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Matrices

INTRODUCTION

Matrices owe their origin to various linear problems, the most important of which consists in the nature of solutions of systems of simultaneous linear equations. In 1857, Arthur Cayley developed the properties of matrices as a pure algebraic structure. These days, matrices are used in a large number of disciplines such as Algebra, Geometry, Statistics, Physics, Chemistry, Psychology etc. In the present chapter, we shall learn about matrices and shall confine ourselves to the study of basic laws of matrix algebra and the applications of these in solving systems of simultaneous linear equations in 2 or 3 unknowns.

2.1 MATRICES

Consider the systems of simultaneous linear equations :

$$\left. \begin{array}{l} 2x - 3y = 7 \\ 5x + 4y = 11 \end{array} \right\} \dots(i)$$

It is a system of two linear equations in two unknowns x, y ; and

$$\left. \begin{array}{l} 2x - 7y + 3z = 8 \\ 3x + 0y + 6z = -11 \end{array} \right\} \dots(ii)$$

is a system of two linear equations in three unknowns x, y, z . In the first case, to know L.H.S.

we need only know the array $\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ consisting of the coefficients of x and y , in this

particular order. Similarly, the array $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$ gives all information about R.H.S. Thus, the system

of equations in (i) is completely specified by the two arrays of numbers $\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$.

Similarly, the system of equations in (ii) is fully specified by the two arrays

$$\begin{bmatrix} 2 & -7 & 3 \\ 3 & 0 & 6 \end{bmatrix} \text{ and } \begin{bmatrix} 8 \\ -11 \end{bmatrix}.$$

Each array of the above type is known as a *matrix*. The numbers, of which each matrix is constituted, are called the *elements* or *entries*. In the above examples, all elements are real numbers. In fact, the elements can be complex numbers, polynomials or even more general functions.

For each element of a matrix, its value as well as its position are both important. It means that a matrix is an *ordered array* of elements.

In the (i) system of equations, the array $[2 \ -3]$ specifies the left side of the first equation and the array $[5 \ 4]$ that of the second equation. Each such horizontal array of a matrix is called a *row* of the matrix.

Similarly, the array $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ specifies the coefficients of x and the array $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$ specifies the coefficients of y in the (i) system of equations. Each such vertical line is called a *column* of the matrix.

Thus, the matrix $\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ consists of two rows and two columns or briefly is a 2×2 matrix, while the matrix $\begin{bmatrix} 2 & -7 & 3 \\ 3 & 0 & 6 \end{bmatrix}$ corresponding to the (ii) system of equations is a matrix with two rows and three columns or briefly a 2×3 matrix. The matrices corresponding to the right side of the above systems (i) and (ii) are 2×1 matrices.

Matrix. A rectangular arrangement of mn elements in the form of an ordered set of m rows, each row consisting of an ordered set of n elements is called an $m \times n$ **matrix** ($m \times n$ is read as m by n).

Such an arrangement is enclosed in the bracket $[\]$ or parenthesis $(\)$. Each of the mn elements constituting the matrix is called an **element** or **entry** of the matrix. To locate the position of a particular element of a matrix, we have to specify the number of row and that of the column in which the element occurs.

An element appearing in the i th row and the j th column is referred as (i, j) th element or (i, j) th entry. Usually, a matrix is denoted by a capital letter and an element of a matrix by a small letter of the *alphabet* along with two suffixes, the first one indicating the number of row and the latter one that of the column in which the element appears. Thus, an $m \times n$ matrix may be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}.$$

In compact form, this matrix is written as

$$A = [a_{ij}]_{m \times n}, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n.$$

A is called a matrix of order (or type) $m \times n$.

If $a_{ij} \in \mathbf{R}$ (set of reals), we say that A is a matrix over \mathbf{R} or A is a real matrix. Similarly, if $a_{ij} \in \mathbf{C}$ (complex numbers) then A is a *complex* matrix.

The array $[a_{i1} \ a_{i2} \ a_{i3} \ \dots \ a_{in}]$ is the i th row and the array $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \dots \\ a_{mj} \end{bmatrix}$ is the j th column of the matrix A .

Comparable Matrices

Two matrices are said to be **comparable** iff each one of them contains as many rows and columns as the other.

Equal Matrices

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be **equal** (written as $A = B$) iff

(i) they are of the same order i.e. number of rows in $A =$ number of rows in B and number of columns in $A =$ number of columns in B , and

(ii) their corresponding elements are equal i.e. $a_{ij} = b_{ij}$ for all i and j .

