MATHEMATICS

(Maximum Marks: 100)

(Candidate are allowed additional 15 minutes for only reading the paper,

They must NOT start writing during the time)

The Question paper consists of three section A, B and C Candidate are required to attempt all question from **Section A** and all questions.

Either from Section **B** **OR** **Section C**

Section B: Internal choice has been provided in two question of four marks each.

Section C: Internal choice has been provided in two question of four marks each.

All working including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of question are given in brackets [ ]

Mathematical tables and graph papers are provided.

Question 1

(i)If f:R→R_+ f(x) = x^3 and g: R→R, g(x) = $2x^2+1$,and R is the set of real number then find fog(x) and gof (f).

Sol.

We have the g(x) = $2x^2+1$

$$fog(x) \rightarrow f(g(x))$$
$$= f(2x^2+1)$$

Since, $f(x) = x^3$

Therefore,

$$= (2x^2+1)^3$$
$$= gof(x) \rightarrow g(f(x)$$
$$= g(x^3)$$

Now, put x^3 in the function of g then we get,

$= 2(x^3)^2 + 1$

$= 2x^6 + 1$

Hence, we get the value. $2x^6 + 1$

(ii) Solve: Sin ($2 \tan^{-1} x$) = 1

Sol.

Simplify the value of *x*.

$\sin(2\tan^{-1} x) = 1$

$2\tan^{-1} x = \sin^{-1} 1 = \frac{\pi}{2}$

$\tan^{-1} x = \frac{\pi}{4}$ Since $\sin^{-1} = \frac{\pi}{2}$

$x = \tan\frac{\pi}{4} = 1$

$x = 1$

Hence, we get the value of x is 1.

(iii) Using determinants, find the values of k; if the area of triangle with verticals $(-2,\ 0)$ $(0,4)$ and $(0,k)$ is 4 square units.

Sol.

Solving the value of *k*.

$$\triangle = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$4 = \frac{1}{2}\left[-2(4-k)\right]$

$8 = -8 + 2k$

$16 = 2k$

$k = 8$

Since, we get the value of $k = 8$

(iv) Show that (A+A$^{'}$) is symmetric matrix, if A=$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

Sol.

We have the value of symmetric matrix $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ Simplify the value of $\left(A + A^{'}\right)$

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$
$$A^{1} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$
$$p = A + A^{'} = \begin{bmatrix} 4 & 7 \\ 7 & 10 \end{bmatrix}$$
$$p' = \begin{bmatrix} 4 & 7 \\ 7 & 10 \end{bmatrix} = p$$

Since, the given value is p and p$^{'}$ is equal and we get the value of $p' = \begin{bmatrix} 4 & 7 \\ 7 & 10 \end{bmatrix} = p$

(v) f(x)=$\dfrac{x^2 - 9}{x - 3}$ is not defined at x=3. What value should be assigned to f(3) for continuity of f(x) at x= 3?

Sol.
Solving the continuity of f(3).

$$f\left(x\right) = \frac{x^2 - 9}{x - 3}$$
$$= \frac{\left(x-3\right)\left(x+3\right)}{x - 3}$$
$$= x + 3$$
$$f(3) = 3 + 3$$
$$f(3) = 6$$

Hence, the value of $f(3) = 6$

(vi) Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on R.

Sol.

We have the function $f(x) = x^3 - 6x^2 + 12x + 5$

By doing first order derivative we get,

$$f(x) = x^3 - 6x^2 + 12x + 5$$
$$f'(x) = 3x^2 - 12x + 12$$
$$=3\left(x^2 - 4x + 4\right)$$
$$=3\left(x-2\right)^2 > 0$$

Therefore, 3 is positive and the square of any thing is always greater than zero.

Hence, the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on R.

(vii) Evaluate $\int \frac{\sec^2 x}{\cos ec^2 x} dx$

Sol.

Simplify the integral.

$$\int \frac{\sec^2 x}{\cos ec^2 x}$$
$$= \int \frac{1}{\cos^2 x} \cdot \frac{1}{\frac{1}{\sin^2 x}} dx$$
$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$
$$= \int \tan^2 x dx$$
$$= \sec^2 \cdot \tan^2 =`1$$
$$= se c^2 x - 1 = \tan^2$$
$$= \int \left(se c^2 x - 1\right) dx$$
$$= \int se c^2 x dx - \int 1 dx$$
$$= \tan x - x + c$$

Since , the obtain value is $\tan x - x + c$

(viii) Using L Hospital Rule evaluate $\lim x \to 0 \ \frac{8^x - 4^x}{4x}$

Sol.

Simplify the expression of L Hospital Rule.

$$\lim x \to 0 \ \frac{8^x - 4^x}{4x}$$

$$= \frac{\left(8^0 - 4^0\right)}{4(0)} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \lim_{\to 0} \ \frac{8^x \log_e 8 - 4^x \log_e 4}{4}$$

$$= \frac{8^0 \log 8 - 4^0 \log 4}{4}$$

$$=\log \frac{\left(\frac{8}{4}\right)}{4} \to \frac{\log e^2}{4}$$

$$= \frac{\log e^2}{4}$$

Hence, the value of $\frac{\log e^2}{4}$

(ix) Two balls are drawn from containing 3 white, 5 red and 2 black ball, one by one without replacement. What is the probability that least one ball is red?

Sol.

Simplify the expression least one ball of red.

We have the white ball is 3, 5 ball is red and 3 ball is black simplify futher.

Therefore, the 2 ball are drown without x placement.

Probability p at least least one ball is red =p(1red)+p(2red).

$$=\frac{5}{10}\times\frac{5}{9}+\frac{5}{10}\times\frac{4}{9}$$

$$=\frac{25}{90}+\frac{20}{90}$$

$$=\frac{45}{90}$$

$$=\frac{1}{2}$$

Since the least one ball is red probability obtain $\frac{1}{2}$.

(x) If event A and B are independent, such that P(A) = $\frac{3}{5}$, P(B) = $\frac{2}{3}$,

Sol.

Solving the expression.

$L.H.S\ \ \sec^{-1}x=t$

x=sect = $\frac{1}{\cos t}$

cos = $\frac{1}{x}$............(1)

$\text{cosec}^{-1}=t$

y = cosect = $\frac{1}{\sin t}$

sint = $\frac{1}{y}$...........(2)

$$\cos^2 t+\sin^2 t=\frac{1}{x^2}+\frac{1}{y^2}$$

1= $\frac{1}{x^2}+\frac{1}{y^2}$

$$\frac{1}{x^2}+\frac{1}{y^2}=1$$

Hence, the value is $\frac{1}{x^2}+\frac{1}{y^2}=1$

Question 2 **[4]**

If f: A$\rightarrow$A and A= R = $\frac{8}{5}$, show that the function f(x)= $\frac{8x+3}{5x-8}$ is one-one onto.

