1. Number System

Exercise 1.1

1. Question

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Answer

Yes, zero is a rational number, It can be written in the form of $\frac{p}{q}$ where q ≠ 0 such as $\frac{0}{3}, \frac{0}{5}, \frac{0}{11}$, etc.

2. Question

Find five rational numbers between 1 and 2.

Answer

Given: to find five rational numbers between 1 and 2, we multiply & divide both the numbers by 6.

Trick: To find "n" rational numbers between any two numbers "a" & "b", just multiply & divide the numbers "a" & "b" by "n+1".

Example,

To find five rational numbers between 1 and 2, we multiply & divide both the numbers by 6, as shown:

$$1 \times \frac{6}{6} = \frac{6}{6}$$

$$And,\ 2 \times \frac{6}{6} = \frac{12}{6}$$

Therefore, five rational numbers between 1 and 2 are:

$$\frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6}$$

3. Question

Find six rational numbers between 3 and 4.

Answer

Given, to find six rational numbers between 3 and 4.

We have,

$3 * \frac{7}{7} = \frac{21}{7}$ and $4 * \frac{7}{7} = \frac{28}{7}$

We know that,

21 < 22 < 23 < 24 < 25 < 26 < 27 < 28

required rational numbers = $\frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

$= 3 < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < 4$

Hence, 6 rational numbers between 3 and 4 are:

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

4. Question

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Answer

Given, to find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

We have, $\frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$ and $\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

We know that,

18 < 19 < 20 < 21 < 22 < 23 < 24

$= \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$

$= \frac{3}{5} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{4}{5}$

$= \frac{3}{5} < \frac{19}{30} < \frac{2}{3} < \frac{7}{10} < \frac{11}{15} < \frac{23}{30} < \frac{4}{5}$

Hence, 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are:

$\frac{19}{30}, \frac{2}{3}, \frac{7}{10}, \frac{11}{15}, \frac{23}{30}$Note: You can multiply and divide with any number you want to find the rational numbers.

5. Question

Are the following statements true or false? Give reasons for your answer.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

(iv) Every natural number is a whole number.

(v) Every integer is a whole number.

(vi) Every rational number is a whole number

Answer

(i) False: As whole numbers include zero, whereas natural numbers doesn't include zero.

(ii) True: As integers are a part of rational numbers.

(iii) False: As integers are a part of natural numbers.

(iv) True: As whole numbers include all the natural numbers.

(v) False: As whole numbers are a part of integers.

(vi) False: As rational numbers include all the whole numbers.

Exercise 1.2

1. Question

Express the following rational numbers as decimals:

(i) $\frac{42}{100}$ (ii) $\frac{327}{500}$ (iii) $\frac{15}{4}$

Answer

(i) By long division

```
      0.42
100)42.00
     400
     ---
      200
      200
     ---
        0
```

$\therefore \frac{42}{100} = 0.42$

(ii) By long division method, we have

```
      0.654
500)327.000
    3000
    ----
     2700
     2500
     ----
      2000
      2000
      ----
         0
```

$\frac{327}{500} = 0.654$

(iii) By long division method, we have

```
      3.75
4)15.00
  12
   30
   28
     20
     20
      0
```

$\therefore \frac{15}{4} = 3.75$

2. Question

Express the following rational numbers as decimals:

(i) $\frac{2}{3}$ (ii) $-\frac{4}{9}$ (iii) $-\frac{2}{15}$ (iv) $-\frac{22}{13}$ (v) $\frac{437}{999}$ (vi) $\frac{33}{26}$

Answer

(i) By long division method, we have

```
   0.6666
3)2.000
  18
   20
   18
    20
    18
     2
```

$\therefore \frac{2}{3} = 0.666... = 0.\bar{6}$

(ii) By long division method, we have


```
   0.4444
9)4.0000
  36
   40
   36
    40
    36
     4
```

$\therefore \frac{4}{9} = 0.4444... = 0.\bar{4}$

(iii) By long division method, we have

```
      0.133
15)2.0000
   15
   ---
    50
    45
    ---
     50
     45
     ---
      6
```

$\therefore \quad \frac{2}{15} = 0.133 = 01\overline{3}$

(iv) By long division method, we have

```
    1.6923076923
13)22.0000
   13
   ---
    90
    78
    ---
    120
    117
    ---
     30
     26
     ---
      40
      39
      ---
       100
        91
       ---
         98
         78
        ---
         120
         117
         ---
           3
```

$\therefore \frac{23}{13} = 1.6923076923... = 1.\overline{692307}$

(v) By long division method, we have

```
     0.43743
999)437.000000
    3996
    ----
     3740
     2997
     ----
      7430
      6993
      ----
       4370
       3996
       ----
        3740
        2997
        ----
         743
```

$\therefore \frac{437}{999} = 0.43743... = 0.\overline{437}$

(vi) By long division method, we have

```
       1.2692307692
26)33.000000000
   26
    70
    52
    180
    156
     240
     234
      60
      52
       80
       78
       200
       182
        180
        156
         24
```

$\therefore \frac{33}{26} = 1.269230769... = 1.\overline{2692307}$

3. Question

Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating, decimal representations. Can you guess what property q must satisfy?

Answer

A rational number $\frac{p}{q}$ is a terminating decimal only, when prime factors of q are 2 and 5 only. Therefore, $\frac{p}{q}$ is a terminating decimal only, when prime factorisation of q must have only powers of 2 or 5 or both.

