Chapter 25

THEOREMS ON AREA

AREA

The ***area*** *of a plane closed figure is the measure of the region (surface) enclosed by its boundary.*

In this section, we shall compare the area of two known figures which have certain relationship between them.

Equal figures

Two plane figures are called ***equal*** *if and only if they have equal areas.*

Difference between congruent and equal figures

Since congruent figures are exact duplicate of each other, so they have equal areas. *Thus, congruent figures are equal figures. However, two equal figures may not be congruent.*

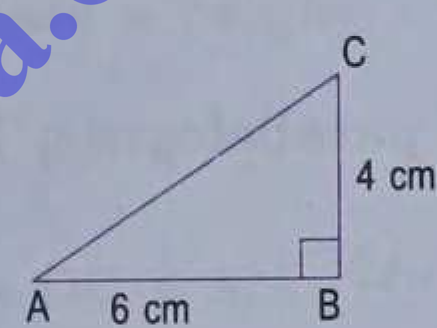
For example, consider the adjoining right angled triangles ABC and PQR.

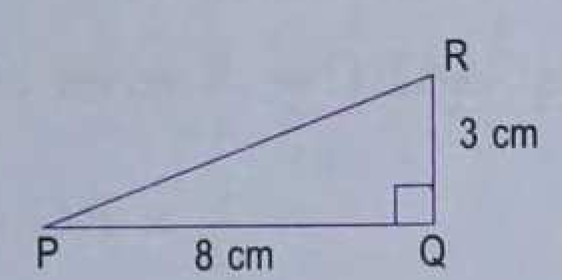
Area of $\Delta ABC = \left(\frac{1}{2} \times 6 \times 4\right) \text{ cm}^2$

$= 12 \text{ cm}^2$

Area of $\Delta PQR = \left(\frac{1}{2} \times 8 \times 3\right) \text{ cm}^2$

$= 12 \text{ cm}^2$





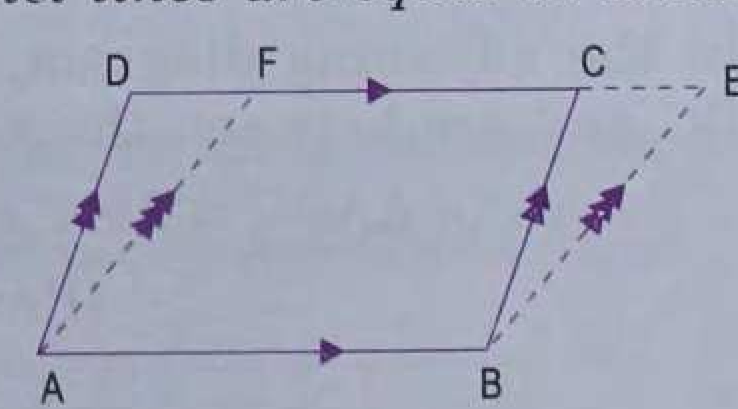
$\therefore$ Area of ΔABC = Area of ΔPQR

$\Rightarrow$ ΔABC and ΔPQR are equal figures. Clearly, ΔABC and ΔPQR are not congruent.

Theorem 1

Parallelograms on the same base and between the same parallel lines are equal in area.

In the adjoining diagram, parallelograms ABCD and ABEF have the same base AB and are between the same parallel lines AB and DE.



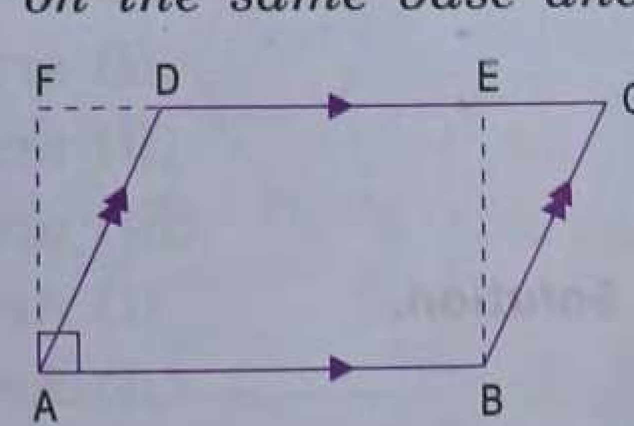
$\therefore$ Area of parallelogram ABCD = Area of parallelogram ABEF

Note. Parallelograms ABCD and ABEF are equal figures but these are not congruent figures.

