ICSE Paper 2013

MATHEMATICS

SECTION A [40 Marks]

(Answer all questions from this Section.)

Question 1.

(a) Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

Find the matrix X such that $A + 2X = 2B + C$. **[3]**

(b) At what rate % p.a. will a sum of ₹ 4000 yield ₹ 1324 as compound interest in 3 years ? **[3]**

(c) The median of the following observations 11, 12, 14, (x – 2), (x + 4), (x + 9), 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. **[4]**

Solution :

(a) *Given :*

$$A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore \quad A + 2X = 2B + C$$

Putting the given values, we get

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix} \qquad \textbf{Ans.}$$

(b) *Given :* Principal = ₹ 4,000, C.I. = ₹ 1,324,

Amount = P + C.I.

= ₹ (4,000 + 1,324) = ₹ 5,324

Time = 3 years

We know that,

$$A = P\left(1 + \frac{r}{100}\right)^T$$

$$5,324 = 4,000\left(1 + \frac{r}{100}\right)^3$$

$$\frac{5,324}{4,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\frac{1{,}331}{1{,}000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3$$

Therefore,

$$1 + \frac{r}{100} = \frac{11}{10}$$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

$$r = 10\% \qquad \text{Ans.}$$

(c) Given observation are 11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38, 47 and median = 24.

$$n = 9 \text{ (odd)}$$

$$\therefore \quad \text{Median} = \frac{n+1}{2} \text{th term}$$

$$= \frac{9+1}{2} \text{th term}$$

$$24 = 5\text{th term}$$

$$\therefore \quad x + 4 = 24$$

$$x = 24 - 4$$

$$x = 20$$

Therefore, 11, 12, 14, $(20-2)$, $(20+4)$, $(20+9)$, 32, 38, 47

$$= 11, 12, 14, 18, 24, 29, 32, 38, 47$$

Now

$$\text{Mean} = \frac{\Sigma x}{n}$$

$$= \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}$$

$$= \frac{225}{9} = 25. \qquad \text{Ans.}$$

Question 2.

(a) What number must be added to each of the number 6, 15, 20 and 43 to make them proportional ? [3]

(b) If $(x-2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x-3)$, it leaves a remainder 52, find the values of a and b. [3]

(c) Draw a histogram from the following frequency distribution and find the mode from the graph : [4]

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	2	5	18	14	8	5

Solution :

(a) Let the number must be added be x, then

$$\text{the new number } = 6 + x, 15 + x, 20 + x, 43 + x$$

∵ These are proportionals.

$$6 + x : 15 + x :: 20 + x : 43 + x$$

or $(6 + x)(43 + x) = (15 + x)(20 + x)$

or $258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$

or $49x - 35x = 300 - 258$

or $14x = 42$

or $x = 3.$ **Ans.**

(b) Let $(x - 2)$ is a factor of the given expression.

∴ $x - 2 = 0$

$x = 2$

Given expression,

$$2x^3 + ax^2 + bx - 14 = 0$$

$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$

$$16 + 4a + 2b - 14 = 0$$

$$4a + 2b + 2 = 0$$

$$4a + 2b = -2$$

$$2a + b = -1 \qquad \text{...(i)}$$

and when given expression is divided by $(x - 3)$

$$x - 3 = 0$$

⇒ $x = 3$

∴ $2x^3 + ax^2 + bx - 14 = 52$

$$2(3)^3 + a(3)^2 + b(3) - 66 = 0$$

$$54 + 9a + 3b - 66 = 0$$

$$9a + 3b = 12$$

$$3a + b = 4 \qquad \text{...(ii)}$$

Solving equation (i) and (ii),

$$2a + b = -1$$
$$3a + b = 4$$
$$\underline{(-) \quad (-) \quad (+)}$$
$$-a = -5$$
$$a = 5$$

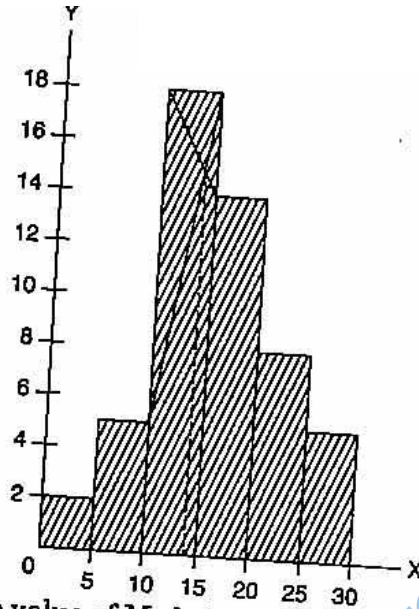from (ii),

$$3 \times 5 + b = 4$$

$$b = 4 - 15$$

$$b = -11$$

∴ $a = 5$ and $b = -11$ **Ans.**

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(c)

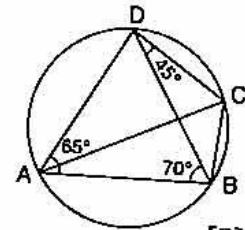From the Histogram the value of Mode is **13·8**.

