10. Congruent Triangles

Exercise 10.1

1. Question

In Fig. 10.22, the sides *BA* and *CA* have been produced such that *BA* = *AD* and *CA* = *AE*. Prove that segment *DE*||*BC*.

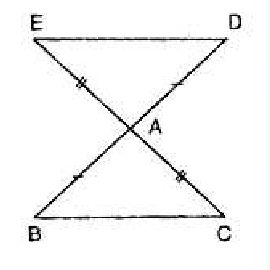


Fig. 10.22

Answer

Given,

The sides BA and CA have been produced, such that:

BA = AD

And, CA = AE

We have to prove that,

DE ∥ BC

Consider ΔBAC and ΔDAE, we have

BA = AD and CA = AE (Given)

∠BAC = ∠DAE (Vertically opposite angle)

So, by SAS congruence rule we have:

$\Delta BAC \cong \Delta DAE$

Therefore, BC = DE and

∠DEA = ∠BCA,

∠EDA = ∠CBA (By c.p.c.t)

Now, DE and BC are two lines intersected by a transversal DB such that,

∠DEA = ∠BCA,

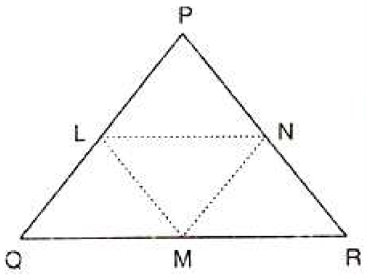
i.e., Alternate angles are equal

Therefore, DE || BC

2. Question

In a Δ *PQR*, if *PQ*=*QR* and *L, M* and *N* are the mid points of the sides *PQ, QR* and *RP* respectively, Prove that *LN*=*MN*.

Answer



Given that in Δ *PQR,*

PQ = QR

And, L, M, N are the mid points of the sides PQ, QR and RP respectively.

We have to prove that,

LN = MN

Here, we can observe that PQR is an isosceles triangle

PQ = QR

And, ∠QPR = ∠QRP (i)

And also, L and M are the mid points of PQ and QR respectively

PL = LQ = $\frac{PQ}{2}$

QM = MR = $\frac{QR}{2}$

And, PQ = QR

PL = LQ = QM = MR = $\frac{PQ}{2} = \frac{QR}{2}$ (ii)

Now, in ΔLPN and ΔMRN

LP = MR (From ii)

∠LPN = ∠MRN (From i)

PN = NR (N is the mid-point of PR)

Hence, By SAS theorem

$\Delta LPN \cong \Delta MRN$

Therefore, LN = MN (By c.p.c.t)

3. Question

In Fig. 10.23, *PQRS* is a square and *SRT* is an equilateral triangle. Prove that

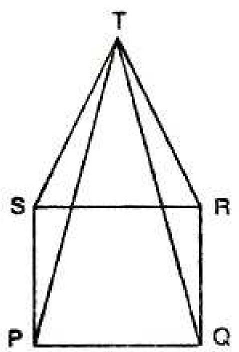


Fig. 10.23

(i) *PT* = *QT* (ii) ∠*TQR* =15°

Answer

Given,

PQRS is a square and SRT is a equilateral triangle

To prove: (i) PT = QT

(ii) $\angle TQR = 15°$

Proof: PQ = QR = RS = SP (As PQRS is a square, all sides will be equal) (i)

And, ∠SPQ = ∠PQR = ∠QRS = ∠RSP = 90^o

And also,

SRT is an equilateral triangle

SR = RT = TS (ii)

And, ∠TSR = ∠SRT = ∠RTS = 60^o

From (i) and (ii)

PQ = QR = SP = SR= RT = TS (iii)

∠TSP = ∠TSR + ∠RSP

= $60^o + 90^o = 150^o$

∠TRQ = ∠TRS + ∠SRQ

= $60^o + 90^o = 150^o$

Therefore, ∠TSR = ∠TRQ = 150^o (iv)

Now, in ΔTSP and ΔTRQ, we have

TS = TR (From iii)

∠TSP = ∠TRQ (From iv)

SP = RQ (From iii)

Therefore, By SAS theorem,

$\Delta TSP \cong \Delta TRQ$

PT = QT (BY c.p.c.t)

In ΔTQR

QR = TR (From iii)

Hence, ΔTQR is an isosceles triangle.

Therefore, ∠QTR = ∠TQR (Angles opposite to equal sides)

Now,

Sum of angles in a triangle is 180^o

∠QTR + ∠TQR + ∠TRQ = 180^O

2∠TQR + 150^O = 180^O (From iv)

2∠TQR = 30^O

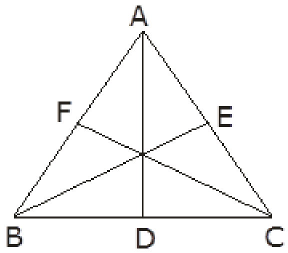
∠TQR = 15^O

Hence, proved

4. Question

Prove that the medians of an equilateral triangle are equal.

Answer



To prove: The medians of an equilateral triangle are equal.

Median = The line joining the vertex and mid-points of opposite sides.

Proof: Let $\Delta\ ABC$ be an equilateral triangle

AD, EF and CF are its medians.

Let,

$AB = AC = BC = x$

In ΔBFC and ΔCEB, we have

AB = AC (Sides of equilateral triangle)

$\frac{1}{2}AB = \frac{1}{2}AC$

$BF = CE$

$\angle ABC = \angle ACB$ (Angles of equilateral triangle)

$BC = BC$ (Common)

Hence, by SAS theorem, we have

$\Delta\ BFC \cong \Delta\ CEB$

BE = CF (By c.p.c.t)

Similarly, $AB = BE$

Therefore, $AD = BE = CF$

Hence, proved

5. Question

In a $\Delta\ ABC$, if $\angle A$ =120° and AB=AC. Find $\angle B$ and $\angle C$.

Answer

Given,

$\angle A$ =120°

$AB = AC$

We have to find $\angle B$ and $\angle C$:

We can observe that $\Delta\ ABC$ is an isosceles triangle since AB = AC

∠B = ∠C (Angle opposite to equal sides are equal) [i]

We know that,

Sum of angles in a triangle is equal to 180^{o}

$\angle A + \angle B + \angle C = 180^o$

$\angle A + \angle B + \angle B = 180^o$

$\angle A + 2\angle B = 180^o$

$120^o + 2\angle B = 180^o$

$2\angle B = 180^o - 120^o$

$2\angle B = 60^o$

$\angle B = 30^o$

$\angle B = \angle C = 30^o$

6. Question

In a Δ *ABC*, if $AB = BC$ 120° and $\angle B = 70°$, Find $\angle A$.

Answer

Consider Δ *ABC,*

We have,

$\angle B = 70°$

And, AB = AC

Therefore, Δ *ABC* is an isosceles triangle.

$\angle B = \angle C$ (Angle opposite to equal sides are equal)

$\angle B = \angle C = 70^o$

And $\angle A + \angle B + \angle C = 180^o$ (Angles of triangle)

$\angle A + 70^o + 70^o = 180^o$

$\angle A = 40^o$

7. Question

The vertical angle of an isosceles triangle is 100°. Find its base angles.

Answer

Consider an isosceles triangle ABC,

Such that:

AB = AC

Given,

Vertical $\angle A$ is 100^o

To find: Base angle

Since, ΔABC is isosceles,

$\angle B = \angle C$ (Angle equal to opposite sides)

And,

$\angle A + \angle B + \angle C = 180^o$ (Angles of triangle)

$100^o + \angle B + \angle B = 180^o$

$\angle B = 40^o$

$\angle B = \angle C = 40^o$

8. Question

In Fig. 10.24, $AB = AC$ and $\angle ACD = 105°$, find $\angle BAC$.

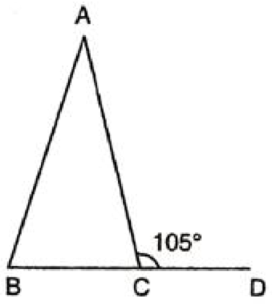


Fig. 10.24

Answer

Given,

$AB = AC$

$\angle ACD = 105^o$

Since, $\angle BCD = 180^o$ (Straight angle)

$\angle BCA + \angle ACD = 180^o$

$\angle BCA + 105^o = 180^0$

$\angle BCA = 75^o$ (i)

Now,

ΔABC is an isosceles triangle

$\angle ABC = \angle ACB$ (Angle opposite to equal sides)

From (i), we have

$\angle ACB = 75^o$

$\angle ABC = \angle ACB = 75^o$

Sum of interior angle of triangle = 180^o

$\angle A + \angle B + \angle C = 180°$

$\angle A = 180° - 75° - 75°$

$= 30°$

Therefore, $\angle BAC = 30^o$

9. Question

Find the measure of each exterior angle of an equilateral triangle.

Answer

To Find: Measure of each exterior angle of an equilateral triangle

Consider an equilateral triangle ABC.

