ICSE Board
Class X Mathematics
Board Question Paper 2016
(Two and a half hours)

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the Question Paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$, leaves a remainder 7.   [3]

(b) Given $A=\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + ml$. Find m.   [4]

(c) The mean of following numbers is 68. Find the value of 'x'.
45, 52, 60, x, 69, 70, 26, 81 and 94
Hence estimate the median.   [3]

Question 2

(a) The slope of a line joining P(6, k) and Q (1− 3k, 3) is $\frac{1}{2}$. Find

   (i)   k
   (ii)   Midpoint of PQ, using the value of 'k' found in (i).   [3]

(b) Without using trigonometrical tables, evaluate:

$$\csc^2 57^o - \tan^2 33^o + \cos 44^o \csc 46^o - \sqrt{2}\cos 45^o - \tan^2 60^o \qquad [4]$$

(c) A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted and recast into a solid sphere of radius 6 cm. Find the number of cones. [3]

Question 3

(a) Solve the following inequation, write the solution set and represent it on the number line. [3]
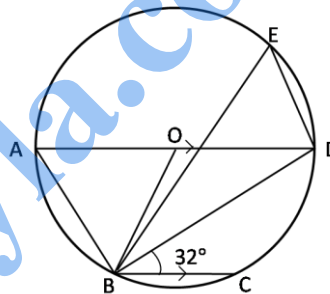
$$-3(x-7) \geq 15 - 7x > \frac{x+1}{3}, x \notin R$$

(b) In the figure given below, AD is a diameter. O is the centre of the circle. AD is parallel to BC and $\angle CBD = 32^o$. Find:

(i) $\angle OBD$ [4]

(ii) $\angle AOB$

(iii) $\angle BED$

(c) If $(3a + 2b) : (5a + 3b) = 18 : 29$. Find $a : b$. [3]

Question 4

(a) A game of numbers has cards marked with 11, 12, 13, ... 40. A card is drawn at random. Find the probability that the number on the card drawn is :

(i)   A perfect square

(ii) Divisible by 7 [3]

(b) Use graph paper for this question. (Take 2 cm = 1 unit along both x and y axis.) Plot the points O(0, 0), A(−4, 4), B(−3, 0) and C(0, −3)

(i)   Reflect points A and B on the y-axis and name them A' and B' respectively. Write down their coordinates.

(ii) Name the figure OABCB'A'.

(iii) State the line of symmetry of this figure [4]

(c)   Mr. Lalit invested Rs. 5000 at a certain rate of interest, compounded annually for two years. At the end of first year it amounts to Rs. 5325. Calculate

(i)   The rate of interest

(ii) The amount at the end of second year, to the nearest rupee.   [3]

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a)   Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct two significant figures.   [3]

(b)   A page from the savings bank account of Mrs. Ravi is given below.

Date	Particulars	Withdrawal (Rs.)	Deposit (Rs.)	Balance (Rs.)
April 3rd 2006	B/F			6000
April 7th	By cash		2300	8300
April 15th	By cheque		3500	11800
May 20th	To self	4200		7600
June 10th	By cash		5800	13400
June 15th	To self	3100		10300
August 13th	By cheque		1000	11300
August 25th	To self	7400		3900
September 6th 2006	By cash		2000	5900

She closed the account on 30th September, 2006. Calculate the interest Mrs. Ravi earned at the end of 30th September, 2006 at 4.5% per annum interest. Hence, find the amount she receives on closing the account.   [4]

(c)   In what time will Rs. 1500 yield Rs. 496.50 as compound interest at 10% per annum compounded annually?   [3]

Question 6

(a) Construct a regular hexagon of side 5 cm. Hence construct all its lines of symmetry and name them. [3]

(b) In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T. [4]

    (i) Prove ΔTPS ~ ΔTRQ.

    (ii) Find SP if TP = 18 cm, RQ = 4 cm and TR = 6 cm.

    (iii) Find area of quadrilateral PQRS if area of ΔPTS = 27 cm^2.
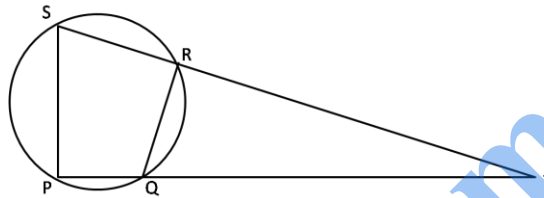
(c) Given matrix $A = \begin{bmatrix} 4\sin 30^o & \cos 0^o \\ \cos 0^o & 4\sin 30^o \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

    If AX = B [3]

    (i) Write the order of matrix X.

    (ii) Find the matrix 'X'.

Question 7

(a) An aeroplane at an altitude of 1500 metres, finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45° and 30° respectively. Find the distance between the two ships. [4]

(b) The table shows the distribution of the scores obtained by 160 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution. (Take 2 cm = 10 scores on the X-axis and 2 cm = 20 shooters on the Y-axis). [6]

Scores	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of shooters	9	13	20	26	30	22	15	10	8	7

Use your graph to estimate the following:

(i) The median.

(ii) The interquartile range.

(iii) The number of shooters who obtained a score of more than 85%.

Question 8

(a) If $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$, show that $\dfrac{x^3}{a^3} + \dfrac{y^3}{b^3} + \dfrac{z^3}{c^3} = \dfrac{3xyz}{abc}$ [3]

(b) Draw a line AB = 5 cm. Mark a point C on AB such that AC = 3 cm. Using a ruler and a compass only, construct :
  (i) A circle of radius 2.5 cm, passing through A and C.
  (ii) Construct two tangents to the circle from the external point B. Measure and record the length of the tangents. [4]

(c) A line AB meets X – axis at A and Y –axis at B. P (4, -1) divides AB in the ratio 1 : 2. [3]
  (i) Find the coordinates of A and B.
  (ii) Find the equation of the line through P and perpendicular to AB.
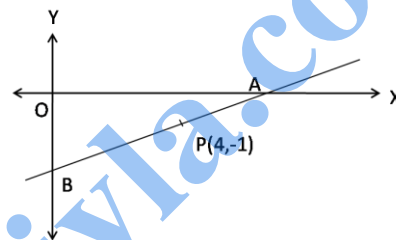
Question 9

(a) A dealer buys an article at a discount of 30% from the wholesaler, the marked price being Rs. 6000. The dealer sells it to a shopkeeper at a discount of 10% on the marked price. If the rate of VAT is 6%, find [3]
 (i) The price paid by the shopkeeper including the tax.
  (ii) The VAT paid by the dealer.

