Chapter 26

CIRCLE

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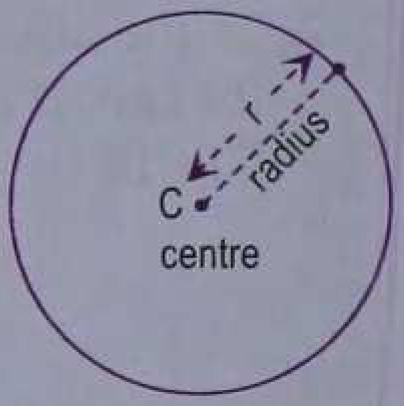
*A **circle** is the set of all those points, say P, in a plane, each of which is at a constant distance from a fixed point in that plane.*



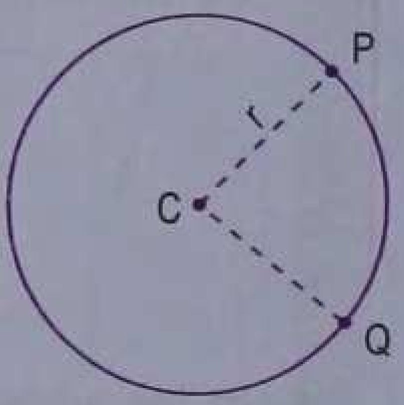
*The fixed point is called the **centre** and the constant distance is called the **radius**.*

The radius of a circle is always positive.

The adjoining figure shows a circle with C as its centre and r as its radius.

Note that the centre of a circle does not lie on the circle.

For convenience, a circle may be named by its centre. In this figure, it is the circle C.



Let C be the centre of a circle and r its radius. If P is a point on the circle, then the line segment CP is a radius of the circle and its length is r. If Q is another point on the circle then CQ is another radius of the circle. Note that all radii (plural of radius) have one point in common, which is the centre of the circle. Also CP = CQ = r. Thus :

All radii of a circle are equal

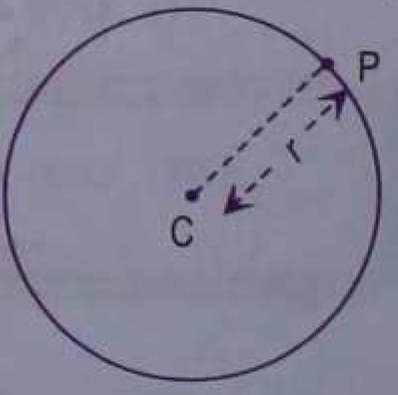
SOME TERMS ASSOCIATED WITH CIRCLE

- **Circle — interior and exterior**

  A circle is a closed curve. It divides the points (or region) of the plane into three parts.

  (*i*) **The circle**

  *A point P lies **on the circle** if and only if its distance from the centre of the circle is equal to the radius of circle.*

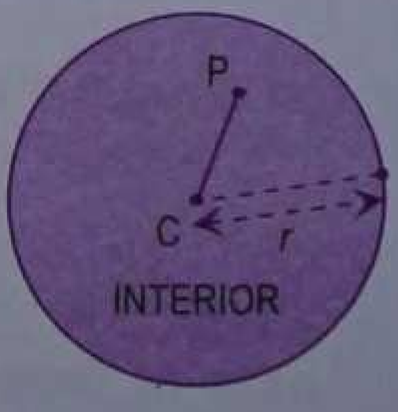
  

  In the adjoining figure, CP = r, so P lies on the circle with centre C and radius r.

  The set of all points P of the plane such that CP = r form a **circle** with centre C and radius r.

  (*ii*) **Interior of a circle**

  *A point P lies **inside a circle** if and only if its distance from the centre of the circle is less than the radius of the circle.*

  In the adjoining figure, CP < r, so P lies inside a circle with centre C and radius r.

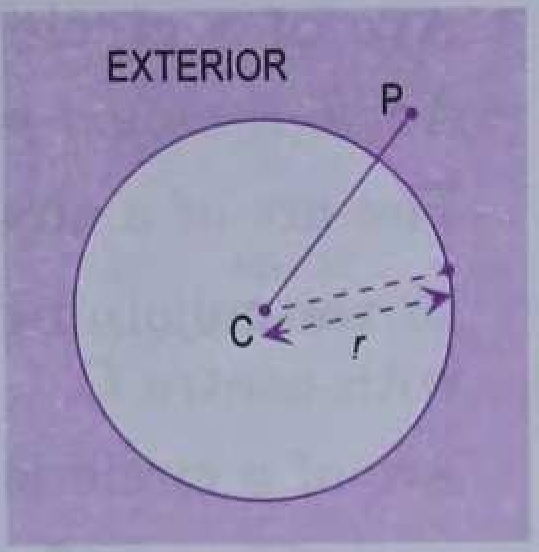
  The set of all points P of the plane such that CP < r form the **interior of the circle**.

