# 1. Relations

## Exercise 1.1

### 1 A. Question

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

R = {(x, y) : x and y work at the same place}

**Answer**

We have been given that,

A is the set of all human beings in a town at a particular time.

Here, R is the binary relation on set A.

So, recall that

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these criteria, we can solve these.

We have,

R = {(x, y): x and y work at the same place}

**Check for Reflexivity:**

Since x & x are the same people then, x & x works at the same place.

Take yourself, for example, if you work at Bloomingdale then you work at Bloomingdale.

Since you can't work in two places at a particular time,

So, ∀ x ∈ A, then (x, x) ∈ R.

**∴ R is Reflexive.**

**Check for Symmetry:**

If x & y works at the same place, then, y and x also work at the same place.

If you & your friend, Chris was working in Bloomindale, then Chris and you are working in Bloomingdale only.

The only difference is in the way of writing, either you write your name and your friend's name or your friend's name and your name, it's the same.

So, if (x, y) ∈ R, then (y, x) ∈ R

∀ x, y ∈ A

**∴ R is Symmetric.**

**Check for Transitivity:**

If x & y works at the same place and y & z works at the same place.

Then, x & z also works at the same place.

Say, if she & I was working in Bloomingdale and she & you were also working in Bloomingdale. Then, you and I are working in the same company.

So, if (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R.

∀ x, y, z ∈ A

**∴ R is Transitive.**

**Hence, R is reflexive, symmetric and transitive.**

**1 B. Question**

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

R = {(x, y) : x and y live in the same locality}

**Answer**

We have been given that,

A is the set of all human beings in a town at a particular time.

Here, R is the binary relation on set A.

So, recall that

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these criteria, we can solve these.

We have,

R = {(x, y): x and y live in the same locality}

**Check for Reflexivity:**

Since x & x are the same people, then, x & x live in the same locality.

Take yourself, for example, if you lived in colony x then you live in colony x.

Since you can't live in two places at a particular time.

So, ∀ x ∈ A, then (x, x) ∈ R.

**∴ R is Reflexive.**

**Check for Symmetry:**

If x & y live in the same locality, then, y & x also lives in the the same locality.

If you & your friend, Chris are neighbors, then you and Chris are neighbors only.

The only difference is in the way of writing, either you write your name and your friend's name or your friend's name and your name, it's the same.

So, if (x, y) ∈ R, then (y, x) ∈ R.

∀ x, y ∈ R

**∴ R is Symmetric.**

**Check for Transitivity:**

If x & y lives in the same locality and y & z lives in the same locality.

Then, x & z also lives in the same locality.

Say, if she and I were living in colony x and she & you were also working in colony x. Then, you and I are living in the same colony.

So, if (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R.

∀ x, y, z ∈ A

**∴ R is Transitive.**

**Hence, R is reflexive, symmetric and transitive.**

**1 C. Question**

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

R = {(x, y) : x is wife of y}

**Answer**

We have been given that,

A is the set of all human beings in a town at a particular time.

Here, R is the binary relation on set A.

So, recall that

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these criteria, we can solve these.

We have,

R = {(x, y): x is wife of y}

**Check for Reflexivity:**

Since x and x are the same people.

Then, x cannot be the wife of itself.

A person cannot be a wife of itself.

Wendy is the wife of Sam; Wendy can't be the wife of herself.

So, ∀ x ∈ A, then (x, x) ∉ R.

**∴ R is not reflexive.**

**Check for Symmetry:**

If x is the wife of y.Then, y cannot be the wife of x.

If Wendy is the wife of Sam, then Sam is the husband of Wendy.

Sam cannot be the wife of Wendy.

So, if (x, y) ∈ R, then (y, x) ∉ R.

∀ x, y ∈ A

**∴ R is not symmetric.**

**Check for Transitivity:**

If x is the wife of y and y is the wife of z, which is not logically possible.

Then, x is not the wife of z.

It's easy, take Wendy, Sam, and Mac.

If Wendy is the wife of Sam, Sam can't be the wife of Mac.

Thus, the possibility of Wendy being the wife of Mac also eliminates.

So, if (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∉ R.

∀ x, y, z ∈ A

**∴ R is not transitive.**

**Hence, R is neither reflexive, nor symmetric, nor transitive.**

**1 D. Question**

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

R = {(x, y) : x is father of y}

**Answer**

We have been given that,

A is the set of all human beings in a town at a particular time.

Here, R is the binary relation on set A.

So, recall that

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these criteria, we can solve these.

We have,

R = {(x, y): x is father of y}

**Check for Reflexivity:**

Since x and x are the same people.

Then, x cannot be the father of itself.

A person cannot be a father of itself.

Leo is the father of Thiago

So, ∀ x ∈ A, then (x, x) ∉ R.

**∴ R is not reflexive.**

**Check for Symmetry:**

If x is the father of y.

Then, y cannot be the father of x.

If Sam is the father of Mac, then Mac is the son of Sam.

Mac cannot be the father of Sam.

So, if (x, y) ∈ R, then (y, x) ∉ R.

∀ x, y ∈ A

**∴ R is not symmetric.**

**Check for Transitivity:**

If x is the father of y and y is the father of z,then, x is not the father of z.

Take Mickey, Sam, and Mac.

If Mickey is the father of Sam, and Sam is the father of Mac.

Thus, Mickey is not the father of Mac, but the grandfather of Mac.

So, if (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∉ R.

∀ x, y, z ∈ A

**∴ R is not transitive.**

**Hence, R is neither reflexive, nor symmetric, nor transitive.**

**2. Question**

Relations $R_1$, $R_2$, $R_3$ and $R_4$ are defined on a set A = {a, b, c} as follows :

$R_1$ = {(a, a) (a, b) (a, c) (b, b) (b, c), (c, a) (c, b) (c, c)}

$R_2$ = {(a, a)}

$R_3$ = {(b, a)}

$R_4$ = {(a, b) (b, c) (c, a)}

Find whether or not each of the relations $R_1$, $R_2$, $R_{3,}$ $R_4$ on A is (i) reflexive (iii) symmetric (iii) transitive.

**Answer**

We have set,

A = {a, b, c}

Here, $R_1$, $R_2$, $R_{3,}$ and $R_4$ are the binary relations on set A.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

So, using these results let us start determining given relations.

We have

$R_1$ = {(a, a) (a, b) (a, c) (b, b) (b, c) (c, a) (c, b) (c, c)}

(i). **Reflexive:**

For all a, b, c ∈ A. [∵ A = {a, b, c}]

Then, (a, a) ∈ $R_1$

(b, b) ∈ A

(c, c) ∈ A

[∵ $R_1$ = {**(a, a)** (a, b) (a, c) **(b, b)** (b, c) (c, a) (c, b) **(c, c)**}]

So, ∀ a, b, c ∈ A, then (a, a), (b, b), (c, c) ∈ R.

**∴ $R_1$ is reflexive.**

(ii). **Symmetric:**

If (a, a), (b, b), (c, c), (a, c), (b, c) ∈ $R_1$

Then, clearly (a, a), (b, b), (c, c), (c, a), (c, b) ∈ $R_1$

∀ a, b, c ∈ A

[∵ $R_1$ = {**(a, a) (a, b) (a, c) (b, b) (b, c) (c, a) (c, b) (c, c)** }]

But, we need to try to show a contradiction to be able to determine the symmetry.

So, we know (a, b) ∈ $R_1$

But, (b, a) $\notin R_1$

So, if (a, b) $\in R_1$, then (b, a) $\notin R_1$.

$\forall$ a, b $\in$ A

**∴ $R_1$ is not symmetric.**

(iii). **Transitive:**

If (b, c) $\in R_1$ and (c, a) $\in R_1$

But, (b, a) $\notin R_1$ [Check the Relation $R_1$ that does not contain (b, a)]

$\forall$ a, b $\in$ A

[∵ $R_1$ = {(a, a) (a, b) (a, c) (b, b) **(b, c) (c, a)** (c, b) (c, c)}]

So, if (b, c) $\in R_1$ and (c, a) $\in R_1$, then (b, a) $\notin R_1$.

$\forall$ a, b, c $\in$ A

**∴ $R_1$ is not transitive.**

Now, we have

$R_2$ = {(a, a)}

(i). **Reflexive:**

Here, only (a, a) $\in R_2$

for a $\in$ A. [∵ A = {a, b, c}]

[∵ $R_2$ = {**(a, a)**}]

So, for a $\in$ A, then (a, a) $\in R_2$.

**∴ $R_2$ is reflexive.**

(ii). **Symmetric:**

For symmetry,

If (x, y) $\in$ R, then (y, x) $\in$ R

$\forall$ x, y $\in$ A.

Notice, in $R_2$ we have

$R_2$ = {**(a, a)**}

So, if (a, a) $\in R_2$, then (a, a) $\in R_2$.

Where a $\in$ A.

**∴ $R_2$ is symmetric.**

(iii). **Transitive:**

Here,

(a, a) $\in R_2$ and (a, a) $\in R_2$

Then, obviously (a, a) $\in R_2$

Where a $\in$ A.

[∵ $R_2$ = {**(a, a)**}]

So, if (a, a) $\in R_2$ and (a, a) $\in R_2$, then (a, a) $\in R_2$, where a $\in$ A.

**∴ $R_2$ is transitive.**

Now, we have

$R_3 = \{(b, a)\}$

(i). **Reflexive:**

$\forall$ a, b $\in$ A [$\because$ A = {a, b, c}]

But, (a, a) $\notin R_3$

Also, (b, b) $\notin R_3$

[$\because R_3 = \{$**(b, a)**$\}$]

So, $\forall$ a, b $\in$ A, then (a, a), (b, b) $\notin R_3$

**∴ $R_3$ is not reflexive.**

(ii). **Symmetric:**

If (b, a) $\in R_3$

Then, (a, b) should belong to $R_3$.

$\forall$ a, b $\in$ A. [$\because$ A = {a, b, c}]

But, (a, b) $\notin R_3$

[$\because R_3 = \{$**(b, a)**$\}$]

So, if (a, b) $\in R_3$, then (b, a) $\notin R_3$

$\forall$ a, b $\in$ A

**∴ $R_3$ is not symmetric.**

(iii). **Transitive:**

We have (b, a) $\in R_3$ but do not contain any other element in $R_3$.

Transitivity can't be proved in $R_3$.

[$\because R_3 = \{$**(b, a)**$\}$]

So, if (b, a) $\in R_3$ but since there is no other element.

**∴ $R_3$ is not transitive.**

Now, we have

$R_4 = \{(a, b)\ (b, c)\ (c, a)\}$

(i). **Reflexive:**

$\forall$ a, b, c $\in$ A [$\because$ A = {a, b, c}]

But, (a, a) $\notin R_4$

Also, (b, b) $\notin R_4$ and (c, c) $\notin R_4$

[$\because R_4 = \{(a, b)\ (b, c)\ (c, a)\}$]

So, $\forall$ a, b, c $\in$ A, then (a, a), (b, b), (c, c) $\notin R_4$

**∴ $R_4$ is not reflexive.**

(ii). **Symmetric:**

If (a, b) ∈ $R_4$, then (b, a) ∈ $R_4$

But (b, a) ∉ $R_4$

[∵ $R_4$ = {**(a, b)** (b, c) (c, a)}]

So, ∀ a, b ∈ A, if (a, b) ∈ $R_4$, then (b, a) ∉ $R_4$.

⇒ $R_4$ is not symmetric.

It is sufficient to show only one case of ordered pairs violating the definition.

**∴ $R_4$ is not symmetric.**

(iii). **Transitivity:**

We have,

(a, b) ∈ $R_4$ and (b, c) ∈ $R_4$

⇒ (a, c) ∈ $R_4$

But, is it so?

No, (a, c) ∉ $R_4$

So, it is enough to determine that $R_4$ is not transitive.

∀ a, b, c ∈ A, if (a, b) ∈ $R_4$ and (b, c) ∈ $R_4$, then (a, c) ∉ $R_4$.

**∴ $R_4$ is not transitive.**

**3 A. Question**

Test whether the following relations $R_1$, $R_2$ and $R_3$ are (i) reflexive (ii) symmetric and (iii) transitive :

$R_1$ on $Q_0$ defined by (a, b) ∊ $R_1$⇔ a = 1/b

**Answer**

Here, $R_1$, $R_2$, $R_3$, and $R_4$ are the binary relations.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

So, using these results let us start determining given relations.

We have

$R_1$ on $Q_0$ defined by (a, b) ∈ $R_1$⇔ $a = \frac{1}{b}$

**Check for Reflexivity:**

∀ a, b ∈ $Q_0$,

(a, a), (b, b) ∈ $R_1$ needs to be proved for reflexivity.

If (a, b) ∈ $R_1$

Then, $a = \frac{1}{b}$ ...(1)

So, for (a, a) ∈ $R_1$

Replace b by a in equation (1), we get

$a = \frac{1}{a}$

But, we know

$a \neq \frac{1}{a}$

⇒ (a, a) ∉ $R_1$

So, ∀ a ∈ $Q_0$, then (a, a) ∉ $R_1$

**∴ $R_1$ is not reflexive.**

**Check for Symmetry:**

If (a, b) ∈ $R_1$

Then, (b, a) ∈ $R_1$

∀ a, b ∈ $Q_0$

If (a, b) ∈ $R_1$

We have, $a = \frac{1}{b}$ …(2)

Now, for (b, a) ∈ $R_1$

Replace a by b & b by a in equation (2), we get

$b = \frac{1}{a}$

⇒ (b, a) ∈ $R_2$

So, if (a, b) ∈ $R_1$, then (b, a) ∈ $R_1$

∀ a, b ∈ $Q_0$

**∴ $R_1$ is symmetric.**

**Check for Transitivity:**

If (a, b) ∈ $R_1$ and (b, c) ∈ $R_1$

$\Rightarrow a = \frac{1}{b} \text{ and } b = \frac{1}{c}$

We need to eliminate b.

We have

$a = \frac{1}{b}$

$\Rightarrow b = \frac{1}{a}$

Putting $b = \frac{1}{a}$ in $b = \frac{1}{c}$, we get

$\Rightarrow \frac{1}{a} = \frac{1}{c}$

$\Rightarrow a = c$

But, $a \neq \frac{1}{c}$

⇒ (a, c) ∉ $R_1$

So, if $(a, b) \in R_1$ and $(b, c) \in R_1$, then $(a, c) \notin R_1$

$\forall$ a, b, c $\in Q_0$

**∴ $R_1$ is not transitive.**

**3 B. Question**

Test whether the following relations $R_1$, $R_2$ and $R_3$ are (i) reflexive (ii) symmetric and (iii) transitive :

$R_2$ on Z defined by $(a, b) \in R_2 \Leftrightarrow |a - b| \leq 5$

**Answer**

Here, $R_1$, $R_2$, $R_3$, and $R_4$ are the binary relations.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

So, using these results let us start determining given relations.

