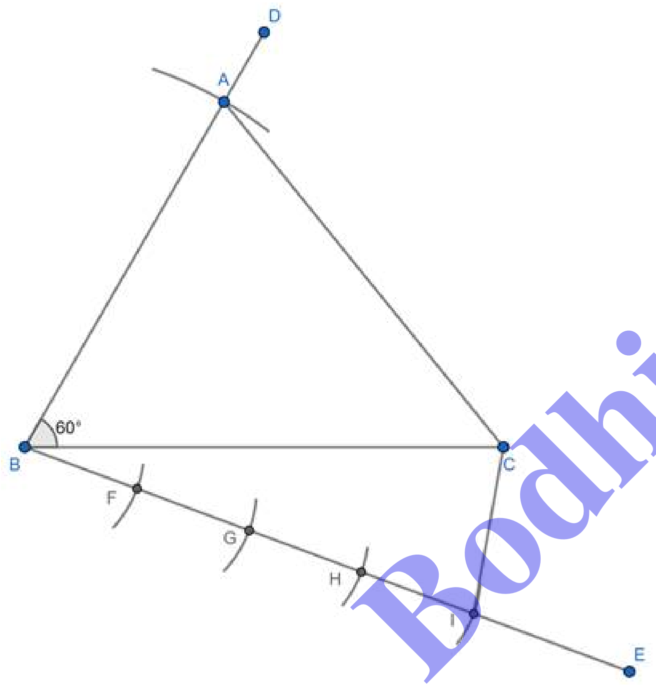
The steps involved in the required construction are:

1) Draw a line segment BC= 6 cm.

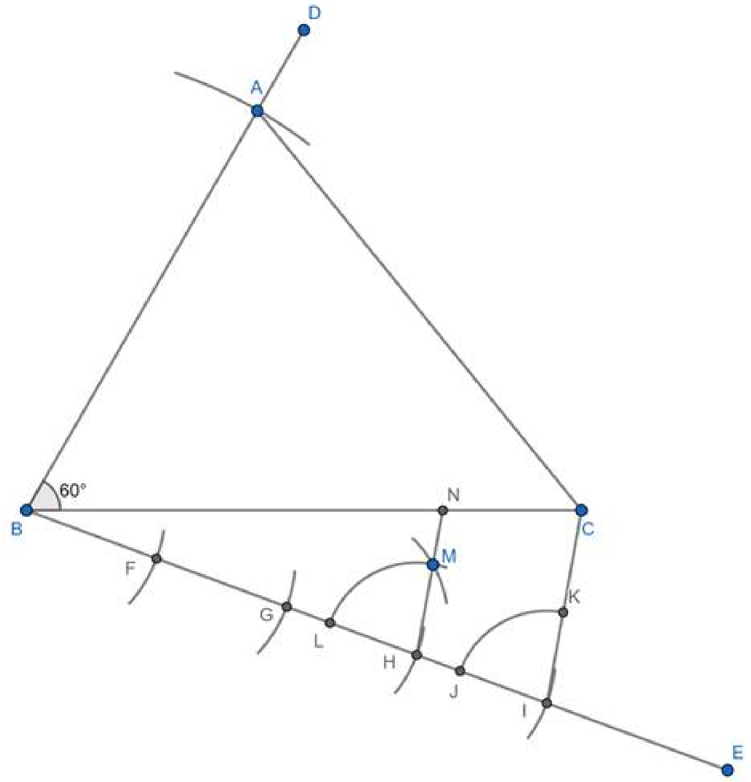
2) Using a protractor, draw ∠CBD=60°. Taking B as the center and radius 5 cm draw an arc, intersecting BD at A. Join AC.



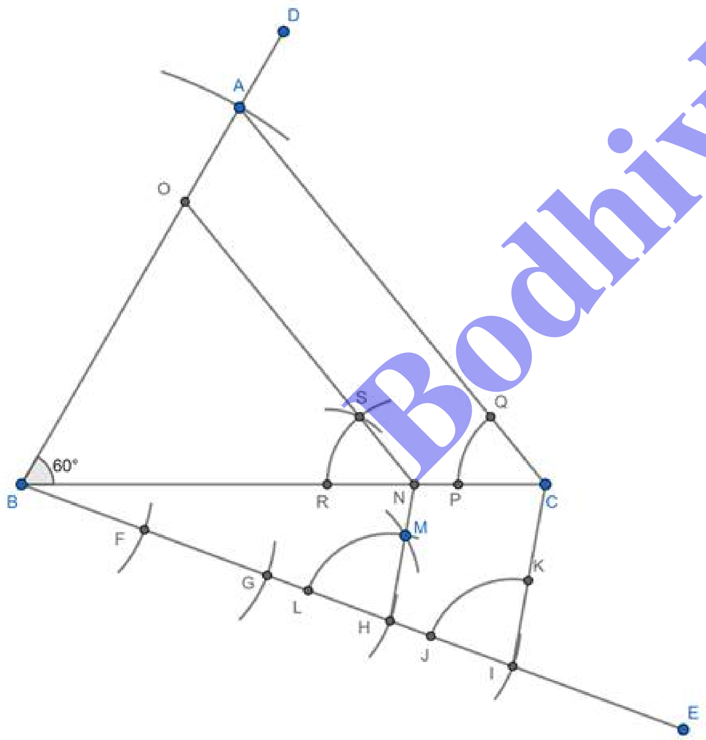
3) Draw any line segment BE, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BE at F. Taking F as the center and radius BF, draw an arc, intersecting BE at G. Similarly, repeat the process 2 more times to get points H and I. Join CI.



4)Taking I as the center and any radius, draw an arc., intersecting BD and CI at J and K respectively. Taking H as the center and radius IJ, draw an arc., intersecting BD at L. Taking L as the center and radius JK, draw an arc, intersecting previous arc at M. Join and extend HM, intersecting BC at N.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at P and Q respectively. Taking N as the center and radius CP, draw an arc., intersecting BC at R. Taking R as the center and radius PQ, draw an arc, intersecting previous arc at S. Join and extend NS, intersecting AB at O.



6) ΔBNO is the required triangle.

11. Question

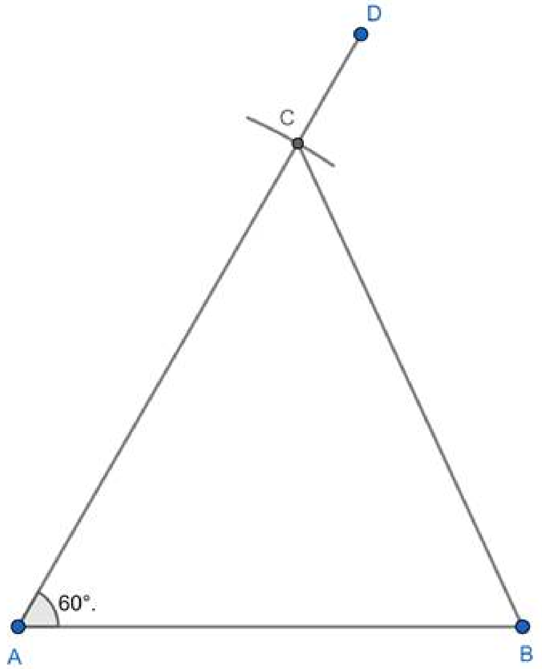
Construct a triangle similar to $\triangle ABC$ in which AB = 4.6 cm, BC = 5.1 cm, $\angle A = 60^0$ with scale factor 4 : 5.

