ICSE Board
Class X Mathematics
Board Paper 2018
(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during the first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

Attempt **all** questions from **Section A** and **any four** questions from **Section B.**
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [].
Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Find the value of and 'y' if: [3]

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) Sonia had a recurring deposit account in a bank and deposited Rs. 600 per month for $2^1/_2$ years. If the rate of interest was 10% p.a., find the maturity value of this account. [3]

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is: [4]

(i) a prime number.
(ii) a number divisible by 4.
(iii) a number that is a multiple of 6.
(iv) an odd number.

Question 2

(a) The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the [3]

(i) radius of the cylinder

(ii) volume of cylinder $(\text{use } \pi = \frac{22}{7})$

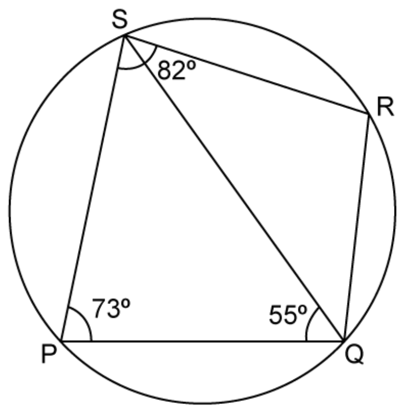
(b) If (k – 3), (2k + l) and (4k + 3) are three consecutive terms of an A.P., find the value of k. [3]

(c) PQRS is a cyclic quadrilateral. Given ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate: [4]

(i) ∠QRS

(ii) ∠RQS

(iii) ∠PRQ



Question 3

(a) If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, find the values of 'a' and `b'. [3]

(b) Prove that $\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta} = \tan\theta + \cot\theta$ [3]

(c) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data: [4]

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsmen	4	18	9	6	7	2	4

Question 4

(a) Solve the following inequation, write down the solution set and represent it on the real number line: [3]

$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$

(b) If the straight lines 3x – 5y = 7 and 4x + ay + 9 = 0 are perpendicular to one another, find the value of a. [3]

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [4]

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) The 4^{th} term of a G.P. is 16 and the 7^{th} term is 128. Find the first term and common ratio of the series. [3]

(b) A man invests Rs. 22,500 in Rs. 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate: [3]

(i) The number of shares purchased

(ii) The annual dividend received.

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

(c) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, –2), C(0, –1) and D(0, 1). [4]

(i) Reflect quadrilateral ABCD on the y-axis and name it as A'B'CD.

(ii) Write down the coordinates of A' and B'.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon A'B'CD.

Question 6

(a) Using properties of proportion, solve for x. Given that x is positive: [3]

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

(b) if $A=\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}, B=\begin{bmatrix}0 & 4\\-1 & 7\end{bmatrix}$ and $c=\begin{bmatrix}1 & 0\\-1 & 4\end{bmatrix}$, find $AC + B^2 - 10C$. [3]

(c) Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1+ \tan\theta + \sec\theta) = 2$ [4]

Question 7

(a) Find the value of k for which the following equation has equal roots. [3]

$x^2 + 4kx + (k^2 - k + 2) = 0$

(b) On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions. AB = 6 cm; BC = 8 cm and all angles are right angles. Find: [3]

(i) the actual length of the diagonal distance AC of the plot in km.

(ii) the actual area of the plot in sq. km.

(c) A(2, 5), B(−1, 2) and C(5, 8) are the vertices of a triangle ABC, `M' is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

Question 8

(a) Rs. 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received Rs. 100 more. Find the original number of children. [3]

(b) If the mean of the following distribution is 24, find the value of 'a '. [3]

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

(c) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and ∠ABC = 120° [4]

(i) Construct a circum-circle of ΔABC

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Question 9

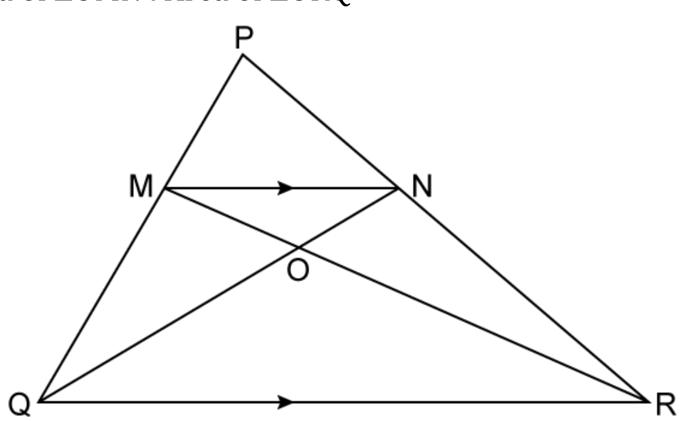
(a) Priyanka has a recurring deposit account of Rs. 1000 per month at 10% per annum. If she gets Rs. 5550 as interest at the time of maturity, find the total time for which the account was held. [3]

(b) In ΔPQR, MN is parrallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$ [3]

