1. Sets

Exercise 1.1

1. Question

What is the difference between a collection and a set? Give reasons to support your answer.

Answer

A *collection* can include different types of elements. Whereas a *set is* a well-defined collection of distinct elements and it *is* enclosed *in* "{}."

E.g.:

The collection of good teachers is a collection but not a set

Whereas when we take all the teachers of the school, then it is set. Because when we take all the teachers, then it is a definite number, so it called a set. But when we say, good teachers, it changes opinion wise, so we can say it is not definite hence we can call it a collection as it is not well defined.

2. Question

Which of the following collections are sets? Justify your answer:

(i) A collection of all natural numbers less than 50.

(ii) The collection of good hockey players in India.

(iii) The collection of all the girls in your class.

(iv) The collection of all talented writers of India.

(v) The collection of difficult topics in Mathematics.

(vi) The collection of novels written by Munshi Prem Chand.

(vii) The collection of all questions of this chapter.

(viii) The collection of all months of a year beginning with the letter J.

(ix) A collection of most dangerous animals of the world.

(x) The collection of prime integers.

Answer

(i) It is a set. As we are saying all numbers less than 50 it is a definite quantity hence it is a set.

(ii) It is not a set. Because when we say good hockey players, it changes opinion wise and cannot be well defined or is not countable.

(iii) It is a set. As we are saying a collection of all the girls in your class it is a definite quantity hence it is a set.

(iv) It is not a set. Because when we say a collection of all talented writers of India, it changes opinion wise and cannot be well defined or is not countable.

(v) It is not a set. Because when we say a collection of all talented writers of India, it changes opinion wise and cannot be well defined or is not countable.

(vi) It is a set. As we are saying a collection of novels written by Munshi Prem Chand.it is a definite quantity hence it is a set.

(vii) It is a set. As we are saying a collection of all the questions in this chapter. It is a definite quantity hence it is a set.

(viii) It is a set. As we are saying a collection of all months of a year beginning with the letter J. It is a definite quantity hence it is a set.

(ix) It is not a set. Because when we say a collection of most dangerous animals of the world it changes opinion wise and we cannot define dangerousness of an animal.

(x) It is a set as we are saying a collection of prime integers. It is a definite quantity hence it is a set.

3. Question

If A={0,1,2,3,4,5,6,7,8,9,10}, then insert the appropriate symbol or in each of the following blank spaces:

i. 4 ……A ii. -4……A

iii. 12…..A iv. 9…..A

v. 0……A vi. -2……A

Answer

First of all it is important that you know what does (ϵ) this symbol means. In terms of mathematics it is used as belongs to () is used as does not belong to.

i.4 ……A

As 4 is there in Set A then we can say 4 belongs to A

∴ 4 ϵ A

ii. -4……A

As -4 is not there in Set A then we can say 4 does not belong to A

∴ 4 A

iii. 12…..A

As 12 is not there in Set A then we can say 12 does not belong to A

∴ 12 A

iv.9 ……A

As 9 is there in Set A then we can say 9 belongs to A

∴ 9 ϵ A

v.0 ……A

As 0 is there in Set A, then we can say 0 belongs to A

∴ 0 ϵ A

vi.-2 ……A

As -2 is not there in Set A then we can say -2 does not belong to A

∴ -2 A

Exercise 1.2

1 A. Question

Describe the following sets in Roster form:

{x:x is a letter before e in the English alphabet}.

Answer

Here x:x it is read as x is such that x

So now when we read whole sentence it becomes x is such that x is a letter before e in the English alphabet. Now letters before e are a,b,c,d.

∴ Roster form will be {a,b,c,d}.

1 B. Question

Describe the following sets in Roster form:

{x ∈N: x^2< 25}.

Answer

First thing analyze the given data. x ϵ N that implies x is a natural number.

$x^2 < 25$

$\Rightarrow x < \pm 5$

As x belongs to the natural number that means x < 5.

All numbers less than 5 are 1,2,3,4.

∴ Roster form will be {1,2,3,4}.

1 C. Question

Describe the following sets in Roster form:

{x N: x is a prime number, 10 < x < 20}

Answer

X is a natural number and is between 10 and 20.

X is such that x is a prime number between 10 and 20.

Prime numbers between 10 and 20 are 11,13,17,19.

∴ Roster form will be {11,13,17,19}.

1 D. Question

Describe the following sets in Roster form:

{x N: x = 2n, n N}.

Answer

X is a natural number also x = 2n

∴ Roster form will be {2,4,6,8…..}.

This an infinite set.

1 E. Question

Describe the following sets in Roster form:

{x R: x > x}.

Answer

Any real number is equal to its value it is neither less nor greater.

According to the question we have to write Roster form of such real numbers which has value less than itself. As there are no such numbers.

∴ Roster form will be ϕ.

This is called as null set.

1 F. Question

Describe the following sets in Roster form:

{x:x is a prime number which is a divisor of 60}.

Answer

All numbers which are divisor of 60 are = 1,2,3,4,5,6,10,12,15,20,30,60.

Now numbers which are prime are = 2,3,5.

∴ Roster form will be {2,3,5}.

1 G. Question

Describe the following sets in Roster form:

{x:x is a two digit number such that the sum of its digits is 8}

Answer

Numbers which have sum as 8 are = 17,26,35,44,53,62,71,80

⇒ Roster form will be {17,26,35,44,53,62,71,80}.

1 H. Question

Describe the following sets in Roster form:

The set of all letters in the word ‘Trigonometry’

Answer

All letters means no letter should be repeated

Trigonometry = t,r,i,g,o,n,m,e,y

∴ Roster form will be {t,r,i,g,o,n,m,e,y}

1 I. Question

Describe the following sets in Roster form:

The set of all letters in the word ‘Better.’

Answer

All letters means no letter should be repeated

Better = b,e,t,r

∴ Roster form will be {b,e,t,r}.

2. Question

Describe the following sets in set-builder form:

(i) A = {1,2,3,4,5,6}

(ii) B={1, 1/2 , 1/3, 1/4, 1/5,.....}

(iii) C={0,3,6,9,12,....}

(iv) D={10,11,12,13,14,15}

(v) E={0}

(vi) {1,4,9,16,...,100}

(vii) {2,4,6,8,....}

(viii) {5,25,125,625}

Answer

(i) {x:x ∈ N,x<7}

This is read as x is such that x belongs to natural number and x is less than 7. It satisfies all condition of roster form.

(ii) {x ∈ z: x=1/n+1,n ∈ W}

This is read as x is such that x is an integer greater than or equal to 0. And it’s value is 1/x+1.

(iii) {x N: x = 3n, n W}

(iv) {x:x ∈ N,9<x<16}

(v) {x:x=0}

(vi) {x ∈ N: $x=n^2$,n≤10, n ∈ N}

(vii) {x N: x = 2n, n N}

(viii) {x N: x = 5^n,0<n<6 N}

3 A. Question

List all the elements of the following sets:

A={x:x^2≤ 10, xZ}

Answer

First of all, x is an integer hence it can be positive and negative also.

$X^2 \leq 10$

X≤ √10

X =±1,±2,±3

A={±1,±2,±3}

3 B. Question

List all the elements of the following sets:

$$b = \left\{ x : x = \frac{1}{2n-1}, 1 \leq n \leq 5 \right\}$$

Answer

Substituting the values of n we will get the solutions

At n=1, $x = \frac{1}{2(1)-1} = \frac{1}{1}$

At n=1, $x = \frac{1}{2(2)-1} = \frac{1}{3}$

At n=1, $x = \frac{1}{2(3)-1} = \frac{1}{5}$

At n=1, $x = \frac{1}{2(4)-1} = \frac{1}{7}$

At n=1, $x = \frac{1}{2(5)-1} = \frac{1}{9}$

$\therefore x = 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$

3 C. Question

List all the elements of the following sets:

C=$b = \left\{ x : x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2} \right\}$

Answer

∴ x is an integer between -1/2 and 9/2

So all integers between given values -0.5<x<4.5

0,1,2,3,4

∴ C = {0,1,2,3,4}

3 D. Question

List all the elements of the following sets:

D={x:x is a vowel in the word “EQUATION”}

Answer

All vowels in equation are E,U,A,I,O

∴ D={A,E,I,O,U}

3 E. Question

List all the elements of the following sets:

e={x:x is a month of a year not having 31 days}

Answer

All months of a year do not have 31 days :

February, April, June, September, November.

E: { February, April, June,September,November}

3 F. Question

List all the elements of the following sets:

F={x:x is a letter of the word “MISSISSIPPI”}

Answer

Letters in word M, I, S, P.

F={M,I,S,P}.

4. Question

Match each of the sets on the left in the roster form with the same set on the right described in the set-builder form:

i.{A,P,L,E} i. {x:x+5=5, x z}

ii.{5,-5} ii. {x:x is a prime natural number and a divisor of 10}

iii. {0} iii. {x:x is a letter of the word “RAJASTHAN”}

iv. {1,2,5,10} iv. {x:x is a natural and divisor of 10}

v. {A,H,j,R,S,T,N} v. {x:x^2 – 25 =0}

vi.{2,5} vi. {x:x is a letter of word “APPLE”}

Answer

When we are dealing with the set builder - roster form match the following it is always better to go in the direction in which we need to convert set builder to roster rather than roster to set builder.

For i. {x:x+5=5, x z}

X is such that x+5 = 5

⇒ x = 5-5

⇒ x = 0

Roster form = {0}

i.e third option

For ii. {x:x is a prime natural number and a divisor of 10}

All natural numbers that are divisor of 10 are = 1,2,5,10

Numbers that are prime are = 2,5

Roster form= {2,5}

i.e. sixth option.

For iii. {x:x is a letter of the word "RAJASTHAN"}

All letters means no letter should be repeated

RAJASTHAN = R,A,J,S,T,H,N

∴ Roster form will be {A,H,J,R,S,T,N}

i.e. fifth option

For iv. {x:x is a natural and divisor of 10}

All natural numbers that are divisor of 10 are = 1,2,5,10.

Roster form= {1,2,5,10}

i.e. Fourth option.

For v. {x:x^2 – 25 =0}

X^2-25 = 0

X =√25

X =±5.

∴ Roster form will be {5,-5}

i.e. second option

for vi. {x:x is a letter of word "APPLE"}

All letters means no letter should be repeated

APPLE = A,P,L,E

∴ Roster form will be {A,E,L,P}

i.e. first option.

5. Question

Write the set of all vowels in the English alphabet which precede q.

Answer

Set of all vowels which precede q.

Precede means to come before.

A, E, I, O these are the vowels they come before q.

B = {A,E,I,O}.

6. Question

Write the set of all positive integers whose cube is odd.

Answer

Every odd number has an odd cube

Odd numbers can be represented as 2n+1.

{2n+1:n ϵ W} or

{1,3,5,7,......}

7. Question

Write the set $\left\{\frac{1}{2},\frac{2}{5},\frac{3}{10},\frac{4}{17},\frac{5}{26},\frac{6}{37},\frac{7}{50}\right\}$ in the set-builder form.

Answer

Here we can see denominator is square of numerator +1.

We can set builder form as

⇒ $\left\{x:\frac{x}{x^2+1}, 0 < x < 8, x \in N\right\}$

Exercise 1.3

1. Question

Which of the following are examples of empty set?

(i) Set of all even natural numbers divisible by 5.

(ii) Set of all even prime numbers.

(iii) {x:x^2–2=0 and x is rational}.

(iv) {x:x is a natural number, x < 8 and simultaneously x > 12}.

(v) {x:x is a point common to any two parallel lines}.

Answer

Note: The Empty set is the set which does not contain any element. Most of the people get confused whether {0} is an empty set or not. It is not because it contains an element 0.

(i) All numbers ending with 0. Except 0 is divisible by 5 and are even. Hence it is not an example of empty set.

(ii) 2 is a prime number and is even, and it is the only prime which is even. So no this not an example of the empty set.

(iii) There is not natural number whose square is 2. So it is an example of empty set.

(iv) Never can a number be simultaneously less than 8 and greater than 12. Hence it is an example of the empty set.

(v) No two parallel lines can never have a common point. Hence it is an example of empty set.