We know that, for an equilateral triangle

AB = AC =CA

And, ∠ABC = ∠BCA = ∠CAB = $\frac{180}{3}$

= 60^o (i)

Now, Extend side BC to D,

CA to E and AB to F

Here,

BCD is a straight line segment

∠BCD = Straight line segment = 180^o

∠BCA + ∠ACD = 180^o

60^o + ∠ACD = 180^o (From i)

∠ACD = 120^o

Similarly, we can find ∠EAB and ∠FBC also as 120^o because ABC is an equilateral triangle.

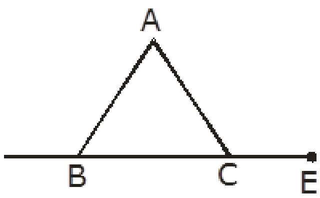
Therefore, ∠ACD = ∠EAB = ∠FBC = 120^o

Hence, the measure of each exterior angle of an equilateral triangle is 120^o

10. Question

If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Answer



To prove: the exterior angles formed are equal to each other

i.e., $\angle ADB = \angle ACE$

Proof: Let ABC be an isosceles triangle

Where BC is the base of the triangle and AB and AC are its equal sides.

$\angle ABC = \angle ACB$

$\angle B = \angle C$ (Angle opposite to equal sides)

Now,

$\angle ADB + \angle ABC = 180°$

$\angle ACB + \angle ACE = 180°$

$\angle ADB = 180° - \angle B$

And

$\angle ACE = 180° - \angle C$

$\angle ADB = 180° - \angle B$

And

$\angle ACE = 180° - \angle B$

$\angle ADB = \angle ACE$

Hence, proved

11. Question

In Fig. 10.25, $AB = AC$ and $DB = DC$, find the ratio $\angle ABD : \angle ACD$.

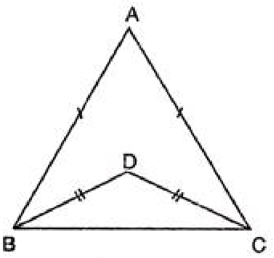


Fig. 10.25

Answer

Consider the figure,

Given,

AB = AC

DB = DC

To find: Ratio $\angle ABD = \angle ACD$

Now, $\Delta ABC\ and\ \Delta DBC$ are isosceles triangles

Since, AB = AC

And,

DB = DC

Therefore, ∠ABC = ∠ACB and,

∠DBC = ∠DCB (Angle opposite equal sides)

Now, consider ∠ABD: ∠ACD

(∠ABC - ∠DBC): (∠ACB - ∠DCB)

(∠ABC - ∠DBC): (∠ABC - ∠DBC) [Since, ∠ABC = ∠ACB and ∠DBC = ∠DCB]

1: 1

Therefore, ∠ABD: ∠ACD = 1:1

12. Question

Determine the measure of each of the equal angles of a right angled isosceles triangle.

OR

ABC is a right-angled triangle in which $\angle A = 90°$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Answer

Given,

ABC is a right-angled triangle

$\angle A = 90°$

And,

$AB = AC$

To find: ∠B and ∠C

Since, AB = AC

Therefore, ∠B = ∠C

And, Sum of angles in a triangle = 180^o

$\angle A + \angle B + \angle C = 180^o$

$90^o + 2\angle B = 180^o$

$2\angle B = 90^o$

$\angle B = 45^o$

Hence, the measure of each angle of the equal angles of a right angle isosceles triangle is 45^o.

13. Question

AB is a line segment. *P* and *Q* are points on opposite sides of *AB* such that each of them is equidistant from the points *A* and *B* (See Fig. 10.26). Show that the line *PQ* is perpendicular bisector of *AB*.

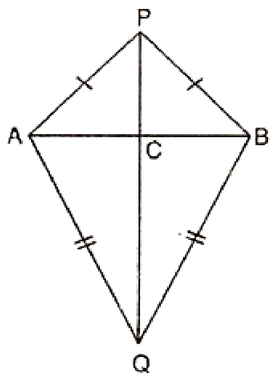


Fig. 10.26

Answer

Consider the figure,

We have AB is a line segment

P, Q are the points on opposite sides on AB

Such that,

AP = BP (i)

AQ = BQ (ii)

To prove: PQ is perpendicular bisector of AB

Proof: Now, consider $\Delta PAQ \text{ and } \Delta PBQ$,

AP = BP (From i)

AQ = BQ (From ii)

PQ = PQ (Common)

Therefore, By SSS theorem

$\Delta PAQ \cong \Delta PBQ$ (iii)

$Now, we\ can\ observe\ that\ \Delta APB\ and\ \Delta ABQ$ are isosceles triangles [From (i) and (ii)]

∠PAB = ∠PBA

And,

∠QAB = ∠QBA

Consider, ΔPAC and ΔPBC

C is the point of intersection of AB and PQ

PA = PB [From (i)]

∠APC = ∠BPC [From (iii)]

PC = PC (Common)

By SAS theorem,

$\Delta PAC \cong \Delta PBC$

AC = CB

And, ∠PCA = ∠PCB (By c.p.c.t) (iv)

And also,

ACB is line segment

∠ACP + ∠BCP = 180^o

But, ∠ACP = ∠PCB

∠ACP = ∠PCB = 90^o (v)

We have,

AC = CB

C is the mid-point of AB

From (iv) and (v), we conclude that

PC is the perpendicular bisector of AB

Since, C is the point on line PQ, we can say that PQ is the perpendicular bisector of AB.

Exercise 10.2

1. Question

In Fig. 10.40, it is given that RT=TS, ∠1 =2∠2 and ∠4 = 2∠3. Prove that Δ *RBT* ≅ Δ *SAT*.

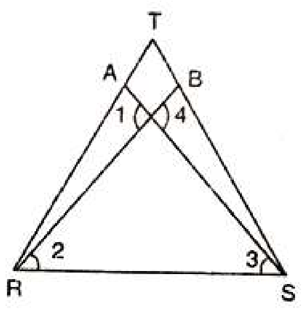


Fig. 10.40

Answer

In the figure, given that:

RT = TS (i)

∠1 = 2∠2 (ii)

And,

∠4 = 2∠3 (iii)

To prove: $\Delta RBT \cong \Delta SAT$

Let the point of intersection of RB and SA be denoted by O.

Since, RB and SA intersect at O.

∠AOR = ∠BOS (Vertically opposite angle)

∠1 = ∠4

2∠2 = 2∠3 [From (ii) and (iii)]

∠2 = ∠3 (iv)

Now, we have in ΔTRS

RT = TS

ΔTRS is an isosceles triangle

Therefore, ∠TRS = ∠TSI (v)

But, we have

∠TRS = ∠TRB + ∠2 (vi)

∠TSR = ∠TSA + ∠3 (vii)

Putting (vi) and (vii) in (v), we get

∠TRB + ∠2 = ∠TSA + ∠3

∠TRB = ∠TSA [From (iv)]

Now, in $\Delta RBT \text{ and } \Delta SAT$

RT = ST [From (i)

∠TRB = ∠TSA [From (iv)

∠RTB = ∠STA (Common angle)

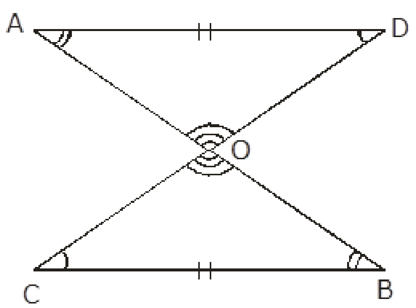
By ASA theorem,

$\Delta RBT \cong \Delta SAT$

2. Question

Two lines *AB* and *CD* intersect at *O* such that *BC* is equal and parallel to *AD*. Prove that the lines *AB* and *CD* bisect at *O*.

Answer



Given that,

Lines AB and CD intersect at O such that:

BC ∥ AD

And, BC = AD (i)

To prove: AB and CD bisect at O

Proof: In Δ *AOD* and Δ *BOC*

$AD = BC$ [From (i)]

$\angle OBC = \angle OAD$ ($AD || BC$ and AB is transversal)

$\angle OCB = \angle ODA$ ($AD || BC$ and CD is transversal)

Therefore, by ASA theorem:

$\Delta\ AOD \cong \Delta\ BOC$

$OA = OB$ (By c.p.c.t)

And,

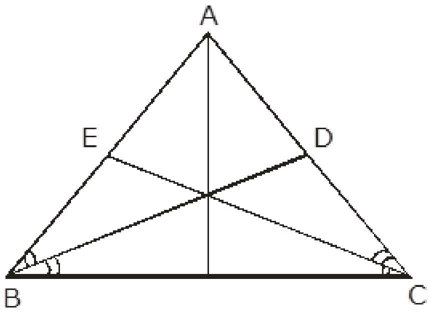
$OD = OC$ (By c.p.c.t)

Hence, AB and CD bisect each other at O.

3. Question

BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles $\Delta\ ABC$ with $AB = BC$. Prove that $BD = CE$.