(b) The given figure represents a kite with a circular and a semicircular motifs stuck on it. The radius of a circle is 2.5 cm and the semicircle is 2 cm. If diagonals AC and BD are of lengths 12 cm and 8 cm respectively, find the area of the: [4]
  (i) Shaded part. Give your answer correct to the nearest whole number.
 (ii) Unshaded part.
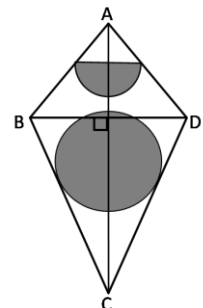
(c) A model of a ship is made to a scale 1 : 300 [3]
   (i) The length of the model of ship is 2 m. Calculate the lengths of the ship.
   (ii) The area of the deck ship is 180,000 m². Calculate the area of the deck of the model.
   (iii) The volume of the model in 6.5 m³. Calculate the volume of the ship.

Question 10

(a) Mohan has a recurring deposit account in a bank for 2 years at 6% p.a. simple interest. If he gets Rs. 1200 as interest at the time of maturity,
   find: [3]
   (i) the monthly installment
   (ii) the amount of maturity

(b) The histogram below represents the scores obtained by 25 students in a mathematics mental test. Use the data to : [4]
   (i) Frame a frequency distribution table
   (ii) To calculate mean
   (iii) To determine the Modal class
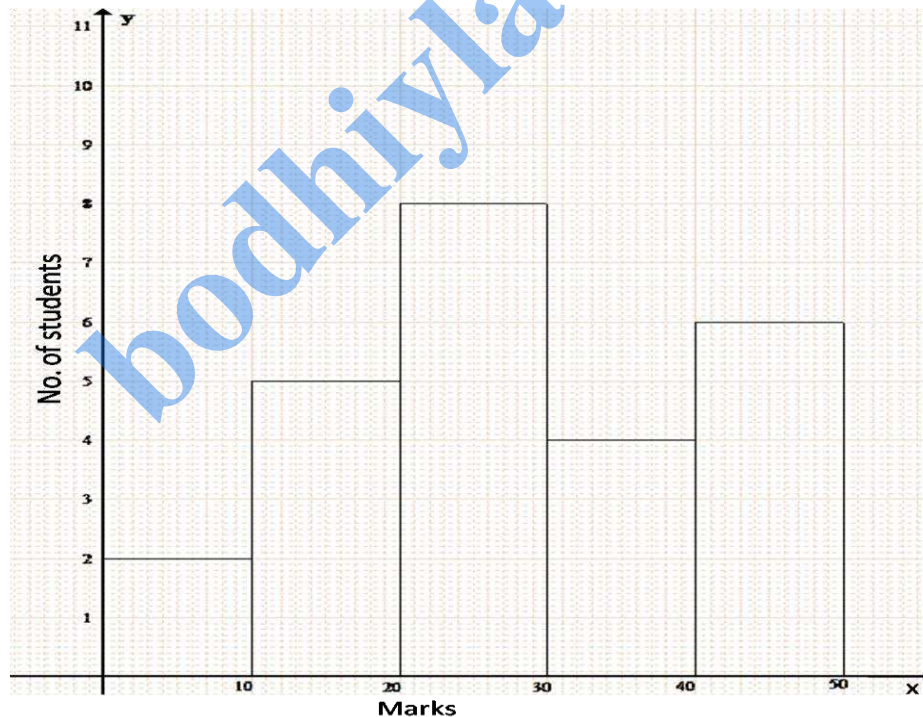
(c) A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/h and as such it takes two hrs longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate 'x'. [3]

Question 11

(a) Prove that $\dfrac{\cos A}{1 + \sin A} + \tan A = \sec A.$ [3]

(b) Use ruler and compasses only for the following questions. All constructions lines and arcs must be clearly shown. [4]
   (i) Construct a $\triangle$ ABC in which BC = 6.5 cm, $\angle$ ABC = 60°, AB = 5 cm.
   (ii) Construct the locus of points at a distance of 3.5 cm from A.
   (iii) Construct the locus of points equidistant from AC and BC.
   (iv) Mark 2 points X and Y which are a distance of 3.5 cm from A and also equidistant from AC and BC. Measure XY.

(c) Ashok invested Rs. 26,400 on 12%, Rs. 25 shares of a company. If he receives a dividend of Rs. 2,475. Find the : [3]
   (i) number of shares he bought
   (ii) Market value of each share

Solution

1. (a) Let $f(x) = 2x^3 + 3x^2 - kx + 5$

Using remainder theorem,

$f(2) = 7$

$\therefore 2(2)^3 + 3(2)^2 - k(2) + 5 = 7$

$\therefore 2(8) + 3(4) - k(2) + 5 = 7$

$\therefore 16 + 12 - 2k + 5 = 7$

$\therefore 2k = 16 + 12 + 5 - 7$

$\therefore 2k = 26$

$\therefore k = 13$

(b) $A^2 = 9A + MI$

$\Rightarrow A^2 - 9A = mI \quad \ldots\ldots (1)$

$A^2 = AA$

$= \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$

Substitute A^2 in (1)

$A^2 - 9A = mI$

$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - 9 \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$

$\Rightarrow m = -14$

(c) Mean $= \dfrac{\text{Sum of all observations}}{\text{Total number of observations}}$

$\therefore 68 = \dfrac{45 + 52 + 60 + x + 69 + 70 + 26 + 81 + 94}{9}$

$\Rightarrow 68 = \dfrac{497 + x}{9}$

$\Rightarrow 612 = 497 + x$

$\Rightarrow x = 612 - 497$

$\Rightarrow x = 115$

Data in ascending order

$26, 45, 52, 60, 69, 70, 81, 94, 115$

Since the number of observations is odd, the median is the $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ observation

$\Rightarrow \text{Median} = \left(\dfrac{9+1}{2}\right)^{\text{th}}$ observation $= 5^{\text{th}}$ observation

Hence, the median is 69.

2. (a) (i) Slope of PQ $= \dfrac{3-k}{1-3k-6}$

$\Rightarrow \dfrac{1}{2} = \dfrac{3-k}{-3k-5}$

$\Rightarrow -3k - 5 = 2(3-k)$

$\Rightarrow -3k - 5 = 6 - 2k$

$\Rightarrow k = -11$

(ii) Substituting k in P and Q, we get

$P(6, k) = P(6, -11)$ and $Q(1 - 3k, 3) = Q(34, 3)$

$\therefore$ Midpoint of PQ $= \left(\dfrac{6+34}{2}, \dfrac{-11+3}{2}\right) = \left(\dfrac{40}{2}, \dfrac{-8}{2}\right) = (20, -4)$

(b) $\csc^2 57° - \tan^2 33° + \cos 44° \csc 46° - \sqrt{2} \cos 45° - \tan^2 60°$

$= \csc^2 (90° - 33°)° - \tan^2 33° + \cos 44° \csc(90° - 44°) - \sqrt{2} \cos 45° - \tan^2 60°$

$= \sec^2 33° - \tan^2 33° + \cos 44° \sec 44° - \sqrt{2} \cos 45° - \tan^2 60$

$= 1 + 1 - \sqrt{2} \cos 45° - \tan^2 60$

$= 1 + 1 - \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right) - \left(\sqrt{3}\right)^2$

$= 2 - 1 - 3$

$= -2$

(c) Let the number of cones be n.

