ISC Paper 2017
Maths

Time Allowed: 3 Hours
Maximum Marks: 100

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time.)

- The Question Paper consists of three sections A, B and C.
- Candidates are required to attempt all questions from Section A and all questions either from Section B or Section C.
- Section A: Internal choice has been provided in three questions of four marks each and two questions of six marks each.
- Section B: Internal choice has been provided in two questions of four marks each.
- Section C: Internal choice has been provided in two questions of four marks each.
- All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
- The intended marks for questions or parts of questions are given in brackets [ ].
- Mathematical tables and graph papers are provided.

Section – A (80 Marks)

Question 1. [10 × 3]

(i) If the matrix $\begin{pmatrix} 6 & -x^2 \\ 2x-15 & 10 \end{pmatrix}$ is symmetric, find the value of x.

(ii) If y – 2x – k = 0 touches the conic $3x^2 - 5y^2 = 15$, find the value of k.

(iii) Prove that $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}\sqrt{x}$

(iv) Using L 'Hospital's Rule, evaluate:
$\underset{x \to \pi/2}{\mathrm{L}} \left(x \tan x - \frac{\pi}{4} \cdot \sec x\right)$

(v) Evaluate: $\int \frac{1}{x^2} \sin^2\left(\frac{1}{x}\right) dx$

(vi) Evaluate: $\int_0^{\pi/4} \log(1 + \tan\theta) d\theta$

(vii) By using the data $\overline{x} = 25, \overline{y} = 30; b_{yx} = 1.6 \text{ and } b_{xy} = 0.4$, find:

(a) The regression equation y on x.
(b) What is the most likely value of y when x = 60?
(c) What is the coefficient of correlation between x and y?

(viii) A problem is given to three students whose chances of solving it are $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{3}$ respectively. Find the probability that the problem is solved.

(*ix*) If $a + ib = \frac{x+iy}{x-iy}$ prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2xy}{x^2 - y^2}$

(x) Solve: $\frac{dy}{dx}$ = 1 − xy + y − x

Solution:

(*i*) Let
$$A = \begin{bmatrix} 6 & -x^2 \\ 2x-15 & 10 \end{bmatrix}$$
$$A' = \begin{bmatrix} 6 & 2x-15 \\ -x^2 & 10 \end{bmatrix}$$
Since given matrix A is symmetric
$\therefore$ $$A = A'$$
$$\begin{bmatrix} 6 & -x^2 \\ 2x-15 & 10 \end{bmatrix} = \begin{bmatrix} 6 & 2x-15 \\ -x^2 & 10 \end{bmatrix}$$
Equating the corresponding terms of equal matrices, we obtain.
$$2x - 15 = -x^2$$
$\Rightarrow$ $$x^2 + 2x - 15 = 0$$
$\Rightarrow$ $$(x+5)(x-3) = 0$$
$\Rightarrow$ $$x = -5 \text{ and } x = 3$$
(*ii*) Given line is $$y - 2x - k = 0$$
or $$y = 2x + k$$
Given conic section is $3x^2 - 5y^2 = 15$
or $$\frac{x^2}{5} - \frac{y^2}{3} = 1$$

Here, $m = 2$, $a^2 = 5$ and $b^2 = 3$

Line $y = mx + c$ touches the conic section $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (Hyperbola),

if $$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$2x + k = 2x \pm \sqrt{5(2)^2 - 3}$$
$$k = \pm\sqrt{20-3}$$
$$k = \pm\sqrt{17}$$

(*iii*) L. H. S. $= \dfrac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$

Put $x = \tan^2\theta \Rightarrow \tan\theta = \sqrt{x} \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta)$$

$$= \frac{1}{2}\times 2\theta = \theta = \tan^{-1}\sqrt{x} = \text{R. H. S.}$$

(*iv*) $\lim\limits_{x\to\pi/2}\left[x\tan x - \dfrac{\pi}{4}\sec x\right]$

$$= \lim_{x\to\pi/2}\left[\frac{x\sin x - \pi/4}{\cos x}\right] \qquad \left[\frac{0}{0}\text{ form, Using L. H. Rule}\right]$$

$$= \lim_{x\to\pi/2}\left[\frac{x\cos x + \sin x.1 - 0}{-\sin x}\right] = \lim_{x\to\pi/2}\frac{x\cos x + \sin x}{-\sin x} = \frac{0+1}{-1} = -1$$

(*v*) Let $\quad I = \displaystyle\int \frac{1}{x^2}\sin^2\left(\frac{1}{x}\right)dx$

Put $\quad \dfrac{1}{x} = t$

$\dfrac{-1}{x^2}dx = dt$

$\therefore \quad I = -\displaystyle\int \sin^2 t\,dt = \int\frac{(\cos 2t - 1)}{2}dt$

$$= \frac{1}{2}\frac{\sin 2t}{2} - t + C = \frac{1}{4}\sin\left(\frac{2}{x}\right) - \frac{1}{2x} + C$$

(*vi*) Let $\quad I = \displaystyle\int_0^{\pi/4}\log(1+\tan\theta)d\theta$

Also, $\quad I = \displaystyle\int_0^{\pi/4}\log\left\{1+\tan\left(\frac{\pi}{4}-\theta\right)\right\}d\theta \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$$= \int_0^{\pi/4}\log\left\{1+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}.\tan\theta}\right\}d\theta = \int_0^{\pi/4}\log\left\{1+\frac{1-\tan\theta}{1+\tan\theta}\right\}d\theta$$

$$= \int_0^{\pi/4} \log\left\{\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right\} d\theta = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$$

$$= \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1+\tan\theta) dx = \int_0^{\pi/4} \log 2\, d\theta - \mathrm{I}$$

$$2\mathrm{I} = \int_0^{\pi/4} \log 2 d\theta = [\theta \log 2]_0^{\pi/4}$$

$$\Rightarrow \quad 2\mathrm{I} = \frac{\pi}{4}\log 2 \Rightarrow \mathrm{I} = \frac{\pi}{8}\log 2$$

(*vii*) Here, given values are :

$\bar{x} = 25$, $\bar{y} = 30$, $b_{yx} = 1.6$ and $b_{xy} = 0.4$

(*a*) Regression equation y on x is given as :