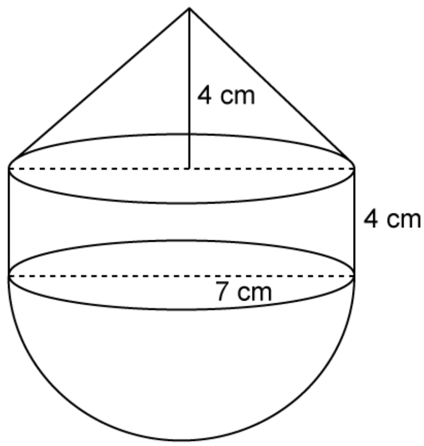
(i) Find $\frac{MN}{QR}$

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find, Area of ΔOMN : Area of ΔORQ



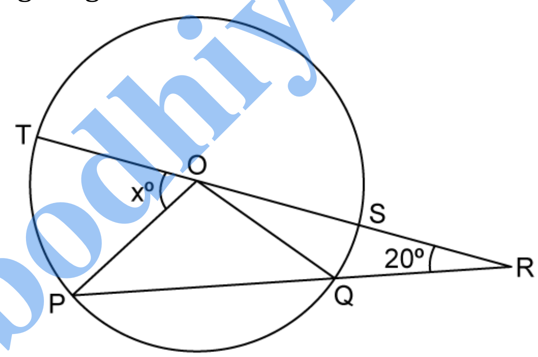
(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4]



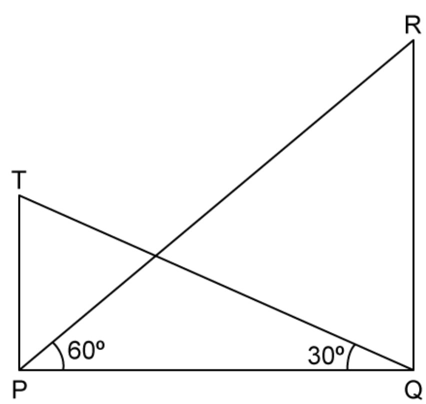
Question 10

(a) Use Remainder theorem to factorize the following polynomial: [3]
$2x^3 + 3x^2 - 9x - 10$.

(b) In the figure given below 'O' is the centre of the circle. If QR = OP and $\angle ORP = 20°$. Find the value of 'x' giving reasons. [3]



(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60^o and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre. [4]



Question 11

(a) The 4th term of an A.P. is 22 and 15th term is 66. Find the first terns and the common difference. Hence find the sum of the series to 8 terms. [4]

(b) Use Graph paper for this question. [6]

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded:

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following:

(i) the median

(ii) lower Quartile

(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

Solution

SECTION A

1.

(a)

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix}+\begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix}+\begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+6 & 14-7 \\ 18+4 & 2y-10+5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow 2x+6=10 \quad \text{and} \quad 2y-5=15$$

$$\Rightarrow 2x=4 \quad \text{and} \quad 2y=20$$

$$\Rightarrow x=2 \quad \text{and} \quad y=10$$

(b)

Given : $P = \text{Rs. } 600$, $n = 30$ months and $r = 10\%$

$$\therefore I = \text{Rs.}\left(600\times\frac{30(30+1)}{2\times 12}\times\frac{10}{100}\right)=\text{Rs. } 2325$$

Since sum deposited $= P\times n = \text{Rs. } 600\times 30 = \text{Rs. } 18000$

Thus, the maturity value $= \text{Rs. } (18000+2325) = \text{Rs. } 20325$

(c)

Total number of cards $= 10$

(i) Prime number card is 2.

$\Rightarrow$ Number of favourable outcomes $= 1$

$\therefore$ Required probability $= \dfrac{1}{10}$

(ii) Cards having number divisible by 4 are 4, 8, 12, 16, 20.

$\Rightarrow$ Number of favourable outcomes $= 5$

$\therefore$ Required probability $= \dfrac{5}{10} = \dfrac{1}{2}$

(iii) Cards having number that is multiple of 6 are 6, 12, 18

$\Rightarrow$ Number of favourable outcomes $= 3$

$\therefore$ Required probability $= \dfrac{3}{10}$

(iv) Odd number card is not there.

$\Rightarrow$ Number of favourable outcomes $= 0$

$\therefore$ Required probability $= 0$

2.

(a)

Let the radius of the cylindrical vessel be r and its height be h.

$\Rightarrow$ Height $= h = 25$ cm

(i) Circumference of the base $= 132$ cm

$\Rightarrow 2\pi r = 132$

$\Rightarrow 2 \times \dfrac{22}{7} \times r = 132$

$\Rightarrow r = 21$ cm

(ii) Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 34650 \text{ cm}^3$$

(b)

$(k-3), (2k+1)$ and $(4k+3)$ are three consecutive terms of an A.P.

$\Rightarrow 2(2k+1) = (k-3)+(4k+3)$

$\Rightarrow 4k+2 = k-3+4k+3$

$\Rightarrow 4k+2 = 5k$

$\Rightarrow k = 2$

(c)

Given : PQRS is a cyclic quadrilateral.

$\angle QPS = 73°, \angle PQS = 55°$ and $\angle PSR = 82°$

(i) Opposite angle of a cyclic quadrilateral are supplementary.

$\Rightarrow \angle QPS + \angle QRS = 180°$

$\Rightarrow 73° + \angle QRS = 180°$

$\Rightarrow \angle QRS = 180° - 73° = 107°$

(ii) Opposite angle of a cyclic quadrilateral are supplementary.