Answer



Given,

In isosceles $\Delta\ ABC$,

BD and CE are bisectors of $\angle B$ and $\angle C$

And,

$AB = AC$

To prove: BD = CE

Proof: In $\Delta\ BEC$ and $\Delta\ CDB$, we have

$\angle B = \angle C$ (Angles opposite to equal sides)

BC = BC (Common)

$\angle BCE = \angle CBD$ (Since, $\angle C = \angle B$ $\frac{1}{2}\angle C = \frac{1}{2}\angle B$ $\angle BCE = \angle CBD$)

By ASA theorem, we have

$\Delta\ BEC \cong \Delta\ CDB$

$EC = BD$ (By c.p.c.t)

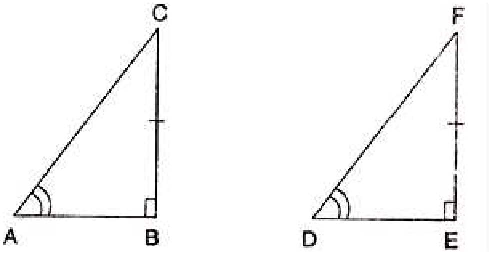
Hence, proved

Exercise 10.3

1. Question

In two triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Answer



Given that in two right angle triangles one side and acute angle of one are equal to the corresponding side and angle of the other.

We have to prove that the triangles are congruent.

Let us consider two right angle triangles. Such that,

∠B = ∠E = 90^{o} (i)

AB = DE (ii)

∠C = ∠F (iii)

Now, observe the two triangles ABC and DEF

∠C = ∠F (iv)

∠B = ∠E [From (i)]

AB = DE [From (ii)]

So, by AAS theorem, we have

$\Delta ABC \cong \Delta DEF$

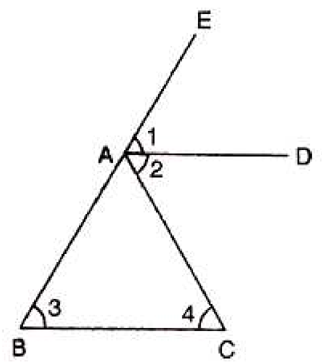
$Therefore, the\ two\ triangles\ are\ congruent$

Hence, proved

2. Question

If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Answer



Given that the bisector of exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles.

Let, ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and AD∥ BC

Let, ∠EAD = 1

∠DAC = 2

∠ABC = 3

∠ACB = 4

We have,

1 = 2 (Therefore, AD is the bisector of ∠EAC)

1 = 3 (Corresponding angles)

And,

2 = 4 (Alternate angles)

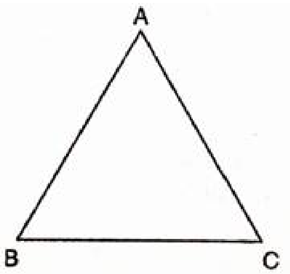
3 = 4 = AB = AC

Since, in $\Delta ABC,$ two sides AB and AC are equal we can say that ΔABC is isosceles.

3. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Answer



Let ΔABC be isosceles

Such that,

AB = BC

∠B = ∠C

Given, that vertex angle A is twice the sum of the base angles B and C.

i.e., ∠A = 2(∠B + ∠C)

∠A = 2(∠B + ∠B)

∠A = 2(2∠B)

∠A = 4∠B

Now,

We know that the sum of all angles of triangle = 180^o

∠A + ∠B + ∠C = 180^o

4∠B + ∠B + ∠B = 180^o (Therefore, ∠A = 4∠B, ∠C = ∠B)

6∠B = 180^o

∠B = $\frac{180}{6}$

= 30^o

Since, ∠B = ∠C = 30^o

And, ∠A = 4∠B

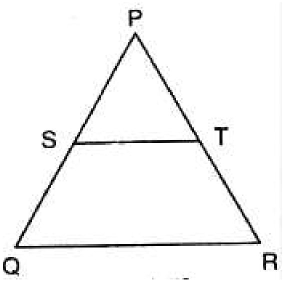
= 4 * 30^o = 120^o

Therefore, the angles of the triangle are 120^o, 30^o, 30^o.

4. Question

PQR is a triangle in which *PQ* = *PR* and *S* is any point on the side *PQ*. Through *S*, a line is drawn parallel to *QR* and intersecting *PR* at *T*. Prove that *PS* = *PT*.

Answer



Given that PQR is a triangle

Such that,

PQ = PR

And, S is any point on side PQ and ST ∥ QR

We have to prove PS = PT

Since,

PQ = PR

PQR is isosceles

∠Q = ∠R

Or, ∠PQR = ∠PRQ

Now,

∠PST = ∠PQR

And,

∠PTS = ∠PRQ (Corresponding angles as ST ∥ QR)

Since,

∠PQR = ∠PRQ

∠PST = PTS

Now, in ΔPST

∠PST = ∠PTS

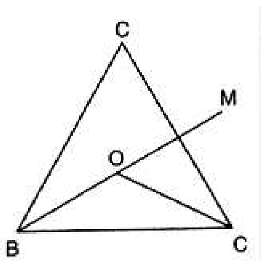
Therefore, ΔPST is an isosceles triangle

PS = PT

5. Question

In a Δ *ABC*, it is given that *AB* = *AC* and the bisectors of ∠*B* and ∠*C* intersect at *O*. If *M* is a point on *BO* produced, prove that ∠*MOC* = ∠*ABC*.

Answer



Given that in Δ *ABC,*

AB = AC and the bisectors of ∠B and ∠C intersect at O and M is a point on BO produced.

We have to prove ∠MOC = ∠ABC

Since,

AB = AC

Δ *ABC* is isosceles

∠B = ∠C

Or,

∠ABC = ∠ACB

Now,

BO and CO are bisectors of ∠ABC and ∠ACB respectively.

∠ABO = ∠OBC = ∠ACO = ∠OCB = $\frac{1}{2}$∠ABC = $\frac{1}{2}$∠ACB (i)

We have, in ΔOBC

∠OCB + ∠OBC + ∠BOC = 180^o (ii)

And also,

∠BOC + ∠COM = 180^o (iii) [Straight angle]

Equating (ii) and (iii), we get

∠OCB + ∠OBC + ∠BOC = ∠BOC + ∠COM

∠OBC + ∠OBC = ∠MOC

2∠OBC = ∠MOC

2($\frac{1}{2}$∠ABC) = ∠MOC [From (i)]

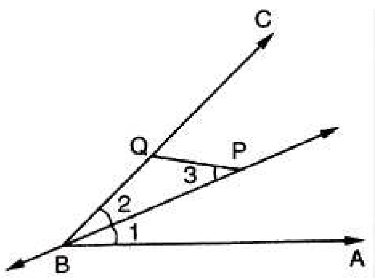
∠ABC = ∠MOC

Therefore, ∠MOC = ∠ABC

6. Question

P is a point on the bisector of an angle ∠*ABC*. If the line through *P* parallel to *AB* meets *BC* at *Q*, prove that the triangle *BPQ* is isosceles.

Answer



Given that P is the point on the bisector of an angle ∠ABC, and PQ ∥ AB

We have to prove that BPQ is isosceles

Since,

BP is the bisector of ∠ABC = ∠ABP = ∠PBC (i)

Now,

PQ ∥ AB

∠BPQ = ∠ABP (ii) [Alternate angles]

From (i) and (ii), we get

∠BPQ = ∠PBC

Or,

∠BPQ = ∠PBQ

Now, in ΔBPQ

∠BPQ = ∠PBQ

ΔBPQ is an isosceles triangle

$Hence, proved$

7. Question

Prove that each angle of an equilateral triangle is 60°.

Answer

Given to prove that each angle of the equilateral triangle is 60^o

Let us consider an equilateral triangle ABC

Such that,

AB = BC = CA

Now,

AB = BC

∠A = ∠C [i] (Opposite angles to equal sides are equal)

BC = AC

∠B = ∠A [ii[(Opposite angles to equal sides are equal)

From [i] and [ii], we get

∠A = ∠B = ∠C [iii]

We know that,

Sum of all angles of triangles = 180^o

∠A + ∠B + ∠C = 180^o

∠A + ∠A + ∠A = 180^o

3∠A = 180^o

∠A = $\frac{180}{3}$

= 60^o

Therefore, ∠A = ∠B = ∠C = 60^o

Hence, each angle of an equilateral triangle is 60^o.

8. Question

Angles *A, B, C* of a triangle *ABC* are equal to each other. Prove that Δ *ABC* is equilateral.

Answer

Given that A, B, C of a triangle ABC are equal to each other.

We have to prove that, Δ ABC is equilateral.

We have,

∠A = ∠B = ∠C

Now,

∠A = ∠B

BC = AC (Opposite sides to equal angles are equal)

∠B = ∠C

AC = AB (Opposite sides to equal angles are equal)

From the above, we get

AB = BC = AC

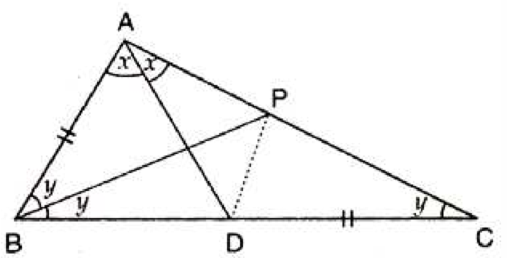
Therefore, ΔABC *is an equilateral triangle.*

Hence, proved

9. Question

ABC is a triangle in which $\angle B = 2\angle C$. D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$. Prove that $\angle BAC = 72°$.

