MATHEMATICS

SECTION A

[10×3]

Question 1

(i) Find the matrix X for which:

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

(ii) Solve for x, if:

$$\tan(\cos^{-1}x) = \frac{2}{\sqrt{5}}$$

(iii) Prove that the line $2x - 3y = 9$ touches the conics $y^2 = -8x$. Also, find the point of contact.

(iv) Using L'Hospital's Rule, evaluate:

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{\cot x}{x}\right)$$

(v) Evaluate: $\int tan^3 x\, dx$

(vi) Using properties of definite integrals, evaluate:

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$$

(vii) The two lines of regressions are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and the variance of x is 12. Find the variance of y and the coefficient of correlation.

(viii) Express $\frac{2 + i}{(1 + i)(1 - 2i)}$ in the form of $a + ib$. Find its modulus and argument.

(ix) A pair of dice is thrown. What is the probability of getting an even number on the first die or a total of 8?

(x) Solve the differential equation:

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Comments of Examiners

(i) Errors were made by candidates while finding inverse and correct $X = A^{-1}B$ form.

(ii) Some errors were made by candidates in converting inverse trigonometric functions (one to another) and also algebraic calculations.

(iii) Some of the candidates made mistakes while finding the point of contact and also proving the condition of tangency.

(iv) Mistakes were made while converting in the $\frac{0}{0}$ form, also applied L'Hospital's Rule without observing the indefinite form $\frac{0}{0}$.

(v) Errors were made while using formula to convert tan^2x to $sec^2x - 1$. A few candidates attempted it by integration by parts and made the steps complicated - they could not proceed further.

(vi) Several candidates made mistakes in applying the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. Some attempted by other methods of integration and could not reach the proper result.

(vii) Many candidates solved the equations unnecessarily and tried to identify b_{yx} arbitrarily. The condition for the two equations to represent regression lines and the tests for identifying them were not used by some.

(viii) Many candidates failed to simplify in the form $a + ib$ also made careless mistakes while computing Modulus and Argument.

(ix) A number of candidates made mistakes in getting all favorable cases. Several candidates had difficulty in calculating the values of $P(A), P(B)$ & $P(A \cap B)$.

(x) Many candidates made errors while identifying the differential equation and attempted to solve it by variable-separable form.

Suggestions for teachers

- Basic operations with Matrices need to be explained. Practice should be given in Inverse and Multiplication of Matrix by a Matrix.
- Explain clearly the conversion of inverse circular functions (one to another form). More practice is required in conversion through diagram and by using formulae.
- Tangency condition must be taught more clearly. Ample practice needs to be given for computing point of contact with and without formulae.
- Applications of L'Hospital's Rule for calculating Limits of Indeterminate Forms $\left(\frac{0}{0}\ or\ \frac{\infty}{\infty}\right)$ should be taught well.
- In the rule the numerator and denominator need to differentiate separately till $\frac{0}{0}$ form is removed.
- Varieties of questions of integration by substitution may be given for practice.
- Teach properties of definite integrals well and see that the candidates learn to apply them appropriately. Properties when correctly used reduce cumbersome calculations into simple ones.
- Emphasize on mutually exclusive events and independent events. Plenty of practice is required to understand the application of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Give ample practice in various types of differential equations.

MARKING SCHEME

Question 1.

(i)

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \qquad AX = B,\ A^{-1}AX = A^{-1}B$$

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

$$X = A^{-1}B$$

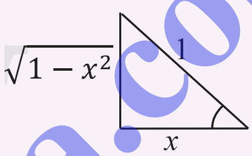
$$X = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

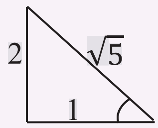
$$X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$$

(ii)

$$\cos^{-1} x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$\tan \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \left(\frac{2}{\sqrt{5}}\right)$$





$$\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

Squaring on both sides

$$\frac{1-x^2}{x^2} = \frac{4}{5}$$

$5 - 5x^2 = 4x^2$

$9x^2 = 5$

$x = \sqrt{5}$ r

$$\cos^{-1} x = \tan^{-1} \frac{2}{\sqrt{5}}$$

$$= \cos^{-1} \frac{\sqrt{5}}{3}$$

$$x = \frac{\sqrt{5}}{3}$$

(iii)

Line $3y = 2x - 9$

$y = \frac{2}{3}x - 3$

$m = {}^{2}/_{3},\ c = -3$

$y^2 = -8x$

$a = -2$

The condition: $a = mc$

$\Rightarrow -2 = \frac{2}{3} \times -3$

	$\Rightarrow\ -2 = -2$ ∴ the line touches the parabola $\left(\frac{2x}{3} - 3\right)^2 = -8x$ *or* $(2x - 9)^2 = -72x$ Point of contact= $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ $\Rightarrow 4x^2 - 36x + 81 = -72x$ Substituting values of a & m $\Rightarrow 4x^2 + 36x + 81 = 0$ $(2x + 9)^2 = 0$ ∴ $x = {-9}/{2}$, $y = \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{-3}{\cancel{9}}}{2} - 3$ $= -6$ ∴ point of contact is $\left(\frac{-9}{2}, -6\right)$
(iv)	$\lim_{x \to o} \left(\frac{1}{x^2} - \frac{\cot x}{x}\right)$ $= \underset{x \to o}{Lt} \left(\frac{1}{x^2} - \frac{1}{x \tan x}\right)$ $= \underset{x \to o}{Lt} \frac{\tan x - x}{x^2 \times \frac{tanx}{x} \times x}$ $= \underset{x \to o}{Lt} \frac{\tan x - x}{x^3}$ $= \underset{x \to o}{Lt} \frac{sec^2 x - 1}{3x^2}$ $= \underset{x \to o}{Lt} \frac{\cancel{2} \sec x \sec x}{\cancel{6}\, 3} \frac{\tan x}{x}$ $= \frac{1}{3}$
(v)	$I = \int tan^3 x\ dx.$ $= \int \tan x \quad \tan^2 x\ \ dx$ $= \int \tan x\ (sec^2 x - 1) dx$ $= \int \tan x\ sec^2 x\ \ dx - \int \tan x\ dx$ $I_1 - (-\log\lvert cos\ x \rvert)$ $I_1 = \int \tan x\ sec^2 x\ \ dx$ $\tan x = t \Rightarrow \sec^2\ x\ \ dx = dt$ $I_1 = \int t\ \ dt = \frac{t^2}{2} = \frac{1}{2}\ tan^2\ x$