2. Question

Which of the following sets are finite and which are infinite?

(i) Set of concentric circles in a plane.

(ii) Set of letters of the English Alphabets.

(iii) {x ∈ N : x > 5}

(iv) {x ∈ N : x < 200}

(v) {x ∈ Z: x < 5}

(vi) {x ∈ R:0 < x < 1}.

Answer

(i) In a plane there can be infinite concentric circles. Hence it is an infinite set.

(ii) There are just 26 letters in English Alphabets and are finite. Hence it is finite set.

(iii) It is an infinite set because, natural numbers greater than 5 is infinite.

(iv) It is a finite set. Natural numbers start from 1 and there are 199 numbers less than 200. Hence it is finite.

(v) It is an infinite set. Because integers less than 5 are infinite.

(vi) It is an infinite set. Because between two real numbers, there are infinite real numbers.

3. Question

Which of the following sets are equal?

i. A = {1, 2, 3}

ii. $B=\{x \in R: x^2-2x+1=0\}$

iii. C= (1, 2, 2, 3}

iv. $D=\{x \in R : x^3 - 6x^2+11x - 6 = 0\}$.

Answer

NOTE: A set is said to be equal with another set if all elements of both the sets are equal and same.

A = {1, 2, 3}

$B =\{x \in R: x^2-2x+1=0\}$

$x^2-2x+1=0$

$(x-1)^2 = 0$

$\therefore x=1.$

B = {1}

C= {1, 2, 2, 3}

In sets we do not repeat elements hence C can be written as {1, 2, 3}

$D = \{x \in R : x^3 - 6x^2+11x - 6 = 0\}$.

For x = 1

$= (1)^3-6(1)^2+11(1)-6$

$= 1-6+11-6$

$= 0$

For x =2

$= (2)^3-6(2)^2+11(2)-6$

$= 8-24+22-6$

$= 0$

For x =3

$= (3)^3-6(3)^2+11(3)-6$

$= 27-54+33-6$

$= 0$

As cubic equation has three roots at max so the roots are 1, 2, 3

∴ D = {1, 2, 3}

Hence Set A, C and D are equal.

1. Question

Are the following sets equal?

A={x:x is a letter in the word reap}, B={x:x is a letter in the word paper},

C={x:x is a letter in the word rope}.

Answer

For A

Letters in word reap

A ={R, E, A, P} ={A, E, P, R}

For B

Letters in word paper

B ={P, A, E, R} = {A, E, P, R}

For C

Letters in word rope

C = {R, O, P, E} = {E, O, P, R}.

Set A = Set B

Because every element of set A is present in set B

But Set C is not equal to either of them because all elements are not present.

1. Question

From the sets given below, pair the equivalent sets:

A= {1, 2, 3}, B={t, p, q, r, s}, C={α, β, γ}, D={a, e, I, o, u}.

Answer

Note: Equivalent set are different from equal sets, Equivalent sets are those which have equal number of elements they do not have to be same.

A = {1, 2, 3}

Number of elements = 3

B={t, p, q, r, s}

Number of elements = 5

C={α, β, γ}

Number of elements = 3

D={a, e, I, o, u}

Number of elements = 5

Set A is equivalent with Set C and Set B is equivalent with Set D.

6 A. Question

Are the following pairs of sets equal? Give reasons.

A={2, 3}, B = {x : x is a solution of $x^2 + 5x + 6= 0$}

Answer

A = {2, 3}

B = {–2, –3}

As A and B do not have exactly same elements hence they are not equal.

6 B. Question

Are the following pairs of sets equal? Give reasons.

A={x:x is a letter of the word “WOLF”}

B={x:x is letter of word “FOLLOW”}

Answer

Every letter in WOLF

A ={W, O, L, F} = {F, L, O, W}

Every letter in FOLLOW

B = {F, O, L, W} = {F, L, O, W}

As A and B have same number of elements which are exactly same hence they are equal sets.

7. Question

From the sets given below, select equal sets and equivalent sets.

A={0, a}, B= {1, 2, 3, 4} C= {4, 8, 12},

D = {3, 1, 2, 4}, E={1, 0}, F = {8, 4, 12}

G={1, 5, 7, 11}, H={a, b}

Answer

We first of all need to manipulate some of the sets

D = {3, 1, 2, 4} = {1, 2, 3, 4, }

F = {8, 4, 12} = {4, 8, 12}

Equivalent sets:

i. A, E, H (all of them have exactly two elements in them)

ii. B, D, G (all of them have exactly four elements in them)

iii. C, F (all of them have exactly three elements in them)

Equal sets :

i. B, D (all of them have exactly the same elements, so they are equal)

ii. C, F (all of them have exactly the same elements, so they are equal)

8. Question

Which of the following sets are equal?

A= {x:x ∈ N, x < 3},

B= {1, 2}, C= {3, 1}

D = {x : x∈ N, x is odd, x < 5},

E= (1, 2, 1, 1}, F= {1, 1, 3}.

Answer

A = x is a natural number. And x is less than 3

So all natural numbers less than 3 constitute set A.

{1, 2}

A ={1, 2}

B = {1, 2}

C = {1, 3}

D = x is a natural number. And x is less than 5 and is odd.

So all odd natural numbers less than 5 constitute set D.

{1, 3}

D = {1, 3}

E ={1, 2, 1, 1}

We don’t repeat same elements in a set.

∴ E = {1, 2}

F ={1, 1, 3}

We don’t repeat same elements in a set.

∴ F = {1, 3}

∴ A = {1, 2}

B = {1, 2}

C = {1, 3}

D = {1, 3}

E = {1, 2}

F = {1, 3}

Now, we can see clearly that set A, B, E are equal and set C, D, F are equal.

9. Question

Show that the set of letters needed to spell “CATARACT” and the set of letters needed to spell “TRACT” are equal.

Answer

For “CATRACTR”

Letters in word are

{C, A, T, R} ={A, C, R, T}

For “TRACT”

Letters in word are

{T, R, A, C} = {A, C, R, T}

As we see letters need to spell cataract is equal to set of letters need to spell tract.

Hence Proved.

Exercise 1.4

1. Question

Which of the following statements are true? Give a reason to support your answer.

(i) For any two sets A and B either A B or B A.

(ii) Every subset of an infinite set is infinite.

(iii) Every subset of a finite set is finite.

(iv) Every set has a proper subset.

(v) {a,b,a,b,a,b,....} is an infinite set.

(vi) {a,b,c} and {1,2,3} are equivalent sets.

(vii) A set can have infinitely many subsets.

Answer

(i) False

No, it is not necessary for any two set A and B to be either A B or B A.

Let A = {1,2} and B ={a,b}

Here neither A B nor B A.

(ii) False

A = {1,2,3} It is subset of infinite set N and is finite.

(iii) True

Even if we think logically then also smaller part of something finite can never be infinite.

(iv) False

Null set or empty set does not have a proper subset.

(v) False

We do not repeat elements in a set, so the given set becomes {a,b} which is a finite set.

(vi) True

In both of the number of the set of elements are same.

(vii) False

In A = {1}

The subsets are ϕ and {1} which are finite.

2. Question

State whether the following statements are true or false:

(i) 1 { 1,2,3}

(ii) a ⊂ {b,c,a}

(iii) {a} {a,b,c}

(iv) {a,b} = {a,a,b,b,a}

(v) The set {x:x + 8 = 8} is the null set.

Answer

(i) True

1 belongs to the given set as it is present in it.

(ii) False

{a} ⊂ {b,c,a}

This is the write form to write it or a ϵ {b,c,a}

(iii) False

{a} ⊂ {b,c,a}

This is the write form to write it or a ϵ {b,c,a}

(iv) True

We do not repeat same elements in a given set.

∴ The given set can be written as {a,b}

(v) False

X+8 = 8

i.e. x = 0

{0} It is not a null set

3. Question

Decide among the following sets, which are subsets of which:

A = {x:x satisfies x^2 – 8x + 12=0}

B= {2,4,6}

C= {2,4,6,8,….}

D={6}

Answer

A = x^2 – 8x + 12=0

= (x–6)(x–2) =0

= x=2 or x=6A = {2,6}

B ={2,4,6}

C= {2,4,6,8}

D ={6}

So we can say

D⊂A⊂B⊂C

4. Question

Write which of the following statements are true? Justify your answer.

(i) The set of all integers is contained in the set of all rational numbers.

(ii) The set of all crows is contained in the set of all birds.

(iii) The set of all rectangles is contained in the set of all squares.

(iv) The set of all rectangle is contained in the set of all squares.

(v) The sets P={a} and B = {{a}} are equal.

(vi) The sets A={x:x is a letter of word "LITTLE"} AND, b = {x:x is a letter of the word "TITLE" } are equal.

Answer

(i) True

A rational number is represented by the form p/q where p and q are integers and (q not equal to 0) keeping q=1 we can place any number as p. Which then will be an integer.

(ii) True

All crows are birds, so they are contained in the set of all birds.

(iii) False

Every square can be a rectangle, but every rectangle cannot be a square.

(iv) False

Every square can be a rectangle, but every rectangle cannot be a square.

(v) False

$P = \{a\}$

$B = \{\{a\}\}$

But $\{a\} = P$

$B = \{P\}$

Hence they are not equal.

(vi) True

A = For "LITTLE"

$A = \{L,I,T,E\} = \{E,I,L,T\}$

B = For "TITLE"

$B = \{T,I,L,E\} = \{E,I,L,T\}$

5. Question

Which of the following statements are correct? Write a correct form of each of the incorrect statements.

(i) $a \subset \{a,b,c\}$

(ii) $\{a\}\ \{a,b,c\}$

(iii) $a\ \{\{a\},b\}$

(iv) $\{a\} \subset \{\{a\},b\}$

(v) $\{b,c\} \subset \{a,\{b,c\}\}$

(vi) $\{a,b\} \subset \{a,\{b,c\}\}$

(vii) $\phi\ \{a,b\}$

(viii) $\phi \subset \{a,b,c\}$

(ix) $\{x : x + 3 = 3\} = \phi$

Answer

(i) In this a isn't subset of given set but belongs to the given set.

∴ The correct form would be

$a \in \{a,b,c\}$

(ii) In this $\{a\}$ is subset of $\{a,b,c\}$

∴ The correct form would be

$\{a\} \subset \{a,b,c\}$

(iii) In this a is not the element of the set.

∴ The correct form would be

$\{a\} \in \{\{a\},b\}$

(iv) In this {a} is not athe subset of given set

∴ The correct form would be

{a} ϵ {{a},b}

(v) {b,c} is not a subset of given set. But it belongs to the given set.

∴ The correct form would be

{b,c} ϵ {a,{b,c}}

(vi) {a,b} is not a subset of given set

∴ The correct form would be

{a,b} {a,{b,c}}

(vii) ϕ does not belong to given set but it is subset

∴ The correct form would be

ϕ ⊂ {a,b}

(viii) True ϕ is subset of every set

(ix) X+3=3

X=0

{0}

It is not ϕ

∴ The correct form would be

{x:x + 3 = 3}= {0}

6. Question

Let A={a,b,{c,d},e}. Which of the following statements are false and why?

(i) {c,d} ⊂ A

(ii) {c, d} A

(iii) {{c,d}} ⊂ A

(iv) a A

(v) a ⊂ A.

(vi) {a,b,e} ⊂ A

(vii) {a,b,e} A

(viii) {a,b,c} ⊂ A

(ix) ϕ A

(x) {ϕ} ⊂ A

Answer

(i) False

{c,d} is not a subset of A but it belong to A.

{c,d} ϵ A

(ii) True

{c,d} ϵ A

(iii) True

{c,d} is a subset of A.

(iv) It is true that a belong to A.

(v) False

a is not a subset of A but it belongs to A

(vi) True

(vii) False

{a,b,e} ⊂ A this is the correct form.

(viii) False

{a,b,c} is not a subset of A

(ix) False

ϕ is a subset of A.

ϕ ⊂ A.

