13 Probability

INTRODUCTION

'It may rain today'.
'Rajesh is quite sure to top his class'.
'It is highly unlikely that Salman will marry Preeti'.
'I have bet 100 rupees on India winning against Pakistan'.
All the above statements indicate *uncertainties* in everyday life.

Probability is a measure of uncertainty.

The theory of probability developed as a result of studies of games of chance or gambling. Suppose you pay ₹ 2 to draw a card out of a pack of 52 cards—if you draw an ace, you get 20 rupees, otherwise you get nothing. Should you play such a game ? Does it give a *fair* chance of winning, or are you going to lose in the long run ? Search for mathematical answers to these types of questions led to the development of the modern theory of probability. Italian mathematician Jerome Cardan, French mathematicians Pascal and Pierre de Fermat, and Swiss mathematician James Bernouli were the pioneers in these studies.

13.1 RANDOM EXPERIMENTS AND SAMPLE SPACES

An experiment

An action (or operation) which results in some (well defined) outcomes is called an ***experiment.***

Sometimes, the result (*i.e.* outcome) is unique. For example, if any triangle is given then without knowing the three angles, we can definitely say that the sum of their measures is 180°. Such experiments are called **deterministic experiments.**

Sometimes the outcome may not be unique. For example, tossing a coin may throw up either heads or tails. Such experiments are called **probabilistic** or **random**.

Random experiment

An experiment is called ***random*** *if it results in two or more outcomes and it is not possible to tell (predict) the outcome in advance.*

For example :

(*i*) tossing a coin
(*ii*) tossing two coins simultaneously
(*iii*) tossing a coin three times
(*iv*) throwing a die
(*v*) drawing a card from a pack of 52 (playing) cards

All these are random experiments.

From now onwards, whenever the word experiment is used it will mean random experiment.

Sample Space

The collection of all possible outcomes of a random experiment is called ***sample space*** *associated with it, it is usually denoted by S.*

For example :

(*i*) When we toss a coin once, it may come up in either of two ways — heads or tails. So there are two possible outcomes of this random experiment. If we denote heads by H and tails by T, then the sample space of this experiment is

S = {H, T}

(*ii*) When we throw (or roll) a die, it may land with any of its 6 faces pointing upward. Thus, the outcome of this experiment is getting any of the six numbers 1, 2, 3, 4, 5, 6. Hence, the sample space for this experiment is

S = {1, 2, 3, 4, 5, 6}

ILLUSTRATIVE EXAMPLES

Example 1. *Describe the sample space when two coins are tossed together.*

Solution. Here we use *ordered pairs.*

Sample space S = {(H, H), (H, T), (T, H), (T, T)} where (H, T) indicates that first coin comes up heads and second coin comes up tails, and so on.

Example 2. *Describe the sample space of a random experiment when a coin and a die are tossed together.*

Solution. Sample space S consists of $2 \times 6 = 12$ outcomes.

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),
(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}.

We may also write

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}.

Example 3. *Find the sample space associated with the experiment of rolling a pair of dice once. Also find the number of elements of the sample space.*

Solution. First die may come up with any of the numbers 1, 2, 3, 4, 5, 6, and the second die may also come up with any of the numbers 1, 2, 3, 4, 5, 6. Together, we may denote the outcome by an ordered pair (x, y) where x appears on first die and y appears on second die. Then, the sample space is given by

$S = \{(x, y) ; x, y \in \{1, 2, 3, 4, 5, 6\}\}$

We can list all possible outcomes as :

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Number of elements (outcomes) of the sample space is $6 \times 6 = 36$.