(*iii*) **Exterior of a circle**

*A point P lies **outside a circle** if and only if its distance from the centre of the circle is greater than the radius of the circle.*

In the adjoining figure, CP > r, so P lies outside a circle with centre C and radius r.

The set of all points P of the plane such that CP > r form the **exterior of the circle**.



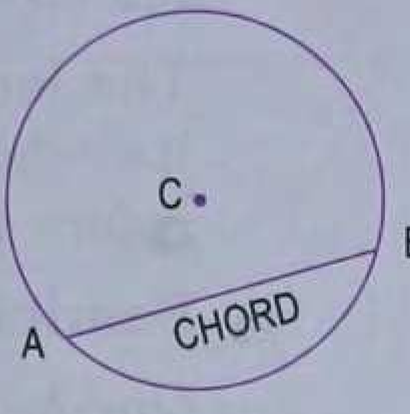
- **Circular region**

  *The set of all points of the plane which either lie on the circle or are inside the circle form the **circular region**.*

- **Chord of a circle**

  *A line segment joining any two points of a circle is called a **chord** of the circle.*

  In the adjoining figure, AB is a chord of the circle with centre C. The distance AB is called the **length of the chord**.



- **Diameter of a circle**

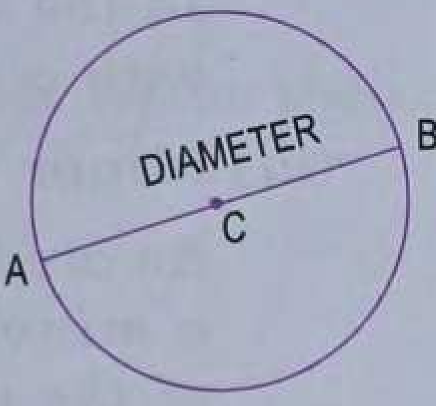
  *A chord of a circle passing through its centre is called a **diameter** of the circle.*

  In the adjoining figure, AB is a diameter of the circle with centre C.

  Notice that CA and CB are both radii of the circle, so CA = CB = r.

  It follows that

  AB = AC + CB = $2r$ = 2 × radius. Thus :



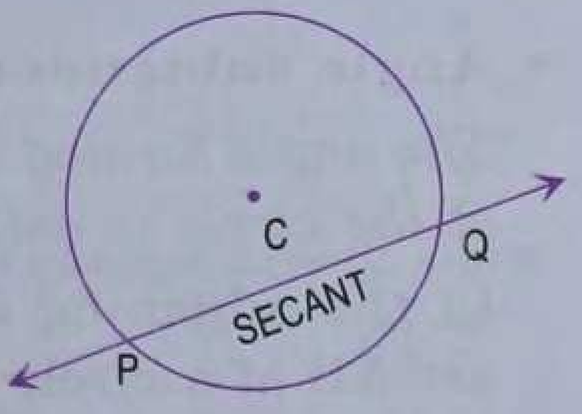
Length of a diameter = 2 × radius

- **Secant of a circle**

  *A line which meets a circle in two points is called a **secant** of the circle.*

  In the adjoining figure, line PQ is a secant of the circle with centre C.

  Note. A line can meet a circle atmost in two points.



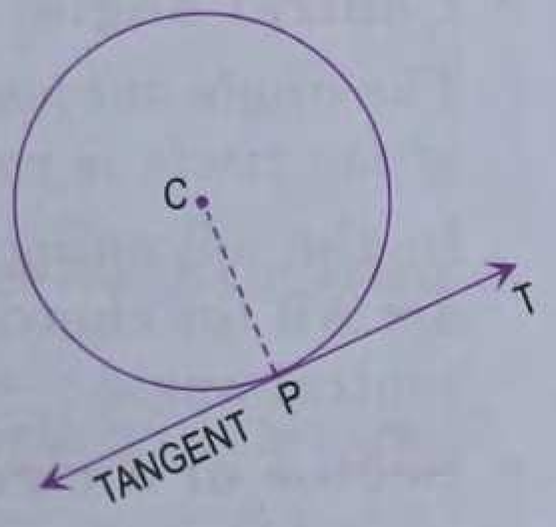
- **Tangent to a circle**

  *A line which meets a circle in one and only one point is called a **tangent** to the circle.*

  In the adjoining figure, the line PT is a tangent to the circle with centre C.

  The point where the line meets (touches) the circle is called its **point of contact**.