Answer



Given that, in ΔABC,

∠B = 2∠C and,

D is a mid-point on BC such that AD bisects ∠BAC and AB = CD.

We have to prove that, ∠BAC = 72^o

Now draw the angular bisector of ∠ABC, which meets AC in P

Join PD

Let, ∠ACB = y

∠B = ∠ABC = 2∠C = 2y

Also, ∠BAD = ∠DAC = x

∠BAC = 2x (Therefore, AD is the bisector of ∠BAC)

Now, in ΔBPC

∠CBP = y (Therefore, BP is the bisector of ∠ABC)

∠PCB = y

∠CBP = ∠PCB = y

Therefore, PC = BP

Consider ΔABP and ΔDCP, we have

∠ABP = ∠DCP = y

AB = DC (Given)

PC = BP (From above)

So, by SAS theorem, we have

$\Delta ABP \cong \Delta DCP$

Now,

∠BAP = ∠CDP

And, AP = DP (By c.p.c.t)

∠BAP = ∠CDP = 2x

Now, in ΔABD

∠ABD + ∠BAD + ∠ADB = 180^o

∠ADB + ∠ADC = 180^o (Straight angle)

2x + 2y + y = 180^o (Therefore, ∠A = 2x, ∠B = 2y, ∠C = y)

2y + 3y = 180^o (Therefore, x = y)

5y = 180^o

y = $\frac{180}{5}$

y = 36^o

Therefore, x = y = 36^o

Now,

∠A = ∠BAC = 2x = 2 * 36^o = 72^o

Therefore, ∠BAC = 72^o

Hence, proved

10. Question

ABC is a right angled triangle in which ∠*A*=90° and *AB*=*AC*. Find ∠*B* and ∠*C*.

Answer

Given that ABC is a right angled triangle

Such that,

∠A = 90^o

And,

AB = AC

Since,

AB = AC

ΔABC is also isosceles triangle

Therefore, we can say that ΔABC is a right angled isosceles triangle.

∠C = ∠B

And,

∠A = 90^o

Now, we have

Sum of angles in a triangle = 180^o

∠A + ∠B + ∠C = 180^o

90^o + ∠B + ∠B = 180^o (From i)

90^o + 2∠B = 180^o

2∠B = 90^o

∠B = 45^o

Therefore, ∠B = ∠C = 45^o

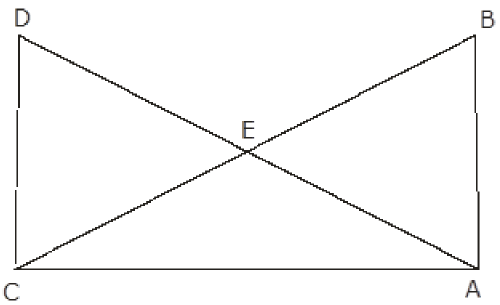
Exercise 10.4

1. Question

In Fig. 10.92, it is given that *AB* = *CD* and *AD* = *BC*. Prove that Δ *ADC* ≅ Δ *CBA*.

Answer

Given, in the figure



AB = CD

And,

AD = BC

To prove: Δ *ADC* ≅ Δ *CBA*

Proof: Consider, $\Delta ADC \text{ and } \Delta CBA$

AB = CD (Given)

BC = AD (Given)

AC = AC (Common)

By SSS theorem,

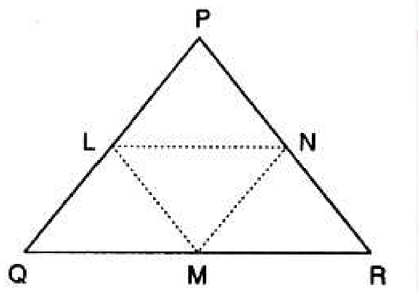
Δ *ADC* ≅ Δ *CBA*

Hence, proved

2. Question

In Δ *PQR*, if *PQ*=*QR* and *L, M* and *N* are the mid-point of the sides *PQ, QR* and *RP* respectively. Prove that *LN*=*MN*.

Answer



Given that,

In Δ *PQR*

PQ = QR

And,

L, M, and N are the mid points of PQ, QR and RP respectively

To prove: LM = MN

Construction: Join L and M, M and N and N and L

Proof: We have,

PL = LQ, QM = MR and RN = NP

Since, L, M and N are mid points of PQ, QR and RP respectively.

And, also PQ = QR

PL = LQ = QM = MR = $\frac{PQ}{2} = \frac{QR}{2}$ (i)

Using mid-point theorem, we have

MN ∥ PQ

And,

MN = $\frac{1}{2}$PQ = MN = PL = LQ (ii)

Similarly, we have

LN ∥ QR

And,

LN = $\frac{1}{2}$QR = LN = QM = MR (iii)

From equations (i), (ii) and (ii), we have

PL = LQ = QM = MR = MN = LN

Therefore, LN = MN

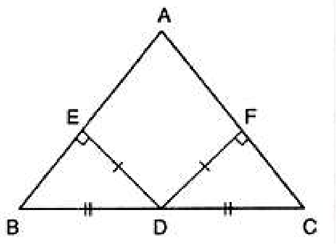
Hence, proved

Exercise 10.5

1. Question

ABC is a triangle and *D* is the mid-point of *BC*. The perpendicular from *D* to *AB* and *AC* are equal. Prove that the triangle is isosceles.

Answer



Given,

ABC is a triangle and D is the mid-point of BC

Perpendicular from D to AB and AC are equal.

To prove: Triangle is isosceles

Proof: Let DE and DF be perpendiculars from A on AB and AC respectively.

In order to prove that $AB = AC$, we will prove that $\Delta BDE \cong \Delta CDF$.

In these two triangles, we have

$\angle BEF = \angle CFD = 90°$

$BD = CD$ (Therefore, D is the mid-point of BC)

$DE = DF$ (Given)

So, by RHS congruence criterion, we have

$\Delta BDE \cong \Delta CDF$

$\angle B = \angle C$ (By c.p.c.t)

$AC = AB$ (By c.p.c.t)

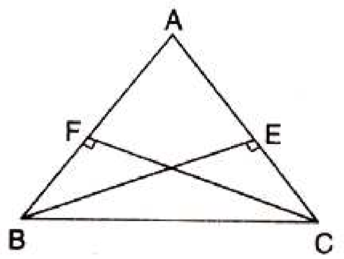
As opposite sides and opposite angles of the triangle are equal.

Therefore, ΔABC is isosceles

2. Question

ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If $BE = CF$, prove that ΔABC is isosceles.

Answer



Given that ABC is a triangle in which BE and CF are perpendiculars to the side AC and AB respectively.

Such that,

BE = CF

We have to prove that, ΔABC is isosceles triangle.

Now, consider $\Delta BCF \text{ and } \Delta CBE$

We have,

$\angle BFC = \angle CEB = 90^{o}$ (Given)

BC = CB (Given)

CF = BE (Given)

So, by RHS congruence rule, we have

$\Delta BFC \cong CEB$

$Now,$

∠FBC = ∠ECB (By c.p.c.t)

∠ABC = ∠ACB (By c.p.c.t)

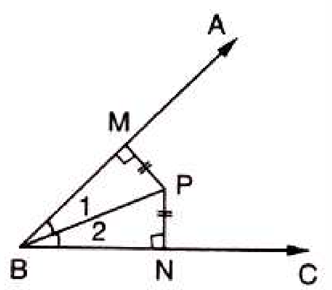
AC = AB (Opposite sides of equal angles are equal in a triangle)

Therefore, ΔABC is isosceles.

3. Question

If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

Answer



Given that perpendiculars from any point within an angle on its arms are congruent.

We have to prove that it lies on the bisector of that angle.

Now, let us consider an ∠ABC and let BP be one of the arm within the angle.

Draw perpendicular PN and PM On the arms BC and BA

Such that,

They meet BC and BA in N and M respectively.

Now, in $\Delta BPM\ and\ \Delta BPN$

We have,

∠BMP = ∠BNP = 90^{o} (Given)

BP = BP (Common)

MP = NP (Given)

So, by RHS congruence rule, we have

$\Delta BPM \cong BPN$

∠MBP = ∠NBP (By c.p.c.t)

BP is the angular bisector of ∠ABC

Hence, proved

4. Question

In Fig. 10.99, $AD \perp CD$ and $CB \perp CD$. If $AQ=BP$ and $DP=CQ$, prove that $\angle DAQ = \angle CBP$.

