ICSE Board
Class X Mathematics
Board Question Paper 2017
(Two and a half hours)

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the Question Paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

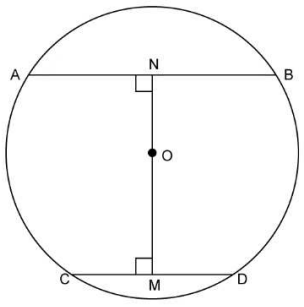
Question 1

(a) If b is the mean proportion between a and c, show that [3]

$$\frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4} = \frac{a^2}{c^2}$$

(b) Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places. [3]

(c) AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm. find the distance between the two chords. [4]



Question 2

(a) Evaluate without using trigonometric tables, [3]

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

(b) [3]

If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find matrix C, where C is a 2 by 2 matrix.

(c) Jaya borrowed Rs. 50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays 33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt. [4]

Question 3

(a) The catalogue price of a computer set is Rs. 42,000. The shopkeeper gives a discount of 10% on the listed price. He further gives an off-season discount of 5% on the discounted price. However, sales tax at 8% is charged on the remaining price after the two successive discounts. Find [3]

(i) the amount of sales tax a customer has to pay

(ii) the total price to be paid by the customer for the computer set.

(b) P(1, -2) is a point on the line segment A(3, -6) and B(x, y) such that AP : PB is equal to 2 : 3. Find the coordinates of B. [3]

(c) The marks of 10 students of a class in an examination arranged in ascending order is as follows: [4]

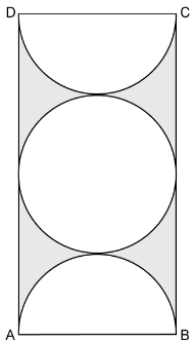
13, 35, 43, x, x + 4, 55, 61, 71, 80

If the median marks is 48, find the value of x. Hence find the mode of the given data.

Question 4

(a) What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor? [3]

(b) In the given figure ABCD is a rectangle. It consists of a circle and two semi-circles each of which are of radius 5 cm. Find the area of the shaded region. Give your answer correct to three significant figures. [3]



(c) Solve the following inequation and represent the solution set on a number line.

$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, x \in I$ [4]

SECTION B (40 Marks)

Attempt any four questions from this section

Question 5

(a)

Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$, find the matrix X if, $X = B^2 - 4B$. [3]

Hence solve for a and b given $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$.

(b) How much should a man invest in Rs. 50 shares selling at Rs. 60 to obtain an income of Rs. 450, if the rate of dividend declared is 10%. Also find his yield percent, to the nearest whole number. [3]

(c) Sixteen cards are labeled as a, b, c, ... m, n, o, p. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is: [4]

(a) a vowel

(b) a consonant

(c) none of the letters of the word median

Question 6

(a) Using a ruler and a compass construct a triangle ABC in which AB = 7 cm, $\angle CAB = 60^o$ and AC = 5 cm. Construct the locus of [3]

(i) points equidistant from AB and AC

(ii) points equidistant from BA and BC

Hence construct a circle touching the three sides of the triangle internally.

(b) A conical tent is to accommodate 77 persons. Each person must have 16 m^3 of air to breathe. Given the radius of the tent as 7 m, find the height of the tent and also its curved surface area. [3]

(c)

If $\frac{7m+2n}{7m-2n}=\frac{5}{3}$, use properties of proportion to find

(i) $m:n$ [4]

(ii) $\frac{m^2+n^2}{m^2-n^2}$

Question 7

(a) A page from a savings bank account passbook is given below: [4]

Date	Particulars	Amount withdrawn (Rs.)	Amount Deposited (Rs.)	Balance (Rs.)
Jan 7, 2016	B/F			3,000.00
Jan 10, 2016	By Cheque		2600.00	5600.00
Feb 8, 2016	To Self	1500.00		4100.00
Apr 6, 2016	By Cheque	2100.00		2000.00
May 4, 2016	By Cash		6500.00	8500.00
May 27, 2016	By Cheque		1500.00	10000.00

(i) Calculate the interest for the 6 months from January to June 2016, at 6% per annum.

(ii) If the account is closed on 1st July 2016, find the amount received by the account holder.

(b) Use a graph paper for this question (Take 2 cms = 1 unit on both x and y axis) [6]

(i) Plot the following points:
A(0, 4), B(2, 3), C(1, 1) and D(2, 0)

(ii) Reflect points B, C, D on the y-axis and write down their coordinates. Name the images as B', C', D' respectively.

(iii) Join the points A, B, C, D, D', C', B' and A in order, so as to form a closed figure. Write down the equation of the line of symmetry of the figure formed.