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 30 = 1.6(x - 25)$$

$$y - 30 = \frac{16}{10}(x - 25)$$

$$5y - 150 = 8x - 200$$

$$8x - 5y - 50 = 0$$

(*b*) Put $x = 60$ in eq. (*i*), we obtain

$$8(60) - 5y - 50 = 0$$

$$5y = 480 - 50$$

$$5y = 430$$

$$y = 86$$

(*c*) Coefficient of correlation between x and y is

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$= \sqrt{1.6 \times 0.4}$$

$$= \sqrt{0.64} = 0.8$$

(*viii*) Let $\quad \mathrm{P(A)} = \frac{1}{4}, \mathrm{P}(\overline{\mathrm{A}}) = 1 - \frac{1}{4} = \frac{3}{4}$

$$\mathrm{P(B)} = \frac{1}{5}, \mathrm{P}(\overline{\mathrm{B}}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\mathrm{P(C)} = \frac{1}{3}, \mathrm{P}(\overline{\mathrm{C}}) = 1 - \frac{1}{3} = \frac{2}{3}$$

Probability (the problem is solved)

$$= 1 - \mathrm{P}(\overline{\mathrm{A}}) \cdot \mathrm{P}(\overline{\mathrm{B}}) \cdot \mathrm{P}(\overline{\mathrm{C}})$$

$$= 1 - \frac{3}{4} \times \frac{4}{5} \times \frac{2}{3}$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

(*ix*) Here, $a + ib = \frac{x+iy}{x-iy}$ ...(*i*)

$\therefore$ $a - ib = \frac{x-iy}{x+iy}$ ...(*ii*)

Multiplying (*i*) and (*ii*), we obtain

$$a^2 + b^2 = \frac{x+iy}{x-iy} \times \frac{x-iy}{x+iy}$$

$$a^2 + b^2 = 1$$

Again $a + ib = \frac{x+iy}{x-iy} \times \frac{x+iy}{x+iy}$

$$= \frac{x^2 - y^2 + 2xyi}{x^2 + y^2}$$

$$a + ib = \frac{x^2 - y^2}{x^2 + y^2} + \frac{2xy}{x^2 + y^2} i$$

Equating real and imaginary part, we obtain

$$a = \frac{x^2 - y^2}{x^2 + y^2} \text{ and } b = \frac{2xy}{x^2 + y^2}$$

Now, $\frac{b}{a} = \frac{2xy}{x^2 + y^2} \times \frac{x^2 + y^2}{x^2 - y^2}$

$$= \frac{2xy}{x^2 - y^2}$$

(*x*) $\frac{dy}{dx} = 1 - xy + y - x$

$$\frac{dy}{dx} = (1 + y) - x(1 + y)$$

$$\frac{dy}{dx} = (1 + y)(1 - x)$$

$$\frac{dy}{1+y} = (1 - x)dx$$

Integrating both sides, we obtain

$$\int \frac{dy}{1+y} = \int (1-x)dx$$

$$\log|1+y| = x - \frac{x^2}{2} + C$$

Question 2.
(a) Using properties of determinants, prove that:

(b) Given that: , find AB.
Using this result, solve the following system of equation: x – y = 3, 2x + 3y + 4z = 17 and y + 2z = 7

Solution:

(a) Let $$\Delta = \begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix}$$

Operating $C_3 \to C_3 - C_2$, we obtain

$$\Delta = \begin{vmatrix} a & b & c \\ c & a & c \\ b & c & a+b-c \end{vmatrix}$$

Operating $R_1 \to R_1 + R_2 + R_3$, we obtain

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c & a & c \\ b & c & a+b-c \end{vmatrix}$$

Taking $(a+b+c)$ common from R_1, we obtain

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c & a & c \\ b & c & a+b-c \end{vmatrix}$$

Operating $C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$, we obtain

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & a-c & c \\ c-a & 2c-a-b & a+b-c \end{vmatrix}$$

Expanding along R_1, we obtain

$$= (a+b+c)\{-(a-c)(a-c)\}$$
$$= (a+b+c)(a-c)^2$$

(b)
$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$AB = 6I_3$$
$$A^{-1}AB = 6A^{-1}I$$
$$\frac{1}{6}B = A^{-1}$$

Given system of equations are :

$$x - y = 3,\ 2x + 3y + 4z = 17 \text{ and } y + 2z = 7$$

Their matrix form is

$$AX = C$$
$$\Rightarrow \quad X = A^{-1}C$$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2$, $y = -1$ and $z = 4$

Question 3.
(a) Solve the equation for x:
$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x, x \neq 0$
(b) If A, B and C are the elements of Boolean algebra, simplify the expression (A’ + B’) (A + C’) + B’ (B + C). Draw the simplified circuit.

Solution:

(*a*) $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$

$$\Rightarrow \sin^{-1}\{x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2}\} = \sin^{-1}\sqrt{1-x^2}$$

$$\left[\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\} \text{ and } \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}\right]$$

$$\Rightarrow x\sqrt{1-1+2x-x^2} + \sqrt{1-x^2} - x\sqrt{1-x^2} = \sqrt{1-x^2}$$

$$\Rightarrow x\sqrt{2x-x^2} - x\sqrt{1-x^2} = 0 \qquad \Rightarrow x(\sqrt{2x-x^2} - \sqrt{1-x^2}) = 0$$

$$\Rightarrow x = 0, \sqrt{2x-x^2} - \sqrt{1-x^2} = 0 \quad \Rightarrow \quad x = 0, \sqrt{2x-x^2} = \sqrt{1-x^2}$$