(x) False

7. Question

Let A = {{1,2,3}, {4,5}, {6,7,8}}. Determine which of the following is true or false:

(i) 1 A

(ii) {1,2,3} ⊂ A

(iii) {6,7,8} A

(iv) {2, ϕ} ⊂ A

(v) 2 ⊂ A

(vi) {2,{1}} A

(vii) {{2}}, {1}} A

(viii) {ϕ,{ϕ}, {1, ϕ}} ⊂ A.

Answer

(i) False

1 is not an element of A.

(ii) True

{1,2,3} ϵ A. this is correct.

(iii) True.

{6, 7, 8} ϵ A.

(iv) False

2 is not an element of A, and ϕ is also not an element of A. Hence, false.

(v) False

2 alone is not an element of A. Hence, false.

(vi) True

{1, 2, 3} is the set belonging to A. {2, {1}} does not belong to A.

(vii) True

$\{\{2\}, \{1\}\}$ does not belongs to A. Hence, true.

(viii) False

Φ does not belong to A.

8. Question

Let $A = \{\phi, \{\phi\}, 1, \{1, \phi\}, 2\}$. Which of the following are true?

(i) $\phi \in A$

(ii) $\{\phi\} \in A$

(iii) $\{1\} \in A$

(iv) $\{2, \phi\} \subset A$

(v) $2 \subset A$

(vi) $\{2, \{1\}\} \not\subset A$

(vii) $\{\{2\}, \{1\}\} \not\subset A$

(viii) $\{\phi, \{\phi\}, \{1, \phi\}\} \subset A$

(ix) $\{\{\phi\}\} \subset A$.

Answer

(i) True

Φ is a member of set A. Hence, true.

(ii) True

{ Φ} is a member of set A. Hence, true.

(iii) False

1 alone is not a member of A. Hence, false.

(iv) False

We can see that 2 is a member of set A, {2, Φ} is not. Hence, false

(v) True

2 is a member of set A. Hence, true.

(vi) True

{1} is not a member of set A.

(vii) True

Neither {2} and nor {1} is a member of set A. Hence, true.

(viii) True

All three are members of set A. Hence, true.

(ix) False

$\{\{\phi\}\}$ is not a member of set A. Hence, false.

9. Question

Write down all possible subsets of each of the following sets:

Total number of the subset of any given set $=2^n$

(i) {a}

(ii) {0,1}

(iii) {a,b,c}

(iv) {1,{1}}

(v) {ϕ}

Answer

(i) Subsets of given set are {a}, ϕ.

(ii) Subsets of given set are {0},{1},{0,1},ϕ.

(iii) Subsets of given set are {a},{b},{c},{a,b},{b,c},{a,c},{a,b,c},ϕ.

(iv) Subsets of given set are {1},{{1}},{1,{1}},ϕ.

(v) Subsets of given set are{ϕ},ϕ.

10. Question

Write down all possible proper subsets each of the following sets.

(i) {1,2}

(ii) {1,2,3}

(iii) {1}

Answer

(i) {1},{2},ϕ

(ii) {1},{2},{3},{1,2},{2,3},{1,3},ϕ.

(iii) ϕ ,{1}.

11. Question

What is the total number of proper subsets of a set consisting of n elements?

Answer

A total number of subsets for any given ser is 2^n.

In every set there is only one improper set i.e. the set itself.

∴ proper subsets would be 2^n-1.

Example : A = {1,2,3}There will be 2^3 - 1 = 8 - 1 = 7 proper subsets.These are [{ ∅},{1},{2},{3},{1,2},{1,3},{2,3}].

12. Question

If A is any set, prove that: A⊆ ϕ ⇔ A=ϕ.

Answer

Let A⊆ ϕ,

If A is a subset of an empty set, then A is the empty set.

∴ A = ϕ

Now let A = ϕ,

This means A is an empty set.

As we know that every set is a subset of itself.

∴ A ⊆ ϕ

Thus, we have,

A⊆ ϕ ⇔ A=ϕ

Hence, Proved.

13. Question

Prove that: A B, B C and C A A = C.

Answer

We have A B, B C and C A \

∴ A B C A.

Now, A is a subset of B and B is a subset of C, so

A is a subset of C, i.e., A C

Also, C A

Hence C= A.

14. Question

How many elements have P (A), if A= ϕ?

Answer

Empty set has zero element.

∴ Power set of ϕ has 2^0 = 1 element.

15. Question

What universal set (s) would you propose for each of the following:

(i) The set of right triangles.

(ii) The set of isosceles triangles

Answer

(i) Set of all triangles.

(ii) Set of all triangles.

Exercise 1.5

1 A. Question

If A and B are two sets such that A ⊂ B, then Find:

A ∩ B

Answer

A ∩ B means A intersection B. Common elements of A and B come in this group.

Given A ⊂ B

i.e. B is having all elements that is present in A.

∴ A ∩ B = A.

Example: A= {1,2}B={1,2,3,4}A ∩ B = { 1,2}   = A

1 B. Question

If A and B are two sets such that A ⊂ B, then Find:

A ∪ B

Answer

A ∪ B means A union B. All elements of A and B come in this set.

Given A ⊂ B

i.e. B is having all elements including elements of A.

∴ A ∪ B = B.

2. Question

If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8}, C = {7, 8, 9, 10, 11} and D = {10, 11, 12, 13 , 14}. Find:

i. A ∪ B

ii. A ∪ C

iii. B ∪ C

iv. B ∪ D

v. A ∪ B ∪ C

vi. A ∪ B ∪ D

vii. B ∪ C ∪ D

viii. A ∩ (B ∪ C)

ix. (A ∩ B) ∩ (B ∩ C)

x. (A ∪ D) ∩ (B ∪ C).

Answer

Note: In general X ∪ Y = {a: a ϵ X or a ϵ Y}

X ∩ Y = {a:a ϵ X and a ϵ Y}.

i. A ∪ B = {x: x ϵ A or x ϵ B}

= {1, 2, 3, 4, 5, 6, 7, 8}

ii. A ∪ C = {x: x ϵ A or x ϵ C}

= {1, 2, 3, 4, 5, 7, 8, 9, 10, 11}

iii. B ∪ C = {x: x ϵ B or x ϵ C}

= {4, 5, 6, 7, 8, 9, 10, 11}

iv. B ∪ D = {x: x ϵ B or x ϵ D}

= {4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

v. A ∪ B = {x: x ϵ A or x ϵ B}

= {1, 2, 3, 4, 5, 6, 7, 8}

A ∪ B ∪ C = {x: x ϵ A ∪ B or x ϵ C}

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

vi. A ∪ B = {x: x ϵ A or x ϵ B}

= {1, 2, 3, 4, 5, 6, 7, 8}

A ∪ B ∪ D = {x: x ϵ A ∪ B or x ϵ D}

= {1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

vii. B ∪ C = {x: x ϵ B or x ϵ C}

$= \{4, 5, 6, 7, 8, 9, 10, 11\}$

$B \cup C \cup D = \{x: x \in B \cup C \text{ or } x \in D\}$

$= \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

viii. $B \cup C = \{x: x \in B \text{ or } x \in C\}$

$= \{4, 5, 6, 7, 8, 9, 10, 11\}$

$A \cap B \cup C = \{x:x \in A \text{ and } x \in B \cup C\}$.

$= \{4, 5\}$

ix. $(A \cap B) = \{x:x \in A \text{ and } x \in B\}$.

$= \{4, 5\}$

$(B \cap C) = \{x:x \in B \text{ and } x \in C\}$

$= \{7, 8\}$

$(A \cap B) \cap (B \cap C) = \{x:x \in (A \cap B) \text{ and } x \in (B \cap C)\}$.

$= \phi$

x. $A \cup D = \{x: x \in A \text{ or } x \in D\}$

$= \{1, 2, 3, 4, 5, 10, 11, 12, 13, 14\}$.

$B \cup C = \{x: x \in B \text{ or } x \in C\}$

$= \{4, 5, 6, 7, 8, 9, 10, 11\}$

$(A \cup D) \cap (B \cup C) = \{x:x \in (A \cup D) \text{ and } x \in (B \cup C)\}$.

$= \{4, 5, 10, 11\}$.

3. Question

Let $A = \{x:x \in N\}$, $B = \{x:x = 2n, n \in N)$, $C = \{x:x = 2n - 1, n \in N\}$ and, $D = \{x:x$ is a prime natural number$\}$ Find:

i. $A \cap B$

ii. $A \cap C$

iii. $A \cap D$

iv. $B \cap C$

v. $B \cap D$

vi. $C \cap D$

Answer

A = All natural numbers i.e. {1, 2, 3.....}

B = All even natural numbers i.e. {2, 4, 6, 8...}

C = All odd natural numbers i.e. {1, 3, 5, 7......}

D = All prime natural numbers i.e. {1, 2, 3, 5, 7, 11, ...}

i. $A \cap B$

A contains all elements of B.

$\therefore B \subset A$

$\therefore A \cap B = B$

ii. $A \cap C$

A contains all elements of C.

∴ C ⊂ A

∴ A ∩ C = C

iii. A ∩ D

A contains all elements of D.

∴ D ⊂ A

∴ A ∩ D = D

iv. B ∩ C

B ∩ C = ϕ

There is no natural number which is both even and odd at same time.

v. B ∩ D

B ∩ D = 2

2 is the only natural number which is even and a prime number.

vi. C ∩ D

C ∩ D = {1, 3, 5, 7...}

Every prime number is odd except 2.

4. Question

Let A = {3, 6, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20}, C = {2, 4, 6, 8, 10, 12, 14, 16} and D = {5, 10, 15, 20}.

Find:

i. A – B

ii. A – C

iii. A – D

iv. B – A

v. C – A

vi. D – A

vii. B – C

viii. B – D

Answer

A – B is defined as {x ϵ A : x ∉ B}

i. A – B is defined as {x ϵ A : x ∉ B}

A = {3, 6, 12, 15, 18, 21}

B = {4, 8, 12, 16, 20}

A – B = {3, 6, 15, 18, 21}

ii. A – C is defined as {x ϵ A : x ∉ C}

A = {3, 6, 12, 15, 18, 21}

C = {2, 4, 6, 8, 10, 12, 14, 16}

A – C = {3, 15, 18, 21}

iii. $A - D$ is defined as $\{x \in A : x \notin D\}$

$A = \{3, 6, 12, 15, 18, 21\}$

$D = \{5, 10, 15, 20\}$.

$A - D = \{3, 6, 12, 18, 21\}$

iv. $B - A$ is defined as $\{x \in B : x \notin A\}$

$A = \{3, 6, 12, 15, 18, 21\}$

$B = \{4, 8, 12, 16, 20\}$

$B - A = \{4, 8, 16, 20\}$

v. $C - A$ is defined as $\{x \in C : x \notin A\}$

$A = \{3, 6, 12, 15, 18, 21\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$C - A = \{2, 4, 8, 10, 14, 16\}$

vi. $D - A$ is defined as $\{x \in D : x \notin A\}$

$A = \{3, 6, 12, 15, 18, 21\}$

$D = \{5, 10, 15, 20\}$.

$D - A = \{5, 10, 20\}$.

vii. $B - C$ is defined as $\{x \in B : x \notin C\}$

$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$B - C = \{4, 8, 20\}$

viii. $B - D$ is defined as $\{x \in B : x \notin D\}$

$B = \{4, 8, 12, 16, 20\}$

$D = \{5, 10, 15, 20\}$

$B - D = \{4, 8, 12, 16\}$

5 A. Question

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find:

A'

Answer

A' means Complement of A with respect to universal set U.

So, $A' = U - A$

$U - A$ is defined as $\{x \in U : x \notin A\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$A' = \{5, 6, 7, 8, 9\}$

5 B. Question

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find:

B'

Answer

B’ means Complement of B with respect to universal set U.

So, B’ = U – B

U – B is defined as {x ∈ U : x ∉ B}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {2, 4, 6, 8}

B’ = {1, 3, 5, 7, 9}

5 C. Question

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find:

(A ∩ C)’

Answer

(A ∩ C) = {x:x ∈ A and x ∈ C}.

= {3, 4}

(A∩C)’ means Complement of (A∩C) with respect to universal set U.