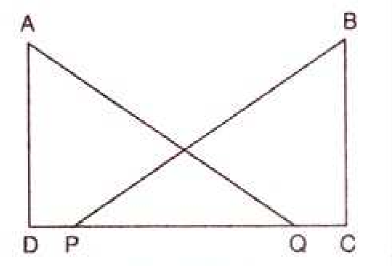


Fig. 10.99

Answer

Given that in figure,

AD ⊥ CD and CB ⊥ CD

And,

AQ = BP, DP = CQ

WE have to prove that,

$\angle DAQ = \angle CBP$

Given that, DP = QC

Adding PQ on both sides, we get

DP + PQ = PQ + QC

DQ = PC (i)

Now consider $\Delta DAQ \text{ and } \Delta CBP$, we have

∠ADQ = ∠BCP = 90^o (Given)

AQ = BP (Given)

And,

DQ = PC (From i)

So, by RHS congruence rule, we have

$\Delta DAQ \cong \Delta CBP$

Now,

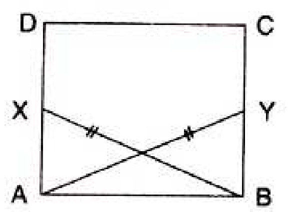
∠DAQ = ∠CBP (By c.p.c.t)

Hence, proved

5. Question

ABCD is a square, *X* and *Y* are points on sides *AD* and *BC* respectively such that *AY* = *BX*. Prove that *BY* = *AX* and ∠*BAY* = ∠*ABX.*

Answer



Given that ABCD is a square, X and Y are points on the sides AD and BC respectively.

Such that,

AY = BX

We have to prove: BY = AX and $\angle BAY = \angle ABX$

Join B and X, A and Y

Since, ABCD is a square

∠DAB = ∠CBA = 90^o

∠XAB = ∠YBA = 90^o (i)

Now, consider $\Delta XAB\ and\ \Delta YBA$

We have,

∠XAB = ∠YBA = 90^o [From (i)]

BX = AY (Given)

AB = BA (Common side)

So, by RHS congruence rule, we have

$\Delta XAB \cong \Delta YBA$

$\mathrm{BY} = \mathrm{AX}$ and $\angle\mathrm{BAY} = \angle\mathrm{ABX}$ (By c.p.c.t)

Hence, proved

6. Question

Which of the following statements are true (T) and which are false (F):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal.

(iii) The measure of each angle of an equilateral triangle is 60°.

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.

(v) The bisectors of two equal angles of a triangle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

(viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.

(ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Answer

(i) False: Sides opposite to equal angles of a triangle are equal.

(ii) True: Since, the sides are equal, the corresponding opposite angles must be equal.

(iii) True: Since, all the three angles of an equilateral triangle are equal and sum of the three angles is 180^o, so each angle will be equal to $\frac{180}{3}$ = 60^o

(iv) False: Here, the altitude from the vertex is also the perpendicular bisector of the opposite side. Here the triangle must be isosceles and may be an equilateral triangle.

(v) True: Since, it is an isosceles triangle, the length of bisector of the two angles are equal.

(vi) False: The angular bisector of the vertex angle is also a median. The triangle must be an isosceles and an equilateral triangle.

(vii) False: Since, two sides are equal the triangle must be an isosceles triangle. The two altitudes corresponding to two equal sides must be equal.

(viii) False: The two right triangles may or may not be congruent.

(ix) True: According to RHS congruence the given statement is true.

7. Question

Fill in the blanks in the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are ....

(i) Sides opposite to equal angles of a triangle are ....

(iii) In an equilateral triangle all angles are ....

(iv) In a Δ *ABC*, if ∠*A*=∠*C*, then *AB* = ...

(v) If altitudes *CE* and *BF* of a triangle *ABC* are equal, then, *AB* = ...

(vi) In an isosceles triangle *ABC* with *AB*=*AC*, if *BD* and *CE* are its altitudes, then *BD* is ....*CE*.

(vii) In right triangles *ABC* and *DEF*, if hypotenuse *AB*=*EF* and side *AC*=*DE*, then Δ *ABC* ≅ Δ ....

Answer

(i) Sides opposite to equal angles of a triangle are equal

(ii) Sides opposite to equal angles of a triangle are equal

(iii) In an equilateral triangle all angles are equal

(iv) In a Δ *ABC*, if ∠*A*=∠*C*, then *AB* = BC

(v) If altitudes *CE* and *BF* of a triangle *ABC* are equal, then, *AB* = AC

(vi) In an isosceles triangle *ABC* with *AB*=*AC*, if *BD* and *CE* are its altitudes, then *BD* is equal to *CE*

(vii) In right triangles *ABC* and *DEF*, if hypotenuse *AB*=*EF* and side *AC*=*DE*, then Δ *ABC* ≅ ΔEFD

Exercise 10.6

1. Question

In Δ *ABC*, if ∠*A*=40° and ∠*B*=60°. Determine the longest and shortest sides of the triangle.

Answer

Given that in Δ *ABC*

$\angle A = 40^o$ and $\angle B = 60^o$

We have to find shortest and longest side.

We know that,

Sum of angles of triangle = 180^o

$\angle A + \angle B + \angle C = 180^o$

$40^o + 60^o + \angle C = 180^o$

$100^o + \angle C = 180^o$

$\angle C = 180^o - 100^o$

$= 80^o$

Now,

$40^o < 60^o < 80^o$

$\angle A < \angle B < \angle C$

$\angle C$ is greater angle and $\angle A$ is smaller angle.

As, $\angle A < \angle B < \angle C$

BC < AC < AB (Therefore, side opposite to greater angle is larger and side opposite to smaller angle is smaller)

Therefore, AB is longest and BC is smallest or shortest side.

2. Question

In a ΔABC, if $\angle B = \angle C = 45°$, which is the longest side?

Answer

Given that in ΔABC,

$\angle B = \angle C = 45^o$

We have to find longest side.

We know that,

Sum of angles in a triangle = 180^o

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 45^o + 45^o = 180^o$

$\angle A + 90^o = 180^o$

$\angle A = 90^o$

Therefore, BC is the longest side because side opposite to greater angle is larger.

3. Question

In ΔABC, side AB is produced to D so that $BD = BC$. If $\angle B = 60°$ and $\angle A = 70°$, prove that:

(i) $AD > CD$ (ii) $AD > AC$

Answer

Given that in ΔABC, side AB is produced to D so that BD = BC and $\angle B = 60^o$, $\angle A = 70^o$

We have to prove that,

(i) AD > CD

And, (ii) AD > AC

First join C and D

Now,

In ΔABC,

$\angle A + \angle B + \angle C = 180^o$ (Sum of all angles of triangle)

$\angle C = 180^o - 70^o - 60^o$

$= 50^o$

$\angle C = 50^o$

$\angle ACB = 50^o$ (i)

And also in ΔBDC,

∠DBC = 180^o - ∠ABC (Therefore, ∠ABD is straight angle)

= 180^o – 60^o

= 120^o

BD = BC (Given)

∠BCD = ∠BDC (Therefore, angle opposite to equal sides are equal)

Now,

∠DBC + ∠BCD + ∠BDC = 180^o (Sum of all sides of triangle)

120^o + ∠BCD + ∠BCD = 180^o

2∠BCD = 180^o – 120^o

2∠BCD = 60^o

∠BCD = 30^o

Therefore, ∠BCD = ∠BDC = 30^o (ii)

Now, consider ΔADC,

∠BAC = ∠DAC = 70^o (Given)

∠BDC = ∠ADC = 30^o [From (ii)]

∠ACD = ∠ACB + ∠BCD

= 50^o + 30^o [From (i) and (ii)]

= 80^o

Now,

∠ADC < ∠DAC < ∠ACD

AC < DC < AD (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

AD > CD

And,

AD > AC

Hence, proved

We have,

∠ACD > ∠DAC

And,

∠ACD > ∠ADC

AD > DC

And,

AD > AC (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

4. Question

Is it possible to draw a triangle with sides of length 2 cm, 3 cm, and 7 cm?

Answer

Given, Length of sides are 2cm, 3cm and 7cm.

We have to check whether it is possible to draw a triangle with the given length of sides.

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

Here,

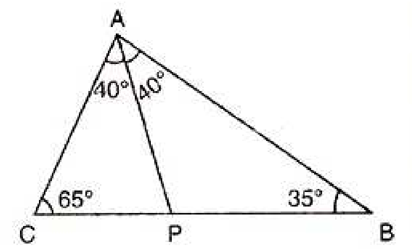
2 + 3>7

So, the triangle does not exist.

5. Question

In Δ *ABC*, ∠*B*=35°, ∠*C*=65° and the bisector of ∠*ABC* meets *BC* in *P*. Arrange *AP, BP* and *CP* in descending order.