Exercise 1.3

1. Question

Express each of the following decimals in the form $\frac{p}{q}$:

(i) 0.39 (ii) 0.750 (iii) 2.15 (iv) 7.010 (v) 9.90 (vi) 1.0001

Answer

To convert decimal into fraction count no of decimal places in decimal number. Let it be x. Then multiply and divide the decimal number with 10^x(i) We have,

$0.39 = \frac{39}{100}$

(ii) We have,

$0.750 = \frac{750}{1000} = \frac{750 \div 250}{1000 \div 250}$

$= \frac{3}{4}$

(iii) We have,

$2.15 = \frac{215}{100} = \frac{215 \div 5}{100 \div 5} = \frac{43}{20}$

(iv) We have,

$7.010 = \frac{7010}{1000} = \frac{7010 \div 10}{1000 \div 10}$

$= \frac{701}{100}$

(v) We have,

$9.90 = \frac{990}{100} = \frac{990 \div 10}{100 \div 10}$

$= \frac{99}{10}$

(vi) We have,

$1.0001 = \frac{10001}{10000}$

2. Question

Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\bar{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{54}$ (iv) $0.\overline{621}$ (v) $125.\bar{3}$ (vi) $4.\bar{7}$ (vii) $0.4\bar{7}$

Answer

(i) Let x = $0.\bar{4}$

Now, x = $0.\bar{4}$ = 0.444… (1)

Multiplying both sides of equation (1) by 10, we get,

10x = 4.444… (2)

Subtracting equation (1) by (2)

10x − x = 4.444… - 0.444…

9x = 4

$x = \frac{4}{9}$

Hence, $0.\bar{4} = \frac{4}{9}$

(ii) Let x = $0.\overline{37}$

Now, x = 0.3737…. (1)

Multiplying equation (1) by 10

10x = 3.737…. (2)

Multiplying equation (2) by 10

100x = 37.3737…. (3)

100x – x = 37

99x = 37

$x = \frac{37}{99}$

Hence, $0.\overline{37} = \frac{37}{99}$

(iii) Now x = $0.\overline{54}$

= 0.5454… (i)

Multiplying both sides of equation (i) by 100, we get

100x = 54.5454…. (ii)

Subtracting (i) by (ii), we get

100x – x = 54.5454…. – 0.5454….

99x = 54

$x = \frac{54}{99}$

(iv) Now x = $0.\overline{621}$

= 0.621621…. (i)

Multiplying both sides by 1000, we get

1000x = 621.621621…. (ii)

Subtracting (i) by (ii), we get

1000x – x = 621.621621…. – 0.621621….

999x = 621

$x = \frac{621}{999} = \frac{23}{37}$

(v) Now x = $125.\overline{3}$

= 125.3333…. (i)

Multiplying both sides of equation (i) by 10, we get

10x = 1253.3333…. (ii)

Subtracting (i) by (ii), we get

10x − x = 1253.3333…. − 125.3333….

9x = 1128

x = 1128 / 9 = 376/3

(vi) Now x = $4.\overline{7}$

= 4.7777…. (i)

Multiplying both sides of equation (i) by 10, we get

10x = 47.7777…. (ii)

10x − x = 47.7777…. − 4.7777….

9x = 43

x = $\frac{43}{9}$

(vii) Now, x = $0.4\overline{7}$

= 0.47777….

Multiplying both sides by 10, we get

10x = 4.7777…. (i)

Multiplying both sides of equation (i) by 10, we get

100x = 47.7777…. (ii)

Subtracting (i) from (ii), we get

100x – 10x = 47.7777…. – 4.7777….

90x = 43

x = $\frac{43}{90}$

Exercise 1.4

1. Question

Define an irrational number.

Answer

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. For example,

1.01001000100001…

2. Question

Explain, how irrational numbers differ from rational numbers?

Answer

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. For example,

0.33033003300033...

On the other hand, every rational number is expressible either as a terminating decimal or as a repeating decimal. For example, $3.2\overline{4}$ and 6.2876 are rational numbers.

3. Question

Examine, whether the following numbers are rational or irrational:

(i) $\sqrt{7}$ (ii) $\sqrt{4}$ (iii) $2+\sqrt{3}$ (iv) $\sqrt{3}+\sqrt{2}$

(v) $\sqrt{3}+\sqrt{5}$ (vi) $(\sqrt{2}-2)^2$ (vii) $(2-\sqrt{2})(2+\sqrt{2})$ (viii) $(\sqrt{2}+\sqrt{3})^2$

(ix) $\sqrt{5}-2$ (x) $\sqrt{23}$ (xi) $\sqrt{225}$ (xii) 0.3796 (xiii) 7.478478

(xiv) 1.101001000100001.....

Answer

(i) $\sqrt{7}$ is not a perfect square root, so it is an irrational number.

(ii) We have,

$\sqrt{4} = 2 = \frac{2}{1}$

$\sqrt{4}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

The decimal expression of $\sqrt{4}$ *is* 2.0

(iii) 2 ia a rational number, whereas $\sqrt{3}$ is an irrational number.

Because, sum of a rational number and an irrational number is an irrational number, so $2 + \sqrt{3}$ is an irrational number

(iv) $\sqrt{2}$ is an irrational number. Also $\sqrt{3}$ is an irrational number. The sum of two irrational numbers is irrational.

Therefore, $\sqrt{3} + \sqrt{2}$ is an irrational number.