We have

$R_2$ on Z defined by $(a, b) \in R_2 \Leftrightarrow |a - b| \leq 5$

**Check for Reflexivity:**

$\forall$ a ∈ Z,

$(a, a) \in R_2$ needs to be proved for reflexivity.

If $(a, b) \in R_2$

Then, $|a - b| \leq 5$ ...(1)

So, for $(a, a) \in R_1$

Replace b by a in equation (1), we get

$|a - a| \leq 5$

$\Rightarrow 0 \leq 5$

$\Rightarrow (a, a) \in R_2$

So, $\forall$ a ∈ Z, then $(a, a) \in R_2$

**∴ $R_2$ is reflexive.**

**Check for Symmetry:**

$\forall$ a, b ∈ Z

If $(a, b) \in R_2$

We have, $|a - b| \leq 5$ ...(2)

Replace a by b & b by a in equation (2), we get

$|b - a| \leq 5$

Since, the value is in mod, $|b - a| = |a - b|$

$\Rightarrow$ The statement $|b - a| \leq 5$ is true.

$\Rightarrow (b, a) \in R_2$

So, if $(a, b) \in R_2$, then $(b, a) \in R_2$

$\forall$ a, b $\in Q_0$

**∴ $R_1$ is symmetric.**

**Check for Transitivity:**

$\forall$ a, b, c $\in$ Z

If $(a, b) \in R_2$ and $(b, c) \in R_2$

$\Rightarrow |a - b| \leq 5$ and $|b - c| \leq 5$

Since, inequalities cannot be added or subtract. We need to take example to check for,

$|a - c| \leq 5$

Take values a = 18, b = 14 and c = 10

Check: $|a - b| \leq 5$

$\Rightarrow |18 - 14| \leq 5$

$\Rightarrow |4| \leq 5$ is true.

Check: $|b - c| \leq 5$

$\Rightarrow |14 - 10| \leq 5$

$\Rightarrow |4| \leq 5$

Check: $|a - c| \leq 5$

$\Rightarrow |18 - 10| \leq 5$

$\Rightarrow |8| \leq 5$ is not true.

$\Rightarrow (a, c) \notin R_2$

So, if $(a, b) \in R_2$ and $(b, c) \in R_2$, then $(a, c) \notin R_1$

$\forall$ a, b, c $\in$ Z

**∴ $R_2$ is not transitive.**

**3 C. Question**

Test whether the following relations $R_1$, $R_2$ and $R_3$ are (i) reflexive (ii) symmetric and (iii) transitive :

$R_3$ on R defined by $(a, b) \in R_3 \Leftrightarrow a^2 - 4\,ab + 3b^2 = 0$.

**Answer**

Here, $R_1$, $R_2$, $R_3$, and $R_4$ are the binary relations.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x $\in$ A, xRx.

R is symmetric if for all x, y $\in$ A, if xRy, then yRx.

R is transitive if for all x, y, z $\in$ A, if xRy and yRz, then xRz.

So, using these results let us start determining given relations.

We have

$R_3$ on R defined by $(a, b) \in R_3 \Leftrightarrow a^2 - 4ab + 3b^2 = 0$

**Check for Reflexivity:**

$\forall$ a $\in$ R,

(a, a) $\in R_3$ needs to be proved for reflexivity.

If (a, b) $\in R_3$, then we have

$a^2 - 4ab + 3b^2 = 0$

Replace b by a, we get

$a^2 - 4aa + 3a^2 = 0$

$\Rightarrow a^2 - 4a^2 + 3a^2 = 0$

$\Rightarrow -3a^2 + 3a^2 = 0$

$\Rightarrow$ 0 = 0, which is true.

$\Rightarrow$ (a, a) $\in R_3$

So, $\forall$ a $\in$ R, (a, a) $\in R_3$

**$\therefore R_3$ is reflexive.**

**Check for Symmetry:**

$\forall$ a, b $\in$ R

If (a, b) $\in R_3$, then we have

$a^2 - 4ab + 3b^2 = 0$

$\Rightarrow a^2 - 3ab - ab + 3b^2 = 0$

$\Rightarrow a (a - 3b) - b (a - 3b) = 0$

$\Rightarrow (a - b) (a - 3b) = 0$

$\Rightarrow (a - b) = 0$ or $(a - 3b) = 0$

$\Rightarrow a = b$ or $a = 3b$ …(1)

Replace a by b and b by a in equation (1), we get

$\Rightarrow b = a$ or $b = 3a$ …(2)

Results (1) and (2) does not match.

$\Rightarrow$ (b, a) $\notin R_3$

**$\therefore R_3$ is not symmetric.**

**Check for Transitivity:**

$\forall$ a, b, c $\in$ R

If (a, b) $\in R_3$ and (b, c) $\in R_3$

$\Rightarrow a^2 - 4ab + 3b^2 = 0$ and $b^2 - 4bc + 3c^2 = 0$

$\Rightarrow a^2 - 3ab - ab + 3b^2 = 0$ and $b^2 - 3bc - bc + 3c^2$

$\Rightarrow a (a - 3b) - b (a - 3b) = 0$ and $b (b - 3c) - c (b - 3c) = 0$

$\Rightarrow (a - b) (a - 3b) = 0$ and $(b - c) (b - 3c) = 0$

$\Rightarrow (a - b) = 0$ or $(a - 3b) = 0$

And $(b - c) = 0$ or $(b - 3c) = 0$

$\Rightarrow a = b$ or $a = 3b$ And $b = c$ or $b = 3c$

What we need to prove here is that, a = c or a = 3c

Take a = b and b = c

Clearly implies that a = c.

[∵ if a = b, just substitute a in place of b in b = c. We get, a = c]

Now, take a = 3b and b = 3c

If a = 3b

$\Rightarrow b = \frac{a}{3}$

Substitute $b = \frac{a}{3}$ in b = 3c. We get

$\frac{a}{3} = 3c$

⇒ a = 9c, which is not the desired result.

⇒ (a, c) ∉ $R_3$

**∴ $R_3$ is not transitive.**

**4. Question**

Let A = {1, 2, 3}, and let $R_1$ = {(1, 1), (1, 3), (3, 1), (2, 2), (2, 1), (3, 3), (2, 2), (2, 1), (3, 3)}, $R_2$={(2,2), (3,1), (1, 3)}, $R_3$ = {(1, 3),(3, 3)}. Find whether or not each of the relations $R_1$, $R_2$, $R_3$ on A is (i) reflexive (ii) symmetric (iii) transitive.

**Answer**

We have been given,

A = {1, 2, 3}

Here, $R_1$, $R_2$, and $R_3$ are the binary relations on A.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

So, using these results let us start determining given relations.

Let us take $R_1$.

$R_1$ = {(1, 1), (1, 3), (3, 1), (2, 2), (2, 1), (3, 3)}

(i). **Reflexive:**

∀ 1, 2, 3 ∈ A [∵ A = {1, 2, 3}]

(1, 1) ∈ $R_1$

(2, 2) ∈ $R_2$

(3, 3) ∈ $R_3$

So, for a ∈ A, (a, a) ∈ $R_1$

**∴ $R_1$ is reflexive.**

(ii). **Symmetric:**

∀ 1, 2, 3 ∈ A

If $(1, 3) \in R_1$, then $(3, 1) \in R_1$

[$\because R_1 = \{(1, 1),$ **$(1, 3), (3, 1)$**$, (2, 2), (2, 1), (3, 3)\}$]

But if $(2, 1) \in R_1$, then $(1, 2) \notin R_1$

[$\because R_1 = \{(1, 1), (1, 3), (3, 1), (2, 2),$**$(2, 1)$**$, (3, 3)\}$]

So, if $(a, b) \in R_1$, then $(b, a) \notin R_1$

$\forall$ a, b $\in$ A

**$\therefore R_1$ is not symmetric.**

(iii). **Transitivity:**

$\forall$ 1, 2, 3 $\in$ A

If $(1, 3) \in R_1$ and $(3, 3) \in R_1$

Then, $(1, 3) \in R_1$

[$\because R_1 = \{(1, 1),$ **$(1, 3)$**$, (3, 1), (2, 2), (2, 1),$**$(3, 3)$**$\}$]

But, if $(2, 1) \in R_1$ and $(1, 3) \in R_1$

Then, $(2, 3) \notin R_1$

So, if $(a, b) \in R_1$ and $(b, c) \in R_1$, then $(a, c) \notin R_1$

$\forall$ a, b, c $\in$ A

**$\therefore R_1$ is not transitive.**

Now, take $R_2$.

$R_2 = \{(2, 2), (3, 1), (1, 3)\}$

(i). **Reflexive:**

$\forall$ 1, 2, 3 $\in$ A [$\because A = \{1, 2, 3\}$]

$(1, 1) \notin R_2$

$(2, 2) \in R_2$

$(3, 3) \notin R_2$

So, for a $\in$ A, $(a, a) \notin R_2$

**$\therefore R_2$ is not reflexive.**

(ii). **Symmetric:**

$\forall$ 1, 2, 3 $\in$ A

If $(1, 3) \in R_2$, then $(3, 1) \in R_2$

[$\because R_2 = \{(2, 2),$ **$(3, 1), (1, 3)$**$\}$]

If $(2, 2) \in R_2$, then $(2, 2) \in R_2$

[$\because R_2 = \{$**$(2, 2),$**$(3, 1), (1, 3)\}$]

So, if $(a, b) \in R_2$, then $(b, a) \in R_2$

$\forall$ a, b $\in$ A

**$\therefore R_2$ is symmetric.**

(iii). **Transitivity:**

∀ 1, 2, 3 ∈ A

If (1, 3) ∈ $R_2$ and (3, 1) ∈ $R_2$

Then, (1, 1) ∉ $R_2$

[∵ $R_2$ = {(2, 2), **(3, 1), (1, 3)** }]

So, if (a, b) ∈ $R_2$ and (b, c) ∈ $R_2$, then (a, c) ∉ $R_2$

∀ a, b, c ∈ A

**∴ $R_2$ is not transitive.**

Now take $R_3$.

$R_3$ = {(1, 3), (3, 3)}

(i). **Reflexive:**

∀ 1, 3 ∈ A [∵ A = {1, 2, 3}]

(1, 1) ∉ $R_3$

(3, 3) ∈ $R_3$

So, for a ∈ A, (a, a) ∉ $R_3$

**∴ $R_3$ is not reflexive.**

(ii). **Symmetric:**

∀ 1, 3 ∈ A

If (1, 3) ∈ $R_3$, then (3, 1) ∉ $R_3$

[∵ $R_3$ = {**(1, 3)**, (3, 3)}]

So, if (a, b) ∈ $R_3$, then (b, a) ∉ $R_3$

∀ a, b ∈ A

**∴ $R_3$ is not symmetric.**

(iii). **Transitivity:**

∀ 1, 3 ∈ A

If (1, 3) ∈ $R_3$ and (3, 3) ∈ $R_3$

Then, (1, 3) ∈ $R_3$

[∵ $R_3$ = {**(1, 3), (3, 3)** }]

So, if (a, b) ∈ $R_3$ and (b, c) ∈ $R_3$, then (a, c) ∈ $R_3$

∀ a, b, c ∈ A

**∴ $R_3$ is transitive.**

**5 A. Question**

The following relations are defined on the set of real numbers :

aRb if a – b > 0

Find whether these relations are reflexive, symmetric or transitive.

**Answer**

Let set of real numbers be $\mathbb{R}$.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all $x \in A$, xRx.

R is symmetric if for all $x, y \in A$, if xRy, then yRx.

R is transitive if for all $x, y, z \in A$, if xRy and yRz, then xRz.

We have

aRb if $a - b > 0$

**Check for Reflexivity:**

For $a \in \mathbb{R}$

If aRa,

$\Rightarrow a - a > 0$

$\Rightarrow 0 > 0$

But $0 > 0$ is not possible.

Hence, aRa is not true.

So, $\forall\ a \in \mathbb{R}$, then aRa is not true.

$\Rightarrow$ R is not reflexive.

**∴ R is not reflexive.**

**Check for Symmetry:**

$\forall\ a, b \in \mathbb{R}$

If aRb,

$\Rightarrow a - b > 0$

Replace a by b and b by a, we get

$\Rightarrow b - a > 0$

[Take a = 12 and b = 6.

$a - b > 0$

$\Rightarrow 12 - 6 > 0$

$\Rightarrow 6 > 0$, which is a true statement.

Now, $b - a > 0$

$\Rightarrow 6 - 12 > 0$

$\Rightarrow -6 > 0$, which is not a true statement as $-6$ is not greater than 0.]

$\Rightarrow$ bRa is not true.

So, if aRb is true, then bRa is not true.

$\forall\ a, b \in \mathbb{R}$

$\Rightarrow$ R is not symmetric.

**∴ R is not symmetric.**

**Check for Transitivity:**

$\forall\ a, b, c \in \mathbb{R}$

If aRb and bRc.

⇒ a – b > 0 and b – c > 0

⇒ a – c > 0 or not.

Let us check.

a – b > 0 means a > b.

b – c > 0 means b > c.

a – c > 0 means a > c.

If a > b and b > c,

⇒ a > b, b > c

⇒ a > b > c

⇒ a > c

Hence, aRc is true.

So, if aRb is true and bRc is true, then aRc is true.

∀ a, b, c ∈ ℝ

⇒ R is transitive.

**∴ R is transitive.**

**5 B. Question**

The following relations are defined on the set of real numbers :

aRb if 1 + ab > 0

Find whether these relations are reflexive, symmetric or transitive.

**Answer**

Let set of real numbers be ℝ.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

We have

aRb if 1 + ab > 0

**Check for Reflexivity:**

For a ∈ ℝ

If aRa,

⇒ 1 + aa > 0

⇒ 1 + $a^2$ > 0

If a is a real number.

[Positive or negative, large or small, whole numbers or decimal numbers are all Real Numbers. Real numbers are so called because they are ‘Real’ not ‘imaginary’.]

This means, even if ‘a’ was negative.

$a^2$ = positive.

$a^2 + 1$ = positive

And any positive number is greater than 0.

Hence, $1 + a^2 > 0$

⇒ aRa is true.

So, ∀ a ∈ ℝ, then aRa is true.

⇒ R is reflexive.

**∴ R is reflexive.**

**Check for Symmetry:**

∀ a, b ∈ ℝ

If aRb,

⇒ 1 + ab > 0

Replace a by b and b by a, we get

⇒ 1 + ba > 0

Whether we write ab or ba, it is equal.

ab = ba

So, 1 + ba > 0

⇒ bRa is true.

So, if aRb is true, then bRa is true.

∀ a, b ∈ ℝ

⇒ R is symmetric.

**∴ R is symmetric.**

**Check for Transitivity:**

∀ a, b, c ∈ ℝ

If aRb and bRc.