Question 8

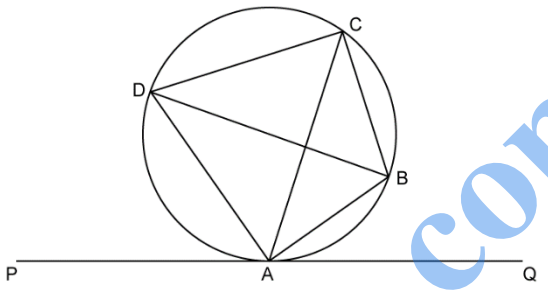
(a) Calculate the mean of the following distribution using step deviation method. [3]

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	10	9	25	30	16	10

(b) In the given figure PQ is a tangent to the circle at A, AB and AD are bisectors of ∠CAQ and ∠PAC. If ∠BAQ = 30°, prove that: [3]

(i) BD is a diameter of the circle

(ii) ABC is an isosceles triangle



(c) The printed price of an air conditioner is Rs. 45000/-. The wholesaler allows a discount of 10% to the shopkeeper. The shopkeeper sells the article to the customer at a discount of 5% of the marked price. Sales tax (under VAT) is charged at the rate of 12% at every stage. Find: [4]

(i) VAT paid by the shopkeeper to the government

(ii) The total amount paid by the customer inclusive of tax.

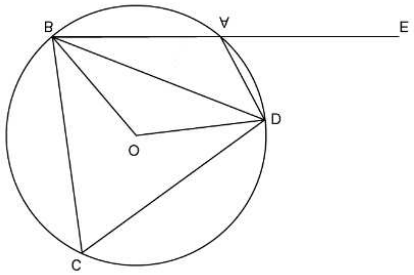
Question 9

(a) In the figure given, O is the centre of the circle. ∠DAE = 70°, Find giving suitable reasons the measure of: [3]

(i) ∠BCD

(ii) ∠BOD

(iii) ∠OBD



(b) A(-1, 3), B(4, 2) and C(3, -2) are the vertices of a triangle. [3]

(i) Find the coordinates of the centroid G of the triangle

(ii) Find the equation of the line through G and parallel to AC

(c) Prove that [4]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

Question 10

(a) The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages. [4]

(b) The daily wages of 80 workers in a project are given below. [6]

Wages (in Rs.)	400-450	450-500	500-550	550-600	600-650	650-700	700-750
No. of Workers	2	6	12	18	24	13	5

Use a graph paper to draw an ogive for the above distribution. (Use a scale of 2 cm = Rs. 50 on x-axis and 2 cm = 10 workers on y-axis). Use your ogive to estimate:

(i) the median wage of the workers

(ii) the lower quartile wage of workers

(iii) the numbers of workers who earn more than Rs. 625 daily

Question 11

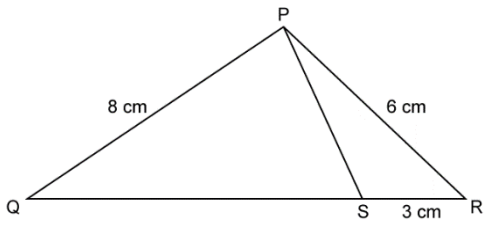
(a) The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60^o and 45^o respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships, Give your answer correct to the nearest whole number. [3]

(b) PQR is a triangle. S is a point on the side QR of ΔPQR such that $\angle PSR = \angle QPR$. Given QP = 8 cm, PR = 6 cm and SR = 3 cm [3]

(i) Prove $\Delta PQR \sim \Delta SPR$

(ii) Find the length of QR and PS

(iii) $\frac{\text{area of } \Delta PQR}{\text{area of } \Delta SPR}$



(c) Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p. a. simple interest. If he gets Rs. 8325 as interest at the time of maturity, find [4]

(i) The monthly deposit

(ii) The maturity value

Solution

SECTION A

1.

(a)

Given, b is the mean proportion between a and c.

$\Rightarrow \frac{a}{b} = \frac{b}{c} = k(\text{say})$

$\Rightarrow a = bk,\ b = ck$

$\Rightarrow a = (ck)k = ck^2, b = ck$

$$\text{L.H.S.} = \frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$$

$$= \frac{(ck^2)^4 + (ck^2)^2(ck)^2 + (ck)^4}{(ck)^4 + (ck)^2c^2 + c^4}$$

$$= \frac{c^4k^8 + (c^2k^4)(c^2k^2) + c^4k^4}{c^4k^4 + (c^2k^2)c^2 + c^4}$$

$$= \frac{c^4k^8 + c^4k^6 + c^4k^4}{c^4k^4 + c^4k^2 + c^4}$$

$$= \frac{c^4k^4(k^4 + k^2 + 1)}{c^4(k^4 + k^2 + 1)}$$

$$= k^4$$

$$\text{R.H.S.} = \frac{a^2}{c^2}$$

$$= \frac{(ck^2)^2}{c^2}$$

$$= \frac{c^2k^4}{c^2}$$

$$= k^4$$

Hence, L.H.S. = R.H.S.

(b)

Given equation is $4x^2 - 5x - 3 = 0$.

Comparing with $ax^2 + bx + c = 0$, we get

$a = 4, b = -5$ and $c = -3$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25 + 48}}{8}$$

$$= \frac{5 \pm \sqrt{73}}{8}$$

$$= \frac{5 \pm 8.54}{8}$$

$$= \frac{13.54}{8} \text{ or } \frac{-3.54}{8}$$

$$= 1.6925 \text{ or } -0.4425$$

$$= 1.69 \text{ or } -0.44$$

(c)

Join OA and OC.

Since the perpendicular from the centre of the circle to a chord bisects the chord.