Hence, find f^{-1}

Sol.

Simplify the expression.

Such that,

$$f(x_1) = \frac{8x_1 + 3}{5x_1 - 8}$$

$$f(x_2) = \frac{8x_2 + 3}{5x_2 - 8}$$

Then, $f(x_1) = f(x_2)$

$$\frac{8x_1 + 3}{5x_1 - 8} = \frac{8x_2 + 3}{5x_2 - 8}$$

$$(8x_1 + 3)(5x_2 - 8) = (8x_2 + 3)(5x_1 - 8)$$

$$= 40x_1x_2 - 64x_1 + 15x_2 - 24 = 40x_1x_2 - 64x_2 + 15x_1 - 24$$

$$= -64x_1 + 15x_2 = -64x_2 + 15x_1$$

$$= -64(x_1 - x_2) = 15(x_1 - x_2)$$

$$x_1 = x_2$$

Then, we get to $x_1 = x_2$ and f(x) one-one function.

$$y = \frac{8x+3}{5x-8}$$

$$5xy - 8y = 8x + 3$$

$$= 5xy - 8x = 3 + 8y$$

$$= x[5y - 8] = 3 + 8y$$

$$x = \frac{3+8y}{5x-8}$$

$$\leftarrow R - \left[\frac{8}{5}\right]$$

Since, the onto function onto is $f^{-1}(x)\frac{3+8x}{5x-8}$.

Question 3 **[4]**

(a) Solve for x:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

Sol.

Simplify the expression.

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

We know that,

$$\tan^{-1}(A+B)=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$=\tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{(x-1)(x+1)}{(x+1)(x+2)}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{x^2-x+2x-2+x^2-2x+x-2}{x^2-4}}{\frac{x^2-4-x^2-1}{x^2-4}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-4}{-3}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{-3}=\tan\left(\frac{\pi}{4}\right)=1$$

$$\Rightarrow 2x^2-4=-3$$

$$\Rightarrow 2x^2=4-3$$

$$\Rightarrow x^2=\frac{1}{2}$$

$$x=\pm\frac{1}{\sqrt{2}}$$

Hence, the value is $x = \pm\frac{1}{\sqrt{2}}$.

OR

If $\sec^{-1}x = \operatorname{cosec}^{-1}y$, show that $\frac{1}{x^2} + \frac{1}{y^2} = 1$

Sol. Simplify the expression.

$L.H.S \quad \sec^{-1}x = t$

$$x = \sec t = \frac{1}{\cos t}$$

$$\cos = \frac{1}{x} \ldots\ldots\ldots\ldots(1)$$

$$\operatorname{cosec}^{-1} = t$$

$$y = \operatorname{cosec} t = \frac{1}{\sin t}$$

$$\sin t = \frac{1}{y} \ldots\ldots\ldots\ldots(2)$$

$$\cos^2 t + \sin^2 t = \frac{1}{x^2} + \frac{1}{y^2}$$

$$1 = \frac{1}{x^2} + \frac{1}{y^2}$$

Hence, we have to prove $1 = \frac{1}{x^2} + \frac{1}{y^2}$

Question 4 **[4]**

Using properties of determinants prove that:

$$\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ y & y\left(y^2+1\right) & y+1 \\ z & z\left(z^2+1\right) & z+1 \end{bmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

Sol.

Simplify the expression properties of determinants.

$$\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ y & y\left(y^2+1\right) & y+1 \\ z & z\left(z^2+1\right) & z+1 \end{bmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - R_1$

$$\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ y-x & y\left(y^2+1\right)-x\left(x^2+1\right) & y+1-(x+1) \\ z-x & z\left(z^2+1\right)-x\left(x^2+1\right) & z+1-(x+1) \end{bmatrix}$$

$$\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ y-x & y^3+y-x^3-x & y-x \\ z-x & z^3+z-x^2-x & z+1-(x+1) \end{bmatrix}$$

$$\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ y-x & (y-x)\left(y^2+x^2+yx\right)+(y-x) & y-x \\ z-x & (z-x)\left(z^2+x^2+zx\right)+(z-x) & z-x \end{bmatrix}$$

$$(y-x)(z-x)\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ 1 & y^2+x^2+yx+1 & 1 \\ 1 & (z-x)\left(z^2+x^2+zx\right)+(z-x) & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$(y-x)(z-x)\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ 0 & y^2-z^2+x(y-z) & 0 \\ 1 & z^2+x^2+zx+1 & 1 \end{bmatrix}$$

$$(y-x)(y-z)(z-x)(x+y+z)\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ 0 & 1 & 0 \\ 1 & z^2+x^2+zx+1 & 1 \end{bmatrix}$$

$$(y-x)(z-x)(y-z)(x+y+z)\begin{bmatrix} x & x\left(x^2+1\right) & x+1 \\ 0 & 1 & 0 \\ 1 & z^2+x^2+zx+1 & 1 \end{bmatrix}$$

$$(y-x)(z-x)(y-z)(x+y+z)\left[+1(x-x-1)\right]$$

$$(y-x)(z-x)(y-z)(x+y+z) = R.H.S$$

Hence, we have the prove is R.H.S

Question 5 **[4]**

(a) Show that the function f(x) = $|x-4|$, $x \varepsilon R$ is continuous, but not differentiable at $x = 4$.

Sol.

Simplify the expression

$f(x) = |x-4|$

At $x = \bar{4}$

$\lim f(x)$

$x \to 4$

L.H.L

Put, x=4 - h

So, $x \to 4$

$h \to 0$

$\lim_{h\to0} f(x)$

$\lim_{h\to0} [4-h-4] = 0$

At x=4

$F(x) = |4-4| = 0$

R.H.L

x=4 + h

$x \to 4, \ h \to 0$

$\lim_{h\to0} (4+h)$

$\lim_{h\to o} (4+h-4)$

$= 0$

L.H.L=R.H.L=Lim $F(x)$ $h \to 4$

F(x) is continuous at x=4

F(x) =(x-4)

L.H.L

$$\lim_{h\to 0} \frac{f(a-h) - f(a)}{-h}$$

$$\lim_{h\to 0} \frac{f(4-h) - f(4)}{-h}$$

$$\lim_{h\to 0} \frac{(4-h-4)-(4-4)}{-h}$$

$$\lim_{h\to 0} \frac{h-0}{-h} = -1$$

$R.H.L$

$$\lim_{h\to 0} \frac{f(a-h) - f(a)}{h}$$

$$\lim_{h\to 0} \frac{f(4-h) - f(4)}{h}$$

$$\lim_{h\to 0} \frac{(4+h-4)-(4-4)}{h}$$

$$\lim_{h\to 0} \frac{h}{h} = 1$$

$L.H..S \neq R.H.S$

f(x) is not differentiable .