Question 3.

(a) *Without using tables evaluate 3 cos 80°.Cosec 10° + 2 sin 59° sec 31°.*

(b) *In the given figure,*

$$\angle BAD = 65°,$$
$$\angle ABD = 70°,$$
$$\angle BDC = 45°$$

(i) *Prove that AC is a diameter of the circle.*

(ii) *Find* $\angle ACB$.

(c) *AB is a diameter of a circle with centre C = (–2, 5). If A = (3, –7). Find :*

(i) *The length of radius AC*

(ii) *The coordinates of B.*

Solution :

Ans.

[3]

[3]

[4]

(a) *Given :*

$$3 \cos 80°·\operatorname{cosec} 10° + 2 \sin 59° \sec 31°$$
$$= 3 \cos 80° \operatorname{cosec} (90° - 80°) + 2 \sin 59° \sec (90° - 59°)$$
$$= 3 \cos 80° \sec 80° + 2 \sin 59° \operatorname{cosec} 59°$$
$$= 3 \cos 80° \times \frac{1}{\cos 80°} + 2 \sin 59° \times \frac{1}{\sin 59°}$$
$$= 3 + 2 = 5.$$

(b) *Given :* $\angle BAD = 65°$, $\angle ABD = 70°$, $\angle BDC = 45°$

Ans.

(i) ∵ ABCD is a cyclic quadrilateral.

In $\triangle ABD$,

$$\angle BDA + \angle DAB + \angle ABD = 180°$$

∴ $$\angle BDA = 180° - (65° + 70°)$$
$$= 180° - 135°$$
$$= 45°$$

By using sum property of $\triangle^s$

Now from $\triangle ACD$,

$$\angle ADC = \angle ADB + \angle BDC$$
$$= 45° + 45° \qquad (\because \angle BDA = \angle ADB = 45°)$$
$$= 90°$$

Hence, $\angle D$ makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

(ii) $\qquad \angle ACB = \angle ADB \qquad$ (Angles in the same segment of a circle)

$\therefore \qquad \angle ACB = 45° \qquad (\because \angle ADB = 45°)$ **Ans.**

(c) (i) The length of radius $AC = \sqrt{(-2-3)^2 + (5+7)^2}$

$$= \sqrt{(-5)^2 + (12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13. \qquad \textbf{Ans.}$$
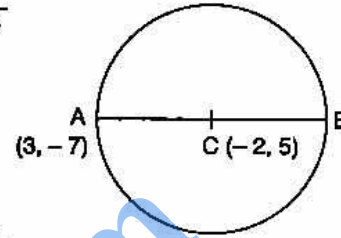
(ii) Let the point of B be (x, y).

Given C is the mid-point of AB. Therefore

$$-2 = \frac{3+x}{2}$$
$$\Rightarrow \qquad 3 + x = -4$$
$$\Rightarrow \qquad x = -4 - 3 = -7$$

and $\qquad 5 = \frac{-7+y}{2}$
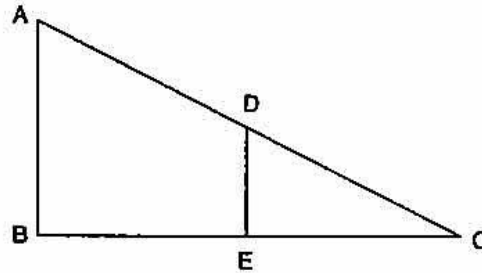
$$\Rightarrow \qquad 10 = -7 + y$$
$$y = 17$$

Hence, the co-ordinate of B $(-7, 17)$. **Ans.**

Question 4.

(a) *Solve the following equation and calculate the answer correct to two decimal places :*

$$x^2 - 5x - 10 = 0. \qquad \textbf{[3]}$$

(b) *In the given figure, AB and DE are perpendicular to BC.*

(i) *Prove that $\triangle ABC \sim \triangle DEC$*

(ii) *If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.*

(iii) *Find the ratio of the area of $\triangle ABC$: area of $\triangle DEC$.* **[3]**

(c) *Using graph paper, plot the points A(6, 4) and B(0, 4).*

(i) *Reflect A and B in the origin to get the images A′ and B′.*

(ii) *Write the co-ordinates of A′ and B′.*

(iii) *State the geometrical name for the figure ABA′B′.*

(iv) *Find its perimeter.* **[4]**

Solution :