Answer

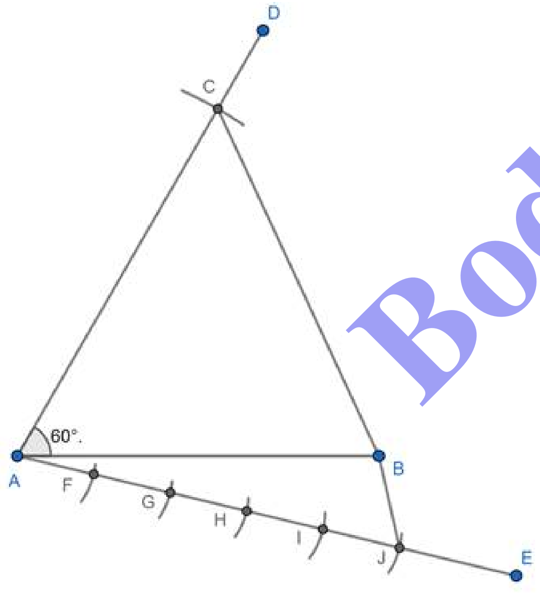
The steps involved in the required construction are:

1) Draw a line segment AB=4.6 cm.

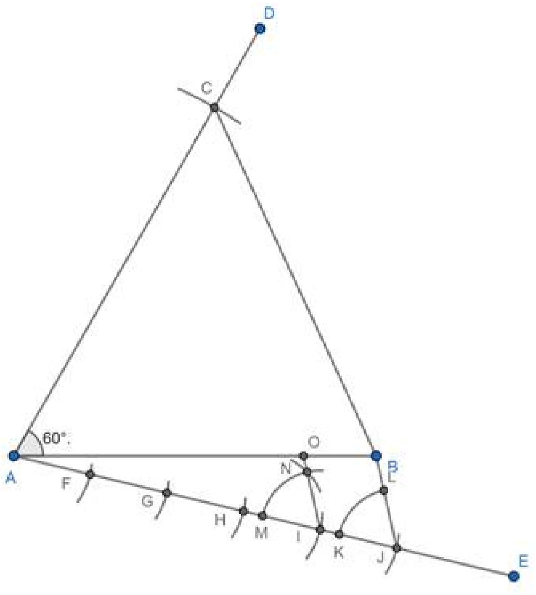
2) Using a protractor, draw ∠BAD=60°. Taking B as the center and radius 5.1 cm, draw an arc, intersecting AD at C. Join BC.



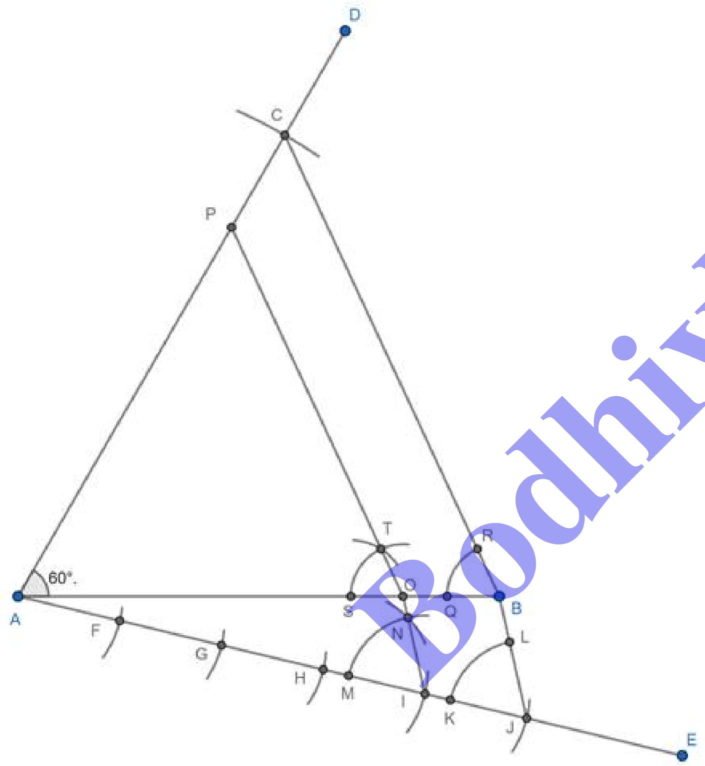
3) Draw any line segment AE, making an acute angle with AB and opposite to the vertex C. Taking A as the center and any radius, draw an arc, intersecting AE at F. Taking F as the center and radius AF, draw an arc, intersecting AE at G. Similarly, repeat the process 3 more times to get points H, I and J. Join BJ.



4) Taking J as the center and any radius, draw an arc., intersecting AE and BJ at K and L respectively. Taking I as the center and radius JK, draw an arc., intersecting AE at M. Taking M as the center and radius KL, draw an arc, intersecting previous arc at N. Join and extend IN, intersecting AB at O.



5) Taking B as the center and any radius, draw an arc., intersecting BA and BC at Q and R respectively. Taking O as the center and radius BQ, draw an arc., intersecting AO at S. Taking S as the center and radius QR, draw an arc, intersecting previous arc at T. Join and extend OT, intersecting AC at P.



6) ΔAOP is the required triangle.

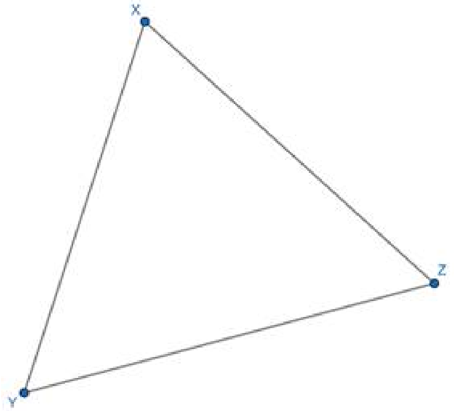
12. Question

Construct a triangle similar to a given ΔXYZ with its sides equal to $(3 / 4)^{th}$ of the corresponding sides of ΔXYZ. Write the steps of construction.

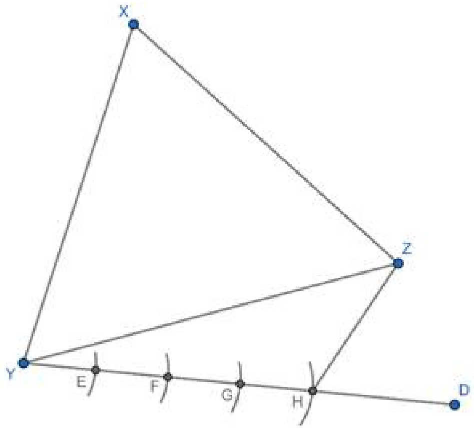
Answer

The steps involved in the required construction are:

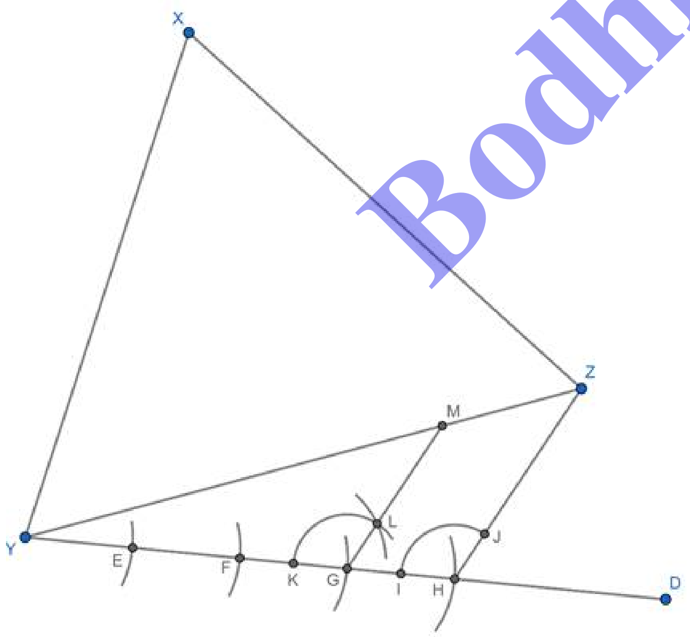
1) Draw any random triangle ΔXYZ.



