ICSE Board

Class X

Mathematics

Board Paper 2019

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately. You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

Attempt **all** questions from **Section A** and **any four questions** from **Section B.**

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

Attempt **all** questions from this Section.

Question 1

(a) Solve the following in equation and write down the solution set: [3]

$11x - 4 < 15x + 4 \leq 13x + 14,\ x \in W$

Represent the solution on a real number line.

Ans. The given inequality is:

$11x - 4 < 15x + 4 \leq 13x + 14$

which forms two cases that are:

Case 1: $15x+4>11x-4$

Case2: $15x+4\le 13x+14$

Solving the case 1, we have

$15x+4>11x-4$

$\Rightarrow 15x-11x>-4-4$
$\Rightarrow 4x>-8$
$\Rightarrow x>\frac{-8}{4}$
$\therefore x>-2$

Solving the case 2, we get

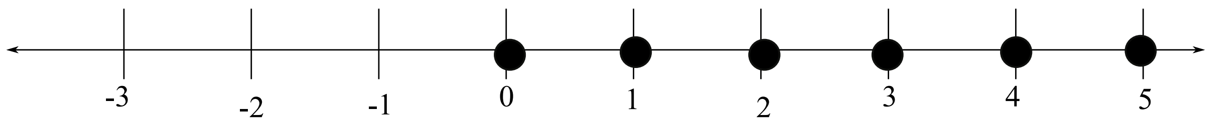
$15x+4\le 13x+14$

$\Rightarrow 15x-13x\le 14-4$
$\Rightarrow 2x\le 10$
$\Rightarrow x\le \frac{10}{2}$
$\therefore x\le 5$

So, we get $x>-2$ and $x\le 5$

$\Rightarrow -2<x\le 5$

Since $x\in w$, Therefore, x= {0, 1, 2, 3, 4, 5}



(b) A man invests Rs. 4500 in shares of a company which is paying 7.5% dividend. If Rs. 100 shares are available at a discount of 10%. Find: [3]

(i) Number of shares he purchases.

(ii) His annual income.

Ans. Given,

Total amount of investment = Rs. 4500

Dividend (%) = 7.5%

Face value = Rs. 100

Discount offered (%) = 10%

Market Value of share = 100 –10% 0f 100

= 100 – 10

= Rs.90

We know that, Numbers of share = $\frac{\text{Investment}}{\text{Market value}}$

$= \frac{4500}{90} = 50\text{ shares}$

Also, Annual Income = $\frac{\text{Number of share} \times \text{Dividend percent} \times \text{Face value}}{100}$

$= \frac{50 \times 7.5 \times 100}{100}$

= Rs. 375.0

Hence, the total number of shares purchased is 50 and the annual income is Rs. 375

(c) In class of 40 students, marks obtained by the students a class test (out of 10) are given below,

Marks	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number of students	**1**	**2**	**3**	**3**	**6**	**10**	**5**	**4**	**3**	**3**

Calculate the following for the given distribution: [4]

(i) Median

(ii) Mode

Ans. Let the marks obtained be represented by x and the number of students be represented by f

Then, we can arrange the data as:

Marks(x)	Number of students(f)	(fx)
1	1	1
2	2	4
3	3	9
4	3	12
5	6	30
6	10	60
7	5	35
8	4	32
9	3	27
10	3	30
Total	$\sum f = 40$	$\sum fx = 240$

i. We know, Mean = $\dfrac{\sum fx}{\sum f}$

$$= \frac{240}{40} = 6$$

ii. Mode = Class with highest frequency

Here in the above table, we see 10 is the highest frequency

Hence, Mode =6

Question 2

(a) Using the factor theorem, show that (x - 2) is a factor of $x^3 + x^2 - 4x - 4$ **, Hence, factorise the polynomial completely. [3]**

Ans. Let's take the given polynomial be P(x).

We have $P(x) = x^3 + x^2 - 4x - 4$

According to question, (x - 2) is a factor of given polynomial

We know that, under factor theorem, x-a is a factor of P(x) only if P(a) = 0

So, to prove that, we substitute the value of x by 2

$$P(2) = 2^3 + 2^2 - 4 \times 2 - 4$$
$$P(2) = 8 + 4 - 8 - 4$$
$$P(2) = 0$$

Therefore, x-2 is a factor of P(x)

Now, factorising the polynomial, we get

$$x^3 + x^2 - 4x - 4$$
$$= x^3 - 4x + x^2 - 4$$
$$= x(x^2 - 4) + 1(x^2 - 4) \quad [\because (a^2 - b^2) = (a+b)(a-b)]$$
$$= (x^2 - 4)(x + 1)$$
$$= (x + 2)(x - 2)(x + 1)$$

(b) Prove that: (cosec θ - sin θ)(sec θ – cos θ) (tan θ + cot θ) = 1 [3]

Ans. Given, L.H.S= (cosec θ - sin θ)(sec θ – cos θ) (tan θ + cot θ)

$$\Rightarrow\left(\frac{1}{\mathrm{Sin}\theta}\text{-Sin}\theta\right)\left(\frac{1}{\mathrm{Cos}\theta}\text{-Cos}\theta\right)\left(\frac{\mathrm{Sin}\theta}{\mathrm{Cos}\theta}+\frac{\mathrm{Cos}\theta}{\mathrm{Sin}\theta}\right)$$

$$=\left(\frac{1\text{-Sin}^2\theta}{\mathrm{Sin}\theta}\right)\left(\frac{1\text{-Cos}^2\theta}{\mathrm{Cos}\theta}\right)\left(\frac{\mathrm{Sin}^2\theta+\mathrm{Cos}^2\theta}{\mathrm{Sin}\theta.\mathrm{Cos}\theta}\right)$$

$$=\frac{\mathrm{Cos}^2\theta}{\mathrm{Sin}\theta}.\frac{\mathrm{Sin}^2\theta}{\mathrm{Cos}\theta}.\frac{1}{\mathrm{Sin}\theta.\mathrm{Cos}\theta}$$

$=1$

$=$R.H.S

Hence, L.H.S = R.H. S proved.