Therefore, N and M are the mid-points of AB and CD respectively.

Consequently,

$$AN = NB = \frac{1}{2}AB = \frac{1}{2} \times 24 = 12 \text{ cm and}$$

$$CM = MD = \frac{1}{2}CD = \frac{1}{2} \times 10 = 5 \text{ cm}$$

In right-angled triangles ANO and CMO, we have

$OA^2 = ON^2 + AN^2$ and $OC^2 = OM^2 + CM^2$

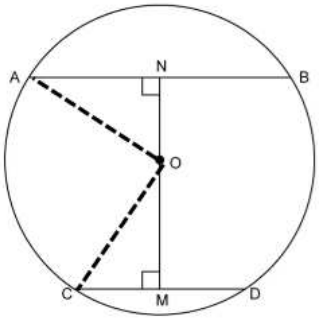
$\Rightarrow 13^2 = ON^2 + 12^2$ and $13^2 = OM^2 + 5^2$

$\Rightarrow ON^2 = 13^2 - 12^2$ and $OM^2 = 13^2 - 5^2$

$\Rightarrow ON^2 = 169 - 144$ and $OM^2 = 169 - 25$

$\Rightarrow ON^2 = 25$ and $OM^2 = 144$

$\Rightarrow ON = 5$ and $OM = 12$



Now, $NM = ON + OM = 5 + 12 = 17$ cm

Hence, the distance between the two chords is 17 cm.

2.

(a)

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

$$= \sin^2 28° + \sin^2(90° - 28°) + \tan^2 38° - \cot^2(90° - 38°) + \frac{1}{4}\sec^2 30°$$

$$= \left(\sin^2 28° + \cos^2 28°\right) + \tan^2 38° - \tan^2 38° + \frac{1}{4} \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 1 + 0 + \frac{1}{4} \times \frac{4}{3}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$

(b)

$$\text{Given}: A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix},\ B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \text{ and } A^2 - 5B^2 = 5C$$

$$\text{Now, } A^2 = A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 3\times3 & 1\times3 + 3\times4 \\ 3\times1 + 4\times3 & 3\times3 + 4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$\text{And, } B^2 = B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times(-2) + 1\times(-3) & -2\times1 + 1\times2 \\ -3\times(-2) + 2\times(-3) & -3\times1 + 2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 - 5B^2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5C$$

$$\text{Hence, } C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

(c)

For 1st year:

$P = \text{Rs. } 50{,}000;\ R = 12\%$ and $T = 1$ year

$\therefore \text{Interest} = \text{Rs. } \frac{50{,}000 \times 12 \times 1}{100} = \text{Rs. } 6{,}000$

And, $\text{Amount} = \text{Rs. } 50{,}000 + \text{Rs. } 6{,}000 = \text{Rs. } 56{,}000$

$\text{Since Money repaid} = \text{Rs. } 33{,}000$

$\therefore \text{Balance} = \text{Rs. } 56{,}000 - \text{Rs. } 33{,}000 = \text{Rs. } 23{,}000$

For 2nd year:

$P = \text{Rs. } 23{,}000;\ R = 15\%$ and $T = 1$ year

$\therefore \text{Interest} = \text{Rs. } \frac{23{,}000 \times 15 \times 1}{100} = \text{Rs. } 3{,}450$

And, $\text{Amount} = \text{Rs. } 23{,}000 + \text{Rs. } 3{,}450 = \text{Rs. } 26{,}450$

Thus, Jaya must pay Rs. 26,450 at the end of 2nd year to clear her debt.

3.

(a)

$\text{List price} = \text{Rs.}42{,}000$

$\text{Discount} = 10\% \text{ of Rs. } 42{,}000$

$= \frac{10}{100} \times \text{Rs. } 42{,}000$

$= \text{Rs. } 4{,}200$

$\Rightarrow \text{Discounted price} = \text{Rs. } 42{,}000 - \text{Rs. } 4{,}200 = \text{Rs. } 37{,}800$

$\text{Off-season discount} = 5\% \text{ of Rs. } 37{,}800$

$= \frac{5}{100} \times \text{Rs. } 37{,}800$

$= \text{Rs. } 1{,}890$

$\therefore \text{Sale-price} = \text{Rs. } 37{,}800 - \text{Rs. } 1{,}890 = \text{Rs. } 35{,}910$

(i) The amount of sales tax a customer has to pay $= 8\% \text{ of Rs. } 35{,}910$

$= \frac{8}{100} \times \text{Rs. } 35{,}910$

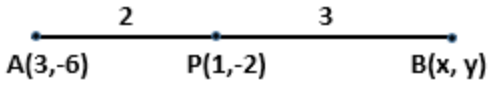
$= \text{Rs. } 2872.80$

(ii) The total price, a customer has to pay for the computer $= \text{Sale-price} + \text{Sales Tax}$

$= \text{Rs. } 35{,}910 + \text{Rs. } 2872.80$

$= \text{Rs. } 38782.80$

(b)



Given, $P(1,-2)$, $A(3,-6)$ and $B(x,y)$

$AP:PB=2:3$

Hence, coordinates of $P=\left(\frac{2\times x+3\times 3}{2+3},\frac{2\times y+3\times(-6)}{2+3}\right)=\left(\frac{2x+9}{5},\frac{2y-18}{5}\right)$

But, the coordinates of P are $(1,-2)$.