Example 4. *An experiment consists of recording boy-girl composition of families with 2 children.*

(*i*) *What is the sample space if we are interested in knowing whether the elder child is a boy or a girl ?*

(*ii*) *What is the sample space if we are interested in knowing whether it is a boy or a girl in the order of their births ?*

(*iii*) *What is the sample space if we are interested in number of girls in the family ?*

Solution. (*i*) The elder child may either be a boy or a girl. Hence, there are only two possible outcomes. The sample space is

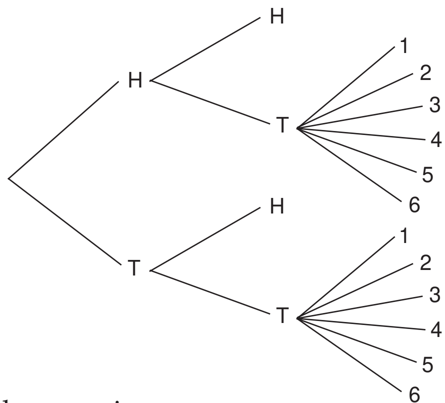
S = {Boy, Girl} or S = {B, G}.

(*ii*) As there are 2 children, there are 4 possible outcomes — boy-boy, boy-girl, girl-boy, girl-girl. Hence, the sample space is
S = {BB, BG, GB, GG}.

(*iii*) As we are interested in knowing only the number of girls, there are 3 possible outcomes — no girl, 1 girl, 2 girls. Hence, the sample space is
S = {0, 1, 2}.

Example 5. *A coin is tossed twice. If the second throw results in a tail, we roll a die. Describe the sample space. How many outcomes are possible in this experiment?*

Solution. We can draw the following diagram.

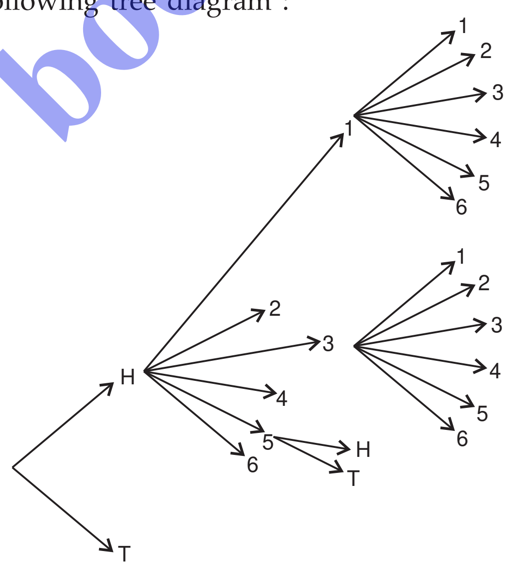


Hence the sample space is
S = {HH, HT1, HT2, HT3, HT4, HT5, HT6, TH, TT1, TT2, TT3, TT4, TT5, TT6}
There are 14 possible outcomes.

Example 6. *A coin is tossed. If the result is a head, a die is thrown. If the die shows up the numbers 1 or 3, the die is thrown again and if it shows the number 5, then a coin is tossed. Write the sample space of this experiment. How many outcomes are possible in this experiment?*

Solution. A coin is tossed, if the outcome is a tail (T). The experiment is over. If the outcome is a head (H), a die is thrown. If the die shows up an even number *i.e.* 2, 4, 6 the experiment is over. If the die shows up the numbers 1 or 3, the die is thrown again and if it shows up the number 5, a coin is tossed.

We draw the following tree diagram :



Here the sample space is

S = {T, H2, H4, H6, H11, H12, H13, H14, H15, H16, H31, H32, H33, H34, H35, H36, H5H, H5T}.

There are 18 possible outcomes.

Example 7. *Consider the experiment in which a coin is tossed repeatedly until a head comes up for the first time. Describe the sample space. What is the number of possible outcomes ?*

Solution. A head may come up on the first toss, or on the second toss or on the third toss, and so on. Hence, the sample space is

$$S = \{H, TH, TTH, TTTH, TTTTH, \ldots\}$$

The number of elements of this sample space is *infinite.*

Example 8. *We wish to choose one child out of 2 boys and 3 girls. A coin is tossed. If it comes up heads, a boy is chosen, otherwise a girl is chosen. Describe the sample space.*

Solution. The outcome of toss of coin may be denoted by H or T. The boys may be called B_1, B_2 and the girls G_1, G_2, G_3. Then the sample space is

$$S = \{HB_1, HB_2, TG_1, TG_2, TG_3\}$$

EXERCISE 13.1

1. Describe the sample space for the following experiments :
(*i*) Two coins are tossed.
(*ii*) One coin is tossed twice.
(*iii*) A coin is tossed three times.
(*iv*) Three coins are tossed simultaneously.
(*v*) A die is rolled twice.
(*vi*) A coin is tossed and a die is thrown.
(*vii*) A coin is tossed and if head comes up, a die is thrown.
(*viii*) A coin is tossed thrice and number of heads is recorded.
(*ix*) A card is drawn from a deck of playing cards, and its colour is noted.
(*x*) A card is drawn from a deck of playing cards, and its suit is noted.