$\Rightarrow \angle PSR + \angle PQR = 180°$

$\Rightarrow \angle PSR + (\angle PQS + \angle RQS) = 180°$

$\Rightarrow 82° + 55° + \angle RQS = 180°$

$\Rightarrow \angle RQS = 180° - 137° = 43°$

(iii) In ΔPQS, by angle sum property, we have

$\Rightarrow \angle PSQ + \angle PQS + \angle QPS = 180°$

$\Rightarrow \angle PSQ + 55° + 73° = 180°$

$\Rightarrow \angle PSQ = 180° - 128° = 52°$

Now, $\angle PRQ = \angle PSQ$ (angles in the same segment of a circle)

$\Rightarrow \angle PRQ = 52°$

3.

(a)

Given $(x+2)$ is a factor of x^3+ax+b;

$\Rightarrow (-2)^3 + a(-2) + b = 0 \qquad (x+2=0 \Rightarrow x=-2)$

$\Rightarrow -8-2a+b=0$

$\Rightarrow -2a+b=8 \quad ....(i)$

Also, given that $(x+3)$ is a factor of x^3+ax+b;

$\Rightarrow (-3)^3 + a(-3) + b = 0$

$\Rightarrow -27-3a+b=0$

$\Rightarrow -3a+b=27 \quad ....(ii)$

Subtracting (i) from (ii), we have

$-a=19 \Rightarrow a=-19$

Substituting $a=-19$ in (i), we have

$-2\times(-19)+b=8$

$\Rightarrow 38+b=8$

$\Rightarrow b=-30$

Hence, $a=-19$ and $b=-30$

(b)

$$\text{L.H.S.} = \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$$

$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$

$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}}$$

$$= \sqrt{\frac{1}{\cos^2\theta\sin^2\theta}}$$

$$= \sqrt{\frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}}$$

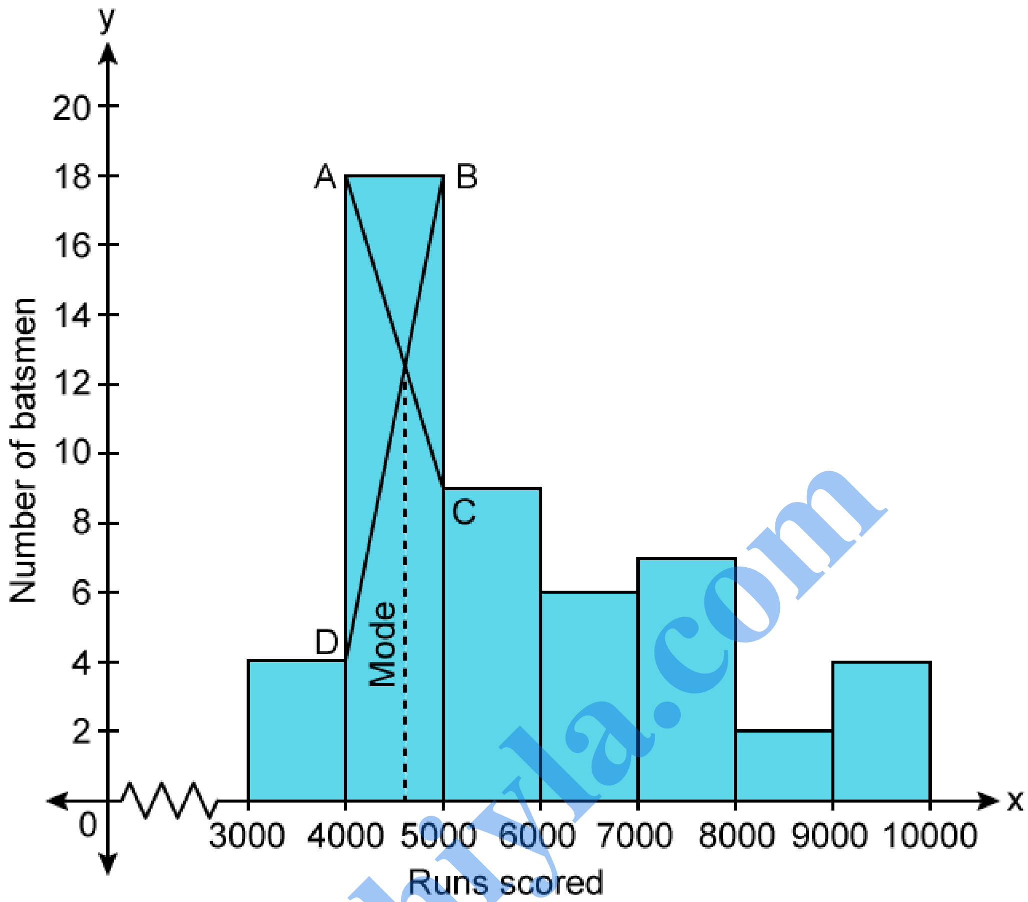
$$= \sqrt{\sec^2\theta \times \operatorname{cosec}^2\theta}$$

$$= \sec\theta \times \operatorname{cosec}\theta$$

$$\text{R.H.S.} = \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta} \times \frac{1}{\sin\theta} = \sec\theta \times \operatorname{cosec}\theta$$

Thus, L.H.S. = R.H.S.

(c) The histogram is as follows:



From histogram, we have mode = 4600

4.