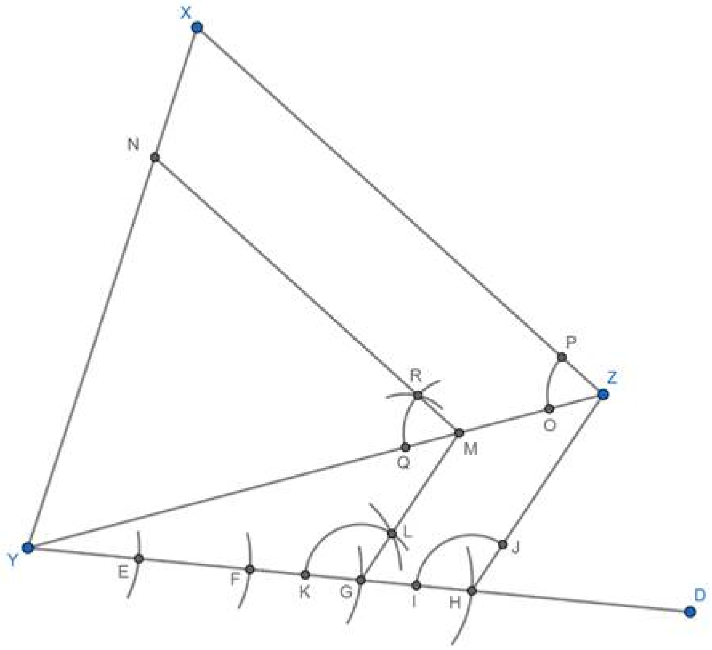
2) Draw any line segment YD, making an acute angle with YZ and opposite to the vertex X. Taking Y as the center and any radius, draw an arc, intersecting YD at E. Taking E as the center and radius YE, draw an arc, intersecting YD at F. Similarly, repeat the process 2 more times to get points G and H. Join ZH.



3) Taking H as the center and any radius, draw an arc., intersecting YD and ZH at I and J respectively. Taking G as the center and radius HI, draw an arc., intersecting YD at K. Taking K as the center and radius IJ, draw an arc, intersecting previous arc at L. Join and extend HL, intersecting YZ at M.



4) Taking Z as the center and any radius, draw an arc., intersecting YZ and ZA at O and P respectively. Taking M as the center and radius ZO, draw an arc., intersecting YZ at Q. Taking Q as the center and radius OP, draw an arc, intersecting previous arc at R. Join and extend MR, intersecting XY at N.



5) ΔYMN is the required triangle.

13. Question

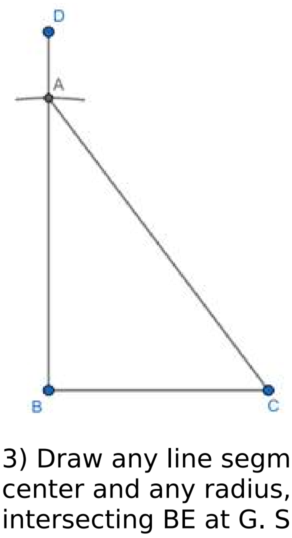
Draw a right triangle in which sides (other than the hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are 3/4 times the corresponding sides of the first triangle.

Answer

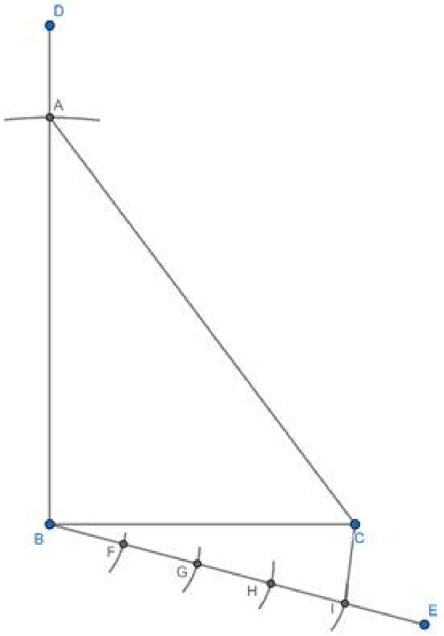
The steps involved in the required construction are:

1) Draw a line segment BC= 6 cm.

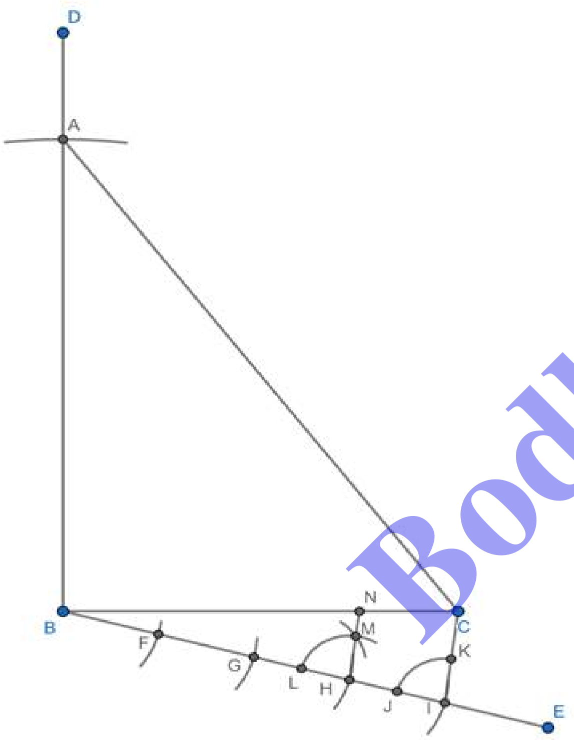
2) Using a protractor, draw ∠CBD=90°. Taking B as the center and radius 8 cm draw an arc, intersecting BD at A. Join AC.



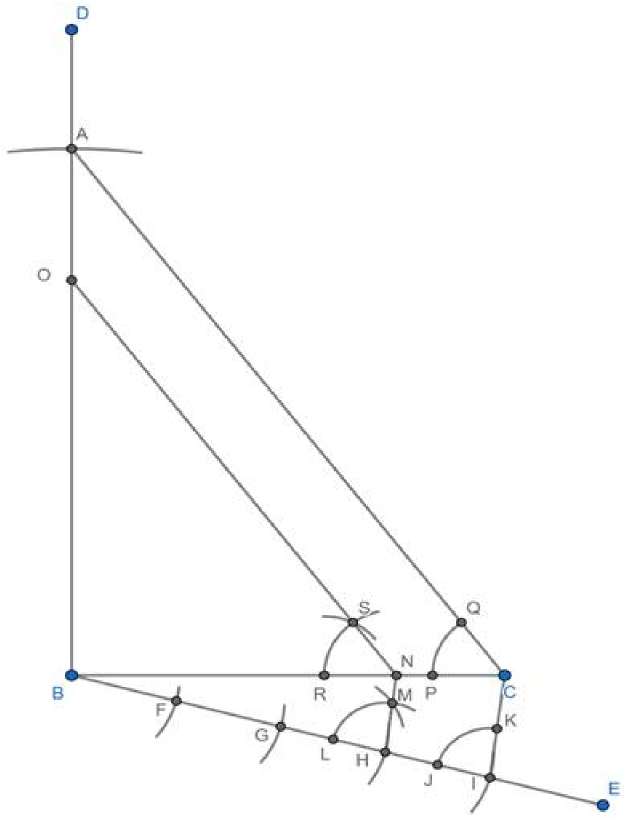
3) Draw any line segment BE, making an acute angle with BC and opposite to the vertex A. Taking B as the center and any radius, draw an arc, intersecting BE at F. Taking F as the center and radius BF, draw an arc, intersecting BE at G. Similarly, repeat the process 2 more times to get points H and I. Join CI.