2. In team A, there are 2 boys and 2 girls. In team B, there is one boy and 3 girls. First a team is chosen, and then a participant. Describe the sample space.

3. A bag contains 2 red and 3 black balls. What is the sample space when the experiment consists of drawing?
(*i*) 1 ball (*ii*) 2 balls (assuming that order is not important)

4. Redo the above problem assuming that the 2 red balls are identical and the three black balls are identical. Assume that after drawing one ball, it is replaced before drawing the second ball.

5. A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 blue and 3 white balls; if it shows tail, we toss the coin again. Describe the sample space.

6. Consider the experiment in which a coin is tossed repeatedly until a tail comes up for the first time. Describe the sample space.

7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

8. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

9. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are then put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space of the experiment.

10. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

11. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Draw the tree diagram for the possible outcomes. Write the sample space for this experiment. How many outcomes are possible in this experiment?

13.2 EVENTS

Event

A subset of the sample space associated with a random experiment is called an ***event.***

For example :

(*i*) When a die is thrown, we can get any number 1, 2, 3, 4, 5, 6. So the sample space is S = {1, 2, 3, 4, 5, 6}. There are a variety of ways in which the outcome could be reported :

Getting a six, corresponding to {6}.

Getting an even number, corresponding to {2, 4, 6}.

Getting an odd number, corresponding to {1, 3, 5}.

Getting a prime number, corresponding to {2, 3, 5}.

Getting a number less than 5, corresponding to {1, 2, 3, 4}, and so on.

All these are the events of the random experiment of throwing a die.

(*ii*) When a pair of coins is tossed, the sample space is S = {HH, HT, TH, TT}. A few events for this could be :

Getting exactly two heads : {HH}

Getting exactly one head : {HT, TH}

Getting no head : {TT}

Getting atleast one head : {HH, HT, TH}, etc.

Occurrence of event

When the outcome of an experiment satisfies the condition mentioned in the event, then we say that ***event has occurred.***

For example :

(*i*) In the experiment of throwing a die, an event E may be defined as 'getting an even number'. If the die comes up with any of the numbers 2, 4 or 6, we say that event E has occurred, otherwise, if we get 1, 3 or 5, we say that event E has not occurred.

(*ii*) In the experiment of tossing a pair of coins, an event E may be getting two heads. If the pair of coins come up with two heads, then we say that event E has occurred, otherwise, we say that event E has not occurred.

13.2.1 Types of Events

Simple event is a single possible outcome of an experiment. For example, if 3 coins are tossed up, {HHH}, {HTH} etc. indicate simple events. *A simple event is also called* ***elementary event*** *or* ***indecomposible event.***

Compound event *is the joint occurrence of two or more simple events.*

In other words, if an event has more than one sample point, it is called a compound event.

For example, when 3 coins are tossed, {HHT, HTH, THH, HHH} is a compound event corresponding to the statement "getting minimum two heads". A compound event is also called ***decomposible event.***

Sure event is *the sample space S itself.* For example, when we throw a die, S = {1, 2, 3, 4, 5, 6}. Then {getting a number less than 7} is a sure event whereas {getting a number less than 5} is not a sure event.

Impossible event *corresponds to null set* ϕ. For example, in the case of dice, {getting a number higher than 6} is an impossible event.

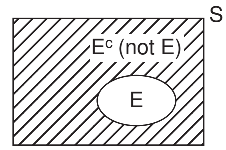
Equally likely outcomes. *If there is no reason for any one outcome to occur in preference to any other outcome, then we say that outcomes are equally likely.* For example, in drawing a card from well shuffled pack, all 52 possible outcomes are equally likely. Getting a spade is as likely as getting a heart (there are 13 cards of each); getting a king is as likely as getting a queen (there are 4 cards of each).