(c) In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively, Find the:[4]

(i) First term

(ii) Common difference

(iii) Sum of the first 20 terms

Ans.

Let the first term of A.P be ‘a’ and common difference be ‘d’.

Then using formula $a_n=a+(n-1)d$ to get n^{th} term, we get

$a_4=a+(4-1)d$

$\Rightarrow 8=a+3d$................(i)

[Given: Fourth term is 8 and sixth term is 14]

Also,

$a_6=a+(6-1)d$

$\Rightarrow 14=a+5d$.............(ii)

Now, solving (i) and (ii) simultaneously, we get a+3d-a-5d = 8-14

$\Rightarrow$ -2d =-6

$\therefore$ d = 3

Putting value of d in (i), we get

a +3×3 = 8

$\Rightarrow$a+9 = 8

$\therefore$ a = -1

We know that, the sum of first n terms = $\frac{n}{2}[2a+(n-1)d]$

So, the sum of first 20 terms $S_{20} = \frac{20}{2}[2(-1)+(20-1)3]$

$$= 10(-2+19\times 3)$$
$$= 10(-2+57)$$
$$= 10\times 55$$
$$= 550$$

Question 3

(a) Simplify: [3]

$$\text{SinA}\begin{bmatrix} \text{SinA} & \text{-CosA} \\ \text{CosA} & \text{SinA} \end{bmatrix} + \text{CosA}\begin{bmatrix} \text{CosA} & \text{SinA} \\ \text{-SinA} & \text{CosA} \end{bmatrix}$$

Ans.

$$\text{SinA}\begin{bmatrix}\text{SinA} & \text{-CosA}\\ \text{CosA} & \text{SinA}\end{bmatrix}+\text{CosA}\begin{bmatrix}\text{CosA} & \text{SinA}\\ \text{-SinA} & \text{CosA}\end{bmatrix}$$

$$=\begin{bmatrix}\text{Sin}^2\text{A} & \text{-SinA.CosA}\\ \text{SinA.CosA} & \text{Sin}^2\text{A}\end{bmatrix}+\begin{bmatrix}\text{Cos}^2\text{A} & \text{CosA.SinA}\\ \text{-CosA.SinA} & \text{Cos}^2\text{A}\end{bmatrix}$$

$$=\begin{bmatrix}\text{Sin}^2\text{A+Cos}^2\text{A} & \text{-SinA.CosA+CosA.SinA}\\ \text{SinA.CosA+(-CosA.SinA)} & \text{Sin}^2\text{A+Cos}^2\text{A}\end{bmatrix}$$

$$=\begin{bmatrix}1 & \text{-SinA.CosA+CosA.SinA}\\ \text{SinA.CosA-CosA.SinA)} & 1\end{bmatrix}[\because \text{Sin}^2\text{A+Cos}^2\text{A=1}]$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

(b) M and N are two points on the X axis and Y axis respectively. P(3, 2) divides the line segment MN in the ratio 2 : 3. Find: [3]

(i) The coordinates of M and N

(ii) Slope of the line MN.

Ans.

Let the co ordinates of M and N be (a,0) and (0,b) on x-axis and y- axis respectively

We know that,

The co ordinates of the point which divides the line segment joining $(x_1, y_1) and (x_2, y_2)$

internally in the ratio m:n is $\left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$

According to question, P(3,2) divides the line joining M and N internally in the ratio 2:3,

Thus, using the above formula, we get

$\left(\frac{3a}{5}, \frac{2b}{5}\right)$=(3,2)

So,

$\frac{3a}{5}$=3

$\Rightarrow$ a=$\frac{3\times5}{3}$

$\therefore$ a=5

And

$\frac{2b}{5}$=2

$\Rightarrow$ b=$\frac{2\times5}{2}$

$\therefore$ b=5

Therefore, the required coordinates of M and N are (5,0) and (0,5)

Now,

M(5,0) and N(0,5)

We know, slope of the line = $\frac{y_2 - y_1}{x_2 - x_1}$

Therefore, Slope of line MN = $\frac{5-0}{0-5} = \frac{5}{-5} = -1$

(c) A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32cm. Find the: [4]

(i) Radius of the cylinder

(ii) Curved surface area of the cylinder

Take π = 3.1

Ans.

(i) Given, radius of metallic sphere(r) = 6cm

We know, volume of sphere = $\frac{4}{3}\pi r^3$

$\therefore$ Volume of metallic sphere = $\frac{4}{3}\pi(6)^3$

Let r_1 be radius of the cylinder

Height of the cylinder (h) = 32cm

We know, volume of cylinder = $\pi r^2 h$

$\therefore$ Volume of given cylinder = $\pi r_1^2 \times 32$

Now, according to question,

Volume of sphere = Volume of cylinder

$$\Rightarrow \frac{4}{3}\pi(6)^3 = \pi r_1^2 \times 32$$
$$\Rightarrow \frac{4}{3}\times 216 = 32 r_1^2$$
$$\Rightarrow \frac{4\times 216}{3\times 32} = r_1^2$$
$$\Rightarrow r_1^2 = 9$$
$$\therefore r_1 = 3$$

Hence, the radius of the cylinder is 3cm

(ii.) Height of cylinder (h) = 32 cm and radius(r) = 3cm

So, The curved surface area of cylinder = $2\pi rh$

$= 2\times 3.1\times 3\times 32$

= 595.2

Question 4

(a) The following numbers, K + 3, K + 2, 3K - 7 and 2K – 3 are in proportion. Find K. [3]

Ans. Given that, K + 3, K + 2, 3K - 7 and 2K – 3 are in proportion.