Now, $\sqrt{2x-x^2} = \sqrt{1-x^2}$

Squaring both sides, we obtain

$$2x - x^2 = 1 - x^2$$

$$\Rightarrow \quad 2x = 1 \quad \Rightarrow \quad x = \frac{1}{2}$$

Hence, $x = 0$ and $x = \frac{1}{2}$

(*b*) $(A' + B')(A + C') + B'(B + C)$

$= A'(A + C') + B'(A + C') + B'B + B'C$

$= A'A + A'C' + B'A + B'C' + B'B + B'C$

$= O + A'C' + B'A + B'C' + O + B'C$ $\qquad [\because A'A = 0]$

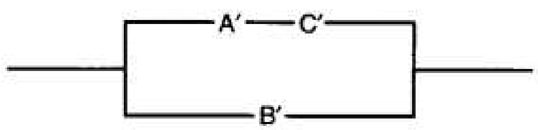
$= A'C' + B'A + B'(C' + C)$

$= A'C' + B'A + B'$ $\qquad [\because A + A' = 1]$

$= A'C' + B'(A + 1)$

$= A'C' + B'$ $\qquad [\because A + 1 = 1]$

Simplified circuit :-



Question 4.
(a) Verify Langrange's mean value theorem for the function: [5]
f(x) = x (1 − log x) and find the value of 'c' in the interval [1, 2]
(b) Find the coordinates of the centre, foci and equation of directrix of the hyperbola
$x^2 - 3y^2 - 4x = 8$. [5]

Solution:
(a) Given function 'f' is continuous in [1, 2] and differentiable in (1, 2)
f(x) = x (1 − log x) = x − x log x
$f'(x) = 1 - x \times \frac{1}{x} - \log x = 1 - 1 - \log x$
f'(x) = − log x
According to Langrange's Mean Value Theorem, E a real number c ∈ (1, 2) s.t.,

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$
$$\frac{f(2)-f(1)}{2-1} = -\log c$$
$$2 - 2\log 2 - (1 - \log 1) = -\log c$$
$$1 - \log 4 = -\log c$$
$$\Rightarrow \quad \log 4 - \log c = 1$$
$$\Rightarrow \quad \log_e\left(\frac{4}{c}\right) = 1$$
$$\Rightarrow \quad e = \frac{4}{c} \Rightarrow c = \frac{4}{e}$$

$$\text{Now, } 2 < e < 4 \Rightarrow \frac{1}{2} > \frac{1}{e} > \frac{1}{4} \Rightarrow \frac{4}{2} > \frac{4}{e} > \frac{4}{4} \Rightarrow 2 > \frac{4}{e} > 1$$
$$\therefore \quad c = \frac{4}{e} \in (1, 2)$$

(b) Here, equation of the given hyporbola is
$$x^2 - 3y^2 - 4x = 8$$
$$\Rightarrow \quad x^2 - 4x + 4 - 3y^2 = 8 + 4$$
$$\Rightarrow \quad (x-2)^2 - 3y^2 = 12$$
$$\Rightarrow \quad \frac{(x-2)^2}{12} - \frac{y^2}{4} = 1$$
Writing $x - 2 = X$ and $y = Y$, the given equation becomes
$$\frac{X^2}{12} - \frac{Y^2}{4} = 1$$

Here, $a^2 = 12$ and $b^2 = 4 \Rightarrow a = 2\sqrt{3}$ and $b = 2$

For Centre put $X = 0$ and $Y = 0$

$x - 2 = 0$ and $y = 0$

$x = 2$ and $y = 0$

$\therefore$ Coordinates of the centre are (2, 0)

For Foci, X = ± ae and Y = 0

Now, $$e = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{12+4}{12}} = \sqrt{\frac{16}{12}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \quad x - 2 = \pm\, 2\sqrt{3} \times \frac{2}{\sqrt{3}} = \pm 4$$

$$x = \pm 4 + 2$$

$$\Rightarrow \quad x = 6,\ x = -2 \text{ and } y = 0$$

$\therefore$ Coordinates of Foci are (6, 0) and (– 2, 0)

The equation of directrices are :

$$X = \pm \frac{a}{e}$$

$$x - 2 = \pm \frac{2\sqrt{3}}{\frac{2}{\sqrt{3}}}$$

$$x - 2 = \pm 3$$

$$x = \pm 3 + 2 \Rightarrow x = 5 \text{ and } x = -1$$

$\therefore$ $x = 5$ and $x = -1$ are the equations of directrices

Question 5.

(a) If y = cos (sin x), show that: [5]

$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$

(b) Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube. [5]

Solution:

(a)
$$y = \cos(\sin x) \quad ...(i)$$

$$\frac{dy}{dx} = -\sin(\sin x)\cos x \quad ...(ii)$$

$$\frac{d^2y}{dx^2} = \sin(\sin x)\sin x - \cos x(\cos(\sin x))\cos x$$

$$= \sin(\sin x)\sin x - \cos^2 x.\, y \quad \text{[using (i)]}$$

$$= -\sin(\sin x)\cos x\left(\frac{-\sin x}{\cos x}\right) - y\cos^2 x$$

$$= \frac{dy}{dx}(-\tan x) - y\cos^2 x \quad \text{[using (ii)]}$$

$$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$$

(b) Let x be the side of square base of cuboid and other side be y.
Then the volume of a cuboid with square base,
V = x × x × y
⇒ V = x^2y
As the volume of the cuboid is given so volume is taken constantly throughout the question, therefore,
$y = \frac{V}{x^2}$ ....(i)
In order to show that surface area is minimum when the given cuboid is a cube, we have to show S” > 0 and x = y.
Let S be the surface area of cuboid, then

$$S = x^2 + xy + xy + xy + xy + x^2$$

$$S = 2x^2 + 4xy \quad ...(ii)$$

$$\Rightarrow \quad S = 2x^2 + 4x.\frac{V}{x^2}$$

$$\Rightarrow \quad S = 2x^2 + \frac{4V}{x} \quad ...(iii)$$

$$\Rightarrow \quad \frac{dS}{dx} = 4x - \frac{4V}{x^2} \quad ...(iv)$$

For maximum / minimum value of S, we have $\frac{dS}{dx} = 0$

$$\Rightarrow \quad 4x - \frac{4V}{x^2} = 0 \Rightarrow 4V = 4x^3$$

$$\Rightarrow \quad V = x^3 \quad \ldots(v)$$

Putting $V = x^3$ in (i), we have

$$y = \frac{x^3}{x^2} = x$$

Here, $y = x \Rightarrow$ cuboid is a cube.

Differentiating (iv) w.r.t. x, we have

$$\frac{d^2S}{dx^2} = \left(4 + \frac{8V}{x^3}\right) > 0$$

Hence, surface area is minimum when given cuboid is a cube.

Question 6.