For example, the matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ are equal iff $a = \alpha$, $b = \beta$, $c = \gamma$ and $d = \delta$.

2.1.1 Types of Matrices

(1) Row Matrix

A matrix having only one row is called a **row matrix**. Thus,

$$A = [1 \quad 5 \quad 3 \quad -7] \text{ is a } 1 \times 4 \text{ row matrix.}$$

(2) Column Matrix

A matrix having only one column is called a **column matrix**. Thus, $A = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}$ is a 3×1 column matrix.

(3) Zero Matrix

An $m \times n$ matrix whose each element is zero is called a **zero matrix** or a **null matrix** of order $m \times n$. Thus

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ is a zero matrix of order } 2 \times 3.$$

(4) Square Matrix

A matrix in which the number of rows equals the number of columns, say n , is called a **square matrix of order n** or an **n -rowed square matrix**.

The elements a_{ij} of a square matrix $A = [a_{ij}]$ for which $i = j$ are called the **diagonal elements** and the line along which these elements lie is called the **principal diagonal** or simply the **diagonal** of the matrix. Thus, the elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ are the diagonal elements of the square matrix $A = [a_{ij}]_{n \times n}$.

For example, $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 7 & 9 \\ 0 & 1 & 3 \end{bmatrix}$ is a square matrix of order 3 and its diagonal elements are

2, 7, 3.

(5) Diagonal Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is called a **diagonal matrix** iff all its non-diagonal elements are zero. The diagonal elements may or may not be zero.

For example, $[4]$, $\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are diagonal matrices of order 1, 2 and 3

respectively.

A diagonal matrix of order n is written as diagonal $[a_{11}, a_{22}, a_{33}, \dots, a_{nn}]$.

(6) Scalar Matrix

A diagonal matrix in which all diagonal elements are equal is called a **scalar matrix**. Thus in a scalar matrix, $A = [a_{ij}]_{n \times n}$, we have

$$a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ k, & \text{when } i = j. \end{cases}$$

For example, $[3]$, $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ are scalar matrices of order 1, 2 and 3 respectively.

(7) Identity Matrix

A scalar matrix in which each diagonal element is unity is called an **identity matrix** or a **unit matrix**. Thus in an identity matrix, $A = [a_{ij}]_{n \times n}$ we have

$$a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ 1, & \text{when } i = j. \end{cases}$$

For example, $[1]$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3 respectively.

An identity matrix of order n is usually denoted by I_n .

(8) Triangular Matrix

(i) **Upper Triangular Matrix.** A square matrix $A = [a_{ij}]_{n \times n}$ is called an **upper triangular matrix** iff $a_{ij} = 0$ for all $i > j$ i.e. iff all elements below the principal diagonal are zero.

(ii) **Lower Triangular Matrix.** A square matrix $A = [a_{ij}]_{n \times n}$ is called a **lower triangular matrix** iff $a_{ij} = 0$ for all $i < j$ i.e. iff all elements above the principal diagonal are zero.

(iii) **Triangular Matrix.** A matrix, upper or lower triangular is called a **triangular matrix**.

A triangular matrix $A = [a_{ij}]_{n \times n}$ is called **strictly triangular** iff

$$a_{ii} = 0 \text{ for all } i = 1, 2, 3, \dots, n.$$

For example, $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 7 & 0 & 0 \\ 4 & 0 & 0 \\ -1 & 2 & 9 \end{bmatrix}$ are upper triangular and lower triangular

matrices of order 3 respectively.

ILLUSTRATIVE EXAMPLES

Example 1. A is a matrix of the type 3×5 and R is a row of A , then what is the type of R as a matrix ?

Solution. Since A is a matrix of type 3×5 , therefore, each row of A contains 5 elements. Hence, R is a row matrix of the type 1×5 .

Example 2. Let $A = \begin{bmatrix} 3 & 7 & -1 \\ 0 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 9 & 0 \\ 3 & -5 & 6 \end{bmatrix}$. Find the sum of (2, 2) entries of A and B .

Solution. Here (2, 2) entry of A is 2 and (2, 2) entry of B is -5 .

\therefore The sum of (2, 2) entries of A and $B = 2 + (-5) = -3$.