$\therefore \frac{K+3}{K+2} = \frac{3K-7}{2K-3}$

$\Rightarrow$ (K+3)(2K-3) = (3K-7)(K+2)

$\Rightarrow 2K^2-3K+6K-9 = 3K^2+6K-7K-14$

$\Rightarrow 2K^2+3K-9=3K^2-K-14$

$\Rightarrow K^2-4K-5=0$

$\Rightarrow K^2-5K+K-5=0$

$\Rightarrow$ (K-5)(K+1) = 0

$\therefore K = 5 or -1$

(b) Solve for x the quadratic equation $x^2 - 4x - 8 = 0$**. Give your answer correct to three significant figures. [3]**

Ans.

Given equation, $x^2 - 4x - 8 = 0$

Comparing it with the quadratic equation $ax^2 + bx+c = 0$, we get

a= 1, b = -4 and c = -8

$\therefore b^2 - 4ac = (-4)^2 - 4\times1\times(-8)$

$= 16+32$

$= 48$

Substituting this value in the formula x = $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

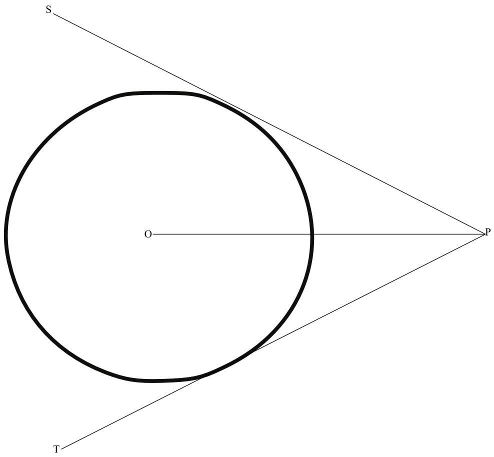
We get,

$$x = \frac{-(-4) \pm \sqrt{48}}{2 \times 1}$$
$$\Rightarrow x = \frac{4 \pm 4\sqrt{3}}{2}$$
$$\Rightarrow x = 2 \pm 2\sqrt{3}$$
$$\Rightarrow x = 2 \pm 2 \times 1.732$$
$$\Rightarrow x = 2 \pm 3.464$$
$$\therefore x = 5.464 \text{ or } -1.464$$

(c) Use ruler and compass only for answering this question. Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre, Construct two tangents to the circle from the external point P. Measure and write down the length of any one tangent.

Ans. Steps for construction are as below:

i. Take measure 4 cm in compass and draw a circle, with center as O.
ii. Draw a straight line from O to P, such that OP = 7cm
iii. Now find the midpoint of OP by drawing a perpendicular bisector
iv. Mark the midpoint as X
v. Take measure of XO in the compass and cut arcs at S and T on the Circle
vi. Join PS and PT
vii. Measure of PS comes out to be 5.74 cm



SECTION B (40 Marks)

Attempt any **four** questions from this Section.

Question 5

(a) There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box. [3]

Find the probability that the number on the disc is:

(i) An odd number

(ii) Divisible by 2 and 3 both.

(iii) A number less than 16.

Ans.

According to question,

Total number of discs numbered 1 to 25 = 25

So, the number of possible outcomes in the sample space is n(S) = 25

We know , Probability of an event $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

i. Let A be the event of getting an odd number

$\therefore$ A = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25}

$\therefore$ n(A) = 13

Therefore, the probability of getting an odd number = $\frac{n(A)}{n(S)} = \frac{13}{25}$

ii. Let B be the event of getting a number divisible by 2 and 3. To get the numbers which are divisible by 2 and 3 both, we need to find the numbers which are divisible by 6.

$\therefore$ B = {6, 12, 18, 24}

$\therefore$ n(B) = 4

Therefore, the probability of getting an odd number = $\frac{n(B)}{n(S)} = \frac{4}{25}$

iii. Let C be the event of getting a number less than 16.

$\therefore$ C = {1, 2, 3, 4, 5, 6,7 , 8, 9, 10, 11, 12, 13, 14,15}

$\therefore$ n(C) = 15

Therefore, the probability of getting a number less than 16 = $\frac{n(C)}{n(S)} = \frac{15}{25} = \frac{3}{5}$

(b) Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives Rs. 441 as interest at the time of maturity. Find the amount Rekha deposited each month. [3]

Ans.

Given, Rate of interest given by bank (r) = 9%

Time period (n) = 20 months

Interest at the time of maturity = Rs.441

Let, the principle deposited every month be P

According to question,

$$P\frac{n(n+1)}{2}\times\frac{r}{12\times100}=441$$
$$\Rightarrow P\frac{20(20+1)}{2}\times\frac{9}{1200}=441$$
$$\Rightarrow P\frac{20\times21}{2}\times\frac{9}{1200}=441$$
$$\Rightarrow P\times1.575=\ 441$$
$$\therefore P=280$$

Therefore, the amount deposited by Rekha each month is Rs 280.