(a)

Given inequation: $-2+10x \le 13x+10 < 24+10x, \ \ x \in Z$

$\Rightarrow -2+10x \le 13x+10 \quad \text{and} \quad 13x+10 < 24+10x$

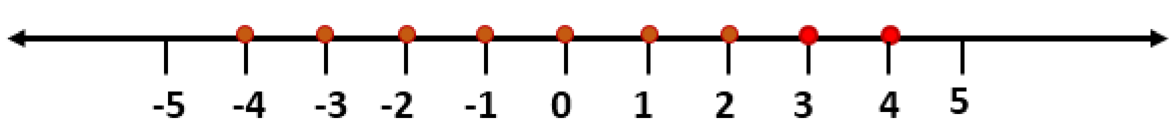
$\Rightarrow -2-10 \le 13x-10x \quad \text{and} \quad 13x-10x < 24-10$

$\Rightarrow -12 \le 3x \quad \text{and} \quad 3x < 14$

$\Rightarrow -4 \le x \quad \text{and} \quad x < 4.6$

$\therefore$ Solution set $= \{x : -4 \le x < 4.6 \text{ and } x \in Z)$

Representation on number line is as follows:



(b)

$$3x - 5y = 7$$
$$\Rightarrow 5y = 3x - 7$$
$$\Rightarrow y = \frac{3}{5}x - \frac{7}{5}$$
$$\Rightarrow \text{Its slope} = \frac{3}{5}$$
$$4x + ay + 9 = 0$$
$$\Rightarrow ay = -4x - 9$$
$$\Rightarrow y = \frac{-4}{a}x - \frac{9}{a}$$
$$\Rightarrow \text{Its slope} = \frac{-4}{a}$$

Since lines are perpendicular to each other,

$$\frac{3}{5} \times \frac{-4}{a} = -1 \Rightarrow \frac{3}{5} \times \frac{4}{a} = 1 \Rightarrow \frac{4}{a} = \frac{5}{3}$$
$$\Rightarrow a = \frac{4 \times 3}{5} = \frac{12}{5}$$

(c)

Given quadratic equation is $x^2 + 7x = 7$

$$\Rightarrow x^2 + 7x - 7 = 0$$

Comparing with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 7$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{77}}{2}$$
$$\Rightarrow x = \frac{-7 \pm 8.77}{2}$$
$$\Rightarrow x = \frac{-7 + 8.77}{2} \text{ and } x = \frac{-7 - 8.77}{2}$$
$$\Rightarrow x = \frac{1.77}{2} \text{ and } x = \frac{-15.77}{2}$$
$$\Rightarrow x = 0.885 \text{ and } x = -7.885$$
$$\Rightarrow x = 0.89 \text{ and } x = -7.89 \text{ (correct to two decimal places)}$$

SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

4^{th} term of G.P. $= 16$

$\Rightarrow ar^{4-1} = 16$

7^{th} term of G.P. $= 128$

$\Rightarrow ar^{7-1} = 128$

so, $\dfrac{ar^3}{ar^6} = \dfrac{16}{128}$

$\Rightarrow \dfrac{1}{r^3} = \dfrac{1}{8}$

$\Rightarrow r = 2$

$ar^3 = 16$

$a \times 2^3 = 16$

$a \times 8 = 16$

$a = 2$

(b)