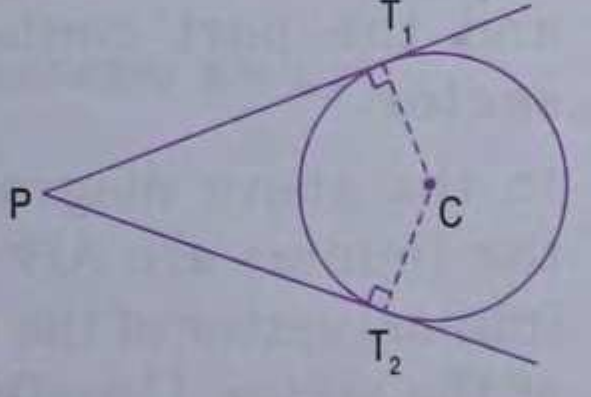
  Notice that CP is perpendicular to PT.



Remarks

- One and only one tangent can be drawn to a circle at a point on the circumference of the circle.
- Two tangents can be drawn to a circle from a point outside the circle.

  In the adjoining diagram, P is a point outside the circle with centre C. PT_1 and PT_2 are two tangents drawn to the circle with centre C and $PT_1 = PT_2$.

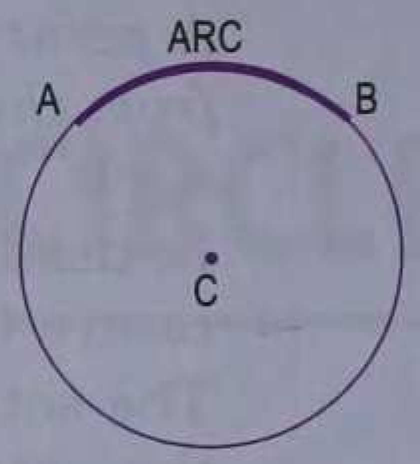


- **Arc of a circle**

A (continuous) part of a circle is called an ***arc*** *of the circle.*

The arc of a circle is denoted by the symbol '⌒'.

In the adjoining diagram, $\overset{\frown}{AB}$ denotes the arc AB of the circle with centre C.

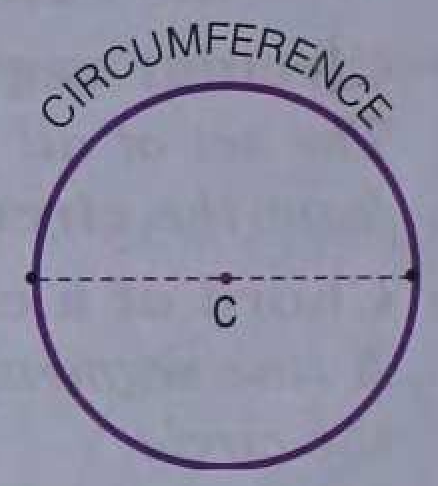


Arc of a circle is divided into following categories:

(*i*) **Circumference**

The whole arc of the circle is called *the* ***circumference*** *of the circle.*

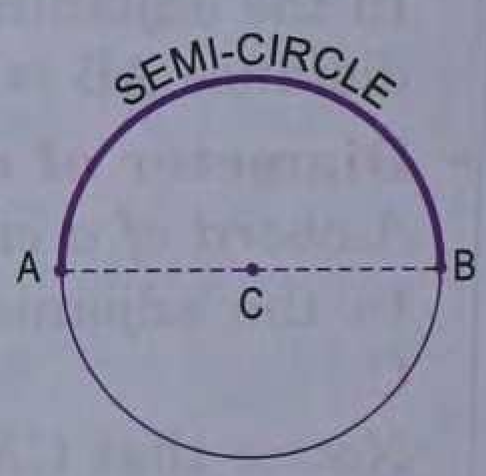
The length of the circumference of a circle is the length of its whole arc. Usually, the term circumference of a circle refers to its length.



(*ii*) **Semicircle**

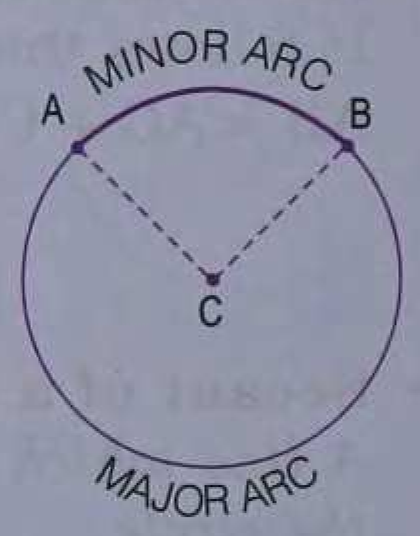
One-half of the whole arc of a circle is called a ***semicircle*** *of the circle.*

In the adjoining figure, arc AB is a semicircle of the circle with centre C.