	$I = \frac{1}{2}\, tan^2 x + \log\, \lvert\cos x\rvert + c$
(vi)	$I = \int_0^{\frac{\pi}{2}} \frac{sin\, x - \cos x}{1 + sin\, x\, cos\, x}\, dx$ (i) $I = \int_0^{\pi/2} \frac{sin\,\left(\frac{\pi}{2} - x\right) - cos\left(\frac{\pi}{2} - x\right) dx}{1 + sin\,\left(\frac{\pi}{2} - x\right)\, cos\left(\frac{\pi}{2} - x\right)}$ $I = \int \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$ (ii) Add (i) + (ii), 2I = 0 ⇒ I = 0
(vii)	$b_{xy} = \frac{-3}{2}$ and $b_{yx} = \frac{-1}{2}$ $\therefore\ r = \sqrt{b_{yx} \times b_{xy}} = -\sqrt{\frac{3}{4}} = -0 \cdot 866$ $-1 \le r \le 1$ $b_{yx} = r \frac{\sigma_y}{\sigma_x}$ and $\sigma_y^2 = 4$
(viii)	$z = \frac{(2+i)}{(1+i)(1-2i)}$ $= \frac{(2+i)\,(3+i)}{(3-i)(3+i)}$ $= \frac{5+5i}{10} = ½ + ½\, i$ $\lvert z\rvert = \sqrt{\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)}$ $= \frac{1}{\sqrt{2}}$ $\frac{1}{2}\big) / \big(\frac{}{2}$ z = $\tan^{-1}$ (1) $= \frac{\pi}{4}$
(ix)	{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6) 6,1),(6,2),(6,3),(6,4),(6,5),(6,6),(3,5),(5,3)} $P(E) = \frac{20}{36} = \frac{5}{9}$ Or P(A)+P(B)-P (A∩B) = $\frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36}$.
(x)	$x\frac{dy}{dx} + y = 3\, x^2 - 2$ $\frac{dy}{dx} + \frac{y}{x} = 3x - \frac{2}{x}$ If = $e^{\int \frac{dx}{x}}$ = e logx = x yx = ∫(3x2 − 2)dx

	$yx = \frac{3\cancel{x}^3}{3} - 2x + c$ $y = x^2 - 2 + \frac{c}{x}$

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} b+c & a & a \\ b & a+c & b \\ c & c & a+b \end{vmatrix} = 4abc$$

(b) Solve the following system of linear equations using matrix method: **[5]**

$$3x + y + z = 1,\ 2x + 2z = 0,\ 5x + y + 2z = 2$$

<u>Comments of Examiners</u>

(a) Many candidates were not able to attempt this part correctly. Several candidates expanded directly. At times, correct cofactors were not used.

(b) Several candidates made errors in finding the cofactors of elements of matrix. For finding the unknown matrix X, some candidates used post-multiplication with inverse of A. A few candidates solved using Cramer's Rule.

<u>Suggestions for teachers</u>

- Explain every property of the determinants with proper examples. This type of questions should be taught in the class by discussion method. Logical and reasoning skills should be used to apply the correct property.
- The basics with regards to cofactors of elements, obtaining adjoint and inverse and meaning of pre-and post-multiplication of matrices must be covered thoroughly in class; e.g. if $AX=B$ then $X=A^{-1}B$ and not BA^{-1} as such multiplication of matrices is not commutative.

MARKING SCHEME **Question 2.**	
(a)	$Let, \Delta = \begin{vmatrix} b+c & a & a \\ b & a+c & b \\ c & c & a+b \end{vmatrix}$ $R_1 \to R_1 - R_2 - R_3$ $\Delta = \begin{vmatrix} 0 & 2c & 2b \\ b & a+c & b \\ c & c & a+b \end{vmatrix}$

$c_2 \rightarrow c_2 - c_1, \; c_3 \rightarrow c_3 - c_1$

$$\Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & a + c - c & 0 \\ c & 0 & a + b - c \end{vmatrix}$$

$= 2c\,(ab + b^2 - bc) - 2b\,(0 - ca - c^2 + bc)$

$= 2abc + \cancel{2b^2c} - \cancel{2bc^2} + 2abc + \cancel{2bc^2} - \cancel{2b^2c}$

$= 4\,abc$ Hence Proved.

(b)

$$Let\ A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ 5 & 1 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ 5 & 1 & 2 \end{vmatrix} = 3(0-2) - 1(4-10) + 1(2-0) = 2$$

$$A^{-1} = \frac{1}{2}\begin{pmatrix} -2 & -1 & 2 \\ 6 & 1 & -4 \\ 2 & 2 & -2 \end{pmatrix}$$

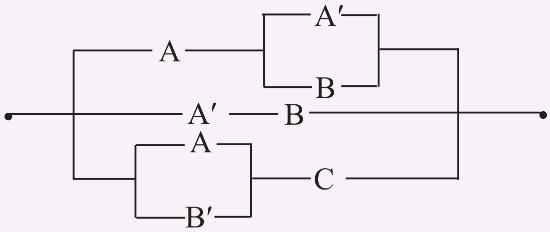
$X = A^{-1}B$

$$X = \frac{1}{2}\begin{pmatrix} -2 & -1 & 2 \\ 6 & 1 & -4 \\ 2 & 2 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \quad \therefore x = 1, y = -1, z = -1$$

Question 3

(a)	If $\sin^{-1}x + \tan^{-1}x = \frac{\pi}{2}$, prove that: $2x^2 + 1 = \sqrt{5}$	**[5]**
(b)	Write the Boolean function corresponding to the switching circuit given below: 	**[5]**

	A, B and C represent switches in ‘on’ position and A′, B′ and C′ represent them in ‘off’ position. Using Boolean algebra, simplify the function and construct an equivalent switching circuit.	

Comments of Examiners

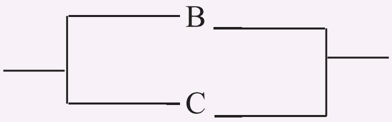
(a) Many candidates attempted this question but errors were made while squaring, simplifying and solving higher degree algebraic equations. Most of the candidates failed to prove the result.

(b) Many candidates were unable to find correct simplification leads to required result (B+C). A few candidates made errors in sketching the simplified circuit.