Total investment $=$ Rs.22,500

Face value $=$ Rs.50

Discount $= \dfrac{10}{100} \times 50 = \text{Rs.}5$

Market value $=$ Face value $-$ discount $=$ Rs.45

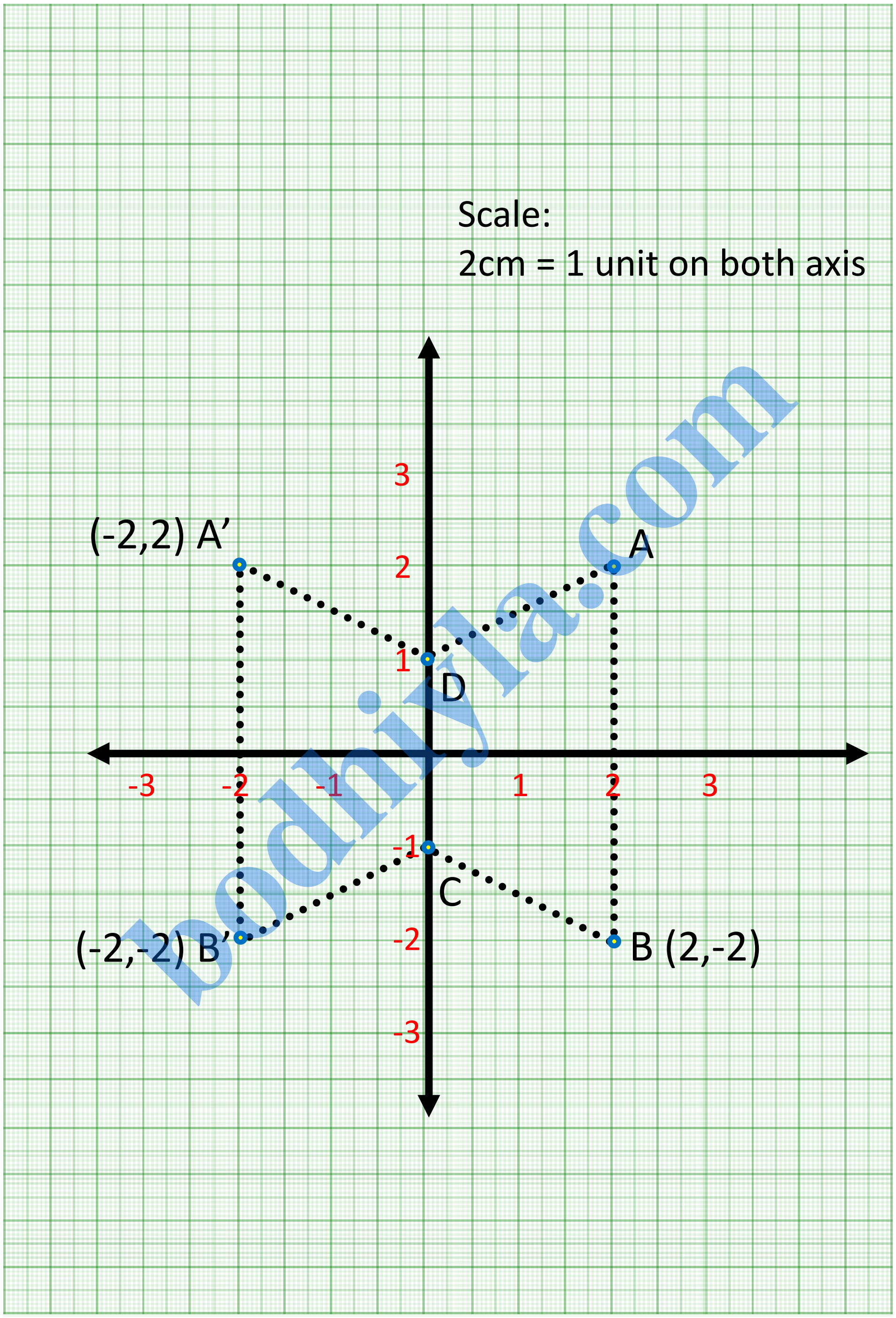
Total shares purchased $= \dfrac{22{,}500}{45} = 500$

Total dividend $= \dfrac{12}{100} \times 50 \times 500 = 3000$

Rate of return $= \dfrac{3000}{22500} \times 100 = 13.33\% \approx 13\%$

(c)

(i) and (ii)



(iii) D and C are invariant points.

(iv) A'B'CD is a trapezium

6.

(a)

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

$$\Rightarrow \frac{2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1}}{2x+\sqrt{4x^2-1}-2x+\sqrt{4x^2-1}}=\frac{4+1}{4-1} \quad \text{(By componendo} - \text{dividendo)}$$

$$\Rightarrow \frac{4x}{2\sqrt{4x^2-1}}=\frac{5}{3}$$

$$\Rightarrow \frac{2x}{\sqrt{4x^2-1}}=\frac{5}{3}$$

$$\Rightarrow \frac{4x^2}{4x^2-1}=\frac{25}{9} \quad \text{(squaring both sides)}$$

$$\Rightarrow \frac{4x^2-4x^2+1}{4x^2-1}=\frac{25-9}{9} \quad \text{(By dividendo)}$$

$$\Rightarrow \frac{1}{4x^2-1}=\frac{16}{9}$$

$$\Rightarrow 9=64x^2-16$$

$$\Rightarrow 64x^2=25$$

$$\Rightarrow x^2=\frac{25}{64}$$

$$\Rightarrow x=\pm\frac{5}{8}$$

$$\Rightarrow x=\frac{5}{8} \quad \text{(x is positive)}$$

(b)

$$\text{Given}: A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix},\ B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix} \text{and } C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$\text{Now, } AC=A\times C=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}\times\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2\times1+3\times(-1) & 2\times0+3\times4\\ 5\times1+7\times(-1) & 5\times0+7\times4\end{bmatrix}$$

$$=\begin{bmatrix}2-3 & 0+12\\ 5-7 & 0+28\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}$$

$$\text{And, } B^2=B\times B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}\times\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$$

$$=\begin{bmatrix}0\times0+4\times(-1) & 0\times4+4\times7\\ -1\times0+7\times(-1) & -1\times4+7\times7\end{bmatrix}$$

$$=\begin{bmatrix}0-4 & 0+28\\ 0-7 & -4+49\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}$$

$$\text{Now, } AC+B^2-10C=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-10\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-\begin{bmatrix}10 & 0\\ -10 & 40\end{bmatrix}$$

$$=\begin{bmatrix}-1-4-10 & 12+28-0\\ -2-7+10 & 28+45-40\end{bmatrix}$$

$$=\begin{bmatrix}-15 & 40\\ 1 & 33\end{bmatrix}$$

(c)

$$\text{L.H.S.} = (1+\cot\theta - \operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$$

$$= \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$

$$= \frac{1}{\sin\theta\cos\theta}\begin{pmatrix}\sin\theta\cos\theta+\sin^2\theta+\sin\theta+\cos^2\theta \\ +\sin\theta\cos\theta+\cos\theta-\cos\theta-\sin\theta-1\end{pmatrix}$$

$$= \frac{1}{\sin\theta\cos\theta}\left(2\sin\theta\cos\theta+(\sin^2\theta+\cos^2\theta)-1\right)$$

$$= \frac{1}{\sin\theta\cos\theta}\left(2\sin\theta\cos\theta+1-1\right)$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2$$

$$= \text{R.H.S.}$$

7.