(*iii*) **Minor and major arc**

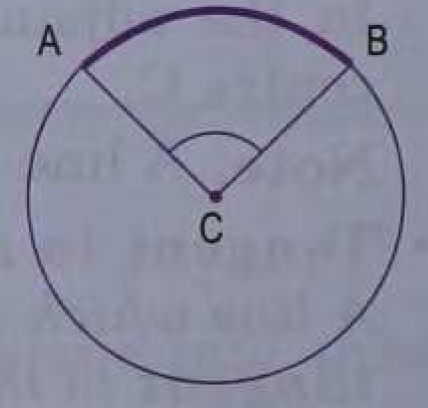
An arc less than one-half of the whole arc of a circle is called a ***minor arc*** *of the circle, and an arc greater than one-half of the whole arc of a circle is called a* ***major arc*** *of the circle.*



- **Angle subtended by an arc**

The angle formed by the two radii of an arc of a circle at the centre of the circle is called the ***angle subtended by the arc.***

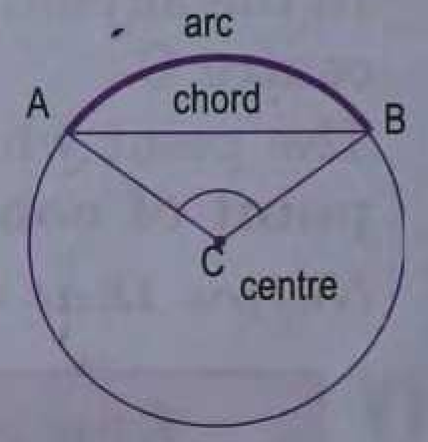
In the adjoining diagram, ∠ACB is the angle subtended by the arc AB of a circle with centre C.



- **Central angle**

The angle subtended by an arc (or chord) of a circle at the centre of the circle is called the ***central angle.***

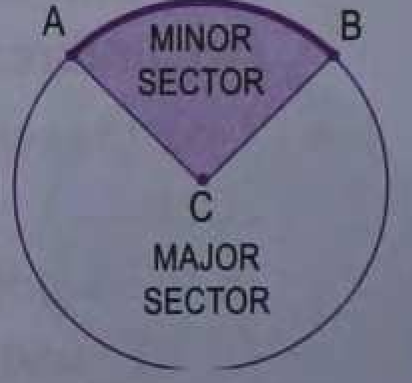
In the adjoining diagram, ∠ACB is the angle subtended by the arc AB (or chord AB) at the centre C of the circle, so ∠ACB is a central angle.



- **Sector of a circle**

The part of the circular region of the plane enclosed by an arc of a circle and its two bounding radii is called a ***sector*** *of the circle.*

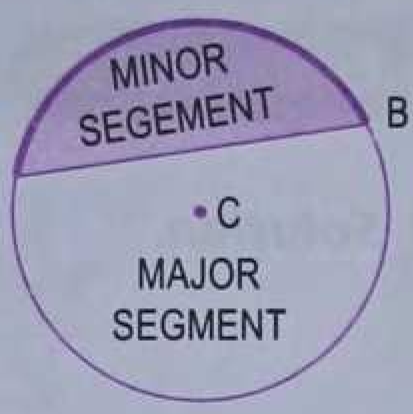
The part containing the minor arc is called the **minor sector**, and the part containing the major arc is called the **major sector**.



In the above diagram, the part of the plane region enclosed by the (minor) arc AB and its two bounding radii CA and CB is a (minor) sector of the circle with centre C. ∠ACB is called the angle of the sector. Usually, the term *sector of a circle* is referred to the area of this region.

- **Segment of a circle**

  A chord of a circle divides its circular region into two parts. Each part is called a ***segment****.*

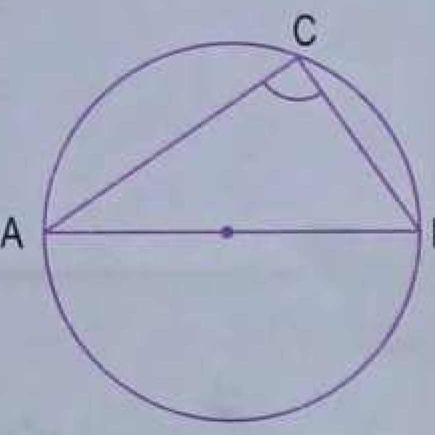


  The part containing the minor arc is called the **minor segment**, and the part containing the major arc is called the **major segment**.

  Usually, the terms '*minor segment*' and '*major segment*' refer to the areas of the regions enclosed by these.