Suggestions for teachers

- Inverse Circular Functions laws needs to be taught thoroughly. The applications need to be illustrated with examples. Inter conversion of functions should also be done. It helps the simplification process.
- Laws of Boolean algebra need to be explained properly and enough practice must be given on simplification and drawing the different types of Boolean expressions.

MARKING SCHEME Question 3.	
(a)	$\sin^{-1}x + \tan^{-1}x = \pi/2$ $\tan^{-1}x = \pi/2 - \sin^{-1} x = \cos^{-1} x$ $\tan^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$ $\Rightarrow \quad x = \frac{\sqrt{1-x^2}}{x}$ $\Rightarrow \quad x^2 = \sqrt{1-x^2}$ $\Rightarrow x^4 = 1 - x^2$ $\Rightarrow x^4 + x^2 - 1 = 0$ $\therefore x^2 = \frac{-1\pm\sqrt{5}}{2}$ But x^2 cannot be negative. $\therefore x^2 = \frac{-1+\sqrt{5}}{2}$ $\Rightarrow 2x^2 + 1 = \sqrt{5}$
(b)	F (A, B, C) = A(A′ + B) + A′B+ (A+B′)C = AA′ + AB + A′B + AC + B′C = O + B(A+A′) + AC + B′C = B + B′C + AC = (B +B′) (B+C) + AC = B + C + AC = B + C



Question 4

(a) Verify the conditions of Rolle's Theorem for the following function: **[5]**

$f(x) = \log(x^2 + 2) - \log 3$ on $[-1,1]$

Find a point in the interval, where the tangent to the curve is parallel to x-axis.

(b) Find the equation of the standard ellipse, taking its axes as the coordinate axes, whose minor axis is equal to the distance between the foci and whose length of latus rectum is 10. Also, find its eccentricity. **[5]**

<u>Comments of Examiners</u>

(a) Several candidates made errors because of confusion between the closed and open interval for continuity and differentiability of the function. Most of the candidates did not find the point of contact of the tangent and the curve.

(b) Many candidates made errors while calculating values of 'a' and 'b' due to application of wrong formula or simplification mistakes. Most of the candidates forgot to find eccentricity and complete the equation of Ellipse.

<u>Suggestions for teachers</u>

- The different criteria for Rolle's and Lagrange's Mean Value theorems need to be understood and differentiated. Difference between 'closed' and 'open' intervals and its significance should be taught by sketching the curve, etc.
- Conics and their equations need to be thoroughly explained. Basics such as axis, directrix, eccentricity and latus rectum, etc. of a conic need to be explained with the help of good number of examples.

MARKING SCHEME

Question 4.

(a)	$f(x) = \log(x^2+2) - \log 3$ is continuous $[-1, 1]$ $f'(x) = \frac{2x}{(x^2+2)}$ $f'(x)$ exists in $(-1, 1)$ $f(-1) = f(1) = 0$ All the conditions of Rolle's theorem are satisfied then there exists 'c' in $(-1, 1)$ such that $f'(c) = 0$

	$\frac{2c}{(1+c^2)} = 0$ c = 0 lies between -1 and 1. Hence, Rolle's theorem is verified. The point where the tangent is parallel to x axis is $(0, \log\frac{2}{3})$
(b)	Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ a > b. Given, $2b = 2ae$ - (i) $\Rightarrow$ $b = ae$ and $\frac{2b^2}{a} = 10$ - (ii) $\Rightarrow$ $b^2 = 5a$ We also know, $b^2 = a^2(1 - e^2)$ – (iii). Substituting (i) and (ii) in (iii), we get $5a = a^2 - b^2$ $= a^2 - 5a$ $\Rightarrow 10a = a^2$ $\Rightarrow a = 10$ and $b^2 = 50$ $\Rightarrow a^2 = 100$ $\therefore$ equation of ellipse is $\frac{x^2}{100} + \frac{y^2}{50} = 1$ or complete equation $x^2 + 2y^2 = 100$ Also, $b^2 = a^2(1 - e^2)$ $50 = 100(1 - e^2)$ $e^2 = 1 - \frac{1}{2} = \frac{1}{2}$ $e = \frac{1}{\sqrt{2}}$

Question 5

(a) If $\log y = \tan^{-1}x$, prove that: **[5]**

$$(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$$

(b) A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle to get maximum area. Also, find the maximum area. **[5]**

Comments of Examiners

(a) First differentiation was done by many candidates and errors were made in finding the second derivative and framing the required result. Use of the rule for composite function as well as chain rule was forgotten by several candidates.

(b) In most of the cases, candidates did not read the question attentively and drew wrong sketch. Due to this, area was not correct in variable. Candidates also made mistakes in calculating the dimensions of rectangle.

Suggestions for teachers

- Derivatives of all forms of functions require continuous practice and review from time to time. Sufficient time needs to be spent on this topic.
- Students need to familiarize themselves with the area, perimeter, surface and volume of 2-Dimensional and 3-Dimensional figures in the syllabus. The function to be optimized needs to be expressed in terms of a single variable by using the given data.

  For maximum value $f'(x)=0$ and $f''(x) < 0$. This needs to be taught well.

MARKING SCHEME

Question 5.

(a) log y = $\tan^{-1}$ x

$$\therefore\ y = e^{tan^{-1}x}$$

$$\frac{dy}{dx} = e^{tan^{-1}x} \times \frac{1}{1+x^2}$$

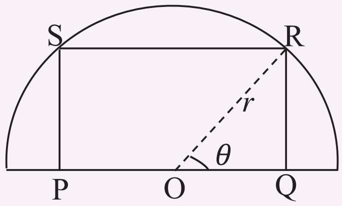
$$(\quad ^{2})\,\frac{dy}{dx} \quad {}^{tan^{-1}x}$$

$(1+x^2)\,\frac{dy}{dx} = y$

Differentiating again,

$(1+x^2)\,\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{dy}{dx}$

$$\therefore (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

(b)	$OQ = r\cos\theta$ $QR = r\sin\theta$ $A(\theta)$ = Area of the Rectangle. $A(\theta) = PQ \times QR$ $= 2OQ \times QR$ $= r^2 \sin 2\theta$  $\frac{dA}{d\theta} = 2r^2 \cos 2\theta = 0$ OR Differentiating area: A=2xy=A=2x $\sqrt{1-x^2}$ $\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{4}$ is the critical point. $\frac{d^2A}{d\theta^2} = -4r^2 \sin 2\theta$ $\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\frac{\pi}{4}} = -4r^2 < 0$ $\Rightarrow$ Area is maximum at $\theta = \frac{\pi}{4}$ So, sides of rectangle are $\sqrt{2}\,r, \frac{\sqrt{2}\,r}{2}$ and Area = r^2 sq. units

Question 6

(a) Evaluate: **[5]**

$$\int \frac{\sin x + \cos x}{\sqrt{9 + 16\sin 2x}}\,dx$$

(b) Find the area of the region bound by the curves $y = 6x - x^2$ and $y = x^2 - 2x$. **[5]**

Comments of Examiners

(a) Some candidates made mistakes in correct substitution. In most of the cases, candidates who used appropriate substitution did not give the final answer in terms of x but left in terms of the new variable.