(a) *Given :* $x^2 - 5x - 10 = 0$

Here, $a = 1$, $b = -5$ and $c = -10$

$\therefore$

$$D = b^2 - 4ac$$
$$= (-5)^2 - 4 \times 1 \times -10$$
$$D = 25 + 40 = 65$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{5 \pm \sqrt{65}}{2 \times 1} = \frac{5 \pm 8.06}{2}$$

$$= \frac{5 + 8.06}{2}, \frac{5 - 8.06}{2}$$

$$= \frac{13.06}{2}, -\frac{3.06}{2}$$

$$x = 6.53, -1.53$$ **Ans.**

(b) (i) From $\triangle ABC$ and $\triangle DEC$,

$$\angle ABC = \angle DEC = 90°$$ (Given)

and $$\angle ACB = \angle DCE = \text{Common}$$

$\therefore$ $$\triangle ABC \sim \triangle DEC$$ (By AA similarity)

(ii) In $\triangle ABC$ and $\triangle DEC$,

$$\triangle ABC \sim \triangle DEC$$ (proved in (i) part)

$\therefore$ $$\frac{AB}{DE} = \frac{AC}{CD}$$

Given : AB = 6 cm, DE = 4 cm, AC = 15 cm,

$\therefore$ $$\frac{6}{4} = \frac{15}{CD}$$

$\Rightarrow$ $$6 \times CD = 15 \times 4$$

$\Rightarrow$ $$CD = \frac{60}{6}$$

$\Rightarrow$ $$CD = 10 \text{ cm.}$$ **Ans.**

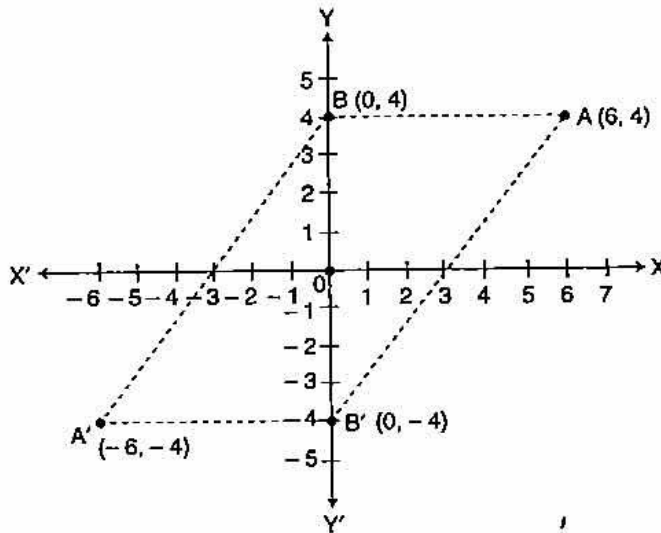(iii) $$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEC} = \frac{AB^2}{DE^2}$$ ($\because \triangle ABC \sim \triangle DEC$)

$$= \frac{(6)^2}{(4)^2}$$

$$= \frac{3.6}{16} = \frac{9}{4}$$

$\therefore$ Area of $\triangle ABC$: Area of $\triangle DEC$ = 9 : 4.

Ans.
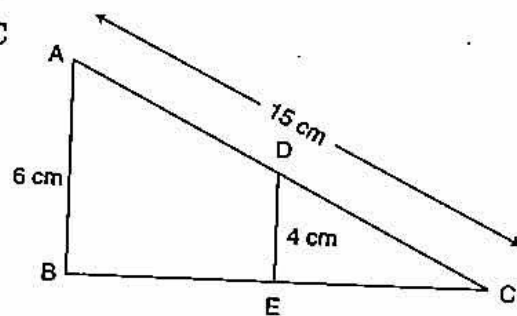
(c) (i) Please See Graph:

(ii) Reflection of A' and B' in the origin = A' (–6, –4) and B' (0, –4)

(iii) The geometrical name for the figure AB A'B' is a parallelogram.

(iv) From the graph, AB = 6 cm, BB' = 8 cm.

In Δ A BB'

$$(AB')^2 = AB^2 + (BB')^2$$
$$= (6)^2 + (8)^2 = 36 + 64$$
$$= 100$$
$$AB' = 10 = A'B \qquad (AB\ A'\ B'\ \text{is a parallelogram})$$

Perimeter of AB A'B' = A'B' + AB' + AB + A'B
$$= 6 + 10 + 6 + 10$$
$$= 32 \text{ units.} \qquad \textbf{Ans.}$$

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *Solve the following inequation, write the solution set and represent it on the number line :*

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R \qquad [3]$$

(b) *Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment.*

[3]

(c) *Salman buys 50 shares of face value ₹ 100 available at ₹ 132.*

(i) *What is his investment ?*

(ii) *If the dividend is 7.5%, what will be his annual income ?*

(iii) *If he wants to increase his annual income by ₹ 150, how many extra shares should he buy ?*
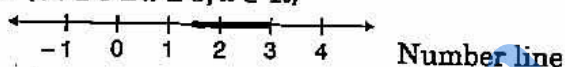
[4]