Let the radius of sphere be $r_s = 6$ cm

Radius of a cone be $r_c = 2$ cm

And height of the cone be h = 3 cm

Volume of sphere = n(Volume of a metallic cone)

$$\Rightarrow \frac{4}{3}\pi r_s^3 = n\left(\frac{1}{3}\pi r_c^2 h\right)$$

$$\Rightarrow \frac{4}{\cancel{3}}\cancel{\pi} r_s^3 = n\left(\frac{1}{\cancel{3}}\cancel{\pi} r_c^2 h\right)$$

$$\Rightarrow \frac{4r_s^3}{r_c^2 h} = n$$

$$\Rightarrow n = \frac{4(6)^3}{(2)^2(3)}$$

$$\Rightarrow n = \frac{\cancel{4} \times 216}{\cancel{4} \times 3}$$

$$\Rightarrow n = 72$$

Hence, the number of cones is 72.

3. (a) $-3(x-7) \geq 15 - 7x > \dfrac{x+1}{3}$

$$\Rightarrow -3(x-7) \geq 15 - 7x \quad \text{and} \quad 15 - 7x > \frac{x+1}{3}$$

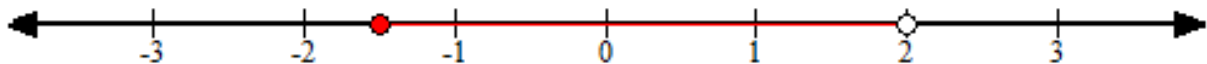$$\Rightarrow -3x + 21 \geq 15 - 7x \quad \text{and} \quad 45 - 21x > x + 1$$

$$\Rightarrow -3x + 7x \geq 15 - 21 \quad \text{and} \quad 45 - 1 > x + 21x$$

$$\Rightarrow 4x \geq -6 \quad \text{and} \quad 44 > 22x$$

$$\Rightarrow x \geq \frac{-3}{2} \quad \text{and} \quad 2 > x$$

$$\Rightarrow x \geq -1.5 \text{ and } 2 > x$$

The solution set is $\{x : x \in R, -1.5 \leq x < 2\}$.

(b) (i) AD is parallel to BC, i.e., OD is parallel to BC and BD is transversal.

$\Rightarrow \angle ODB = \angle CBD = 32°$ ....(Alternate angles)

In $\triangle OBD$,

OD = OB ....(Radii of the same circle)

$\Rightarrow \angle ODB = \angle OBD = 32°$

(ii) AD is parallel to BC, i.e., AO is parallel to BC and OB is transversal.

$\Rightarrow \angle AOB = \angle OBC$ ....(Alternate angles)

$\Rightarrow \angle OBC = \angle OBD + \angle DBC$

$\Rightarrow \angle OBC = 32° + 32°$

$\Rightarrow \angle OBC = 64°$

$\Rightarrow \angle AOB = 64°$

(iii) In $\triangle OAB$,

OA = OB ....(Radii of the same circle)

$\Rightarrow \angle OAB = \angle OBA = x \text{ (say)}$

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow x + x + 64° = 180°$

$\Rightarrow 2x = 180° - 64°$

$\Rightarrow 2x = 116°$

$\Rightarrow x = 58°$

$\Rightarrow \angle OAB = 58°$

i.e., $\angle DAB = 58°$

$\Rightarrow \angle DAB = \angle BED = 58°$ ....(Angles inscribed in the same arc are equal.)

(c) $\dfrac{3a + 2b}{5a + 3b} = \dfrac{18}{29}$

$\Rightarrow 29(3a + 2b) = 18(5a + 3b)$

$\Rightarrow 87a + 58b = 90a + 54b$

$\Rightarrow 58b - 54b = 90a - 87a$

$\Rightarrow 4b = 3a$

$\Rightarrow \dfrac{a}{b} = \dfrac{4}{3}$

4. (a) Total number of outcomes = 30

(i) The perfect squares lying between 11 and 40 are 16, 25 and 36.