(v) $\sqrt{5}$ is an irrational number. Also, $\sqrt{3}$ is an irrational number. The sum of two irrational numbers is irrational.

Therefore, $\sqrt{3} + 5$ is an irrational number.

(vi) We have,

$(\sqrt{2} - \sqrt{2})^2 = (\sqrt{2})^2 - 2 * \sqrt{2} * 2 + (2)^2$

$= 2 - 4\sqrt{2} + 4$

$= 6 - 4\sqrt{2}$

Now 6 is a rational number, whereas $4\sqrt{2}$ is an irrational number

The difference of a rational number and an irrational number is an irrational number.

So, it is an irrational number.

(vii) We have,

$(2 - \sqrt{2})(2 + \sqrt{2}) = (2)^2 - (\sqrt{2})^2$ [Therefore, $(a - b)(a + b) = a^2 - b^2$]

$= 4 - 2 = 2 = \frac{2}{1}$

Since 2 is a rational number

Therefore, $(2 - \sqrt{2})(2 + \sqrt{2})$ is a rational number

(viii) We have,

$(\sqrt{2} + \sqrt{3})^2 = (\sqrt{2})^2 + 2 \times \sqrt{2} \times \sqrt{3} + (\sqrt{3})^2$

$= 2 + 2\sqrt{6} + 3$

$= 5 + 2\sqrt{6}$

The sum of a rational number and an irrational number is irrational number. Therefore, it is an irrational number.

(ix) The difference of a rational number and an irrational number is an irrational number.

Therefore, $5 - \sqrt{2}$ is an irrational number.

(x) $\sqrt{23} = 4.79583152331....$

Therefore, it is an irrational number

(xi) $\sqrt{225} = 15 = \frac{15}{1}$

Therefore, it is a rational number as it is represented in the form of $\frac{p}{q}$, where $q \neq 0$

(xii) 0.3796, as a decimal expansion of this number is terminating, so it is an irrational number.

(xiii) $7.478478.... = 7.\overline{478}$

As, decimal expansion of this number is non – terminating recurring so it is a rational number

(xiv) 1.101001000100001.....

It is an irrational number

4. Question

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{1.44}$ (iv) $\sqrt{\frac{9}{27}}$ (v) $-\sqrt{64}$ (vi) $\sqrt{100}$

Answer

(i) $\sqrt{4}$ = 2 = $\frac{2}{1}$

$\sqrt{4}$ can be written in the form of $\frac{p}{q}$, so it is a rational number.

Its decimal expansion is 2.0

(ii) $3\sqrt{18}$ = 3 $\sqrt{2*3*3}$

= 3 * $3\sqrt{2}$

= $9\sqrt{2}$

Since, the product of a rational and an irrational is an irrational number.

Therefore, $9\sqrt{2}$ is an irrational;

$3\sqrt{18}$ is an irrational number

(iii) We have,

$\sqrt{1.44}$ = $\frac{12}{10}$

= 1.2

Every terminating decimal is a rational number, so 1.2 is a rational number.

(iv) we have,

$\sqrt{\frac{9}{27}} = \frac{3}{\sqrt{27}} = \frac{3}{\sqrt{3*3*3}}$

= $\frac{1}{3}$

Quotient of a rational and an irrational number is irrational number. Therefore, it is an irrational number.

(v) - $\sqrt{64}$ = - $\sqrt{8}*8$

= - 8 = -8/1

AS it can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(vi) $\sqrt{100}$ = 10 = $\frac{10}{1}$

Thus it can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

5. Question

In the following equations, find which variables *x, y, z* etc. represent rational or irrational numbers:

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = 0.04$ (iv) $u^2 = \frac{17}{4}$ (v) $v^2 = 3$ (vi) $w^2 = 27$ (vii) $t^2 = 0.4$

Answer

(i) We have,

$x^2 = 5$

Taking square root on both sides,

$= \sqrt{x^2} = \sqrt{5}$

$= x = \sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) We have,

$y^2 = 9$

$y = \sqrt{9}$

$= 3 = \frac{3}{1}$

$\sqrt{9}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iii) We have,

$z^2 = 0.04$

Taking square root on both the sides, we get,

$\sqrt{z^2} = \sqrt{0.04}$

$z = \sqrt{0.04}$

$= 0.2 = \frac{2}{10}$

$= \frac{1}{5}$

z can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iv) We have,

$u^2 = \frac{17}{4}$

Taking square root on both the sides, we get

$\sqrt{u^2} = \frac{\sqrt{17}}{\sqrt{4}}$

$u = \frac{\sqrt{17}}{\sqrt{2}}$

Quotient of an rational number is irrational, so u is an irrational number.

(v) We have,

$v^2 = 3$

Taking square roots on both the sides, we get,

$\sqrt{v}^2 = \sqrt{3}$

$v = \sqrt{3}$

$\sqrt{3}$ is not a perfect square root, so v is an irrational number.

(vi) We have,

$w^2 = 27$

Taking square roots on both the sides, we get,

$\sqrt{w}^2 = \sqrt{27}$

$w = \sqrt{3} * \sqrt{3} * \sqrt{3} = 3\sqrt{3}$

Product of a rational number and an irrational number is irrational number. So, it is an irrational number.

(vii) We have,

$t^2 = 0.4$

Taking square roots on both the sides, we get,

$\sqrt{t}^2 = \sqrt{0.4} = \frac{\sqrt{4}}{\sqrt{10}}$

$= \frac{2}{\sqrt{10}}$

Since, quotient of a rational number and an irrational number is irrational number, so t is an irrational number.