Hence, the value is L.H.D is not equal to R.H.D and f(x) is not differential $x = 4$

(b) Verify the Language's mean value theorem for the function.

$f(x)$ =x+$\frac{1}{x}$ in the interval c

Sol.

Simplify the expression.

We have the function below is.

$f(x)$ =x+$\frac{1}{x}$

F(x) is continuous in $[1,\ 3]$

F(x) is the differentiable is $[1,\ 3]$

$$f^{'}(c) = \frac{f(b)-f(b)}{b-a}$$

Where c is the point lying a&b

$$f'(x) = 1-\frac{1}{x^2}$$

$$f'(x) = 1-\frac{1}{c^2}$$

$$f(a) = f(1) = 1+\frac{1}{1} = 2$$

$$f'(x) = 3+\frac{1}{3} = \frac{10}{3}$$

$$f'(c) = 1-\frac{1}{c^2} = \frac{\frac{10}{3}-2}{3-1}$$

$$1-\frac{1}{c^2} = \frac{4}{3\times 2} = \frac{2}{3}$$

$$\frac{1}{c^2} \quad 1-\frac{2}{3} = \frac{1}{3}$$

$$c^2 = 3 \quad c = \pm\sqrt{3}$$

Hence, the obtained the value is $c = \pm\sqrt{3}$ but (-) is not lying and we take is value of $\sqrt{3}$.

Question 6

If $y = e^{\sin^{-1} x}$ and $z = e^{-\cos^{-1} x}$ prove that $\frac{dy}{dx} = e^{\pi/2}$

Sol. Solve this expression.

$$y = e^{\sin^{-1} x}$$

$$\frac{dy}{dx} = e^{\sin^{-1} x} . \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dz} = e^{-\cos^{-1} x} . \frac{d}{dx}\left(-cox^{-1}\right)$$

Then,

$$\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{e^{-\sin^{-1} x} \frac{1}{\sqrt{1-x^2}}}{e^{-\cos^{-1} x} \frac{1}{\sqrt{1-x^2}}}$$

$$= e^{\sin^{-1} + \cos^{-1} x}$$

$$= e^{\frac{\pi}{2}}$$

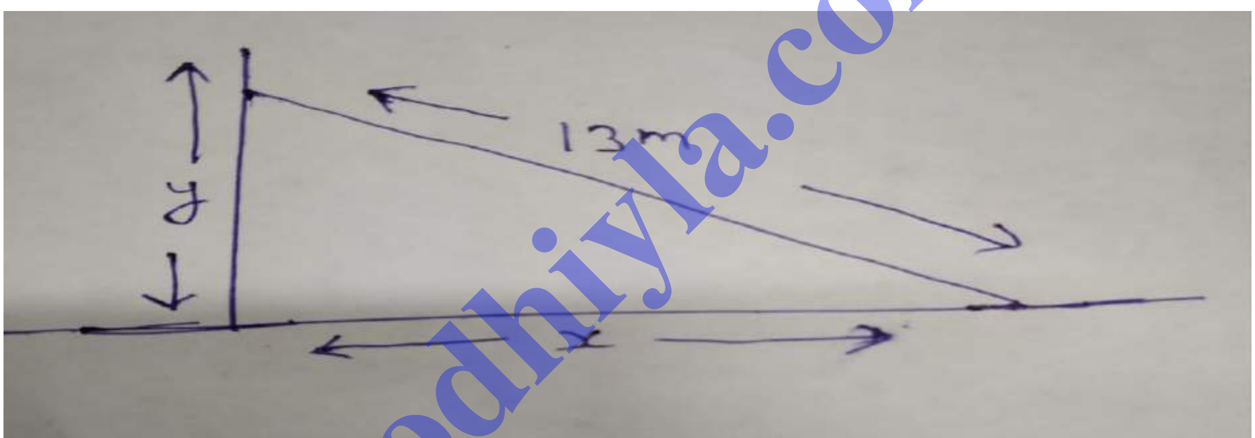
Hence, the prove this $\frac{dy}{dx} = e^{\frac{\pi}{2}}$

Question 7

A 13 in long ladder in leaning against launching the wall at a contain height from the ground level. The bottom of the ladder in pulled away from the wall, along the ground, at the rate of 2 m/s .how fast is the height on the wall decreasing when the foot of the ladder is 5 on form the wall?

Sol.

Simplify the expression.



There for the given is $\frac{dx}{dt} = 2m/s$.

Then x=5 m,

$$x^2 + y^2 = (13)^2$$

$$x = 5m$$

$$y = 12m$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2x\frac{dx}{dt} = \text{-2y}\frac{dy}{dt}$$

$$-\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

$$= \frac{5}{12} \times 2$$

$$= \frac{5}{6} m / s$$

Hence, the value is $\frac{5}{6} m / s$

Question 8 **[4]**

(a) Evaluate : $\int \frac{x(1+x^2)}{1+x^4} dx$

Sol. Simplify the expression.

$$x^2 = t$$

$$2xdx = dt$$

$$\frac{1}{2}\int \frac{(1+t)}{(1+t^2)} dt \rightarrow \frac{1}{2}\left[\int \frac{dt}{1+t^2} + \frac{1}{2}\int \frac{2t}{1+t^2} dt\right]$$

$$= \frac{1}{2}\left[\tan^{-1} t + \frac{1}{2}\log(1+t^2)\right] + c$$

Replacing

$$t \rightarrow x^2$$

$$= \frac{1}{2}\left[\tan^{-1}(x^2) + \frac{1}{2}\log|1+x^4|\right] + c$$

Hence, we get the value of $\frac{1}{2}\left[\tan^{-1}(x^2) + \frac{1}{2}\log|1+x^4|\right] + c$.

OR

(b) Evaluate: $\int_{-6}^{3}(x+3)dx$

Sol.

Simplify the expression.