$\therefore \frac{2x+9}{5}=1$ and $\frac{2y-18}{5}=-2$

$\Rightarrow 2x+9=5$ and $2y-18=-10$

$\Rightarrow 2x=-4$ and $2y=8$

$\Rightarrow x=-2$ and $y=4$

Hence, the coordinates of B are $(-2,4)$.

(c)

Data in ascending order:

$13,\ 35,\ 43,\ 46,\ x,\ x+4,\ 55,\ 61,\ 71,\ 80$

Median $=48$

Number of observations $=n=10$ (even)

$\therefore \text{Median}=\frac{\left(\frac{n}{2}\right)^{th}\text{ term}+\left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$

$\Rightarrow 48=\frac{\left(\frac{10}{2}\right)^{th}\text{ term}+\left(\frac{10}{2}+1\right)^{th}\text{ term}}{2}$

$\Rightarrow 48=\frac{5^{th}\text{ term}+6^{th}\text{ term}}{2}$

$\Rightarrow 48=\frac{x+x+4}{2}$

$\Rightarrow 48=\frac{2x+4}{2}$

$\Rightarrow 48=x+2$

$\Rightarrow x=46$

$\Rightarrow x+4=46+4=50$

Thus, the observations are $13,\ 35,\ 43,\ 46,\ 46,\ 50,\ 55,\ 61,\ 71,\ 80$

Observation 46 is appearing twice.

Hence, the mode of the data is 46.

4.

(a)

Let the number to be subtracted from the given polynomial be k.

Let $f(y) = 16x^3 - 8x^2 + 4x + 7 - k$

It is given that $(2x+1)$ is a factor of $f(y)$.

$\therefore f\left(-\frac{1}{2}\right) = 0$

$\Rightarrow 16\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - k = 0$

$\Rightarrow 16 \times \left(-\frac{1}{8}\right) - 8 \times \frac{1}{4} - 2 + 7 - k = 0$

$\Rightarrow -2 - 2 - 2 + 7 - k = 0$

$\Rightarrow 1 - k = 0$

$\Rightarrow k = 1$

Thus, 1 should be subtracted from the given polynomial.

(b)

Length of a rectangle = Radius of two semi-circles + Diameter of a circle

$= 5 + 5 + 10$

$= 20$ cm

Breadth of a rectangle = Diameter of a circle $= 2 \times 5 = 10$ cm

$\therefore$ Area of a rectangle = Length $\times$ Breadth

$= 20 \times 10$

$= 200$ sq. cm

Area of a circle $= \frac{22}{7} \times 5 \times 5 = 78.571$ sq. cm

And, area of two semi-circles each of radius 5 cm $= 2\left(\frac{1}{2} \times 78.571\right) = 78.571$ sq. cm

Now,

Area of shaded region = Area of a rectangle – Area of a circle – Area of two semi-circles

$= 200 - 78.571 - 78.571$

$= 200 - 157.142$

$= 42.858$ sq. cm

(c)

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$$

$$\Rightarrow -\frac{17}{2} < -\frac{1}{2} - 4x \le \frac{15}{2},\ x \in I$$

Take $-\frac{17}{2} < -\frac{1}{2} - 4x$ and $-\frac{1}{2} - 4x \le \frac{15}{2}$

$-\frac{17}{2} + \frac{1}{2} < -4x$ and $-4x \le \frac{15}{2} + \frac{1}{2}$

$-\frac{16}{2} < -4x$ and $-4x \le \frac{16}{2}$

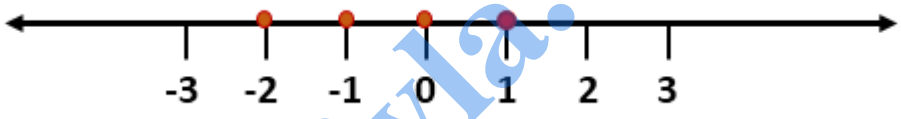
$-8 < -4x$ and $-4x \le 8$

$2 > x$ and $x \ge -2$

Thus, on simplifying, the given inequation reduces to $-2 \le x < 2$.

Since $x \in I$, the solution set is $\{-2, -1, 0, 1\}$.

The required graph on number line is as follows:



SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

$$\text{Given: } B=\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix} \text{ and } X=B^2-4B$$

$$\text{Now, } B^2=B\times B$$

$$=\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}\times\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+1\times8 & 1\times1+1\times3\\ 8\times1+3\times8 & 8\times1+3\times3\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 1+3\\ 8+24 & 8+9\end{bmatrix}$$

$$=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}$$

$$X=B^2-4B=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}-4\begin{bmatrix}1 & 1\\ 8 & 3\end{bmatrix}=\begin{bmatrix}9 & 4\\ 32 & 17\end{bmatrix}-\begin{bmatrix}4 & 4\\ 32 & 12\end{bmatrix}=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

$$\text{Now, } X\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5a+0b\\ 0a+5b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5a\\ 5b\end{bmatrix}=\begin{bmatrix}5\\ 50\end{bmatrix}$$