Answer



Given: ∠*B* = 35°

∠*C* = 65°

The bisector of ∠*ABC* meets *BC* in *P*

We have to arrange AP, BO and CP in descending order

In Δ *ACP*, we have

∠*ACP* > ∠*CAP*

AP > *CP* (i)

In Δ *ABP*, we have

∠*BAP* > ∠*ABP*

BP > *AP* (ii)

From (i) and (ii), we have

BP > *AP* > *CP*

6. Question

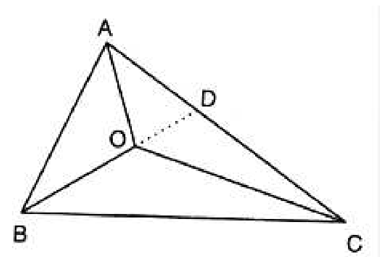
O is any point in the interior of Δ *ABC*. Prove that

(i) *AB* + *AC* > *OB* + *OC*

(ii) *AB* + *BC* + *CA* > *OA* + *OB* + *OC*

(iii) *OA* + *OB* + *OC* > $\frac{1}{2}$ (*AB* + *BC* + *CA*)

Answer



Given that, O is any point in the interior of ΔABC

We have to prove:

(i) $AB + AC > OB + OC$

(ii) $AB + BC + CA > OA + OB + OC$

(iii) $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

We know that,

In a triangle sum of any two sides is greater than the third side.

So, we have

In ΔABC

AB + BC > AC

BC + AC > AB

AC + AB > BC

In ΔOBC,

OB + OC > BC (i)

In ΔOAC,

OA + OC > AC (ii)

In ΔOAB,

OA + OB > AB (iii)

Now, extend BO to meet AC in D.

In ΔABD, we have

AB + AD > BD

AB + AD > BO + OD (iv) [Therefore, BD = BO + OD]

Similarly,

In ΔODC, we have

OD + DC > OC (v)

(i) Adding (iv) and (v), we get

AB + AD + OD + DC > BO + OD + OC

AB + (AD + DC) > OB + OC

AB + AC > OB + OC (vi)

Similarly, we have

BC + BC > OA + OC (vii)

And,

CA + CB > OA + OB (viii)

(ii) Adding (vi), (vii) and (viii), we get

AB + AC + BC + BA + CA + CB > OB + OC + OA + OC + OA + OB

2AB + 2BC + 2CA > 2OA + 2OB + 2OC

2 (AB + BC + CA) > 2 (OA + OB + OC)

AB + BC + CA > OA + OB + OC

(iii) Adding (i), (ii) and (iii), we get

OB + OC + OA + OC + OA + OB > BC + AC + AB

2OA + 2OB + 2OC > AB + BC + CA

2 (OA + OB + OC) > AB + BC + CA

Therefore, (OA + OB + OC) > $\frac{1}{2}$ (AB+ BC + CA)

7. Question

Prove that the perimeter of a triangle is greater than the sum of its altitudes.

Answer

Given: A ΔABC in which AD perpendicular BC and BE perpendicular AC and CF perpendicular AB.

To prove: AD + BE + CF < AB + BC + AC

Proof: We know that all the segments that can be drawn into a given line, from a point not lying on it, perpendicular distance i.e. the perpendicular line segment is the shortest. Therefore,

AD perpendicular BC

AB > AD and AC > AD

AB + AC > 2AD (i)

Similarly,

BE perpendicular AC

BA > BE and BC > BE

BA + BC > 2BE (ii)

And also

CF perpendicular AB

CA > CF and CB > CF

CA + CB > 2CF (iii)

Adding (i), (ii) and (iii), we get

AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF

2AB + 2BC + 2CA > 2 (AD + BE + CF)

2 (AB + BC + CA) > 2 (AD + BE + CF)

AB + BC + CA > AD + BE + CF

The perimeter of the triangle is greater than the sum of its altitudes.

Hence, proved

8. Question

Prove that in a quadrilateral the sum of all the sides is greater than the sum of its diagonals.

Answer

Given: Let ABCD is a quadrilateral with AC and BD as its diagonals

To Prove: Sum of all the sides of a quadrilateral is greater than the sum of its diagonals

Proof: Consider a quadrilateral ABCD where AC and BD are the diagonals

AB+BC > AC (i) (Sum of two sides is greater than the third side)

AD+DC > AC (ii)

AB+AD > BD (iii)

DC+BC > BD (iv)

Adding (i), (ii), (iii), and (iv)

AB+BC+AD+DC+AB+AD+DC+BC > AC+AC+BD+BD

2(AB+BC+CD+DA) > 2(AC+BD)

AB+BC+CD+DA > AC+BC

Hence, proved that the Sum of all the sides of a quadrilateral is greater than the sum of its diagonals

9. Question

In Fig. 10.131, prove that:

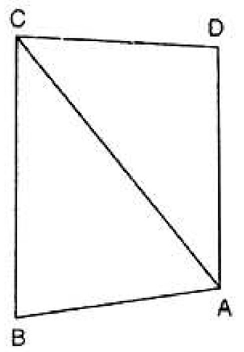


Fig. 10.131

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD + DA + AB > BC$

Answer

Given to prove,

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD + DA + AB > BC$

From the given figure,

We know that,

In a triangle sum of any two sides is greater than the third side.

(i) So,

In ΔABC, we have

AB + BC > AC (1)

In ΔADC, we have

CD + DA > AC (2)

Adding (1) and (2), we get

AB + BC + CD + DA > AC + AC

CD + DA + AB + BC > 2 AC

(ii) Now, in ΔABC, we have

CD + DA > AC

Add AB on both sides, we get

CD + DA +AB > AC + AB > BC

CD + DA + AB > BC

Hence, proved

10. Question

Which of the following statements are true (T) and which are false (F)?

(i) Sum of the three sides of a triangle is less than the sum of its three altitudes.

(ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.

(iii) Sum of any two sides of a triangle is greater than the third side.

(iv) Difference of any two sides of a triangle is equal to the third side.

(v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.

(vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Answer

(i) False: Sum of three sides of a triangle is greater than sum of its three altitudes.

(ii) True

(iii) True

(iv) False: The difference of any two sides of a triangle is less than the third side.

(v) True: The side opposite to greater angle is longer and smaller angle is shorter in a triangle.

(vi) True: The perpendicular distance is the shortest distance from a point to a line not containing it.

11. Question

Fill in the blanks to make the following statements true:

(i) In a right triangle the hypotenuse is the ....side.

(ii) The sum of three altitudes of a triangle is ...... than its perimeter.

(iii) The sum of any two sides of a triangle is ..... than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the .... side opposite to it.

(v) Difference of any two sides of a triangle is .... than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has .... angle opposite to it.

Answer

(i) In a right triangle the hypotenuse is the **largest** side**.**

(ii) The sum of three altitudes of a triangle is **less** than its perimeter.

(iii) The sum of any two sides of a triangle is **greater** than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the **smaller** side opposite to it.

(v) Difference of any two sides of a triangle is **less** than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has **greater** angle opposite to it.

CCE - Formative Assessment

1. Question

In two congruent triangles ABC and DEF, if $AB = DE$ and $BC = EF$. Name the pairs of equal angles.

Answer

By c.p.c.t. that is corresponding part of congruent triangles, the pair of equal angles are:

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

2. Question

In two triangles ABC and DEF, it is given that $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. Are the two triangles necessarily congruent?

Answer

No, the two triangles are not necessarily congruent in the given case as knowing only angle-angle-angle (AAA) does not work because it can produce similar but not congruent triangles.

3. Question

If ABC and DEF are two triangles such that $AC = 2.5$ cm, $BC = 5$ cm, $\angle C = 75°$, $DE = 2.5$ cm, $DF = 5$ cm and $\angle D = 75°$. Are two triangles congruent?

Answer

Yes, the given triangles are congruent as AC=DE, BC = DF and $\angle D$ is equal to $\angle C$. Hence, By SAS theorem triangle ABC is congruent to triangle EDF.

4. Question

In two triangles ABC and ADC, if $AB = AD$ and $BC = CD$. Are they congruent?

Answer

Yes, the given triangles are congruent as AB= AD, BC= CD and AC is a common side. Hence, by SSS theorem triangle ABC is congruent to triangle ADC.

5. Question

In triangles ABC and CDE, if $AC = CE$, $BC = CD$, $\angle A = 60°$, $\angle C = 30°$ and $\angle D = 90°$. Are two triangles congruent?

Answer

Yes, the two given triangles are congruent because AC = CE, BC= CD

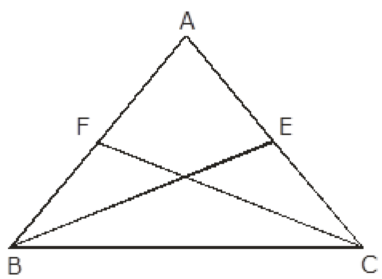
And $\angle B = \angle D = 90°$.

Therefore by SSA criteria triangle ABC is congruent to triangle CDE.

6. Question

ABC is an isosceles triangle in which $AB = AC$. BE and CF are its two medians. Show that $BE = CF$.

Answer



Given,

ABC is an isosceles triangle

AB = AC

BE and CF are two medians

To prove: BE = CF

Proof: In $\Delta BEC \text{ and } \Delta CFB$

CE = BF (Since, AC = AB = $\frac{1}{2}AC$ = $\frac{1}{2}$AB = CE = BF)

∠ECB = ∠FBC (Angle opposite to equal sides are equal)

BC = BC (Common)

Therefore, By SAS theorem

ΔBEC ≅ CFB

$BE = CF$ (By c.p.c.t)

7. Question

Find the measure of each angle of an equilateral triangle.