Solution :

(a) *Given :*
$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

Taking L.C.M. of 3, 2 and 6 is 6.

$$-\frac{x}{3} \times 6 \le \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6$$

$$-2x \le 3x - 8 < 1$$

$\Rightarrow$	$-2x \le 3x - 8$	and	$3x - 8 < 1$
$\Rightarrow$	$8 \le 3x + 2x$	$\Rightarrow$	$3x < 1 + 8$
$\Rightarrow$	$8 \le 5x$	$\Rightarrow$	$3x < 9$
$\Rightarrow$	$\dfrac{8}{5} \le x$	$\Rightarrow$	$x < 3$

$\therefore$ The solution set is $\{x : 1.6 \le x \le 3, x \in R\}$

Number line

(b) Let the monthly instalment be ₹ x

Given : Maturity amount = ₹ 8,088, Time (n) = 3 years = 3 × 12 months = 36 months, Rate (R) = 8% p.a.

$$\text{Principle for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{x \times 36 \times 37}{2}$$

$$= 18 \times 37\, x$$

$$\text{Interest} = \frac{18 \times 37x \times 8 \times 1}{100 \times 12} \qquad \left[\because I = \frac{PRT}{100} \right]$$

$$= \frac{444\, x}{100}$$

$$\text{Actual sum deposited} = 36\, x$$

$$\text{Maturity amount} = \text{Interest} + \text{Actual sum deposited}$$

$$8,088 = \frac{444\, x}{100} + 36\, x$$

$$8,088 = \frac{4,044\, x}{100}$$

$$\therefore \qquad x = \frac{8,088 \times 100}{4,044} = 200$$

Hence, the monthly instalment be ₹ 200.   **Ans.**

(c)

$$\text{Number of shares} = 50$$
$$\text{Face value of each share} = ₹\, 100$$
$$\text{Market value of each share} = ₹\, 132$$
$$\text{Total face value} = ₹\, 100 \times 50$$
$$= ₹\, 5,000$$

(i)

$$\text{Total investment} = ₹\, 132 \times 50$$
$$= ₹\, 6,600$$ **Ans.**

(ii)

$$\text{Rate of dividend} = 7 \cdot 5\%$$

$$\text{Annual income} = ₹\frac{5{,}000 \times 7 \cdot 5}{100}$$

$$= ₹\,375 \qquad \textbf{Ans.}$$

(iii) Let extra share should he buy be x.

$$\text{then total number of shares} = 50 + x$$

$$\text{Total face value} = ₹\,100 \times (50 + x)$$

$$\therefore \quad \text{Annual income} = ₹\frac{100 \times (50 + x) \times 7 \cdot 5}{100}$$

$$= (50 + x) \times 7 \cdot 5$$

$$\therefore \quad (50 + x) \times 7 \cdot 5 = 375 + 150$$

$$50 + x = \frac{525}{7 \cdot 5} = 70$$

$$x = 70 - 50$$

$$x = 20$$

Hence, the extra shares should be buy $= 20$. **Ans.**

Question 6.

(a) Show that $\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \dfrac{\sin A}{1 + \cos A}$ **[3]**

(b) In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.   **[3]**
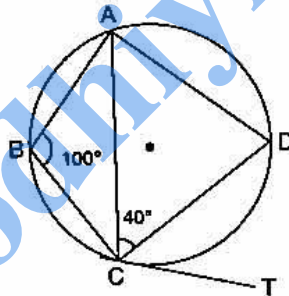
(c) Given below are the entries in a Savings Bank A/c pass book :

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8.	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	—
April 12	By cash	—	₹ 2,230	—
June 15	To self	₹ 5,000	—	—
July 8	By cash	—	₹ 6,000	—

Calculate the interest for six months from February to July at 6% p.a.   **[4]**

Solution :

(a) L.H.S. $= \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

Multiplying by $\sqrt{1 + \cos A}$ in numerator and denominator

$$= \sqrt{\frac{1 - \cos A}{1 + \cos A}} \times \sqrt{\frac{1 + \cos A}{1 + \cos A}}$$

$$= \sqrt{\frac{(1-\cos A)(1+\cos A)}{(1+\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A} = \text{R.H.S.}$$

Proved

(b) *Given* : $\angle ABC = 100°$

We know that,

$$\angle ABC + \angle ADC = 180°$$

∴ $\quad 100° + \angle ADC = 180°$ 　　(The sum of opposite angles in a cyclic quadrilateral = 180°)

$$\angle ADC = 180° - 100°$$

$$\angle ADC = 80°$$

Join OA and OC, we have a isosceles △ OAC,

∵ $\quad OA = OC$ 　　(Radii of a circle)