$$\Rightarrow 5a=5 \text{ and } 5b=50$$

$$\Rightarrow a=1 \text{ and } b=10$$

(b)

$$\text{Since Dividend on 1 share} = 10\% \text{ of Rs. } 50 = \frac{10}{100}\times \text{Rs. } 50 = \text{Rs. } 5$$

$$\therefore \text{Number of shares bought} = \frac{\text{Total dividend}}{\text{Dividend on 1 share}} = \frac{\text{Rs. } 450}{\text{Rs. } 5} = 90$$

$$\text{Since market value of each share} = \text{Rs. } 60$$

$$\therefore \text{Sum invested by the man} = 90\times \text{Rs. } 60 = \text{Rs. } 5,400$$

$$\text{Percentage return} = \frac{\text{Total return}}{\text{Sum invested}}\times 100\% = \frac{\text{Rs. } 450}{\text{Rs. } 5400}\times 100\% = 8.33\% = 8\%$$

(c)

Outcomes: a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p

Total number of all possible outcomes $= 16$

(i) When the selected card has a vowel,

the possible outcomes are a, e, i, o.

Number of favourable outcomes $= 4$

$\therefore$ Required probability $= \frac{4}{16} = \frac{1}{4}$

(ii) When the selected card has a consonant,

Number of favourable outcomes $= 16 - 4 = 12$

$\therefore$ Required probability $= \frac{12}{16} = \frac{3}{4}$

(iii) When the selected card has none of the letters from the word median,

the possible outcomes are b, c, f, g, h, j, k, l, o, p.

Number of favourable outcomes $= 10$

$\therefore$ Required probability $= \frac{10}{16} = \frac{5}{8}$

6.

(a) **Steps of construction:**

(i) Draw line AC = 5 cm and $\angle CAB = 60^\circ$. Cut off AB = 7 cm. Join BC, ΔABC is the required triangle.

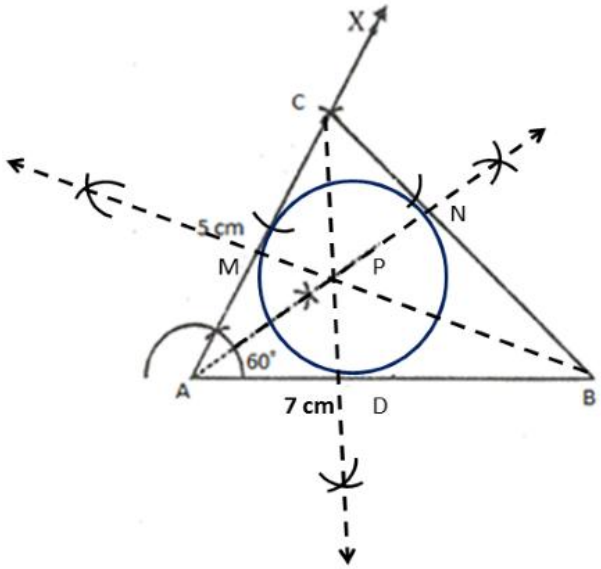
(ii) Draw angle bisectors of $\angle A$ and $\angle B$.

(iii) Bisector of $\angle B$ meets AC at M and bisector of $\angle A$ meets BC at N.

(iv) Similarly, draw the angle bisector of $\angle C$ which meets AB at D.

(v) P is the point which is equidistant from AB, BC and AC.

(vi) With DP as the radius, draw a circle touching the three sides of the triangle (incircle.)



(b)

Let h be the height and r be the radius of the base of the conical tent.

According to the given information,

$$77\times16=\frac{1}{3}\pi r^2h$$

$$\Rightarrow 77\times16=\frac{1}{3}\times\frac{22}{7}\times7\times7\times h$$

$$\Rightarrow 77\times16=\frac{1}{3}\times22\times7\times h$$

$$\Rightarrow h=\frac{77\times16\times3}{22\times7}\Rightarrow h=24\text{ m}$$

Now, $l^2=r^2+h^2$

$$\Rightarrow l^2=7^2+24^2=625$$

$$\Rightarrow l=25\text{ m}$$

$$\therefore \text{Curved surface area}=\pi rl=\frac{22}{7}\times7\times25=550\text{ m}^2$$

Hence, the height of the tent is 24 m and the curved surface area of the tent is 550 m^2.

(c)

(i) $\frac{7m+2n}{7m-2n}=\frac{5}{3}$

By Componendo – Divinendo, we get

$$\frac{7m+2n+(7m-2n)}{7m+2n-(7m-2n)}=\frac{5+3}{5-3}$$

$$\Rightarrow\frac{14m}{4n}=\frac{8}{2}$$

$$\Rightarrow\frac{7m}{2n}=\frac{4}{1}$$

$$\Rightarrow\frac{m}{n}=\frac{8}{7}$$

$$\Rightarrow m:n=8:7$$

(ii) $\frac{m}{n}=\frac{8}{7}\Rightarrow\frac{m^2}{n^2}=\frac{8^2}{7^2}$

Applying Componendo – Divinendo, we get

$$\Rightarrow\frac{m^2+n^2}{m^2-n^2}=\frac{8^2+7^2}{8^2-7^2}$$

$$\Rightarrow\frac{m^2+n^2}{m^2-n^2}=\frac{64+49}{64-49}$$

$$\Rightarrow\frac{m^2+n^2}{m^2-n^2}=\frac{113}{15}$$

7.