(c) Use a graph sheet for this question. [4]

Take 1 cm = 1 unit along both x and y axis.

(i) Plot the following points:

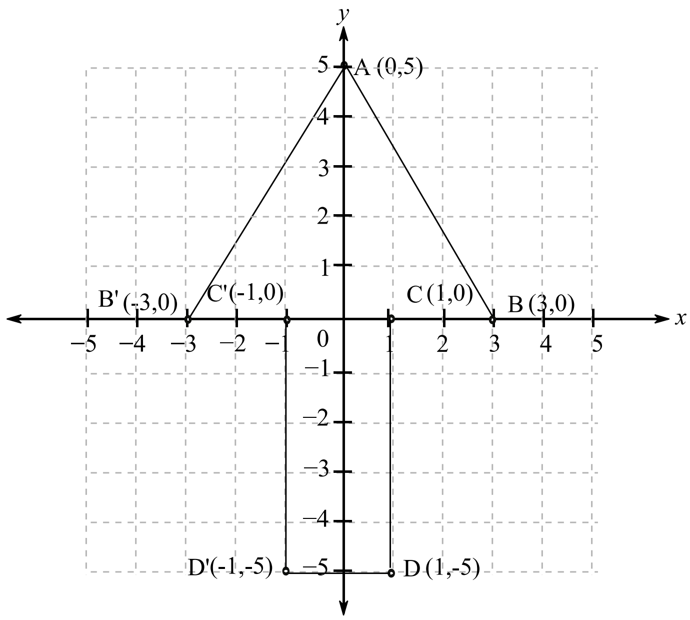
A(0,5), B(3,0), C(1,0), and D(1,-5)

(ii) Reflect the points B, C, and D on the y axis and name them as B', C' and D' respectively.

(iii) Write down the coordinates of B', C' and D'.

(iv) Join the points A, B, C, D, D', C', B', A in order and give a name to the closed figure ABCDD'C'B'.

Ans.

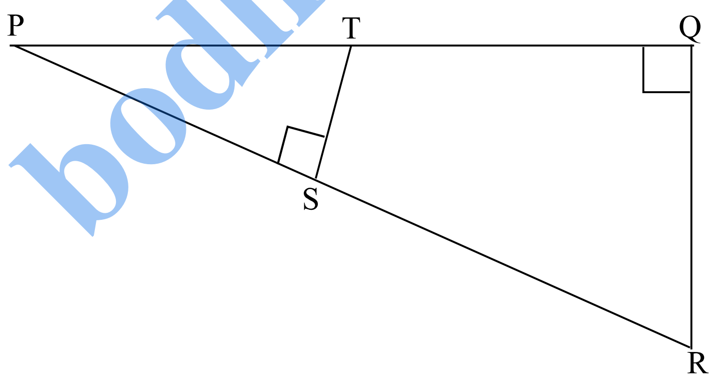


Question 6

(a) In the given figure ∠PQR = ∠PST = 90^0, PQ = 5 cm and PS = 2 cm. **[3]**

(i) Prove that △PQR = △PST

(ii) Find Area of △PQR: Area of quadrilateral SRQT.



Ans.

Given, PQ = 5cm and PS = 2cm

(i). Here, in △PQR and △PST

∠PQR = ∠PST = 90^0

And ∠RPQ = ∠SPT [∵ Being common angle]

$\therefore \Delta PQR \sim \Delta PST$ [Being AA similarity]

So, $\frac{PQ}{PS} = \frac{QR}{ST} = \frac{PR}{PT} = \frac{5}{2}$

$\therefore \frac{ar(\Delta PQR)}{ar(\Delta PST)} = \frac{5^2}{2^2} = \frac{25}{4}$

$\therefore ar(\Delta PST) = \frac{4}{25} ar(\Delta PQR)$.........................$eq(1)$

(ii)

$$\frac{ar(\Delta PQR)}{ar(\Delta PST)} = \frac{5^2}{2^2} = \frac{25}{4}$$

$$\frac{ar(\Delta PQR) - ar(\Delta PST)}{ar(\Delta PST)} = \frac{25-4}{4} = \frac{21}{4}$$

$$\frac{ar(\square SRQT)}{ar(\Delta PST)} = \frac{21}{4}$$

$$\frac{ar(\square SRQT)}{\frac{4}{25} ar(\Delta PQR)} = \frac{21}{4} \text{.....................} \textit{From equation } 1$$

$$\frac{ar(\square SRQT)}{ar(\Delta PQR)} = \frac{21}{4} \times \frac{4}{25}$$

$$\frac{ar(\square SRQT)}{ar(\Delta PQR)} = \frac{21}{25}$$

(b) The first and last term of a Geometrical Progression (G.P.) are 3 and 96 respectively. If the common ratio is 2, find: [3]

(i) 'n' the number of terms of the G.P.

(ii) Sum of the n terms.

Ans. Given first term (a) = 3,last term (l) =96 and common difference (r) = 2.