We show that integral function is $\int_{-6}^{3}(x+3)dx$

$$\int_{-6}^{3}(x+3)$$

$$\int_{-6}^{3}-(x+3)dx+\int_{-3}^{3}(x+3)dx$$

$$=-\left[\frac{x^2}{2}+3x\right]_{-6}^{-3}+\left[\frac{x^2}{2}+3x\right]_{-3}^{3}$$

$$= =-\left[\frac{(-3)^2}{2}+3(-3)\right]-\left[\frac{(-6)^2}{2}+3(-6)\right]+\left[\frac{(3)^2}{2}+3(3)\right]-\left[\frac{(-3)^2}{2}-9\right]$$

$$=-\left[\frac{9}{2}-9-18+18\right]+\left[\frac{9}{2}+9-\frac{9}{2}+9\right]$$

$$=-\frac{9}{2}+9+9+9=27-\frac{9}{2}=\frac{54-9}{2}$$

$$=\frac{45}{2}$$

Hence, we get the solution is $\frac{45}{2}$

Question 9 **[4}**

Solve the differential equation: $\frac{dy}{dx}=\frac{x+y+2}{2(x+y)-1}$

Sol. simplify the expression.

$$\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-y}$$

let x+y =v

$$1+\frac{dy}{dx} = \frac{dv}{dx}, \qquad \frac{dy}{dx} = \frac{d\theta}{dx} = -1$$

$$\frac{dv}{dx} - 1 = \frac{v+2}{2v-1}$$

$$\frac{dv}{dx} = \frac{v+2+2v-1}{2v-1} = \frac{3v+1}{2v-1}$$

$$= \left(\frac{2v-1}{3v+1}\right)dv$$

Therefore, integral both side.

$$\int \frac{2v-1}{3v+1} d\theta = \int dx$$

$$\int \left[\frac{2}{3} - \frac{5}{3}\left(\frac{1}{3v+1}|\ |\right)\right] dv = \int dx$$

$$\int \frac{2}{3} dv - \frac{5}{3}\int \frac{1}{3v+1} dv = \int dx$$

$$\int \frac{2}{3} dv - \frac{5}{3}\frac{\log|3v+1|}{3} = x$$

$$\frac{2}{3}(x+y) - \frac{5}{9}\log|3(x+y)+1| = x$$

Hence, the value get is $\frac{2}{3}(x+y) - \frac{5}{9}\log|3(x+y)+1| = x$

Question 10 **[4]**

Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two ball are drown from Bag A and placed in bag B. Then, what is the probability of drawing a white ball from Bag B?

Sol.

Simplify the probability expression.

BAG A $\begin{matrix} W & B \\ 4 & 3 \end{matrix}$

BAG B 3 5

The two ball are drawn from Bag A& placed in B.

Case 1. Both white ball are drawn from Bag B.

P (drawn white ball) = $\frac{4c_2}{7c_2}$.

P (drawn a white Ball from Bag B = $\frac{5}{10}$.

We get $\frac{4c_2}{7c_2} \times \frac{5}{10}$

Case 2. Both black balls one drawn from A

White ball from Bag 2.

The required probability $= \frac{3c_2}{7c_2} \times \frac{3}{10}$.

Case3. One white $ one black.

The required probability $= \frac{4c_2}{7c_2} \times \frac{4}{10}$

Adding the 1 case and 2 case, and 3 case that is the solution.

Hence, the solution is case 1, case 2 and case 3 adding we get the solution.

Question 11 **[6]**

Solve the following system of linear equation using matrix method:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$$
$$\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$$
$$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

Sol. Simplify the expression.

We have the given value below is .

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9,\ \frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52,\ \frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

Let $\frac{1}{x}=a,\ \frac{1}{y}=b,\ \frac{1}{z}=c$

$$a+b+c=9$$
$$2a+5b+7c=52$$
$$2a+b-c=0$$

AX= B where,

$$A=\begin{bmatrix}1 & 1 & 1\\ 2 & 5 & 7\\ 2 & 1 & -1\end{bmatrix}\qquad X=\begin{bmatrix}a\\ b\\ c\end{bmatrix}\qquad B=\begin{bmatrix}9\\ 52\\ 0\end{bmatrix}$$

$X = A^{-1}B$

Then,

$$\begin{bmatrix}1 & 1 & 1\\ 2 & 5 & 7\\ 2 & 1 & -1\end{bmatrix}$$

$$\text{Adj A}=\begin{bmatrix}+(-12) & +16 & (-8)\\ +2 & +(-3) & +1\\ +2 & -5 & +3\end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA$$

$$|A| = 1(-5-7) - 1(-2-14) + 1(2-10)$$
$$= -12 + 16 - 8 = -4$$

$$\frac{1}{-4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 8 & 1 & 3 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{-4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 8 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

$$\frac{1}{-4}\begin{bmatrix} -12\times 9 & 52\times 2 & +0 \\ 16\times 9 & -3\times 52 & +0 \\ -8\times 9 & +1\times 52 & +0 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -108 & +104 & \\ 144 & -156 & \\ -72 & +52 & \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$= a = 1$, b=3, c=5

$x=1$, $y=\frac{1}{3}$, $z=\frac{1}{5}$

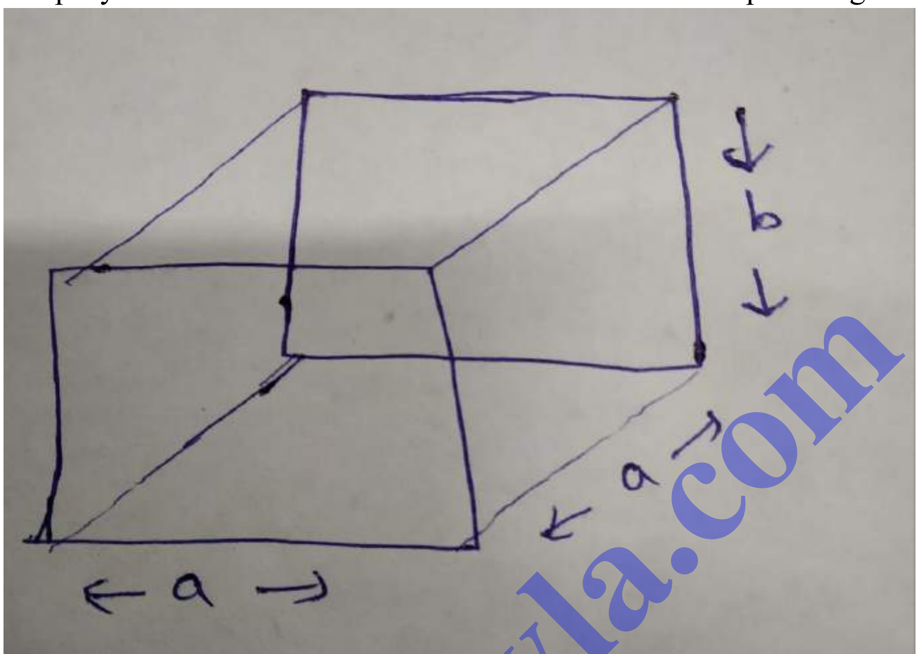
Hence, we obtained the value is $x=1$, $y=\frac{1}{3}$, $z=\frac{1}{5}$.