13.2.2 Algebra of Events

Since events can be represented as sets, the set operations like union, intersection, complement, difference etc. can be applied to events.

Complement of an event

The ***complement of an event*** *E, denoted by* $\overline{E}$ *or* E' *or* E^c*, is the set of all points of the sample space other than the points occurring in E.*



In other words, $E^c = \{w : w \in S \text{ and } w \notin E\} = S - E$.

Note that $E \cup E^c = S$ and $E \cap E^c = \phi$.

Take the example of tossing two coins. The sample space is

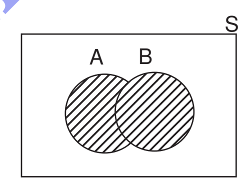
$S = \{HH, HT, TH, TT\}$.

If E is the event "atleast one tail appears" = {HT, TH, TT}, then

E^c or "not E" or $\overline{E}$ or E' = {HH} = {no tail appears}.

The event "A or B"

Recall that union of two sets A and B, denoted by $A \cup B$, also called "A or B" is the set which contains all those elements which are either in A or in B or in both.



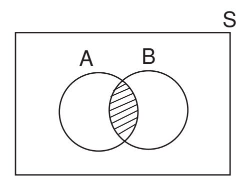
Shaded portion is $A \cup B$

Similarly, event "A or B" is

$A \cup B = \{w : w \in A \text{ or } w \in B\}$.

The event "A and B"

Intersection of two sets A and B, denoted by $A \cap B$, also called "A and B" is the set of all those elements which are common to A and B *i.e.* which belong to both A and B.



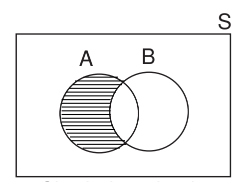
Shaded portion is $A \cap B$

Similarly, event "A and B" is

$A \cap B = \{w : w \in A \text{ and } w \in B\}$.

The event A but not B

The difference of two sets A and B, denoted by $A - B$ is the set of those elements which are in A but not in B.



Shaded portion is $A - B = A \cap B'$

Similarly, event A but not B is

$A - B = \{w : w \in A \text{ and } w \notin B\} = A \cap B'$.

Exhaustive events

The events $E_1, E_2, \ldots, E_n$ associated with a random experiment *are called* ***exhaustive*** *if* $E_1 \cup E_2 \cup \ldots \cup E_n = S$.

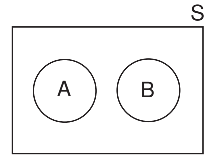
For example, if S = {1, 2, 3, 4, 5, 6}, A = {1, 3, 5}, B = {2, 4, 6}, C = {1, 2, 3, 4}, then $A \cup B = S$, so A and B are exhaustive events. But $A \cup C = \{1, 2, 3, 4, 5\} \neq S$, so A and C are not exhaustive events.

Note that when events $E_1, E_2, \ldots, E_n$ are exhaustive, then (atleast) one of them must necessarily occur every time the experiment is performed.

Mutually exclusive events

Two or more events associated with a random experiment are called ***mutually exclusive*** *if the occurrence of any one of them excludes the occurrence of the other events.*

If A and B are mutually exclusive, then $A \cap B = \phi$ (the null set). In above example, $A \cap B = \{1, 3, 5\} \cap \{2, 4, 6\} = \phi$, so A and B are mutually exclusive, whereas $A \cap C = \{1, 3, 5\} \cap \{1, 2, 3, 4\} = \{1, 3\} \neq \phi$, so A and C are not mutually exclusive. Similarly in a deck of cards, let A = {getting a spade}, B = {getting a heart}, C = {getting a king}, then A and B are mutually exclusive as $A \cap B = \phi$, whereas A and C are not mutually exclusive as $A \cap C$ = {king of spade} $\neq \phi$.