(i). We know, formula to find nth term of a G.P is $a_n = ar^{n-1}$ or $l = ar^{n-1}$

So, according to question,

$96 = 3 \times 2^{n-1}$

$\Rightarrow 2^{n-1} = 32$

$\Rightarrow 2^{n-1} = (2)^5$

$\Rightarrow n - 1 = 5$

$\therefore n = 6$

(ii). We have, n= 6, r= 2 and a=3

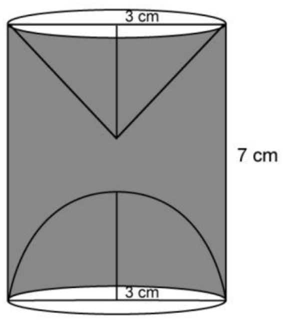
Formula for the sum of n terms of a G.P $(S_n) = \dfrac{a(r^n - 1)}{r - 1}$

$\therefore$ Sum of 6 terms $(S_6) = \dfrac{3(2^6 - 1)}{2 - 1}$

$= \dfrac{3(64 - 1)}{1}$

$= 3 \times 63 = 189$

(c) A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm. Give your answer correct to the nearest whole number Take $\pi = \dfrac{22}{7}$ **[4]**



Ans.

Given, Height of cylinder (h) = 7cm

Radius (r) = 3cm

$\therefore$ Volume of cylinder $(V_1) = \pi r^2 h$

$= \pi(3)^2 7 = \pi.63cm^3$

Height of cone (H) = 3cm

$\therefore$ Volume of cone (V_2) = $\frac{1}{3}\pi r^2 H$

$= \frac{1}{3}\pi(3)^2 3 = \pi 9cm^3$

Volume of hemisphere (V_3) = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi(3)^3 = \pi 18cm^3$

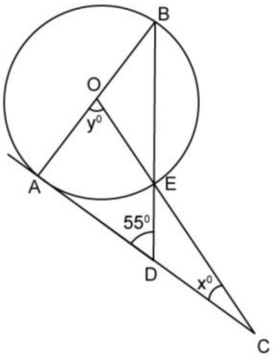
According to question,

Volume of remaining solid = V_1-V_2-V_3

$$\therefore \text{Remaining volume} = \begin{array}{l} \pi.63 - \pi 9 - \pi 18 \\ = \pi(63 - 9 - 18) \\ = \frac{22}{7} \times 36 \\ = 113.14cm^3 \end{array}$$

Question 7

(a) In the given figure AC is a tangent to the circle with centre O. If ADB = 55^0**, find x and y, Give reasons for your answer. [3]**



Ans.

Given that, AC is a tangent to the circle with centre O

And $\angle$ADB= 55^0

Here, ΔABD is a right angled triangle, so $\angle BAD = 90^0$

We know,

Sum of interior angles of a triangle = 180^0

$\angle \text{ADB} + \angle BAD + \angle ABD = 180^0$

$\Rightarrow 55^0 + 90^0 + \angle ABD = 180^0$

$\Rightarrow \angle ABD = 180^0 - 145$

$\therefore \angle ABD = 35^0$

Since, $\angle AOE$ is subtended at the centre and $\angle EBA$ on the circle by the arc AE,

Thus, 2 $\angle ABD = \angle AOE$

$\angle AOE = 2 \times 35$

$\therefore \text{y} = 70^0$

In ΔAOC, $\angle OAC = 90^0$

$\because$ Sum of interior angles of triangle = 180^0

$\therefore \angle OAC + \angle AOC + \angle ACO = 180^0$

$\Rightarrow 90^0 + y + x = 180^0$

$\Rightarrow x + y = 90^0$

$\Rightarrow x - 35^0 = 90$

$\therefore x = 55^0$

(b) The model of a building is constructed with scale factor 1: 30. [3]

(i) If the height of the model is 80 cm, find the actual height of the building in meters.

(ii) If the actual volume of a tank at the top of the building is $27m^3$, find the volume of the tank on the top of the model.

Ans. Given scale factor is 1:30

(i). The ratio of height of the model and building is 1:30

So, if height of the model is 80cm

Then, the actual height of the building = 80×30= 2400 cm = 24m

(i)i. Given, actual volume of tank = $27m^3$

Given scale factor =1:30

Here,

$$1\text{m} = \frac{10}{3}\text{cm}$$

$$(1\text{m})^3 = \left(\frac{10}{3}\right)^3$$

$$1\text{m}^3 = \frac{1000}{27}cm^3$$

Then, $27m^3 = \dfrac{1000}{27} \times 27 = 1000cm^3$

Now,

$1cm^3 = 0.001 litres$

$\therefore 1000cm^3 = 0.001 \times 1000 = 1 l\text{itres}$

Therefore, the required volume of model tank is 1litres.

(c)Given, $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \text{M=6I}$ **, where M is a matrix and 1 is unit matrix of order 2 x 2. [4]**

(i) State the order of matrix M.

(ii) Find the matrix M

Ans.

(i) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \text{M=6I}$

For matrix multiplication, the number of columns in first matrix should be equal to the number of rows in the other matrix and the resulting matrix will have order as below:

$$|\ |_{m \times n} \times |\ |_{n \times q} = |\ |_{m \times q}$$

As given:

$$\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \text{M} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

So, the order of M should be 2 x 2

Let M = $\begin{vmatrix} p & q \\ r & s \end{vmatrix}$

$$\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \text{M} = \begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \times \begin{vmatrix} p & q \\ r & s \end{vmatrix} = \begin{vmatrix} 4p+2r & 4q+2s \\ -p+r & -q+s \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 4p+2r & 4q+2s \\ -p+r & -q+s \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

$4p + 2r = 6................eq1$

$-p + r = 0................eq2$

$4q + 2s = 0................eq3$

$-q + s = 6................eq4$

From eq2 we get,

$r = p$

Substituting the value of r in eq1

$4r + 2r = 6$

$6r = 6$

$r = 1$

$\Rightarrow p = 1$

From eq4 we get,

$s = 6 + q$

Substituting the value of r in eq3

$4q + 2(6 + q) = 0$

$4q + 12 + 2q = 0$

$6q = -12$

$q = -2$

$\Rightarrow s = 6 - 2 = 4$

Therefore, M = $\begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix}$

Question 8

(a) The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference. [3]

Ans. Let the first term of A.P be a and the common difference be d.