Theorem 2

The area of a parallelogram is equal to the area of a rectangle on the same base and between the same parallel lines.

In the adjoining diagram, parallelogram ABCD and rectangle ABEF have the same base AB and are between the same parallel lines AB and FC.



$\therefore$ Area of parallelogram ABCD = Area of rectangle ABEF.

Corollary

Area of a parallelogram = base × height

By the above theorem,

area of parallelogram ABCD = area of rectangle ABEF

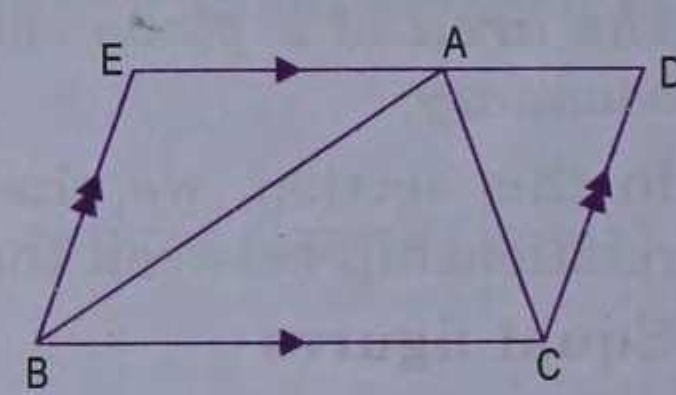
But area of rectangle = length × breadth = AB × BE

∴ Area of parallelogram ABCD = AB × BE = base × height.

Theorem 3

Area of a triangle is half that of a parallelogram on the same base and between the same parallels.

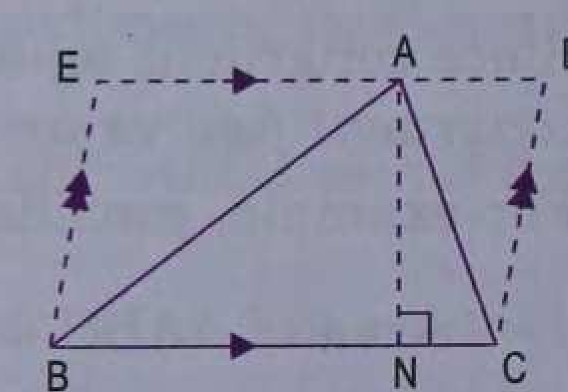
In the adjoining diagram, ΔABC and parallelogram BCDE have the same base AB and are between the same parallel lines BC and ED.



∴ Area of ΔABC = $\frac{1}{2}$ area of parallelogram BCDE

Corollary

Area of a triangle = $\frac{1}{2}$ *base × height*



In the adjoining diagram, note that

base of ΔABC = BC = base of parallelogram ABDE and height of ΔABC = AN = height of parallelogram gm BCDE

By the above theorem,

area of ΔABC = $\frac{1}{2}$ area of parallelogram BCDE

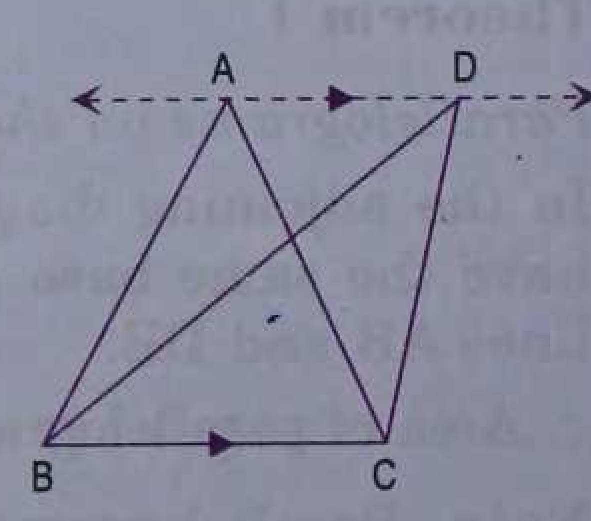
= $\frac{1}{2}$ × BC × AN

= $\frac{1}{2}$ base × height.

Theorem 4

Triangles on the same base and between the same parallel lines are equal in area.