⇒ 1 + ab > 0 and 1 + bc > 0

⇒ 1 + ac > 0 or not.

Let us check.

1 + ab > 0 means ab > –1.

1 + bc > 0 means bc > –1.

1 + ac > 0 means ac > –1.

If ab > –1 and bc > –1.

⇒ ac > –1 should be true.

Take a = –1, b = 0.9 and c = 1

ab > –1

⇒ (–1)(0.9) > –1

⇒ –0.9 > –1, is true on the number line.

bc > –1

⇒ (0.9)(1) > –1

⇒ 0.9 > –1, is true on the number line.

ac > –1

⇒ (–1)(1) > –1

⇒ –1 > –1, is not true as –1 cannot be greater than itself.

⇒ ac > –1 is not true.

⇒ 1 + ac > 0 is not true.

⇒ aRc is not true.

So, if aRb is true and bRc is true, then aRc is not true.

∀ a, b, c ∈ ℝ

⇒ R is not transitive.

**∴ R is not transitive.**

**5 C. Question**

The following relations are defined on the set of real numbers :

aRb if | a | ≤ b.

Find whether these relations are reflexive, symmetric or transitive.

**Answer**

Let set of real numbers be ℝ.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

We have

aRb if |a| ≤ b

**Check for Reflexivity:**

For a ∈ ℝ

If aRa,

⇒ |a| ≤ a, is true

If a is a real number.

[Positive or negative, large or small, whole numbers or decimal numbers are all Real Numbers. Real numbers are so called because they are ‘Real’ not ‘imaginary’.]

This means, even if ‘a’ was negative.

|a| = positive.

& |a| ≤ a

Hence, |a| ≤ a.

⇒ aRa is true.

So, ∀ a ∈ ℝ, then aRa is true.

⇒ R is reflexive.

**∴ R is reflexive.**

**Check for Symmetry:**

$\forall$ a, b $\in \mathbb{R}$

If aRb,

⇒ $|a| \leq b$

Replace a by b and b by a, we get

⇒ $|b| \leq a$, which might be true or not.

Let a = 2 and b = 3.

$|a| \leq b$

⇒ $|2| \leq 3$, is true

$|b| \leq a$

⇒ $|3| \leq 2$, is not true

⇒ bRa is not true.

So, if aRb is true, then bRa is not true.

$\forall$ a, b $\in \mathbb{R}$

⇒ R is not symmetric.

**∴ R is not symmetric.**

**Check for Transitivity:**

$\forall$ a, b, c $\in \mathbb{R}$

If aRb and bRc.

⇒ $|a| \leq b$ and $|b| \leq c$

⇒ $|a| \leq c$ or not.

Let us check.

If $|a| \leq b$ and $|b| \leq c$

$b \neq |b|$

Say, if b = –2

⇒ $-2 \neq |-2|$

⇒ $-2 \neq 2$

But, from $|a| \leq b$

$b \geq 0$ in every case otherwise the statement would not hold true.

⇒ b can only accept positive values including 0.

⇒ b is a whole number.

∴ if $|a| \leq b$ and $|b| \leq c$

⇒ $|a| \leq b$, $b \leq c$

⇒ $|a| \leq b \leq c$

⇒ $|a| \leq c$

⇒ aRc is true.

So, if aRb is true and bRc is true, then aRc is true.

$\forall$ a, b, c $\in \mathbb{R}$

$\Rightarrow$ R is transitive.

**∴ R is transitive.**

**6. Question**

Check whether the relation R defined on the set A={1,2,3,4,5,6} as R= {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

**Answer**

We have the set A = {1, 2, 3, 4, 5, 6}

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x $\in$ A, xRx.

R is symmetric if for all x, y $\in$ A, if xRy, then yRx.

R is transitive if for all x, y, z $\in$ A, if xRy and yRz, then xRz.

We have

R = {(a, b): b = a + 1}

∵ Every a, b $\in$ A.

And A = {1, 2, 3, 4, 5, 6}

The relation R on set A can be defined as:

Put a = 1

$\Rightarrow$ b = a + 1

$\Rightarrow$ b = 1 + 1

$\Rightarrow$ b = 2

$\Rightarrow$ (a, b) $\equiv$ (1, 2)

Put a = 2

$\Rightarrow$ b = 2 + 1

$\Rightarrow$ b = 3

$\Rightarrow$ (a, b) $\equiv$ (2, 3)

Put a = 3

$\Rightarrow$ b = 3 + 1

$\Rightarrow$ b = 4

$\Rightarrow$ (a, b) $\equiv$ (3, 4)

Put a = 4

$\Rightarrow$ b = 4 + 1

$\Rightarrow$ b = 5

$\Rightarrow$ (a, b) $\equiv$ (4, 5)

Put a = 5

$\Rightarrow$ b = 5 + 1

$\Rightarrow$ b = 6

⇒ (a, b) ≡ (5, 6)

Put a = 6

⇒ b = 6 + 1

⇒ b = 7

⇒ (a, b) ≠ (6, 7) [∵ 7 ∉ A]

Hence, R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

**Check for Reflexivity:**

For 1, 2, …, 6 ∈ A [∵ A = {1, 2, 3, 4, 5, 6}]

(1, 1) ∉ R

(2, 2) ∉ R

…

(6, 6) ∉ R

So, ∀ a ∈ A, then (a, a) ∉ R.

⇒ R is not reflexive.

**∴ R is not reflexive.**

**Check for Symmetry:**

∀ 1, 2 ∈ A [∵ A = {1, 2, 3, 4, 5, 6}]

If (1, 2) ∈ R

Then, (2, 1) ∉ R

[∵ R = {**(1, 2)**, (2, 3), (3, 4), (4, 5), (5, 6)}]

So, if (a, b) ∈ R, then (b, a) ∉ R

∀ a, b ∈ A

⇒ R is not symmetric.

**∴ R is not symmetric.**

**Check for Transitivity:**

∀ 1, 2, 3 ∈ A

If (1, 2) ∈ R and (2, 3) ∈ R

Then, (1, 3) ∉ R

[∵ R = {**(1, 2), (2, 3)**, (3, 4), (4, 5), (5, 6)}]

So, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∉ R.

∀ a, b, c ∈ A

⇒ R is not transitive.

**∴ R is not transitive.**

**7. Question**

Check whether the relation R on **R** defined by R = {(a, b) : a ≤ $b^3$} is reflexive, symmetric or transitive.

**Answer**

We have the set of real numbers, **R**.

So, recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

We have

R = {(a, b): $a \leq b^3$}

**Check for Reflexivity:**

For a ∈ **R**

If (a, a) ∈ R,

⇒ $a \leq a^3$, which is not true.

Say, if a = – 2.

$a \leq a^3$

⇒ – 2 ≤ – 8

⇒ –2 ≤ –8, which is not true as – 2 > – 8.

Hence, (a, a) ∉ R

So, ∀ a ∈ **R**, then (a, a) ∉ R.

⇒ R is not reflexive.

**∴ R is not reflexive.**

**Check for Symmetry:**

∀ a, b ∈ **R**

If (a, b) ∈ R

⇒ $a \leq b^3$

Replace a by b and b by a, we get

⇒ $b \leq a^3$

[Take a = –2 and b = 3.

$a \leq b^3$

⇒ $-2 \leq 3^3$

⇒ –2 ≤ 27, which is a true statement.

Now, $b \leq a^3$

⇒ $3 \leq (-2)^3$

⇒ 3 ≤ –8, which is not a true statement as 3 > –8]

⇒ (b, a) ∉ R

So, if (a, b) ∈ R, then (b, a) ∉ R

∀ a, b ∈ **R**

⇒ R is not symmetric.

**∴ R is not symmetric.**

**Check for Transitivity:**

∀ a, b, c ∈ **R**

If (a, b) ∈ R and (b, c) ∈ R

⇒ $a \leq b^3$ and $b \leq c^3$

⇒ $a \leq c^3$ or not.

Let us check.

Take a = 3, $b = \frac{3}{2}$ and $c = \frac{6}{5}$.

$a \leq b^3$

$$\Rightarrow 3 \leq \left(\frac{3}{2}\right)^3$$

$$\Rightarrow 3 \leq \frac{27}{8}$$

⇒ 3 ≤ 3.37, which is true.

$b \leq c^3$

$$\Rightarrow \frac{3}{2} \leq \left(\frac{6}{5}\right)^3$$

$$\Rightarrow \frac{3}{2} \leq \frac{216}{125}$$

⇒ 1.5 ≤ 1.728

$a \leq c^3$

$$\Rightarrow 3 \leq \left(\frac{6}{5}\right)^3$$

$$\Rightarrow 3 \leq \frac{216}{125}$$

⇒ 3 ≤ 1.728, which is not true as 3 > 1.728.

Hence, (a, c) ∉ R.

So, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∉ R.

∀ a, b, c ∈ ℝ

⇒ R is not transitive.

**∴ R is not transitive.**

**Hence, R is neither reflexive, nor symmetric, nor transitive.**

**8. Question**

Prove that every identity relation on a set is reflexive, but the converse is not necessarily true.

**Answer**

To Prove: Every identity relation on a set is reflexive, but every reflexive relation is not identity relation.

Proof:

Let us first understand what 'Reflexive Relation' is and what 'Identity Relation' is.

Reflexive Relation: A binary relation R over a set A is reflexive if every element of X is related to itself. Formally, this may be written as ∀ x ∈ A: xRx.

Identity Relation: Let A be any set.

Then the relation R= {(x, x): x ∈ A} on A is called the identity relation on A. Thus, in an identity relation, every element is related to itself only.

Let A = {a, b, c} be a set.

Let R be a binary relation defined on A.

Let $R_A$ = {(a, a): a ∈ A} is the identity relation on A.

Hence, every identity relation on set A is reflexive by definition.

Converse: Let A = {a, b, c} is the set.

Let R = {(a, a), (b, b), (c, c), (a, b), (c, a)} be a relation defined on A.

R is reflexive as per definition.

[∵ (a, a) ∈ R, (b, b) ∈ R & (c, c) ∈ R]

But, (a, b) ∈ R

(c, a) ∈ R

⇒ R is not identity relation by definition.

**Hence, proved that every identity relation on a set is reflexive, but the converse is not necessarily true.**

**9 A. Question**

If A = {1, 2, 3, 4}, define relations on A which have properties of being

reflexive, transitive but not symmetric.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these properties, we can define R on A.

A = {1, 2, 3, 4}

We need to define a relation (say, R) which is reflexive, transitive but not symmetric.

Let us try to form a small relation step by step.

The relation must be defined on A.

Reflexive relation:

R = {(1, 1), (2, 2), (3, 3), (4, 4)} …(1)

Transitive relation:

R = {(1, 2), (2, 1), (1, 1)}, is transitive but also symmetric.

So, let us define another relation.

R = {(1, 3), (3, 2), (1, 2)}, is transitive and not symmetric. …(2)

Let us combine (i) and (ii) relation.

R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 2), (1, 2)} …(A)

(A) can be shortened by eliminating (3, 2) and (1, 2) from R.

R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 3)} …(B)

Further (B) can be shortened by eliminating (2, 2) and (4, 4).

R = {(1, 1), (3, 3), (1, 3)} …(C)

All the results (A), (B) and (C) is correct.

Thus, we have got the relation which is reflexive, transitive but not symmetric.

**9 B. Question**

If A = {1, 2, 3, 4}, define relations on A which have properties of being

symmetric but neither reflexive nor transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these properties, we can define R on A.

A = {1, 2, 3, 4}

We need to define a relation (say, R) which is symmetric but neither reflexive nor transitive.

The relation R must be defined on A.

Symmetric relation:

R = {(1, 2), (2, 1)}

Note that, the relation R here is neither reflexive nor transitive, and it is the shortest relation that can be form.

Similarly, we can also write:

R = {(1, 3), (3, 1)}

Or R = {(3, 4), (4, 3)}

Or R = {(2, 3), (3, 2), (1, 4), (4, 1)}

And so on…

All of these are right answers.

Thus, we have got the relation which is symmetric but neither reflexive nor transitive.

**9 C. Question**

If A = {1, 2, 3, 4}, define relations on A which have properties of being

reflexive, symmetric and transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Using these properties, we can define R on A.

A = {1, 2, 3, 4}

We need to define a relation (say, R) which is reflexive, symmetric and transitive.

The relation must be defined on A.

Reflexive Relation:

R = {(1, 1), (2, 2), (3, 3), (4, 4)}

Or simply shorten it and write,

R = {(1, 1), (2, 2)} …(1)

Symmetric Relation:

R = {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3)}

Or simply shorten it and write,

R = {(1, 2), (2, 1)} …(2)

Combine results (1) and (2), we get

R = {(1, 1), (2, 2), (1, 2), (2, 1)}

It is reflexive, symmetric as well as transitive as per definition.

Similarly, we can find other combinations too.

Thus, we have got the relation which is reflexive, symmetric as well as transitive.

**10. Question**

Let R be a relation defined on the set of natural numbers N as

R = {(x, y) : x, y ∈ N, 2x + y = 41}

Find the domain and range of R. Also, verify whether R is

(i) reflexive,

(ii) symmetric

(iii) transitive.

**Answer**

First let us define what range and domain are.

Range: The range of a function is the complete set of all possible resulting values of the dependent variable (y, usually) after we have substituted the domain. In plain English, the definition means: The range is the resulting y-values we get after substituting all the possible x-values.

Domain: The domain of definition of a function is the set of "input" or argument values for which the function is defined. That is, the function provides an "output" or value for each member of the domain.

We have been given that, R is a relation defined on N.

N = set of natural numbers

R = {(x, y): x, y ∈ N, 2x + y = 41}

We have the function as

2x + y = 41

⇒ y = 41 – 2x

As y ∈ N (Natural number)

⇒ 41 – 2x ≥ 1

⇒ –2x ≥ 1 – 41

⇒ –2x ≥ –40

⇒ 2x ≤ 40

$\Rightarrow x \leq 20$

**So, the domain is first 20 natural numbers.**

As, $2x + y = 41$

$\Rightarrow 2x = 41 - y$

$$\Rightarrow x = \frac{41 - y}{2}$$

As $x \in N$ (Natural number)

$$\Rightarrow \frac{41 - y}{2} \geq 1$$

$\Rightarrow 41 - y \geq 2$

$\Rightarrow -y \geq 2 - 41$

$\Rightarrow -y \geq -39$

$\Rightarrow y \leq 39$

**So, the range is first 39 natural numbers.**

We have relation R defined on set N.

$R = \{(x, y): x, y \in N, 2x + y = 41\}$

**Check for Reflexivity:**

$\forall\, x \in N$

If $(x, x) \in R$

$\Rightarrow 2x + x = 41$

$\Rightarrow 3x = 41$

$$\Rightarrow x = \frac{41}{3}$$

$\Rightarrow x = 13.67$

But, $x \neq 13.67$ as $x \in N$.