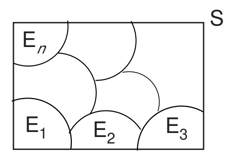


Remark. The simple events of a sample space are always mutually exclusive.

Mutually exclusive and exhaustive events

Let S be the sample space associated with a random experiment, then the events $E_1, E_2, \ldots, E_n$ *of S are called* ***mutually exclusive and exhaustive*** *if* $E_1 \cup E_2 \cup \ldots \cup E_n = S$ *i.e.*

$$\bigcup_{i=1}^{n} E_i = S \text{ and } E_i \cap E_j = \phi \text{ for all } i \neq j.$$



For example, let A = {getting a spade}, B = {getting a club}, C = {getting a heart}, D = {getting a diamond}, then A, B, C, D, are mutually exclusive and exhaustive events.

Exhaustive number of cases

It is the total number of possible outcomes of an experiment. Thus, when a coin is thrown, the exhaustive number of cases is 2 as we may get either heads or tails. When a die is thrown, the exhaustive number of cases is 6. When three dice are thrown simultaneously, the exhaustive number of cases is $6^3 = 216$. When 2 cards are drawn simultaneously from a pack of playing cards, the exhaustive number of cases is ${}^{52}C_2$, and so on.

Favourable number of cases

The number of cases favourable to an event is known as the favourable number of cases. Thus if the event is simultaneous throw of three coins, the number of cases favourable to the event 'getting minimum two heads' is 4 as the cases HHH, HHT, HTH, THH are favourable to it. Can you find the favourable number of cases for the event 'getting a sum of 10' when two dice are thrown simultaneously ?

ILLUSTRATIVE EXAMPLES

Example 1. *(i) From a group of 3 boys and 2 girls, we select two children. Describe the sample space. How would you represent the events "both selected children are boys" and "one boy and one girl is selected" ?*

(ii) A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 red and 4 black balls; if it shows tails, we throw a die. What is the sample space of this experiment ? How would you represent the events "coin came up head" and "the throw of the die resulted in an even number" ?

Solution. (*i*) Two children out of five can be selected in ${}^5C_2 = 10$ ways. If the three boys are represented by B_1, B_2, B_3 and the two girls are represented by G_1, G_2, then the sample space is

$$S = \{B_1 B_2, B_1 B_3, B_1 G_1, B_1 G_2, B_2 B_3, B_2 G_1, B_2 G_2, B_3 G_1, B_3 G_2, G_1 G_2\}.$$

The event "both selected children are boys" can be represented as

$$E_1 = \{B_1 B_2, B_1 B_3, B_2 B_3\}$$

The event "one boy and one girl is selected" can be represented as

$$E_2 = \{B_1 G_1, B_1 G_2, B_2 G_1, B_2 G_2, B_3 G_1, B_3 G_2\}$$

(*ii*) If we denote the red balls by r_1, r_2, r_3 and the black balls by b_1, b_2, b_3, b_4, then the sample space is

$$S = \{Hr_1, Hr_2, Hr_3, Hb_1, Hb_2, Hb_3, Hb_4, T1, T2, T3, T4, T5, T6\}$$

The event "coin came up head" can be represented as

$$E_1 = \{Hr_1, Hr_2, Hr_3, Hb_1, Hb_2, Hb_3, Hb_4\}$$

The event "the throw of the die resulted in even number" can be represented as

$$E_2 = \{T2, T4, T6\}$$

Example 2. *A die is thrown twice. Describe the sample space of this experiment. Let E_1 = {both numbers are even}, E_2 = {both numbers are odd}, E_3 = {sum is less than 6}. Describe E_1, E_2, E_3, $E_1 \cup E_2$, $E_1 \cap E_2$, $E_1 \cup E_3$, $E_1 \cap E_3$, E_1^c, E_2^c.*

Solution. $S = \{(x, y);\ x, y \in \{1, 2, 3, 4, 5, 6\}\}$

or $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

E_1 = {both numbers are even}

$= \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$

E_2 = {both numbers are odd}

$= \{(1, 1), (1, 3), (1, 5)\ (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

E_3 = {sum is less than 6}

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$

$E_1 \cup E_2$ = {both numbers are even or both numbers are odd}

$= \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6), (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