(b) Many candidates got the correct solution for this question. A few candidates solved the question with incorrect limits. Some candidates made mistakes in sketching the curve and finding limits.

Suggestions for teachers

- Practice needs to be given in Integration using substitution and special integrals. Teachers must instruct students not to leave their answers in terms of the new variable.
- Sketching of curves may not be necessary always but a rough sketch will always help the student to understand the requirements, the area required to be found, the points of intersection and the limits of the definite integral. Since the functions given was simple algebraic the integration of such functions will not cause problems and the solution can be easily found.

MARKING SCHEME

Question 6.

(a) Let $I = \int \frac{\sin x+\cos x}{\sqrt{9+16\sin 2x}}\,dx$

Put $u = \sin x - \cos x$

$du = (\sin x + \cos x)dx$

$u^2 = 1 - \sin 2x$

$\therefore \sin 2x = 1 - u^2$

$$I = \int \frac{du}{\sqrt{9+16\,(1-u^2)}}$$

$$= \int \frac{du}{\sqrt{25-16u^2}}$$

$$= \int \frac{du}{\sqrt{5^2-(4u)^2}}$$

$$= \frac{1}{4}\sin^{-1}\left(\frac{4u}{5}\right) + c$$

$$= \frac{1}{4}\sin^{-1}\left(\frac{4(\sin x-\cos x)}{5}\right) + c$$

(b) The curve $y = 6x - x^2$

$y = -(x-3)^2 + 9$

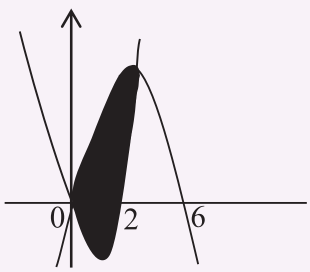
represents a parabola with vertex at (3, 9) and it opens downward.

The curve $y = x^2 - 2x$

$y = (x-1)^2 - 1$

represents a parabola with vertex at (1 -1) and it opens upward.

Both the curves pass through origin and intersect in the first quadrant at (4, 8)



	Required area $= \int_0^4 (6x - x^2)\,dx - \int_0^4 (x^2 - 2x)dx$ $= \int_0^4 (8x - 2x^2)dx = \left[8.\frac{x^2}{2} - 2.\frac{x^3}{3}\right]_0^4$ $= 64 - \frac{128}{3}$ $= \frac{64}{3}$ sq units

Question 7

(a) Calculate Karl Pearson's coefficient of correlation between *x* and *y* for the following data and interpret the result: **[5]**

(1, 6), (2, 5), (3,7), (4, 9), (5, 8), (6, 10), (7, 11), (8, 13), (9, 12)

not clear about the different *Suggestions for teachers*

(b) The marks obtained by 10 candidates in English and Mathematics are given below: **[5]**

Marks in English	20	13	18	21	11	12	17	14	19	15
Marks in Mathematics	17	12	23	25	14	8	19	21	22	19

Estimate the probable score for Mathematics if the marks obtained in English are 24.

Comments of Examiners

(a) Some candidates w ir about the different formulae for the calculation of correlation coefficient. Most of the candidates did not write the interpretation of the result. A few candidates solved the problem by Spearman's Rank correlation coefficient method.

(b) Candidates were not clear about the formulae used for b_{yx} and b_{xy}. In most of the cases, candidates applied wrong formula to evaluate b_{yx}. Some candidates used the equation for the regression line of 'x on y' instead of 'y on x' to find y when x is given.

for te...

- Plenty of practice is required in order to solve problems on correlation.
- Teachers should make sure that the students understand the need for the appropriate formula at appropriate situations. Accuracy levels should be maintained at highest level as such simplifications involve simple calculations only.

MARKING SCHEME

Question 7.

(a)

x	Y	Xy	x^2	y^2
1	6	6	1	36
2	5	10	4	25
3	7	21	9	49
4	9	36	16	81
5	8	40	25	64
6	10	60	36	100
7	11	77	49	121
8	13	104	64	169
9	12	108	81	144
45	81	462	285	789

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

$$= \frac{9\times462 - 45\times81}{\sqrt{[9\times285-(45)^2][9\times789-(81)^2}}$$

$$= \frac{513}{\sqrt{540\times540}} \quad = 0{\cdot}95$$

There is a high +ve correlation between the two variables.

(b)

Eng (x)	Maths (y)	dx=x-$\bar{x}$	dy=y-$\bar{y}$	$(dx)^2$	$(dy)^2$	dxdy
20	17	4	-1	16	1	-4
13	12	-3	-6	9	36	18
18	23	2	5	4	25	10
21	25	5	7	25	49	35
11	14	-5	-4	25	16	20
12	8	-4	-10	16	100	40
17	19	1	1	1	1	1
14	21	-2	3	4	9	-6
19	22	3	4	9	16	12
15	19	-1	1	1	1	-1

	$\sum x = 160 \ \sum y = 180 \ \sum dxdy = 125 \ \sum dx^2 = 110 \ \sum dy^2 = 254$ $b_{yx} = \frac{\sum(dxdy)}{(dx)^2}$ $= \frac{125}{110}$ $= \frac{25}{22} = 1{\cdot}13$ Regression equation y on x is $y - 18 = \frac{25}{22}(x - 16)$ Probable score $y - 18 = \frac{25}{22}\ (24 - 16)$ $y = 9.0909 + 18 = 27{\cdot}0909 = 27{\cdot}1$

Question 8

Explain the probability laws and its

(a)	A committee of 4 persons has to be chosen from 8 boys and 6 girls, consisting of at least one girl. Find the probability that the committee consists of more girls than boys.	**[5]**
(b)	An urn contains 10 white and 3 black balls while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. Find the probability that the ball drawn from the second urn is a white ball.	**[5]**

Comments of Examiners

(a) Very few candidates understood the question and answered it correctly. Most of the candidates could not write the complete combinations correctly. A few candidates answered based on the definition of probability and others approached on conditional probability.