Question 12 **[6]**

(a) The volume of a closed rectangular metal box with a square is $4096 cm^3$.The cost of polishing the outer surface of the box is Rs 4 per cm^2. Find the dimensions of the box for the minimum cost of polishing it.

Sol.

Simplify the dimensions of the box for the minimum cost polishing.



We have the formula volume of cube.

$$V = a^2 b = 4096$$
$$A = 2\left(a^2 + ab\right)$$

$$4 \times 2\left[a^2 + 2ab\right]$$
$$= 8\left[a^2 + 2ab\right]$$
$$= 8a^2 + 16ab$$
$$= 8a^2 + 16a\left(\frac{4096}{a^2}\right)$$
$$= 8a^2 + \frac{16 \times 4096}{a^2}$$
$$y = 8a^2 + \frac{65536}{a}$$
$$0 = \frac{dy}{dx} = 16a - \frac{65536}{a^2}$$
$$16a = \frac{65536}{a^2}$$
$$a^3 = \frac{65536}{16} = 4096$$
$$a = (4096)^{\frac{1}{3}} = 16$$
$$a = 16$$
y= Total cost = $\frac{d^2y}{dx^2} = 16 + \frac{2 \times 65536}{a^3} > 0$
$$a^2 b = 4096$$
$$(16)^2 b = 4096$$
$$b = 16$$

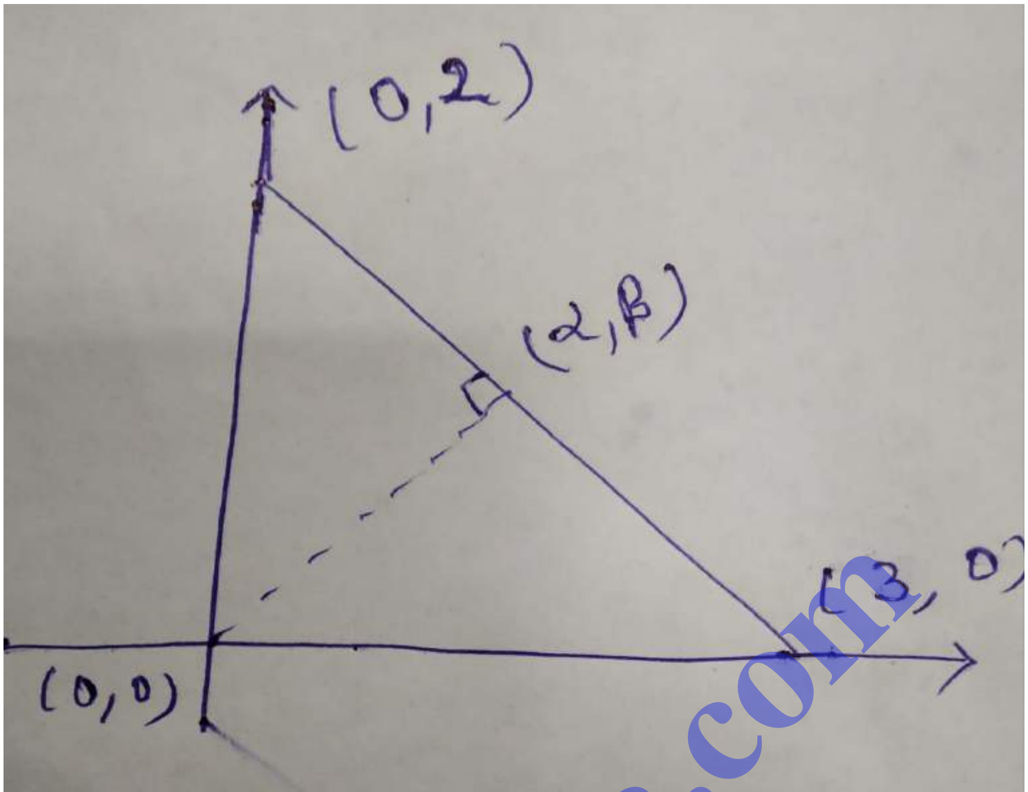
Since, the, minimum cost is 16.

OR

(b) Find the point on the straight line 2x+3y=6, which is closest to the origin.

Sol.

Simplify the expression.


(0,2)
(α,β)
(3, 0)
(0,0)

$2\alpha + 3\beta = 6$

$2\alpha = \frac{6-3\beta}{2}$

$\alpha = 3 - \frac{3}{2}R$

$2x + 3y = 6$

$(\alpha - 0)^2 + (\beta - 0)^2 \quad D^2 = y$

$\alpha^2 + \beta^2 = y$

$0 + 2\left(\frac{9}{2}\right)\beta - 9 + 2\beta = \frac{dy}{d\beta} = 0$

$\frac{13\beta}{2} = 9$

$\frac{13\beta}{2} = 9$

$\beta = \frac{18}{13}$

$\alpha = 3 - \frac{3}{2}\left(\frac{18}{13}\right)$

$= 3 - \frac{27}{13}$

$\alpha = \frac{12}{13}$

$\frac{d^2y}{d\beta^2} = \frac{13}{12} > 0$

Hence, we get the origin point is $\alpha = \frac{12}{13}$ and $\frac{13}{12}$.

Question 13 **[6]**

Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

Sol.

Simplify the value is.

We have this expression.

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

$$\int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x)} dx \quad =$$

$$\int_0^{\pi} \frac{(\pi - x)\tan x}{(\sec x + \tan x)} dx \quad = \quad 2$$

$$\int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx + \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x} dx \; = 2$$

$$\int_0^{\pi} \frac{\tan x(\sec x - \tan x)dx}{\sec^2 x - \tan^2 x} \quad = 2$$

$$_0^{\pi}\left[\sec x - [\tan x - x]\right]_0^{\pi} = \frac{2}{\pi} .....(1)$$

$$[\sec \pi - \sec 0] - [\tan \pi - \pi + \tan 0 + 0] = \frac{2}{\pi} ....(1)$$

$$(-1-1) - (0 - \pi) = \frac{2}{\pi} .....(1)$$

$$-2 + \pi \;\; = \frac{2}{\pi} ....(1)$$

$$\frac{\pi}{2}(\pi - 2) \quad =(1)$$

Hence, the integral value is $\frac{\pi}{2}(\pi - 2) \quad =(1)$.