$\Rightarrow (x, x) \notin R$

So, $\forall\, x \in N$, then $(x, x) \notin R$

$\Rightarrow$ R is not reflexive.

**∴ R is not reflexive.**

**Check for Symmetry:**

$\forall\, x, y \in N$

If $(x, y) \in R$

$\Rightarrow 2x + y = 41$ …(i)

Now, replace x by y and y by x, we get

$\Rightarrow 2y + x = 41$ …(ii)

Take $x = 20$ and $y = 1$.

Equation (i) $\Rightarrow 2(20) + 1 = 41$

$\Rightarrow 40 + 1 = 41$

$\Rightarrow 41 = 41$, holds true.

Equation (ii) ⇒ 2(1) + 20 = 41

⇒ 2 + 20 = 41

⇒ 22 = 41, which is not true as 22 ≠ 41.

⇒ (y, x) ∉ R

So, if (x, y) ∈ R, then (y, x) ∉ R.

∀ x, y ∈ N

⇒ R is not symmetric.

**∴ R is not symmetric.**

**Check for Transitivity:**

∀ x, y, z ∈ N

If (x, y) ∈ R and (y, z) ∈ R

⇒ 2x – y = 41 and 2y – z = 41

⇒ 2x – z = 41, may be true or not.

Let us sole these to find out.

We have

2x – y = 41 …(iii)

2y – z = 41 …(iv)

Multiply 2 by equation (i), we get

4x – 2y = 82 …(v)

Adding equation (v) and (iv), we get

(4x – 2y) + (2y – z) = 82 + 41

⇒ 4x – z = 123

⇒ 2x + 2x – z = 123

⇒ 2x – z = 123 – 2x

Take x = 40 (as x ∈ N)

⇒ 2x – z = 123 – 2(40)

⇒ 2x – z = 123 – 80

⇒ 2x – z = 43 ≠ 41

⇒ (x, z) ∉ R

So, if (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∉ R.

∀ x, y, z ∈ N

⇒ R is not transitive.

**∴ R is not transitive.**

**11. Question**

Is it true that every relation which is symmetric and transitive is also reflexive? Give reasons.

**Answer**

It is not true that every relation which is symmetric and transitive is also reflexive.

Take for example:

Take a set A = {1, 2, 3, 4}

And define a relation R on A.

Symmetric relation:

R = {(1, 2), (2, 1)}, is symmetric on set A.

Transitive relation:

R = {(1, 2), (2, 1), (1, 1)}, is the simplest transitive relation on set A.

⇒ R = {(1, 2), (2, 1), (1, 1)} is symmetric as well as transitive relation.

But R is not reflexive here.

If only (2, 2) ∈ R, had it been reflexive.

Thus, it is not true that every relation which is symmetric and transitive is also reflexive.

**12. Question**

An integer m is said to be related to another integer n if m is a multiple of n. Check if the relation is symmetric, reflexive and transitive.

**Answer**

According to the question,

m is related to n if m is a multiple of n.

∀ m, n ∈ **I** (**I** being set of integers)

The relation comes out to be:

R = {(m, n): m = kn, k ∈ $\mathbb{Z}$}

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

**Check for Reflexivity:**

∀ m ∈ **I**

If (m, m) ∈ R

⇒ m = k m, holds.

As an integer is always a multiple of itself, So, ∀ m ∈ **I**, then (m, m) ∈ R.

⇒ R is reflexive.

**∴ R is reflexive.**

**Check for Symmetry:**

∀ m, n ∈ **I**

If (m, n) ∈ R

⇒ m = k n, holds.

Now, replace m by n and n by m, we get

n = k m, which may or not be true.

Let us check:

If 12 is a multiple of 3, but 3 is not a multiple of 12.

⇒ $n = km$ does not hold.

So, if $(m, n) \in R$, then $(n, m) \notin R$.

$\forall$ m, n $\in$ I

⇒ R is not symmetric.

**∴ R is not symmetric.**

**Check for Transitivity:**

$\forall$ m, n, o $\in$ I

If $(m, n) \in R$ and $(n, o) \in R$

⇒ $m = kn$ and $n = ko$

Where $k \in \mathbb{Z}$

Substitute $n = ko$ in $m = kn$, we get

$m = k(ko)$

⇒ $m = k^2o$

If $k \in \mathbb{Z}$, then $k^2 \in \mathbb{Z}$.

Let $k^2 = r$

⇒ $m = ro$, holds true.

⇒ $(m, o) \in R$

So, if $(m, n) \in R$ and $(n, o) \in R$, then $(m, o) \in R$.

$\forall$ m, n $\in$ I

⇒ R is transitive.

**∴ R is transitive.**

**13. Question**

Show that the relation "≥" on the set R of all real numbers is reflexive and transitive but not symmetric.

**Answer**

We have

The relation "≥" on the set R of all real numbers.

Recall that for any binary relation R on set A. We have,

R is reflexive if for all $x \in A$, xRx.

R is symmetric if for all $x, y \in A$, if xRy, then yRx.

R is transitive if for all $x, y, z \in A$, if xRy and yRz, then xRz.

So, let the relation having "≥" be P.

We can write

$P = \{(a, b): a \geq b, a, b \in R\}$

**Check for Reflexivity:**

$\forall$ a $\in$ R

If $(a, a) \in P$

⇒ $a \geq a$, which is true.

Since, every real number is equal to itself.

So, $\forall a \in R$, then $(a, a) \in P$.

⇒ P is reflexive.

**∴ P is reflexive.**

**Check for Symmetry:**

$\forall a, b \in R$

If $(a, b) \in P$

⇒ $a \geq b$

Now, replace a by b and b by a. We get

$b \geq a$, might or might not be true.

Let us check:

Take $a = 7$ and $b = 5$.

$a \geq b$

⇒ $7 \geq 5$, holds.

$b \geq a$

⇒ $5 \geq 7$, is not true as $5 < 7$.

⇒ $b \geq a$, is not true.

⇒ $(b, a) \notin P$

So, if $(a, b) \in P$, then $(b, a) \notin P$

$\forall a, b \in R$

⇒ P is not symmetric.

**∴ P is not symmetric.**

**Check for Transitivity:**

$\forall a, b, c \in R$

If $(a, b) \in P$ and $(b, c) \in P$

⇒ $a \geq b$ and $b \geq c$

⇒ $a \geq b \geq c$

⇒ $a \geq c$

⇒ $(a, c) \in P$

So, if $(a, b) \in P$ and $(b, c) \in P$, then $(a, c) \in P$

$\forall a, b, c \in R$

⇒ P is transitive.

**∴ P is transitive.**

Thus, shown that the relation "≥" on the set R of all the real numbers are reflexive and transitive but not symmetric.

**14 A. Question**

Give an example of a relation which is

reflexive and symmetric but not transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Let there be a set A.

A = {1, 2, 3, 4}

We need to define a relation on A which is reflexive and symmetric but not transitive.

Let there be a set A.

A = {1, 2, 3, 4}

Reflexive relation:

R = {(1, 1), (2, 2), (3, 3), (4, 4)} …(1)

Symmetric relation:

R = {(3, 4), (4, 3)} …(2)

Combine results (1) and (2), we get

**R = {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 3)}**

Check for Transitivity:

If (3, 4) ∈ R and (4, 3) ∈ R

Then, (3, 3) ∈ R

∀ 3, 4 ∈ A [∵ A = {1, 2, 3, 4}]

So eliminate (3, 3) from R, we get

**R = {(1, 1), (2, 2), (4, 4), (3, 4), (4, 3)}**

Check for Transitivity:

If (4, 3) ∈ R and (3, 4) ∈ R

Then, (4, 4) ∈ R

∀ 3, 4 ∈ A

So, eliminate (4, 4) from R, we get

**R = {(1, 1), (2, 2), (3, 4), (4, 3)}**

Thus, the relation which is reflexive and symmetric but not transitive is:

**R = {(1, 1), (2, 2), (3, 4), (4, 3)}**

**14 B. Question**

Give an example of a relation which is

reflexive and transitive but not symmetric.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Let there be a set A.

A = {1, 2, 3, 4}

We need to define a relation on A which is reflexive and transitive but not symmetric.

Let there be a set A.

A = {1, 2, 3, 4}

Reflexive relation:

R = {(1, 1), (2, 2), (3, 3), (4, 4)} …(1)

Transitive relation:

R = {(3, 4), (4, 1), (3, 1)} …(2)

Combine results (1) and (2), we get

**R = {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 1), (3, 1)}**

Check for Symmetry:

If (3, 4) ∈ R

Then, (4, 3) ∉ R

∀ 3, 4 ∈ A [∵ A = {1, 2, 3, 4}]

One example is enough to prove that R is not symmetric.

Thus, the relation which is reflexive and transitive but not symmetric is:

**R = {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 1), (3, 1)}**

**14 C. Question**

Give an example of a relation which is

symmetric and transitive but not reflexive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

Let there be a set A.

A = {1, 2, 3, 4}

We need to define a relation on A which is symmetric and transitive but not reflexive.

It is not possible to define such relation which is symmetric and transitive but not reflexive. As every relation which is symmetric and transitive will use identity ordered pair of the form (x, x) to balance the relation (to make the relation symmetric and transitive). Without such identity pair both, symmetry and transitivity will not be possible.

**14 D. Question**

Give an example of a relation which is

symmetric but neither reflexive nor transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all $x \in A$, xRx.

R is symmetric if for all $x, y \in A$, if xRy, then yRx.

R is transitive if for all $x, y, z \in A$, if xRy and yRz, then xRz.

Let there be a set A.

$A = \{1, 2, 3, 4\}$

We need to define a relation which is symmetric but neither reflexive nor transitive.

Let there be a set A.

$A = \{1, 2, 3, 4\}$

Symmetric Relation:

$\{(1, 3), (3, 1)\}$

This is neither reflexive nor transitive.

$\because (1, 1) \notin R$

$(3, 3) \notin R$

Hence, R is not reflexive.

$\because (1, 3) \in R$ and $(3, 1) \in R$

Then, $(1, 1) \notin R$

Hence, R is not transitive.

Thus, the relation which is symmetric but neither nor transitive is:

$R = \{(1, 3), (3, 1)\}$

**14 E. Question**

Give an example of a relation which is

transitive but neither reflexive nor symmetric.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all $x \in A$, xRx.

R is symmetric if for all $x, y \in A$, if xRy, then yRx.

R is transitive if for all $x, y, z \in A$, if xRy and yRz, then xRz.

Let there be a set A.

$A = \{1, 2, 3, 4\}$

We need to define a relation which is transitive but neither reflexive nor symmetric.

Let there be a set A.

$A = \{1, 2, 3\}$

Transitive Relation:

$R = \{(2, 4), (4, 1), (2, 1)\}$

This is neither reflexive nor symmetric.

$\because (1, 1) \notin R$

(2, 2) ∉ R

(4, 4) ∉ R

Hence, R is not reflexive.

∵ if (2, 4) ∈ R

Then, (4, 2) ∉ R

Hence, R is not symmetric.

Thus, the relation which is transitive but neither reflexive nor symmetric is:

R = {(2, 4), (4, 1), (2, 1)}

**15. Question**

Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, add a minimum number ordered pairs so that the enlarged relation is symmetric, transitive and reflexive.

**Answer**

Given is:

R = {(1, 2), (2, 3)} on the set A.

A = {1, 2, 3}

Right now, we have

R = {(1, 2), (2, 3)}

Symmetric Relation:

We know (1, 2) ∈ R

Then, (2, 1) ∈ R

Also, (2, 3) ∈ R

Then, (3, 2) ∈ R

So, add (2, 1) and (3, 2) in R, so that we get

R’ = {(1, 2), (2, 1), (2, 3), (3, 2)}

Transitive Relation:

We need to make the relation R’ transitive.

So, we know (1, 2) ∈ R and (2, 1) ∈ R

Then, (1, 1) ∈ R

Also, (2, 3) ∈ R and (3, 2)

Then, (2, 2) ∈ R

Also, (2, 1) ∈ R and (1, 2) ∈ R

Then, (2, 2) ∈ R

Also, (3, 2) ∈ R and (2, 3) ∈ R

Then, (3, 3) ∈ R

Add (1, 1), (2, 2) and (3, 3) in R’, we get

R’’ = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

Thus, we have got a relation which is reflexive, symmetric and transitive.

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

The ordered pair added are (1, 1), (2, 2), (3, 3), (3, 2).

**16. Question**

Let A = {1, 2, 3} and R = {(1, 2), (1, 1), (2, 3)} be a relation on A. What minimum number of ordered pairs may be added to R so that it may become a transitive relation on A.

**Answer**

We have the relation R such that

R = {(1, 2), (1, 1), (2, 3)}

R is defined on set A.

A = {1, 2, 3}

Recall that,

A relation R defined on a set A is called transitive if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R, ∀ a, b, c ∈ A.

For transitive relation:

Note in R,

(1, 2) ∈ R and (2, 3) ∈ R

Then, (1, 3) ∈ R

So, add (1, 3) in R.

R = {(1, 2), (1, 1), (2, 3), (1, 3)}

Now, we can see that R is transitive.

Hence, the ordered pair to be added is (1, 3).

**17. Question**

Let A = {a, b, c} and the relation R be defined on A as follows R={(a,a), (b, c), (a, b)}. Then, write a minimum number of ordered pairs to be added in R to make it reflexive and transitive.

**Answer**

Recall that,

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

We have relation R = {(a, a), (b, c), (a, b)} on A.

A = {a, b, c}

For Transitive:

If (a, b) ∈ R and (b, c) ∈ R

Then, (a, c) ∈ R

∀ a, b, c ∈ A

For Reflexive:

∀ a, b, c ∈ R

Then, (a, a) ∈ R

(b, b) ∈ R

(c, c) ∈ R

We need to add (b, b), (c, c) and (a, c) in R.

We get

$R = \{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$

**18 A. Question**

Each of the following defines a relation on N :

$x > y$, $x, y \in N$

Determine which of the above relations are reflexive, symmetric and transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all $x \in A$, xRx.

R is symmetric if for all $x, y \in A$, if xRy, then yRx.

R is transitive if for all $x, y, z \in A$, if xRy and yRz, then xRz.

We have

$x > y$, $x, y \in N$

This relation is defined on N (set of Natural Numbers)

The relation can also be defined as

$R = \{(x, y): x > y\}$ on N

**Check for Reflexivity:**

$\forall x \in N$

We should have, $(x, x) \in R$

$\Rightarrow x > x$, which is not true.

1 can't be greater than 1.

2 can't be greater than 2.

16 can't be greater than 16.

Similarly, x can't be greater than x.

So, $\forall x \in N$, then $(x, x) \notin R$

$\Rightarrow$ R is not reflexive.