4)Taking I as the center and any radius, draw an arc., intersecting BD and CI at J and K respectively. Taking H as the center and radius IJ, draw an arc., intersecting BD at L. Taking L as the center and radius JK, draw an arc, intersecting previous arc at M. Join and extend HM, intersecting BC at N.



5) Taking C as the center and any radius, draw an arc., intersecting BC and CA at P and Q respectively. Taking N as the center and radius CP, draw an arc., intersecting BC at R. Taking R as the center and radius PQ, draw an arc, intersecting previous arc at S. Join and extend NS, intersecting AB at O.



6) ΔBNO is the required triangle.

14. Question

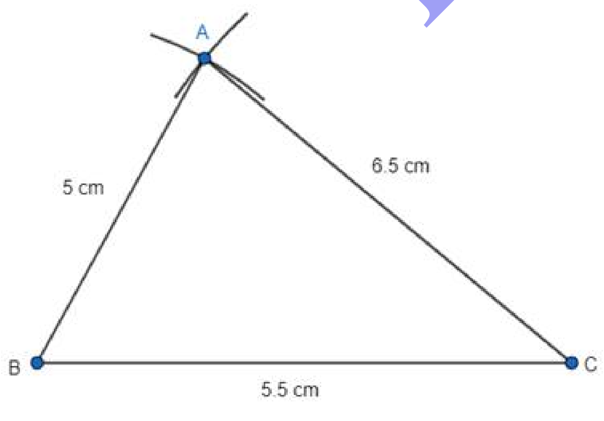
Construct a triangle with sides 5 cm, 5.5 cm and 6.5 cm. Now construct another triangle, whose sides are 3/5 times the corresponding sides of the given triangle.

Answer

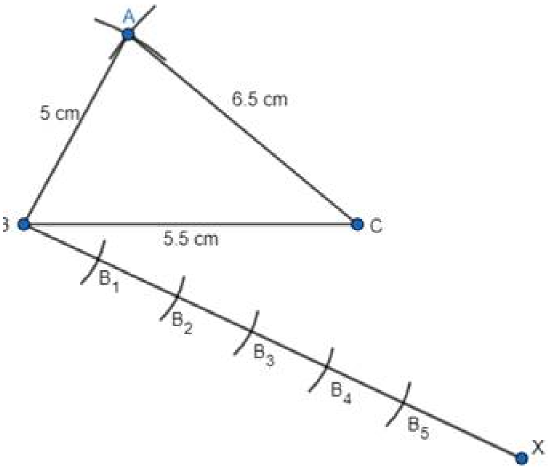
Step 1. At first drawn a base line BC of length 5.5 cm with the help of scale.

Step 2. Taking B as center draw an arc of radius 5 cm with the help of compass. Similarly taking C as center draw a arc of radius 6.5 cm with the help of compass.

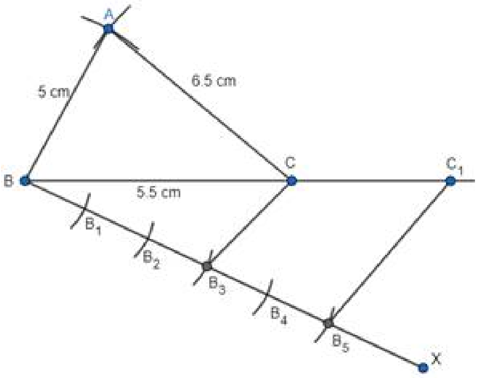
Join AB and AC thus completing the triangle ABC.



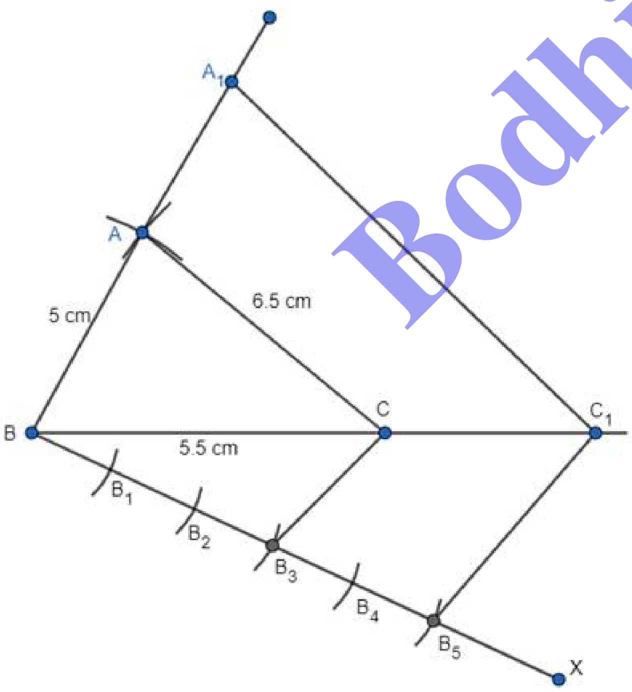
Step 3. A ray BX is drawn making an acute angle with BC opposite to vertex A. Five points B_1 , B_2 , B_3, B_4, and B_5 at equal distance is marked on BX.



Step 4. B_3 is joined with C to form B_3 C as 3 point is smaller. B_5 C_1 is drawn parallel to B_3 C as 5 point is greater.



Step 5:$C_1A_1$is drawn parallel to CA.



Thus, A_1BC_1 is the required triangle.

Justification:

Since the scale factor is $\frac{3}{5}$,

We need to prove,

$$\frac{A_1B}{AB} = \frac{A_1C_1}{AC} = \frac{BC_1}{BC} = \frac{3}{5}$$

By construction,

$\frac{BC_1}{BC} = \frac{BB_5}{BB_3} = \frac{3}{5}$ … (1)

Also, A_1C_1 is parallel to AC.

So, this will make same angle with BC.

∴ $\angle A_1C_1B = \angle ACB$ …. (2)

Now,

In ΔA_1BC_1 and ΔABC

∠ B = ∠ B (common)

$\angle A_1C_1B = \angle ACB$ (from 2)

$\Delta A_1BC_1 \sim \Delta ABC$

Since corresponding sides of similar triangles are in same ratio.

$$\frac{A_1B}{AB} = \frac{A_1C_1}{AC} = \frac{BC_1}{BC}$$

From (1)

$$\frac{A_1B}{AB} = \frac{A_1C_1}{AC} = \frac{BC_1}{BC} = \frac{3}{5}$$

Hence construction is justified.

15. Question

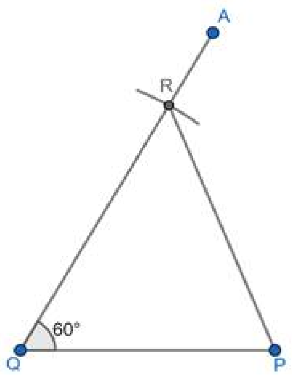
Construct a triangle PQR with sides QR = 7 cm, PQ = 6 cm and $\angle PQR = 60^0$. Then construct another triangle whose sides are 3/5 of the corresponding sides of ΔPQR.