(a) Evaluate: $\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$ [5]

(b) Draw a rough sketch of the curve $y^2 = 4x$ and find the area of region enclosed by the curve and the line $y = x$. [5]

Solution:

(a) Let $\quad I = \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$

$$I = \int \frac{2\sin x \cos x}{(1+\sin x)(2+\sin x)} dx$$

Put $\quad \sin x = t \Rightarrow \cos x \, dx = dt$

$$\therefore \quad I = \int \frac{2t}{(1+t)(2+t)} dt$$

Let $\quad \frac{2t}{(1+t)(2+t)} = \frac{A}{t+1} + \frac{B}{t+2} \Rightarrow 2t = A(t+2) + B(t+1)$

Put $\quad t + 2 = 0 \Rightarrow t = -2$

$$-4 = B(-2+1) \Rightarrow B = 4$$

Put $\quad t + 1 = 0 \Rightarrow t = -1$

$$-2 = A(-1+2) \Rightarrow A = -2$$

$\therefore \qquad I = \int \left[\frac{-2}{t+1} + \frac{4}{t+2} \right] dt$

$= -2\log(t+1) + 4\log(t+2) + C$

$= -2\log(\sin x + 1) + 4\log(\sin x + 2) + C$

(*b*) $y^2 = 4x$ is a right handed parabola with vertex at O(0, 0) and axis of parabola is x-axis.

$y = x$ is a line passing through origin O(0, 0)

Now, finding their intersection

$$y^2 = 4x$$

$$\Rightarrow \quad x^2 = 4x$$

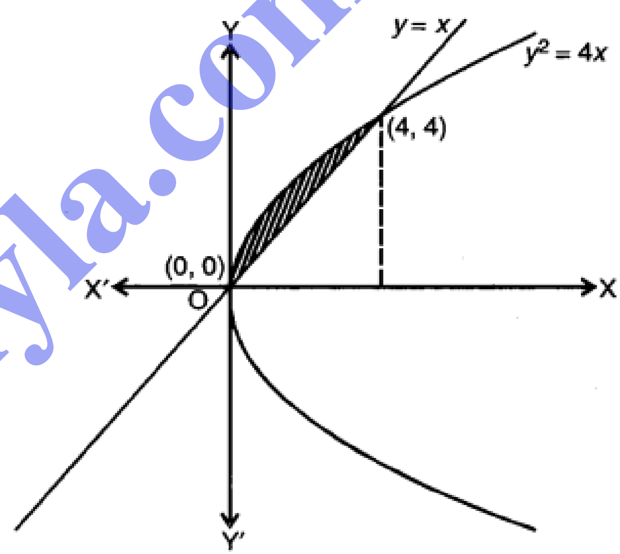
$$\Rightarrow \quad x^2 - 4x = 0$$

$$\Rightarrow \quad x(x-4) = 0$$

$$\Rightarrow \quad x = 0 \quad \text{and} \quad x = 4$$

Also, $y = x \Rightarrow y = 0$ and $y = 4$

$\therefore$ Points of Intersection are (0, 0) and (4, 4)



Required area $= 2\int_0^4 \sqrt{x}\,dx - \int_0^4 x\,dx$

$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^4 - \left[\frac{x^2}{2}\right]_0^4$

$= 2 \times \frac{2}{3}\left|(4)^{3/2} - 0\right| - \left|\frac{4^2}{2} - 0\right|$

$= \frac{4}{3} \times 8 - 8$

$= \frac{32-24}{3} = \frac{8}{3}$

$= 2\frac{8}{3}$ sq.units

Question 7.
(a) Calculate the Spearman's rank correlation coefficient for the following data and interpret the result: [5]

X	35	54	80	95	73	73	35	91	83	81
Y	40	60	75	90	70	75	38	95	75	70

(b) Find the line of best fit for the following data, treating x as the dependent variable (Regression equation x on y): [5]

X	14	12	13	14	16	10	13	12
Y	14	23	17	24	18	25	23	24

Hence, estimate the value of x when y = 16.

Solution:
(a) To determine Spearman's Rank Correlation:

X	Y	R_1	R_2	$d = R_1 - R_2$	d^2
35	40	9.5	9	0.5	0.25
54	60	8	8	0	0
80	75	5	4	1	1
95	90	1	2	–1	1
73	70	6.5	6.5	0	0
73	75	6.5	4	2.5	6.25
35	38	9.5	10	–0.5	0.25
91	95	2	1	1	1
83	75	3	4	– 1	1
81	70	4	6.5	– 2.5	6.25
Total					$\Sigma d^2 = 17$

$$r = 1 - \frac{6\left[\Sigma d^2 + \Sigma\left(\frac{t^3 - t}{12}\right)\right]}{N(N^2 - 1)}$$

where t is the number of individual involved in a tie.

Here, $\Sigma d^2 = 17, N = 10, t = 3, 2, 2, 2$

$$\sum \frac{t^3 - t}{12} = \frac{(3)^3 - 3}{12} + 3 \times \frac{(2)^3 - 2}{12}$$

$$= \frac{27 - 3}{12} + 3 \times \frac{8 - 2}{12}$$

$$= \frac{24}{12} + \frac{18}{12} = \frac{42}{12} = \frac{7}{2} = 3.5$$

$$r = 1 - \frac{6[17+3.5]}{10(100-1)}$$

$$= 1 - \frac{123}{990}$$

$$= \frac{990-123}{990}$$

$$= \frac{867}{990} = 0.876$$

(*b*)