So, (A∩C)’ = U – (A∩C)

U – (A∩C)’ is defined as {x ∈ U : x ∉ (A∩C)’}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(A∩C)’ = {3, 4}

U – (A∩C)’ = {1, 2, 5, 6, 7, 8, 9}

5 D. Question

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find:

(A∪B)’

Answer

A ∪ B = {x: x ∈ A or x ∈ B}

= {1, 2, 3, 4, 6, 8}

(A∪B)’ means Complement of (A∪B) with respect to universal set U.

So, (A∪B)’ = U – (A∪B)’

U – (A∪B)’ is defined as {x ∈ U : x ∉ (A∪B)’}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(A∪B)’ = {1, 2, 3, 4, 6, 8}

U – (A∪B)’ = {5, 9}

5 E. Question

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find:

(A’)’

Answer

A’ = U – A

((A’)’ = (U – A)’

= U – (U – A)

= A

A = {1, 2, 3, 4}

5 F. Question

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find:

(B – C)’

Answer

B – C is defined as {x ϵ B : x ∉ C}

B = {2, 4, 6, 8}

C = {3, 4, 5, 6}

B – C = {2, 8}

Now, (B – C)’ = U – (B – C)

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B – C = {2, 8}

(B – C)’ = {1, 3, 4, 5, 6, 7, 9}.

6 A. Question

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that:

(A ∪ B)’ = A’ ∩ B’

Answer

A ∪ B = {x: x ϵ A or x ϵ B}

= {2, 3, 4, 5, 6, 7, 8}

(A∪B)’ means Complement of (A∪B) with respect to universal set U.

So, (A∪B)’ = U – (A∪B)’

U – (A∪B)’ is defined as {x ϵ U : x ∉ (A∪B)’}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(A∪B)’ = {2, 3, 4, 5, 6, 7, 8}

U – (A∪B)’ = {1, 9}

Now

A’ means Complement of A with respect to universal set U.

So, A’ = U – A

U – A is defined as {x ϵ U : x ∉ A}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {2, 4, 6, 8}

A’ = {1, 3, 5, 7, 9}

B’ means Complement of B with respect to universal set U.

So, B’ = U – B

U – B is defined as {x ϵ U : x ∉ B}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {2, 3, 5, 7}.

B’ = {1, 4, 6, 8, 9}

A’ ∩ B’ = = {x:x ϵ A’ and x ϵ C’}.

= {1, 9}

Hence verified.

6 B. Question

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that:

(A ∩ B}’ = A’ ∪ B’.

Answer

(A ∩ B) = {x:x ϵ A and x ϵ B}.

= {2}

(A∩B)’ means Complement of (A∩B) with respect to universal set U.

So, (A∩B)’ = U – (A∩B)

U – (A∩B)’ is defined as {x ϵ U : x ∉ (A∩B)’}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(A∩B)’ = {2}

U – (A∩B)’ = {1, 3, 4, 5, 6, 7, 8, 9}

A’ means Complement of A with respect to universal set U.

So, A’ = U – A

U – A is defined as {x ϵ U : x ∉ A}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {2, 4, 6, 8}

A’ = {1, 3, 5, 7, 9}

B’ means Complement of B with respect to universal set U.

So, B’ = U – B

U – B is defined as {x ϵ U : x ∉ B}

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {2, 3, 5, 7}.

B’ = {1, 4, 6, 8, 9}

A’ ∪ B’ = {x: x ϵ A or x ϵ B}

= {1, 3, 4, 5, 6, 7, 8, 9}

Hence verified.

Exercise 1.6

1. Question

Find the smallest set A such that A ∪ {1, 2} = {1, 2, 3, 5, 9}.

Answer

A ∪ {1, 2} = {1, 2, 3, 5, 9}

Elements of A and {1, 2} together give us the result

So smallest se of A can be

A = {1, 2, 3, 5, 9} – {1, 2}

A = {3, 5, 9}.

2 A. Question

Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7}. Verify the following identities:

A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Answer

L.H.S =

(B ∩ C) = {x:x ϵ B and x ϵ C}

= {5, 6}

A ∪ (B ∩ C) = {x: x ϵ A or x ϵ (B ∩ C) }

= {1, 2, 4, 5, 6}.

R.H.S =

(A ∪ B) = {x: x ϵ A or x ϵ B }

= {1, 2, 4, 5, 6}.

(A ∪ C) = {x: x ϵ A or x ϵ C }

= {1, 2, 4, 5, 6, 7}.

(A ∪ B) ∩ (A ∪ C) = {x:x ϵ (A ∪ B) and x ϵ (A ∪ C) }.

= {1, 2, 4, 5, 6}.

Hence Verified.

2 B. Question

Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7}. Verify the following identities:

A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Answer

L.H.S.

(B ∪ C) = {x: x ϵ B or x ϵ C }

= {2, 3, 4, 5, 6, 7}.

(A ∩ (B ∪ C)) = {x:x ϵ A and x ϵ (B ∪ C)}

= {2, 4, 5}

R.H.S:

(A ∩ B) = {x:x ϵ A and x ϵ B}

= {2, 5}

(A ∩ C) = {x:x ϵ A and x ϵ C}

= {4, 5}

(A ∩ B) ∪ (A ∩ C) = {x:x ϵ (A∩B) and x ϵ (A∩C) }.

$= \{2, 4, 5\}$.

Hence verified.

2 C. Question

Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$. Verify the following identities:

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Answer

$B–C$ is defined as $\{x \in B : x \notin C\}$

$B = \{2, 3, 5, 6\}$

$C = \{4, 5, 6, 7\}$

$B–C = \{2, 3\}$

$(A \cap (B - C)) = \{x : x \in A \text{ and } x \in (B - C)\}$

$= \{2\}$

R.H.S:

$(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

$= \{2, 5\}$

$(A \cap C) = \{x : x \in A \text{ and } x \in C\}$

$= \{4, 5\}$

$(A \cap B) - (A \cap C)$ is defined as $\{x \in (A \cap B) : x \notin (A \cap C)\}$

$= \{2\}$.

Hence Verified.

2 D. Question

Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$. Verify the following identities:

$A– (B \cup C) = (A - B) \cap (A–C)$

Answer

L.H.S.

$(B \cup C) = \{x : x \in B \text{ or } x \in C\}$

$= \{2, 3, 4, 5, 6, 7\}$.

$A–(B \cup C)$ is defined as $\{x \in A : x \notin (B \cup C)\}$

$A = \{1, 2, 4, 5\}$

$(B \cup C) = \{2, 3, 4, 5, 6, 7\}$.

$A–(B \cup C) = \{1\}$

R.H.S

$(A - B) =$

$A–B$ is defined as $\{x \in A : x \notin B\}$

$A = \{1, 2, 4, 5\}$

$B = \{2, 3, 5, 6\}$

$A–B = \{1, 4\}$

$(A–C) =$

$A–C$ is defined as $\{x \in A : x \notin C\}$

$A = \{1, 2, 4, 5\}$

$C = \{4, 5, 6, 7\}$

$A–C = \{1, 2\}$

$(A – B) \cap (A–C) = \{x : x \in (A – B) \text{ and } x \in (A – C)\}$.

$= \{1, 2\}$

Hence verified.

2 E. Question

Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$. Verify the following identities:

$A– (B \cap C) = (A – B) \cup (A – C)$

Answer

$(B \cap C) = \{x : x \in B \text{ and } x \in C\}$

$= \{5, 6\}$

$(A – (B \cap C)) =$

$A–(B \cap C)$ is defined as $\{x \in A : x \notin (B \cap C)\}$

$A = \{1, 2, 4, 5\}$

$(B \cap C) = \{5, 6\}$

$(A – (B \cap C)) = \{1, 2, 4\}$

R.H.S:

$(A – B) =$

$A–B$ is defined as $\{x \in A : x \notin B\}$

$A = \{1, 2, 4, 5\}$

$B = \{2, 3, 5, 6\}$

$A–B = \{1, 4\}$

$(A–C) =$

$A–C$ is defined as $\{x \in A : x \notin C\}$

$A = \{1, 2, 4, 5\}$

$C = \{4, 5, 6, 7\}$

$A–C = \{1, 2\}$

$(A – B) \cup (A–C) = \{x : x \in (A – B) \text{ OR } x \in (A – C)\}$.

$= \{1, 2, 4\}$

Hence verified.

2 F. Question

Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$. Verify the following identities:

$A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$.

Answer

$A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$.

Now,

$B \cup C = \{5, 6\}$

$A \cap (B \cup C) = \{5\}$

Similarly finding out R.H.S we get,

$A \cap B = \{2, 5\}$

$A \cap C = \{4, 5\}$

$(A \cap B) \cap (A \cap C) = \{5\}$

L.H.S = R.H.S

Hence, Verified.

3 A. Question

If $U = \{2, 3, 5, 7, 9\}$ is the universal set and $A = \{3, 7\}$, $B = \{2, 5, 7, 9\}$, then prove that:

$(A \cup B)' = A' \cap B'$

Answer

$A \cup B = \{x: x \in A \text{ or } x \in B\}$

$= \{2, 3, 5, 7, 9\}$

$(A\cup B)'$ means Complement of $(A\cup B)$ with respect to universal set U.

So, $(A\cup B)' = U - (A\cup B)'$

$U-(A\cup B)'$ is defined as $\{x \in U : x \notin (A\cup B)'\}$

$U = \{2, 3, 5, 7, 9\}$

$(A\cup B)' = \{2, 3, 5, 7, 9\}$

$U-(A\cup B)' = \phi$

Now

A' means Complement of A with respect to universal set U.

So, $A' = U-A$

$U-A$ is defined as $\{x \in U : x \notin A\}$

$U = \{2, 3, 5, 7, 9\}$

$A = \{3, 7\}$

$A' = \{2, 5, 9\}$

B' means Complement of B with respect to universal set U.

So, $B' = U-B$

$U-B$ is defined as $\{x \in U : x \notin B\}$

$U = \{2, 3, 5, 7, 9\}$

$B = \{2, 5, 7, 9\}$.

$B' = \{3\}$

$A' \cap B' = = \{x: x \in A' \text{ and } x \in C'\}$.

$= \phi$.

Hence verified.

3 B. Question

If U = {2, 3, 5, 7, 9} is the universal set and A = {3, 7}, B = {2, 5, 7, 9}, then prove that:

Answer

(A ∩ B}' = A' ∪ B.'

(A ∩ B) = {x:x ∊ A and x ∊ B }.

= {7}

(A∩B)' means Complement of (A∩B) with respect to universal set U.

So, (A∩B)' = U–(A∩B)

U–(A∩B)' is defined as {x ∊ U : x ∉ (A∩B)'}

U = {2, 3, 5, 7, 9}

(A∩B)' = {7}

U–(A∩B)' = {2, 3, 5, 9}

A' means Complement of A with respect to universal set U.

So, A' = U–A

U–A is defined as {x ∊ U : x ∉ A}

U = {2, 3, 5, 7, 9}

A = {3, 7}

A' = {2, 5, 9}

B' means Complement of B with respect to universal set U.

So, B' = U–B

U–B is defined as {x ∊ U : x ∉ B}

U = {2, 3, 5, 7, 9}

B = {2, 5, 7, 9}.

B' = {3}

A' ∪ B' = {x: x ∊ A or x ∊ B }

= {2, 3, 5, 9}

Hence verified.

4 A. Question

For any two sets A and B, prove that

B ⊂ A ∪ B

Answer

Let an element be p such that it belongs to B

∴ p ∊ B

⇒ p ∊ B ∪ A

⇒B ⊂ A ∪ B

4 B. Question

For any two sets A and B, prove that

$A \cap B \subset A$

Answer

Let an element be p such that it belongs to B

$\therefore p \in A \cap B$

$\Rightarrow p \in A$ and $p \in B$

$\Rightarrow A \cap B \subset A$

4 C. Question

For any two sets A and B, prove that

$A \subset B$ $A \cap B = A$

Answer

Let

$p \in A \subset B$.

$\Rightarrow x \in B$

Let and $p \in A \cap B$

$\Leftrightarrow x \in A$ and $x \in B$

$\therefore (A \cap B) = A$.

