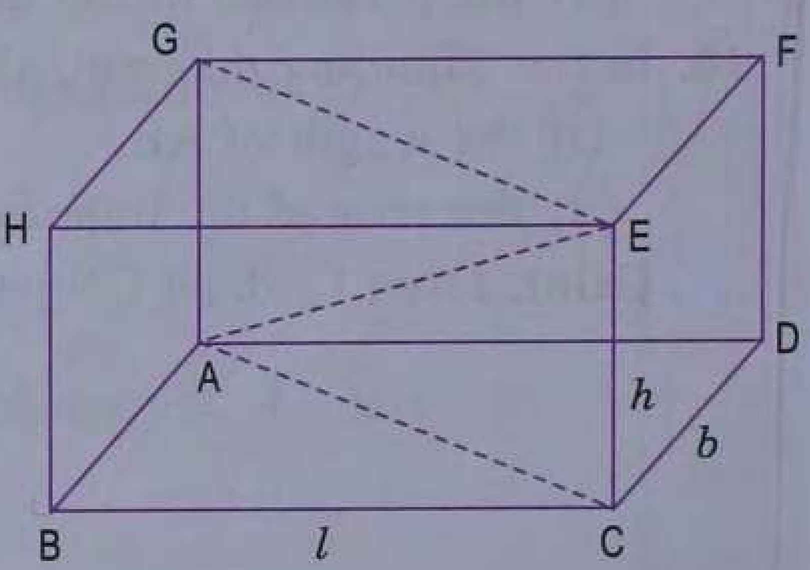
Chapter 29

VOLUME AND SURFACE AREA OF CUBOID

CUBOID

The following diagram shows a cuboid. It has 6 rectangular faces, 8 vertices and 12 edges.

Let its length, breadth and height be denoted by l, b and h respectively.



Volume

Volume of a cuboid $= l \times b \times h$

Surface area

Surface area of a cuboid $= 2\,(lb + lh + bh)$

Lateral surface area

Lateral surface of a cuboid (area of four walls)

$$= 2\,(l + b) \times h$$

Length of diagonal

A cuboid has four diagonals. In the above cuboid, the four diagonals are AE, BF, CG and DH. All these four diagonals have equal length.

Consider the diagonal AE.

In rectangle ABCD, the length of its diagonal AC $= \sqrt{l^2 + b^2}$.

Notice that ACEG is a rectangle with length AC and breadth

$= \text{CE} = h$, so the length of its diagonal AE $= \sqrt{\text{AC}^2 + \text{CE}^2}$.

$$\Rightarrow \text{AE} = \sqrt{\left(\sqrt{l^2 + b^2}\right)^2 + h^2} = \sqrt{l^2 + b^2 + h^2}.$$

Hence, the length of a diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$.

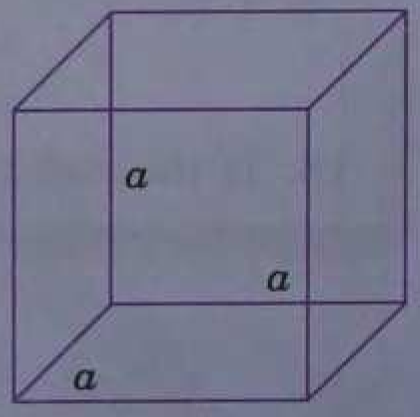
It is the length of the longest rod that can be placed in the cuboid.

CUBE

It is a particular case of a cuboid where

length = breadth = height

Let a be the length of an edge of a cube.



Volume

Volume of a cube $= a^3$

Surface area

Surface area of a cube $= 6a^2$

Lateral surface area

Lateral surface area = $4a^2$

Length of diagonal

Length of diagonal = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3}\, a$

Hence, the length of a diagonal of a cube with edge a = $\sqrt{3}$ a.

Example 1. The length, breadth and height of a cuboid are 30 cm, 25 cm and 20 cm respectively. Find its

(*i*) volume (*ii*) surface area

(*iii*) lateral surface area (*iv*) length of a diagonal.

Solution. (*i*) Volume of the cuboid = $(30 \times 25 \times 20)$ cm³ = 15000 cm³.

(*ii*) Surface area of the cuboid = $2\,(lb + lh + bh)$

$= 2\,(30 \times 25 + 30 \times 20 + 25 \times 20)$ cm²

$= 3700$ cm².

(*iii*) Lateral surface area $= 2\,(l + b) \times h$

$= 2\,(30 + 25) \times 20$ cm² $= 2200$ cm².

(*iv*) Length of a diagonal $= \sqrt{l^2 + b^2 + h^2} = \sqrt{30^2 + 25^2 + 20^2}$ cm

$= \sqrt{1925}$ cm $= 5\sqrt{77}$ cm.

Example 2. If the length of a diagonal of a cube is $8\sqrt{3}$ cm, find its surface area and volume.

Solution. Let the length of an edge of the cube be a cm, then the length of its diagonal = $\sqrt{3}\,a$ cm.