- **Angle in a semicircle**

  An angle inscribed in a semicircle is called an ***angle in the semicircle****.*

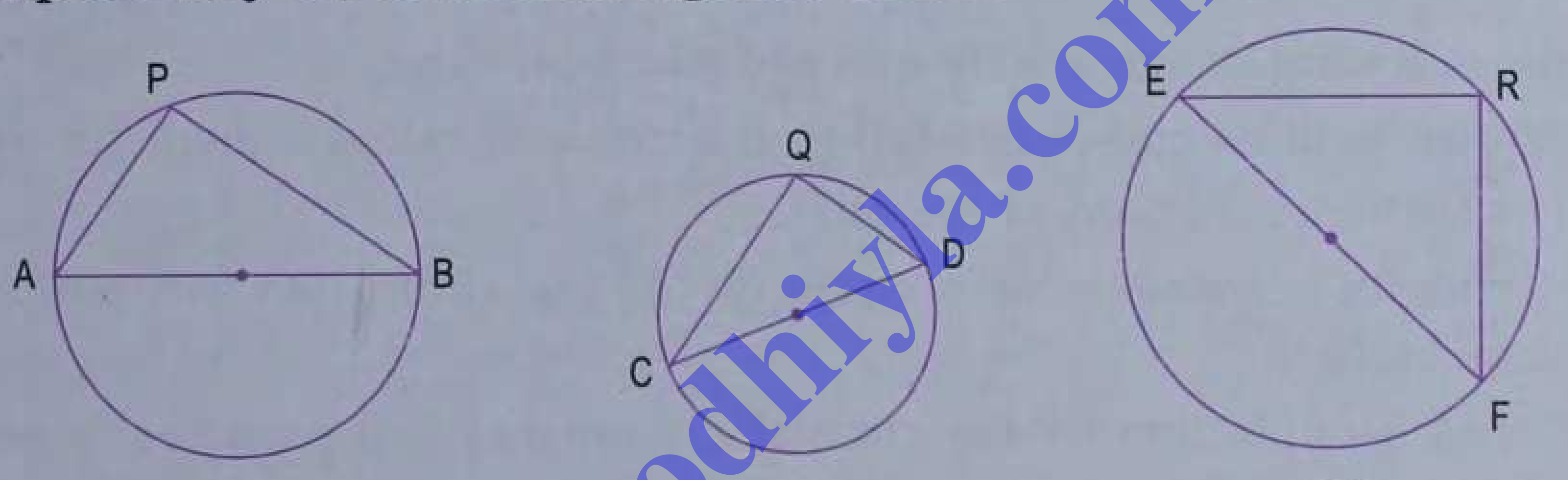


  In the adjoining figure, AB is a diameter of a circle and C is a point on the semicircle. ∠ACB is an angle in the semicircle.

An angle in a semicircle is a right angle

Verification

Let us take three circles (*i*), (*ii*) and (*iii*) of different radii and AB, CD and EF be their diameters respectively as shown in figures below.



(*i*) (*ii*) (*iii*)

Let P, Q and R be *any points* on the three semicircles (*i*), (*ii*) and (*iii*) respectively. Join AP and BP; CQ and DQ; ER and FR.

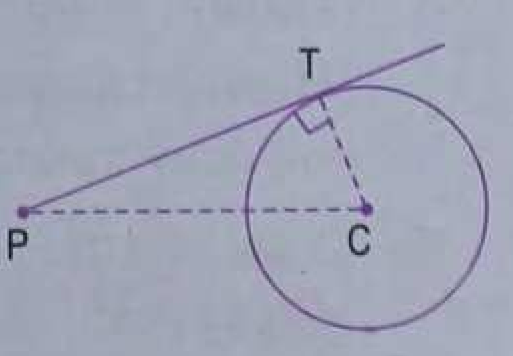
Now measure ∠APB, ∠CQD and ∠ERF with the help of a protractor.

It will be found that each angle is equal to 90°.

Hence, 'angle in a semicircle is a right angle'.

Example 1. Find the length of the tangent drawn to a circle of radius 5 cm from a point distant 13 cm from the centre.

Solution. Let PT be the tangent drawn from the point P to a circle with centre C.



Given CP = 13 cm,

CT = radius of circle = 5 cm.

Since PT is tangent to the circle, CT ⊥ PT

$\Rightarrow \quad \angle CTP = 90°$.