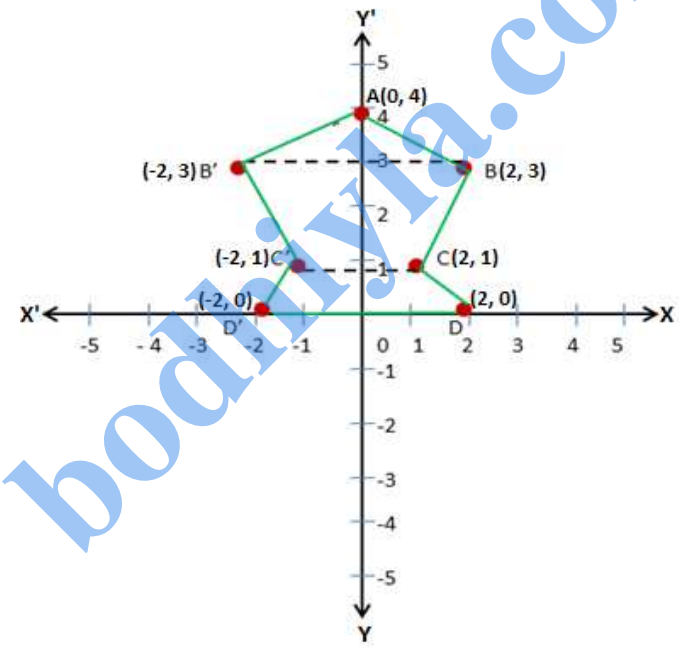
(a) Principal for the month of Jan = Rs. 5600
Principal for the month of Feb = Rs. 4100
Principal for the month of Mar = Rs. 4100
Principal for the month of Apr = Rs. 2000
Principal for the month of May = Rs. 8500
Principal for the month of June = Rs. 10000
Total Principal for one month = Rs. 34300

Rate of interest = 6% pa

(i) Simple interest $= \frac{PRT}{100} = \frac{34300 \times 6 \times 1}{100 \times 12} = \text{Rs.}171.50$

(ii) Total amount $= \text{Rs.}10000 + \text{Rs.}171.50 = \text{Rs.}10171.50$

(b)



The image of point (x, y) on Y-axis has the coordinates (–x, y).
Thus, we have
Coordinates of B' = (–2, 3)
Coordinates of C' = (–1, 1)
Coordinates of D' = (–2, 0)

Since, Y-axis is the line of symmetry of the figure formed, the equation of the line of symmetry is x = 0.

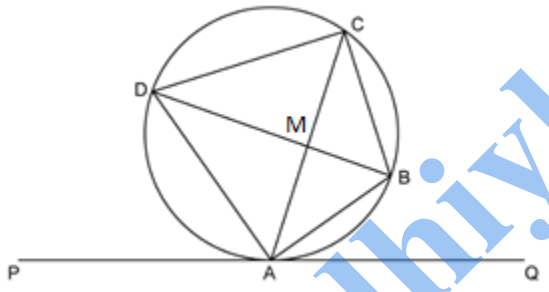
8.

(a) Let the assumed mean A = 25

Marks	Mid-value x	f	$d = x - A$	$t = \frac{x-A}{i} = \frac{x-25}{10}$	ft
$0-10$	5	10	-20	-2	-20
$10-20$	15	9	-10	-1	-9
$20-30$	25	25	0	0	0
$30-40$	35	30	10	1	30
$40-50$	45	16	20	2	32
$50-60$	55	10	30	3	30
		$\sum f = 100$			$\sum ft = 63$

$$\therefore \text{Mean} = A + \frac{\sum ft}{\sum f} \times i = 25 + \frac{63}{100} \times 10 = 25 + \frac{63}{10} = 25 + 6.3 = 31.3$$

(b)



(i) $\angle BAQ = 30°$

Since AB is the bisector of $\angle CAQ$

$\Rightarrow \angle CAB = \angle BAQ = 30°$

AD is the bisector of $\angle CAP$ and $P - A - Q$,

$\angle DAP + \angle CAD + \angle CAQ = 180°$

$\Rightarrow \angle CAD + \angle CAD + 60° = 180°$

$\Rightarrow \angle CAD = 60°$

So, $\angle CAD + \angle CAB = 60° + 30° = 90°$

Since angle in a semi-circle $= 90°$

$\Rightarrow$ Angle made by diameter to any point on the circle is $90°$

So, BD is the diameter of the circle.

(ii) Since BD is the diameter of the circle, so it will pass through the centre.

By Alternate segment theorem,

$\angle ABD = \angle DAP = 60°$

So, in ΔBMA,

$\angle AMB = 90°$ ....(Use Angle Sum Property)

We know that perpendicular drawn from the centre to a chord of a circle bisects the chord.

$\Rightarrow \angle BMA = \angle BMC = 90°$

In ΔBMA and ΔBMC,

$\angle BMA = \angle BMC = 90°$

$BM = BM$ (common side)

$AM = CM$ (perpendicular drawn from the centre to a chord of a circle bisects the chord.)