$E_1 \cap E_2$ = {both numbers are even and both numbers are odd}

$= \phi$ (the null set)

$E_1 \cup E_3$ = {both numbers are even or sum is less than 6}

$= \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6), (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (3, 1), (3, 2), (4, 1)\}$

$E_1 \cap E_3$ = {both numbers are even and sum is less than 6}

$= \{(2, 2)\}$

E_1^c = [those cases except where both numbers are even}

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 5), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 3), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 3), (6, 5)\}$

E_2^c = {those cases except where both numbers are odd}

$= \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

Example 3. *Two dice are rolled. A is the event that the sum of the numbers shown on the two dice is 5. B is the event that atleast one of the dice shows up a 3. Are the two events A and B (i) mutually exclusive (ii) exhaustive ? Give arguments in support of your answer.*

Solution. Here the sample space S consists of 36 points, so $n(S) = 36$.

$A = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$ and

$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 3), (2, 3), (4, 3), (5, 3), (6, 3)\}$

As $A \cap B = \{(2, 3), (3, 2)\} \neq \phi$, events A and B are not mutually exclusive.

Also $n(A \cup B) = 13 < n(S)$. Hence, A and B are not exhaustive as $A \cup B \neq S$.

Example 4. *A coin is tossed three times. Consider the following events :*
A : 'No head appears', B : 'exactly one head appears', C : 'Atleast two heads appear'.
Do they form a set of mutually exclusive and exhaustive events?

Solution. The sample space of the experiment is
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT},
A = {TTT}, B = {HTT, THT, TTH}, C : {HHT, HTH, THH, HHH}
Now $A \cup B \cup C$ = {TTT, HTT, THT, TTH, HHT, HTH, THH, HHH} = S,
So A, B and C are exhaustive events.
Also $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$,
So events are pairwise disjoint *i.e.* they are mutually exclusive.
Hence A, B and C form a set of mutually exclusive and exhaustive events.

EXERCISE 13.2

1. (*i*) A coin is tossed once. Write its sample space and all possible events.
   (*ii*) A coin is tossed twice. Write its sample space and all possible events.
   (*iii*) A coin is tossed n times. What is the number of elements in its sample space ?
   Also write the number of all possible events.
   Hint. Let S be the sample space. As a sequence of n items XX...X occurs when 'X' may either be H or T, so there are 2^n possible outcomes. Hence O(S) = 2^n. Number of possible events = number of subsets of S = $2^{O(S)}$.
2. If three dice are thrown together, how many outcomes would the sample space have ? Represent the event when all dice come up with same number.
3. A coin and a die are tossed. Describe the sample space and the following events.
   (*i*) A = getting a head and an even number.
   (*ii*) B = getting a prime number.
   (*iii*) C = getting a tail and an odd number.
   (*iv*) D = getting a head or a tail.
   Now answer the following questions.
   (*a*) Are A and B mutually exclusive ?
   (*b*) Are A and C mutually exclusive ?
   (*c*) Are A and C exhaustive ?
   (*d*) Are A and D exhaustive ?
4. Three coins are tossed.
   (*i*) Describe two events A and B which are mutually exclusive.
   (*ii*) Describe three events A, B and C which are mutually exclusive and exhaustive.
   (*iii*) Describe two events which are not mutually exclusive.
   (*iv*) Describe three events which are not mutually exclusive.
   (*v*) Describe two events which are mutually exclusive but not exhaustive.
   (*vi*) Describe three events which are mutually exclusive but not exhaustive.
5. Two dice are thrown. If E is the event "both dice come up with same number" and F is the event "product of the numbers on the two dice is odd", then describe
   (*i*) E (*ii*) F (*iii*) E or F (*iv*) E and F (*v*) E but not F.
6. A pair of dice is rolled. Consider the following events
   A : the sum is greater than 8
   B : 2 occurs on either die
   C : the sum is at least 7 and a multiple of 3.
   (*i*) Find A, B, C, $A \cap B$, $B \cap C$ and $A \cap C$.
   (*ii*) Which pairs of events are mutually exclusive ?