Answer

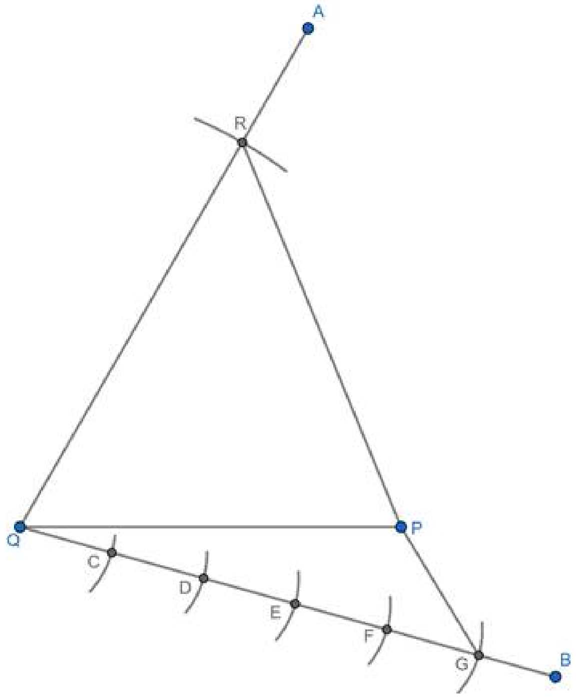
The steps involved in the required construction are:

1) Draw a line segment PQ=6 cm.

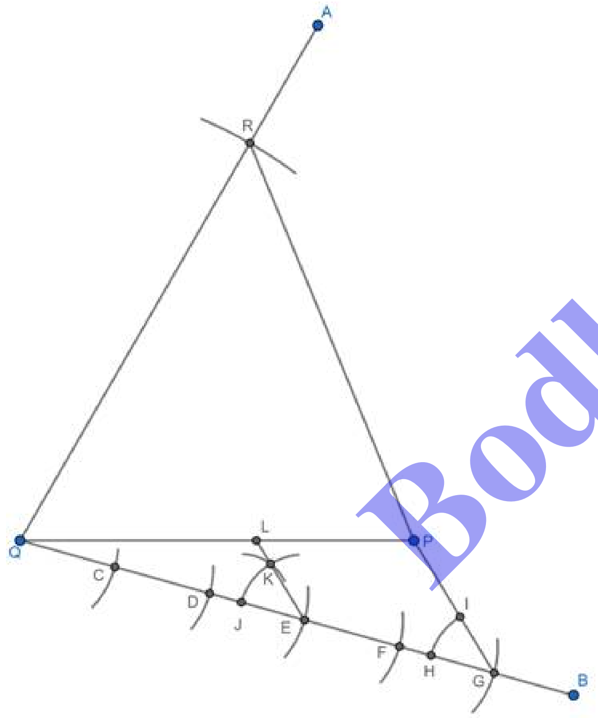
2) Using a protractor, draw ∠PQA=60°. Taking Q as the center and radius 7 cm, draw an arc, intersecting QA at R. Join PR.



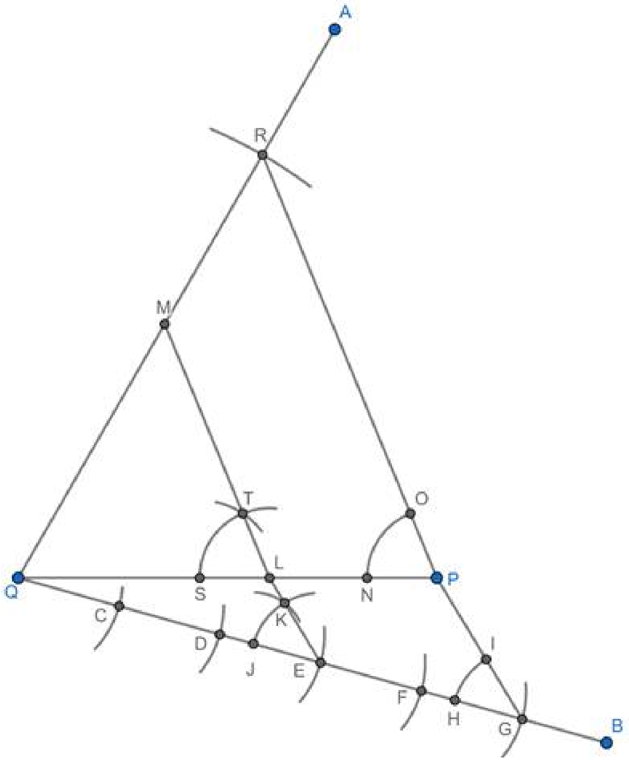
3) Draw any line segment QB, making an acute angle with PQ and opposite to the vertex R. Taking Q as the center and any radius, draw an arc, intersecting QB at C. Taking C as the center and radius QC, draw an arc, intersecting QB at D. Similarly, repeat the process 3 more times to get points E, F and G. Join PG.



4) Taking G as the center and any radius, draw an arc., intersecting QB and PG at H and I respectively. Taking E as the center and radius GH, draw an arc, intersecting QB at J. Taking J as the center and radius HI, draw an arc, intersecting previous arc at K. Join and extend EK, intersecting extended PQ at L.



5) Taking P as the center and any radius, draw an arc., intersecting PQ and PR at N and O respectively. Taking L as the center and radius PN, draw an arc., intersecting PQ at S. Taking S as the center and radius NO, draw an arc, intersecting previous arc at T. Join and extend LT, intersecting QR at M.



6) ΔQLM is the required triangle.

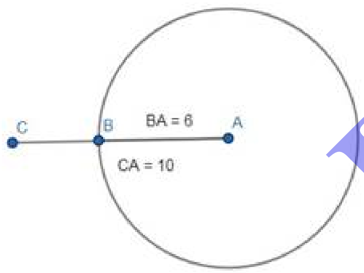
Exercise 11.3

1. Question

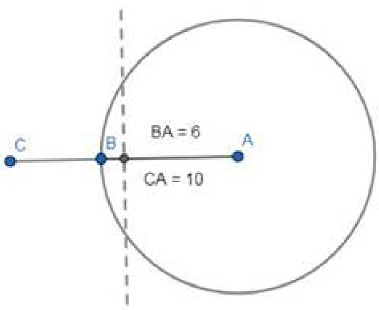
Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Answer

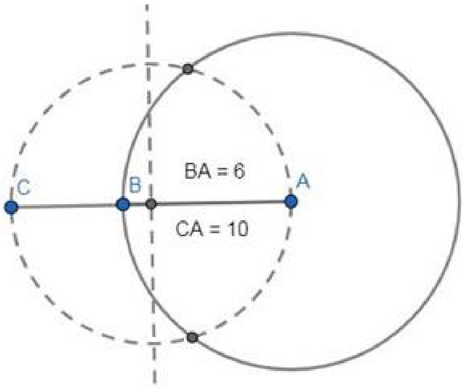
Step1: Draw circle of radius 6cm with center A, mark point C at 10 cm from center



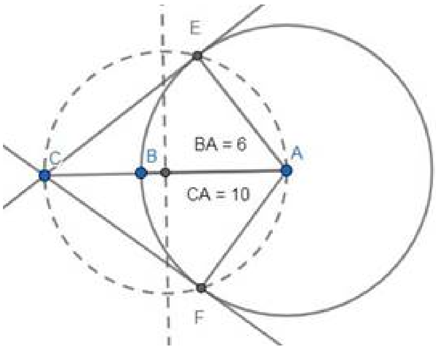
Step 2: find perpendicular bisector of AC



Step3: Take this point as center and draw a circle through A and C



Step4:Mark the point where this circle intersects our circle and draw tangents through C



Length of tangents = 8cm

AE is perpendicular to CE (tangent and radius relation)