5 A. Question

For any two sets A and B, show that the following statements are equivalent:

$A \subset B$

Answer

To show that the following four statements are equivalent, we need to show that $1 \Rightarrow 2$, $2 \Rightarrow 3$, $3 \Rightarrow 4$, $4 \Rightarrow 1$

We first show that $1 \Rightarrow 2$

Now $A–B = \{x \in A: x \notin B\}$ .As $A \subset B$,

$\therefore$ Each element of A is an element of B,

$\therefore A–B = \phi$

Hence we proved $1 \Rightarrow 2$.

5 B. Question

For any two sets A and B, show that the following statements are equivalent:

$A – B = \phi$

Answer

We need to show that $2 \Rightarrow 3$

So assume that $A–B = \phi$

To show: $A \cup B = B$.

$\because A – B = \phi$

$\therefore$ Every element of A is an element of B.

So $A \subset B$ and therefore $A \cup B = B$

So (2) ⇒ (3) is true.

5 C. Question

For any two sets A and B, show that the following statements are equivalent:

$A \cup B = B$

Answer

We need to show that 3 ⇒ 4

Assume that $A \cup B = B$

To show: $A \cap B = A$.

$\because A \cup B = B$

$\therefore A \subset B$ and so $A \cap B = A$.

So (3) ⇒ (4) is true.

5 D. Question

For any two sets A and B, show that the following statements are equivalent:

$A \cap B = A$.

Answer

Finally, now we need to show 4 ⇒ 1.

So, assume $A \cap B = A$.

To show: $A \subset B$

$\because A \cap B = A$, therefore $A \subset B$, and so (4) ⇒ (1) is true.

6 A. Question

For three sets A, B, and C, show that

$A \cap B = A \cap C$ need not imply $B = C$.

Answer

Let $A = \{1, 2\}$

$B = \{2, 3\}$

$C = \{2, 4\}$

Then,

$A \cap B = \{2\}$

$A \cap C = \{2\}$

Hence, $A \cap B = A \cap C$, but clearly B is not equal to C.

6 B. Question

For three sets A, B, and C, show that

$A \subset B$ $C - B \subset C - A$

Answer

Given: $A \subset B$

To show: $C–B \subset C–A$

Let $x \in C– B$

$\Rightarrow x \in C$ and $x \notin B$ [by definition C–B]

$\Rightarrow x \in C$ and $x \notin A$

$\Rightarrow x \in C–A$

Thus $x \in C–B \Rightarrow x \in C–A$. This is true for all $x \in C–B$.

$C – B \subset C – A$.

7 A. Question

For any two sets, prove that:

$A \cup (A \cap B) = A$

Answer

$A \cup (A \cap B)$ [$\because$ union is distributive over intersection]

$(A \cup A) \cap (A \cup B)$[$\because A \cup A = A$]

$= A \cap (A \cup B)$

$= A$.

7 B. Question

For any two sets, prove that:

$A \cap (A \cup B) = A$

Answer

$= (A \cap A) \cup (A \cap B)$ [$\because$intersection is distributive over union]

$= A \cup (A \cap B)$ [$\because A \cap A = A$]

$= A$.

8. Question

Find sets A, B, and C such that $A \cap B$, $A \cap C$ and $B \cap C$ are non–empty sets and $A \cap B \cap C = \phi$.

Answer

To find sets A, B and C such that $A \cap B = \phi$, $A \cap C = \phi$ and $A \cap B \cap C = \phi$

Take $A = \{1, 2, 3\}$

$B = \{2, 4, 6\}$

$C = \{3, 4, 7\}$

Then,

$A \cap B = \{2\}$

$\therefore A \cap B \neq \phi$

$A \cap C = \{3\}$

$\therefore A \cap C \neq \phi$

$C \cap B = \{4\}$

$\therefore C \cap B \neq \phi$

But A, B and C do not have no elements in common,

$\therefore A \cap B \cap C = \phi$.

9. Question

For any two sets A and B, prove that: A ∩ B = ϕ A B'.

Answer

Let x be any element of Set A

And y be any element of Set B

Now x≠y A ∩ B = ϕ

This means no element of B should be in A.

Thus, x is an element of A and an element of B'

As x ϵ B'

A B'

Hence, Proved.

10. Question

If A and B are sets, then prove that A – B, A ∩ B and B – A are pair wise disjoint.

Answer

Let x ϵ A and y ϵ B

A – B = The set of values of A that are not in B.

A ∩ B = The set containing common values of A and B

B – A = The set of values of B that are not in A.

Two sets X and Y are called disjoint if,

X ∩ Y = ϕ

(A – B) ∩ (A ∩ B) = ((A – B) ∩ A) ∪ ((A – B) ∩B)

(A – B) ∩ (A ∩ B) = ϕ ∪ ϕ

(A – B) ∩ (A ∩ B) = ϕ

Similarly,

(B – A) ∩ (A ∩ B) = ((B – A) ∩ A) ∪ ((B – A) ∩B)

(B – A) ∩ (A ∩ B) = ϕ

Hence, the three sets are pair wise disjoint.

11. Question

Using properties of sets, show that for any two sets A and B, (A ∪ B) ∩ (A ∪ B') = A.

Answer

We need to show (A ∪ B) ∩ (A ∪ B') = A.

Now,

(A ∪ B) ∩ (A ∪ B') = ((A ∪ B) ∩ A) ∩ B'.

= (A ∩ A) ∪ (B ∩ A)) ∩B'

= A ∩ B'

= A.

12 A. Question

For any two sets of A and B, prove that:

$A' \cup B = U$ $A \subset B$

Answer

To show: $A \subset B$

$\Rightarrow x \notin A$

$\Rightarrow x \in A$ and $A \subset U$

$\Rightarrow x \in A$

$\Rightarrow x \in (A' \cup B)$ [$\because U = A' \cup B$]

$\Rightarrow x \in A'$ or $x \in B$

But, $x \notin A'$,

$\therefore x \in B$

Thus, $x \in A \Rightarrow x \in B$

This is true for all $x \in A$

$\therefore A \subset B$

12 B. Question

For any two sets of A and B, prove that:

$B' \subset A'$ $A \subset B$

Answer

We have $B' \subset A'$

To Show: $A \subset B$

Let, $x \in A$

$\Rightarrow x \notin A'$ [$\because A \cap A' = \phi$]

$\Rightarrow x \notin B'$ [$\because B' \subset A'$]

$\Rightarrow x \in B$ [$\because B \cap B' = \phi$]

Thus, $x \in A \Rightarrow x \in B$

This is true for all $x \in A$

$\therefore A \subset B$.

13. Question

Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Answer

It is a False statement.

Let, $A = \{3\}$ and $B = \{4\}$

Then,

$P(A) = \{\phi , \{3\}\}$

And $P(B) = \{\phi , \{4\}\}$

$\therefore P(A) \cup P(B) = \{\phi , \{1\}, \{2\}\}$

Now,

$A \cup B = \{1, 2\}$

And $P(A \cup B) = \{\phi , \{1\}, \{2\}, \{1, 2\}\}$

Hence, $P(A) \cup P(B) \neq P(A \cup B)$

14 A. Question

Show that for any sets A and B,

$A = (A \cap B) \cap (A - B)$

Answer

We know that $(A \cap B) \subset A$ and $(A - B) \subset A$

$\Rightarrow (A \cap B) \cap (A - B) \subset A$....(1)

Let and $x \in (A \cap B) \cap (A - B)$

$\Rightarrow x \in (A \cap B)$ and $x \in (A–B)$

$\Rightarrow x \in A$ and $x \in B$ and $x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in A$ [$\because x \in B$ and $x \notin B$ are not possible simultaneously]

$\rightarrow x \in A$

$\therefore (A \cap B) \cap (A - B) \subset A$...(2)

From (1) and (2), we get

$A = (A \cap B) \cap (A - B)$

14 B. Question

Show that for any sets A and B,

$A \cup (B - A) = A \cup B$

Answer

Let $x \in A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B–A)$

$\Rightarrow X \in A$ or $x \in B$ or $x \notin A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in (A \cup B)$

$\therefore A \cup (B - A) \subset (A \cup B)$.......(1)

Let and $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in A$ or $x \in B–A$

$\Rightarrow x \in A \cup (B–A)$

$\therefore (A \cup B) \subset A \cup (B - A)$.......(2)

From (1) and(2), we get

$A \cup (B - A) = A \cup B$.

Exercise 1.7

1. Question

For any two sets A and B, prove that: $A' - B' = B - A$

Answer

To show, A' – B' = B – A

We need to show

A' – B' ⊆ B – A

B – A ⊆ A' – B'

Let, x ϵ A' – B.'

⇒ x ϵ A' and x ∉ B.'

⇒ x ∉ A and x ϵ B

⇒ x ϵ B – A

It is true for all x x ϵ A' – B'

∴ A' – B' = B – A

Hence Proved.

2 A. Question

For any two sets A and B, prove the following:

A ∩ (A' ∪ B) = A ∩ B

Answer

Expanding

(A ∩ A') ∪ (A∩ B)

(A ∩ A') =ϕ

⇒ ϕ ∪ (A∩ B)

(A ∪ ϕ =A)

⇒ (A∩ B)

2 B. Question

For any two sets A and B, prove the following:

A – (A – B) = A ∩ B

Answer

For any sets A and B we have De morgans law

(A ∪B)' =A' ∩B' , (A ∩ B)' = A' ∪ B'

=A – (A–B)

= A ∩ (A–B)'

= A∩(A∩B')'

= A∩(A'∪ B')')

= A ∩ (A'∪B)

= (A ∩ A') ∪ (A ∩ B)

= ϕ ∪ (A ∩ B)

= (A ∩ B)

2 C. Question

For any two sets A and B, prove the following:

$A \cap (A \cup B') = \phi$

Answer

$= A\cap (A \cup B')$

$= A\cap(A'\cap B')$ [By De–morgan's law]

$= (A \cap A') \cap B'$ [$\therefore A \cap A' = \phi$]

$= \phi \cap B'$

$= \phi$

=RHS

2 D. Question

For any two sets A and B, prove the following:

$A - B = A \Delta (A \cap B)$

Answer

$= A \Delta (A \cap B)$ [$\because E \Delta F =(E-F) \cup (F-E)$]

$= (A-(A \cap B)) \cup (A \cap B - A)$ [$\because E - F = E \cap F'$]

$= (A \cap (A \cap B)') \cup (A\cap B\cap A')$

$= (A \cap (A'\cup B')) \cup (A\cap A'\cap B)$

$= \phi \cup (A \cap B') \cup \phi$

$= A \cap B'$ [$\because A \cap B' = A-B$]

$= A-B$

=LHS

$\therefore$ LHS=RHS Proved.

3. Question

If A, B, C are three sets such that $A \subset B$, then prove that $C - B \subset C - A$.

Answer

We have, ACB.