∴ $\quad \angle AOC = 2 \times \angle ADC$ 　　(by theorem)

or $\quad \angle AOC = 2 \times 80° = 160°$

In △ AOC,

$$\angle AOC + \angle OAC + \angle OCA = 180°$$

$$160° + \angle OCA + \angle OCA = 180° \; [\because \angle OAC = \angle OCA]$$

$$2\angle OCA = 20°$$

$$\angle OCA = 10°$$

$$\angle OCA + \angle OCD = 40°$$

$$10° + \angle OCD = 40°$$

∴ $\quad \angle OCD = 30°$

Hence, $\quad \angle OCD + \angle DCT = \angle OCT$

∵ $\quad \angle OCT = 90°$

(The tangent at a point to circle is ⊥ to the radius through the point to contant)

$$30° + \angle DCT = 90°$$

∴ $\quad \angle DCT = 60°$ 　　　**Ans.**

(c)
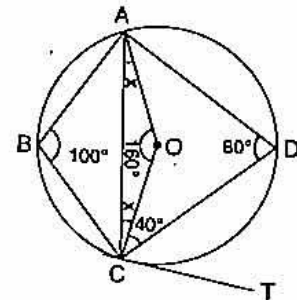
Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	₹ 4,500
April 12	By cash	—	₹ 2,230	₹ 6,730
June 15	To self	₹ 5,000	—	₹ 1,730
July 8	By cash	—	₹ 6,000	₹ 7,730

Principal for the month of Feb. = ₹ 4,500

Principal for the month of March = ₹ 4,500

$$\text{Principal for the month of April} = ₹ \quad 4,500$$
$$\text{Principal for the month of May} = ₹ \quad 6,730$$
$$\text{Principal for the month of June} = ₹ \quad 1,730$$
$$\text{Principal for the month of July} = ₹ \quad 7,730$$
$$\text{Total principal from the month of Feb. to July} = ₹ \quad 29,690$$

$$\text{Time} = \frac{1}{12} \text{ years}$$

$$\text{Rate of interest} = 6\%$$

$$\text{Interest} = \frac{P \times R \times T}{100}$$

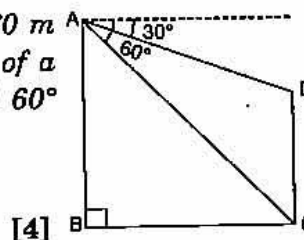$$= \frac{29690 \times 6 \times 1}{100 \times 12}$$

$$= ₹ \, 148·45 \qquad \textbf{Ans.}$$

Question 7.

(a) *In △ ABC, A(3, 5), B(7, 8) and C(1, −10). Find the equation of the median through A.* [3]

(b) *A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler ?* [3]

(c) *In the figure given, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :*

   (i) *The horizontal distance between AB and CD.*

   (ii) *The height of the lamp post.* [4]

Solution :

(a) Here D is mid point of BC.

$$\therefore \quad \text{The co-ordinate of D} = \left(\frac{7+1}{2}, \frac{8-10}{2} \right)$$

$$= (4, -1)$$

(Given)

Now equation of median AD,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
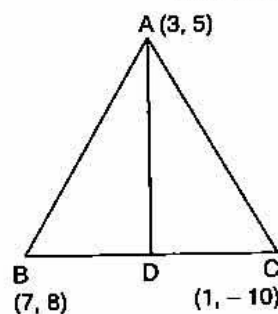
Here, $x_1 = 3, y_1 = 5, x_2 = 4, y_2 = -1$

$$y - 5 = \frac{-1-5}{4-3}(x-3)$$

$$y - 5 = \frac{-6}{1}(x-3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 18 + 5$$
$$y = -6x + 23$$
$$6x + y - 23 = 0 \qquad \text{Ans.}$$

(b)

$$\text{Listed price of an article} = ₹1,500$$
$$\text{Rate of VAT} = 12\%$$
$$\text{VAT on the article} = \frac{12}{100} \times 1500$$
$$= ₹180$$

Let C.P. of this article be x, then

$$\text{VAT} = \frac{12}{100} \times x$$
$$= ₹\frac{12x}{100}$$

If the shopkeeper pays a VAT $= ₹36$

Then
$$180 - \frac{12x}{100} = 36$$
$$\frac{18000 - 12x}{100} = 36$$
$$18000 - 12x = 3600$$
$$\therefore \qquad 12x = 18000 - 3600 = 14,400$$
$$x = ₹1,200$$

$\therefore$ The price at which the shopkeeper purchased the article inclusive of sales tax

$$= 1,200 + \frac{12}{100} \times 1,200$$
$$= 1,200 + 144$$
$$= ₹1,344 \qquad \text{Ans.}$$