So the number of possible outcomes = 3

∴ Probability that the number on the card drawn is a perfect square

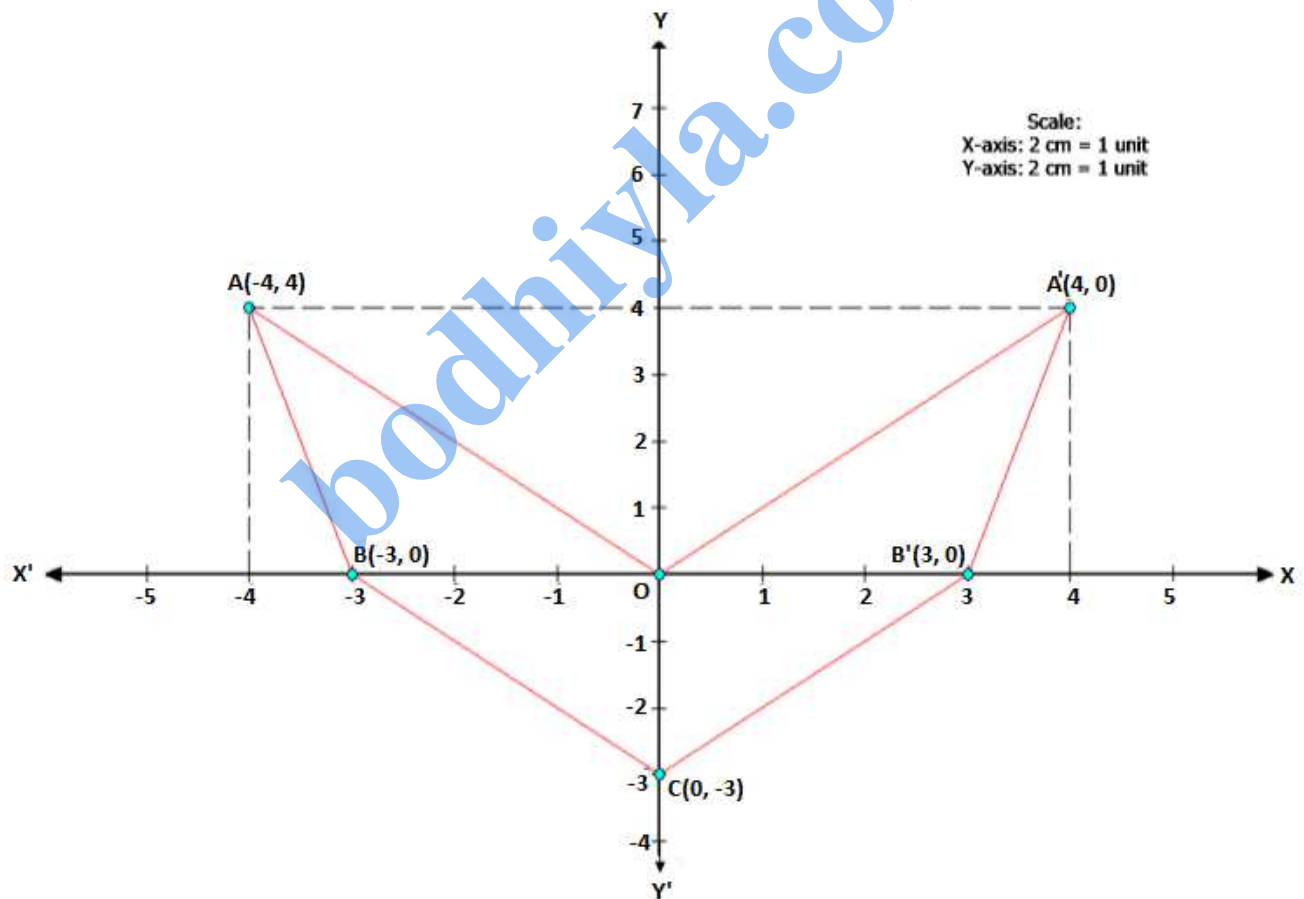$$= \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}} = \frac{3}{30} = \frac{1}{10}$$

(ii) The numbers from 11 to 40 that are divisible by 7 are 14, 21, 28 and 35.

So the number of possible outcomes = 4

Probability that the number on the card drawn is divisible by 7

$$= \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}} = \frac{4}{30} = \frac{2}{15}$$

(b) (i)

A' = (4, 4) AND B' = (3, 0)

(ii) The figure is an arrow head.

(iii) The y-axis is the line of symmetry of figure OABCB'A'.

(c) (i) $P = Rs. 5000, T = 1 \, year, A = Rs. 5325$

$I = A - P$

$\Rightarrow I = 5325 - 5000$

$\Rightarrow I = 325$

So, the interest at the end of first year is Rs. 325.

$I = \dfrac{PRT}{100}$

$\Rightarrow R = \dfrac{I \times 100}{P \times T}$

$\Rightarrow R = \dfrac{325 \times 100}{5000 \times 1}$

$\Rightarrow R = \dfrac{32500}{5000} = 6.5\%$

So, the rate of interest at the end of the first year is 6.5%.

(ii) The amount at the end of the first year will be the principal for the second year.

$P = Rs. 5325, T = 1 \, year, R = 6.5\%$

$I = \dfrac{PRT}{100}$

$\Rightarrow I = \dfrac{5325 \times 6.5 \times 1}{100}$

$\Rightarrow I = 346.125$

$A = P + I$

$\Rightarrow A = 5325 + 346.125$

$\Rightarrow A = 5671.125$

$\Rightarrow A \approx Rs. 5671$

So, the amount at the end of the second year is Rs. 5671.

5. (a) $x^2 - 3(x + 3) = 0$

$\Rightarrow x^2 - 3x - 9 = 0$

Comparing with $ax^2 + bx + c$, we get

$a = 1, b = -3, c = -9$

$\therefore\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$

$\Rightarrow x = \dfrac{3 \pm \sqrt{9 + 36}}{2}$

$\Rightarrow x = \dfrac{3 \pm \sqrt{45}}{2}$

$\Rightarrow x = \dfrac{3 \pm \sqrt{9 \times 5}}{2}$

$\Rightarrow x = \dfrac{3 \pm 3\sqrt{5}}{2}$

$\Rightarrow x = \dfrac{3 + 3\sqrt{5}}{2}$ or $x = \dfrac{3 - 3\sqrt{5}}{2}$

$\Rightarrow x = \dfrac{3 + 3 \times 2.236}{2}$ or $x = \dfrac{3 - 3 \times 2.236}{2}$

$\Rightarrow x = \dfrac{3 + 6.708}{2}$ or $x = \dfrac{3 - 6.708}{2}$

$\Rightarrow x = \dfrac{9.708}{2}$ or $x = \dfrac{-3.708}{2}$

$\Rightarrow x = 4.85$ or $x = -1.85$

(b) Since the interest is earned on the minimum balance between 10th day and the last day of the month as per entries, we have

Minimum balance for April = Rs. 8300

Minimum balance for May = Rs. 7600

Minimum balance for June = Rs. 10300

Minimum balance for July = Rs. 10300

Minimum balance for August = Rs. 3900

Minimum balance for September = Rs. 0

Total balance = Rs. 40400

$\Rightarrow$ Total amount qualifying for interest = Rs. 40400

$\Rightarrow$ Principle for 1 month $\left(\dfrac{1}{12}\ year\right)$ = Rs. 40400,

Rate of interest = 4.5% per annum

$\Rightarrow$ Interest = Rs. $\dfrac{40400 \times 4.5 \times \dfrac{1}{12}}{100}$ = Rs. 151.50

Amount = Balance in the account in last month + Interest

$= 5900 + 151.5$

$= $ Rs. 6051.50

Thus, Mrs. Ravi receives Rs. 6051.50 on closing the account.