Answer

Let ABC is an equilateral triangle

We know that all the angles in an equilateral triangle are equal.

Therefore,

∠A = ∠B = ∠C (i)

Now,

∠A + ∠B + ∠C = 180^o (Sum of all angles of a triangle)

∠A + ∠A + ∠A = 180^o

3∠A = 180^o

∠A = $\frac{180}{3}$

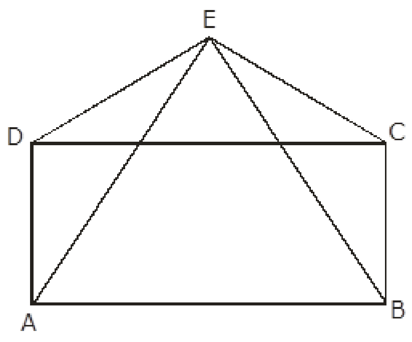
= 60^o

Hence, the measure of each angle of an equilateral triangle is 60^o.

8. Question

CDE is an equilateral triangle formed on a side *CD* of a square *ABCD*. Show that Δ *ADE* ≅ Δ *BCE*.

Answer



Given: An equilateral triangle CDE is on side CD of square ABCD

To prove: $\Delta ADE \cong \Delta BCE$

Proof: ∠EDC = ∠DCE = ∠CED = 60^o (Angles of equilateral triangle)

$\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^o$ (Angles of square)

$\angle EDA = \angle EDC + \angle CDA$

$= 60^o + 90^o$

$= 150^o$ (i)

Similarly,

$\angle ECB = 150^o$ (ii)

In $\Delta ADE \text{ and } \Delta BCE$

ED = EC (Sides of equilateral triangle)

AD = BC (Sides of square)

$\angle EDA = \angle ECB$ [From (i) and (ii)]

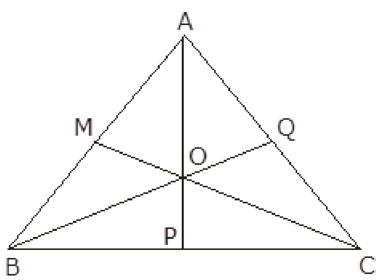
Therefore, By SAS theorem

$\Delta ADE \cong \Delta BCE$

9. Question

Prove that the sum of three altitudes of a triangle is less than the sum of its sides.

Answer



In Δ APB,

AP is a median

Angle APB is greater than angle ABP

So angle opposite to greater side is longer

Therefore, AB is greater than AP

Hence proved that in triangle APB the side of the triangle is more than the median or altitude AP of the triangle.

Similarly, in the same manner the other triangles can also be proved.

10. Question

In Fig. 10.134, if $AB = AC$ and $\angle B = \angle C$. Prove that $BQ = CP$.

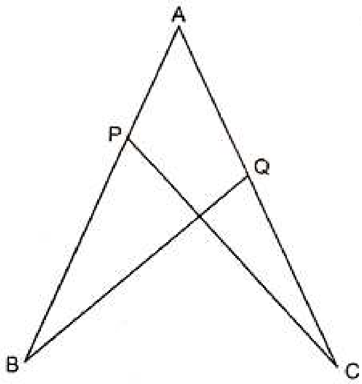


Fig. 10.134

Answer

Given,

∠B = ∠C

AB = AC

To Prove: BQ = CP

Proof: In △ABQ and △ACP

∠B = ∠C (Given)

AB = AC (Given)

∠A = ∠A (Common)

Hence, by A.S.A. Theorem

△ABQ ≅ △ACP

BQ = CP (By c.p.c.t)

1. Question

If Δ *ABC* ≅ Δ *LKM*, then side of Δ *LKM* equal to side *AC* of Δ *ABC* is

A. *LK*

B. *KM*

C. *LM*

D. None of these

Answer

Since, by corresponding part of congruent triangle AC of △ABC is equal to the LM of △LKM.

2. Question

If Δ *ABC* ≅ Δ *ACB*, then Δ *ABC* is isosceles with

A. *AB* = *AC*

B. *AB* = *BC*

C. *AC* = *BC*

D. None of these

Answer

AB and AC are the equal sides of equilateral triangle ABC because only then after reverting it to form △ACB, the two triangles can be proved congruent.

3. Question

If $\Delta\ ABC \cong \Delta\ PQR$ and $\Delta\ ABC$ is not congruent to $\Delta\ PQR$, then which of the following not true:

A. $BC = PQ$

B. $AC = PR$

C. $AB = PQ$

D. $QR = BC$

Answer

Since, BC and PQ are the non c.p.c.t. part of the two given triangles, hence we cannot judge them to be equal or not from the given information about them.

4. Question

In triangle ABC and PQR three equality relations between some parts are as follows:

$AB = QP$, $\angle B = \angle P$ and $BC = PR$

State which of the congruence conditions applies:

A. SAS

B. ASA

C. SSS

D. RHS

Answer

Since, two adjacent sides and the so formed angle with the two sides are shown equal while proving them congruent. Hence, by S.A.S. theorem the two triangles can be proved congruent.

5. Question

In triangles ABC and PQR, if $\angle A = \angle R$, $\angle B = \angle P$ and $AB = RP$, then which one of the following congruence conditions applies:

A. SAS

B. ASA

C. SSS

D. RHS

Answer

Since, the two adjacent angles and the contained side are shown equal while proving the two triangles congruent. Hence, by A.S.A. theorem the two triangles can be proved congruent.

6. Question

In $\Delta\ PQR \cong \Delta\ EFD$ then $ED =$

A. PQ

B. QR

C. PR

D. None of these

Answer

Since, by corresponding part of congruent triangle ED of ΔEFD is equal to the PR of ΔPQR.

7. Question

In $\Delta\ PQR \cong \Delta\ EFD$ then $\angle E =$

A. $\angle P$

B. $\angle Q$

C. $\angle R$

D. None of these

Answer

Since, by corresponding part of congruent triangle ∠E of ∆EFD is equal to the ∠P of ∆PQR.

8. Question

In a $\Delta\ ABC$, if $AB = AC$ and BC is produced to D such that $\angle ACD$ =100°, then $\angle A$ =

A. 20°

B. 40°

C. 60°

D. 80°

Answer

∠ACB + ∠ACD = 180°

On solving we get,

∠ACB = 80°

∠ABC = ∠ACB = 80° (Angles opposite to equal sides are equal)

In ∆ABC,

∠A + ∠B + ∠C= 180°

∠A + 80° + 80° = 180°

∠A = 20°

9. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, then the measure of vertex angle of the triangle is

A. 100°

B. 120°

C. 110°

D. 130°

Answer

Let the base angles be x each,

Vertex angle = 2(x + x) = 4x

Now, since the sum of all the angles of a triangle is 180°

x + x + 4x = 180°

6x = 180°

x = 30°

Therefore, vertex angle= 4x = 120°

10. Question

D, E, F are the mid-point of the sides *BC*, *CA* and *AB* respectively of $\Delta\ ABC$. Then $\Delta\ DEF$ is congruent to triangle

A. *ABC*

B. *AEF*

C. *BFD, CDE*

D. *AFE, BFD, CDE*

Answer

Since, the so formed triangle divides the complete triangle ABC into four congruent triangles.

11. Question

Which of the following is not criterion for congruence of triangles?

A. SAS

B. SSA

C. ASA

D. SSS

Answer

Since the two triangles with two adjacent sides and an angle adjacent to any one side among them are shown equal, then the two triangles will be similar but not necessarily congruent.

12. Question

In Fig. 10.135, the measure of $\angle B'AC'$ is

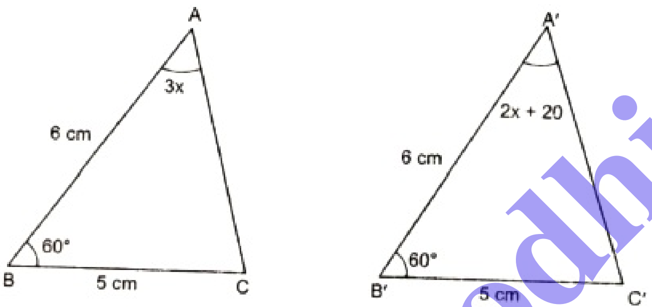


Fig. 10.135

A. 50°

B. 60°

C. 70°

D. 80°

Answer

In ΔABC and ΔA'B'C',

AB= A'B' (Given)

$\angle$B = $\angle$B'(Given)

BC = B'C' (Given)

Hence, by S.A.S. theorem,

$\Delta ABC \cong \Delta A'B'C'$

Therefore,

By c.p.c.t

$\angle A = \angle A'$

$3x = 2x + 20$

$x = 20°$

Therefore angle A' = 2x +20

= 60°

13. Question

If *ABC* and *DEF* are two triangles such that $\Delta ABC \cong \Delta FDE$ and $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$, Then, which of the following is true?