(c) *Given* : $AB = 60$ m

$$\angle PAC = 60°$$
$$\angle PAC = \angle BCA$$

(i) Now in $\triangle ABC$,

$$\tan 60° = \frac{AB}{BC}$$
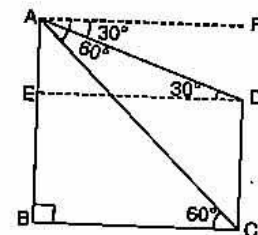$$\sqrt{3} = \frac{60}{BC}$$
$$\Rightarrow \qquad \sqrt{3}\, BC = 60$$
$$\Rightarrow \qquad BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Hence, the horizontal distance between AB and CD $= 20\sqrt{3}$ m. **Ans.**

(ii) Let $AE = x$ and proved above $BC = 20\sqrt{3}$m

$$\therefore \qquad BC = ED = 20\sqrt{3}$$

Now in $\triangle$ AED,

$$\tan 30° = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$$\Rightarrow \qquad \sqrt{3}\,AE = 20\sqrt{3}$$

$$\Rightarrow \qquad AE = 20 \text{ m}$$

now $\qquad EB = AB - AE$

$\therefore \qquad EB = 60 - 20 \quad \Rightarrow \quad 40 \text{ m}$

$\because \qquad EB = CD$

$\therefore \qquad CD = 40 \text{ m}$

Hence, the height of the lamp post = 40 m. **Ans.**

Question 8.

(a) *Find x and y if* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ [3]

(b) *A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.* [3]

(c) *Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots :*

$$x^2 + (p - 3)\,x + p = 0$$ [4]

Solution :

(a) *Given :* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$\therefore \qquad 5x = 5 \quad \Rightarrow \quad x = 1$

and $\qquad 6y = 12 \quad \Rightarrow \quad y = 2$

Hence, $x = 1$ and $y = 2$ **Ans.**

(b) $\qquad$ Radius of a solid sphere, $r = 15$ cm

Volume of a solid sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi\,(15)^3 \text{ cm}^3.$$

Now, $\qquad$ radius of right circular cone = 2·5 cm

and $\qquad$ height, $h = 8$ cm.

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\,(2·5)^2 \times 8$$

$$\therefore \quad \text{The number of cones} = \frac{\text{Volume of a sphere}}{\text{Volume of a cone}}$$

$$= \frac{\frac{4}{3}\pi \times (15)^3}{\frac{1}{3}\pi (2 \cdot 5)^2 \times 8}$$

$$= \frac{15 \times 15 \times 15}{2 \cdot 5 \times 2 \cdot 5 \times 2}$$

$$= 270 \qquad \textbf{Ans.}$$

(c) Given equation $\qquad x^2 + (p - 3)x + p = 0$

$\therefore$ Roots are real and equal, then

$$b^2 - 4ac = 0$$

Here we compare the coefficients of a, b and c with the equation $ax^2 + bx + c = 0$.

$$a = 1, b = p - 3 \text{ and } c = p$$

Now putting the values of a, b and c in equation

$$(p - 3)^2 - 4 \times 1 \times p = 0$$

$$p^2 + 9 - 6p - 4p = 0$$

$$p^2 + 9 - 10p = 0$$

$$p^2 - 10p + 9 = 0$$

$$p^2 - 9p - p + 9 = 0$$

$$p(p - 9) - 1(p - 9) = 0$$

$$\Rightarrow \qquad (p - 9)(p - 1) = 0$$

Hence,

$$p = 9 \text{ or } 1 \qquad \textbf{Ans.}$$

Question 9.

(a) In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$ [3]
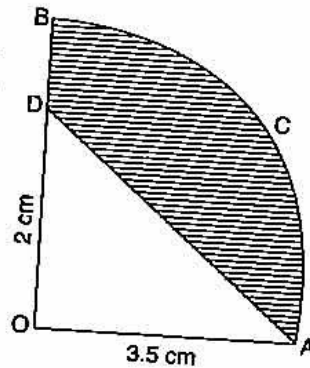
(b) A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. [3]

(c) Find the mean of the following distribution by step deviation method : [4]

Class Interval	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	10	6	8	12	5	9

Solution :

(a) Radius of quadrant OACB, $r = 3\cdot5$ cm

$\therefore$ $\qquad$ Area of quadrant OACB $= \dfrac{1}{4}\pi r^2$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{1}{4} \times \dfrac{22}{7} \times 3\cdot5 \times 3\cdot5$

$\qquad\qquad\qquad\qquad\qquad = 9\cdot625$ cm^2.

Here, $\qquad\qquad\qquad\qquad \angle \text{AOD} = 90°$

Then $\qquad\qquad$ area of Δ AOD $= \dfrac{1}{2} \times$ base $\times$ height

Base $= 3\cdot5$ cm and height $= 2$ cm

$\therefore$ $\qquad\qquad\qquad\qquad\qquad = \dfrac{1}{2} \times 3\cdot5 \times 2 = 3\cdot5$ cm^2.