**Check for Symmetry:**

$\forall x, y \in N$

If $(x, y) \in R$

$\Rightarrow x > y$

Now, replace x by y and y by x. We get

$y > x$, which may or not be true.

Let us take $x = 5$ and $y = 2$.

$x > y$

$\Rightarrow 5 > 2$, which is true.

$y > x$

$\Rightarrow 2 > 5$, which is not true.

$\Rightarrow y > x$, is not true as $x > y$

$\Rightarrow (y, x) \notin R$

So, if $(x, y) \in R$, but $(y, x) \notin R\ \forall\ x, y \in N$

$\Rightarrow$ R is not symmetric.

**Check for Transitivity:**

$\forall\ x, y, z \in N$

If $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow x > y$ and $y > z$

$\Rightarrow x > y > z$

$\Rightarrow x > z$

$\Rightarrow (x, z) \in R$

So, if $(x, y) \in R$ and $(y, z) \in R$, and then $(x, z) \in R$

$\forall\ x, y, z \in N$

$\Rightarrow$ R is transitive.

**Hence, the relation is transitive but neither reflexive nor symmetric.**

**18 B. Question**

Each of the following defines a relation on N :

$x + y = 10,\ x, y \in N$

Determine which of the above relations are reflexive, symmetric and transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all $x \in A$, xRx.

R is symmetric if for all $x, y \in A$, if xRy, then yRx.

R is transitive if for all $x, y, z \in A$, if xRy and yRz, then xRz.

We have

$x + y = 10,\ x, y \in N$

This relation is defined on N (set of Natural Numbers)

The relation can also be defined as

$R = \{(x, y): x + y = 10\}$ on N

**Check for Reflexivity:**

$\forall\ x \in N$

We should have, $(x, x) \in R$

$\Rightarrow x + x = 10$, which is not true everytime.

Take $x = 4$.

$x + x = 10$

$\Rightarrow 4 + 4 = 10$

$\Rightarrow 8 = 10$, which is not true.

That is $8 \neq 10$.

So, $\forall$ x $\in$ N, then (x, x) $\notin$ R

$\Rightarrow$ R is not reflexive.

**Check for Symmetry:**

$\forall$ x, y $\in$ N

If (x, y) $\in$ R

$\Rightarrow x + y = 10$

Now, replace x by y and y by x. We get

$y + x = 10$, which is as same as $x + y = 10$.

$\Rightarrow y + x = 10$

$\Rightarrow$ (y, x) $\in$ R

So, if (x, y) $\in$ R, and then (y, x) $\in$ R $\forall$ x, y $\in$ N

$\Rightarrow$ R is symmetric.

**Check for Transitivity:**

$\forall$ x, y, z $\in$ N

If (x, y) $\in$ R and (y, z) $\in$ R

$\Rightarrow x + y = 10$ and $y + z = 10$

$\Rightarrow x + z = 10$, may or may not be true.

Let us take x = 6, y = 4 and z = 6

$x + y = 10$

$\Rightarrow 6 + 4 = 10$

$\Rightarrow 10 = 10$, which is true.

$y + z = 10$

$\Rightarrow 4 + 6 = 10$

$\Rightarrow 10 = 10$, which is true.

$x + z = 10$

$\Rightarrow 6 + 6 = 10$

$\Rightarrow 12 = 10$, which is not true

That is, $12 \neq 10$

$\Rightarrow x + z \neq 10$

$\Rightarrow$ (x, z) $\notin$ R

So, if (x, y) $\in$ R and (y, z) $\in$ R, and then (x, z) $\notin$ R

$\forall$ x, y, z $\in$ N

$\Rightarrow$ R is not transitive.

**Hence, the relation is symmetric but neither reflexive nor transitive.**

**18 C. Question**

Each of the following defines a relation on N :

xy is square of an integer, x, y ∈ N

Determine which of the above relations are reflexive, symmetric and transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

We have

xy is the square of an integer. x, y ∈ N.

This relation is defined on N (set of Natural Numbers)

The relation can also be defined as

$R = \{(x, y): xy = a^2, a = \sqrt{(xy)}, a \in N\}$ on N

**Check for Reflexivity:**

$\forall x \in N$

We should have, $(x, x) \in R$

$\Rightarrow xx = a^2$, where $a = \sqrt{(xx)}$

$\Rightarrow x^2 = a^2$, where $a = \sqrt{(x^2)}$

which is true every time.

Take x = 1 and y = 4

$xy = a^2$

$\Rightarrow 1 \times 4 = (\sqrt{(1 \times 4)})^2$ [$\because a = \sqrt{(xy)}$]

$\Rightarrow 4 = (\sqrt{4})^2$

$\Rightarrow 4 = (2)^2$

$\Rightarrow 4 = 4$

So, $\forall x \in N$, then $(x, x) \in R$

⇒ R is reflexive.

**Check for Symmetry:**

$\forall x, y \in N$

If $(x, y) \in R$

$\Rightarrow xy = a^2$, where $a = \sqrt{(xy)}$

Now, replace x by y and y by x. We get

$yx = a^2$, which is as same as $xy = a^2$

where $a = \sqrt{(yx)}$

$\Rightarrow yx = a^2$

$\Rightarrow (y, x) \in R$

So, if $(x, y) \in R$, and then $(y, x) \in R$ $\forall x, y \in N$

⇒ R is symmetric.

**Check for Transitivity:**

∀ x, y, z ∈ N

If (x, y) ∈ R and (y, z) ∈ R

⇒ xy = $a^2$ and yz = $a^2$

⇒ xz = $a^2$, may or may not be true.

Let us take x = 8, y = 2 and z = 50

xy = $a^2$, where a = √(xy)

⇒ (8)(2) = $(\sqrt{(8 \times 2)})^2$

⇒ 16 = $(4)^2$

⇒ 16 = 16, which is true.

yz = $a^2$

⇒ (2)(50) = $(\sqrt{(2 \times 50)})^2$

⇒ 100 = $(10)^2$

⇒ 100 = 100, which is true

xz = $a^2$

⇒ (8)(50) = $(\sqrt{(8 \times 50)})^2$

⇒ 400 = $(20)^2$

⇒ 400 = 400

We won't be able to find a case to show a contradiction.

⇒ xz = $a^2$

⇒ (x, z) ∈ R

So, if (x, y) ∈ R and (y, z) ∈ R, and then (x, z) ∈ R

∀ x, y, z ∈ N

⇒ R is transitive.

**Hence, the relation is symmetric and transitivity, but not reflexive.**

**18 D. Question**

Each of the following defines a relation on N :

x + 4y = 10, x, y ∈ N

Determine which of the above relations are reflexive, symmetric and transitive.

**Answer**

Recall that for any binary relation R on set A. We have,

R is reflexive if for all x ∈ A, xRx.

R is symmetric if for all x, y ∈ A, if xRy, then yRx.

R is transitive if for all x, y, z ∈ A, if xRy and yRz, then xRz.

We have

$x + 4y = 10$, $x, y \in N$

This relation is defined on N (set of Natural Numbers)

The relation can also be defined as

$R = \{(x, y): 4x + y = 10\}$ on N

**Check for Reflexivity:**

$\forall x \in N$

We should have, $(x, x) \in R$

$\Rightarrow 4x + x = 10$, which is obviously not true everytime.

Take $x = 4$.

$4x + x = 10$

$\Rightarrow 16 + 4 = 10$

$\Rightarrow 20 = 10$, which is not true.

That is $20 \neq 10$.

So, $\forall x \in N$, then $(x, x) \notin R$

$\Rightarrow$ R is not reflexive.

**Check for Symmetry:**

$\forall x, y \in N$

If $(x, y) \in R$

$\Rightarrow 4x + y = 10$

Now, replace x by y and y by x. We get

$4y + x = 10$, which may or may not be true.

Take $x = 1$ and $y = 6$

$4x + y = 10$

$\Rightarrow 4(1) + 6 = 10$

$\Rightarrow 4 + 6 = 10$

$\Rightarrow 10 = 10$

$4y + x = 10$

$\Rightarrow 4(6) + 1 = 10$

$\Rightarrow 24 + 1 = 10$

$\Rightarrow 25 = 10$, which is not true.

$\Rightarrow 4y + x \neq 10$

$\Rightarrow (y, x) \notin R$

So, if $(x, y) \in R$, and then $(y, x) \notin R$ $\forall x, y \in N$

$\Rightarrow$ R is not symmetric.

**Check for Transitivity:**

$\forall x, y, z \in N$

If $(x, y) \in R$ and $(y, z) \in R$

Then, $(x, z) \in R$

We have

$4x + y = 10$

$\Rightarrow y = 10 - 4x$

Where $x, y \in N$

So, put $x = 1$

$\Rightarrow y = 10 - 4(1)$

$\Rightarrow y = 10 - 4$

$\Rightarrow y = 6$

Put $x = 2$

$\Rightarrow y = 10 - 4(2)$

$\Rightarrow y = 10 - 8$

$\Rightarrow y = 2$

We can't take $y > 2$, because if we put $y = 3$

$\Rightarrow y = 10 - 4(3)$

$\Rightarrow y = 10 - 12$

$\Rightarrow y = -2$

But, $y \neq -2$ as $y \in N$

So, only ordered pairs possible are

$R = \{(1, 6), (2, 2)\}$

This relation R can never be transitive.

Because if $(a, b) \in R$, then $(b, c) \notin R$

$\Rightarrow$ R is not reflexive.

**Hence, the relation is neither reflexive nor symmetric nor transitive.**



## Exercise 1.2

**1. Question**

Show that the relation R defined by $R = \{(a, b): a - b \text{ is divisible by } 3; a, b \in Z\}$ is an equivalence relation.

**Answer**

We have,

$R = \{(a,b) : a-b \text{ is divisible by } 3; a, b \in Z\}$

To prove : R is an equivalence relation

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

$(a, a) \in R$

Let $a \in Z$

$\Rightarrow a - a = 0$

⇒ a – a is divisible by 3

(∵ 0 is divisible by 3).

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let a, b ∈ Z and (a, b) ∈ R

⇒ a – b is divisible by 3

⇒ a – b = 3p(say) For some p ∈ Z

⇒ –( a – b) = –3p

⇒ b – a = 3 × (–p)

⇒ b – a ∈ R

⇒ R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let a, b, c ∈ Z and such that (a, b) ∈ R and (b, c) ∈ R

⇒ a – b = 3p(say) and b – c = 3q(say) For some p, q ∈ Z

⇒ a – c = 3 (p + q)

⇒ a – c = 3 (p + q)

⇒ (a, c) ∈ R

⇒ R is transitive

Since, R is reflexive, symmetric and transitive

⇒ R is an equivalence relation.

**2. Question**

Show that the relation R on the set Z of integers, given by

R = {(a, b) : 2 divides a – b}, is an equivalence relation.

**Answer**

We have,

R = {(a, b) : a – b is divisible by 2; a, b ∈ Z}

To prove : R is an equivalence relation

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a ∈ Z

⇒ a – a = 0

⇒ a – a is divisible by 2

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let a, b ∈ Z and (a, b) ∈ R

⇒ a – b is divisible by 2

⇒ a – b = 2p For some p ∈ Z

⇒ b – a = 2 × (–p)

⇒ b – a ∈ R

⇒ R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let a, b, c ∈ Z and such that (a, b) ∈ R and (b, c) ∈ R

⇒ a – b = 2p(say) and b – c = 2q(say) , For some p, q ∈ Z

⇒ a – c = 2 (p + q)

⇒ a – c is divisible by 2

⇒ (a, c) ∈ R

⇒ R is transitive

Now, since R is symmetric, reflexive as well as transitive-

⇒ R is an equivalence relation.

**3. Question**

Prove that the relation R on Z defined by

(a, b) ∈ R ⇔ a – b is divisible by 5

is an equivalence relation on Z.

**Answer**

We have,

R = {(a, b) : (a – b) is divisible by 5} on Z.

We want to prove that R is an equivalence relation on Z.

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a ∈ Z

⇒ a – a = 0

⇒ a – a is divisible by 5.

∴ (a, a) ∈ R so R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (a, b) ∈ R

⇒ a – b = 5p(say) For some p ∈ Z

⇒ b – a = 5 × (–p)

⇒ b – a is divisible by 5

⇒ (b, a) ∈ R, so R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (a, b) ∈ R and (b, c) ∈ R

⇒ a – b = 5p(say) and b – c = 5q(say), For some p, q ∈ Z

⇒ a – c = 5 (p + q)

⇒ a – c is divisible by 5.

⇒ R is transitive

∴ R being reflexive, symmetric and transitive on Z.

⇒ R is equivalence relation on Z.

**4. Question**

Let n be a fixed positive integer. Define a relation R on Z as follows :

(a, b) ∈ R ⇔ a – b is divisible by n.

Show that R is an equivalence relation on Z.

**Answer**

R = {(a, b) : a – b is divisible by n} on Z.

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a ∈ Z

⇒ a – a = 0 × n

⇒ a – a is divisible by n

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (a, b) ∈ R

⇒ a – b = np For some p ∈ Z

⇒ b – a = n(–p)

⇒ b – a is divisible by n

⇒ (b, a) ∈ R

⇒ R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (a, b) ∈ R and (b, c) ∈ R

⇒ a – b = np and b – c = nq For some p, q ∈ Z

⇒ a – c = n (p + q)

⇒ a – c = is divisible by n

⇒ (a, c) ∈ R

⇒ R is transitive

∴ R being reflexive, symmetric and transitive on Z.

⇒ R is an equivalence relation on Z

**5. Question**

Let Z be the set of integers. Show that the relation R = {(a, b) : a, b∈ Z and a + b is even} is an equivalence relation on Z.

**Answer**

We have,

Z = set of integers and

R = {(a, b) : a, b ∈ Z and a + b is even} be a relation on Z.

To prove: R is an equivalence relation on Z.

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a ∈ Z

⇒ a + a is even

$$\begin{bmatrix} \text{if a is even} \Rightarrow a + a \text{ is even} \\ \text{if a is odd} \Rightarrow a + a \text{ is even} \end{bmatrix}$$

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric: For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let a, b ∈ Z and (a, b) ∈ R

⇒ a + b is even

⇒ b + a is even

⇒ (b, a) ∈ R

⇒ R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (a, b) ∈ R and (b, c) ∈ R For some a, b, c ∈ Z

⇒ a + b is even and b + c is even

[if b is odd, then a and c must be odd ⇒ a + c is even,

If b is even, then a and c must be even ⇒ a + c is even]

⇒ a + c is even

⇒ (a, c) ∈ R

⇒ R is transitive

Hence, R is an equivalence relation on Z

**6. Question**

m is said to be related to n if m and n are integers and m – n is divisible by 13. Does this define an equivalence relation?