X	Y	$x = X - \overline{X}$	$y = Y - \overline{Y}$	x^2	y^2	xy
14	14	14 – 13 = 1	14 – 21= – 7	1	49	– 7
12	23	12 – 13 = – 1	23 – 21 = 2	1	4	– 2
13	17	13 – 13 = 0	17 – 21 = – 4	0	16	0
14	24	14 – 13 = 1	24 – 21 = 3	1	9	3
16	18	16 – 13 = 3	18 –21 = – 3	9	9	– 9
10	25	10 – 13 = – 3	25 –21 = 4	9	16	– 12
13	23	13 – 13 = 0	23 – 21 = 2	0	4	0
12	24	12 – 13 = – 1	24 –21 = 3	1	9	– 3
$\Sigma X = 104$	$\Sigma Y = 168$			$\Sigma x^2 = 22$	$\Sigma y^2 = 116$	$\Sigma xy = -30$

$$\overline{X} = \frac{104}{8} = 13, \quad \overline{Y} = \frac{168}{8} = 21$$

$$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{-30}{116} = -0.259$$

Regression equation of x on y is

$$x - \overline{X} = b_{xy}\left(y - \overline{Y}\right)$$

$$x - 13 = \frac{-30}{116}(y - 21)$$

$$58x - 754 = -15y + 315$$

$$58x + 15y = 1069$$

Put $y = 16$,

$$58x + 15(16) = 1069$$

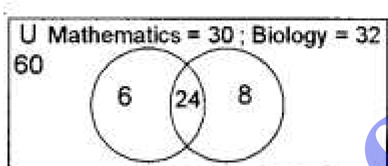
$$58x + 240 = 1069$$

$$58x = 829$$

$$x = 14.29$$

Question 8.
(a) In a class of 60 students, 30 opted for Mathematics, 32 opted for Biology and 24 opted for both Mathematics and Biology. If one of these students is selected at random, find the probability that: [5]
(i) The student opted for Mathematics or Biology.
(ii) The student has opted neither Mathematics nor Biology.
(iii) The student has opted Mathematics but not Biology.
(b) Bag A contains 1 white, 2 blue and 3 red balls. Bag B contains 3 white, 3 blue and 2 red balls. Bag C contains 2 white, 3 blue and 4 red balls. One bag is selected at random and then two balls are drawn from the selected bag. Find the probability that the balls draw n are white and red. [5]

Solution:



U = 60
n(M) = 30
n(B) = 32
n(M ∩ B) = 24
n(M ∪ B) = n(M) + n(B) – n(M ∩ B) = 30 + 32 – 24 = 38
n(M ∪ B)' = n(∪) – n(M ∪ B) = 60 – 38 = 22
Only Mathematics = n(M) – n(M ∩ B) = 30 – 24 = 6
(i) P(student opted for Mathematics or Biology) = $\frac{24}{60}$ = $\frac{2}{5}$
(ii) P(student opted neither Mathematics nor Biology) = $\frac{22}{60}$ = $\frac{11}{30}$
(iii) P(student opted Mathematics but not Biology) = $\frac{6}{60}$ = $\frac{1}{10}$
(b) Given:
Bag A: 1 white, 2 blue and 3 red balls
Bag B: 3 white, 3 blue and 2 red balls

Bag C: 2 white, 3 blue and 4 red balls
Let B_1, B_2, B_3 and E be the events defined as
B_1: Bag A is selected
B_2: Bag B is selected
B_3: Bag C is selected
And E: 1 white and 1 red ball is drawn

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(E/B_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1}{5}$$

$$P(E/B_2) = \frac{{}^3C_1 \times {}^2C_1}{{}^8C_2} = \frac{3}{14}$$

$$P(E/B_3) = \frac{{}^2C_1 \times {}^4C_1}{{}^9C_2} = \frac{2}{9}$$

$$\text{Required probability} = P(B_1) \times P(E/B_1) + P(B_2) \times P(E/B_2) + P(B_3) \times P(E/B_3)$$

$$= \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{3}{14} + \frac{1}{3} \times \frac{2}{9}$$

$$= \frac{1}{3}\left[\frac{1}{5} + \frac{3}{14} + \frac{2}{9}\right]$$

$$= \frac{1}{3}\left[\frac{126+135+140}{5 \times 14 \times 9}\right]$$

$$= \frac{401}{3 \times 5 \times 14 \times 9} = \frac{401}{1890}$$

Question 9.

(a) Prove that locus of z is circle and find its centre and radius if $\frac{z-i}{z-1}$ is purely imaginary. [5]

(b) Solve: $(x^2 - yx^2)\,dy + (y^2 + xy^2)\,dx = 0$ [5]

Solution:

(*a*) Now, $\dfrac{z-i}{z-1} = \dfrac{x+iy-i}{x+iy-1} = \dfrac{x+i(y-1)}{(x-1)+iy}$ $(\because\ z = x+iy)$

Rationalising

$$= \frac{\{x+i(y-1)\}\{(x-1)-iy\}}{\{(x-1)+iy\}\{(x-1)-iy\}}$$

$$= \frac{x(x-1)+y(y-1)+i\{(y-1)(x-1)-xy\}}{(x-1)^2+y^2}$$

$\Rightarrow$ $\dfrac{x^2+y^2-x-y}{x^2+y^2-2x+1} = 0$, (Real part) $[\because$ it is purely imaginary$]$

$\Rightarrow$ $x^2+y^2-x-y = 0$

which is a circle with centre $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ *i.e.*, $\dfrac{1}{2}(1+i)$ and radius is

$$r = \sqrt{\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-0} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

(*b*) Given differential equation is :

$$(x^2-yx^2)dy + (y^2+xy^2)dx = 0$$

$$\Rightarrow \quad x^2(1-y)dy + y^2(1+x)dx = 0$$

$$\Rightarrow \quad \int\frac{1-y}{y^2}dy + \int\frac{1+x}{x^2}dx = 0$$

$$\Rightarrow \quad \int\left(\frac{1}{y^2}-\frac{1}{y}\right)dy + \int\left(\frac{1}{x^2}+\frac{1}{x}\right)dx = 0$$

$$\Rightarrow \quad \int y^{-2}dy - \log y + \int x^{-2}dx + \log x = 0$$

$$\Rightarrow \quad \frac{y^{-1}}{-1} - \log y + \frac{x^{-1}}{-1} + \log x = C$$

$$\Rightarrow \quad -\frac{1}{y} - \log y - \frac{1}{x} + \log x = C$$

$$\Rightarrow \quad -\frac{1}{x} - \frac{1}{y} + \log\left(\frac{x}{y}\right) = C$$

Section – B (20 Marks)

Question 10.

(a) If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined with vectors $\vec{a}, \vec{b}$ and $\vec{c}$. [5]

(b) Find the value of λ for which the four points with position vectors $6\hat{i}-7\hat{j}, 16\hat{i}-19\hat{j}-4\hat{k}, \lambda\hat{j}-6\hat{k}$ and $2\hat{i}-5\hat{j}+10\hat{k}$ are coplanar. [5]

Solution:

(a) *Given* : $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors and $|\vec{a}| = |\vec{b}| = |\vec{c}|$.