$\qquad$ Area of shaded portion $=$ Area of quadrant $-$ Area of triangle

$\qquad\qquad\qquad\qquad\qquad = 9\cdot625 - 3\cdot5$

$\qquad\qquad\qquad\qquad\qquad = 6\cdot125$ cm^2. $\qquad\qquad$ **Ans.**

(b) Let the number of black balls be x, then

$\qquad\qquad$ Total number of balls $= 30 + x$

Thus, $\qquad$ the probability of black balls $= \dfrac{x}{30 + x}$

and $\qquad$ the probability of white balls $= \dfrac{30}{30 + x}$

Given : $\qquad$ Probability of black ball $= \dfrac{2}{5} \times$ probability of white ball

$\qquad\qquad\qquad \dfrac{x}{30 + x} = \dfrac{2}{5} \times \dfrac{30}{x + 30}$

$\qquad\qquad\qquad\qquad 5x = 60$

$\qquad\qquad\qquad\qquad\quad x = 12$ $\qquad\qquad$ **Ans.**

Hence, the number of black balls $= 12$.

(c)

C.I.	Frequency (f_i)	Mid-value (x)	$d_i = \dfrac{x-a}{h}$	$f_i d_i$
20–30	10	25	−2	−20
30–40	6	35	−1	−6
40–50	8	45	0	0
50–60	12	55	1	12
60–70	5	65	2	10
70–80	9	75	3	27
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$

Here, $a = 45$ and $h = 10$

$\therefore$ $\qquad\qquad\qquad$ Mean $= a + \dfrac{\Sigma f_i d_i}{\Sigma f_i} \times h$

$\qquad\qquad\qquad\qquad\quad = 45 + \dfrac{23}{50} \times 10$

$\qquad\qquad\qquad\qquad\quad = 45 + 4\cdot6 = 49\cdot6$. $\qquad\qquad$ **Ans.**

Question 10.

(a) *Using a ruler and compasses only :*

  (i)   *Construct a triangle ABC with the following data :*
        AB = 3.5 cm, BC = 6 cm and ∠ABC = 120°

  (ii) *In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.*

  (iii) *Measure ∠BCP.*   [3]

(b) *The mark obtained by 120 students in a test are given below :*

Marks	No. of Students
0–10	5
10–20	9
20–30	16
30–40	22
40–50	26
50–60	18
60–70	11
70–80	6
80–90	4
90–100	3

Draw an ogive for the given distribution on a graph sheet.

Using suitable scale for ogive to estimate the following :

  (i)   *The median.*

  (ii) *The number of students who obtained more than 75% marks in the test.*

  (iii) *The number of students who did not pass the test if minimum marks required to pass is 40.*   [6]

Solution :

(a) **Steps of Construction :**

  (i)   Draw a line BC = 6 cm.

  (ii)   With the help of the point B, draw ∠ABC = 120°
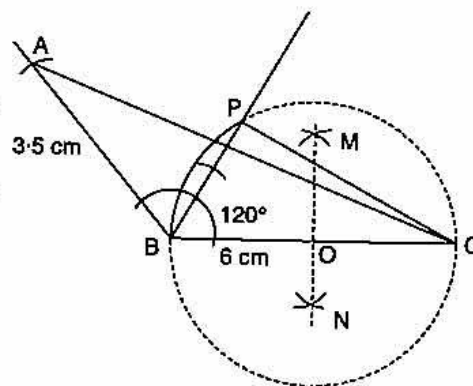
  (iii) Taking radius 3·5 cm cut BA = 3·5 cm.

  (iv)   Join A to C.

  (v)   Draw ⊥ bisector MN of BC.

  (vi)   Draw a circle O as centre and OC as radius.

  (vii) Draw angle bisector of ∠ABC which intersects circle at P.

  (viii) Join BP and CP.
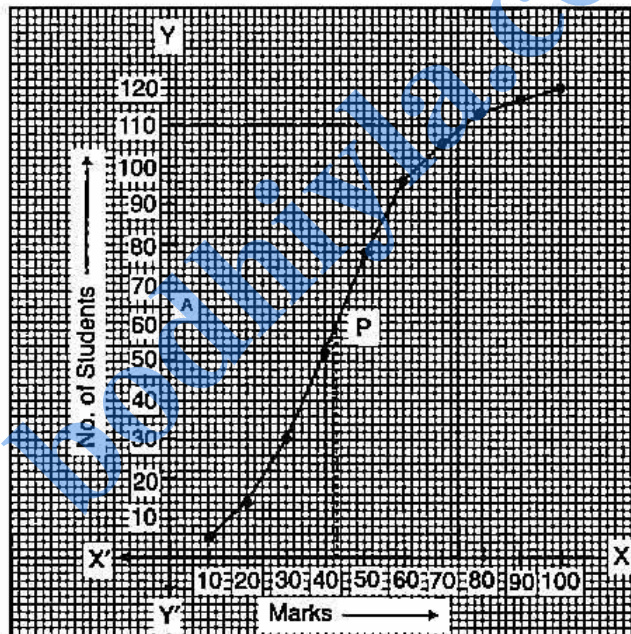
  (ix)   Now, ∠BCP = 30°.