**Answer**

To check that relation is equivalence, we need to check that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let m ∈ Z

⇒ m – m = 0

⇒ m – m is divisible by 13

⇒ (m, m) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let m, n ∈ Z and (m, n) ∈ R

⇒ m – n = 13p For some p ∈ Z

⇒ n – m = 13 × (–p)

⇒ n – m is divisible by 13

⇒ (n – m) ∈ R,

⇒ R is symmetric

Transitive:: For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (m, n) ∈ R and (n, q) ∈ R For some m, n, q ∈ Z

⇒ m – n = 13p and n – q = 13s For some p, s ∈ Z

⇒ m – q = 13 (p + s)

⇒ m – q is divisible by 13

⇒ (m, q) ∈ R

⇒ R is transitive

Hence, R is an equivalence relation on Z.

**7. Question**

Let R be a relation on the set A of ordered pairs of non-zero integers defined by (x, y) R (u, v) iff xv = yu.

Show that R is an equivalence relation.

**Answer**

(x, y) R (u, v) ⇔ xv = yu

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

∵ xy = yu

∴ (x, y) R (x, y)

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (x, y) R (u, v)

TPT (u, v) R (x, y)

Given xv = yu

⇒ yu = xv

⇒ uy = vx

∴ (u, v) R (x, y)

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (x, y) R (u, v) and (u, v) R (p, q) ...(i)

TPT (x, y) R (p, q)

TPT (xq = yp

From (1) xv = yu & uq = vp

xvuq = yuvp

xq = yp

∴ R is transitive

Since R is reflexive, symmetric & transitive

⇒ R is an equivalence relation.

**8. Question**

Show that the relation R on the set A = {x∈ Z ; 0 ≤ x ≤ 12}, given by R = {(a, b) : a = b}, is an equivalence relation. Find the set of all elements related to 1.

**Answer**

We have,

A = {x ∈ Z : 0 ≤ x ≤ 12} be a set and

R = {(a, b) : a = b} be a relation on A

Now,

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a ∈ A

⇒ a = a

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let a, b ∈ A and (a, b) ∈ R

⇒ a = b

⇒ b = a

⇒ (b, a) ∈ R

⇒ R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let a, b & c ∈ A

and Let (a, b) ∈ R and (b, c) ∈ R

⇒ a = b and b = c

⇒ a = c

⇒ (a, c) ∈ R

⇒ R is transitive

Since, R is being reflexive, symmetric and transitive, so R is an equivalence relation.

Also, we need to find the set of all elements related to 1.

Since the relation is given by, R = {(a, b) : a = b}, and 1 is an element of A,

R = {(1, 1) : 1 = 1}

Thus, the set of all element related to 1 is 1.



**9. Question**

Let L be the set of all lines in XY-plane and R be the relation in L defined as R = {($L_1$, $L_2$): $L_1$ is parallel to $L_2$}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

**Answer**

We have, L is the set of lines.

R = {($L_1$, $L_2$) : $L_1$ is parallel to $L_2$} be a relation on L

Now,

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Since a line is always parallel to itself.

∴ $(L_1, L_2) \in R$

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If $(a, b) \in R$, then $(b, a) \in R$

Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

⇒ $L_1$ is parallel to $L_2$

⇒ $L_2$ is parallel to $L_1$

⇒ $(L_1, L_2) \in R$

⇒ R is symmetric

Transitive: For Transitivity, we need to prove that-

If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$

Let $L_1$, $L_2$ and $L_3 \in L$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

⇒ $L_1$ is parallel to $L_2$ and $L_2$ is parallel to $L_3$

⇒ $L_1$ is parallel to $L_3$

⇒ $(L_1, L_3) \in R$

⇒ R is transitive

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

And, the set of lines parallel to the line $y = 2x + 4$ is $y = 2x + c$ For all $c \in R$

where R is the set of real numbers.

**10. Question**

Show that the relation R, defined on the set A of all polygons as

$R = \{(P_1, P_2) : P_1$ and $P_2$ have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

**Answer**

$R = \{(P_1, P_2)$: $P_1$ and $P_2$ have same the number of sides$\}$

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity: For Reflexivity, we need to prove that-

$(a, a) \in R$

R is reflexive since $(P_1, P_1) \in R$ as the same polygon has the same number of sides with itself.

Symmetric: For Symmetric, we need to prove that-

If $(a, b) \in R$, then $(b, a) \in R$

Let $(P_1, P_2) \in R$.

⇒ $P_1$ and $P_2$ have the same number of sides.

⇒ $P_2$ and $P_1$ have the same number of sides.

⇒ $(P_2, P_1) \in R$

∴ R is symmetric.

Transitive: For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Now, (P1, P2), (P2, P3) ∈ R

⇒ $P_1$ and $P_2$ have the same number of sides. Also, $P_2$ and $P_3$ have the same number of sides.

⇒$P_1$ and $P_3$ have the same number of sides.

⇒ $(P_1, P_3)$ ∈ R

∴ R is transitive.

Hence, R is an equivalence relation.

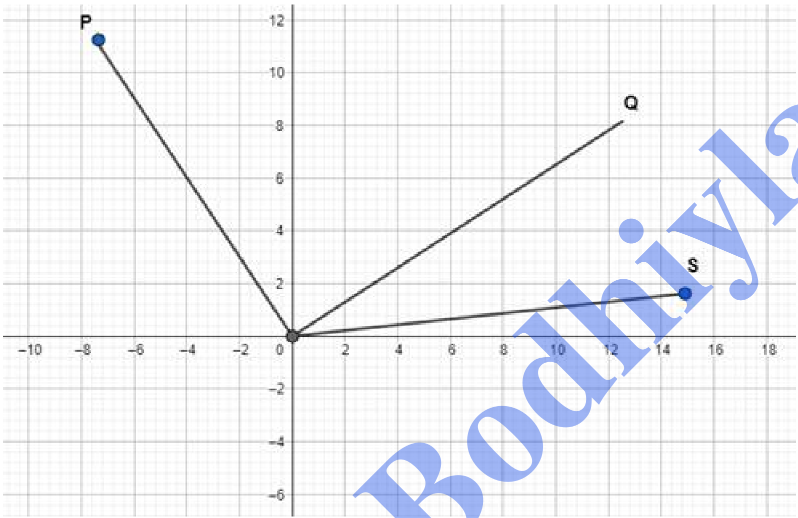
And, now the elements in A related to the right-angled triangle (T) with sides 3, 4 and 5 are those polygons which have three sides (since T is a polygon with three sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

**11. Question**

Let O be the origin. We define a relation between two points P and Q in a plane if OP = OQ. Show that the relation, so defined is an equivalence relation.

**Answer**






Let A be set of points on the plane.

Let R = {(P, Q) : OP = OQ} be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A.

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let p ∈ A

Since OP = OP ⇒ (P, P) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (P, Q) ∈ R for P, Q ∈ R

Then OP = OQ

⇒ Op = OP

⇒ (Q, P) ∈ R

⇒ R is symmetric

Transitive: For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (P, Q) ∈ R and (Q, S) ∈ R

⇒ OP = OQ and OQ = OS

⇒ OP = OS

⇒ (P, S) ∈ R

⇒ R is transitive

Thus, R is an equivalence relation on A

**12. Question**

Let R be the relation defined on the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7} are related to each other, and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

**Answer**

Given A = {1, 2, 3, 4, 5, 6, 7} and R = {(a, b) : both a and b are either odd or even number}

Therefore,

R = {(1, 1), (1, 3), (1, 5), (1, 7), (3, 3), (3, 5), (3, 7), (5, 5), (5, 7), (7, 7), (7, 5), (7, 3), (5, 3), (6, 1), (5, 1), (3, 1), (2, 2), (2, 4), (2, 6), (4, 4), (4, 6), (6, 6), (6, 4), (6, 2), (4, 2)}

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Here (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7) all ∈ R

From the relation R it is seen that R is reflexive.

Symmetric: For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

From the relation R, it is seen that R is symmetric.

Transitive: For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

[I (a, b) are odd and (b, c) are odd then (a, c) are also odd numbers]

From the relation R, it is seen that R is transitive too.

Also, from the relation R, it is seen that {1, 3, 5, 7} are related with each other only and {2, 4, 6} are related with each other .

**13. Question**

Let S be a relation on the set R of all real numbers defined by S = {(a, b)∈ R × R : $a^2 + b^2 = 1$}. Prove that S is not an equivalence relation on R.

**Answer**

S = {(a, b) : $a^2$ + $b^2$ =1}

Proof :

To prove that relation is not equivalence, we need to prove that it is either not reflexive, or not symmetric or not transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a = 1/2, a ∈ R

Then,

$$a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$$

⇒ (a, a) ∉ S

⇒ S is not reflexive

Hence, S is not an equivalence relation on R.

**14. Question**

Let Z be the set of all integers and $Z_0$ be the set of all non-zero integers. Let a relation R on Z × $Z_0$ be defined as follows :

(a, b) R (c, d) ⇔ ad = bc for all (a, b), (c, d) ∈ Z × $Z_0$

Prove that R is an equivalence relation on Z × $Z_0$

**Answer**

We have, Z be set of integers and $Z_0$ be the set of non-zero integers.

R = {(a, b) (c, d) : ad = bc} be a relation on Z and $Z_0$.

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

(a, b) ∈ Z × $Z_0$

⇒ ab = ba

⇒ ((a, b), (a, b)) ∈ R

⇒ R is reflexive

Symmetric : For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let ((a, b), (c, d) ∈ R

⇒ ad = bc

⇒ cd = da

⇒ ((c, d), (a, b)) ∈ R

⇒ R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (a, b), (c, d) ∈ R and (c, d), (e, f) ∈ R

⇒ ad = bc and cf = de

$\Rightarrow \frac{a}{d} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \frac{a}{b} = \frac{e}{f}$

⇒ af = be

⇒ (a, c) (e, f) ∈ R

⇒ R is transitive

Hence, R is an equivalence relation on Z × $Z_0$

**15. Question**

If R and S are relations on a set A, then prove the following :

(i) R and S are symmetric ⇔ R ⋂ S, and R ⋃ S is symmetric

(ii) R is reflexive, and S is any relation ⇔ R ⋃ S is reflexive.

**Answer**

R and S are two symmetric relations on set A

(i) To prove: R ⋂ S is symmetric

Symmetric: For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (a, b) ∈ R ⋂ S

⇒ (a, b) ∈ R and (a, b) ∈ S

⇒ (b, a) ∈ R and (b, a) ∈ S

[∴ R and S are symmetric]

⇒ (b, a) ∈ R ⋂ S

⇒ R ⋂ S is symmetric

To prove: R ⋃ S is symmetric

Symmetric: For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (a, b) ∈ R ⋃ S

⇒ (a, b) ∈ R or (a, b) ∈ S

⇒ (b, a) ∈ R or (b, a) ∈ S

[∴ R and S are symmetric]

⇒ (b, a) ∈ R ⋃ S

⇒ R ⋃ S is symmetric

(ii) R and S are two relations on a such that R is reflexive.

To prove : R ⋃ S is reflexive

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Suppose R ⋃ S is not reflexive.

This means that there is a ∈ R ⋃ S such that (a, a) ∉ R ⋃ S

Since a ∈ R ⋃ S,

∴ a ∈ R or a ∈ S

If a ∈ R, then (a, a) ∈ R

[∵ R is reflexive]

⇒ (a, a) ∈ R ⋃ S

Hence, R ⋃ S is reflexive

**16. Question**

If R and S are transitive relations on a set A, then prove that R ⋃ S may not be a transitive relation on A.

**Answer**

We will prove this using an example.

Let A = {a, b, c} be a set and

R = {(a, a) (b, b) (c, c) (a, b) (b, a)} and

S = {(a, a) (b, b) (c, c) (b, c) (c, d)} are two relations on A

Clearly R and S are transitive relation on A

Now,

R ⋃ S = {(a, a) (b, b) (c, c) (a, b) (b, a) (b, c) (c, b)}

Here, (a, b) ∈ R ⋃ S and (b, c) ∈ R ⋃ S

but (a, c) ∉ R ⋃ S

∴ R ⋃ S is not transitive

**17. Question**

Let C be the set of all complex numbers and $C_0$ be the set of all non-zero complex numbers. Let a relation R on $C_0$ be defined as

$z_1$ R $z_2$ ⇔ $\frac{z_1 - z_2}{z_1 + z_2}$ is real for all $z_1$, $z_2$ ∈ $C_0$.

Show that R is an equivalence relation.

**Answer**

We have,

$$R = \{(z_1, z_2) : \frac{z_1 - z_2}{z_1 + z_2} \text{ is real}\}.$$

We want to prove that R is an equivalence relation on Z.

Now,

Proof :

To prove that relation is equivalence, we need to prove that it is reflexive, symmetric and transitive.

Reflexivity : For Reflexivity, we need to prove that-

(a, a) ∈ R

Let a ∈ $C_0$

$\Rightarrow \frac{z_1 - z_2}{z_1 + z_2} = \frac{0}{2a} = 0$

And, 0 is real

∴ (a, a) ∈ R, so R is reflexive

Symmetric: For Symmetric, we need to prove that-

If (a, b) ∈ R, then (b, a) ∈ R

Let (a, b) ∈ R

$\Rightarrow \frac{a - b}{a + b} = p(say)$

⇒ p is real.

$\Rightarrow \frac{b - a}{b + a} = (-p)$

And ∵ p is real

⇒ -p is also a real no.

⇒ (b, a) ∈ R, so R is symmetric

Transitive : : For Transitivity, we need to prove that-

If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R

Let (a, b) ∈ R and (b, c) ∈ R

$\Rightarrow \frac{a - b}{a + b} = p(say)$

⇒ p is real no.

$\Rightarrow \frac{a - b + (a + b)}{a - b - (a + b)} = \frac{p + 1}{p - 1}$

$\Rightarrow \frac{2a}{-2b} = \frac{p + 1}{p - 1}$

$\Rightarrow \frac{a}{b} = \frac{p+1}{1-p}$ is a real no. .....(1)

And, $\frac{b - c}{b + c} = q(say)$

⇒ q is real.

$\Rightarrow \frac{b - c + (b + c)}{b - c - (b + c)} = \frac{q + 1}{q - 1}$

$\Rightarrow \frac{2b}{-2c} = \frac{q + 1}{q - 1}$

$\Rightarrow \frac{b}{c} = \frac{q+1}{1-q}$ is a real no. .....(2)

Dividing (1) by (2), we get-

$\frac{a}{c} = \frac{p + 1}{1 - p} \times \frac{1 - q}{q + 1} = Q$

Where, Q is a rational number.

⇒ Q is real number

Now, by componendo dividendo-

$\Rightarrow \frac{a-c}{a+c} = \frac{Q-1}{Q+1}$ is a real no.

$\Rightarrow$ (a, c) $\in$ R.

$\Rightarrow$ R is transitive

Thus, R is reflexive, symmetric and, transitive on $C_0$.

Hence, R is an equivalence relation on $C_0$.