Example 3. Construct a 2×3 matrix A whose elements a_{ij} are given by

(i) $a_{ij} = 2i - 3j$ (ii) $a_{ij} = i \cdot j$

Solution. (i) $a_{ij} = 2i - 3j$, $i = 1, 2$ and $j = 1, 2, 3$

\therefore $a_{11} = 2.1 - 3.1 = -1$, $a_{12} = 2.1 - 3.2 = -4$, $a_{13} = 2.1 - 3.3 = -7$;
 $a_{21} = 2.2 - 3.1 = 1$, $a_{22} = 2.2 - 3.2 = -2$, $a_{23} = 2.2 - 3.3 = -5$.

Hence, the required matrix $A = \begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \end{bmatrix}$.

(ii) $a_{ij} = i \cdot j$, $i = 1, 2$ and $j = 1, 2, 3$

\therefore $a_{11} = 1.1 = 1$, $a_{12} = 1.2 = 2$, $a_{13} = 1.3 = 3$;
 $a_{21} = 2.1 = 2$, $a_{22} = 2.2 = 4$, $a_{23} = 2.3 = 6$.

Hence, the required matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$.

Example 4. For what values of x and y are the following matrices equal?

$$A = \begin{bmatrix} 2x + 1 & y^2 + 2 \\ 5 & y^2 - 5y \end{bmatrix}, \quad B = \begin{bmatrix} x + 3 & 3y \\ 5 & -6 \end{bmatrix}.$$

Solution. By definition of equality of matrices, we get

$2x + 1 = x + 3$...(i)

$y^2 + 2 = 3y$...(ii)

$y^2 - 5y = -6$...(iii)

From (i), $2x - x = 3 - 1 \Rightarrow x = 2$.

From (ii), we get

$$y^2 - 3y + 2 = 0$$

$$\Rightarrow (y - 1)(y - 2) = 0 \Rightarrow y = 1, 2.$$

From (iii), we get

$$y^2 - 5y + 6 = 0$$

$$\Rightarrow (y - 2)(y - 3) = 0 \Rightarrow y = 2, 3.$$

Since (ii) and (iii) must be true simultaneously, we take the common value of y .
Therefore, $y = 2$.

Hence, $A = B$ if $x = 2$ and $y = 2$.

Example 5. Find x , y , a and b if

$$\begin{bmatrix} 3x + 4y & 6 & x - 2y \\ a + b & 2a - b & -3 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 5 & -5 & -3 \end{bmatrix}.$$

Solution. By definition of equality of matrices, we get

$$3x + 4y = 2 \quad \dots(i)$$

$$x - 2y = 4 \quad \dots(ii)$$

$$a + b = 5 \quad \dots(iii)$$

$$2a - b = -5 \quad \dots(iv)$$

To find x , y ;

Multiply (ii) by 2 and then add it to (i), we get

$$5x = 10 \Rightarrow x = 2.$$

$$\therefore \text{From (i), } 3(2) + 4y = 2 \Rightarrow 4y = -4 \Rightarrow y = -1.$$

To find a , b ;

Adding (iii) and (iv), we get

$$3a = 0 \Rightarrow a = 0.$$

$$\therefore \text{From (iii), } 0 + b = 5 \Rightarrow b = 5.$$

Hence, $x = 2$, $y = -1$, $a = 0$, $b = 5$.

EXERCISE 2.1

1. If A is a matrix of type $p \times q$ and R is a row of A , then what is the type of R as a matrix?
2. If A is a column matrix with 5 rows, then what type of matrix is a row of A ?
3. If a matrix has 12 elements, what are the possible orders it can have? What if it has 7 elements?

4. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by

$$(i) a_{ij} = \frac{(i+2j)^2}{2} \quad (ii) a_{ij} = \frac{1}{2} |2i - 3j|.$$

5. Construct a 2×3 matrix whose elements a_{ij} are given by

$$(i) a_{ij} = 2i - j \quad (ii) a_{ij} = \frac{i-j}{i+j} \quad (iii) a_{ij} = \frac{(i-2j)^2}{2}.$$

6. (i) If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$, then find the value of $(y-x)$.

(ii) If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then find the values of x and y .

7. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find the values of a and b .

8. If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find x , y , z and w .

9. Find x , y , a and b if $\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$.

10. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$,

find the values of a , b , c , x , y and z .

2.2 OPERATIONS ON MATRICES

2.2.1 Addition of Matrices

If A and B are two matrices of the same order, we say that these are **compatible** (or **conformable**) for addition and their sum $A + B$ is the matrix obtained by adding the corresponding elements of A and B .

Thus, if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, then $A + B = [a_{ij} + b_{ij}]_{m \times n}$, $1 \leq i \leq m$, $1 \leq j \leq n$.