7. From a group of 2 men and 3 women, two persons are selected. Describe the sample space of the experiment. If E is the event in which one man and one woman are selected, then which are the cases favourable to E ?

8. A coin is tossed thrice. If E denotes the 'number of heads is odd' and F denotes the 'number of tails is odd,' then find the cases favourable to the event $E \cap F$.

9. If E, F, G denote the subsets representing the events of a sample space S, what are the sets representing the following events ?

(*i*) Out of the three events at least two events occur.
(*ii*) Out of the three events only one occurs.
(*iii*) Out of the three events only E occurs.
(*iv*) Out of the three events exactly two events occur.

10. Two dice are rolled. Let A, B and C be the events of getting a sum 2, sum 3, and a sum 4, respectively.

(*i*) Is event A simple ?
(*ii*) Is event B simple ?
(*iii*) Is event C compound ?
(*iv*) Are events A and B mutually exclusive ?

11. Three coins are tossed once. Let A denote the event "three heads show", B denote the event "two heads and one tail show", C denote the event "three tails show" and D denote the event "a head shows on the first coin".

Which events are

(*i*) mutually exclusive (*ii*) simple (*iii*) compound ?

12. From a group of 2 boys and 3 girls, two children are selected at random. Describe the events

(*i*) A : both selected children are girls.
(*ii*) B : the selected group consists of one boy and one girl.
(*iii*) C : atleast one boy is selected.

Which pair (s) of events is (are) mutually exclusive ?

13. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are then put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the following events :

A : The number on the first slip is larger than the number on the second slip.
B : The number on the second slip is greater than 2.
C : The sum of the numbers on the two slips is 6 or 7.
D : The number on the second slip is twice that on the first slip.

Which pair (s) of events is (are) mutually exclusive ?

13.3 MATHEMATICAL DEFINITION OF PROBABILITY (CLASSICAL OR A PRIORI PROBABILITY)

If an experiment has n exhaustive, mutually exclusive and equally likely outcomes, then the sample space S has n sample points. If an event A consists of m sample points, $0 \le m \le n$, then the ***probability*** *of event A, denoted by P(A) is defined as*

$$P(A) = \frac{m}{n} = \frac{\text{number of outcomes favourable to } A}{\text{total number of outcomes}}.$$

21. If the probability of a football match going into overtime is 0·1, find the probability that atmost one of the five games go into overtime.

22. The probability of a student passing an examination is $\frac{3}{7}$ and of student B passing is $\frac{5}{7}$. Assuming the two events 'A passes' and 'B passes' as independent, find the probability of :
(*i*) only A passing the examination (*ii*) only one of them passing the examination.

23. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles ?

24. Assume that on an average, one telephone number out of 15 called between 2 p.m. and 3 p.m. on weekdays is busy. What is the probability that if six randomly selected telephone numbers are called, atleast 3 of them will be busy ?

25. A and B play a game in which A's chance of winning the game is $\frac{3}{5}$. In a series of 6 games, find the probability that A will win atleast 4 games.

26. A bag contains 3 white, 3 black and 2 red balls. Three balls are drawn one by one without replacement. Find the probability that the third ball is red.

27. There are three urns A, B and C. A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 3 white balls and 6 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white and one is blue ?

28. Eight coins are thrown simultaneously. Find the chance of obtaining atleast six heads.

29. One card is drawn from a well-shuffled pack of 52 cards. If E is the event 'the card drawn is a king or a queen' and F is the event 'the card drawn is a queen or an ace', then find the probability of the conditional event $E|F$.

30. India plays two one-day cricket matches each with Australia and South Africa. In any one match, the probabilities of India getting points 0, 1, 2 are respectively 0·45, 0·5 and 0·5. Assuming that the outcomes are independent, find the probability of India getting atleast 7 points.

31. A factory has two machines M_1 and M_2. Past records show that the machine M_1 produces 60% of the items and machine M_2 produces the remaining items of the output of the factory. If 2% of the items produced by machine M_1 and 1% of the items produced by the machine M_2 are defective, what is the probability that an item selected is defective?