According to given information, $\sqrt{3}\,a = 8\sqrt{3} \Rightarrow a = 8$

$\therefore$ The surface area of the cube = $6a^2$ cm²

$= (6 \times 8 \times 8)$ cm² $= 384$ cm².

The volume of the cube = a^3 cm³

$= (8 \times 8 \times 8)$ cm³ $= 512$ cm³.

Example 3. If the surface area of a cube is 2166 cm², find

(*i*) the length of a diagonal (*ii*) volume of the cube.

Solution. Let the length of an edge of the given cube be a cm.

Surface area of the cube = $6a^2$ cm²

According to given information, $6a^2 = 2166 \Rightarrow a^2 = 361$

$\Rightarrow \quad a = \sqrt{361} = 19$

(*i*) The length of diagonal of the cube = $\sqrt{3}\,a$ cm = $19\sqrt{3}$ cm

$= (19 \times 1{\cdot}73)$ cm $= 32{\cdot}9$ cm.

(*ii*) The volume of cube = a^3 cm³

$= (19 \times 19 \times 19)$ cm³ $= 6859$ cm³.

Example 4. The area of three faces of a box are 120 cm², 72 cm² and 60 cm². What is the volume of the box?

Solution. Let the dimensions of the box be l cm, b cm and h cm.

According to given information,

$l \times b = 120$

$l \times h = 72$

$b \times h = 60$

Multipying these equations, we get

$l^2 \times b^2 \times h^2 = 120 \times 72 \times 60$

$\Rightarrow l \times b \times h = \sqrt{120 \times 72 \times 60} = \sqrt{60^2 \times 12^2}$

$\Rightarrow l \times b \times h = 60 \times 12 = 720$

Hence, the volume of the box = 720 cm^3.

Example 5. The length, breadth and height of a cuboid are in the ratio 7 : 6 : 5. If the surface area of the cuboid is 1926 cm^2, find

(*i*) volume (*ii*) length of a diagonal of the cuboid.

Solution. As the length, breadth and height of a cuboid are in the ratio 7 : 6 : 5, let its length = $7x$ cm, breadth = $6x$ cm and height = $5x$ cm.

Surface area $= 2\,(7x \times 6x + 7x \times 5x + 6x \times 5x)$ cm^2

$= (2 \times 107)\, x^2$ cm^2 $= 214\, x^2$ cm^2.

According to given information, $214\, x^2 = 1926$

$\Rightarrow \quad x^2 = 9 \quad \Rightarrow \quad x = 3$

$\therefore$ Length = (7 × 3) cm = 21 cm, breadth = 18 cm and height = 15 cm

(*i*) The volume of the cuboid = (21 × 18 × 15) cm^3 = 5670 cm^3.

(*ii*) The length of a diagonal $= \sqrt{21^2 + 18^2 + 15^2}$ cm

$= \sqrt{441 + 324 + 225}$ cm

$= \sqrt{990}$ cm $= 3\sqrt{110}$ cm.

Example 6. The internal dimensions of a rectangular room are 6 m, 5 m and 3·5 m. It has two doors of size 1·2 m by 2 m and three windows of size 1 m by 1·9 m. The walls of the room are to be papered with a wall paper of width 70 cm. Find the cost of the paper at the rate of ₹6·50 per metre.

Solution. The surface area of the walls of the room $= 2\,(l + b) \times h$

$= 2\,(6 + 5) \times 3{\cdot}5$ m^2

$= (22 \times 3{\cdot}5)$ m^2 $= 77$ m^2.

Surface area of 2 doors = (2 × 1·2 × 2) m^2 = 4·8 m^2.

Surface area of 3 windows = (3 × 1 × 1·9) m^2 = 5·7 m^2.

$\therefore$ Surface area to be papered

= surface area of walls – surface area of doors and windows

= 77 m^2 – (4·8 m^2 + 5·7 m^2) = 77 m^2 – 10·5 m^2 = 66·5 m^2.

Width of the wall paper = 70 cm = 0·7 m.

$\therefore$ The length of the wall paper required $= \frac{66{\cdot}5}{0{\cdot}7}$ m = 95 m.

$\therefore$ The cost of the wall paper = ₹(95 × 6·50) = ₹617·50.

Example 7. The external dimensions of an open rectangular wooden box are 98 cm by 84 cm by 77 cm. If the wood is 2 cm thick all around, find

(*i*) the internal dimensions of the box.

(*ii*) the capacity of the box.

(*iii*) the volume of the wood used in making the box.

(*iv*) the weight of the box in kilogram correct to 1 decimal place, given that 1 cm^3 of wood weighs 0·8 gm.

Solution. (*i*) The internal dimensions of the box are :

length = 98 cm − 4 cm = 94 cm,

breadth = 84 cm − 4 cm = 80 cm,

height = 77 cm − 2 cm = 75 cm.

(Since the box is open at the top, so only the thickness of bottom is to be reduced from the height.)