To show: $C - B \subset C - A$

Let, $x \in C-B$

$\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A$

$\Rightarrow x \in C - A$

Thus, $x \in C-B \Rightarrow x \in C - A$

This is true for all $x \in C-B$

$\therefore C - B \subset C - A$

4 A. Question

For any two sets A and B, prove that

$(A \cup B) - B = A - B$

Answer

$= (A–B) \cup (B–B)$

$= (A–B) \cup \phi$

$= A–B$

4 B. Question

For any two sets A and B, prove that

$A – (A \cap B) = A – B$

Answer

$= (A–A) \cap (A–B)$

$= \phi \cap (A – B)$

$= A – B.$

4 C. Question

For any two sets A and B, prove that

$A – (A – B) = A \cap B$

Answer

Let $x \in A–(A–B) \Leftrightarrow x \in A$ and $x \notin (A–B)$

$\Leftrightarrow x \in A$ and $x \notin (A \cap B)$

$\Leftrightarrow x \in A \cap (A \cap B)$

$\Leftrightarrow x \in (A \cap B)$

$\therefore A–(A–B) = (A \cap B)$

4 D. Question

For any two sets A and B, prove that

$A \cup (B – A) = A \cup B$

Answer

Let $x \in A \cup (B –A) \Rightarrow x \in A$ or $x \in (B – A)$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in B$

$\Rightarrow x \in (A \cup B)$ $[\because B \subset (A \cup B)]$

This is true for all $x \in A \cup (B–A)$

$\therefore A\cup(B–A)\subset(A\cup B)$......(1)

Conversely,

Let $x \in (A \cup B) \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ or $x \in (B–A)$ $[\because B \subset (A \cup B)]$

$\Rightarrow x \in A \cup (B–A)$

$\therefore (A\cup B)\subset A\cup(B–A)$......(2)

From 1 and 2 we get…

$A \cup (B – A) = A \cup B$

4 E. Question

For any two sets A and B, prove that

$(A - B) \cup (A \cap B) = A$

Answer

Let $x \in A$

Then either $x \in (A–B)$ or $x \in (A \cap B)$

$\Rightarrow x \in (A–B) \cup (A \cap B)$

$\therefore A \subset (A - B) \cup (A \cap B)$....(1)

Consverly,

Let $x \in (A–B) \cup (A \cap B)$

$\Rightarrow x \in (A–B)$ or $x \in (A \cap B)$

$\Rightarrow x \in A$ and $x \notin B$ or $x \in B$

$\Rightarrow x \in A$

$\therefore (A–B) \cup (A \cap B) \subset A$..........(2)

$\therefore$ From (1) and (2), We get

$(A–B) \cup (A \cap B) = A$

Exercise 1.8

1. Question

If A and B are two sets such that $n(A \cup B) = 50$, $n(A) = 28$ and $n(B) = 32$, find $n(A \cap B)$.

Answer

Given:

$n(A \cup B) = 50$

$n(A) = 28$

$n(B) = 32$

To Find:

$n(A \cap B) = ?$

We know,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Substituting the values we get

$50 = 28+32 - n(A \cap B)$

$50 = 60 - n(A \cap B)$

$-10 = - n(A \cap B)$

$\therefore n(A \cap B) = 10$.

2. Question

If P and Q are two sets such that P has 40 elements, $P \cup Q$ has 60 elements and $P \cap Q$ has 10 elements, how many elements does Q have?

Answer

Given:

$n(P) = 40$

$n(P \cup Q) = 60$

$n(P \cap Q) = 10$

$n(Q) = ?$

We know,

$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

Substituting the values we get

$60 = 40 + n(Q) - 10$

$60 = 30 + n(Q)$

$n(Q) = 30$

∴ Q =has 30 elements.

3. Question

In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics, and 4 teach physics and mathematics. How many teach physics?

Answer

Given :

20 teachers teach physics or math.

4 teachers teach physics and math.

12 teach maths

Let us denote teachers who teach physics as n(P) and for Maths n(M).

Now ,

20 teachers teach physics or math = $n(P \cup M) = 20$

4 teachers teach physics and math = $n(P \cap M) = 4$

12 teach maths = $n(M) = 12$

We know,

$n(P \cup M) = n(M) + n(P) - n(P \cap M)$

Substituting the values we get

$20 = 12 + n(P) - 4$

$20 = 8 + n(P)$

$n(P) = 12$

∴ There are 12 physics teachers.

4. Question

In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many like both coffee and tea?

Answer

A total number of people = 70.

Number of people liking Coffee = $n(C) = 37$.

Number of people liking Tea = $n(T) = 52$

Total number = $n(C \cup T) = 70$

Person who likes both would be $n(C \cap T)$

We know,

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

Substituting the values we get

$70 = 37+52- n(C \cap T)$

$70 = 89 - n(C \cap T)$

$n(C \cap T) = 19.$

∴ There are 19 persons who like both coffee and tea.

5 A. Question

Let A and B be two sets such that : $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find

$n(B)$

Answer

We know,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Substituting the values we get

$42 = 20 + n(B) - 4$

$42 = 16 + n(B)$

$n(B) = 26$

5 B. Question

Let A and B be two sets such that : $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find

$n(A - B)$

Answer

$n(A - B) = n(A \cup B) - n(B)$

$= 42 - 26$

$= 16.$

5 C. Question

Let A and B be two sets such that : $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find

$n(B - A)$

Answer

$n(B - A) = n(B) - n(A \cap B)$

$= 26 - 4$

$= 22.$

6. Question

A survey shows that 76% of the Indians like oranges, whereas 62% like bananas. What percentage of the Indians like both oranges and bananas?

Answer

Let us denote the people who like oranges by $n(O) = 76$

Let us denote the people who like oranges by n(B) = 62

Total number of people will be those who like oranges or banana = n (O ∪ B) =100

People who like both oranges and banana = n (O ∩ B)

We know,

n (O ∪ B) = n(O) + n(B) – n (O ∩ B)

Substituting the values we get

100 = 76+62 – n (O ∩ B)

100= 138 – n (O ∩ B)

n (O ∩ B) = 38

People who like both oranges and banana = 38%.

7. Question

In a group of 950 persons, 750 can speak Hindi and 460 can speak English. Find:

i. How many can speak both Hindi and English.

ii. How many can speak Hindi only.

iii. how many can speak English only.

Answer

Let , Total number of people be n(P) = 950

People who can speak English n(E) = 460

People who can speak Hindi n(H) = 750

i. How many can speak both Hindi and English.

People who can speak both Hindi and English = n (H ∩ E)

We know,

n (P) = n(E) + n(H) – n (H ∩ E)

Substituting the values we get

950 = 460+750 – n (H ∩ E)

950= 1210 – n (H ∩ E)

n (H ∩ E)=260.

Number of people who can speak both English and Hindi are 260.

ii. How many can speak Hindi only.

We can see that H is disjoint union of n(H–E) and n (H ∩ E).

(If A and B are disjoint then n (A ∪ B) = n(A) + n(B))

∴ H = n(H–E) ∪ n (H ∩ E).

⇒ n(H) = n(H–E) + n (H ∩ E).

⇒ 750 = n (H – E)+ 260

⇒ n(H–E) = 490.

Only, 490 people speak Hindi.

iii. how many can speak English only.

We can see that E is disjoint union of n(E–H) and n (H ∩ E).

(If A and B are disjoint then n (A ∪ B) = n(A) + n(B))

∴ E = n(E–H) ∪ n (H ∩ E).

⇒ n(E) = n(E–H) + n (H ∩ E).

⇒ 460 = n (H – E)+ 260

⇒ n(H–E) = 200.

Only, 200 people speak English.

8. Question

In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find:

i. how may drink tea and coffee both.

ii. how many drink coffee but not tea.

Answer

Let total number of people n(P) = 50

A number of people who drink Tea n(T) = 30.

A number of people who drink coffee n(C).

n(T–C) = 14

i. how may drink tea and coffee both.

We can see that T is disjoint union of n(T–C) and n (T ∩ C).

(If A and B are disjoint then n (A ∪ B) = n(A) + n(B))

∴ T = n(T–C) ∪ n (T ∩ C).

⇒ n(T) = n(T–C) + n (T ∩ C).

⇒ 30 = 14 + n (T ∩ C).

⇒ n(T ∩ C) = 16.

16 People drink both coffee and tea.

ii. how many drink coffee but not tea.

We know

n (P) = n(T) + n(C) – n (T ∩ C)

Substituting the values we get

50 = 30+n(C) – 16

n(C) = 36.

We can see that T is disjoint union of n(C–T) and n (T ∩ C).

(If A and B are disjoint then n (A ∪ B) = n(A) + n(B))

∴ C = n(C–T) ∪ n (T ∩ C).

⇒ n(C) = n(C–T) + n (T ∩ C).

⇒ 36 = n(C–T) + 16.

⇒ n(C–T) = 20.

20 People drink coffee but not tea.

9 A. Question

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

the numbers of people who read at least one of the newspapers.

Answer

Total number of People n(P) = 60.

n(H) =25.

n(T) = 26.

n(I) = 26.

$n\ (H \cap I) = 9$

$n\ (H \cap T) = 11$

$n\ (T \cap I) = 8$

$n\ (H \cap T \cap I) = 3$

The people who read at least one newspaper would be n(H or I or T) = $n(H \cup I \cup T)$

We know,

$n(H \cup I \cup T) = n(H)+n(I)+n(T) - n\ (H \cap I) - n\ (H \cap T) - n\ (T \cap I) + n\ (H \cap T \cap I)$

$n(H \cup I \cup T) = 25+26+26-9-11-8+3$

$n(H \cup I \cup T) = 52.$

There are 52 people who read at least one newspaper.

9 B. Question

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

the number of people who read exactly one newspaper.

Answer

No venn diagram provided.

10. Question

Of the members of three athletic teams in a certain school, 21 are in the basketball team, 26 in the hockey team and 29 in the football team. 14 play hockey and basket ball, 15 play hockey and football, 12 play football and basketball and 8 play all the three games. How many members are there in all?

Answer

A total number of People n(P) = ?.

People who play Basketball n(B) =21.

People who play Football n(F) = 29.

People who play Hockey n(H) = 26.

People who play Basketball and Hockey $n(B \cap H) = 14$

People who play Football and Hockey $n(H \cap F) = 15$

People who play Basketball and Football $n(B \cap F) = 12$

People who play all games. $n(H \cap B \cap F) = 8$

Total number of people would be n(H or B or F) = $n(H \cup B \cup F)$

We know,

$n(H \cup B \cup F) = n(H)+n(B)+n(F) – n(H \cap B)– n(H \cap F)– n(B \cap F)+ n(H \cap B \cap F)$

$n(H \cup B \cup F) = 26+21+29–14–15–12+8$

$n(H \cup B \cup F) = 43$.

Hence, there are 43 members in all.

11. Question

In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali? How many can speak both Hindi and Bengali?

Answer

Let

Total number of People n(P) = 1000

People who speak Hindi n(H) = 750

People who speak Bengali n(B) = 400

We have $P = n(H \cup B)$

$\Rightarrow n(P) = n(H)+n(B)– n(H \cap B)$

$= 1000 = 750+400 – n(H \cap B)$

$\Rightarrow n(H \cap B) = 150$.

∴ 150 People can speak both languages.

$H = (H–B) \cup (H \cap B)$ (union is disjoint)

$\therefore n(H) = n(H–B) +n(H \cap B)$

$750 = n(H–B)+150$

$n(H–B) = 600$

∴ 600 people speak Hindi.

$B = (B–H) \cup (H \cap B)$ (union is disjoint)

$\therefore n(B) = n(B–H) +n(H \cap B)$

$400 = n(B–H)+150$

$n(B–H) = 250$

∴ 250 people speak Bengali.

12. Question

A survey of 500 television viewers produced the following information;285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games? How many watch exactly one of the three games?

Answer

Let ,

Total number of People n(P) = 500.

People who watch Basketball n(B) =115.

People who watch Football n(F) = 285.

People who watch Hockey n(H) = 195.

People who watch Basketball and Hockey $n(B \cap H) = 50$

People who watch Football and Hockey $n(H \cap F) = 70$

People who watch Basketball and Football $n(B \cap F) = 45$

People who do not watch any games. $n(H \cup B \cup F) = 50$

Now,

$n(H \cup B \cup F)' = n(P) - n(H \cup B \cup F)$

$50 = 500 - (n(H)+n(B)+n(F) - n(H \cap B) - n(H \cap F) - n(B \cap F) + n(H \cap B \cap F))$

$50 = 500 - (285+195+115-70-50-45 + n(H \cap B \cap F))$

$50 = 500 - 430 + n(H \cap B \cap F))$

$n(H \cap B \cap F) = 70 - 50$

$n(H \cap B \cap F)) = 20$

∴ 20 People watch all three games.

Number of people who only watch football

$= 285-(50+20+25)$

$= 285-95$

$= 190.$

Number of people who only watch Hockey

$= 195-(50+20+30)$

$= 195-100$

$= 95.$

Number of people who only watch Basketball

$= 115-(25+20+30)$

$= 115-75$

$= 40.$

Number of people who watch exactly one of the three games

As the sets are pairwise disjoint we can write

= number of people who watch either football only or hockey only or Basketball only

$=190+95+40$

$=325$

∴ 325 people watch exactly one of the three games.

13 A. Question

In a survey of 100 persons, it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find:

How many read none of three magazines?