From right angled triangle CPT, by Pythagoras theorem, we get

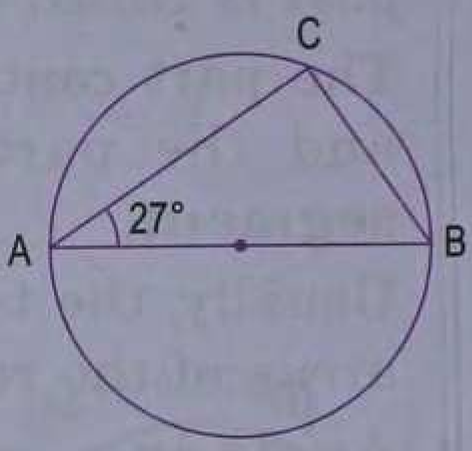
$$CP^2 = PT^2 + CT^2$$

$$\Rightarrow \quad PT^2 = CP^2 - CT^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$\Rightarrow \quad PT = \sqrt{144} = 12.$$

Hence, the length of tangent = 12 cm.

Example 2. In the adjoining figure, AB is a diameter of the circle. If $\angle CAB = 27°$, find $\angle CBA$.



Solution. We know that angle in a semicircle is 90°.

$\therefore \quad \angle ACB = 90°$

$\angle CBA + \angle CAB + \angle ACB = 180°$

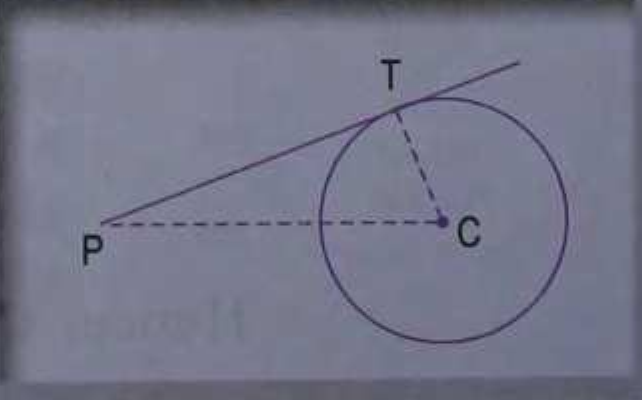
(sum of angles in a triangle = 180°)

$\Rightarrow \quad \angle CBA + 27° + 90° = 180°$

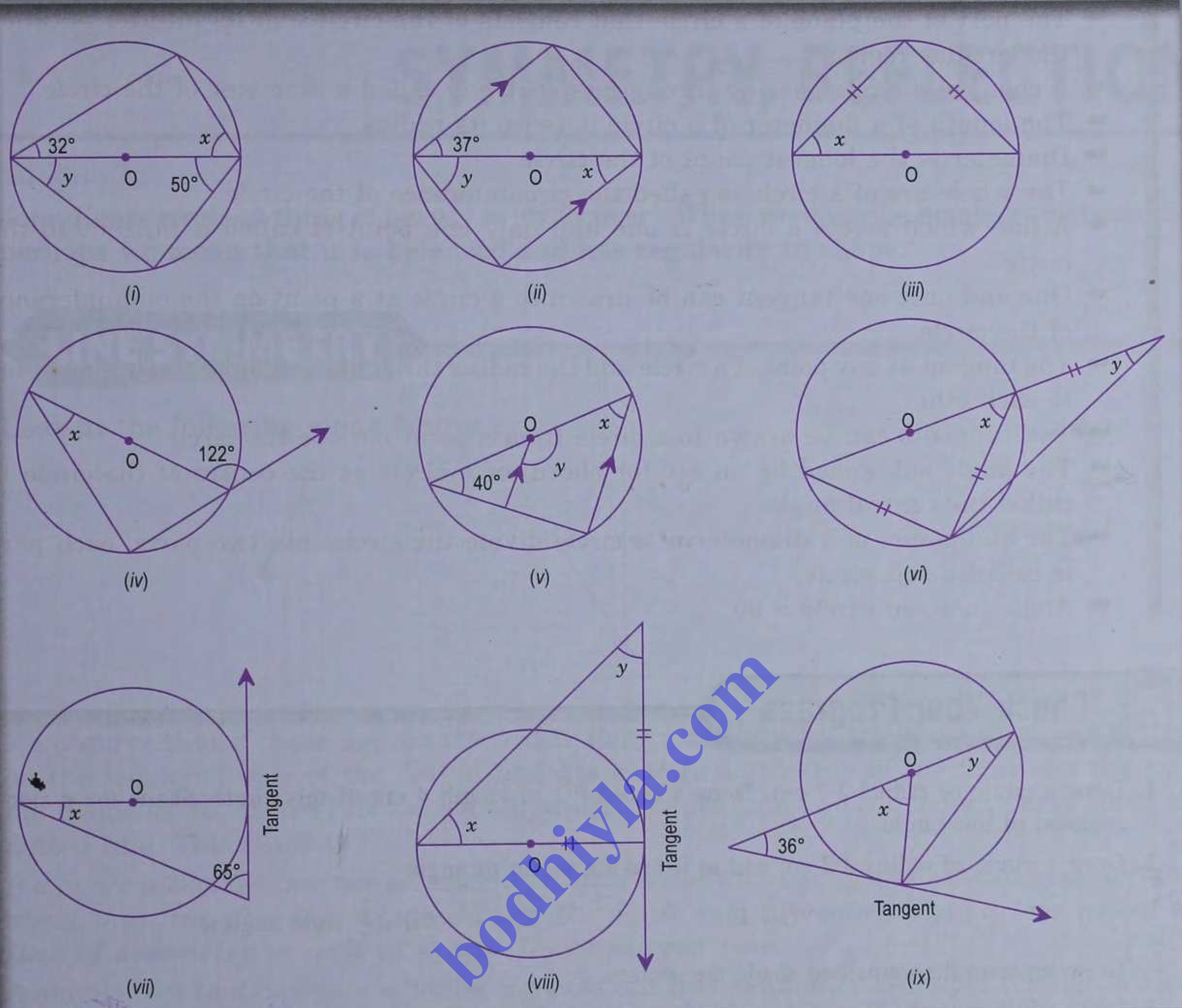
$\Rightarrow \quad \angle CBA = 180° - 27° - 90° = 63°.$