(b) Few candidates not considered the possibility of "one white and one black" when two balls are drawn from the first urn. Most of the candidates answered correctly.

Suggestions for teachers

the probability laws and applications thoroughly. Teachers must explain the concepts with the help of addition and multiplication rule and its applications including conditional probability and condition based problems.

– Explain and make students to understand the problem and identify the cases that satisfy the situation and conditions.

MARKING SCHEME

Question 8.

(a) Number of ways the committee can be selected:

$= 14_{c_4} - 8_{c_4} = 1001 - 70 = 931$ or $\left(6_{c_1}.8_{c_3}\right) + \left(6_{c_2}.8_{c_2}\right) + \left(6_{c_3}.8_{c_1}\right) + 6_{c_4}$

No. of Committees consists of more girls than boys= $6_{c_4} + 6_{c_3} \times 8_{c_1}$

$$P(E) = \frac{6_{c_4} + 6_{c_3} \times 8_{c_1}}{931}$$

$$= \frac{15+160}{931} = \frac{175}{931} = 0 \cdot 188$$

(b) P(transferring 2 white balls to urn 2 and drawing a white ball from urn 2)

$$= \frac{10C_2 \times 5C_1}{13C_2 \times 10C_1}$$

$$= \frac{45}{13\times12}$$

p (transferring 2 black balls to urn 2 land drawing a white from urn 2)

$$= \frac{13C_2 \times 3C_1}{13C_2 \times 10C_1}$$

$$= \frac{9}{13\times12\times5}$$

P(transferring1white and a black ball to urn 2 and drawing a white ball from urn 2)

$$= \frac{10C_1 \times 3C_1 \times 4C_1}{13C_2 \times 10C_1}$$

$$= \frac{}{13\times12}$$

Required probability $= \frac{1}{13\times12}\left[45 + 24 + \frac{9}{5}\right] = \frac{59}{130}$

Question 9

(a) Find the locus of a complex number, $z = x + iy$, satisfying the relation $\left|\frac{z-3i}{z+3i}\right| \leq \sqrt{2}$. Illustrate the locus of z in the Argand plane. **[5]**

(b) Solve the following differential equation: **[5]**

$x^2\, dy + (xy + y^2)\, dx = 0$, when $x = 1$ and $y = 1$.

Comments of Examiners

(a) Most of the candidates did not sketch the locus in the Argand plane. Made mistakes while marking centre and with correct radius.

(b) Candidates made errors while solving the integration part due to lack of clear understanding about the formula of special integrals and sufficient practice.

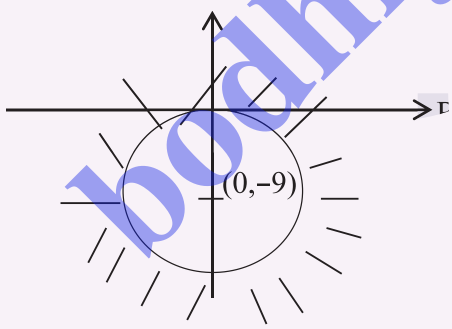
Suggestions for teachers

– Sketching of straight lines and curves (circle, conics etc.) should be practiced regularly.

– Post differentiation and integration solving of differential equations needs a lot of practice.

MARKING SCHEME

Question 9.

(a)	z = x + iy $\left\lvert\frac{x+iy-3i}{x+iy+3i}\right\rvert \le \sqrt{2}$ $\Rightarrow \sqrt{x^2+(y-3)^2} \le \sqrt{2}\,\sqrt{x+(y+3)^2}$ Squaring $x^2 + y^2 + -6y + 9 \le 2\,(x^2 + y^2 + 6y + 9)$ $\Rightarrow x^2 + y^2 + 18y + 9 \ge 0$ $\Rightarrow x^2 + (y+9)^2 - 81 + 9 \ge 0$ $\Rightarrow x^2 + (y+9)^2 \ge 72$ The locus is the region outside the circle with centre (0, –9) and the radius=$\sqrt{72}$ 
(b)	$\frac{dy}{dx} = -\frac{(xy+y^2)}{x^2}$ put $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = -v - v^2$ $x\frac{dv}{dx} = -2v - v^2$ $\int \frac{dv}{(v^2+2v)} = \int -\frac{dx}{x}$

	$\frac{1}{2}\, log \left\vert \frac{v}{v+2} \right\vert = -\log x + \log c$ $\frac{v}{(v+2)} = \frac{c}{x^2}$ $\frac{y}{(y+2x)} = \frac{c}{x^2}$ $3x^2y = y + 2x$ (when $x = 1, y = 1$) Or $\sqrt{\frac{v}{v+2}} = \frac{c}{x}$ $c = \frac{1}{\sqrt{3}}$ Or $log\, x\sqrt{\frac{v}{v+2}} = c$ $log\ \frac{1}{\sqrt{3}} = c$

SEC 20 Marks)

Question 10

(a) For any three vectors $\vec{a}, \vec{b}, \vec{c}$, show that $\vec{a} - \vec{b},\ \vec{b} - \vec{c}, \vec{c} - \vec{a}$ are coplanar. **[5]**

(b) Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ **[5]**

Comments of Examiners

(a) Some of the candidates failed to apply condition of coplanarity. Many candidates made errors while simplifying the scalar triple product $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}]$. A few candidates failed to write the correct order of scalar triple product.

(b) Some candidates used cross product without evaluating $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. Some candidates made mistakes while evaluating the unit vector in the final answer.

Suggestions for teachers

– Vector notations, usage, dot and cross products in terms of the vectors or their components need to be taught well and in detail. Unit vector and its properties must be taught with a number of examples.

– Scalar triple product and its applications need to be taught with the help of practical examples.

– Students must be asked to read the instructions given in the question carefully.

MARKING SCHEME

Question 10.