So, the first three terms could be a-d, a, and a+d.

According to question,

$a-d+a+a+d= 42$

$\Rightarrow 3a = 42$

$\therefore a = \frac{42}{3} = 14$..........(1)

And,

$(a-d)(a+d) = 52$

$\Rightarrow a^2 - d^2 = 52 \qquad [\because (a+b)(a-b) = a^2 - b^2]$

Putting value of a from (1), we get

$\Rightarrow (14)^2 - d^2 = 52$

$\Rightarrow 196 - 52 = d^2$

$\Rightarrow d^2 = 144$

$\therefore d = 12$

(b) The vertices of a $\triangle ABC$ are A(3, 8), B(-1, 2) and C(6, -6), Find: [3]

(i) Slope of BC.

(ii) Equation of a line perpendicular to BC and passing through A.

Ans. Given, the vertices of triangle ABC are A(3, 8), B(-1, 2) and C(6, -6)

i.e. know, slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of BC = $\frac{-6-2}{6-(-1)} = \frac{-8}{7}$

ii. Let AE be the line which is perpendicular to BC and passes through A

So, Slope of AE = $-\frac{1}{\text{Slope of BC}}$ [$\because$ AE is perpendicular to BC]

$\therefore$ Slope of AE = $-\left(\frac{1}{-\frac{8}{7}}\right) = \frac{7}{8}$

Now, we have

Slope of AE (m) = $\frac{7}{8}$ and A (3,8)

$\therefore$ Equation of line AE

$= y-8=\frac{7}{8}(x-3)$

$\Rightarrow 8y-64=7x-21$

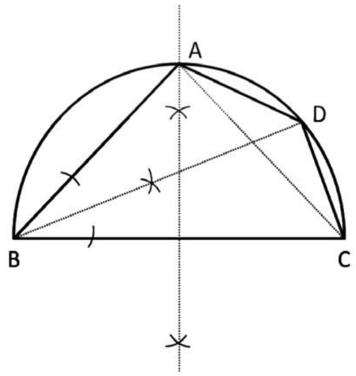
$\Rightarrow 7x-8y-21+64=0$

$\Rightarrow 7x-8y+43=0$

(c) Using ruler and a compass only construct a semi-circle with diameter BC = 7cm. Locate a point A on the circumference of the semicircle such that A is equidistant from B and C. Complete the cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. Measure ∠ADC and write it down. [4]

Ans.

1) Construct a semi-circle with diameter BC = 7 cm i.e. radius 3.5 cm
2) Draw the perpendicular bisector of BC and extend it to touch the semi-circle
3) Mark this point as A (Since A should be equidistant from B and C)
4) Join AB and AC
5) Draw the angle bisector of angle ABC and extend it to meet the semi-circle
6) Mark this point as D (D is equidistant from AB and BC)
7) Join AD and CD
8) Measure of $\angle ADC$ is 135^0



Question 9

(a) The data on the number of patients attending a hospital in a month are given below. Find the average (mean) number of patients attending the hospital in a month by using the shortcut method. [3]

Take the assumed mean as 45. Give your answer correct to 2 decimal places.

Number of patients	10-20	20-30	30-40	40-50	50-60	60-70
Number of days	5	2	7	9	2	5

Ans. Given, Assumed mean (A) = 45

The frequency table can be obtained using the data above:

Number of patients	Mid-point(x)	Number of days(f)	d = x-A =x- 45	fd
10-20	15	5	-30	-150
20-30	25	2	-20	-40
30-40	35	7	-10	-70
40-50	45	9	0	0
50-60	55	2	10	20
60-70	65	5	20	100
		$\sum f = 30$		$\sum fd = -140$

We have formula for mean = $A+\frac{\sum fd}{\sum f}$

$\therefore$ Mean number of patients attending the hospital in a day

$= 45+\frac{-140}{30} = 45 - 4.667 = 40.33$

Hence, the mean or average number of patients attending the hospital in a month

$= 40.33 \times 30$

$= 1209.9$

(b) Using properties of proportion solve for x, given. [3]

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} = 4$$

Ans. Given, $\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} = 4$

Using rationalization, we get

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} \times \frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}} = 4$$

$$\Rightarrow \frac{\left(\sqrt{5x}\right)^2+\left(\sqrt{2x-6}\right)^2}{\left(\sqrt{5x}\right)^2-\left(\sqrt{2x-6}\right)^2} = 4$$

$$\Rightarrow \frac{5x+2\sqrt{5x}\sqrt{2x-6}+2x-6}{5x-2x+6} = 4$$

$$\Rightarrow 7x+2\sqrt{5x}\sqrt{2x-6}-6 = 4(3x+6)$$

$$\Rightarrow 7x+2\sqrt{5x}\sqrt{2x-6}-6 = 12x+24$$

$$\Rightarrow 2\sqrt{5x}\sqrt{2x-6} = 5x+30$$

Squaring both the sides,we get

$$\left(2\sqrt{5x}\sqrt{2x-6}\right)^2 = (5x+30)^2$$

$$\Rightarrow 4\times 5x(2x-6) = 25x^2+300x+900$$

$$\Rightarrow 20x(2x-6) = 25x^2+300x+900$$

$$\Rightarrow 40x^2-120x-25x^2-300x = 900$$

$$\Rightarrow 15x^2-420x-900 = 0$$

$$\Rightarrow x^2-28x-60 = 0$$

$$\Rightarrow x^2-30x+2x-60 = 0$$

$$\Rightarrow x(x-30)+2(x-30) = 0$$

$$\Rightarrow (x-30)(x+2) = 0$$

$$\therefore x = 30 \text{ or } -2$$

(c) Sachin invests Rs. 8500 in 10%, Rs. 100 shares at Rs. 170 He sells the shares when the price of each share rises by Rs. 30. He invests the proceeds in 12% Rs. 100 shares at Rs. 125, Find:[4]