Exercise 26

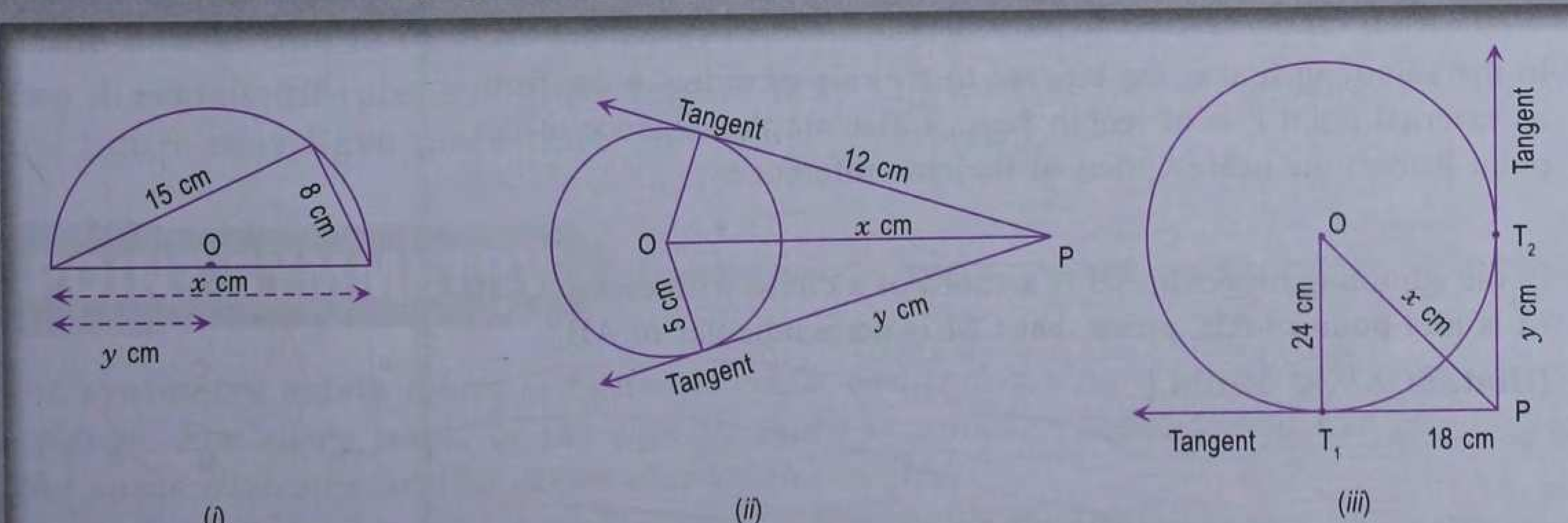
1. Fill in the blanks with correct word(s) to make the statement true :
   (*i*) Radius of a circle is one-half of its.....
   (*ii*) A radius of a circle is a line segment with one end point at..... and the other end on.....
   (*iii*) A chord of a circle is a line segment with its end points....
   (*iv*) A diameter of a circle is a chord that..... the centre of the circle.
   (*v*) All radii of a circle are.....
   (*vi*) The angle in a semicircle is.....
2. State which of the following statements are true and which are false :
   (*i*) A line segment with its end-points lying on a circle is called a diameter of the circle.
   (*ii*) Diameter is the longest chord of the circle.
   (*iii*) The end-points of a diameter of a circle divide the circle into two parts; each part is called a semicircle.
   (*iv*) A diameter of a circle divides the circular region into two parts; each part is called a semicircular region.
   (*v*) The diameters of a circle are concurrent. The centre of the circle is the point common to all diameters.
   (*vi*) Every circle has unique centre and it lies inside the circle.
   (*vii*) Every circle has unique diameter.
   (*viii*) From a given point in the exterior of a circle, two tangents can be drawn to it and these two tangents are equal in length.
3. Draw a circle with centre O and radius 2·5 cm. Draw two radii OA and OB such that $\angle AOB = 60°$. Measure the length of the chord AB.
4. Draw a circle of radius 3·2 cm. Draw a chord AB of this circle such that AB = 5 cm. Shade the minor segment of the circle.
   [Hint. To draw chord AB of length 5 cm, take a point A on the circle. With A as centre and radius 5 cm, draw an arc to meet the circle at B. Join AB.**]**
5. Draw a circle of radius 4 cm with C as its centre. Draw two radii CP and CQ such that $\angle PCQ = 45°$. Shade the minor sector of the circle.
6. Find the length of the tangent drawn to a circle of radius 3 cm, from a point at a distance 5 cm from the centre.
7. In the adjoining figure, PT is a tangent to the circle with centre C. Given CP = 20 cm and PT = 16 cm, find the radius of the circle.