(a)

$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}]$

$= (\vec{a}-\vec{b}) \,.\, [(\vec{b}-\vec{c}) \times (\vec{c}-\vec{a})]$

$= (\vec{a}-\vec{b}) \,.\, [\vec{b}\times\vec{c} - \vec{b}\times\vec{a} - \vec{c}\times\vec{c} + \vec{c}\times\vec{a}]$

$= (\vec{a}-\vec{b}) . [\vec{b}\times\vec{c} + \vec{a}\times\vec{b} + \vec{c}\times\vec{a}]$

$= \vec{a}\,.\left(\vec{b}\times\vec{c}\right) + \vec{a}.(\vec{a}\times\vec{b}) + \vec{a}\,.(\vec{c}\times\vec{a}) - \vec{b}\,.\left(\vec{b}\times\vec{c}\right) - \vec{b}.(\vec{a}\times\vec{b}) - \vec{b}\,.(\vec{c}\times\vec{a})$

$= [\vec{a} \quad \vec{b} \quad \vec{c}\,] - [\vec{b} \quad \vec{c} \quad \vec{a}\,]$

$= [\vec{a} \quad \vec{b} \quad \vec{c}\,] - [\vec{a} \quad \vec{b} \quad \vec{c}\,] = 0$, hence vectors are coplannar.

OR

If a+b+c=0 , one vector can be represented as liner combination of other two

∴ a,b,c are coplanar

Hence proved that a-b, b-c, c-a are coplanar

Since (a-b)+(b-c)+(c-a)=0

(b)

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}, \ \vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$\vec{a} + \vec{b} = 4\hat{\imath} + 4\hat{\jmath}$

$\vec{a} - \vec{b} = 2\hat{\imath} + 4\hat{k}$

$\vec{r} = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = (4\hat{\imath} + 4\hat{\jmath}) \times (2\hat{\imath} + 4\hat{k})$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16\hat{\imath} - 16\hat{\jmath} - 8\hat{k}$$

$$= 8\,(2\hat{\imath} - 2\hat{\jmath} - \hat{k})$$

$\therefore$ *unit vector*

$$= \frac{\vec{r}}{|\vec{r}\,|} = \frac{8(2\hat{\imath}-2\hat{\jmath}-\hat{k})}{8 \times 3}$$

$$= \frac{2}{3}\,\hat{\imath} - \frac{2}{3}\,\hat{\jmath} - \frac{1}{3}\hat{k}$$

Question 11

(a) Find the image of the point (2, −1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ **[5]**

Also, find the length of the perpendicular from the point (2, −1, 5) to the line.

(b) Find the Cartesian equation of the plane, passing through the line of intersection of the planes: $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) + 5 = 0$ and $\vec{r}.(\hat{\imath} - 5\hat{\jmath} + 7\hat{k}) + 2 = 0$ and intersecting y-axis at (0, 3). **[5]**

<u>Comments of Examiners</u>

(a) Candidates made mistakes in calculating a point M on the line PQ and then applying the condition to prove that AM ⊥ PQ ie $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Some made errors while calculating the length of perpendicular.

(b) Most of the candidates were able to score marks in this part.

<u>Suggestions for teachers</u>

– Need to emphasize on complete understanding of perpendicularity condition at different stages. Give more practice in solving problems based on the concept of Image of a given point with reference to a line.

MARKING SCHEME

Question 11.

(a) Let A (2, -1, 5) be [illegible] nd PQ be the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.

Let M be any point on the line PQ is

M (10r + 11, −4r −2, −11r – 8)

DR's of AM

10r + 9, −4r – 1, −11r – 13

AM is perpendicular to PQ

$\therefore a_1, a_2 + b_1b_2 + c_1c_2 = 0$

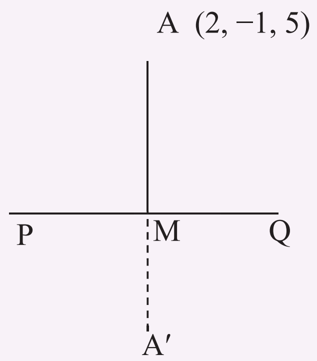
10 (10r + 9) −4 (−4r −1) −11 (−11r – 13) = 0

100 r + 90 + 16r + 4 + 121r + 143 = 0

237r = −237

$\therefore r = -1$

$\therefore$ M is (1, 2, 3)



	M is the midpoint of A,A′ $\frac{2+x}{2}=1,\ \ \frac{-1+y}{2}=2,\ \ \frac{5+z}{s}=3$ Coordinates of A′ are (0, 5, 1) Length of the perpendicular = $\sqrt{(-1)^2+3^2+(-2)^2}$ = $\sqrt{14}$ units

Question 12

(a) In an automobile factory, certain parts are to be fixed into the chassis in a section before it moves into another section. On a given day, one of the three persons A, B and C carries out this task. A has 45% chance, B has 35% chance and C has 20% chance of doing the task. The probability that A, B and C will take more than the allotted time is $\frac{1}{6}, \frac{1}{10}$ *and* $\frac{1}{20}$ respectively. If it is found that the time taken is more than the allotted time, what is the probability that A has done the task? **[5]**

(b) The difference between mean and variance of a binomial distribution is 1 and the difference of their squares is 11. Find the distribution. **[5]**

Comments of Examiners

(a) Many candidates used Baye's theorem correctly but took the probabilities of A, B, C, as 45, 35, 20, instead of percentages. Some candidates did not implement the theorem correctly.

(b) Several candidates made errors in solving the equations np-npq=1, $(np)^2$-$(npq)^2$=11. Most of the candidates were not clear in writing the correct Binomial Distribution.

Suggestions for teachers

- Teachers need to emphasize that probabilities are ratios and not numbers. Conditional probability concept is to be taught clearly which helps the students to understand the advanced concept Baye's theorem.
- Revision must be done on concepts mean and variance of binomial probability distribution. While solving for n, p, and q it must be noted that p+q=1 . More practice in solving equations in two /three unknowns is essential.