(*ii*) The capacity of the box = internal volume of the box

$= (94 \times 80 \times 75)\ cm^3 = 564000\ cm^3$.

(*iii*) External volume of the box $= (98 \times 84 \times 77)\ cm^3 = 633864\ cm^3$

The volume of the wood used in making the box

= external volume − internal volume

$= 633864\ cm^3 - 564000\ cm^3 = 69864\ cm^3$.

(*iv*) The weight of the box = the weight of the wood

$= (69864 \times 0{\cdot}8)$ gm = 55891·2 gm

$= \frac{55891{\cdot}2}{1000}$ kg = 55·8912 kg = 55·9 kg.

Exercise 29

1. Find the volume, surface area and the length of diagonal of a cube of an edge
   (*i*) 7 cm (*ii*) 4·5 cm.
2. If the length of a diagonal of a cube is $12\sqrt{3}$ cm, find its surface area and volume.
3. If the surface area of a cube is 486 cm^2, find
   (*i*) the length of a diagonal (*ii*) volume of the cube.
4. Find the volume, surface area, lateral surface area and length of diagonal of a cuboid with dimensions :
   (*i*) length = 10 cm, breadth = 7 cm and height = 8 cm
   (*ii*) length = 1·5 m, breadth = 90 cm and height = 70 cm.
5. For a cuboid, fill in the following blanks :

	Volume	*Length*	*Breadth*	*Height*	*Surface area*	*Length of diagonal*
(*i*)	90 cm^3	...	5 cm	3 cm	...	...
(*ii*)	840 cm^3	15 cm	...	7 cm	...	...

6. The volume of a cuboid is 448 cm^3. Its height is 7 cm and the base is a square. Find
   (*i*) a side of the square base (*ii*) surface area of the cuboid.
7. The length, breadth and height of a rectangular solid are in the ratio 5 : 4 : 2. If its total surface area is 1216 cm^3, find the volume of the solid.
8. A rectangular reservoir contains 42000 litres of water. Find the depth of the water in the reservoir if its base measures 6 m by 3·5 m.
9. A rectangular pit 1·4 m long, 90 cm broad and 70 cm deep was dug and 1000 bricks of base 21 cm by 10·5 cm were made from the earth dug out. Find the height of each brick.
10. A rectangular playground is of dimensions 120 m by 40 m. Find the cost of covering the ground with gravel 1 cm deep, given that the gravel costs ₹ 240 per cubic metre.

11. A rectangular room is 6 m long, 5 m wide and 3·5 m high. It has 2 doors of size 1·1 m by 2 m and 3 windows of size 1·5 m by 1·4 m.
Find the cost of white washing the walls and the ceiling of the room at the rate of ₹ 5·30 per square metre.

12. A cuboidal block of metal has dimensions 36 cm by 32 cm by 0·25 m. It is melted and recast into cubes with an edge of 4 cm.
(*i*) How many such cubes can be made?
(*ii*) What is the cost of silver coating the surfaces of the cubes at the rate of ₹ 0·75 per square centimetre?

13. Three cubes of silver with edges 3 cm, 4 cm and 5 cm are melted and recast into a single cube. Find the cost of coating the surface of the new cube with gold at the rate of ₹ 3·50 per square centimetre.
[**Hint.** If a cm is the edge of a new cube, then $a^3 = 3^3 + 4^3 + 5^3$
$\Rightarrow \quad a^3 = 216 = (6)^3 \qquad \Rightarrow \quad a = 6.$]

Summary

➡ **Cuboid**

If l, b and h are length, breadth and height of a cuboid, then
- ❑ Volume = $l \times b \times h$
- ❑ Surface area = $2\,(lb + lh + bh)$
- ❑ Lateral surface area = $2\,(l + b) \times h$
- ❑ Length of a diagonal = $\sqrt{l^2 + b^2 + h^2}$.

➡ **Cube**

If a be the length of an edge of a cube, then
- ❑ Volume = a^3
- ❑ Surface area = $6a^2$
- ❑ Length of a diagonal = $\sqrt{3}\,a$.

Check Your Progress

1. If the surface area of a cube is 1176 cm², find
(*i*) the length of a diagonal (*ii*) volume of the cube.

2. A closed rectangular wooden box has inner dimensions 90 cm by 80 cm by 70 cm. Compute its capacity and the area of the tin foil needed to line its inner surface.

3. The lateral surface area of a cuboid is 224 cm². Its height is 7 cm and the base is a square. Find
(*i*) a side of the square base (*ii*) the volume of the cuboid.

4. The inner dimensions of a closed wooden box are 2 m by 1·2 m by 0·75 m. The thickness of the wood is 2·5 cm. Find the cost of wood required to make the box if 1 m³ of wood costs ₹ 5400.

5. A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base of the vessel are 15 cm × 12 cm, find the rise in the water level in centimetres correct to 2 decimal places, assuming that no water overflows.