For example, if $A = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 0 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 9 & -11 \\ 3 & 1 & -2 \end{bmatrix}$

then $A + B = \begin{bmatrix} 2+4 & 3+9 & -1+(-11) \\ 5+3 & 0+1 & 7+(-2) \end{bmatrix} = \begin{bmatrix} 6 & 12 & -12 \\ 8 & 1 & 5 \end{bmatrix}$.

Note. If A and B are not of the same order, then $A + B$ is not defined, and we say that A and B are not compatible for the sum $A + B$.

2.2.2 Subtraction of Matrices

If A and B are two matrices of the same order, we say that these are **compatible for subtraction** and their difference $A - B$ is the matrix obtained by subtracting the elements of B from the corresponding elements of A .

Thus, if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, then $A - B = [a_{ij} - b_{ij}]_{m \times n}$, $1 \leq i \leq m$, $1 \leq j \leq n$.

For example, if $A = \begin{bmatrix} 4 & -7 & -2 \\ 5 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & -5 \\ -9 & 6 & -3 \end{bmatrix}$

then $A - B = \begin{bmatrix} 4-2 & -7-4 & -2-(-5) \\ 5-(-9) & 3-6 & 0-(-3) \end{bmatrix} = \begin{bmatrix} 2 & -11 & 3 \\ 14 & -3 & 3 \end{bmatrix}$.

Properties of Matrix Addition

(1) Addition of matrices is commutative

i.e. if A and B are matrices of the same order then $A + B = B + A$.

Proof. Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ be two matrices of the same order $m \times n$, then

$$\begin{aligned} A + B &= [a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} = [a_{ij} + b_{ij}]_{m \times n} && \text{(by def. of sum)} \\ &= [b_{ij} + a_{ij}]_{m \times n} && (\because \text{addition of numbers is commutative}) \\ &= [b_{ij}]_{m \times n} + [a_{ij}]_{m \times n} = B + A. \end{aligned}$$

(2) Addition of matrices is associative

i.e. if A , B and C are matrices of the same order then $(A + B) + C = A + (B + C)$.

Proof. Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$ and $C = [c_{ij}]_{m \times n}$ be three matrices of the same order, then

$$\begin{aligned} (A + B) + C &= ([a_{ij}]_{m \times n} + [b_{ij}]_{m \times n}) + [c_{ij}]_{m \times n} \\ &= [a_{ij} + b_{ij}]_{m \times n} + [c_{ij}]_{m \times n} = [(a_{ij} + b_{ij}) + c_{ij}]_{m \times n} \\ &= [a_{ij} + (b_{ij} + c_{ij})]_{m \times n} \quad (\because \text{addition of numbers is associative}) \\ &= [a_{ij}]_{m \times n} + [b_{ij} + c_{ij}]_{m \times n} \\ &= [a_{ij}]_{m \times n} + ([b_{ij}]_{m \times n} + [c_{ij}]_{m \times n}) \\ &= A + (B + C). \end{aligned}$$

(3) Existence of Additive Identity

For any matrix A , there exists a null matrix O of the same order such that

$$A + O = A = O + A.$$

Proof. Let $A = [a_{ij}]_{m \times n}$ be any matrix and O be the null matrix of the same order $m \times n$, then

$$\begin{aligned} O &= [0_{ij}]_{m \times n} \text{ where } 0_{ij} = 0 \text{ for all } i, j. \\ \text{Now } A + O &= [a_{ij}]_{m \times n} + [0_{ij}]_{m \times n} = [a_{ij} + 0_{ij}]_{m \times n} \\ &= [a_{ij} + 0]_{m \times n} = [a_{ij}]_{m \times n} \quad (\because a + 0 = a \text{ for all numbers } a) \\ &= A \end{aligned}$$

$$\text{Also } A + O = O + A \quad (\because \text{addition of matrices is commutative})$$

$$\therefore A + O = A = O + A.$$

(4) Existence of Additive Inverse

For any matrix $A = [a_{ij}]_{m \times n}$, there exists a matrix $-A = [-a_{ij}]_{m \times n}$ such that

$$A + (-A) = O = (-A) + A.$$

$$\begin{aligned} \text{Proof. } A + (-A) &= [a_{ij}]_{m \times n} + [-a_{ij}]_{m \times n} \\ &= [a_{ij} + (-a_{ij})]_{m \times n} = [0]_{m \times n} = [0_{ij}]_{m \times n} = O_{m \times n} \end{aligned}$$

$$\text{Also } (-A) + A = A + (-A) \quad (\because \text{addition of matrices is commutative})$$

$$\therefore A + (-A) = O = (-A) + A.$$

The matrix $-A = [-a_{ij}]_{m \times n}$ is called the **additive inverse** of the matrix $A = [a_{ij}]_{m \times n}$.