## Very short answer

### 1. Question

Write the domain of the relation R defined on the set Z of integers as follows:

(a, b) $\epsilon$R $\Leftrightarrow$ $a^2 + b^2 = 25$

**Answer**

Given a and b are integers, i.e. a,b $\in$ Z.

$\therefore$ Domain of R = Set of all first elements in the relation.

= Values of 'a' which are in the relation.

=Z (Integers)

Range of R=Set of all second elements in the relation.

=Values of 'b' which are in the relation.

=Z(Integers)

Since, $a^2 + b^2 = 25$ and a, b are integers;

$\Rightarrow$ R= {(5,0), (0,5), (-5,0), (0, -5), (3,4), (4,3), (-3, -4), (-4, -3),

(-3,4), (4, -3), (-4,3), (3, -4)}

$\Rightarrow$ Domain of R= {-5, -4, -3,0,3,4,5}

### 2. Question

If R = {(x,y): $x^2 + y^2 \leq 4$; x,y $\epsilon$ Z} is a relation of Z, write the domain of R.

**Answer**

Given x and y are integers, i.e x,y $\in$ Z.

$\therefore$ Domain of R = Set of all first elements in the relation.

= Values of 'x' which are in the relation.

=Z (Integers)

Range of R=Set of all second elements in the relation.

=Values of 'y' which are in the relation.

=Z(Integers)

Since, $x^2 + y^2 \leq 4$ and x,y are integers;

$\Rightarrow$ R= {(0,0), (1,0), (0,1), (-1,0), (0,-1), (1,1), (-1,-1), (-1,1),

(1,-1), (0,2), (0,-2), (2,0), (-2,0)}

$\Rightarrow$ Domain of R= {-2,-1,0,1,2}

### 3. Question

Write the identity relation of set A = {a, b, c}.

**Answer**

⇒ Identity relation of a set refers to the relation in which every element on the set is related to itself.

Thus the Identity relation of set A is as under:

⇒ R={(a,a),(b,b),(c,c)}

**4. Question**

Write the smallest reflexive relation of set A = {1, 2, 3, 4}.

**Answer**

The smallest reflexive relation of set A = {1, 2, 3, 4} is as under:

As Relation R on a set A is said to be a reflexive relation on A if:

⇒ $(a,a) \in R\ \forall\ a \in A$

⇒ R={(1,1),(2,2),(3,3),(4,4)}

**5. Question**

If R = {(x,y) : x + 2y = 8} is a relation on N, then write the range of R.

**Answer**

Given x and y are natural numbers, i.e. $x,y \in N$.

∴ Range of R=Set of all second elements in the relation.

=Values of 'y' which are in relation.

=N (Natural Numbers)

Since, x+ 2y = 8 and x,y are Natural numbers;

⇒ R= {(2,3), (4,2), (6,1)}

⇒ Range of R= {1,2,3}

NOTE: 0 is a whole number that's why it is not considered in this set.



**6. Question**

If R is a symmetric relation on a set A, then write a relation between R and $R^{-1}$.

**Answer**

The relation between R and $R^{-1}$ on a Set A is as under:

⇒ $R^{-1} = \{(b,a):(a,b) \in R\}$

⇒ Clearly $(a,b) \in R$

⇒ $(b,a) \in R^{-1}$

i.e Domain(R)=Range($R^{-1}$)

and Domain($R^{-1}$)=Range(R)

**7. Question**

Let R = {(x,y): $|x^2 - y^2| < 1$} be a relation on set A = {1, 2, 3, 4, 5}. Write R as a set of ordered pairs.

**Answer**

Given a relation R = {(x,y): $|x^2 - y^2| < 1$} on set A = {1, 2, 3, 4, 5}.

Now according to the condition $|x^2 - y^2| < 1$ .

⇒ $x^2 - y^2 = 0$

⇒ Describing R as set of ordered pairs:

⇒ R={(1,1),(2,2),(3,3),(4,4),(5,5)}

**8. Question**

If A = {2, 3, 4}, B = {1, 3, 7} and R = {(x,y) : x∊ A, y ∊B and x < y} is a relation from A to B, then write $R^{-1}$.

**Answer**

Given A = {2, 3, 4}, B = {1, 3, 7} and R = {(x,y) : x∊ A, y ∊B and x < y}

According to the condition x < y

⇒ R= A × B

⇒ R={(2,3), (2,7), (3,7), (4,7)}

⇒ $R^{-1}$= B × A

⇒ $R^{-1}$={ (3,2), (7,2), (7,3), (7,4)}

**9. Question**

Let A = {3, 5, 7}, B = {2, 6, 10} and R be a relation from A to B defined by R = {(x, y) : x and y are relatively prime}. Then, write R and $R^{-1}$.

**Answer**

Given A = {3, 5, 7}, B = {2, 6, 10} and R = {(x, y) : x and y are relatively prime }

According to the condition, x and y are prime numbers.

⇒ R= A × B

⇒ R= {(3,2), (5,2), (7,2)}

⇒ $R^{-1}$= B × A

⇒ $R^{-1}$= { (2,3), (2,5), (2,7)}

**10. Question**

Define a reflexive relation.

**Answer**

Relation R on a set A is said to be a reflexive relation on A if:

⇒ (a,a) ∈ R ∀ a ∈ A

Eg A={1,2,3}

⇒ R={(1,1),(2,2),(3,3)}

**11. Question**

Define a symmetric relation.

**Answer**

Relation R on a set A is said to be a symmetric relation on A if:

(a,b) ∈ R

⇒ (b,a) ∈ R ∀ a,b∈ A

eg A={1,2,3}

⇒ R={(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)}

**12. Question**

Define a transitive relation.

**Answer**

Relation R on a set A is said to be a transitive relation on A iff:

(a,b) ∈ R and (b,c) ∈ R

⇒ (a,c) ∈ R ∀

a,b,c ∈ A

A={1,2,3}

⇒ R={(1,2),(2,3),(1,3)}

**13. Question**

Define an equivalence relation.

**Answer**

A relation R on a set A is said to be an equivalence relation on A iff:

1. Its reflexive i.e (a,a) ∈ R ∀ a ∈ A

2. Its symmetric i.e (a,b) ∈ R → (b,a) ∈ R ∀ a,b ∈ A

3. Its transitive i.e (a,b) ∈ R and (b,c) ∈ R → (a,c) ∈ R ∀

a,b,c ∈ A

**14. Question**

If A = {3, 5, 7} and B = {2, 4, 9} and R is a relation given by "is less than", write R as a set ordered pairs.

**Answer**

Given A = {3, 5, 7} and B = {2, 4, 9}

Now according to the condition:

R is a relation given by "is less than."

⇒ R= A × B

⇒ R={(3,4),(3,9),(5,9),(7,9)}

**15. Question**

A = {1, 2, 3, 4, 5, 6, 7, 8} and if R = {(x,y) : y is one half of x; x, y∈A} is a relation on A, then write R as a set of ordered pairs.

**Answer**

Given A = {1, 2, 3, 4, 5, 6, 7, 8} and a relation R = {(x,y) : y is one half of x; x, y∈A}

Now according to the condition $y = \frac{x}{2}$

⇒ R={(2,1),(4,2),(6,3),(8,4)}

**16. Question**

Let A = {2, 3, 4, 5} and B = {1, 3, 4}. If R is the relation from A to B given by a R b iff "a is a divisor of b." Write R as a set of ordered pairs.

**Answer**

Given A = {2, 3, 4, 5} and B = {1, 3, 4} and R is a relation from A to B which is true only iff "a is a divisor of b" i.e b is divisible by a.

⇒ R =A × B

⇒ R={(2,4),(4,4),(3,3)}

**17. Question**

State the reason for the relation R on the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

**Answer**

Given set {1,2,3} and relation R = {(1, 2), (2, 1)}.

For relation R to be transitive:

R={(1,2),(2,3),(1,3)}

But in the given relation:

⇒ (2,3) and (1,3) ∉ R

Hence, Given Relation R is not transitive.

**18. Question**

Let R = {(a, $a^3$) : a is a prime number less than 5} be a relation. Find the range of R.

**Answer**

Given a relation R = {(a, $a^3$) : a is a prime number less than 5}

Now according to the question ‘a’ is a prime number < 5:

⇒ a = {2,3}

⇒ R={(2,8),(3,27)}

⇒ Range of R={8,27)

**19. Question**

Show that R is an equivalence relation on the set Z of integers given by R = {(a, b) : 2 divides a – b}.Write the equivalence class [0].

**Answer**

Given R = {(a, b) : 2 divides a – b}

For equivance relation we have to check three parameters:

(i) Reflexive:

If (a-b) is divisible by 2 then,

⇒ (a-a)=0 is also divisible by 2

⇒ (a,a) ∈ R

Hence R is Reflexive ∀ (a,b) ∈ Z

(ii)Symmetric:

If (a-b) is divisible by 2 then,

⇒ (b-a)=-(a-b) is also divisible by 2

⇒ (a,b) ∈ R and (b,a) ∈ R

Hence R is Symmetric ∀ (a,b) ∈ Z

(iii)Transitive:

If (a-b) and (b-c) are divisible by 2 then,

⇒ a-c=(a-b)+(b-c) is also divisible by 2

⇒ (a,b) ∈ R, (b,c) ∈ R and (a,c) ∈ R

Hence R is Transitive ∀ (a,b) ∈ Z

⇒ As Relation R is satisfying all the three parameters, hence R is an equivalence relation.

Now equivalence class [0] is the set of all those elements in A which are related to 0 under relation.

Now,

(a,0) ∈ R

⇒ a – 0 is divisible by 2 and a ∈ A.

⇒ a ∈ A such that 2 divides a.

⇒ a = 0, 2,4

Thus [0] = {0,2,4}

**20. Question**

For the set A = {1, 2, 3}, define a relation R on the set A as follows:

R = {(1, 1), (2, 2), (3, 3), (1, 3)}

Write the ordered pairs to be added to R to make the smallest equivalence relation.

**Answer**

Given set A = {1, 2, 3} and relation R = {(1, 1), (2, 2), (3, 3), (1, 3)}

A relation is an equivalence relation if and only if it is reflexive, symmetric and transitive.

⇒ R = {(1, 1), (2, 2), (3, 3), (1, 3)} is reflexive and transitive but not symmetric.

⇒ If (3,1) is added to the ordered pairs, R will become symmetric.

Thus the new R:

⇒ R = {(1, 1), (2, 2), (3, 3), (1, 3),(3,1)} is the obtained smallest equivalence relation.

**21. Question**

Let A = {0, 1, 2, 3} and R be a relation on A defined as

F = {(0, 0) (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}

Is R reflexive? Symmetric” transitive?

**Answer**

Given A = {0, 1, 2, 3} and a relation R on A defined as

F = {(0, 0) (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}

A relation is an equivalence relation if and only if it is reflexive, symmetric and transitive:

(i) The set contains (0,0),(1,1),(2,2),(3,3):

Hence (a,a) ∈ R → R is reflexive.

(ii)The set also contains (0,1),(1,0) and (0,3),(3,0)

Hence (a,b) ∈ R and (b,a) ∈ R → R is symmetric.

(ii) The set also contains (0,0),(0,1) and (0,0),(0,3)

Hence (a,b) ∈ R, (b,c) ∈ R and (a,c) ∈ R → R is transitive.

Hence R is reflexive,symmetric and transitive.

**22. Question**

Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = {(a, b) : $|a^2 – b^2| < 8$}. Write R as a set of ordered pairs.

**Answer**

Given a set A = {1, 2, 3, 4, 5} and relation R = {(a, b) : $|a^2 – b^2| < 8$}

Now according to the question $|a^2 – b^2| < 8$

⇒ R = {(1,1),(2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (2,3), (3,2), (3,4), (4,3),}

**23. Question**

Let the relation R be defined on N by a Rb iff 2a + 3b = 30. Then write R as a set of ordered pairs.

**Answer**

Given R = {(a, b) : 2a + 3b = 30} ∀ (a,b) ∈ N

Now according to the question 2a + 3b = 30:

⇒ R={(3,8),(6,6),(9,4),(12,2)}

NOTE: 0 is a whole number that’s why its not considered in this set, Although if we consider 0 as a natural number then the answer would be:

⇒ R={(0,10),(3,8),(6,6),(9,4),(12,2),(15,0)}

**24. Question**

Write the smallest equivalence relation on the set A = {1, 2, 3}.

**Answer**

A relation is an equivalence relation if and only if it is reflexive, symmetric and transitive:

The smallest equivalence relation on the set A={1,2,3} is :

R={(1,1),(1,3),(3,1)}

∵ (1,1) ∈ R → Reflexive

(1,3) ∈ R and (3,1) ∈ R → Symmetric

(1,3) ∈ R and (3,1) ∈ R and (1,1) ∈ R → Transitive



## MCQ

**1. Question**

Let R be a relation on the set N given by R = {a, b) : a = b – 2, b > 6}. Then

A. (2, 4) ∊R

B. (3, 8) ∊R

C. (6, 8) ∊R

D. (8, 7) ∊R

**Answer**

Given R = {(a, b) : a = b – 2, b > 6}

Now according to the question a = b – 2 and b > 6

⇒ R={(5,7),(6,8),(7,9).......∞ }

**2. Question**

Which of the following is not an equivalence relation on Z?

A. a R b ⇔ a + b is an even integer

B. a R b ⇔ a – b is a even integer

C. a R b ⇔ a < b

D. a R b ⇔ a = b

**Answer**

a R b ⇔ a – b is a even integer

Given R = {(a, b) : a – b is an even integer(i.e divisible by 2)}

For equivance relation we have to check three parameters:

(iii) Reflexive:

If (a-b) is divisible by 2 then,

⇒ (a-a)=0 is also divisible by 2

⇒ (a,a) ∈ R

Hence R is Reflexive ∀ (a,b) ∈ Z

(ii)Symmetric:

If (a-b) is divisible by 2 then,

⇒ (b-a)=-(a-b) is also divisible by 2

⇒ (a,b) ∈ R and (b,a) ∈ R

Hence R is Symmetric ∀ (a,b) ∈ Z

(iii)Transitive:

If (a-b) and (b-c) are divisible by 2 then,

⇒ a-c=(a-b)+(b-c) is also divisible by 2

⇒ (a,b) ∈ R, (b,c) ∈ R and (a,c) ∈ R

Hence R is Transitive ∀ (a,b) ∈ Z

→ As Relation R is satisfying all the three parameters, hence R is an equivalence relation.