To prove : $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Proof : $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{c} = \vec{c}\,.\,\vec{a}$ because $\vec{a}\perp\vec{b}$, $\vec{b}\perp\vec{c}$ and $\vec{c}\perp\vec{a}$

Now, $\vec{a}\,.\,(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\,.\,\vec{a} + \vec{a}\,.\,\vec{b} + \vec{a}\,.\,\vec{c}$

$= |\vec{a}|^2 + 0 + 0 = |\vec{a}|^2$

Also, $\vec{b}\,.\,(\vec{a}+\vec{b}+\vec{c}) = |\vec{b}|^2$

and $\vec{c}\,.\,(\vec{a}+\vec{b}+\vec{c}) = |\vec{c}|^2$

As $|\vec{a}| = |\vec{b}| = |\vec{c}|$

$\therefore$ $\vec{a}\,.(\vec{a}+\vec{b}+\vec{c}) = \vec{b}\,.(\vec{a}+\vec{b}+\vec{c}) = \vec{c}\,.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}|^2$

Hence, $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(b) Let the four points be A, B, C and D, whose position vectors are

P. V. of A = $6\hat{i}-7\hat{j}$, P. V. of B = $16\hat{i}-19\hat{j}-4\hat{k}$, P. V. of C = $\lambda\hat{j}-6\hat{k}$

and P. V. of D = $2\hat{i}-5\hat{j}+10\hat{k}$

$\therefore$ $\overline{AB} = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$\overline{AC} = -6\hat{i} + (\lambda+7)\hat{j} - 6\hat{k}$

$\overline{AD} = -4\hat{i} + 2\hat{j} + 10\hat{k}$

Since A, B, C and D are coplanar

$\Rightarrow \quad \overline{AB}, \overline{AC}, \overline{AD}$ are coplanar

$\Rightarrow \quad [\overline{AB}\ \overline{AC}\ \overline{AD}] = 0$

$$\Rightarrow \quad \begin{vmatrix} 10 & -12 & -4 \\ -6 & \lambda+7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 0$$

$$\Rightarrow \quad 10(10\lambda + 70 + 12) + 12(-60 - 24) - 4(-12 + 4\lambda + 28) = 0$$

$$\Rightarrow \quad 100\lambda + 820 - 1008 - 64 - 16\lambda = 0$$

$$\Rightarrow \quad 84\lambda = 252$$

$$\Rightarrow \quad \lambda = 3$$

Hence, the value of λ is 3.

Question 11.

(a) Show that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect. Find the coordinates of their point of intersection. [5]

(b) Find the equation of the plane passing through the point (1, -2, 1) and perpendicular to the line joining the points A(3, 2, 1) and B(1, 4, 2). [5]

Solution:

(a) Given lines are:

$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7} \quad \ldots(i)$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \quad \ldots(ii)$$

Let two lines (*i*) and (*ii*) intersect at P(α, β, γ)

$\therefore$ P(α, β, γ) satisfy line (*i*)

$$\Rightarrow \quad \frac{\alpha-4}{1} = \frac{\beta+3}{-4} = \frac{\gamma+1}{7} = \lambda$$

$\Rightarrow \quad \alpha = \lambda + 4, \beta = -4\lambda - 3$ and $\gamma = 7\lambda - 1$

Again, P(α, β, γ) also lie on (*ii*)

$$\Rightarrow \quad \frac{\lambda+4-1}{2} = \frac{-4\lambda-3+1}{-3} = \frac{7\lambda-1+10}{8}$$

$$\Rightarrow \quad \frac{\lambda+3}{2} = \frac{-4\lambda-2}{-3} = \frac{7\lambda+9}{8}$$

$$\Rightarrow \quad -3\lambda - 9 = -8\lambda - 4 \text{ and } -32\lambda - 16 = -21\lambda - 27$$

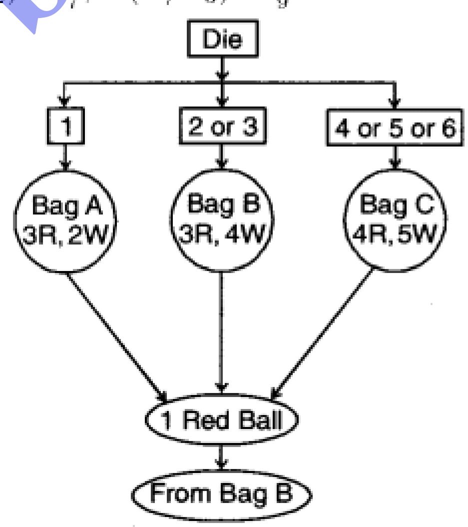
$$\Rightarrow \quad \lambda = 1 \quad \text{and} \quad \lambda = 1$$

Since value of λ in both the cases is same.

Thus, both lines (i) and (ii) intersect each other at a point.
And P(λ + 4, -4λ – 3, 7λ – 1) is P(5, -7, 6).
Hence, the coordinates of the point of intersection are (5, -7, 6).
(b) Here, given that the plane passes through the point (1, -2, 1) and it is perpendicular to the line joining the points A(3, 2, 1) and B(1, 4, 2).
Direction ratios of its normal are < 1 – 3, 4 – 2, 2 – 1 > i.e., < -2, 2, 1 >
Hence, the required equation of the plane is
$-2(x - 1) + 2(y + 2) + 1(z - 1) = 0$
$\Rightarrow -2x + 2 + 2y + 4 + z - 1 = 0$
$\Rightarrow -2x + 2y + z + 5 = 0$
or $2x - 2y - z - 5 = 0$

Question 12.
(a) A fair die is rolled. If face 1 turns up, a ball is drawn from Bag A. If face 2 or 3 turns up, a ball is drawn from Bag B. If face 4 or 5 or 6 turns up, a ball is drawn from Bag C. Bag A contains 3 red and 2 white balls, Bag B contains 3 red and 4 white balls and Bag C contains 4 red and 5 white balls. The die is rolled, a Bag is picked up and a ball is drawn. If the drawn ball is red; what is the probability that it is drawn from Bag B? [5]
(b) An urn contains 25 balls of which 10 balls are red and the remaining green. A ball is drawn at random from the urn, the colour is noted and the ball is replaced. If 6 balls are drawn in this way, find the probability that: [5]
(i) All the balls are red.
(ii) Not more than 2 balls are green.
(iii) The number of red balls and green balls is equal.