(i) The sale proceeds

(ii) The number of Rs. 125 shares he buys.

(iii) The change in his annual income.

Ans.

Purchase price of each share = Rs. 170

Then, number shares purchased for Rs.8500 = $\frac{8500}{170} = 50$

As the price of share increases by Rs. 30,

So,

Selling price of each share = Rs. (170 +30) = Rs.200

i. Sale proceeds shares = Rs. (200 × 50) = Rs. 10000

ii. The number of shares purchased at Rs. 125 each = $\frac{10000}{125} = 80$

iii. Initially,

Total face value of share = Face value of each share× numbers of share

= 100× 50= Rs.5000

Dividend = 10% of total face value

$= \frac{10}{100} \times 5000 = \text{Rs.}500$

After the price of share rises by Rs.30,

Total face value of share = 100×80= Rs. 8000

And, Dividend = 12% of 8000 = $\frac{12}{100} \times 8000 = Rs.960$

Hence, the change in annual income = Rs.(960-500) =Rs. 460

Question 10

(a) Use graph paper for this question. The marks obtained by 120 students in an English test are given below. [6]

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	**5**	**9**	**16**	**22**	**26**	**18**	**11**	**6**	**4**	**3**

Draw the ogive and hence, estimate:

(i) The median marks.

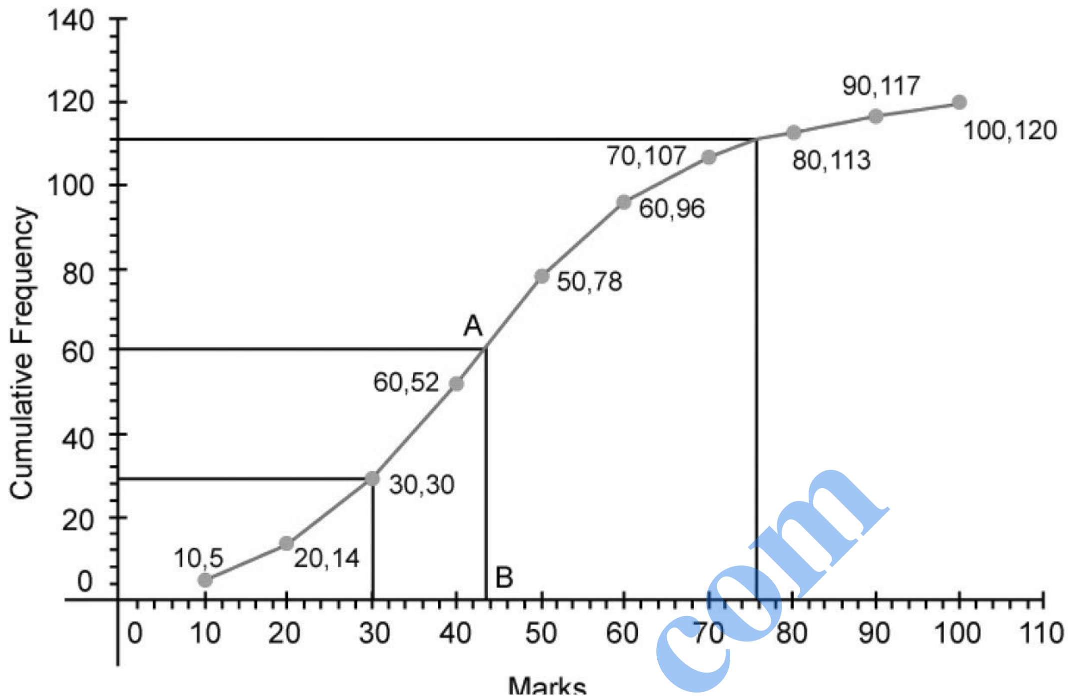
(ii) The number of students who did not pass the test if the pass percentage was 50.

(iii) The upper quartile marks.

Ans. We have to prepare a frequency table first,

Class Interval	Frequency	Cumulative frequency
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

Here, n =120 and $\frac{n}{2} = \frac{120}{2} = 60$



i. Through mark 60, draw a line segment parallel to x-axis which meets the curve at A. From A, draw a line parallel to y- axis which meets the x-axis at point B. Therefore,

Median = 43

ii. Number of students who did not pass the test = the students who obtained less than 50 % marks =5+9+16+22+26=78

iii. The formula for upper quartile = $\left(\frac{3n}{4}\right)^{th}$ term.

Therefore, the upper quartile marks = $\left(\frac{3\times 120}{4}\right)^{th}$ term = 90^{th} term= 117

(b) A man observes the angle of elevation of the top of the lower to be 45^0. He walks towards it in a horizontal line through its base. On covering 20 m the angle of elevation changes to 60^0. Find the height of the tower correct to 2 significant figures. [4]

Ans.