(a)

For the given equation $x^2+4kx+(k^2-k+2)=0$

$a=1,\ b=4k$ and $c=k^2-k+2$

Since the roots are equal,

$$b^2-4ac=0$$

$$\Rightarrow (4k)^2-4\times1\times(k^2-k+2)=0$$

$$\Rightarrow 16k^2-4k^2+4k-8=0$$

$$\Rightarrow 12k^2+4k-8=0$$

$$\Rightarrow 3k^2+k-2=0$$

$$\Rightarrow 3k^2+3k-2k-2=0$$

$$\Rightarrow 3k(k+1)-2(k+1)=0$$

$$\Rightarrow (k+1)(3k-2)=0$$

$$\Rightarrow k+1=0 \text{ or } 3k-2=0$$

$$\Rightarrow k=-1 \text{ or } k=\frac{2}{3}$$

(b)

Scale: $1:50000$

1 cm represents 50000 cm $= \dfrac{50000}{1000 \times 100} = 0.5$ km

(i) In ΔABC, by pythagoras theorem

$$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$\Rightarrow AC = 10$ cm

$\Rightarrow$ Actual length of diagonal $AC = 10 \times 0.5 = 5$ km

(ii) 1 cm $= 0.5$ km

$\Rightarrow 1\ \text{cm}^2 = 0.25$ km

Area of rectangle $ABCD = AB \times BC = 6 \times 8 = 48\ \text{cm}^2$

$\Rightarrow$ Actual area of a plot $= 48 \times 0.25 = 12\ \text{km}^2$

(c)

Let the co-ordinates of M be (x, y).

Thus, we have

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times (-1) + 2 \times 2}{1+2} = \frac{-1+4}{3} = \frac{3}{3} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times (2) + 2 \times 5}{1+2} = \frac{2+10}{3} = \frac{12}{3} = 4$$

$\Rightarrow$Co-ordinates of M are (1, 4).

Slope of line passing through C and M $= m = \dfrac{4-8}{1-5} = \dfrac{-4}{-4} = 1$

$\therefore$ Required equation is given by

$y - 8 = 1(x - 5)$

$\Rightarrow y - 8 = x - 5$

$\Rightarrow y = x + 3$

8.

(a)

Let the original number of children be x.

It is given that Rs. 7500 is divided among x children.

$\Rightarrow$ Money received by each child $= \text{Rs.}\dfrac{7500}{x-20}$

If there were 20 less children, then money received by each child $= \text{Rs.}\ \dfrac{7500}{x-20}$

From the given information, we have

$$\frac{7500}{x-20}-\frac{7500}{x}=100$$

$$\Rightarrow \frac{75}{x-20}-\frac{75}{x}=1$$

$$\Rightarrow \frac{75x-75x+1500}{x^2-20x}=1$$

$$\Rightarrow 1500=x^2-20x$$

$$\Rightarrow x^2-20x-1500=0$$

$$\Rightarrow x^2-50x+30x-1500=0$$

$$\Rightarrow x(x-50)+30(x-50)=0$$

$$\Rightarrow (x-50)(x+30)=0$$

$$\Rightarrow x=50 \text{ or } x=-30$$

Since number of children cannot be negative, we reject $x=-30$.

$\Rightarrow x=50$

Thus, the original number of children $=50$

(b)

We have,

C.I.	f	Class mark x	fx
$0-10$	7	5	35
$10-20$	a	15	15a
$20-30$	8	25	200
$30-40$	10	35	350
$40-50$	5	45	225
	$\sum f=30+a$		$\sum fx=810+15a$

Mean $=24$ (given)

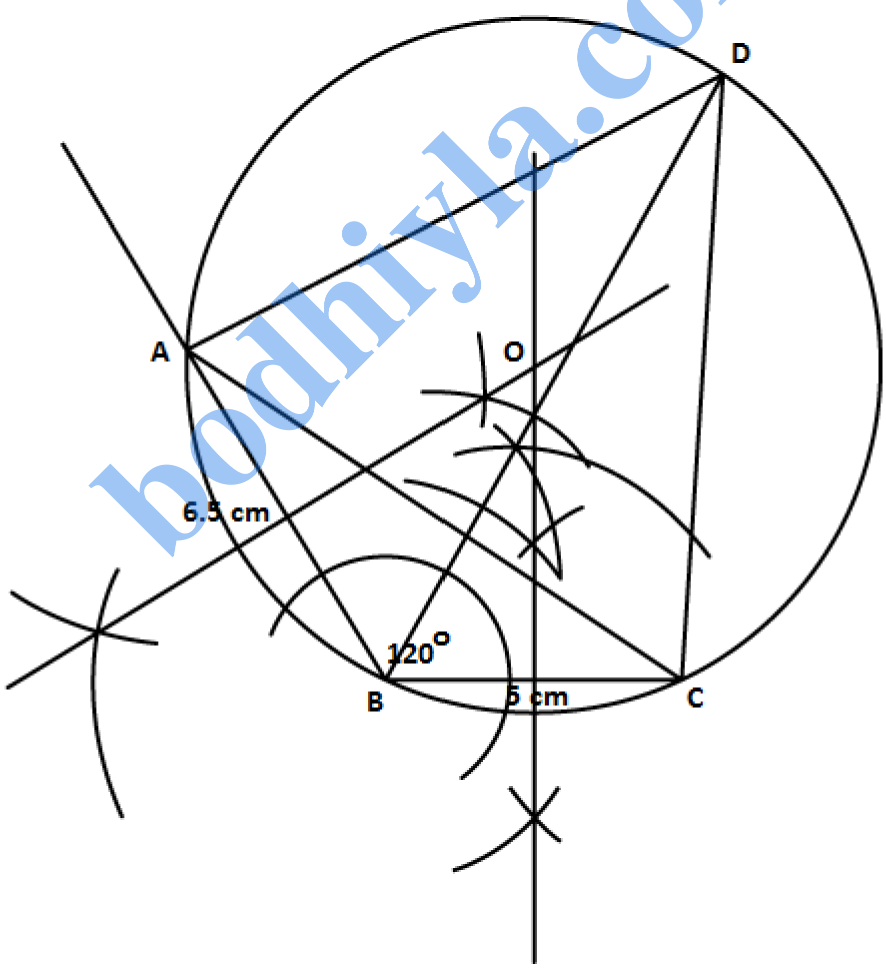
$$\Rightarrow \frac{\sum fx}{\sum f}=24 \Rightarrow \frac{810+15a}{30+a}=24 \Rightarrow 810+15a=720+24a$$