$$\text{For example, if } A = \begin{bmatrix} 2 & -3 & 0 \\ 5 & 7 & -6 \end{bmatrix}, \text{ then } -A = \begin{bmatrix} -2 & 3 & 0 \\ -5 & -7 & 6 \end{bmatrix}.$$

2.2.3 Multiplication of a matrix by a scalar (number)

If k is any scalar (number) and A is any matrix, then the matrix obtained by multiplying each element of A by k is called the **scalar multiple** of A by k and is denoted by kA .

Thus, if $A = [a_{ij}]_{m \times n}$, then $kA = [k a_{ij}]_{m \times n}$.

$$\text{For example, if } A = \begin{bmatrix} 2 & 5 & 0 \\ -3 & 7 & 9 \end{bmatrix},$$

$$\text{then } 3A = \begin{bmatrix} 3 \times 2 & 3 \times 5 & 3 \times 0 \\ 3 \times (-3) & 3 \times 7 & 3 \times 9 \end{bmatrix} = \begin{bmatrix} 6 & 15 & 0 \\ -9 & 21 & 27 \end{bmatrix}.$$

Properties of Scalar Multiplication

If A, B are two matrices of the same order and k, l are any scalars (numbers), then

$$(i) (k + l)A = kA + lA \quad (ii) k(A + B) = kA + kB \quad (iii) (kl)A = k(lA) = l(kA)$$

$$(iv) 1A = A \quad (v) (-1)A = -A.$$

29. If $A = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$ and $A + A' = I$, where I is the unity matrix, find the values of x .

30. Find the number of all possible matrices of order 3×3 with each entry 0 or 1.

31. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric, find the values of a , b and c .

32. If a matrix is both symmetric and skew-symmetric, show that it is null matrix.

33. If $A + B + C = \pi$, find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin(A+C) & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & \tan(B+C) & 0 \end{vmatrix}$$

34. Use Martin's rule to solve the following system of equations :

$$x + y + 2z = 0, \quad x + 2y - z = 9, \quad x - 3y + 3z = -14.$$

35. Using matrices, solve the following system of equations :

$$x + 5y = 3$$

$$2x + 10y = 6.$$

36. Using matrices, find k so that the equations :

$$3x - 2y + 2z = 1, \quad 2x + y + 3z = -1, \quad x - 3y + kz = 0$$

may have a unique solution.

37. Using matrices, examine whether the following systems of equations are consistent or not. If consistent, then solve them :

$$(i) \quad x + y + z = 6$$

$$(ii) \quad 2x - y + 3z = 1$$

$$x + 2y + 3z = 14$$

$$x + 2y - z = 2$$

$$x + 4y + 7z = 30$$

$$5y - 5z = 3.$$

ANSWERS

EXERCISE 2.1

1. $1 \times q$.

2. 1×1 .

3. $1 \times 12, 12 \times 1, 2 \times 6, 6 \times 2, 3 \times 4, 4 \times 3; 1 \times 7, 7 \times 1$.

4. (i) $\begin{bmatrix} 9 & 25 \\ 2 & 2 \\ 8 & 18 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$.

5. (i) $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & -\frac{1}{3} & -\frac{1}{2} \\ 1 & 0 & -\frac{1}{5} \\ 3 & 0 & -\frac{1}{5} \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 9 & 25 \\ 2 & 2 & 2 \\ 0 & 2 & 8 \end{bmatrix}$.

6. (i) 2 (ii) $x = 2, y = 3$.

7. $a = 2, b = 4$ or $a = 4, b = 2$.

8. $x = 1, y = 2, z = 3, w = 4$.

9. $x = 2, y = 1, a = 3$ and $b = 5$.

10. $a = -2, b = -7, c = -1, x = -3, y = -5, z = 2$.

EXERCISE 2.2

1. (i) $x = 3, y = -3; x = -7, y = -3$ (ii) $x = 2, y = -8$ (iii) 11 (iv) 4, -1.

2. (i) $x = 3, y = 6, z = 9, t = 6$ (ii) $a = 2, b = 4, c = 1, d = 3$.