Solution:
(a) Let E_1, E_2, E_3 be the events that a die is thrown and getting 1, 2 or 3 and 4 or 5 or 6 respectively.
$P(E_1) = \frac{1}{6}, P(E_2) = \frac{2}{6}, P(E_3) = \frac{3}{6}$
Let A be the event that drawn ball is red
$P(A/E_1) = \frac{3}{5}, P(A/E_2) = \frac{3}{7}, P(A/E_3) = \frac{4}{9}$



Required probability *i.e.*, drawn ball is red from bag B is :

$$P(E_2/A) = \frac{P(E_2)\times P(A/E_2)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)}$$

$$= \frac{\frac{2}{6}\times\frac{3}{7}}{\frac{1}{6}\times\frac{3}{5}+\frac{2}{6}\times\frac{3}{7}+\frac{3}{6}\times\frac{4}{9}} = \frac{\frac{1}{7}}{\frac{1}{10}+\frac{1}{7}+\frac{2}{9}} = \frac{1}{7}\times\frac{7\times 9\times 10}{293} = \frac{90}{293}$$

(*b*) Number of red balls $= 10$

Number of green balls $= 25 - 10 = 15$

Total number of balls $= 25$

Number of balls drawn is 6 with replacement

(*i*) P(all the balls are red) $= \frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25} = \frac{64}{15625}$ [Fixed case]

(*ii*) P(not more than 2 balls are green)

= P(2 green balls and 4 red balls) + P(1 green ball and 5 red balls) + P(all 6 red balls)

= P(first two green balls and next four red balls) $\times \frac{6!}{2!4!}$

+ P(first green ball and next 5 red balls) $\times \frac{6!}{5!}$ + P(all 6 red balls)

$$= \left(\frac{15}{25}\times\frac{15}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{6!}{2!4!}\right) + \left(\frac{15}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{6!}{1!5!}\right)$$

$$+ \left(\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\right) = \frac{112}{625}$$

(*iii*) P(number of red balls and green balls are equal)

= P(3 red balls and 3 green balls)

= P(first three red balls and next three green balls) $\times \frac{6!}{3!3!}$

$$= \frac{10}{25}\times\frac{10}{25}\times\frac{10}{25}\times\frac{15}{25}\times\frac{15}{25}\times\frac{15}{25}\times\frac{6!}{3!3!} = \frac{864}{3125}.$$

Section – C (20 Marks)

Question 13.
(a) A machine costs ₹ 60000 and its effective life is estimated to be 25 years. A sinking fund is to be created for replacing the machine at the end of its life when its scrap value is estimated as ₹ 5000. The price of the new machine is estimated to be 100% more than the price of the present one. Find the amount that should be set aside at the end of each year, out of the profits, for the sinking fund it accumulates at an interest of 6% per annum compounded annually. [5]
(b) A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% nitrogen and 10% phosphoric acid. After the soil test, it is found that at least 7 kg of nitrogen and the same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs ₹ 5.00 per kg and the type B costs ₹ 8.00 per kg. Using Linear programming, find how many kilograms of each type of fertilizer should be bought to meet the requirement and for the cost to be minimum. Find the feasible region in the graph. [5]

Solution:

(a) We have $A = \frac{P}{i}\left\{(1+i)^n - 1\right\}$ ...(i)

Here, $A = 60000 + 100\% \text{ of } 60000 - 5000 = 60000 + 60000 - 5000 = 115000$

$n = 25, i = \frac{6}{100} = 0.06, P = ?$

Substituting these values in (i), we obtain

$$115000 = \frac{P}{0.06}\left\{(1+0.06)^{25} - 1\right\}$$

$$\Rightarrow \quad 0.06 \times 115000 = P\left\{(1.06)^{25} - 1\right\}$$

$$6900 = P\left\{(1.06)^{25} - 1\right\} \quad ..(ii)$$

Let $x = (1.06)^{25}$

Taking log of both sides, we have

$$\log x = 25 \log(1.06) = 25 \times 0.0253 = 0.6325$$

$$x = \text{Antilog}(0.6325) = 4.29$$

Now, from (*ii*), $6900 = P\{4.29 - 1\}$

$$P = \frac{6900}{3.29} = ₹\, 2097.26 = ₹\, 2097 \text{ correct to nearest rupee}$$

(*b*) Let the farmer use x kg of fertilizer of type A and y kg of fertilizer of type B.

The L.P.P. as per given statement of the question is :

We have to minimise $Z = 5x + 8y$, subject to the constraints :

$$\frac{10}{100} \times x + \frac{5}{100} \times y \geq 7$$

or $$10x + 5y \geq 700$$

$$2x + y \geq 140 \quad \ldots(i)$$

$$\frac{6}{100} \times x + \frac{10}{100} \times y \geq 7$$

or $$6x + 10y \geq 700$$

$$3x + 5y \geq 350 \quad \ldots(ii)$$

$$x \geq 0, y \geq 0$$

Consider $2x + y = 140$

Table of solutions is :