8. In each of the following figure, O is the centre of the circle. Find the size of each lettered angle :



9. In each of the following figures, O is the centre of the circle. Find the values of x and y.



Summary

- A circle is the set of all those points in a plane each of which is at a constant distance from a fixed point in that plane.
- The fixed point is called the centre and constant distance is called radius.

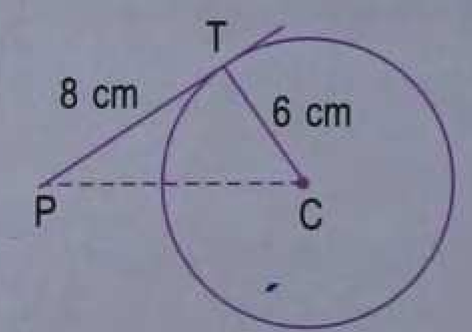
- All radii of a circle are equal.
- The centre of a circle lies in the interior of the circle.
- The part of the plane of a circle that consists of the circle and its interior is called the circular region.
- A chord of a circle passing through its centre is called a diameter of the circle.
- The length of a diameter of a circle is twice its radius.
- Diameter is the longest chord of the circle.
- The whole arc of a circle is called the circumference of the circle.
- A line which meets a circle is one and only one point is called a tangent to the circle.
- One and only one tangent can be drawn to a circle at a point on the circumference of the circle.
- The tangent at any point of a circle and the radius through the point are perpendicular to each other.
- Two tangents can be drawn to a circle from a point outside the circle.
- The angle subtended by an arc (or chord) of a circle at the centre of the circle is called the central angle.
- The end-points of a diameter of a circle divide the circle into two parts; each part is called a semicircle.
- Angle in a semicircle = 90°.

Check Your Progress

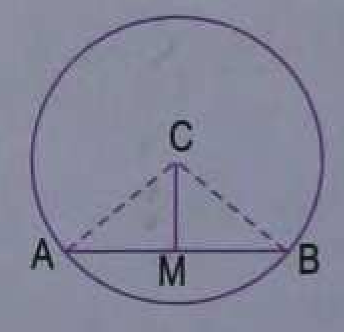
1. Draw a circle of radius 2·7 cm. Draw a chord PQ of length 4 cm of this circle. Shade the major segment of this circle.
2. Draw a circle of radius 3·2 cm and in it make a sector of angle

   (*i*) 30° (*ii*) 135° (*iii*) $2\frac{2}{3}$ right angles.

   Draw separate diagrams and shade the sectors.
3. Draw a line segment PQ = 6·4 cm. Construct a circle on PQ as diameter. Take any point R on this circle and measure ∠PRQ.
4. In the adjoining figure, the tangent to a circle of radius 6 cm from an external point P is of length 8 cm. Calculate the distance of the point P from the nearest point of the circumference.



5. In the adjoining diagram, AB is a chord of a circle with centre C. If M is mid-point of AB, prove that CM is perpendicular to AB.
   [Hint. ΔCAM ≅ ΔCBM.**]**



6. In the adjoining figure, O is the centre of the circle. If ∠ABP = 35° and ∠BAQ = 65°, find
   (*i*) ∠PAB
   (*ii*) ∠QBA.