6. Question

Give an example of each, of two irrational numbers whose:

(i) Difference is a rational number.

(ii) Difference is an irrational number.

(iii) Sum is a rational number.

(iv) Sum is an irrational number.

(v) Product is a rational number.

(vi) Product is an irrational number.

(vii) Quotient is a rational number.

(viii) Quotient is an irrational number.

Answer

(i) $\sqrt{3}$ is an irrational number.

Now, $(\sqrt{3}) - (\sqrt{3})$ = 0

0 is the rational number.

(ii) Let two irrational numbers are $5\sqrt{2}$ and $\sqrt{2}$

Now, $(5\sqrt{2}) - (\sqrt{2}) = 4\sqrt{2}$

$4\sqrt{2}$ is an irrational number.

(iii) Let two irrational numbers be $\sqrt{11}$ and - $\sqrt{11}$

Now, $(\sqrt{11}) + (-\sqrt{11})$ = 0

0 is a rational number

(iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

Now, $(4\sqrt{6}) + (\sqrt{6}) = 5\sqrt{6}$

$5\sqrt{6}$ is an irrational number.

(v) Let two irrational numbers are $2\sqrt{3}$ and $\sqrt{3}$

Now, $2\sqrt{3} * \sqrt{3}$ = 2 * 3

= 6

6 is a rational number.

(vi) Let two irrational numbers are $\sqrt{2}$ and $\sqrt{5}$

Now, $\sqrt{2} * \sqrt{5} = \sqrt{10}$

$\sqrt{10} is\ a\ irrational\ number.$

(vii) Let two irrational numbers are $3\sqrt{6}$ and $\sqrt{6}$

Now, $\frac{3\sqrt{6}}{\sqrt{6}}$ = 3

is a rational number.

(viii) Let two irrational numbers are $\sqrt{6}$ and $\sqrt{2}$

Now, $\frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{3} * \sqrt{2}}{\sqrt{2}}$

$= \sqrt{3}$

$\sqrt{3}$ is an irrational number.

7. Question

Give two rational numbers lying between 0.232332333233332…. and 0.212112111211112.

Answer

Let a = 0.212112111211112

And, b = 0.232332333233332….

Clearly a < b because in the second decimal place a has digit 1 and b has digit 3

If we consider rational numbers in which decimal place has the digit 2, then they will lie between a and b

Let,

x = 0.22

y = 0.22112211…

Then,

a < x < y < b

Hence, x and y are required rational numbers.

8. Question

Give two rational numbers lying between 0.515115111511115…. and 0.5353353335….

Answer

Let a = 0.515115111511115….

b = 0.5353353335….

We observe that the second decimal place a has digit 1 and b has digit 3, therefore, a < b. So if we consider rational numbers

x = 0.52

y = 0.52052052

Then,

a < x < y < b

Hence, x and y are required rational numbers.

9. Question

Find one irrational number between 0.2101 and 0.2222 ….= $0.\bar{2}$.

Answer

Let a = 0.2101

And, b = 0.2222….

We observe that the second decimal place a has digit 1 and b has digit 2, therefore, a < b. In the third decimal place a has digit 0. So, if we consider irrational number

x = 0.211011001100011…

We find that,

a < x < b

Hence, x is required irrational number.

10. Question

Find a rational number and also irrational number lying between the numbers 0.3030030003…..and 0.3010010001….

Answer

Let a = 0.3010010001….

And, b = 0.3030030003…..

We observe that the second decimal place a has digit 1 and b has digit 3, therefore, a < b. In the third decimal place a has digit 1. So, if we consider rational and irrational numbers

x = 0.302

y = 0.302002000200002…

We find that,

a < x < b

And, a < y < b

Hence, x and y are required rational and irrational numbers respectively.

11. Question

Find two irrational numbers between 0.5 and 0.55.

Answer

Let a = 0.5 = 0.50

And, b = 0.55

We observe that in the second decimal place a has digit 0 and b has digit 5. Therefore a < b. So, if we consider irrational numbers

x = 0.51051005100051…

y = 0.530534305535353 0…

We find that,

a < x < y < b

Hence, x and y are required irrational numbers.

12. Question

Find two irrational numbers lying between 0.1 and 0.12.

Answer

Let, a = 0.1 = 0.10

And, b = 0.12

We observe that in the second decimal place a has digit 0 and b has digit 2. Therefore a < b. So, if we consider irrational numbers

x = 0.11011001100011…

y = 0.111011110111110…

We find that,

a < x < y < b

Hence, x and y are required irrational numbers.

13. Question

Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.

Answer

If possible, let $\sqrt{3}+\sqrt{5}$ be a rational number equal to x. Then,

$x = \sqrt{3}+\sqrt{5}$

$x^2 = (\sqrt{3}+\sqrt{5})^2$

$= (\sqrt{3})^2 + (\sqrt{5})^2 + 2 * \sqrt{3} * \sqrt{5}$

$= 3 + 5 + 2\sqrt{15}$

$= 8 + 2\sqrt{15}$

$x^2 - 8 = 2\sqrt{15}$

$\frac{x*x-8}{2} = \sqrt{15}$

Now, x is rational

x^2 is rational

$\frac{x*x-8}{2}$ is rational

$\sqrt{15}$ is rational

But, $\sqrt{15}$ is irrational

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3} + \sqrt{5}$ is rational is wrong.