$\Rightarrow a=10$

(c)

Steps of construction:

1) Draw a line segment BC of length 5 cm.
2) At B, draw a ray BX making an angle of 120° with BC.
3) With B as centre and radius 6.5 cm, draw an arc to cut the ray BX at A. Join AC.
   ΔABC will be obtained.
4) Draw the perpendicular bisectors of AB and BC to meet at point O.
5) With O as centre and radius OA, draw a circle. The circle will circumscribe ΔABC.
6) Draw the angle bisector of $\angle$ABC.
7) The angle bisector of $\angle$ABC and let it meet circle at point D.
8) Join AD and DC to obtain the required cyclic quadrilateral ABCD such that point D is equidistant from AB and BC.



9.

(a)

Given : $P = \text{Rs. } 1000$, $r = 10\%$ and $I = \text{Rs. } 5550$

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 5550 = 1000 \times \frac{n(n+1)}{24} \times \frac{10}{100}$$

$$\Rightarrow 1332 = n(n+1)$$

$$\Rightarrow n^2 + n - 1332 = 0$$

$$\Rightarrow n^2 + 37n - 36n - 1332 = 0$$

$$\Rightarrow n(n+37) - 36(n+37) = 0$$

$$\Rightarrow (n+37)(n-36) = 0$$

$$\Rightarrow n = -37 \text{ or } n = 36$$

Since number of months cannot be negative, we reject $n = -37$

$$\Rightarrow n = 36$$

Thus, total time is 36 months.

(b)

(i) In ΔPMN and ΔPQR, $MN \parallel QR$

$\Rightarrow \angle PMN = \angle PQR$ (alternate angles)

$\Rightarrow \angle PNM = \angle PRQ$ (alternate angles)

$\Rightarrow \Delta PMN \sim \Delta PQR$ (AA postulate)

$$\Rightarrow \frac{PM}{PQ} = \frac{MN}{QR}$$

$$\Rightarrow \frac{2}{5} = \frac{MN}{QR} \quad \left[\frac{PM}{MQ} = \frac{2}{3} \Rightarrow \frac{PM}{PQ} = \frac{2}{5}\right]$$

(ii) In ΔOMN and ΔORQ,

$\angle OMN = \angle ORQ$ (alternate angles)

$\angle MNO = \angle OQR$ (alternate angles)

$\Rightarrow \Delta OMN \sim \Delta ORQ$ (AA postulate)

(iii) $$\frac{\text{Area of } \Delta OMN}{\text{Area of } \Delta ORQ} = \frac{MN}{RQ} = \frac{2}{5}$$

(c)

$$\text{Volume of solid} = \text{Volume of cone} + \text{Volume of cylinder} + \text{Volume of hemisphere}$$

$$\text{Volume of cone} = \frac{\pi r^2 h}{3} = \frac{22\times7\times7\times4}{7\times3} = \frac{616}{3}\ \text{cm}^3$$

$$\text{Volume of cylinder} = \pi r^2 h = \frac{22\times7\times7\times4}{7} = 616\ \text{cm}^3$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = \frac{2\times22\times7\times7\times7}{3\times7} = \frac{2156}{3}\ \text{cm}^3$$

$$\text{Total volume} = \frac{616}{3} + 616 + \frac{2156}{3} = 1540\ \text{cm}^3$$

10.

(a) Let P(x) = $2x^3 + 3x^2 - 9x - 10$
P(2) = 16 + 12 – 18 – 10
P(2) = 0

So, (x – 2) is a factor.