(b)

Marks	No. of Students (*f*)	Cumulative Frequency
0–10	5	5
10–20	9	14
20–30	16	30
30–40	22	52
40–50	26	78
50–60	18	96
60–70	11	107
70–80	6	113
80–90	4	117
90–100	3	120
	n = 120	

On the graph paper, we plot the following points :

(10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120).

(i)
$$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{term} \qquad [\because n = 120, \text{ even}]$$

$$= \frac{120}{2} = 60^{th} \text{ term}$$

From the graph 60th term = 42 **Ans.**

(ii) The number of students who obtained more than 75% marks in test
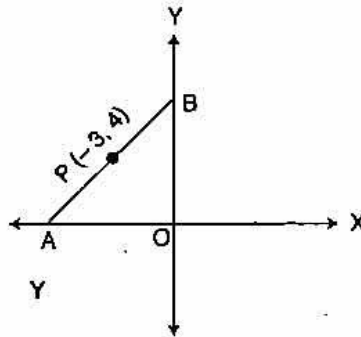
$$= 120 - 110$$

$$= 10. \qquad \textbf{Ans.}$$

(iii) The number of students who did not pass the test if the minimum pass marks 40 = 52. **Ans.**

Question 11.

(a) *In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(–3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.*

(b) *Using the properties of proportion, solve for x, given*

$$\frac{x^4 + 1}{2x^2} = \frac{17}{8}$$ [3]

(c) *A shopkeeper purchases a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books.* [4]

Solution :

(a) Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$

∵ The co-ordinates of a point P $(–3, 4)$ on AB divides it in the ratio 2 : 3.

i.e., $\qquad$ AP : PB $= 2 : 3$

By using section formula, we get

$$-3 = \frac{2 \times 0 + 3 \times x}{2 + 3} \qquad \left[\because x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \right]$$

$$-3 = \frac{3x}{5} \implies 3x = -15$$

$$\implies \qquad x = -5$$

and $\qquad 4 = \frac{2 \times y + 3 \times 0}{2 + 3} \qquad \left[\because y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right]$

$$4 = \frac{2y}{5} \implies 2y = 20$$

$$\implies \qquad y = 10$$

Hence, the co-ordinates of A and B are $(–5, 0)$ and $(0, 10)$. **Ans.**

(b) *Given :* $\qquad \dfrac{x^4 + 1}{2x^2} = \dfrac{17}{8}$

By using componendo and dividendo, we get

$$\frac{x^4 + 1 + 2x^2}{x^4 + 1 - 2x^2} = \frac{17 + 8}{17 - 8}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \frac{25}{9}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

Taking square root on both sides, we get

$$\frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$$\Rightarrow \qquad 5x^2 - 5 = 3x^2 + 3$$

$$\Rightarrow \qquad 5x^2 - 3x^2 = 3 + 5$$

$$\Rightarrow \qquad 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4$$

$$\Rightarrow \qquad x = \pm 2 \qquad\qquad\qquad \textbf{Ans.}$$

(c) Given the original cost of each book be ₹ x.

$$\text{Total cost} = ₹960 \qquad\qquad \text{(Given)}$$

$$\therefore \qquad \text{Number of books for 960} = \frac{960}{x}$$

If the cost per book was ₹ 8 less, (*i.e.*, $x - 8$) then

$$\text{Number of books} = \frac{960}{x-8}$$

According to question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960\left[\frac{x - x + 8}{x(x-8)}\right] = 4$$

$$\frac{8}{x^2 - 8x} = \frac{1}{240}$$

$$\Rightarrow \qquad x^2 - 8x = 1,920$$

$$\Rightarrow \qquad x^2 - 8x - 1,920 = 0 \qquad\qquad \textbf{Ans.}$$

$$\Rightarrow \qquad x^2 - 48x + 40x - 1,920 = 0$$

$$\Rightarrow \qquad x(x - 48) + 40(x - 48) = 0$$

$$\Rightarrow \qquad (x - 48)(x + 40) = 0$$

$$x - 48 = 0 \qquad \text{or} \quad x + 40 = 0$$

$$x = 48 \qquad \text{or} \qquad x = -40$$

∵ -40 is not possible.

Hence, the original cost of each book = ₹ 48. **Ans.**

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