In ΔACE

AC becomes hypotenuse

$AC^2 = CE^2 + AE^2$

$10^2 = CE^2 + 6^2$

$CE^2 = 100-36$

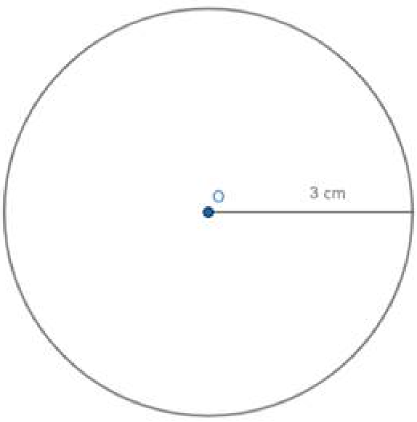
$CE^2 = 64$

CE = 8cm

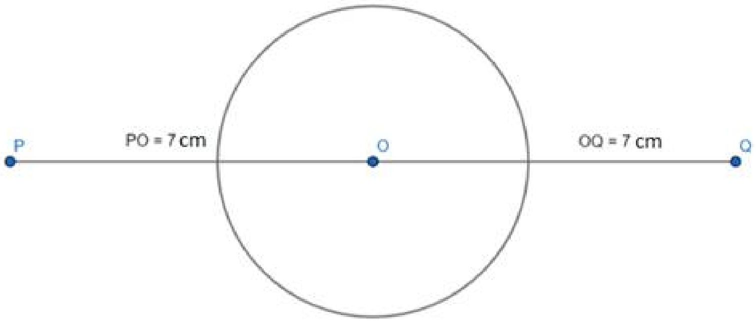
2. Question

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

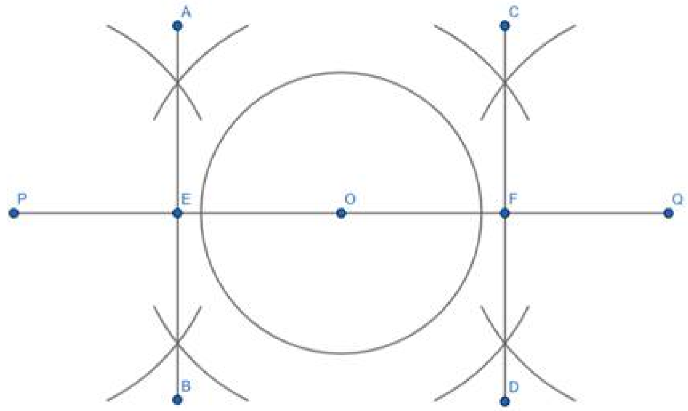
Answer



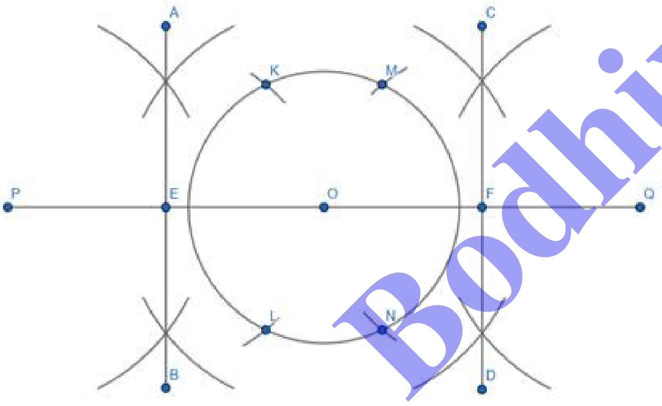
<u>Step 1</u>: We construct a circle with centre O and radius 3 cm.



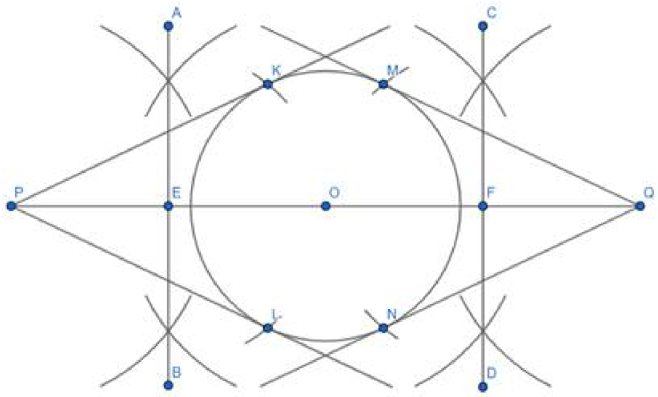
Step 2: We draw a diameter through O and extend it from both ends to points P and Q such that OP = OQ = 7 cm.



Step 3: We construct perpendicular bisectors AB and CD of segments OP and OQ respectively. E and F are the corresponding intersection points.



Step 4: We take OE as radius and construct arcs taking E as centre to cut the circle at points K and L. Similarly, we take OF as radius and construct arcs taking F as centre to cut the circle at points M and N.



Step 5: We join points P and K, P and L, Q and M, and Q and N to get the tangents PK, PL, QM, QN from points

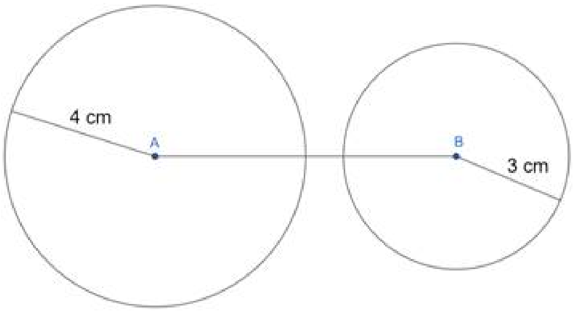
P and Q to the circle.

3. Question

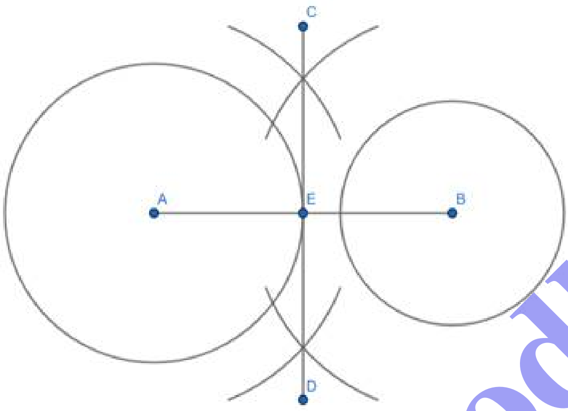
Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

Answer

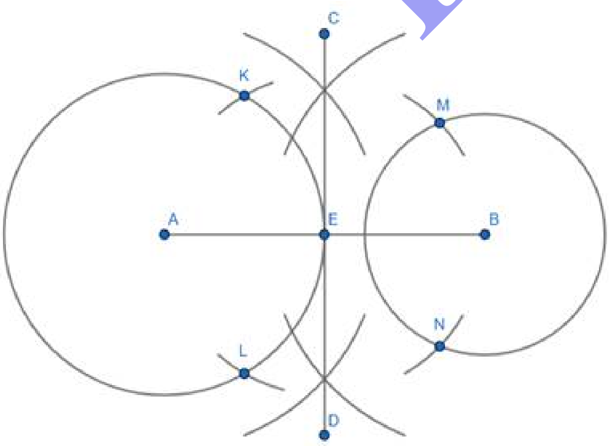
Step 1: We construct line segment AB of length 8 cm.