(c) Given P = Rs. 1500, I = 496.50, R = 10%

$A = P + I$

$\Rightarrow A = $ Rs. $1500 + $ Rs. $496.50 = $ Rs. 1996.50

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$\Rightarrow 1996.50 = 1500\left(1 + \dfrac{10}{100}\right)^n$

$\Rightarrow \dfrac{1996.50}{1500} = \left(1 + \dfrac{1}{10}\right)^n$

$\Rightarrow 1.331 = (1.1)^n$

$\Rightarrow (1.1)^3 = (1.1)^n$

$\Rightarrow n = 3$

6. (a) Steps of construction:

1. Draw AF measuring 5 cm using a ruler.
2. With A as the centre and radius equal to AF, draw an arc above AF.
3. With F as the centre, and same radius cut the previous arc at Z
4. With Z as the centre, and same radius draw a circle passing through A and F.
5. With A as the centre and same radius, draw an arc to cut the circle above AF at B.
6. With B as the centre and same radius, draw ar arc to cut the circle at C.
7. Repeat this process to get remaining vertices of the hexagon at D and E.
8. Join consecutive arcs on the circle to form the hexagon.
9. Draw the perpendicular bisectors of AF, FE and DE.
10. Extend the bisectors of AF, FE and DE to meet CD, BC and AB at X, L and O respectively.
11. Join AD, CF and EB.
12. These are the 6 lines of symmetry of the regular hexagon.
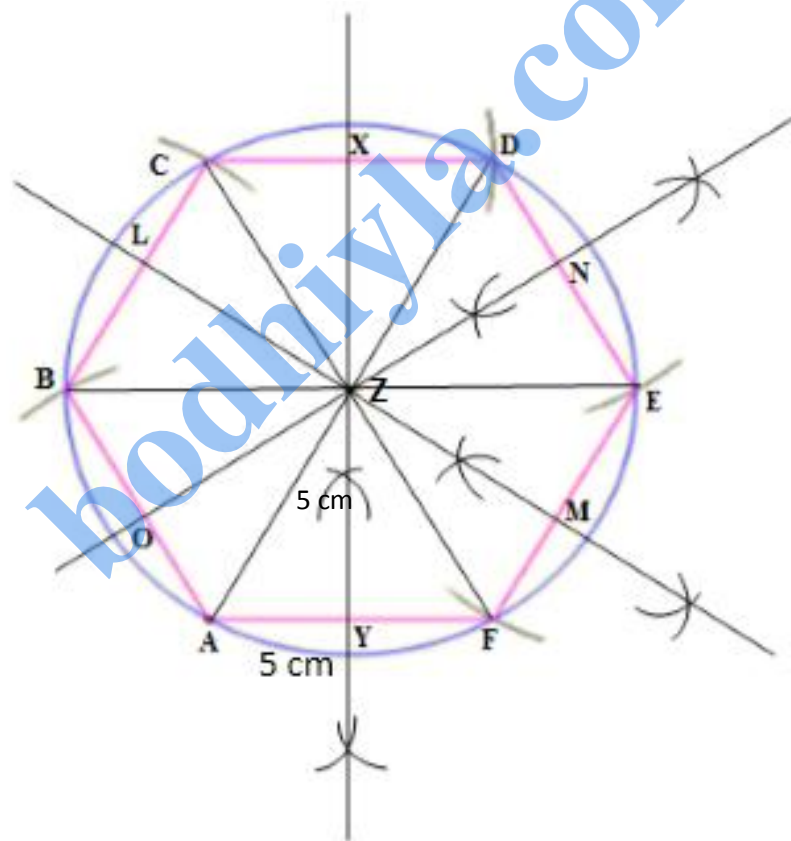
(b) (i) Since □PQRS is a cyclic quadrilateral,

$\angle QRT = \angle SPT$ ....(1)(exterior angle is equal to interior opposite angle)

In $\triangle TPS$ and $\triangle TRQ$,

$\angle PTS = \angle RTQ$ ....(common angle)

$\angle QRT = \angle SPT$ ....(from 1)

$\Rightarrow \triangle TPS \sim \triangle TRQ$ ....(AA similarity criterion)

(ii) Since $\triangle TPS \sim \triangle TRQ$, implies that corresponding sides are proportional

i.e., $\dfrac{SP}{QR} = \dfrac{TP}{TR}$

$\Rightarrow \dfrac{SP}{4} = \dfrac{18}{6}$

$\Rightarrow SP = \dfrac{18 \times 4}{6}$

$\Rightarrow SP = 12\,cm$

(iii) Since $\triangle TPS \sim \triangle TRQ$,

$\dfrac{Ar(\triangle TPS)}{Ar(\triangle TRQ)} = \dfrac{SP^2}{RQ^2}$

$\Rightarrow \dfrac{27}{Ar(\triangle TRQ)} = \dfrac{12^2}{4^2}$

$\Rightarrow Ar(\triangle TRQ) = \dfrac{27 \times 4 \times 4}{12 \times 12}$

$\Rightarrow Ar(\triangle TRQ) = 3\,cm^2$

Now, $Ar(\square PQRS) = Ar(\triangle TPS) - Ar(\triangle TRQ) = 27 - 3 = 24\,cm^2$

(c) $A = \begin{bmatrix} 4\sin 30^o & \cos 0^o \\ \cos 0^o & 4\sin 30^o \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(i) Let the order of matrix $X = m \times n$

Order of matrix $A = 2 \times 2$

Order of matrix $B = 2 \times 1$

Now, $AX = B$

$\Rightarrow A_{2 \times 2} \bullet X_{m \times n} = B_{2 \times 1}$

$\Rightarrow$ Order of matrix X = m × n = 2 × 2

(ii) Let the matrix $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$AX = B$

$\Rightarrow \begin{bmatrix} 4\sin 30^o & \cos 0^o \\ \cos 0^o & 4\sin 30^o \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4\left(\dfrac{1}{2}\right) & 1 \\ 1 & 4\left(\dfrac{1}{2}\right) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x + y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow 2x + y = 4$ ....(1)

$\quad x + 2y = 5$ ....(2)

Multiplying (1) by 2, we get

$4x + 2y = 8$ ....(3)
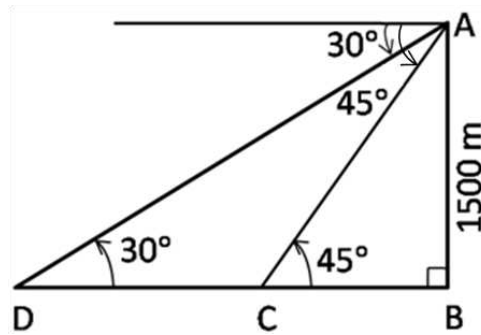
Subtracting (2) from (3), we have

$3x = 3$

$\Rightarrow x = 1$

Substitute x in (1), we get

$2 \times 1 + y = 4$

$\Rightarrow y = 2$

Hence, the matrix $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

7. (a)

A is the aeroplane, D and C are the ships sailing towards A.
Ships are sailing towards the aeroplane in the same direction.
In the figure, height AB=1500 m

To find: Distance between the ships, that is CD.