$\Rightarrow \Delta BMA \cong \Delta BMC$

$\Rightarrow AB = BC$ (SAS congruence criterion)

$\Rightarrow \Delta ABC$ is an isosceles triangle.

(c)

(i) Printed price of an air conditioner = Rs. 45000

Discount = 10%

$\therefore$ C.P. of the air conditioner = Rs. $\frac{45000 \times (100-10)}{100}$

$= \text{Rs.} \frac{45000 \times 90}{100}$

$= \text{Rs.}\, 40500$

VAT (12%) $= 40500 \times \frac{12}{100} = \text{Rs.}\, 4860$

So, the shopkeeper paid VAT of Rs. 4860 to the government.

(ii) Discount = 5% of the marked price

$\therefore$ C.P. of the air conditioner = Rs. $\frac{45000 \times (100-5)}{100}$

$= \text{Rs.} \frac{45000 \times 95}{100}$

$= \text{Rs.}\, 42750$

VAT (12%) $= 42750 \times \frac{12}{100} = \text{Rs.}\, 5130$

So, the total amount paid by the customer inclusive of tax

$= \text{Rs.}\, 42750 + \text{Rs.}\, 5130$

$= \text{Rs.}\, 47880$

9.

(a)

(i) $\angle DAE = 70°$ ....(given)

$\angle BAD + \angle DAE = 180°$ ....(linear pair)

$\Rightarrow \angle BAD + 70° = 180°$

$\Rightarrow \angle BAD = 110°$

Since ABCD is a cyclic quadrilateral, sum of the measures of the opposite angles are supplementary.

So, $\angle BCD + \angle BAD = 180°$

$\Rightarrow \angle BCD + 110° = 180°$

$\Rightarrow \angle BCD = 70°$

(ii) $\angle BOD = 2\angle BCD$ (Inscribed angle theorem)

$\Rightarrow \angle BOD = 2(70°) = 140°$

(iii) In ΔOBD,

$OB = OD$ ....(radii of same circle)

$\Rightarrow \angle OBD = \angle ODB$

By Angle Sum property,

$\angle OBD + \angle ODB + \angle BOD = 180°$

$\Rightarrow 2\angle OBD + \angle BOD = 180°$

$\Rightarrow 2\angle OBD + 140° = 180°$

$\Rightarrow 2\angle OBD = 40°$

$\Rightarrow \angle OBD = 20°$

(b)

Given vertices: A(−1, 3), B(4, 2) and C(3, −2)

(i) Coordinates of the centroid G of ΔABC are given by

$$G = \left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$

(ii) Since the line through G is parallel to AC, the slope of the lines are the same.

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$$

So, equation of the line passing through G(2, 1) and with slo pe $\frac{-5}{4}$ is given by,

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = \frac{-5}{4}(x - 2)$

$\Rightarrow 4y - 4 = -5x + 10$

$\Rightarrow 5x + 4y = 14$ is the required equation.

(c)

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta\,[2(1-\sin^2\theta)-1]}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}$$

$$= \tan\theta$$

$$= \text{R.H.S.} \quad \text{(proved)}$$

10.

(a)

Let Vivek's age be x years and Amit's age be (47-x) years.

According to the given information,

$x(47-x) = 550$

$\Rightarrow 47x - x^2 = 550$

$\Rightarrow x^2 - 47x + 550 = 0$

$\Rightarrow (x-25)(x-22) = 0$

$\Rightarrow x = 25 \text{ or } x = 22$

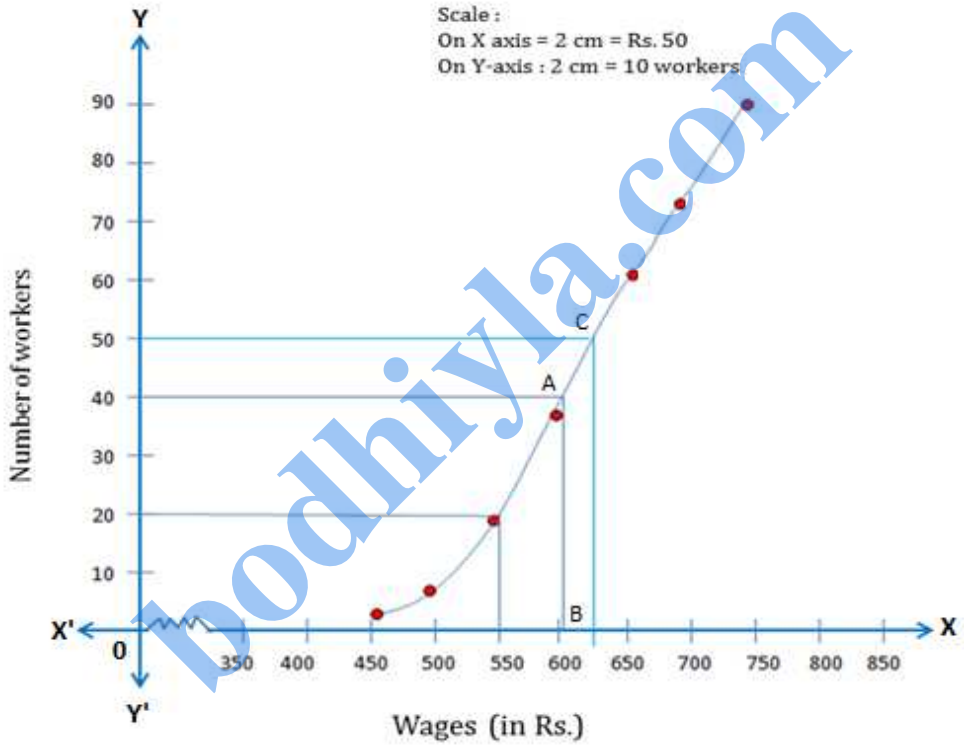
So, Vivek's age is 25 years and Amit's age is 22 years.