Question 14 **[6]**

(a) Given three identical Boxes A, and B and C, Box A contains 2 gold and 1 silver coin, Box B contains 1 gold and 2 silver coins and Box C contains 3 silver coins. A person chooses a Box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver.

Sol: Simplify the expression.

A		B		C
gold	silver	gold	silver	silver
2	1	1	2	3

$$P(A) = \text{P(B)} = \text{P(C)} = \frac{1}{3}$$

$$P(S/A) = \frac{1}{3}$$

$$P(S/B) = \frac{2}{3}$$

$$P(S/B) = \frac{3}{3}$$

$$P(C/S) = \frac{P(C)\times P(S/C)}{P(A)P(S/A)+P(B)P(S/B)+P(C)P(S/C)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\times 3}$$

$$= \frac{1/3}{\frac{1}{9}+\frac{2}{9}+\frac{9}{9}} = \frac{1/3}{12/9} = \frac{1}{4}$$

Hence, the probability also get the two coin of silver is $\frac{1}{4}$.

OR

(b) Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$. Also, find the probability of obtaining at most one success.

Sol.

Simplify the expression.

$np = 9$

$\sigma = \frac{3}{2} = \sqrt{npq} = \frac{3}{2}$

$9 \times q = \frac{9}{4}$

$q = \frac{1}{4}$

$p = 1 - \frac{3}{4}$

$n \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{4}$

$n = \frac{1}{4}$

${}^{n}C_{r} p^{0} q^{n-r}$

$P(r \leq 1) = P_0 + P_1$

$= {}^{12}C_{1} \left(\frac{3}{4}\right)^{0} \left(\frac{1}{4}\right)^{12} + {}^{12}C_{0} \left(\frac{3}{4}\right)^{1} \left(\frac{1}{4}\right)^{11}$

$= \left(\frac{1}{4}\right)^{12} + 12 \times \frac{3}{4^{12}}$

$= \frac{37}{(4)^{12}}$

Hence, we get the solution is $\frac{37}{(4)^{12}}$.

Question 15 **[3×2]**

(a) If $\bar{a}$ and $\bar{b}$ are perpendicular vectors, $\left[\bar{a} + \bar{b}\right]$=13 and $\left|\bar{a}\right| = 5$, find the value of $\left|\bar{b}\right|$.

Sol.
Simplify the expression.

We have the vector is.
So, here

$$\left|\vec{a}+\vec{b}\right|=13$$

$$\left(\vec{a}+\vec{b}\right)^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\vec{a}.\vec{b}$$

$$(13)^2=(5)^2+b^2+2ab\cos 90^0$$

$$169 = 25 + b^2 + 0$$

$$b^2 = 144$$

$$b = 12$$

Hence, we get the value is $b=12$

(b) Find the length of the perpendicular form origin to the place $\overline{r}.(3i-4j-12k)+3=0$.

Sol.
Simplify the expression.
We have the equation is.

$$\vec{r}\cdot(3i-4j-12k)+39=0$$

$$3x-4y-12z+39=0$$

Perpendicular from orgin.

$$x^2=y$$

$$x^2+x-2=0$$

$$x^2+2x-x-2=0$$

$$x(x+2)-1(x+2)=0$$

$$x=1,\ x=-2$$

$$\int_{-2}^{1}(2-x)-x^2dx$$

$$=\left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_2^1$$

$$=\left[2-\frac{1}{2}-\frac{1}{3}\right]-\left[-4-\frac{4}{2}+\frac{8}{3}\right]$$

$$=\left[\frac{7}{6}\right]-\left[\frac{-20}{6}\right]$$

$$=\frac{27}{6}=\frac{9}{2}$$

$$=\frac{9}{2}$$

Hence, we get the value of $\frac{9}{2}$.

(c) Find the angle between the two lines $2x-3y=-z$ and $\vec{a}\times\vec{b}$.

Sol.
Simplify the expression.

$$2x = 3y = -z$$

$$\frac{x-0}{\frac{1}{2}} = \frac{4-0}{\frac{1}{3}} = \frac{z-0}{-1}$$

$$a_1 = \frac{1}{2},\ b_1 = \frac{1}{3},\ c_1 = -1$$

$$6x = -y = -4z$$

$$\frac{x-0}{1/6} = \frac{y-0}{-1} = \frac{z-0}{-1/4}$$

$$a_2 = 1/6,\ b_2 = -1,\ c_2 = -1/4$$

Suppose, θ be angle between two lines.
We have formula below in.

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a^2{}_1 + b^2{}_1 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{\frac{1}{2}\times\frac{1}{6} + \frac{1}{3}(-1) + (-1)\times\left(-\frac{1}{4}\right)}{\sqrt{(1/2)^2 + (1/3)^2 + (-1)^2}\sqrt{(1/6) + (-1)^2 + (-1/4)^2}}$$

$$\cos\theta = \frac{\frac{1-4+3}{12}}{\sqrt{\frac{9+4+36}{36}}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Hence, we getting $\cos\theta = 0$. So the value is $\theta = \frac{\pi}{2}$.

Question 16 **[4]**

(a) If $\overline{\alpha} = i - 2j + 3k$, $\overline{b}$=2i+3j-5k, prove that and $\vec{a}\times\vec{b}$ are perpendicular.

Sol.
Simplify the expression.

$\vec{a} = i - 2j + 3k,\ \vec{b} = 2i + 3j - 5k$

$\vec{a} \times \vec{b} = \begin{vmatrix} i & \text{j} & \text{k} \\ 1 & \text{-2} & \text{-5} \end{vmatrix}$

$= i(10-9)$ -j $(-5-6)$ +k $(3+4)$

$= i + 11j + 7k$

$\vec{a} \cdot (\vec{a} \times \vec{b})$

$(i - 2j + 3k) \cdot (i + 11j + 7k) = 1 - 22 + 21 = 0$

$\vec{a} \perp \vec{a} \times \vec{b}$

Hence, the value of a perpendicular of b.

OR

(b) If $\vec{a}$ and $\vec{b}$ are not collinear vectors, find the value of x such that the vectors $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.

Sol.

Simplify the expression.

$(2x + 2y - 3z - 7) + k(2x + 5y + 3 - 9) = 0$

$(2+2k)x + (2+5k)y + (-3+3k)z - 4 - 9k = 0$

$(2+2k)x + (2+5k)y + (-3+3k)z = 7 + 9k$

$\dfrac{x}{\dfrac{7+9k}{2+2k}} + \dfrac{y}{\dfrac{7+9k}{2+5k}} + \dfrac{x}{\dfrac{7+9k}{3k-3}}$

$x \operatorname{int} erpl$ = z interpl

$\dfrac{7+9k}{2+2k} = \dfrac{7+9k}{3k-3}$

$21k - 21 + 27k^2 - 27k = 14 + 14k + 18k + 18k^2$

$= 9k^2 - 38k - 35 > 0$

Hence, we get the solution is $9k^2 - 38k - 35 > 0$.