32. A bag P contains 8 white and 7 red balls while another bag Q contains 5 white and 4 red balls. One ball is randomly picked up from bag P and mixed up with the balls in bag Q. Then a ball is drawn randomly from the bag Q, what is the probability that the ball drawn is white?

ANSWERS

EXERCISE 13.1

1. (*i*) {HH, HT, TH, TT}
(*ii*) {HH, HT, TH, TT}
(*iii*) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
(*iv*) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
(*v*) {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(*vi*) {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

(*vii*) {H1, H2, H3, H4, H5, H6, T}

(*viii*) {0, 1, 2, 3}

(*ix*) {Red, Black}

(*x*) {Spade, Heart, Diamond, Club}.

2. {Ab_1, Ab_2, Ag_1, Ag_2, Bb_3, Bg_3, Bg_4, Bg_5},
where team A has two boys b_1, b_2 and two girls g_1, g_2, while team B has one boy b_3, and three girls g_3, g_4, g_5.

3. (*i*) S = {r_1, r_2, b_1, b_2, b_3}

(*ii*) S = {$r_1 r_2$, $r_1 b_1$, $r_1 b_2$, $r_1 b_3$, $r_2 b_1$, $r_2 b_2$, $r_2 b_3$, $b_1 b_2$, $b_1 b_3$, $b_2 b_3$}

4. (*i*) S = {red, black}

(*ii*) S = {(red, red), (red, black), (black, red), (black, black)}

5. {HB_1, HB_2, HW_1, HW_2, HW_3, TH, TT}

6. {T, HT, HHT, HHHT, ...}

7. {R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6}.

8. {DDD, DDN, DND, DNN, NDD, NDN, NND, NNN}.

9. {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}.

10. {RW, WR, WW}.

11. {2H, 2T, 4H, 4T, 6H, 6T, 1HH, 1HT, 1TH, 1TT, 3HH, 3HT, 3TH, 3TT, 5HH, 5HT, 5TH, 5TT}; 18.

EXERCISE 13.2

1. (*i*) S = {H, T}. There are four possible events — {H, T}, {H}, {T}, ϕ.

(*ii*) S = {HH, HT, TH, TT}. There are 16 possible events — {HH, HT, TH, TT}, {HH, HT, TH}, {HH, HT, TT}, {HH, TH, TT}, {HT, TH, TT}, {HH, HT}, {HH, TH}, {HH, TT}, {HT, TH}, {HT, TT}, {TH, TT}, {HH}, {HT}, {TH}, {TT}, ϕ.

(*iii*) 2^n ; 2^{2^n}.

2. Sample space has $6 \times 6 \times 6 = 216$ points. The event when all dice come up with same number can be represented as

E = {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}.

3. S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

A = {H2, H4, H6}

B = {H2, H3, H5, T2, T3, T5}

C = {T1, T3, T5}

D = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

(*a*) No, as $A \cap B = \{H2\} \neq \phi$

(*b*) Yes, as $A \cap C = \phi$

(*c*) No, as $A \cup C \neq S$

(*d*) Yes, as $A \cup D = S$.

4. S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(*i*) Let A = {HHH, HHT, HTH, THH} and B = {TTT}

(*ii*) Let A = {HHH, HHT, HTH, THH}, B = {HTT, THT, TTH}, and C = {TTT}

(*iii*) Let A = {HHT, HTH} and B = {HTH, TTT}

(*iv*) Let A = {HHH, HHT}, B = {HHH, HTT} and C = {HHH, TTT}

(*v*) Let A = {HHH, HHT, HTH, THH} and B = {HTT, THT, TTH}

(*vi*) Let A = {HHH, HHT}, B = {HTH, THH} and C = {HTT, THT}.

5. (*i*) E = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

(*ii*) F = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

(*iii*) E or F = $E \cup F$ = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 3), (1, 5), (3, 1), (3, 5), (5, 1), (5, 3)}

(*iv*) E and F = $E \cap F$ = {(1, 1), (3, 3), (5, 5)}

(*v*) E but not F = $E - F = E \cap F^c$ = {(2, 2), (4, 4), (6, 6)}.