(b)

The cumulative frequency table of the given distribution is as follows:

Wages in Rs.	Upper Limit	No. of workers	Cumulative frequency
400-450	450	2	2
450-500	500	6	8
500-550	550	12	20
550-600	600	18	38
600-650	650	24	62
650-700	700	13	75
700-750	750	5	80

The ogive is as follows:



Number of workers = n = 80

(i) Median $= \left(\frac{n}{2}\right)^{th}$ term $= 40^{th}$ term

Through mark 40 on the Y-axis, draw a horizontal line which meets the curve at point A.
Through point A, on the curve draw a vertical line which meets the X-axis at point B.
The value of point B on the X-axis is the median, which is 605.

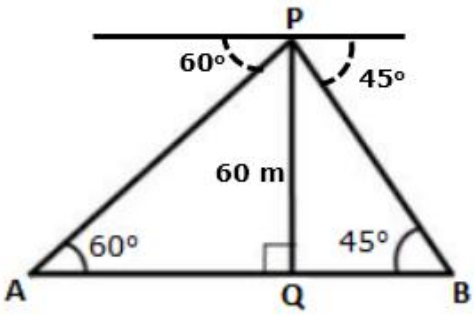
(ii) Lower quartile $(Q_1) = \left(\frac{80}{4}\right)^{th}$ term $= 20^{th}$ term $= 550$

(ii) Through mark of 625 on X-axis, draw a verticle line which meets the graph at point C.
Then through point C, draw a horizontal line which meets the Y-axis at the mark of 50.
Thus, number of workers that earn more than Rs. 625 daily $= 80 - 50 = 30$

11.

(a)

Let PQ be the lighthouse.



$\Rightarrow PQ = 60$

In ΔPQA,

$\tan 60° = \frac{PQ}{AQ}$

$\Rightarrow \sqrt{3} = \frac{60}{AQ}$

$\Rightarrow AQ = \frac{60}{\sqrt{3}}$

$\Rightarrow AQ = \frac{20 \times 3}{\sqrt{3}}$

$\Rightarrow AQ = \frac{20 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}}$

$\Rightarrow AQ = 20\sqrt{3}$ m

In ΔPQB,

$\tan 45° = \frac{PQ}{QB}$

$\Rightarrow 1 = \frac{60}{QB}$

$\Rightarrow QB = 60$ m

Now,

$AB = AQ + QB$

$= 20\sqrt{3} + 60$

$= 20 \times 1.732 + 60$

$= 94.64$

$= 95$ m

(b)

(i) In ΔPQR and ΔSPR, we have

$\angle QPR = \angle PSR$ ....(given)

$\angle PRQ = \angle PRS$ ....(common)

So, by AA-axiom similarity, we have

$\Delta PQR \sim \Delta SPR$ ....(proved)

(ii) Since $\Delta PQR \sim \Delta SPR$ ....(proved)

$$\Rightarrow \frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR}$$

Consider $\frac{QR}{PR} = \frac{PR}{SR}$ ....[From (1)]

$$\Rightarrow \frac{QR}{6} = \frac{6}{3}$$

$$\Rightarrow QR = \frac{6 \times 6}{3} = 12 \text{ cm}$$

Also, $\frac{PQ}{SP} = \frac{PR}{SR}$

$$\Rightarrow \frac{8}{SP} = \frac{6}{3}$$

$$\Rightarrow \frac{8}{SP} = 2$$

$$\Rightarrow SP = \frac{8}{2} = 4 \text{ cm}$$

(iii) $\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta SPR} = \frac{PQ^2}{SP^2} = \frac{8^2}{4^2} = \frac{64}{16} = 4$

(c)

(i) Let the deposit per month = Rs. P

Number of months (n) = 36

Rate of interest (r) = 7.5% p.a.

$$\therefore \text{S.I.} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 8325 = P \times \frac{36 \times 37}{2 \times 12} \times \frac{7.5}{100}$$

$$\Rightarrow 8325 = P \times \frac{3 \times 37}{2} \times \frac{7.5}{100}$$

$$\Rightarrow P = \frac{8325 \times 2 \times 100}{3 \times 37 \times 7.5} = \text{Rs. } 2000$$

(ii) Maturity value $= P \times n + \text{S.I.} = \text{Rs.}(2000 \times 36 + 8325) = \text{Rs. } 80{,}325$