A. $DF = 5$ cm, $\angle F = 60°$

B. $DE = 5$ cm, $\angle E = 60°$

C. $DF = 5$ cm, $\angle E = 60°$

D. $DE = 5$ cm, $\angle D = 40°$

Answer

In ΔABC,

$\angle A + \angle B + \angle C = 180$

$80° + 40° + \angle C = 180°$

$\angle C = 60°$

Now, by c.p.c.t

AB = DF = 5cm

And

$\angle C = \angle E = 60°$

14. Question

In Fig. 10.136, $AB \perp BE$ and $FE \perp BE$. If $BC = DE$ and $AB = EF$, then ΔABD is congruent to

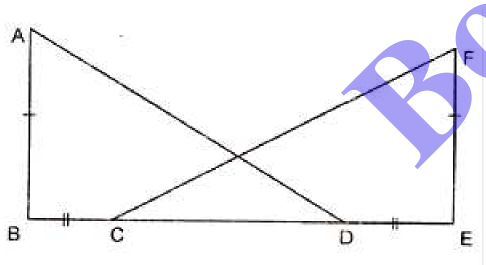


Fig. 10.136

A. Δ *EFC*

B. Δ *ECF*

C. Δ *CEF*

D. Δ *FEC*

Answer

In ΔABD and ΔFEC,

AB= FE (Given)

$\angle B = \angle E$ (Each 90°)

BC = DE (Given)

Add CD both sides, we get

BD = EC

Therefore, by S.A.S. theorem,

$\triangle ABD \cong \triangle FEC$

18. Question

ABC, is an isosceles triangle such that *AB*=*AC* and *AD* is the median to base *BC*. Then, $\angle BAD$=

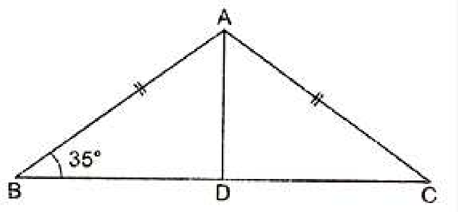


Fig. 10.137

A. 55°

B. 70°

C. 35°

D. 110°

Answer

For an isosceles triangle ∠ABC = ∠ACB = 35°

Let the ∠ADB be x

Then,

$\angle ADC = 180^o - x$

As AD is median so BD = CD

And for isosceles triangle AB = AC

So,

$\frac{AB}{AC} = \frac{BD}{CD} = 1$

By angle bisector theorem,

∠BAD = ∠CAD = y (Let)

For $\triangle$BAD

35 + x + y =180 (i)

For $\triangle$DAC

35 + 180 – x + y = 180 (ii)

35 + y = x

Therefore,

$35 + 34 + y + y = 180^o$

$2y + 70 = 180^o$

$2y = 100^o$

$y = 55^o$

Therefore,

∠BAD = ∠CAD = 55°

16. Question

In Fig. 10.138, if $AE||DC$ and AB=AC, the value of $\angle ABD$ is

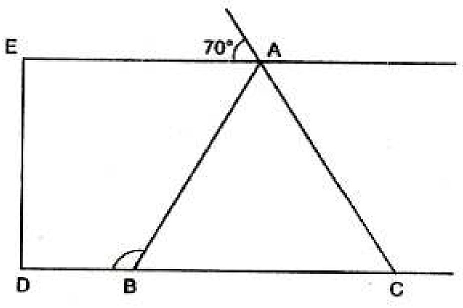


Fig. 10.138

A. 75°

B. 110°

C. 120°

D. 130°

Answer

∠EAP = ∠BCA (Corresponding angles)

∠BCA = 70°

∠CBA = ∠BCA (Angles opposite to equal sides are equal)

∠CBA = 70°

Now,

∠ABD + ∠CBA = 180°

∠ABD + 70 = 180°

∠ABD = 110°

17. Question

In Fig. 10.139, ABC is an isosceles triangle whose side AC is produced to E. Through C, CD is drawn parallel to BA. The value of x is

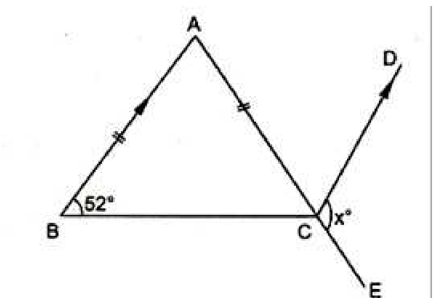


Fig. 10.139

A. 52°

B. 76°

C. 156°

D. 104°

Answer

∠B = ∠C (Angles opposite to equal sides are equal)

In △ABC,

∠A + ∠B+ ∠C= 180°

∠A = 76°

Now,

∠BAC= ∠ACD (Alternate angles)

∠ACD = 76°

Now,

∠ACD + ∠ECD= 180°

x = 180- 76°

x = 104°

18. Question

In Fig. 10.140, *X* is a point in the interior of square *ABCD.AXYZ* is also a square. If *DY* = 3 cm and *AZ* = 2 cm, then *BY* =

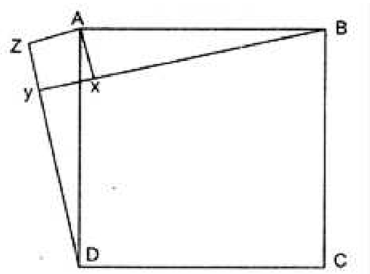


Fig. 10.140

A. 5 cm

B. 6 cm

C. 7 cm

D. 8 cm

Answer

∠Z = 90° (Angle of square)

Therefore, AZD is a right angle triangle,

By Pythagoras theorem,

$AD^2 = AZ^2 + ZD^2$

$AD^2 = 2^2 + (2+3)^2$

$AD^2 = 4 + 25$

AD = √29

In △AXB, withX as right angle,

By Pythagoras theorem,

$AB^2 = AX^2 + XB^2$

$XB^2 = 29-4$

XB = 5

BY = XB + XY

= 5 + 2

= 7cm

19. Question

In Fig. 10.141, *ABC* is a triangle in which $\angle B = 2\angle C$. *D* is a point on side such that *AD* bisects $\angle BAC$ and $AB = CD$. BE is the bisector of $\angle B$. The measure of $\angle BAC$ is

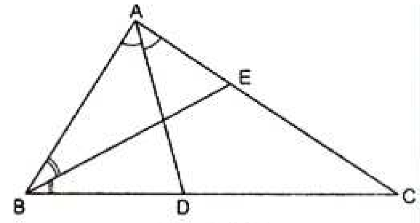


Fig. 10.141

[Hint: $\Delta\ ABE \cong \Delta\ DCE$]

A. 72°

B. 73°

C. 74°

D. 95°

Answer

Given that ΔABC

BE is bisector of ∠Band AD is bisector of ∠BAC

∠B = 2 ∠C

By exterior angle theorem in triangle ADC

∠ADB = ∠DAC + ∠C (i)

In ΔADB,

∠ABD + ∠BAD + ∠ADB = 180^o

2 ∠C + ∠BAD + ∠DAC + ∠C = 180^o [From (i)]

3 ∠C + ∠BAC = 180^o

∠BAC = 180^o – 3 ∠C (ii)

Therefore,

AB = CD

∠C = ∠DAC

∠C = 1/2 ∠BAC (iii)

Putting value of Angle C in (ii), we get

∠BAC = 180^o – 1/2 ∠BAC

∠BAC + $\frac{3}{2}$∠BAC = 180^o

$\frac{5}{2}$∠BAC = 180^o

∠BAC = $\frac{180*2}{5}$

= 72^o

∠BAC = 72^o

20. Question

In Fig. 10.142, if *AC* is bisector of ∠*BAD* such that *AB*=3 cm and *AC*=5 cm, then *CD*=

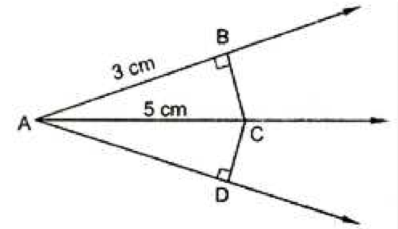


Fig. 10.142

A. 2 cm

B. 3 cm

C. 4 cm

D. 5 cm

Answer

In ΔABC using Pythagoras theorem, we get

$AB^2 + BC^2 = AC^2$

$9 + BC^2 = 25$

BC = 4 cm

In $\Delta ABC \text{ and } \Delta ADC$

∠BAC = ∠CAD (Therefore, AC is bisector of ∠A)

∠B = ∠D = 90^o

∠ABC + ∠BCA + ∠CAB = 180^o

∠CAD + ∠ADC + ∠DCA = 180^o

∠ABC + ∠BCA + ∠CAB = ∠CAD + ∠ADC + ∠DCA

∠BCA = ∠DCA (i)

In $\Delta ABC \text{ and } \Delta ADC$

∠CAB = ∠CAD (Therefore, AC is bisector of ∠A)

∠BCA = ∠DCA [From (i)

AC = AC (Common)

By ASA theorem, we have

$\Delta ABC \cong \Delta ADC$

BC = CD (By c.p.c.t)

CD = 4cm