Hence, $\sqrt{3} + \sqrt{5}$ is an irrational number.

14. Question

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Answer

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are:

0.73073007300073000073…..

0.75075007500075000075….

0.79079007900079000079….

Exercise 1.5

1. Question

Complete the following sentences:

(i) Every point on the number line corresponds to a ….number which many be either …… or ……

(ii) The decimal from of an irrational number is neither……. nor…..

(iii) The decimal representation of a rational number is either ……… or ……….

(iv) Every real number is either …………… number or ……………. number.

Answer

(i) Every point on the number line corresponds to a REAL number which many be either RATIONAL or IRRATIONAL

(ii) The decimal from of an irrational number is neither TERMINATING Nor REPEATING.

(iii) The decimal representation of a rational number is either TERMINATING or NON TERMINATING

(iv) Every real number is either RATIONAL number or IRRATIONAL Number

2. Question

Represent $\sqrt{6}, \sqrt{7}, \sqrt{8}$ on the number line.

Answer

Draw a number line and mark point O, representing zero, on it.

Then, for representing $\sqrt{6}$.

Step 1: On point A, 2 unit distance away from O,

Step 2:Draw a perpendicular to number line of 1 unit distance (as taken on number line).

Step 3:Mark the point as A.

Step 4: Join B to O. line OB represent $\sqrt{5}$.

Step 5: Put compass on O and B and by taking the radius cut number line with one needle still on O.

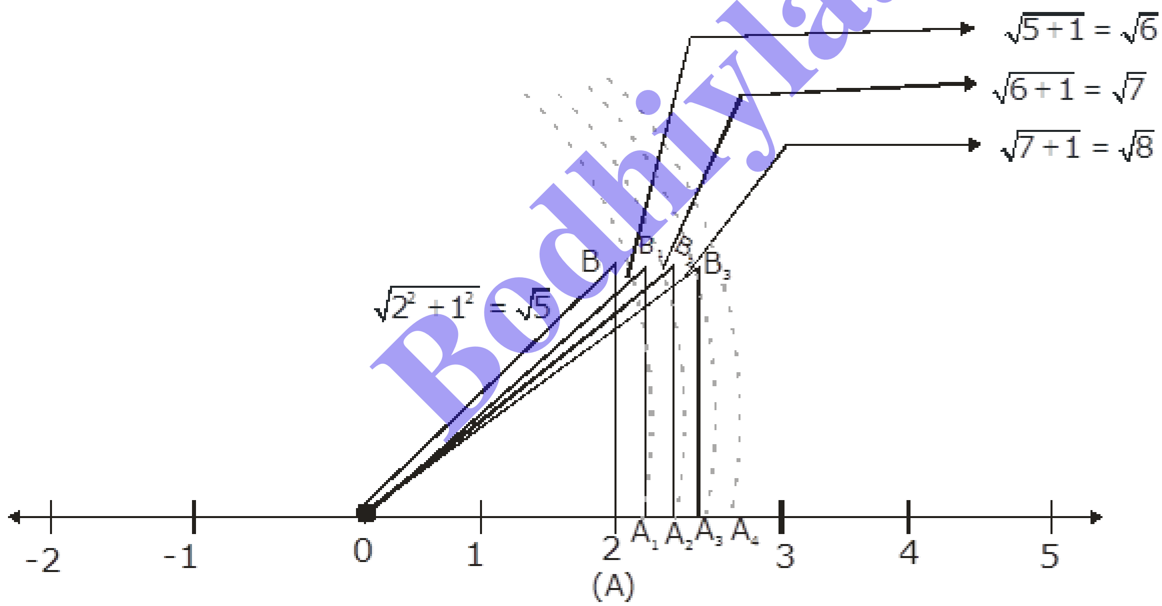
Step 6: Mark point as A_1. This line OA_1 represent $\sqrt{6}$.

Step 7: Now draw 1 unit long perpendicular on A_1. And mark the end point as B_1.

Step 8: Now OB_1 represent $\sqrt{7}$.

Step 9: By following Step 5 now cut $\sqrt{7}$ on number line.

Step 10: For $\sqrt{8}$ Draw perpendicular on $\sqrt{7}$ on number line and follow above steps .



3. Question

Represent $\sqrt{3.5}, \sqrt{9.4}, \sqrt{10.5}$ on the real number line.

Answer

To represent $\sqrt{3.5}$ on number line follow the following steps:

Step 1- Draw a line and mark a point A on it.