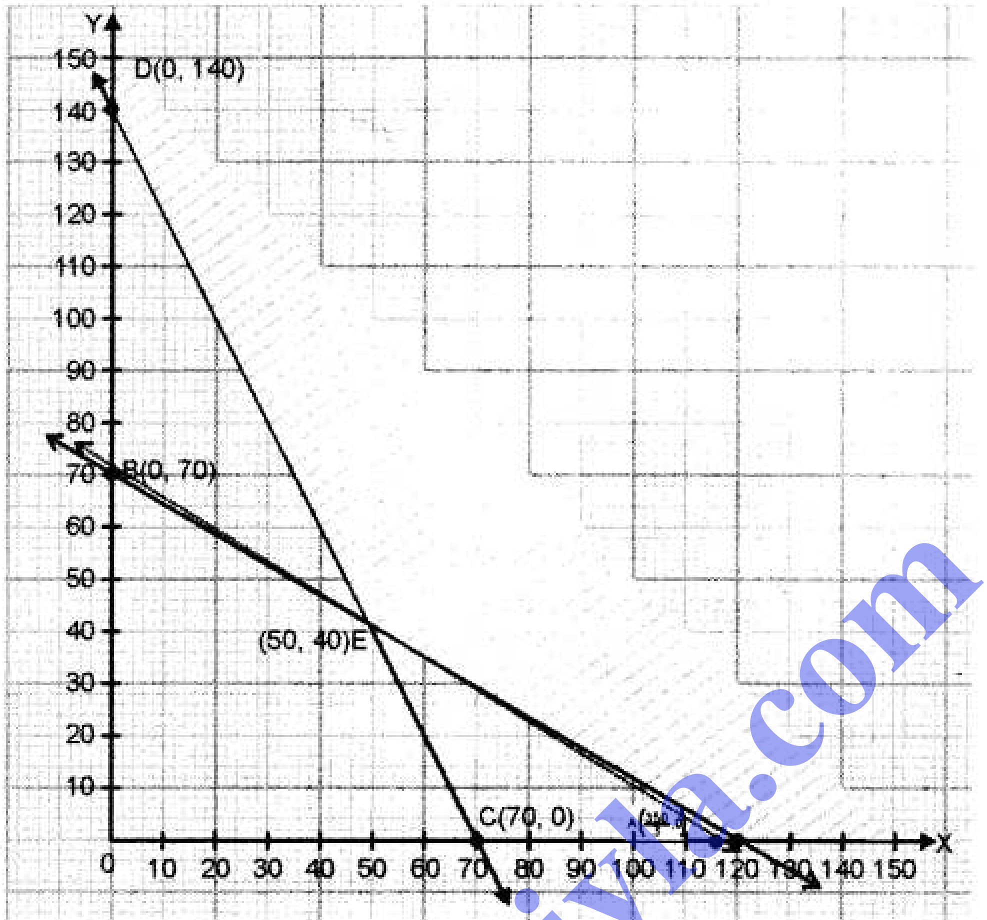
x	70	0
y	0	140

Consider $3x + 5y = 350$

Table of solutions is :

x	0	115
y	70	1

On plotting graph of above constraints or inequalities, we obtained the shaded region having corner points A, E, D as feasible region.



Now, the value of Z is evaluated at corner points in the following table :

Corner point	$Z = 5x + 8y$
$A\left(\frac{350}{3}, 0\right)$	$Z = 5 \times \frac{350}{3} + 0 = 583.33$
E(50, 40)	$Z = 5 \times 50 + 8 \times 40$ $= 570$ (Minimum)
D(0, 140)	$Z = 0 + 8 \times 140$ $= 1120$

Since feasible region is unbounded

$\therefore$ We have to draw the graph of the inequality $5x + 8y < 570$.

The graph of the above inequality does not have any common point except (50, 40), so minimum value of Z = ₹ 570 at (50, 40).

Question 14.
(a) The demand for a certain product is represented by the equation $p = 500 + 25x - \frac{x^2}{3}$ in rupees, where x is the number of units and p is the price per unit. Find:
(i) Marginal revenue function.
(ii) The marginal revenue when 10 units are sold. [5]
(b) A bill of ₹ 60000 payable 10 months after the date was discounted for ₹ 57300 on 30th June 2007. If the rate of interest was $11\frac{1}{4}$ % per annum, on what date was the bill drawn? [5]

Solution:
(a) The demand function for a certain product is represented as:
$p = 500 + 25x - \frac{x^2}{3}$, p being price per unit
If R be the total revenue for x units, then
$\mathrm{R} = p \cdot x = 500x + 25x^2 - \frac{x^3}{3}$
The Marginal Revenue (MR) is given as:
$\mathrm{MR} = \frac{d(\mathrm{R})}{dx} = 500 + 50x - x^2$
Marginal Revenue when 10 units are sold i.e., put x = 10
$(\mathrm{MR})_{10} = 500 + 50(10) - (10)^2 = 500 + 500 - 100 = 900.$
(b) Banker discount (BD) = Face value – Amount received = 60000 – 57300 = ₹ 2700
Now, BD is interested in face value for the remaining period

$$2700 = 60000 \times 11\frac{1}{4}\% \times \text{remaining period}$$

$$\text{Remaining period} = \frac{2700 \times 400}{60000 \times 45} = 0.4 \text{ years} = \frac{4}{10} \times 365 = 146 \text{ days}$$

Now, the bill was drawn on 30 June 2007 for 10 months so legally due date is 3 April 2008.
The bill was enhanced 146 days before discounting days backwards.
April – 3 days
March – 31 days
Feb. – 28 days
Jan. – 31 days
Dec. – 31 days
Nov. – 19 days
Bill was encashed on (30 – 19) = 11th Nov, 2007.

Question 15.

(a) The price relatives and weights of a set of commodities are given below: [5]

Commodity	A	B	C	D
Price relatives	125	120	127	119
Weights	x	$2x$	y	$y+3$

If the sum of the weights is 40 and the weighted average of price relatives index number is 122, find the numerical values of x and y.

(b) Construct 3 yearly moving averages from the following data and show on a graph against the original data: [5]

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Annual sales in lakhs	18	22	20	26	30	22	24	28	32	35

Solution:

(a) Given x + 2x + y + (y + 3) = 40
or 3x + 2y = 37 ...(i)

and I = the index for the set = 122

We have I = weighted average of price-relatives

$$= \frac{125 \times x + 120 \times 2x + 127 \times y + 119 \times (y+3)}{x + 2x + y + (y+3)}$$

or $$122 = \frac{125x + 240x + 127y + 119y + 357}{3x + 2y + 3}$$

or $$122 = \frac{365x + 246y + 357}{40} \quad \text{[using } (i)]$$

or $$365x + 246y + 357 = 4880$$

or $\quad 365x + 246y = 4523 \qquad \ldots(ii)$

Solving (*i*) and (*ii*), we have $x = 7$ and $y = 8$

(*b*)

Year	Annual sale (in Lakhs)	3 years moving sale	3 years moving average
2000	18	– – –	– – –
2001	22	60	20
2002	20	68	22.67
2003	26	76	25.33
2004	30	78	26
2005	22	76	25.33
2006	24	74	24.67
2007	28	84	28
2008	32	95	31.67
2009	35	– – –	– – –