Let us divide P(x) with (x-2), we get

$(x-2)\ (2x^2 + 7x + 5)$

This can be further factored to

$(x-2)\ (2x^2 + 5x + 2x + 5)$ ……… (Split 7x into two terms, whose sum is 7x and product is$10x^2$)

$(x-2)\ (2x^2 + 5x + 2x + 5)$

$(x-2)\ (x(2x+5) + 1(2x+5))$

$(x-2)(2x+5)(x+1)$

(b)

Now,

OP = QR....given

So, OP = OT = OQ = QR

In ΔRQP

RQ = QO

So $\angle QRO = \angle QOR = 20^{\circ}$

So by sum of angles in ΔRQP

$\angle RQO = 140^{\circ}$

Now

$\angle RQO + \angle OQP = 180^{\circ}$.....linear pair

$\angle OQP = 40^{\circ}$

In ΔPOQ

OQ = PO...radii

So $\angle QPO = \angle OQP = 40^{\circ}$

So by sum of angles in ΔOQP

$\angle POQ = 100^{\circ}$

Now,

$\angle POT + \angle POQ + \angle QOR = 180^{\circ}$......angles in straight line

$x = 60^{\circ}$

(c)

In ΔPQR

$$\tan 60^{\circ} = \frac{RQ}{PQ}$$

$$\sqrt{3} = \frac{50}{PQ}$$

$$PQ = \frac{50}{\sqrt{3}}$$

In ΔPQT

$$\tan 30^{\circ} = \frac{PT}{PQ}$$

$$\frac{1}{\sqrt{3}} = \frac{PT}{\frac{50}{\sqrt{3}}}$$

$$PT = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \frac{50}{3}$$

11.

(a)

Let a be the first term and d be the common difference of given A.P.

Now,

4^{th} term $= 22$

$\Rightarrow a + 3d = 22$ ....(i)

15^{th} term $= 66$

$\Rightarrow a + 14d = 66$ ....(ii)

Subtracting (i) from (ii), we have

$11d = 44$

$\Rightarrow d = 4$

Substituting the value of d in (i), we get

$a = 22 - 3 \times 4 = 22 - 12 = 10$

$\Rightarrow$ First term $= 10$

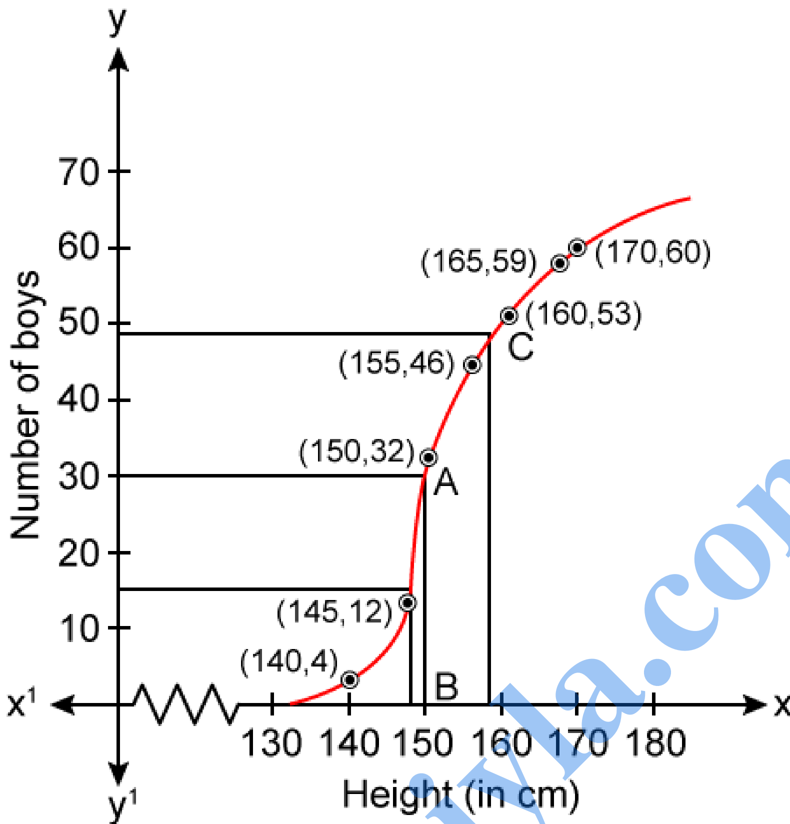
Now,

Sum of 8 terms $= \frac{8}{2}[2 \times 10 + 7 \times 4] = 4[20 + 28] = 4 \times 48 = 192$

(b) The cumulative frequency table of the given distribution table is as follows:

Height in cm	No. of boys (f)	Cumulative frequency
135-140	4	4
140-145	8	12
145-150	20	32
150-155	14	46
155-160	7	53
160-165	6	59
165-170	1	60

Plot the points (140, 4), (145, 12), (150, 32), (155, 46), (160, 53), (165, 59) and (170, 60) on a graph paper and join them to get an ogive.



Number of boys = N = 60

(i) $\text{Median} = \left(\frac{N}{2}\right)^{th} \text{term} = \left(\frac{60}{2}\right)^{th} \text{term} = 30^{th} \text{ term}$

Through mark 30 on the Y-axis, draw a horizontal line which meets the curve at point A. Through point A, on the curve draw a vertical line which meets the X-axis at point B. The value of point B on the X-axis is the median, which is 152.

(ii) $\text{Lower quartile } (Q_1) = \left(\frac{N}{4}\right)^{th} \text{term} = \left(\frac{60}{4}\right)^{th} \text{term} = 15^{th} \text{ term} = 148$

(iii) Through mark of 158 on X-axis, draw a verticle line which meets the graph at point C. Then through point C, draw a horizontal line which meets the Y-axis at the mark of 48. Thus, number of boys in the class who are tall $= 60 - 48 = 12$