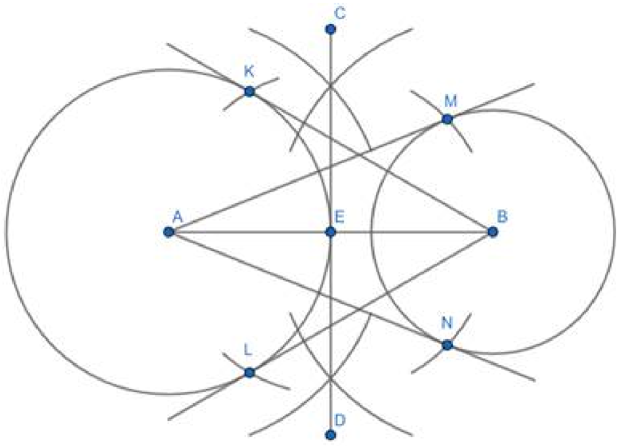
Step 2: Taking A as centre, we construct a circle of radius 4 cm and taking B as centre, we construct a circle of radius 3 cm.



Step 3: We construct perpendicular bisector CD of line segment AB. They intersect at point E.



Step 4: Taking AE as radius and E as centre, we construct arcs to cut circle A at points K and L. Similarly, Taking BE as radius and E as centre, we construct arcs to cut circle B at points M and N.

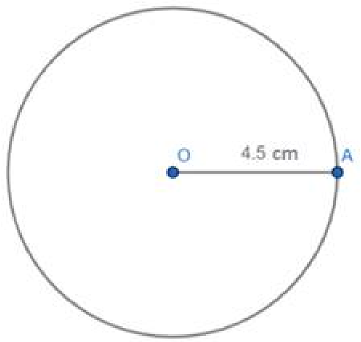


Step 5: We join points A and M, A and N, B and K, B and L to get tangents AM and AN from point A to circle B, and tangents BK and BL from point B to circle A.

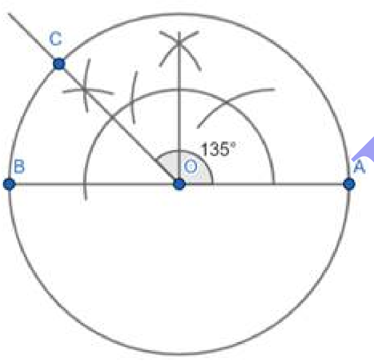
4. Question

Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

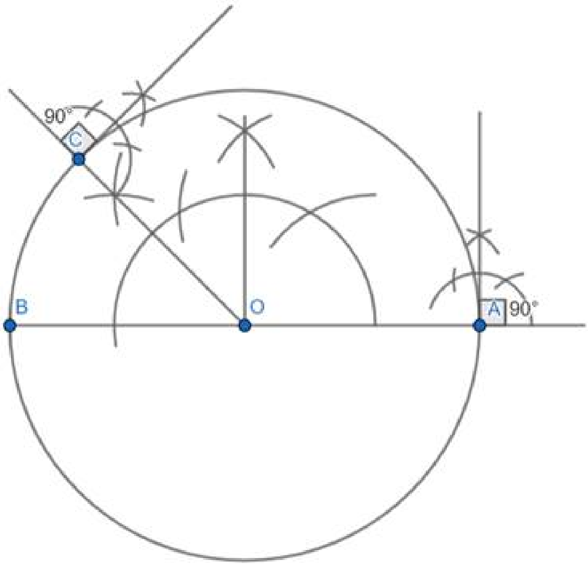
Answer



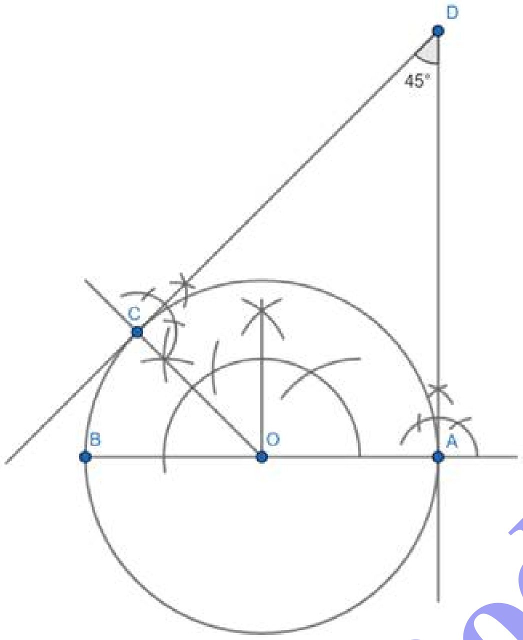
Step 1: We construct a circle with radius 4.5 cm, centred at O.



Step 2: We construct an angle of 135° at centre O, such that ∠AOC = 135°, where C is another point on the circle.



<u>Step 3</u>: We construct perpendiculars to OC and OA at points C and A respectively.

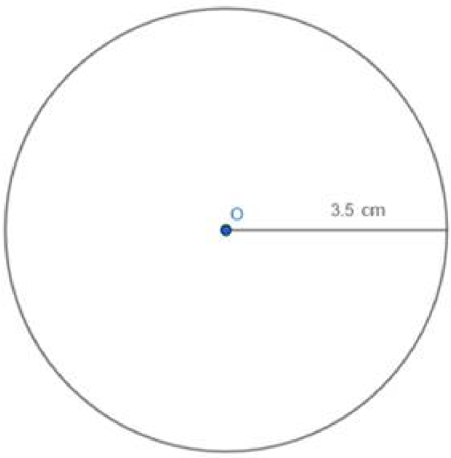


<u>Step 4</u>: We extend the perpendiculars to meet at point D, and we get tangents AD and CD to the circle, enclosing 45° between them.

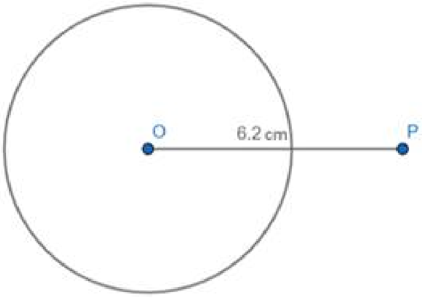
5. Question

Draw two tangents to a circle of radius 3.5 cm from a point P at a distance of 6.2 cm from its centre.

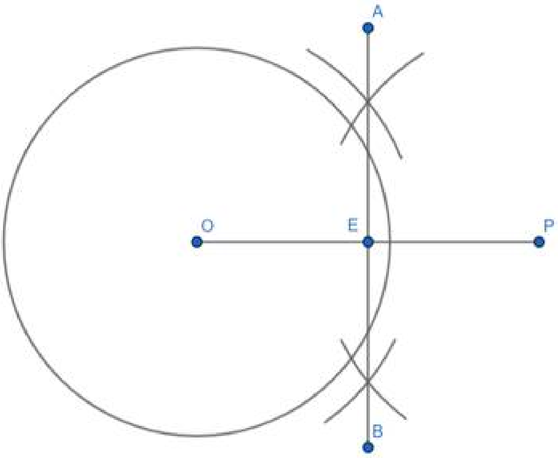
Answer



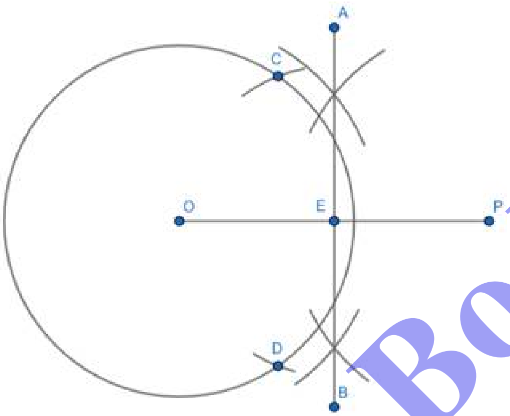
Step 1: Construct a circle of radius 3.5 cm, centred at a point O.