MARKING SCHEME **Question 12.**	
(a)	Let E_1, E_2, E_3, denote the events of carrying out the task by A, B and C respectively. Let H denote the event of taking more time. Then $P(E_1) = 0.45$ $P(E_2) = 0.35$ $P(E_3) = 0.20$ $P(H/E_1) = \frac{1}{6}$ $P(H/E_2) = \frac{1}{10}$ $P(H/E_3) = \frac{1}{20}$

	$P(\frac{E_1}{H}) = \frac{P(E_1).P(H/E_1)}{P(E_1).P\left(\frac{H}{E_1}\right)+P(E_2).P\left(\frac{H}{E_2}\right)+P(E_3).P(H/E_3}$ $= \frac{0.075}{0.075+0.035+0.01} = 0.625 = \frac{5}{8}$
(b)	$np - npq = 1, \quad np(1-q) = 1$ (i) $(np)^2 - (npq)^2 = 11, \quad (np)^2\,(1 - q^2) = 11$ (ii) dividing (ii) by (i), we get $(1+q)/(1-1) = 11$ $1 + q = 11 - 11q$ $12q = 10$ $q = 5/6,\ p = 1/6$ we get $n = 36$ The distribution is given by $(\frac{1}{6} + \frac{5}{6})^{36}$ or $x \sim B\,(36, \frac{1}{6})$

SECTION C (20 Marks)

Question 13

(a) A man borrows ₹20,000 at 12% per annum, compounded semi-annually and agrees to pay it in 10 equal semi-annual installments. Find the value of each installment, if the first payment is due at the end of two years. **[5]**

(b) A company manufactures two types of products A and B. Each unit of A requires 3 grams of nickel and 1 gram of chromium, while each unit of B requires 1 gram of nickel and 2 grams of chromium. The firm can produce 9 grams of nickel and 8 grams of chromium. The profit is ₹40 on each unit of product A and ₹50 on each unit of type B. How many units of each type should the company manufacture so as to earn maximum profit? Use linear programming to find the solution. **[5]**

Comments of Examiners

(a) Many candidates made errors while identifying the total time and deferred time. Deferred annuity formula was noted wrongly in most of the cases. Some substitutions were also not correct. Very few candidates solved this part correctly.

(b) In some cases, all the constraints were not used and hence coordinates of all feasible points were not obtained.

Suggestions for teachers

- Help students to distinguish between total time and deferred time, the concept of deferred annuity and its application in solving problems.
- The optimum function and all possible constraints in the form of inequations have to be put down from what is stated in the problem. Students must be made to solve the constraints equations in pairs to obtain all feasible points leading to maximum or minimum value of desired function.

MARKING SCHEME

Question 13.

(a) P = ₹ 20,000, n = 10, m = 3 and i = 0·06

$$P = \frac{a}{0\cdot06}\left\{\frac{1}{(1+i)^m} - \frac{1}{(1+i)^{n+m}}\right\}$$

$$20{,}000 = \frac{a}{0\cdot06}\left\{\frac{1}{(1\cdot06)^3} - \frac{1}{(1\cdot06)^{13}}\right\}$$

$\Rightarrow a = 3235\cdot90$

Size of the installment is ₹ 3235·90

(b) Let x units of prod y units of product B.

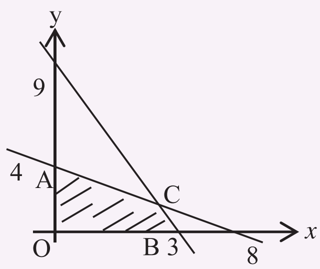
Maximize Z = 40x + 50y

Subject to constraints

$3x + y \leq 9$

$x + 2y \leq 8$

$x \geq 0,\ y \geq 0$



Solving, we get A(0, 4), B(3, 0), C(2, 3)

At A, z = 40×0 + 50× 4 = ₹ 200

B, z = 40×3 + 50×0 = ₹ 120

C, z = 40×2 + 50×3 = ₹ 230

Maximum profit is ₹ 230, when 2 units of type A and 3 units of type B are produced.

Question 14

(a) The demand function is $x = \frac{24 - 2p}{3}$ where x is the number of units demanded and p is the price per unit. Find: **[5]**

(i) The revenue function R in terms of p.

(ii) The price and the number of units demanded for which the revenue is maximum.

(b) A bill of ₹ 1,800 drawn on 10th September, 2010 at 6 months was discounted for ₹1,782 at a bank. If the rate of interest was 5% per annum, on what date was the bill discounted? **[5]**

<u>Comments of Examiners</u>

(a) Some of the candidates made mistakes in obtaining Revenue function in terms of p. Several candidates forgot to apply the condition for the maxima $\frac{d^2R}{dp^2} < 0$ for R.

(b) Some candidates used True Discount formula instead of Banker's Discount or relevant formula for calculating 'n'. Several candidates had no idea how to get the Date of Discounting.

<u>Suggestions for teachers</u>

- Question should be answered as asked in the paper and Revenue function must be found in terms of p. Application of Maxima and Minima should be taught by taking up more examples.
- All relevant terms and formulae need to be taught well for complete understanding.

MARKING SCHEME

Question 14.

(a) Total revenue function = px

Revenue function = $\frac{p(24-2p)}{3}$

$= \frac{24p - 2p^2}{3}$

$dR/dp = (24 - 4p)/3$

For maximum or minimum $dR/dp = 0$

$(24 - 4p) / 3 = 0$

$p = 6$

$d^2R / dp^2 = -4/3 < 0 \quad \therefore$ R is maximum

Number of units $x = (24 - 12)/3$

$x = 4$

Hence, R is maximum when 4 units are demanded at the price of □ 6 per unit.

(b)	A = 1800; i= 5% p.a. BD = 1800 – 1782 = 18 BD = Ani $\Rightarrow$ 18 = 1800 × n × $\frac{5}{100}$ $\Rightarrow$ n = $\frac{1}{5}$ year = 73 days Date of expiry: March 13, 2011 Date of discounting: December 30, 2010.

Question 15

(a) The index number by the method of aggregates for the year 2010, taking 2000 as the base year, was found to be 116. If sum of the prices in the year 2000 is □300, find the values of x and y in the data given below: **[5]**

Commodity	A	B	C	D	E	F
Price in the year 2000 (□)	50	x	30	70	116	20
Price in the year 2010 (□)	60	24	y	80	120	28

(b) From the details given below, calculate the five yearly moving averages of the number of candidates who have studied in a school. Also, plot these and original data on the same graph paper. **[5]**

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Number of Students	332	317	357	392	402	405	410	427	405	438

Comments of Examiners

(a) Most of the candidates attempted this part correctly. Errors were committed by a few candidates while calculating value of 'y'.