Solution:
In the right-angled ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{1500}{BC}$$

$$\Rightarrow BC = 1500 \text{ m}$$

In the right-angled ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500}{BD}$$

$$\Rightarrow BD = 1500\sqrt{3} \text{ m}$$

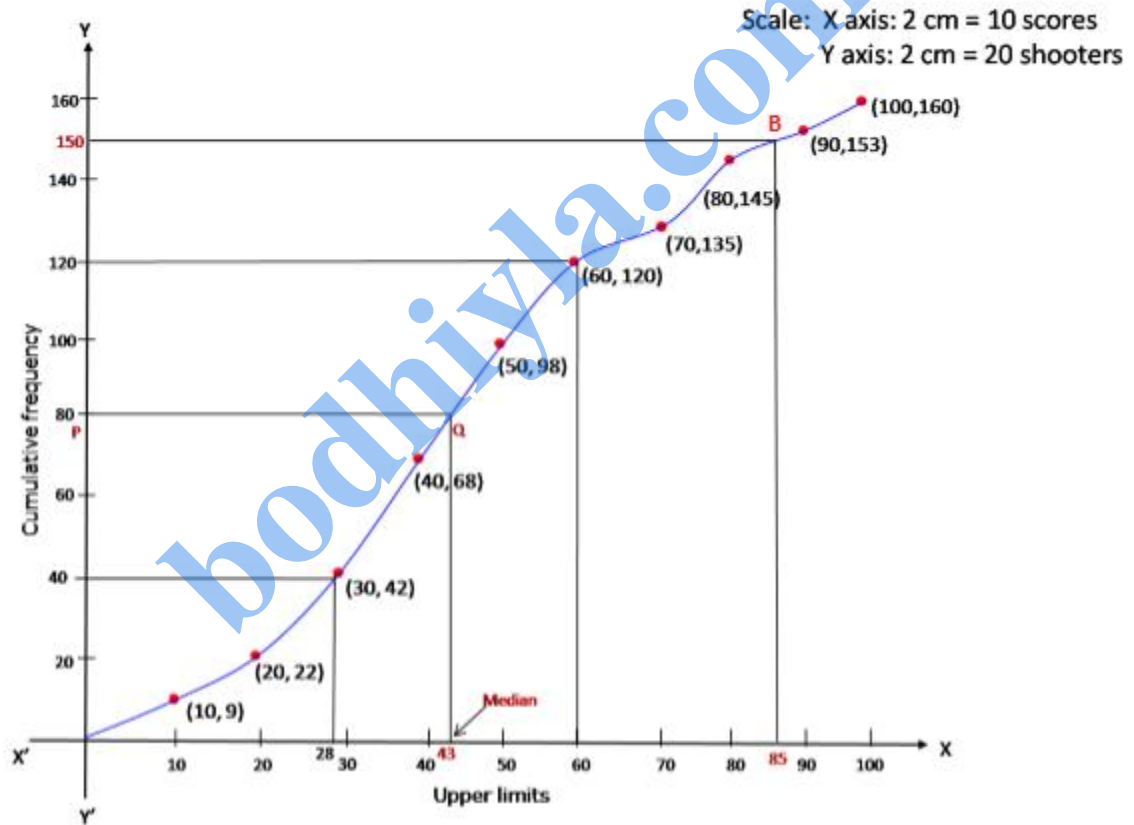$$\Rightarrow BD = 1500(1.732) = 2598 \text{ m}$$

∴ Distance between the ships = CD = BD − BC

$$= 2598 - 1500$$

$$= 1098 \text{ m}$$

(b)

Scores	f	c.f.
0 – 10	9	9
10 – 20	13	22
20 – 30	20	42
30 – 40	26	68
40 – 50	30	98
50 – 60	22	120
60 – 70	15	135
70 – 80	10	145
80 – 90	8	153
90 – 100	7	160
	n = 160	

Scale: X axis: 2 cm = 10 scores
Y axis: 2 cm = 20 shooters

(i) Median $= \left(\dfrac{n}{2}\right)^{\text{th}}$ term $= \left(\dfrac{160}{2}\right)^{\text{th}}$ term $= 80^{\text{th}}$ term

Through mark 80 on y-axis, draw a horizontal line which meets the ogive drawn at point Q.

Through Q, draw a vertical line which meets the x-axis at the mark of 43.

$\Rightarrow$ Median = 43

(ii) Since the number of terms = 160

$$\text{Lower quartile } (Q_1) = \left(\frac{160}{4}\right)^{th} \text{term} = 40^{th}\text{term} = 28$$

$$\text{Upper quartile } (Q_3) = \left(\frac{3 \times 160}{4}\right)^{th} \text{term} = 120^{th}\text{term} = 60$$

$$\therefore \text{ Inter-quartile range} = Q_3 - Q_1 = 60 - 28 = 32$$

(iii) Since 85% scores = 85% of 100 = 85

Through mark for 85 on x-axis, draw a vertical line which meets the ogive drawn at point B.

Through the point B, draw a horizontal line which meets the y-axis at the mark of 150.

$\Rightarrow$ Number of shooters who obtained more than 85% score = 160 − 150 = 10

8. (a) Let $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c} = k$

$\Rightarrow x = ak, y = bk, z = ck$

$$\text{L.H.S.} = \frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3}$$

$$= \frac{(ak)^3}{a^3} + \frac{(bk)^3}{b^3} + \frac{(ck)^3}{c^3}$$

$$= \frac{a^3k^3}{a^3} + \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3}$$

$$= k^3 + k^3 + k^3$$

$$= 3k^3$$

$$\text{R.H.S.} = \frac{3xyz}{abc}$$

$$= \frac{3(ak)(bk)(ck)}{abc}$$

$$= 3k^3$$

$$= \text{L.H.S.}$$

$\Rightarrow \text{L.H.S.} = \text{R.H.S.}$

$$\Rightarrow \frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

(b) Steps for construction:

(i)   Draw AB = 5 cm using a ruler.

(ii)   With A as the centre cut an arc of 3 cm on AB to obtain C.

(iii) With A as the centre and radius 2.5 cm, draw an arc above AB.

(iv)   With same radius, and C as the centre draw an arc to cut the previous arc and mark the intersection as O.

(v)   With O as the centre and radius 2.5 cm, draw a circle so that points A and C lie on the circle formed.

(vi)   Join OB.

(vii) Draw the perpendicular bisector of OB to obtain the mid-point of OB, M.

(viii) With the M as the centre and radius equal to OM, draw a circle to cut the previous circle at points P and Q.