Question 17 **[4]**

(a) Find the equation of three plane through the intersection of the planes $2x + 2y - 3z - 7 = 0$ and $2x + 5y + 3z - 7 = 9$ such that the intersection made by the resulting plane on the x –axis and the z-axis re equal.

OR

(b) Find the equation of the lines passing through the point $(2,1,3)$ and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

Sol. suppose equation of line passing through the point.

We have the equation below is.

$$\frac{x-2}{a}=\frac{y-1}{b}=\frac{z-3}{c}$$

$$a+2b+3c=0$$

$$=-3a+2b+5c$$

$$=\frac{a}{10-6}=\frac{b}{-9-5}=\frac{c}{2+6}$$

$$\frac{a}{4}=\frac{b}{-14}=\frac{c}{8}$$

$$\frac{a}{2}=\frac{b}{-7}=\frac{c}{4}=k$$

$a=2k$, b=-7k, c=4k

equation of line

$$\frac{x-2}{2k}=\frac{y-1}{-7}=\frac{z-3}{4k}$$

$$\frac{x-2}{2}=\frac{y-1}{-7}=\frac{z-3}{4}$$

Hence, we get the solution is $\frac{x-2}{2}=\frac{y-1}{-7}=\frac{z-3}{4}$.

Question 18 **[6]**

Draw a rough sketch and find the area bounded by the curve $x^2=y$ and $x+y=2$.

Sol.

Simplify the expression.

$x^2 = y$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$x = 1$, x= -2

$\int_{-2}^{1} (2-x) - x^2 dx$

$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_2^1$

$= \left[2 - \frac{1}{2} - \frac{1}{3} \right] - \left[-4 - \frac{4}{2} + \frac{8}{3} \right]$

$= \left[\frac{7}{6} \right] - \left[\frac{-20}{6} \right]$

$= \frac{27}{6} = \frac{9}{2}$

$= \frac{9}{2}$

Hence, the solution is $\frac{9}{2}$.

SECTION C (20Marks)

Question 19 **[3×2]**

(a) A company produces a commodity with Rs $Rs24000$ as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of Rs 8 per unit. Find the break-even point.

Sol.

FC = $Rs24000$ SP per unit $= Rs8$

Suppose number of units are x

Therefore,

$TR = P \times Q \qquad 8 \times x$

VC=25% of TR $= \frac{1}{4} \times 8x = 2x$

$TC = FC + VC = 24000 + 2x$

We show that the breakeven point TC=TR

$$24000 + 2x = 8x$$

$$24000 = 6x$$

$$x = 4,000 units$$

Hence, the value of breakeven point is $x = 4,000 units$.

(b) The total cost function for a production is given by C(x) = $\frac{3x^2}{4} - 7x + 27$. Find the number of units produced for which M.C = A.C
(M.C= Marginal Cost and A.C =Average Cost)

Sol.

Simplify the expression.

C(x) =

$$\frac{3}{4}x^2 - 7x + 27$$

$$Ac(x) = \frac{C(x)}{x} = \frac{3}{4}x - 7 + \frac{27}{x}$$

$$MC = C'(x) = \frac{3}{4}x - 7$$

We have that MC =AC

$$\frac{3}{2}x - 7 = \frac{3}{4}x - 7 + \frac{27}{x}$$

$$\frac{3}{2}x - \frac{3}{4}x = \frac{27}{x}$$

$$\frac{6x - 3x}{4} = \frac{27}{x}$$

$$\frac{3x}{4} = \frac{27}{x}$$

$$x^2 = 36$$

$$x = 6 units$$

Hence, the number of production units is $x = 6 units$.

(c) If $\vec{x} = 18$, $\bar{y} = 100$, $\sigma_x = 14$, $\sigma_y = 20$ and correction coefficient $r_{xy} = 0.8$ find the regression equation of y and x.

Sol.

Simplify the regression equation.

$\bar{x} = 18$ $\quad \bar{y} = 100$ $\quad \sigma_x = 14,$ $\quad \sigma_y = 20$

$r_{xy} = 0.8$

The regression equation is y on x.

$$y - \bar{y} = x\left(x - \bar{x}\right)$$

$$x = \frac{r\sigma_y}{\sigma_x} = \frac{0.8 \times 20}{10 \times 14} = \frac{8}{7}$$

$$y - 100 = \frac{8}{7}(x - 18)$$

$$7y - 700 = 8x - 144$$

$$y - 100 = \frac{8}{7}(x - 8)$$

$$7y - 700 = 8x - 144$$

$$7y - 700 = 8x - 144$$

$$7y - 8x - 556 = 0$$

Hence, the solution is $7y - 8x - 556 = 0$.

Question 20 **[4]**

(a) The following results were obtained with respect to two variable x and y.

$\sum x = 15, \sum y = 25, \sum xy = 83, \sum y^2 = 135$ And $n = 5$

(i) Find the regression coefficient b_{xy}..

(ii) Find the regression equation of x and y.

(i) Sol. solving the regression coefficient b_{xy}.

$$\frac{N\sum xy - \sum x \sum y}{N\sum y^2 - \left(\sum y\right)^2}$$

Then, $bxy = \dfrac{5 \times 83 - 15 \times 25}{5 \times 135 - (25)^2}$

$$= \frac{415 - 375}{675 - 625} = \frac{40}{50} = \frac{4}{5}$$

(ii)Sol.

Solving the regression equation of x and y.

$$x-\bar{x}=b_{xy}\left(y-\bar{y}\right)$$

$$\bar{x}=\frac{\sum x}{n}=\frac{15}{5}=3,\quad \bar{y}=\frac{\sum y}{n}=\frac{25}{5}$$

$$x-3=\frac{4}{5}(y-5)$$

$$5x-15=4y-20$$

$$5x-4y+5=0$$

Hence the solution is $5x-4y+5=0$.

OR

(b) Find the equation of the regression line of y and x, if the observation (x,y) are as follows:
(1, 4), (2, 8), (3, 2), (4, 12), (5, 10) (6, 14), (7, 16), (8, 6) (9, 18)
Also, find the estimated value of y when $x=4$.