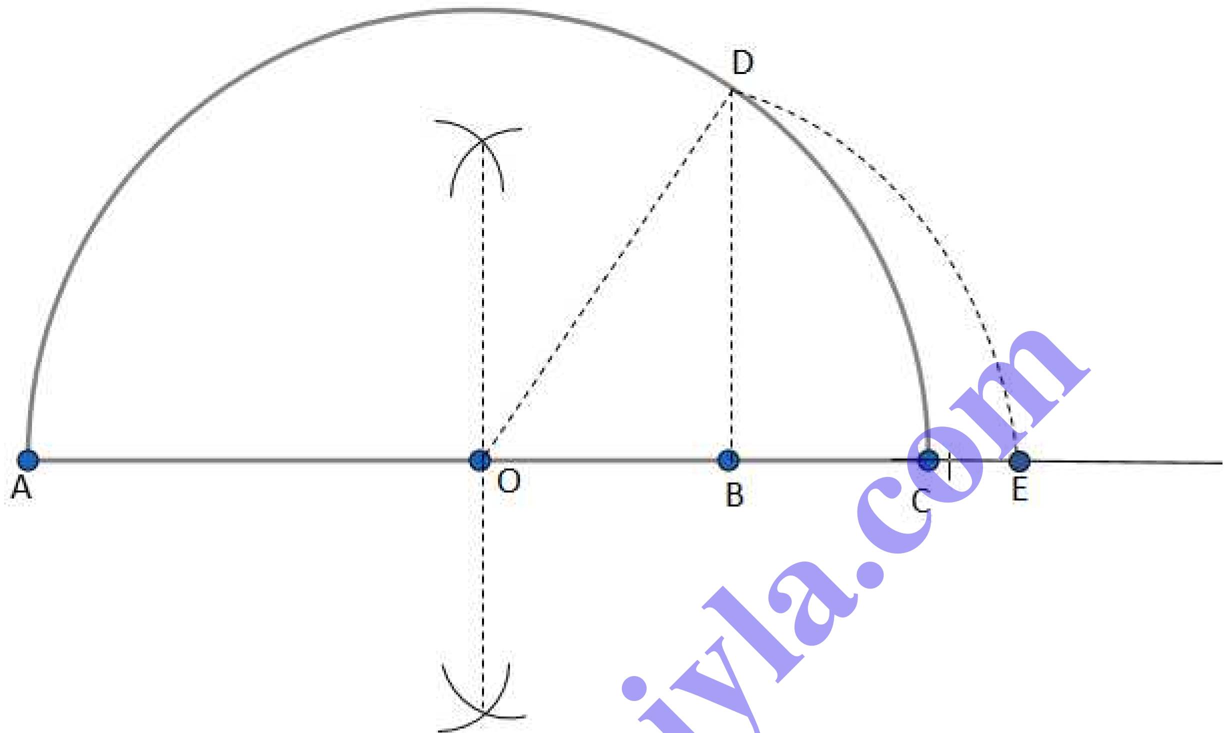
Step 2- Mark a point B on the line such that AB=3.5 units.

Step 3- Mark a point C on AB produced such that BC=1 unit.

Step 4- Find the mid-point of AC. Let it be O

Step 5- Taking O as the centre and OC=OA as radius draw a semi circle. Also draw a line passing trough B perpendicular to OB. Suppose it cuts the semi circle at D.

Step 6- Taking B as centre and BD as radius draw an arc cutting OC produced at E. Point E so obtain represent $\sqrt{3.5}$



Similarly, represnt other square root numbers on number line.

4. Question

Find whether the following statements are true or false.

(i) Every real number is either rational or irrational.

(ii) π is an irrational number.

(iii) Irrational numbers cannot be represented by points on the number line.

Answer

(i) True: As we know that rational and irrational numbers taken form the set of real numbers.

(ii) True: As, π is ratio of the circumference of a circle to its diameter, it is an irrational number.

$\pi = \frac{2\pi r}{2r}$

(iii) False: Irrational numbers can be represented by point on the number line.

Exercise 1.6

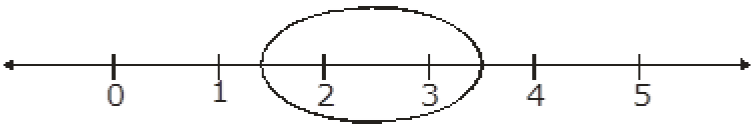
1. Question

Visualise 2.665 on the number line, using successive magnification.

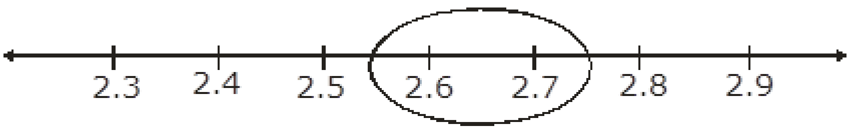
Answer

The following steps for successive magnification to visualise 2.665 are:

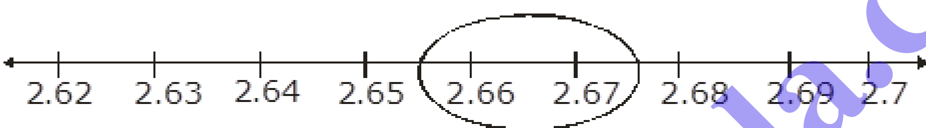
(1) We observe that 2.665 is located somewhere between 2 and 3 on the number line. So, let us look at the portion of the number line between 2 and 3.



(2) We divide this portion into 10 equal parts and mark each point of division. The first mark to the right of 2 will represent 2.1, the next 2.2 and soon. Again we observe that 2.665 lies between 2.6 And 2.7.

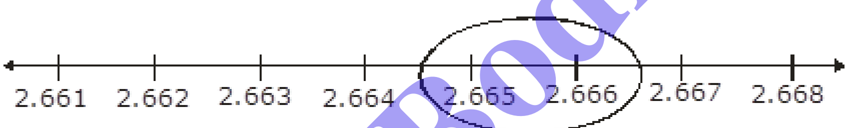


(3) We mark these points A_1 and A_2 respectively. The first mark on the right side of A_1, will represent 2.61, the number 2.62, and soon. We observe 2.665 lies between 2.66 and 2.67.



(4) Let us mark 2.66 as B_1 and 2.67 as B_2. Again divide the B_1B_2 into ten equal parts. The first mark on the right side of B_1 will represent 2.661, then next 2.662, and so on.