**3. Question**

R is a relation on the set Z of integers and it is given by (x, y) ϵ R ⇔ |x – y| ≤ 1. Then, R is

A. reflective and transitive

B. reflexive and symmetric

C. symmetric and transitive

D. an equivalence relation

**Answer**

∵ According to the condition |x – y| ≤ 1

⇒ R={(1,1),(2,1), (1,2), (2,2), … (n,n), (n+1,n), (n,n+1) …∞ }

⇒ (a,a) ∈ R → Reflexive

⇒ (a,b) ∈ R and (b,a) ∈ R →Symmetric

**4. Question**

The relation R defined on the set A = {1, 2, 3, 4, 5} by R = $\left\{(a,b) : |a^2 - b^2| < 16\right\}$, is given by

A. {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}

B. {(2, 2), (3, 2), (4, 2), (2, 4)}

C. {(3, 3), (4, 3), (5, 4), (3, 4)

D. none of these

**Answer**

Given set A = {1, 2, 3, 4, 5} and relation R ={(a,b):$|a^2-b^2|<16$}

According to the condition $|a^2-b^2|<16$:

⇒ R={(1, 1), (2, 1), (3, 1), (4, 1), (2, 3),(2, 2), (3, 2), (4, 2), (2, 4),(3, 3), (4, 3), (5, 4), (3, 4)}

**5. Question**

Let R be the relation over the set of all straight lines in a plane such that $\ell_1$ R $\ell_2 \Leftrightarrow \ell_1 \perp \ell_2$. Then, R is

A. symmetric

B. reflexive

C. transitive

D. an equivalence relation

**Answer**

Think of line as a vector quantity:

As $\ell_1 \perp \ell_2$;

And $\ell_2 \perp \ell_1$

Hence R is symmetric.

Also Given a relation R over straight lines such that $\ell_1 \perp \ell_2$

As $\ell_1 \perp \ell_2$:

⇒ $\ell_1 . \ell_2 = 0$(DOT PRODUCT)

∵ cos θ =0 as θ =90°;

⇒ This thing is possible if $\ell_1$ and $\ell_2$ are symmetric.

E.g. A= 2i-4j and B=-4i-2j

⇒ A. B=-8+8=0

**6. Question**

If A = {a, b, c}, then the relation R = {(b, c)} on A is

A. reflexive only

B. symmetric only

C. transitive only

D. reflexive and transitive only

**Answer**

According to the question:

R={(a,a), (b,b), (c,c), (a,b), (b,c), (a,c)}

Thus R = {(b, c)} can only be transitive.

i.e (a,b)∈ R and (b,c)∈ R → (a,c)∈ R

**7. Question**

Let A = {2, 3, 4, 5, ..., 17, 18}. Let '≃' be the equivalence relation on A × A, cartesian product of A with itself, defined by (a, b) ≃ (c, d) iff ad = bc. Then, the number of ordered pairs of the equivalence class of (3, 2) is

A. 4

B. 5

C. 6

D. 7

**Answer**

Let (3,2) ≃ (x,y)

⇒ 3y = 2x

This is possible in the cases:

x = 3, y = 2

x = 6, y = 4

x= 9, y =6

x=12, y = 3

x=15, y = 10

x=18,y=12

Hence total pairs are 6.

**8. Question**

Let A = {1, 2, 3}. Then, the number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

**Answer**

R={(1,2),(2,1),(1,3),(3,1),(1,1)} is the only relation.

The addition of (2,2) or (3,3) will make R transitive.

**9. Question**

The relation 'R' in N × N such that (a, b) R (c, d)⇔ a + d = b + c is

A. reflexive but of symmetric

B. reflexive and transitive but not symmetric

C. an equivalence relation

D. none of these

**Answer**

Check:

(a,b)R (a,b) ∈ R

a + b = b + a

hence R is reflexive.

Now,

(a, b) R (c, d) ∈ R

a + d = b + c

⇒ c + b = d + a

⇒ (c,d) R (a,b) ∈ R

R is symmetric

Now,

(a, b) R (c, d) ∈ R

a + d = b + c

(c,d) R (e,f) ∈ R

⇒ c + f = d + e

Now,

a + d + c + f = b + c + d + e

⇒ a + f = b + e

So (a,b) R (e,f)

R is transitive.

Hence R is an equivalence relation.

**10. Question**

If A = {1, 2, 3}, B = {1, 4, 5, 9} and R is a relation from A to B defined by 'x is greater than y'. Then, Range of R is

A. {1, 2, 6, 9}

B. {4, 6, 9}

C. {1}

D. none of these

**Answer**

Here

⇒ R={(2,1), (3,1)}

Hence Range(R)={1}

**11. Question**

A relation R is defined form {2, 3, 4, 5} to {3, 6, 7, 10} by xRy ⇔ x is relatively prime to y. Then, domain of R is

A. {2, 3 5}

B. {3, 5}

C. {2, 3, 4} D. {2, 3, 4, 5}

**Answer**

R: x R y⟺ x is relatively prime to y.

Two numbers are relatively prime if their Highest Common Factor is 1.

Then, R= {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}

Therefore, the domain of R is {2, 3, 4, 5}

**12. Question**

A relation ϕ from C to R is defined by x ϕ ⇔ |x| = y. Which one is correct?

A. (2 + 3i) ϕ 13

B. 3 ϕ (–3)

C. (1 + i) ϕ 2

D. i ϕ 1

**Answer**

In complex numbers the y-axis is taken as the imaginary axis,

Thus i ϕ 1

**13. Question**

Let R be a relation on N defined by x + 2y = 8. The domain of R is

A. {2, 4, 8}

B. {2, 4, 6, 8}

C. {2, 4, 6}

D. {1, 2, 3, 4}

**Answer**

{2, 4, 6}

∵ R={(2,3),(4,2),(6,1)}

Hence domain(R)={2,4,6}

**14. Question**

R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x – 3. Then, $R^{-1}$ is

A. {(8, 11), (10, 13)}

B. {(11, 8), (13, 10)}

C. {(10, 13), (8, 11), (8, 11)}

D. none of these

**Answer**

{(8, 11), (10, 13)}

Acoording to the question R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x – 3:

⇒R={(11,8),(13,10)}

⇒$R^{-1}$={(8, 11), (10, 13)}

**15. Question**

Let R = {(a, a), (b, b), (c, c), (a, b)} be a relation on set A = {a, b, c}. Then, R is

A. identity relation

B. reflexive

C. symmetric

D. equivalence

**Answer**

reflexive

∵ R = {(a, a), (b, b), (c, c), (a, b)} is satisfuing only the property of reflexive relation.

i.e Its reflexive i.e (a,a) ∈ R ∀ a ∈ A

**16. Question**

Let A = {1, 2, 3} and R = {(1, 2), (2, 3), (1, 3)} be a relation on A. Then, R is

A. neither reflexive nor transitive

B. neither symmetric nor transitive

C. transitive

D. none of these

**Answer**

∵ R = {(1, 2), (2, 3), (1, 3)} satisfying only the property of transitive relation.

i.e Its transitive i.e (a,b) ∈ R and (b,c) ∈ R → (a,c) ∈ R ∀

a,b,c ∈ A

**17. Question**

If R is the largest equivalence relation on a set A and S is any relation on A, then

A. R ⊂ S

B. S ⊂ R

C. R = S

D. none of these

**Answer**

∵ Understood property

**18. Question**

If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by x R y⇔ y = 3x, then R =

A. {(3, 1), (6, 2), (8, 2), (9, 3)}

B. {(3, 1), (6, 2), (9, 3)}

C. {(3, 1), (2, 6), (3, 9)}

D. none of these

**Answer**

∵ for A = {1, 2, 3, 4, 5, 6, 7, 8, 9} the satisfying complete relation is:

R={(1, 3), (2, 6), (3, 9)}

**19. Question**

If R is a relation on the set A = {1, 2, 3} given by R = (1, 1), (2, 2), (3, 3), then R is

A. reflexive

B. symmetric

C. transitive

D. all the three options

**Answer**

∵ An important property of equivalence is that it divides the set into pairwise disjoint subsets called equivalent classes whose collection is called a partition of the set. Note that the union of all equivalence classes gives the complete set.

**20. Question**

If A = {a, b, c, d}, then a relation R = {(a, b), (b, a), (a, a)} on A is

A. symmetric and transitive only

B. reflexive and transitive only

C. symmetric only

D. none of these

**Answer**

∵ R is symmetric and reflexive

**21. Question**

If A = {1, 2, 3}, then a relation R = {(2, 3)} on A is

A. symmetric and transitive only

B. symmetric only

C. transitive only

D. none of these

**Answer**

∵ R={(1,1),(2,2),(3,3),(1,2),(1,3),(2,3),(3,2),(2,1),(3,1)}

Symmetric- {(2,3),(3,2)}

Transitive- {(1,2),(2,3),(1,3)}

**22. Question**

Let R be the relation on the set A = {1, 2, 3, 4} given by

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Then,

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalence relation

**Answer**

.

**23. Question**

Let A = {1, 2, 3}. Then, the number of equivalence relations containing (1, 2) is

A. 1

B. 2

C. 3

D. 4

**Answer**

.

**24. Question**

The relation R = {(1, 1), (2, 2), (3, 3)} on the set {1, 2, 3} is

A. symmetric only

B. reflexive only

C. an equivalence relation

D. transitive only

**Answer**

∵ An important property of equivalence is that it divides the set into pairwise disjoint subsets called equivalent classes whose collection is called a partition of the set. Note that the union of all equivalence classes gives the complete set.

**25. Question**

S is a relation over the set R of all real numbers and its is given by $(a, b) \in S \Leftrightarrow ab \geq 0$. Then, S is

A. symmetric and transitive only

B. reflexive and symmetric only

C. antisymmetric relation

D. an equivalence relation

**Answer**

.

**26. Question**

If the set Z of all integers, which of the following relation R is not an equivalence relation?

A. x R y : if $x \leq y$

B. x R y : if $x = y$

C. x R y : if $x - y$ is an even integer

D. x R y : if $x \equiv y \pmod{3}$

**Answer**

.

**27. Question**

Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}. Then, R is

A. reflexive but not symmetric

B. reflexive but not transitive

C. symmetric and transitive

D. neither symmetric nor transitive.

**Answer**

.

**28. Question**

The relation S defined on the set R of all real number by the rule a Sb iff $a \geq b$ is

A. an equivalence relation

B. reflexive, transitive but not symmetric

C. symmetric, transitive but not reflexive

D. neither transitive nor reflexive but symmetric

**Answer**

S: a S b $\Longleftrightarrow$ a $\geq$ b

Since a=a $\forall$a $\in$ R, therefore a $\geq$ a always. Hence (a, a) always belongs to S $\forall$a $\in$ R. Therefore, S is reflexive.

If a $\geq$ b then b $\leq$ a $\nRightarrow$ b $\geq$ a. Hence if (a, b) belongs to S, then (b, a) does not always belongs to S. Hence S is not symmetric.

If a $\geq$ b and b $\geq$ c, therefore a $\geq$ c. Hence if (a, b) and (b, c) belongs to S, then (a, c) will belong to S$\forall$a, b, c$\in$R. Hence, S is transitive.

**29. Question**

The maximum number of equivalence relations on the set A = {1, 2, 3} is

A. 1

B. 2

C. 3

D. 5

**Answer**

A= {1, 2, 3}

Then the equivalence relations would be,

P = {(1, 1), (2, 2), (3, 3)}

Q = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

R = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)}

S = {(1, 1), (2, 2), (3, 3), (2, 3), (3, 1)}

T = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 1)}

Hence, total 5 equivalence relations.

**30. Question**

Let R be a relation on the set N of natural numbers defined by n R m iff n divides m. Then, R is

A. Reflexive and symmetric

B. Transitive and symmetric

C. Equivalence

D. Reflexive, transitive but Not symmetric

**Answer**

This question is quite self explanatory:

Reflexive: eg. m=5 and n=5 $\rightarrow$ 5 is divisible by 5

Transitive: 2 divides 4, 4 divides 40 and 2 also divedes 40.

**31. Question**

Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\ell$ R m iff $\ell$ is perpendicular to m for all $\ell$, m $\in$ L. Then, R is

A. reflexive

B. symmetric

C. transitive D. none of these

**Answer**

Consider $6^{th}$ part of MCQs

**32. Question**

Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as a R b if a is congruent to b for all a, b ϵ T. Then, R is

A. reflexive but not symmetric

B. transitive but not symmetric

C. equivalence

D. none of these

**Answer**

R: a R b ⟺ a ≅ b

Since, every triangle a∈T is congruent to itself, therefore (a, a)∈R ∀a∈T. Hence, R is reflexive.

If a ≅ b, then b ≅ a. Hence if (a, b)∈R, then (b, a)∈R ∀a, b∈T. Hence, R is symmetric.

If a ≅ b and b ≅ c, then a ≅ c. Hence if (a, b) and (b, c) belongs to R, then (a, c) will belong to R ∀a, b, c∈T. Hence, R is transitive.

Since R is reflexive, symmetric and transitive, therefore R is equivalence relation.



**33. Question**

Consider a non-empty set consisting of children in a family and a relation R defined as a R B if a is brother of b. Then, R is

A. symmetric but not transitive

B. transitive but not symmetric

C. Neither symmetric nor transitive

D. both symmetric and transitive

**Answer**

R: a R b ⟺ a is a brother of b.

If a is a brother of b, then that does not necessarily mean b is a brother of a, since b can be a sister of a too. Hence, if (a, b) ∈ R, then (b, a)∉R always. Hence R is not symmetric.

If a is a brother of b and b is a brother of c, then a has to be a brother of c. Hence if (a, b) and (b, c) belongs to R, then (a, c) will belong to R. Hence, R is transitive.

**34. Question**

For real numbers x and y, define x R y iff x – y+ √2 is an irrational number. Then the relation R is

A. reflexive

B. symmetric

C. transitive

D. none of these

**Answer**

R: x R y ⟺ $x - y + \sqrt{2}$ is irrational.

Since, x-x=0 always, therefore $x - x + \sqrt{2} = \sqrt{2}$ is irrational. Hence, (x, x)∈R ∀ x ∈ R. Hence, R is reflexive.

R is not symmetric. Proof by counter-example:

Let $x - y = \sqrt{2}$. Then $x - y + \sqrt{2} = 2\sqrt{2}$ is irrational, Therefore (x, y)∈R. But

$y - x + \sqrt{2} = -(x - y) + \sqrt{2} = -\sqrt{2} + \sqrt{2} = 0$ is rational, therefore (y, x)∉R. Hence, R is not symmetric.

R is not transitive. Proof by counter-example:

Let $x - y = -\frac{\sqrt{2}}{2}$ and $y - z = -\frac{\sqrt{2}}{2}$, then $x - z = (x - y) + (y - z) = -\sqrt{2}$

$x - y + \sqrt{2} = \frac{\sqrt{2}}{2}$ is irrational, therefore (x, y)∈R

$y - z + \sqrt{2} = \frac{\sqrt{2}}{2}$ is irrational, therefore (y, z)∈R

$x - z + \sqrt{2} = 0$ is rational, therefore (x, z)∉R. Hence, R is not transitive.