(ix)   Join PB and QB. PB and QB are the required tangents to the given circle from exterior point B.
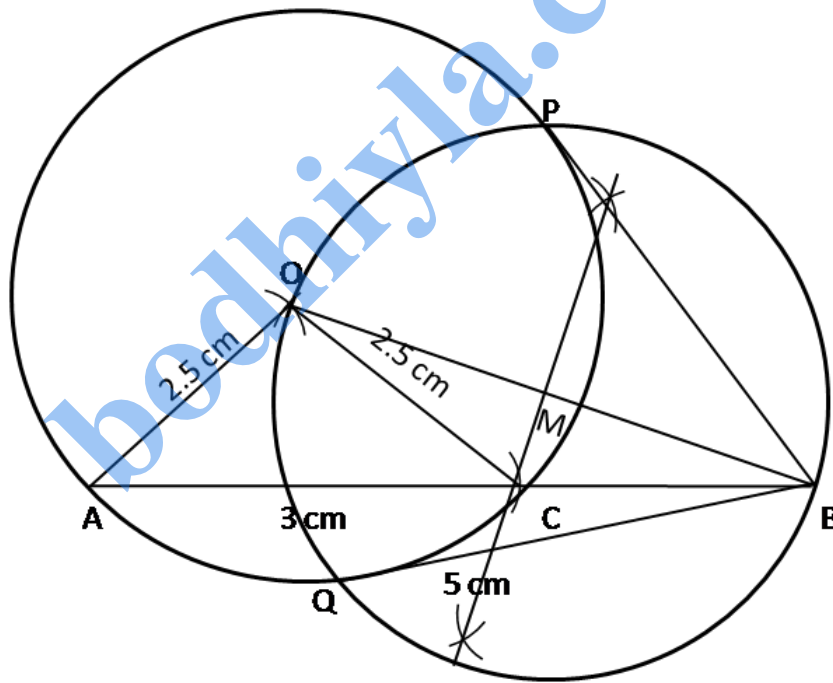
QB = PB = 3 cm
That is, length of the tangents is 3 cm.

(c) (i) Since, A lies on the x-axis, let the coordinates of A be (x, 0).

Since B lies on the y - axis, let the coordinates of B be $(0, y)$.

Let $m = 1$ and $n = 2$.

Using section formula,

Coordinates of $P = \left(\dfrac{1(0) + 2(x)}{1 + 2}, \dfrac{1y + 2(0)}{1 + 2} \right)$

$\Rightarrow (4, -1) = \left(\dfrac{2x}{3}, \dfrac{y}{3} \right)$

$\Rightarrow \dfrac{2x}{3} = 4 \text{ and } \dfrac{y}{3} = -1$

$\Rightarrow x = 6 \text{ and } y = -3$

So, the coordinates of A are $(6, 0)$ and that of B are $(0, -3)$.

(ii) Slope of $AB = \dfrac{-3 - 0}{0 - 6} = \dfrac{-3}{-6} = \dfrac{1}{2}$

$\Rightarrow$ Slope of line perpendicular to $AB = m = -2$

$P = (4, -1)$

$\Rightarrow$ Required equation is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-1) = -2(x - 4)$

$\Rightarrow y + 1 = -2x + 8$

$\Rightarrow 2x + y = 7$

9. (a) Marked price of the article = Rs. 6,000.

A dealer buys an article at a discount of 30% from the wholesaler.

∴ Price of the article which the dealer paid to the wholesaler

$= 6000 - 30\%$ of 6000

$= 6000 - \dfrac{30}{100} \times 6000$

$= Rs. 4200$

Sales tax paid by dealer $= 6\%$ of $4200 = \dfrac{6}{100} \times 4200 = Rs.252$

∴ Amount of the article inclusive of sales tax at which the dealer bought it

$= Rs. 4200 + Rs. 252 = Rs.4452$

Dealer sells the article at a discount of 10% to the shopkeeper.

∴ Price of the article which the shopkeeper paid to the dealer

$= 6000 - 10\%$ of 6000

$= 6000 - \dfrac{10}{100} \times 6000$

$= Rs.5400$

Sales tax paid by shopkeeper $= 6\%$ of $5400 = \dfrac{6}{100} \times 5400 = Rs.324$

∴ Amount of the article inclusive of sales tax at which the shopkeeper bought it

$= Rs.5400 + Rs.324 = Rs.5724$

The value added by dealer $= Rs.5400 - Rs.4200 = Rs.1200$

Amount of VAT paid by dealer

$= 6\%$ of 1200

$= \dfrac{6}{100} \times 1200$

$= Rs. 72$

∴ Price paid by shopkeeper including tax is Rs 5724.

∴ VAT paid by dealer is Rs.72.

(b) (i) Area of the shaded part = Area of the circle + area of the semicircle

$$= \pi(2.5)^2 + \frac{\pi(2)^2}{2}$$

$$= \pi[6.25 + 2]$$

$$= \frac{22}{7}[8.25]$$

$$\approx 26 \, cm^2$$

(ii) Area of kite $= \dfrac{\text{product of the diagonals}}{2} = \dfrac{AC \times BD}{2} = \dfrac{12 \times 8}{2} = 48 \, cm^2$

Area of the unshaded part = Area of the kite − Area of the shaded part

$$= 48 - 26$$

$$= 22 \, cm^2$$

(c) (i) Scale factor $k = \dfrac{1}{300}$

Length of the model of the ship = k × Length of the ship

$$\Rightarrow 2 = \frac{1}{300} \times \text{Length of the ship}$$

$$\Rightarrow \text{Length of the ship} = 600 \text{ m}$$

(ii) Area of the deck of the model $= k^2 \times$ Area of the deck of the ship

$$\Rightarrow \text{Area of the deck of the model} = \left(\frac{1}{300}\right)^2 \times 180,000$$

$$= \frac{1}{90000} \times 180,000$$

$$= 2 \, m^2$$

(iii) Volume of the model $= k^3 \times$ Volume of the ship

$$\Rightarrow 6.5 = \left(\frac{1}{300}\right)^3 \times \text{Volume of the ship}$$

$$\Rightarrow \text{Volume of the ship} = 6.5 \times 27000000 = 175500000 \, m^3$$

10. (a) (i) $I = Rs.\, 1200, n = 2 \times 12 = 24\, months, r = 6\%$

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 1200 = P \times \frac{24 \times 25}{24} \times \frac{6}{100}$$

$$\Rightarrow 1200 = P \times \frac{3}{2}$$

$$\Rightarrow P = \frac{1200 \times 2}{3}$$

$$\Rightarrow P = Rs.\, 800$$

So the monthly instalment is Rs. 800.

(ii) Total sum deposited = $P \times n$ = Rs. 800×24 = Rs. 19200

$\therefore$ Amount of maturity = Total sum deposited + Interest on it

= Rs. 19200 + Rs. 1200

= Rs. 20400

(b) (i)

Class interval	Frequency
0-10	2
10- 20	5
20-30	8
30-40	4
40-50	6

(ii)

Class interval	Frequency (f)	Mean value (x)	fx
0-10	2	5	10
10- 20	5	15	75
20-30	8	25	200
30-40	4	35	140
40-50	6	45	270
	$\Sigma f = 25$		$\Sigma f = 695$

$$\therefore Mean = \frac{\Sigma fx}{\Sigma f} = \frac{695}{25} = 27.8$$

(iii) Here the maximum frequency is 8 which is corresponding to class 20 – 30.
Hence, the modal class is 20 – 30.