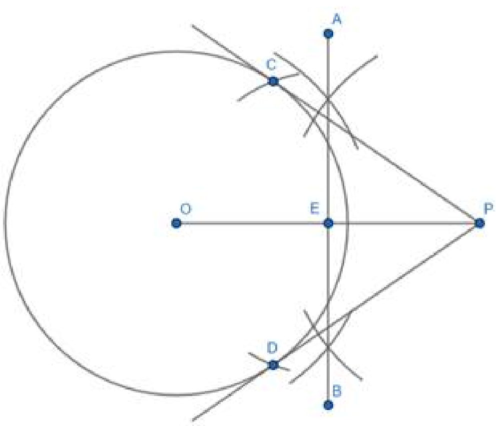
Step 2: Construct line segment OP, such that point P is at a distance of 6.2 cm from O.



Step 3: Construct perpendicular bisector AB of line segment OP. Point E is where they intersect.



Step 4: Taking OE as the radius and centre at E, we draw arcs to cut the circle at points C and D.

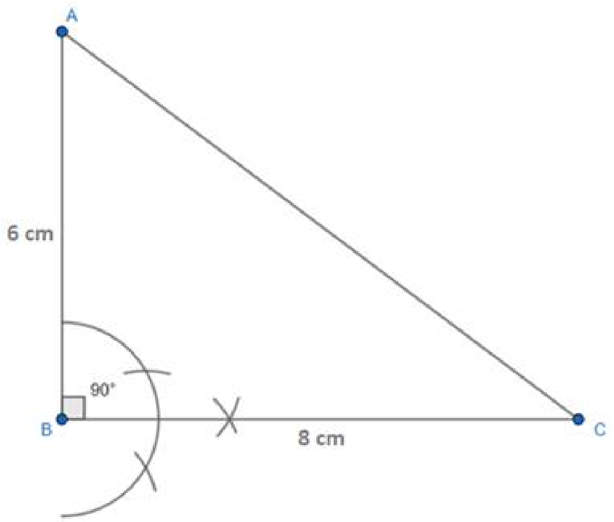


Step 5: We join points P and C, and P and D to get tangents PC and PD to the circle.

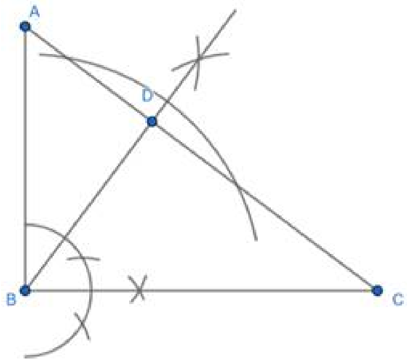
6. Question

Draw a right triangle ABC in which AB = 6 cm, BC = 8 cm and $\angle B = 90^0$. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

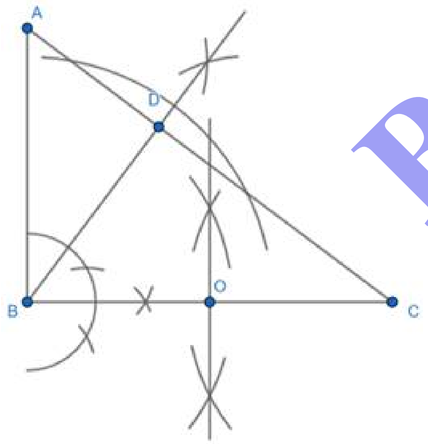
Answer



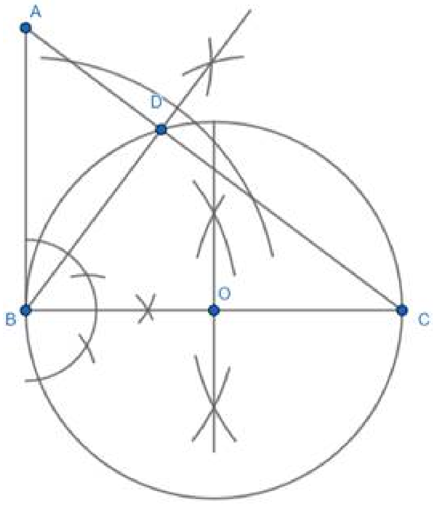
Step 1: Construct line segment AB of length 6 cm. Then construct perpendicular BC of length 8 cm. Finally join points A and C to complete the triangle.



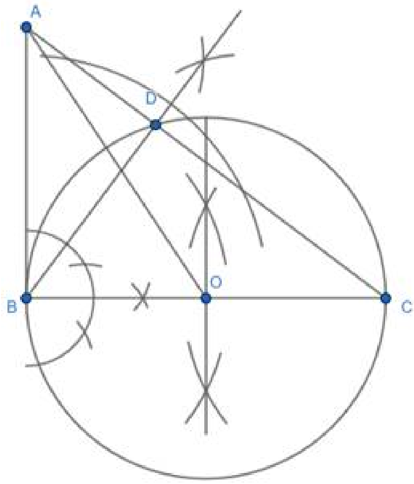
Step 2: Construct a perpendicular from point B onto AC. Point D is where this perpendicular meets AC.



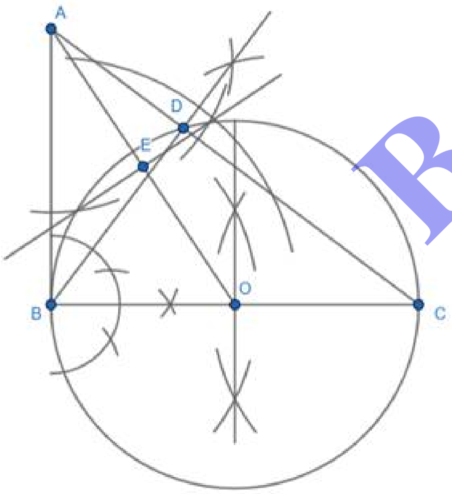
Step 3: Construct perpendicular bisector of line segment BC, which intersects it at pt. O.



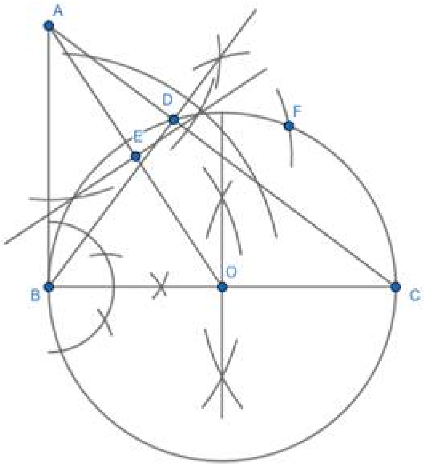
Step 4: Taking centre at O and radius as OB, we construct a circle. It passes through points C and D too.



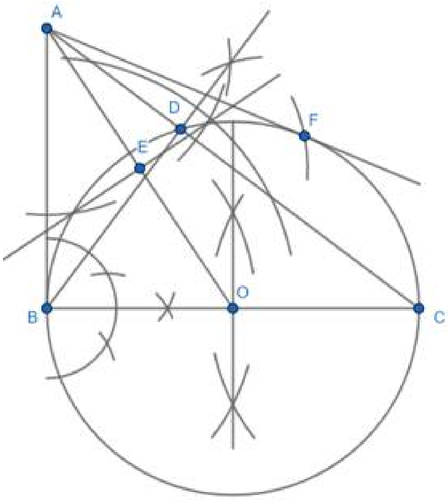
Step 5: We join points A and O.



Step 6: We construct the perpendicular bisector of AO, which intersects it at point E.



Step 7: Taking centre at E and radius as OE, we draw an arc cutting the circle at F. (We cut it at one point only because AB is already a tangent to the circle)



Step 8: We join points A and F, thus obtaining tangents AB and AF to the circle.