Let PQ be the height of the tower and R and S be two positions of man from where the angle of elevation was formed.

According to question, RS = 20m, $\angle QSP = 45^0$ and $\angle QRP = 60^0$

Let PQ = h and RP = x

Then, in ΔQSP,

$$\tan 45^0 = \frac{PQ}{SP}$$

$$\Rightarrow \tan 45^0 = \frac{h}{20+x} \qquad [\because \text{Being right angled triangle } \& \ \tan 45^0 = 1]$$

$$\Rightarrow 20 + x = h$$

$$\Rightarrow x = h - 20$$

Similarly, in ΔPQR

$$\tan 60^0 = \frac{PQ}{RP}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

Now, we can write, $h - 20 = \frac{h}{\sqrt{3}}$ [Combining the above equations]

$$\Rightarrow \sqrt{3}(h-20) = h$$
$$\Rightarrow \sqrt{3}h - 20\sqrt{3} = h$$
$$\Rightarrow \sqrt{3}h - h = 20\sqrt{3}$$
$$\Rightarrow h(\sqrt{3}-1) = 20\sqrt{3}$$
$$\Rightarrow h = \frac{20\sqrt{3}}{(\sqrt{3}-1)}$$
$$\therefore h = 47.32$$

Hence, the height of the tower is 47.32m.

Question 11

(a) Using the Remainder Theorem find the remainders obtained when $x^3 + (kx + 8)x + k$ **is divided by x + 1 and x – 2.Hence find k if the sum of the two remainders is 1.** **[3]**

Ans.

Let P(x) be a polynomial

So, we have P(x) = $x^3 + (kx + 8)x + k$

When P(x) is divided by (x+1) and (x-2), the remainder is P(-1) and P(2) respectively.

$$\text{Now, P(-1)} \begin{aligned} &= (-1)^3 + [k(-1)+8]-1+k \\ &= -1 + (k-8) + k \\ &= 2k - 9 \end{aligned}$$

And,

$$\text{P(2)} \begin{aligned} &= (2)^3 + [k(2)+8]2+k \\ &= 8 + (2k+8)2 + k \\ &= 8 + 4k + 16 + k \\ &= 5k + 24 \end{aligned}$$

According to question, the sum of the two remainders is 1

i.e.

$P(-1)+P(2)=1$

$\Rightarrow$(2k-9) + (5k+24) =1

$\Rightarrow$2k-9+5k+24=1

$\Rightarrow$7k+15=1

$\Rightarrow$7k = -14

$\therefore k=-2$

(b) The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [3]

Ans. Let the two consecutive natural numbers which are multiple of 3 be 3x and (3x+3)

Then, according to question,

Product of the numbers = 810

$$\Rightarrow 3x \times (3x+3) = 810$$
$$\Rightarrow 9x^2 + 9x = 810$$
$$\Rightarrow x^2 + x = 90$$
$$\Rightarrow x^2 + x - 90 = 0$$
$$\Rightarrow x^2 + 10x - 9x + 90 = 0$$
$$\Rightarrow x(x+10) - 9(x+10) = 0$$
$$\Rightarrow (x+10)(x-9) = 0$$
$$\therefore x = -10 \text{or } 9$$

Taking x=9, we get 3x = 3×9 = 27

And (3x+3) = (3×9+3) = 30

Taking x= -10, we get 3x= 3×-10 = −30

And, (3x+3) = [3×(-10)+3]=−27

Hence, the required numbers are 27 and 30 or −27 and −30

(c) In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC||AE. If ∠BAC = 50°, find giving reasons: [4]

(i) ∠ACB

(ii) ∠EDC

(iii) ∠BEC

Hence prove that BE is also a diameter.

Ans. Given that, ABCDE is a pentagon inscribed in a circle such that AC is the diameter of circle and BC||AE.

And , $\angle BAC = 50^0$

Here, $\triangle$ABC is right angled triangle , so $\angle ABC = 90^0$

Now,

$\angle ABC + \angle BAC + \angle ACB = 180^0$

$\Rightarrow 50^0 + 90^0 + \angle ACB = 180$ [Sum of interior angles of triangle $=180^0$]

$\Rightarrow \angle ACB = 180^0 - 140^0$

$\therefore \angle ACB = 40^0$

Since, BC||AE

$\therefore \angle ACB = \angle EAC = 40^0$ [Being alternate angle]

Again, in $\Delta AEC, \angle AEC = 90^0$

[Sum of interior angles of triangle $=180^0$]

$\angle AEC + \angle EAC + \angle ACE = 180^0$

$\Rightarrow 90^0 + 40^0 + \angle ACE = 180$

$\Rightarrow \angle ACE = 180^0 - 130^0$

$\therefore \angle ACE = 50^0$

And, $\angle BAC = 50^0$

Therefore, AB||CE

[$\because$ Being alternate angles made by transversal AC with lines CE&AB,equal]

Now, $\angle BAE = 50^0 + 40^0 = 90^0$

Also, since $\angle$EAB is subtended by EB on the circle
So,BE is a diameter of the given circle.

Here, AEDC is cyclic quadrilateral
So,
$\angle \text{EAC} + \angle \text{ EDC} = 180^0$
$\Rightarrow 40^0 + \angle \text{ EDC} = 180^0$
$\therefore \angle \text{ EDC} = 140^0$

Now,
We see that Arc BC subtends angles $\angle$BAC and $\angle$BEC on the same side of the circle
Therefore,
$\angle \text{BAC} = \angle \text{BEC} = 50^0$