(c) Time taken by bus to cover total distance with speed x km/h $= \dfrac{240}{x}$

Time taken by bus to cover total distance with speed $(x-10)$ km/h $= \dfrac{240}{x-10}$

According to the given condition,

$$\dfrac{240}{x-10} - \dfrac{240}{x} = 2$$

$$\Rightarrow 240\left(\dfrac{1}{x-10} - \dfrac{1}{x}\right) = 2$$

$$\Rightarrow \dfrac{1}{x-10} - \dfrac{1}{x} = \dfrac{1}{120}$$

$$\Rightarrow \dfrac{x-x+10}{x(x-10)} = \dfrac{1}{120}$$

$$\Rightarrow \dfrac{10}{x^2-10x} = \dfrac{1}{120}$$

$$\Rightarrow x^2 - 10x = 1200$$

$$\Rightarrow x^2 - 10x - 1200 = 0$$

$$\Rightarrow (x-40)(x+30) = 0$$

$$\Rightarrow x - 40 = 0 \text{ or } x + 30 = 0$$

$$\Rightarrow x = 40 \text{ or } x = -30$$

Since, the speed cannot be negative, the uniform speed is 40 km/h.

11. (a) L.H.S. $= \dfrac{\cos A}{1 + \sin A} + \tan A$

$$= \dfrac{\cos A(1 - \sin A)}{(1 + \sin A)(1 - \sin A)} + \dfrac{\sin A}{\cos A}$$

$$= \dfrac{\cos A - \sin A \cos A}{1 - \sin^2 A} + \dfrac{\sin A}{\cos A}$$

$$= \dfrac{\cos A - \sin A \cos A}{\cos^2 A} + \dfrac{\sin A}{\cos A}$$

$$= \dfrac{1}{\cos A} - \dfrac{\sin A}{\cos A} + \dfrac{\sin A}{\cos A}$$

$$= \dfrac{1}{\cos A}$$

$$= \sec A$$

$$= \text{R.H.S}$$

(b) (i) Steps of construction:
1. Draw BC = 6.5 cm using a ruler.
2. With B as the centre and radius equal to approximately half of BC, draw an arc that cuts the segment BC at Q.
3. With Q as the centre, and same radius, cut the previous arc at P.
4. Join BP and extend it.
5. With B as the centre and radius 5 cm, draw an arc that cuts the arm PB to obtain point A.
6. Join AC to obtain ΔABC.
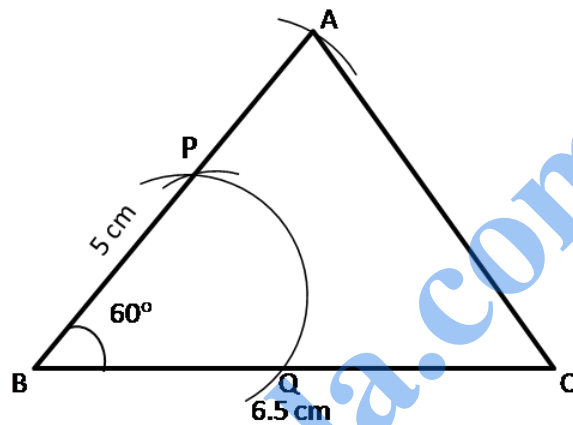
(ii) Steps for construction :
1. With A as the centre and radius 3.5 cm, draw a circle.
2. The circumference of a circle is the required locus.
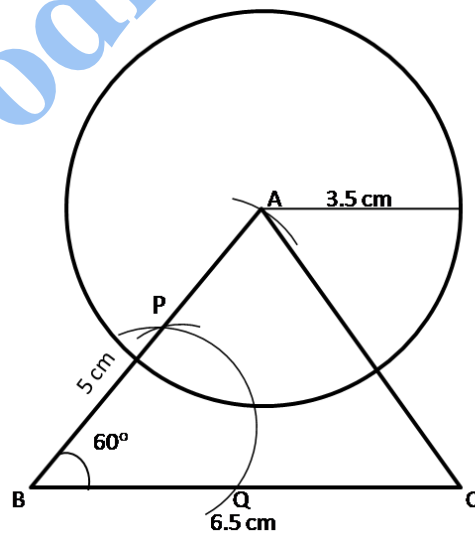
(iii) Steps for construction :
1. With C as the centre and with radius of a length less than CA or BC, draw an arc to cut the line segments AC and BC at D and E respectively.
2. With the same radius or a suitable radius and with D as the centre, draw an arc of a circle.
3. With the same radius and with E as the centre draw an arc such that the two arcs intersect at H.
4. Join C and H.
5. CH is the bisector of ÐACB and is the required locus.
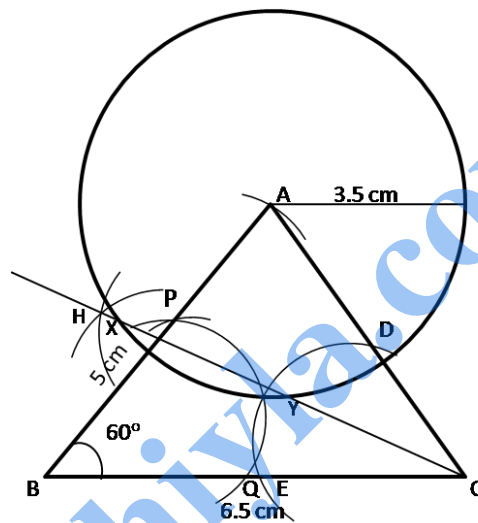
(iv) Steps for construction :
1. We known that the points at a distance of 3.5 cm from A will surely lie on the circle with centre A.
2. Also, the points on the angle bisector CH are the points equidistant from AC and BC.
3. Mark X and Y which are at the intersection of the circle and the angle bisector CH.
4. Measure XY. XY = 5 cm

(c)
(i) Total dividend = Rs.2,475

Dividend on each share $= 12\%$ of Rs. $25 = \dfrac{12}{100} \times 25 =$ Rs. 3

$\therefore$ Number of shares bought $= \dfrac{\text{Total dividend}}{\text{Dividend on 1 share}} = \dfrac{2475}{3} = 825$

(ii) Market value of each share $= \dfrac{\text{Total Investment}}{\text{Number of shares bought}} = \dfrac{26400}{825} =$ Rs. 32