Answer

Total number of People n(P) = 100.

People who Read Magazine A n(A) =28.

People who Read Magazine B n(B) = 30.

People who Read Magazine C n(C) = 42.

People who Read Magazine A and B $n(A \cap B) = 8$

People who Read Magazine B and C $n(B \cap C) = 10$

People who v Read Magazine A and C $n(A \cap C) = 5$

$n(A \cap B \cap C) = 3$

Now,

$n(A \cup B \cup C) = n(A)+n(B)+n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 28+30+42-8-10-5+3$

$= 100-20$

$= 80$

People who read none of the three magazines

$= n(A \cup B \cup C)'$

$= n(P) - n(A \cup B \cup C)$

$= 100 - 80$

$= 20.$

∴ People who read no magazine = 20

13 B. Question

In a survey of 100 persons, it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find:

How many read magazine C only?

Answer

$n(C) = 42 -(7+3+2)$

$= 42-12.$

$= 30.$

14 A. Question

In a survey of 100 students, the number of students studying the various languages was found to be: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find:

How many students were studying Hindi?

Answer

Let us denote,

Total number of students by n(P)

Students studying English n(E)

Students Studying Hindi n(H)

Students studying Sanskrit n(S)

According to the question,

$n(P) = 100$, $n(E–H) = 23$, $n(E \cap S) = 8$, $n(E) = 26$, $n(S) = 48$, $n(H \cap S) = 8$, $n(E \cup H \cup S)' = 24$

Number of students studying English only = 18

Now,

$n(E \cup H \cup S)' = 24$

$n(P) – n(E \cup H \cup S) = 24$

$100 – 24 = n(E \cup H \cup S)$

$n(E \cup H \cup S) = 76$

$n(E \cup H \cup S) = n(E) + n(H) + n(S) – n(E \cap S) – n(E \cap H) – n(H \cap S) + (E \cap H \cap S)$

$76 = 26 + n(H) + 48 – 3 – 8 – 8 + 3$

$n(H) = 76 – 58$

$n(H) = 18$

18 Students are studying Hindi.

14 B. Question

In a survey of 100 students, the number of students studying the various languages was found to be: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find:

How many students were studying English and Hindi?

Answer

From q1 we have $n(E \cap H) = 3$

∴ 3 students study both hindi and English.

15. Question

In a survey it was found that 21 persons liked product P_1, 26 liked product P_2 and 29 liked product P_3. If 14 persons liked products P_1 and P_2; 12 persons liked product P_3 and P_1; 14 persons liked products P_2, and P_3 and 8 liked all the three products. Find how many liked product P_3 only.

Answer

Let $n(P_1)$ be a number of people liking product P_1.

Let $n(P_2)$ be a number of people liking product P_2.

Let $n(P_3)$ be a number of people liking product P_3.

Then, According to the question:

$n(P_1) = 21$, $n(P_2) = 26$, $n(P_3) = 29$, $n(P_1 \cap P_2) = 14$

$n(P_1 \cap P_3) = 12$, $n(P_2 \cap P_3) = 14$, $n(P_1 \cap P_2 \cap P_3) = 8$.

∴ Number of people liking product P_3 only:

$= 29 – (4 + 8 + 6)$

$= 29 – 18$

$= 11$

Very Short Answer

1. Question

If a set contains n elements, then write the number of elements in its power set.

Answer

Let A be the set with n elements.

Then power set of A contains all the subsets of A.

Each member of A has two possibilities either present or absent.

⇒ Total possible subsets of A are 2×2×2×...n times= 2^n

∴ number of elements in power set of A are 2^n.

2. Question

Write the number of elements in the power set of null set.

Answer

Power set contains all the subsets of a set.

Null set has no member in it and each set is a subset of itself (i.e. null set is the only subset of null set)

∴ number of elements in power set of null set is 1.

3. Question

Let A = {x : x∈ N, x is a multiple of 3} and B = {x : x∈ N and x is a multiple of 5}. Write $A \cap B$.

Answer

Here, A={X:X∈N,X is a multiple of 3} and

B={X:X∈N,X is a multiple of 5}.

A⋂B={X:X∈N,X is a multiple of 3} and{X:X∈N,X is a multiple of 5}

={X:X∈N,X is a multiple of 3 and 5}

={X:X∈N,X is a multiple of 3*5=15}

={X:X∈N,X is a multiple of 15}

4. Question

Let A and B be two sets having 3 and 6 elements respectively. Write the minimum number of elements that $A \cup B$ can have.

Answer

Here, n(A)=3 and n(B)=6

Now, n(A⋃B)=n(A)+n(B)-n(A⋂B)

=3+6-n(A⋂B)

=9-n(A⋂B)

So,n(A⋃B) is minimum whenever n(A⋂B) is maximum and it is possible only when A⊂B

Now,A⊂B then max(n(A⋂B))=n(A)=3.

∴ min(n(A⋃B))=9-3=6

5. Question

If A = {x∈ C : x^2 = 1} and B = {x∈ C : x^4 = 1}, then write A − B and B − A.

Answer

Here, A={X∈C:X^2=1}

={1 ,-1}

$B=\{x \epsilon C: X^4=1\}$

$=\{1,-1,i,-i\}$

Now, $A \setminus B = A \setminus (A \cap B)$

$=\{1,-1\} \setminus \{1,-1\}$

$=\phi$

And $B \setminus A = B \setminus (A \cap B)$

$=\{1,-1,i,-i\} \setminus \{1,-1\}$

$=\{i,-i\}$

6. Question

If A and B are two sets such that $A \subset B$, then write B' − A' in terms of A and B.

Answer

Here, $A \subset B \Rightarrow B' \subset A'$

$\therefore A' \cap B' = B'$

$B' \setminus A' = B' \setminus (A' \cap B')$

$=B' \setminus B'$

$=\phi$

7. Question

Let A and B be two sets having 4 and 7 elements respectively. Then write the maximum number of elements that $A \cup B$ can be.

Answer

Here, n(A)=4 and n(B)=7

Now, $n(A \cup B) = n(A)+n(B)-n(A \cap B)$

$=4+7-n(A \cap B)$

$=11-n(A \cap B)$

So, $n(A \cup B)$ is maximum whenever $n(A \cap B)$ is minimum and it is possible only when $A \cap B = \phi$

Now, $A \cap B = \phi$ then $\min(n(A \cap B)) = 0$.

$\therefore \min(n(A \cup B)) = 11-0 = 11$

8. Question

If $A = \left\{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\right\}$ and $B = \{(x, y) : y = -x, x \in R\}$, then write $A \cap B$.

Answer

Here, $A = \left\{(x,y) : y = \frac{1}{x}, 0 \neq x \epsilon R\right\}$ and

$B=\{(x,y):y=-x, x \epsilon R$

$A \cap B = \left\{(x,y) : y = \frac{1}{x}, 0 \neq x \epsilon R\right\}$ and $\{(x,y) : y = -x, x \epsilon R\}$

$= \{(x,y) : y = \frac{1}{x}$ and $y = -x, x \epsilon R\}$

$= \left\{(x,y) : -x = \frac{1}{x}, x \epsilon R\right\}$

$=\{(x, y):-x^2 = 1, x\epsilon R\}$

$=\{(x, y):x^2 = -1, x\epsilon R\}$

$=\phi$ ($\because$ there is no $x\epsilon R$ such that $x^2 = -1$)

9. Question

If A = {(x, y) : y = e^x, x$\in$R} and B = {(x, y) : y = e^{-x}, xR}, then write $A \cap B$.

Answer

Here, $A = \{(x,y): y = e^x, x\epsilon R\}$ and

$B = \{(x,y): y = e^{-x}, x\epsilon R\}$

$A \cap B = \{(x,y): y = e^x, x\epsilon R\}$ and $\{(x,y): y = e^{-x}, x\epsilon R\}$

$=\{(x,y): y = e^x$ and $y = e^{-x}$ $x\epsilon R\}$

$=\{(x,y): e^x = e^{-x}$ $x\epsilon R\}$

$=\left\{(x,y): e^x = \frac{1}{e^x}, x\epsilon R\right\}$

$=\{(x,y): e^{2x} = 1, x\epsilon R\}$

$=\{(0,1)\}$ ($\because$ $e^0 = 1$)

10. Question

If A and B are two sets such that n(A) = 20, n(B) = 25 and $n(A \cup B) = 40$, then write $n(A \cap B)$.

Answer

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$\therefore n(A \cap B) = 20 + 25 - 40 = 5$

$\therefore n(A \cap B) = 5$

11. Question

If A and B are two sets such that n(A) = 115, n(B) = 326, n (A – B) = 47, then write $(A \cup B)$.

Answer

Here, n(A)=115 , n(B)=326 , n(A\B)=47

n(A\B)=n(A)-n(A⋂B)

$\therefore$ n(A⋂B)=n(A)-n(A\B)

=115-47

=68

Now, n(A⋃B)=n(A)+n(B)-n(A⋂B)

=115+326-68

=373

MCQ

1. Question

Mark the correct alternative in the following:

For any set A, (A’) is equal to

A. A’

B. A

C. φ

D. none of these

Answer

Here, Question is $(A')' = ?$

Now, A'=U\A

⇒(A')'=(U\A)'=U'\A'

⇒(A')'=U'\(U\A)

⇒(A')'=ϕ\U+A

⇒(A')'=A

2. Question

Mark the correct alternative in the following:

Let A and B be two sets in the same universal set. Then, A − B =

A. $A \cap B$

B. $A' \cap B$

C. $A \cap B'$

D. none of these

Answer

A\B is the set of elements which are in A but not in B

⇒A\B is the set of elements which are in A and in B'

⇒A\B is the set of elements of A⋂B'

$\therefore A \setminus B = A \cap B'$

3. Question

Mark the correct alternative in the following:

The number of subsets of a set containing n elements is

A. n

B. $2^n - 1$

C. n^2

D. 2^n

Answer

Let A be the set with n elements.

Each member of A has two possibilities either present or absent.

⇒ Total possible subsets of A are 2×2×2×…n times = 2^n

4. Question

Mark the correct alternative in the following:

For any two sets A and B , $A \cap (A \cap B) =$

A. A

B. B

C. φ

D. none of these

Answer

Here correct answer is D. none of this as A⋂(A⋂B)=(A⋂A)⋂B)

=A⋂B

Or we have to give one condition that $A \subset B$ then correct answer is A because A⋂(A⋂B)=(A⋂A)⋂B)

=A⋂B

=A (∵ A⊂B⇒A⋂B=A)

5. Question

Mark the correct alternative in the following:

If A = {1, 3, 5, B} and B = {2, 4}, then

A. 4∈A

B. {4}∈A

C. $B \subset A$

D. none of these

Answer

A is not correct answer because 4 is not in A.

B is not correct answer because {4} is not in A.

C is not correct answer because 2 is in B but not in A.

So, D is correct answer.