Clearly, fifth point will represent 2.665.

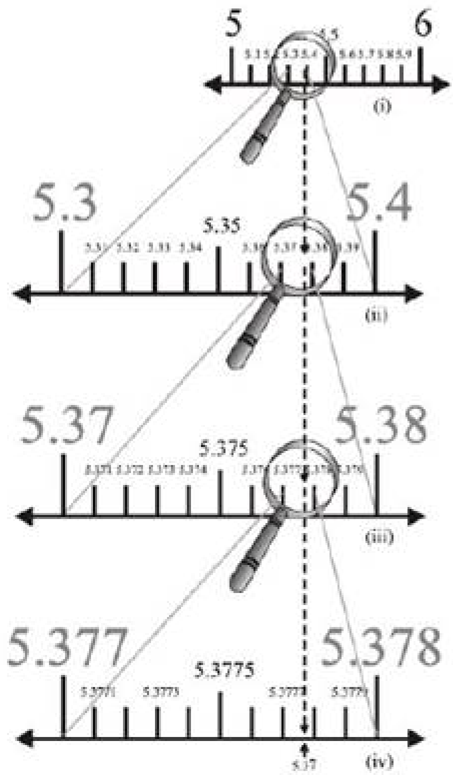


2. Question

Visualise the representation of $5.3\overline{7}$ on the number line up to 5 decimal places that is up to 5.37777.

Answer

Once again we proceed by successive magnification, and successively decrease the lengths of the portions of the number line in which $5.3\overline{7}$ is located. First, we see that $5.3\overline{7}$ is located between 5 and 6. In the next step, we locate $5.3\overline{7}$ between 5.3 and 5.4. To get a more accurate visualisation of the representation, we divide this portion of the number line into ten equal parts and use a magnifying glass to visualize that $5.3\overline{7}$ lies between 5.37 and 5.38. To visualize $5.3\overline{7}$ more accurately, we again divide the portion between 5.37 and 5.38 in ten equal parts and use a magnifying glass to visualize that $5.3\overline{7}$ lies 5.377 and 5.378. Now to visualize $5.3\overline{7}$ still more accurately, we divide the portion between 5.377 and 5.378 into ten equal parts, and visualize the representation of $5.3\overline{7}$ as in the fig. (iv) Notice that $5.3\overline{7}$ is located closer to 5.3778 than to 5.3777 (iv)



CCE - Formative Assessment

1. Question

Which one of the following is a correct statement?

A. Decimal expansion of a rational number is terminating.

B. Decimal expansion of a rational number is non-terminating.

C. Decimal expansion of an irrational number is terminating.

D. Decimal expansion of an irrational number is non-terminating and non-repeating.

Answer

D)The decimal expansion of an irrational number never repeats or terminates (essentially, that is repeating zeroes), unlike any rational number does.

2. Question

Which one of the following statements is true?

A. The sum of two irrational numbers is always an irrational number.

B. The sum of two irrational numbers is always a rational number.

C. The sum of two irrational numbers may be a rational number or an irrational number.

D. The sum of two irrational numbers is always an integer.

Answer

If the irrational parts on adding forms a rational term then the whole number will be rational and if the irrational parts on adding gives again an irrational term then the complete number will be irrational.

3. Question

Which of the following is a correct statement?

A. Sum of two irrational numbers is always irrational.

B. Sum of a rational and irrational number is always an irrational number

C. Square of an irrational number is always a rational number

D. Sum of two rational numbers can never be an integer.

Answer

Let the rational number be of the form $\frac{p}{q}$, where p ϵ Z, while the rational number be r. If r + $\frac{p}{q}$ is a rational then we have that,

r + $\frac{p}{q}$ = $\frac{a}{b}$ for some a ϵ Z and b ϵ Z \ {0}. This means that r = $\frac{a}{b}$ - $\frac{p}{q}$ = $\frac{aq-bp}{bq}$ where aq − bp ϵ Z and this contradicts the facts that r is irrational. Hence, our assumption that r + $\frac{p}{q}$ is a rational is false. Hence, it is an irrational number.

4. Question

Which of the following statement is true?

A. Product of two irrational numbers is always irrational.

B. Product of a rational and an irrational number is always irrational.

C. Sum of two irrational numbers can never be irrational.

D. Sum of an integer and a rational number can never be an integer.

Answer

Example: Take any rational number other than Zero let's take 2 as rational number and√2 as irrational number than product as:

= 2 × √2 = 2 √2

2 √2 is irrational number so if we can say that if rational number other than zero product with any irrational number the result is also irrational.

5. Question

Which of the following is irrational?

A. $\sqrt{\frac{4}{9}}$

B. $\frac{4}{5}$

C. $\sqrt{7}$

D. $\sqrt{81}$

Answer

Proof: let us assume that √7 be rational.

then it must in the form of p / q [q ≠ 0] [p and q are co-prime]

√7 = p / q

√7 x q = p

squaring on both sides

$7q^2 = p^2$ (i)

p^2 is divisible by 7

p is divisible by 7

p = 7c [c is a positive integer] [squaring on both sides]

$p^2 = 49\ c^2$ (ii)

Substitute p^2 in eq (i), we get,

$7q^2 = 49\ c^2$

$q^2 = 7c^2$

q is divisible by 7

Thus q and p have a common factor 7.

There is a contradiction

As our assumption p & q are co - prime but it has a common factor.

So that √7 is an irrational.

6. Question

Which of the following is irrational?