(b) Moving averages were mostly correctly calculated but for plotting, centered moving averages were required. Some candidates did not use the centered averages. In some cases, the graphs were not neat.

Suggestions for teachers

- Help the students to identify whether the question is based on aggregate or average method. Thorough knowledge of the formula is required.
- Moving Averages need to be calculated correct to two decimal places. For plotting, centered averages correct to one decimal place are sufficient. The axes should be labelled and the plotting and sketching should be as neat as possible; the graph should be given a caption.

MARKING SCHEME

Question 15.

(a)

Commodity	**Price in ₹**	
	2000	**2010**
A	50	60
B	x	24
C	30	y
D	70	80
E	116	120
F	20	28
	$286+x$	$312+y$

$286 + x = 300$

$x = 14$

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$$

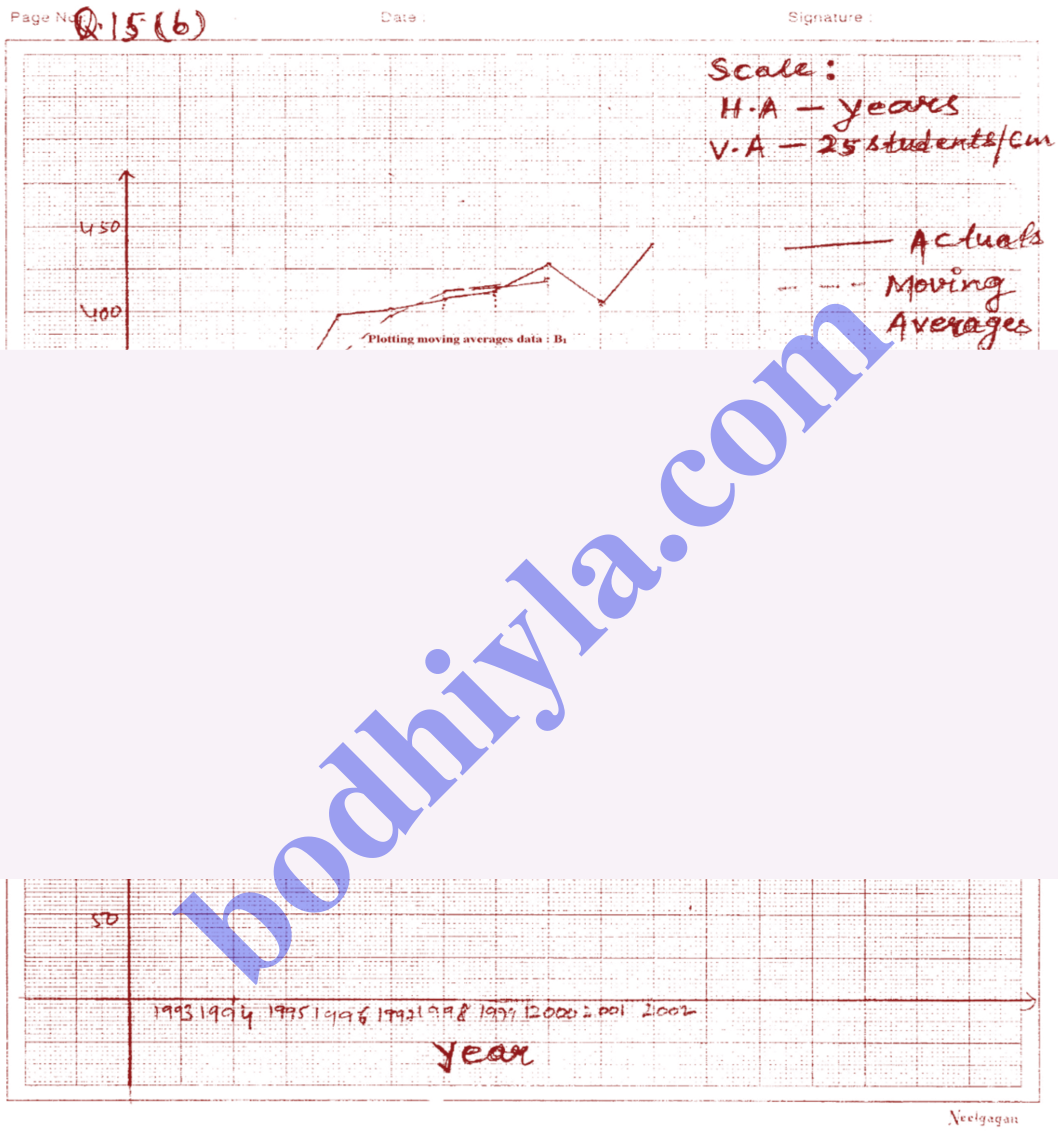
$$116 = \frac{12+y}{300} \times 100$$

$36 = y$

(b)

Year	**Number**	**5yrs moving total**	**5yr moving average**

		1993	332	--	--	
		1994	317	--	--	
		1995	357	1800	360	
		1996	392	1873	374·6	
		1997	402	1966	393·2	
		1998	405	2036	407·2	
		1999	410	2049	409·8	
		2000	427	2085	417	
		2001	405	--	--	
		2002	438	--	--	



Note: For questions having more than one correct solution, alternate correct solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS:

(a) Topics found difficult by candidates:

- Indefinite Integrals (use of substitution or integration by parts)
- Definite Integrals – use of properties.
- Inverse Circular Functions (formulae and relations)
- Differential Equations (solving Homogeneous and Linear Differential Equations)
- Vectors – in general
- Annuity (Deferred annuities)
- Conics in general
- Probability – use of sum and product laws and identifying all cases.
- Maxima and Minima

(b) Concepts in which candidates got confused:

- Regression lines: y on x and x on y
- Sum and product laws of probability
- 3 – D: Image of a given point and perpendicular distance
- Conditional probability property in Baye's theorem
- Price Index by aggregate and Price Relative methods
- Differences between and usage of formulae for BD, TD, BG, DV, etc.

(c) Suggestions for candidates:

- Study the entire syllabus thoroughly and revise from time to time. Concepts of Class XI must be revised and integrated with the Class XII syllabus.
- Develop logical and reasoning skills to have a clear understanding.
- Revise all topics and formulae involved and make a chapter wise or topic-wise list of these.
- Make wise choices from the options available in the question paper and management time wisely.
- Be methodical and neat in working.