6. Question

Mark the correct alternative in the following:

The symmetric difference of A and B is not equal to

A. $(A - B) \cap (B - A)$

B. $(A - B) \cup (B - A)$

C. $(A \cup B) - (A \cap B)$

D. $\{(A \cup B) - A\} \cup \{(A \cup B) - B\}$

Answer

A\B=A⋂B'

≠(A\B)⋂(B\A)

7. Question

Mark the correct alternative in the following:

The symmetric difference of A = {1, 2, 3} and B = {3, 4, 5} is

A. {1, 2}

B. {1, 2, 4, 5}

C. {4, 3}

D. {2, 5, 1, 4, 3}

Answer

Here,$A=\{1,2,3\}$ and $B=\{3,4,5\}$

$A \setminus B=\{1,2,3\} \setminus \{3,4,5\}$

$=\{1,2,3\} \setminus \{3\}$

$=\{1,2\}$

$B \setminus A=\{3,4,5\} \setminus \{1,2,3,4\}$

$=\{3,4,5\} \setminus \{3\}$

$=\{4,5\}$

$A \Delta B=(A \setminus B) \cup (B \setminus A)$

$=\{1,2\} \cup \{4,5\}$

$=\{1,2,4,5\}$

8. Question

Mark the correct alternative in the following:

For any two sets A and B, $(A - B) \cup (B - A) =$

A. $(A - B) \cup A$

B. $(B - A) \cup B$

C. $(A \cup B) - (A \cap B)$

D. $(A \cup B) \cap (A \cap B)$

Answer

$(A \setminus B) \cup (B \setminus A)=[A \cap B'] \cup [B \cap A']$

$=[A \cup (B \cap A')] \cap [B' \cup (B \cap A')]$

$=[(A \cap B) \cup (A \cap A')] \cap [(B' \cap B) \cup (B' \cap A')]$

$=[(A \cap B) \cup \phi] \cap [\phi \cup (B' \cap A')]$

$=[(A \cap B)] \cap [(B' \cap A')$

$=[(A \cap B)] \cap [(B \cup A)']$

$=(B \cup A)' \cap (A \cap B)$

$=(A \cup B) \setminus (A \cap B)$

9. Question

Mark the correct alternative in the following:

Which of the following statement is false:

A. $A - B = A \cap B'$

B. $A - B = A - (A \cap B)$

C. $A - B = A - B'$

D. $A - B = (A \cup B) - B$

Answer

A is true statement because

A\B is the set of elements which are in A but not in B

⇒A\B is the set of elements which are in A and in B'

⇒A\B is the set of elements of A⋂B'

∴A\B=A⋂B'

B is true statement because

A\B is the set of elements which are in A but not in B

⇒A\B is the set of elements which are in A but not in A⋂B'

⇒ A\B=A\(A⋂B')

C is false statement because

A\B=A⋂B' and A\B^'=A⋂B

∴A⋂B'≠A⋂B⇒A\B≠B\A.

10. Question

Mark the correct alternative in the following:

For any three sets A, B and C

A. $A \cap (B - C) = (A \cap B) - (A \cap C)$

B. $A \cap (B - C) = (A \cap B) - C$

C. $A \cup (B - C) = (A \cup B) \cap (A \cup C')$

D. $A \cup (B - C) = (A \cup B) - (A \cup C)$

Answer

(A⋂B)\(A⋂C)=(A⋂B)\A⋃(A⋂B)\C (∵ De Morgen’s law with difference and intersection.)

=ϕ⋃(A⋂B)\C

=(A⋂B)\C

=A⋂(B\C) (∵ intersection with difference is difference with intersection)

11. Question

Mark the correct alternative in the following:

Let A = {x : x∈R, x > 4} and B = {x∈R : x < 5}. The, $A \cap B =$

A. (4, 5)

B. (4, 5)

C. [4, 5)

D. [4, 5]

Answer

Here, A= $\{x: x \in R, x>4\}$ and B= $\{x: x \in R, x<5\}$

$A \cap B = \{x: x \in R, x>4\}$ and $\{x: x \in R, x<5\}$

$= \{x \in R: x>4, x<5\}$

$= \{x \in R: 4<x<5\}$

$= (4,5)$

12. Question

Mark the correct alternative in the following:

Let U be the universal set containing 700 elements. If A, B are sub-sets of U such that n (A) = 200, n (B) = 300 and $n(A \cap B)$=100. Then, $n(A' \cap B')$=

A. 400

B. 600

C. 300

D. none of these

Answer

Here, $n(A)=200$, $n(B)=300$, $n(A \cap B)=100, n(U)=700$

$n(A \cup B)=n(A)+n(B)-n(A \cap B)$

$=200+300-100$

$=400$

$n(A' \cap B')=n(U) \setminus n(A \cup B)$

$=700-400$

$=300$

13. Question

Mark the correct alternative in the following:

Let A and B be two sets such that n (A) = 16, n (B) = 25. Then, $n(A \cap B)$ is equal to

A. 30

B. 50

C. 5

D. none of these

Answer

In this question we cannot find $n(A \cap B)$ as we haven't required values and information about their union or about universal set.

So, correct answer is D

14. Question

Mark the correct alternative in the following:

If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is

A. 120

B. 30

C. 31

D. 32

Answer

Here, n(A)=5

⇒n(P(A))=2^5=32

So,A has total 32 subsets but one of them is A itself.

∴ Proper subsets of A are 32-1=31.

15. Question

Mark the correct alternative in the following:

In set-builder method the null set is represented by

A. { }

B. Φ

C. {x : x ≠ x}

D. {x : x = x}

Answer

We know x≠x is false for any x.

∴ the set builder form for null set is {x:x≠x}.

16. Question

Mark the correct alternative in the following:

If A and B are two disjoint sets, then n$\left(A \cup B\right)$ is equal to

A. n(A) + n(B)

B. n(A) + n(B) − n$\left(A \cap B\right)$

C. n(A) + n(B) + n$\left(A \cap B\right)$

D. n(A) n(B)

Answer

Here, A and B are two disjoint sets.

⇒A⋂B=ϕ

⇒n(A⋂B)=0

n(A⋃B)=n(A)+n(B)-n(A⋂B)

=n(A)+n(B)-0

=n(A)+n(B)

17. Question

Mark the correct alternative in the following:

For two sets $A \cup B = A$ iff

A. $B \subseteq A$

B. $A \subseteq B$

C. $A \neq B$

D. $A = B$

Answer

If A⋃B=A

Then, any element is in B then it is in A⋃B=A

So, B⊂A

If B⊂A

Then, any element of A⋃B is in A.

So, A⋃B=A

Hence, A⋃B=A iff B⊂A

18. Question

Mark the correct alternative in the following:

If A and B are two sets such that n(A) = 70, n(B) = 60, n$(A \cup B)$= 110, then n$(A \cap B)$ is equal to

A. 240

B. 50

C. 40

D. 20

Answer

n(A⋃B) =n(A)+n(B)-n(A⋂B)

∴n(A⋂B) =n(A)+n(B)-n(A⋃B)

∴n(A⋂B) =70+60-110

∴n(A⋂B) =20

19. Question

Mark the correct alternative in the following:

If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to

A. A

B. B

C. Φ

D. $A \cap B^c$

Answer

$A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$

$= (A \cap A^c) \cup (A \cap B^c)$

$= \phi \cup (A \cap B^c)$

$= A \cap B^c$

20. Question

Mark the correct alternative in the following:

If A = {x : x is a multiple of 3} and, B = {x : x is a multiple 5}, then A− B is

A. $A \cap B$

B. $A \cap \overline{B}$

C. $\overline{A} \cap \overline{B}$

D. $\overline{A \cap B}$

Answer

A \ B is the set of elements which are in A but not in B

⇒A \B is the set of elements which are in A and in B'

⇒A \ B is the set of elements of $A \cap B'$

∴$A \setminus B = A \cap B'$

$= A \cap \overline{B}$

21. Question

Mark the correct alternative in the following:

In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus is

A. 80%

B. 40%

C. 60%

D. 70%

Answer

Let A be the set of population travels by car(in percentage) and

B be the set of population travels by bus(in percentage).

Then $A \cap B$ is the set of population travels by car and bus and

$A \cup B$ is the set of population travels by car or bus.

Then, $n(A)=20$, $(B)=50$,$n(A \cap B)=10$

$n(A \cup B)=n(A)+n(B)-n(A \cap B)$

=20+50-10

=60

∴ The set of population travels by car or bus is 60%

22. Question

Mark the correct alternative in the following:

If $A \cap B = B$, then

A. $A \subseteq B$

B. $B \supseteq A$

C. $A = \Phi$

D. $B = \Phi$

Answer

A⋂B=B which means elements of B are in the both sets A and B.

⇒ All the elements of B are contained in the intersection of A and B which is equal to B.

⇒B⊂A.

23. Question

Mark the correct alternative in the following:

An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee and tea. The investigator reporter that 10 students take all three drinks milk, coffee and tea; 20 students take milk coffee; 25 students take milk and tea; 20 students take coffee and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of three drinks is

A. 10

B. 20

C. 25

D. 30

Answer

Let U be the set of students which are interviewed by investigator

A be the set of students which take milk

B be the set of students which take coffee

C be the set of students which take tea

Then ,n(U)=100 ,n(A)=12, n(B)=5 ,n(C)=8

Also, (A⋂B⋂C) means students who drink all of three=10

(A⋂B) means students who drink milk and coffee =20-10=10

(B⋂C) means students who drink coffee and tea =20-10=10

(A⋂C) means students who drink milk and tea =25-10=15

A⋃B⋃C is the total of above =12+5+8+10+10+15+10=70

The number of students who didn't take any drink =100-70=30

24. Question

Mark the correct alternative in the following:

Two finite sets have m and n elements. The number of elements in the power set of first set is 48 more than the total number of elements in power in power set of the second set. Then, the values of m and n are:

A. 7, 6

B. 6, 8

C. 6, 4

D. 7, 4

Answer

Let A and B be the set which contain m and n elements respectively.

Then n(P(A))=2^m and n(P(B))=2^n

Also given that,n(P(A))=n(P(B))+48

$\Rightarrow 2m = 2n + 48$

Above equation is only true when $m = 6$ and $n = 4$.

25. Question

Mark the correct alternative in the following:

In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone?

A. 35

B. 48

C. 60

D. 22

Answer

Let M,P and C denote the set of students who opting mathematics ,physics and chemistry respectively.

Number of students who opted.

mathematics only =n(M)-n(M⋂P)-n(M⋂C)+n(M⋂P⋂C)

=100-30-28+18

=60

26. Question

Mark the correct alternative in the following:

Suppose A_1, A_2,...,A_{30} are thirty sets each having 5 elements and B_1, B_2,...,B_n are n sets each with 3 elements, let $\bigcup_{i=1}^{30} = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the $A_i^{'s}$ and exactly 9 of the $B_j^{'s}$, then n is equal to

A. 15

B. 3

C. 45

D. 35

Answer

If elements are not repeated then number of elements in $A_1 \cup A_2 \cup A_3 \cup \ldots . \cup A_{30}$=30*5=150

But, each element of S is in 10 of A_i

$\therefore S = \frac{150}{10} = 15.$

If elements are not repeated then number of elements in $B_1 \cup B_2 \cup B_3 \cup \ldots . \cup B_n$=3*n

But, each element of S is in 9 of B_i

$\therefore S = \frac{3 \times n}{9}$

$\therefore \frac{3 \times n}{9} = 15$

$\therefore 3\times$ n=15$\times$ 9

∴n=45

27. Question

Mark the correct alternative in the following:

Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second. The values of m and n are respectively.

A. 4, 7

B. 7, 4

C. 4, 4

D. 7, 7

Answer

Let A and B be the set which contain m and n elements respectively.

Then $n(P(A))=2^m$ and $n(P(B))=2^n$

Also given that, $n(P(A))=n(P(B))+112$

$\Rightarrow 2^m = 2^n + 112$

Above equation is only true when m=7 and n=4.

28. Question

Mark the correct alternative in the following:

For any two sets A and B, $A \cap (A \cup B)'$ is equal to

A. A

B. B

C. φ

D. $A \cap B$

Answer

$A \cap (A \cup B)' = A \cap (A' \cap B')$

$= (A \cap A') \cap (A \cap B')$

$= \phi \cap (A \cap B')$

$= \phi$

29. Question

Mark the correct alternative in the following:

The set $(A \cup B')' \cup (B \cap C)$ is equal to

A. $A' \cup B \cup C$

B. $A' \cup B$

C. $A' \cup C'$

D. $A' \cap B$

Answer

$(A \cup B')' \cup (B \cap C) = (A' \cap (B')') \cup (B \cap C)$

$= (A' \cap B) \cup (B \cap C)$

$= B \cap (A' \cup C)$

30. Question

Mark the correct alternative in the following:

Let F_4 be the set of all parallelograms, F_2 the set of all rectangles, F_3 the set of all rhombuses, F_4 the set of all squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to

A. $F_2 \cap F_3$

B. $F_3 \cap F_4$

C. $F_2 \cup F_3$

D. $F_2 \cup F_3 \cup F_4 \cup F_1$

Answer

Here, F_1 is the set of all parallelograms

We know that every rectangle, rhombus and square in plane are parallelogram but trapezium is not parallelogram

So, $F_1 = F_1 \cup F_2 \cup F_3 \cup F_4$.