A. 0.14

B. $0.14\overline{16}$

C. $0.\overline{1416}$

D. 0.1014001400014….

Answer

Since, it is non - terminating and non - repeating decimal.

7. Question

Which of the following is rational?

A. $\sqrt{3}$

B. π

C. $\frac{4}{0}$

D. $\frac{0}{4}$

Answer

Since it is in the form of p/q , and where q ≠ 0.

8. Question

The number 0.318564318564318564……. is:

A. A natural number

B. An integer

C. A rational number

D. An irrational number

Answer

Since it is a non - terminating repeating decimal, hence it is a rational number.

9. Question

In *n* is a natural number, than $\sqrt{n}$ is

A. Always a natural number

B. Always an irrational number

C. Always an irrational number

D. Sometimes a natural number and sometimes an irrational number

Answer

If n can be written in the form of p/q, where q≠0, then it is a rational number else irrational.

10. Question

Which of the following numbers can be represented as non-terminating, repeating decimals?

A. $\frac{39}{24}$

B. $\frac{3}{16}$

C. $\frac{3}{11}$

D. $\frac{137}{25}$

Answer

Since, it can be represented as 0.27272727... which is a non - terminating repeating decimal.

11. Question

Every point on a number line represents

A. A unique real number

B. A natural number

C. A rational number

D. An irrational number

Answer

A real number is a value that represents a quantity along a line.

12. Question

An irrational number between 2 and 2.5 is

A. $\sqrt{11}$

B. $\sqrt{5}$

C. $\sqrt{22.5}$

D. $\sqrt{12.5}$

Answer

Note: *If a and b are two positive numbers, such that the product of a and b is not a perfect square of a rational number, then,* $\sqrt{ab}$ is an irrational number lying between "a" and "b".

So, now we have two numbers 2 and 2.5.

Now the product of these two number is 5, which is not a perfect square. So, we can say that √5 is an irrational number lying between "2" and "2.5"

√5= 2.23606797749978969, Which is a non– terminating and non- repeating decimal.

Thus, option B is the correct answer.

13. Question

Which of the following is irrational?

A. 0.15

B. 0.01516

C. $0.\overline{1516}$

D. 0.5015001500015…

Answer

Since, it is non - terminating and non - repeating decimal.

14. Question

The number of consecutive zeroes in $2^3 \times 3^4 \times 5^4 \times 7$, is

A. 3

B. 2

C. 4

D. 5

Answer

3As the expression has $2^3 \times 5^3$ which yields zeroes in expression.as this would make 1000 so 3 zeroes will be there

15. Question

The number $1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

A. $\frac{14}{9}$

B. $\frac{14}{11}$

C. $\frac{14}{13}$

D. $\frac{14}{15}$

Answer

Since, after dividing 14 from 11, we get that number.

16. Question

The number $0.\overline{3}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

A. $\frac{33}{100}$

B. $\frac{3}{10}$

C. $\frac{1}{3}$

D. $\frac{3}{100}$

Answer

Since, among the following only the division of 1 by 3 gives that specified number.

17. Question

The smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal, is

A. $\frac{1}{10}$

B. $\frac{3}{10}$

C. 3

D. 30

Answer

Since, among them 3/10 is the only number which when multiplied, then its decimal expansion terminates after one place of decimal.

18. Question

$0.3\bar{2}$ when expressed in the form $\frac{p}{q}$ (p, q are integers, $q \neq 0$), is

A. $\frac{8}{25}$

B. $\frac{29}{90}$

C. $\frac{32}{99}$

D. $\frac{32}{199}$

Answer

Let x = $0.3\bar{2}$

10x = $3.\bar{2}$ (i)

100x = $32.\bar{2}$ (ii)

Now, subtracting (i) from (ii), we get

100x – 10x = $32.\bar{2}$ - $3.\bar{2}$

90x = 29

x = $\frac{29}{90}$

19. Question

$23.\overline{43}$ when expressed in the form $\frac{p}{q}$ (p, q are integers, $q \neq 0$), is

A. $\frac{2320}{99}$

B. $\frac{2343}{100}$

C. $\frac{2343}{999}$

D. $\frac{2320}{199}$

Answer

x = $23.\overline{43}$ (i)

100x = 2343. $.\overline{43}$ (ii)

Subtracting (i) from (ii), we get

100x – x = 2343. $23.\overline{43}$ - 23. . $23.\overline{43}$

99x = 2320

x = $\frac{2320}{99}$

20. Question

$0.\overline{001}$ when expressed in the form $\frac{p}{q}$ (p, q are integers, $q \neq 0$), is

A. $\frac{1}{1000}$

B. $\frac{1}{100}$

C. $\frac{1}{1999}$

D. $\frac{1}{999}$

Answer

x = $0.\overline{001}$ (i)

1000x = $001.\overline{001}$ (ii)

Subtracting (i) from (ii), we get

1000x – x = $001.\overline{001}$ - 0. $\overline{001}$

999x = 1

x = $\frac{1}{999}$

21. Question

The value of $0.\overline{23}+0.\overline{22}$ is

A. $0.\overline{45}$

B. $0.\overline{43}$

C. $0.\overline{45}$

D. 0.45

Answer

$0.\overline{23}$ = 0.232323....

$0.\overline{22}$ = 0.222222....

Now, $0.\overline{23}+0.\overline{22}$ = 0.23232323.... + 0.22222222....

= 0.45454545....

= $0.\overline{45}$