Sol.
Simplify the expression.
Therefore,

x	y	xy	x^2
1	4	4	1
2	8	16	4
3	2	6	9
4	12	48	16
5	10	50	25
6	14	84	36
7	16	112	49
8	6	48	64
9	18	162	81
45	90	530	285

Simplify the further regression equation *y* on *x*.

$$y-\bar{y}=byx\left(x-\bar{x}\right)$$

$$\bar{y}=\frac{\sum y}{n}=\frac{90}{9}=10$$

$$\bar{x}=\frac{\sum}{n}=\frac{45}{9}=5$$

$$byx=\frac{N\sum xy-\sum x\sum y}{N\sum x^2-\left(\sum x\right)^2}$$

$$=\frac{9\times 530-45\times 90}{9\times 285-\left(45\right)^2}$$

$$=\frac{4770-4050}{2565-2025}$$

$$=\frac{720}{540}=\frac{4}{3}$$

$$3y-30=4x-20$$

$$4x-3y+10=0$$

Hence, the solution is $4x-3y+10=0$.

Question 21 **[4]**

(a) The cost function of a product is given by C(x)= $\frac{x^2}{3}-45x^2-900x+36$ where x is the number of units produced. How many units should be produced to minimize the marginal cost?

OR

(b) The marginal cost function of x units of a product is given by MC=$3x^2-10x+3$. the cost of producing one unit is Rs 7. Find the total cost function and average cost function.

(a) Sol.

Simplify the expression.

MC = $3x^2-10x+3$.

Solving the integral function.

$$MC = \int \left(3x^2 - 10x + 3\right).dx$$

$$= \int \frac{3x^3}{3} - 10\frac{x^2}{2} + 3x + c$$

$$= x^3 - 5x^2 + 3x + c$$

$$x = 1, \ \ \mathrm{C}\left(y\right) = 7$$

$$7 = 1 - 5 + 3 + c$$

$$7 = -1 + c$$

$$c = 8$$

$$C\left(x\right) = x^3 - 5x^2 + 5x + 8$$

$$AvC\left(x\right) = \frac{C\left(x\right)}{x}$$

$$= \frac{x^3 - 5x^2 + 5x + 8}{x}$$

$$= x^2 - 5x^2 + 3x + \frac{8}{x}$$

Hence, we get the solution is $4x - 3y + 10 = 0$.

Question 22 **[6]**

A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for Rs 48 per unit and product B is sold for Rs 40 per unit of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income?

Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph.

Sol. Simplify the expression.

Suppose x and y units of produce A and B to be produced in order to obtain the maximum gross income.

Then, subject to constraints.

$x, \ \geq 0$

$2x + y \ \leq 90$ ....(i)

$x \ + 2y \ \ \leq 80$ ....(ii)

$x + y \ \ \ \leq 50$ ....(iii)

The object is z then the corresponding equation are –

$$= 48x + 40y \ .... \ (\text{max.}) \qquad \begin{aligned} 2x + y &= 90 \quad \text{or} \quad \frac{x}{45} + \frac{y}{90} = 1 \\ x + 2y &= 80 \quad \text{or} \quad \frac{x}{80} + \frac{y}{40} = 1 \\ x + y &= 50 \quad \text{or} \quad \frac{x}{50} + \frac{y}{50} = 1 \end{aligned}$$

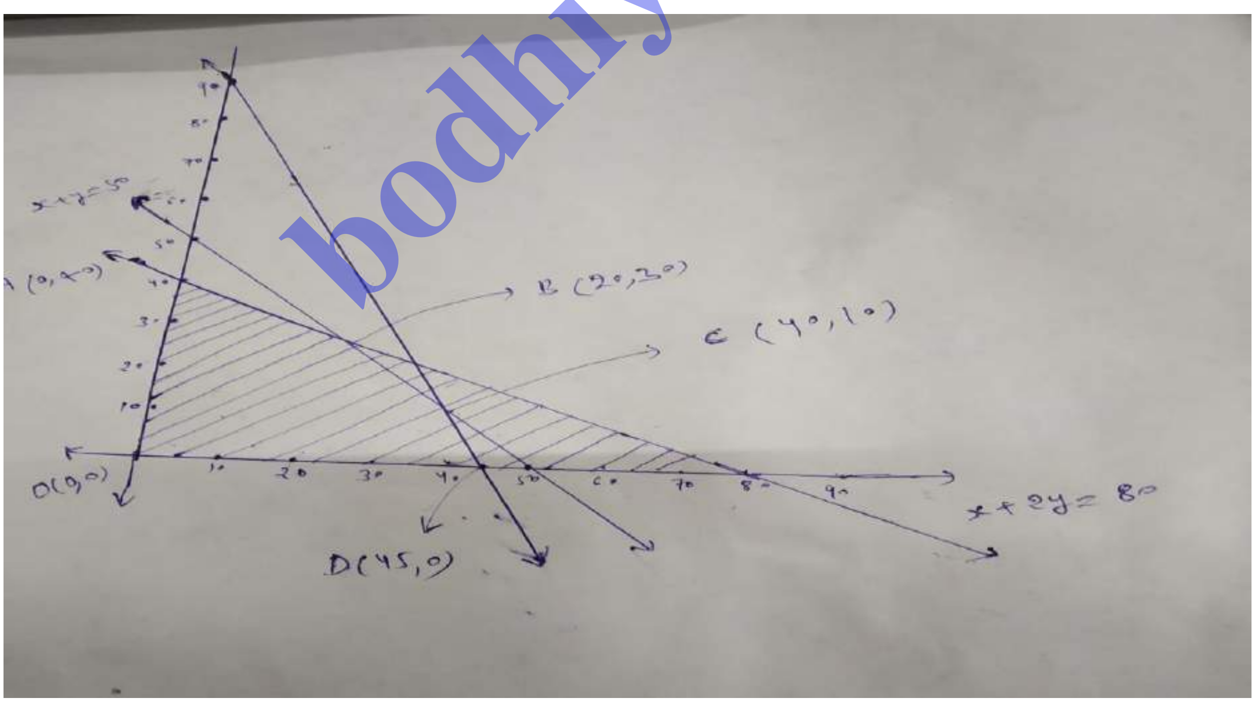
$2x + y = 90$ & x+2y

$x = \dfrac{100}{3}$ & $\dfrac{90}{3}$

$2x + y = 90$ x+y =50

We have $x = 20$ & y $y = 30$

$B(20, \ 30)$ & $A(40, \ 10)$



Corner prints	Value of z=48x+40y
0 (0,0)	0
A (0, 40)	1600
B (20, 30)	960+120=1080
C (40 ,10)	1920 +400=2320 (max).
D (45, 0)	2160

Since,sss the maximum gross income is 2320 .

When, the 40 units is produces A $ 10 units of product B to be product.

Hence, the maximum gross income is 2320 .