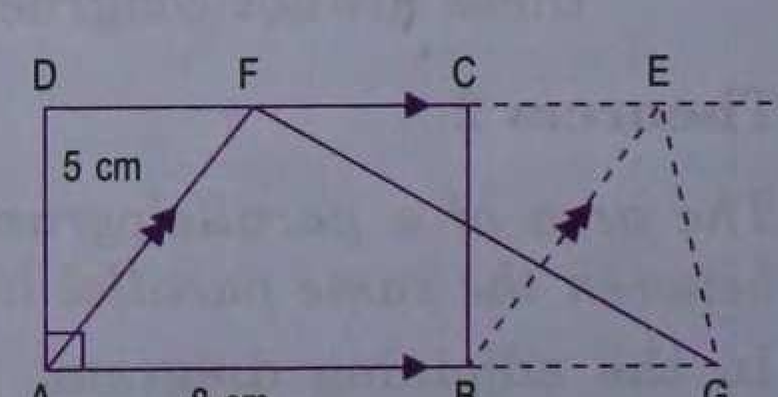
In the adjoining diagram, ΔABC and ΔDBC have the same base BC and are between the same parallel lines BC and AD.



∴ Area of ΔABC = Area of ΔDBC

(Proofs of the above theorems are deferred to the next class.)

Example 1. In this diagram, ABCD is a rectangle with side AB = 8 cm and AD = 5 cm.



Find

(*i*) area of rectangle ABCD

(*ii*) area of parallelogram ABEF

(*iii*) area of ΔEFG

Solution. (*i*) Area of rectangle ABCD = base × height = (8 × 5) cm^2 = 40 cm^2.

(*ii*) Area of parallelogram ABEF = area of rectangle ABCD

(on the same base AB and between same parallels AB and DE)

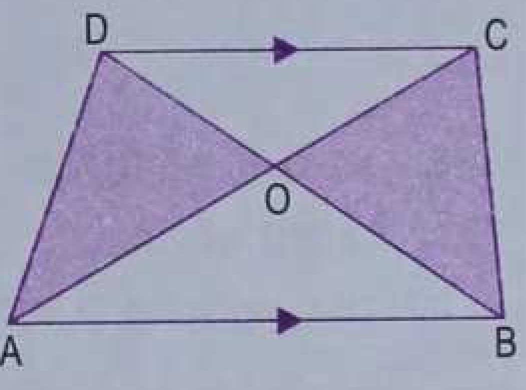
= 40 cm^2.

(*iii*) Area of ΔEFG = $\frac{1}{2}$ area of parallelogram ABEF

(on the same base EF and between same parallels EF and AG)

$$= \left(\frac{1}{2} \times 40\right) \text{ cm}^2 = 20 \text{ cm}^2.$$

Example 2. ABCD is a trapezium with AB || DC, and diagonals AC and BD intersect at O. Prove that area of ΔODA = area of ΔOBC.



Solution. Area of ΔDAB = Area or ΔCAB

(triangles on the same base AB and between the same parallel lines AB and DC)

$\Rightarrow$ area of ΔODA + area of ΔOAB = area of ΔOBC + area of ΔOAB

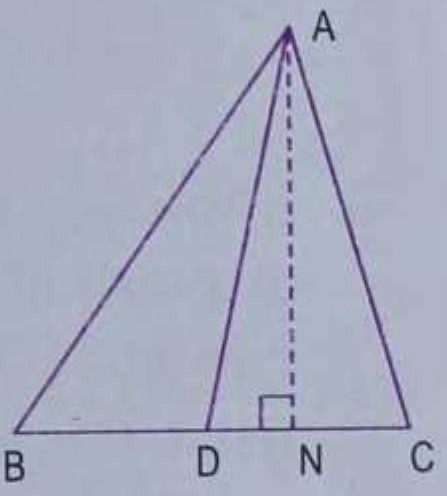
(from figure, area axiom)

$\Rightarrow$ area of ΔODA = area of ΔOBC.

(subtracting same area of ΔOAB from both sides)

Example 3. Prove that a median divides a triangle into two triangles of equal area.

Solution. Let ABC be any triangle and AD be its median *i.e.* D is mid-point of BC, so BD = DC. From A, draw AN perpendicular to BC.



Since area of a triangle = $\frac{1}{2}$ base $\times$ height,

area of ΔABD = $\frac{1}{2} \times$ BD $\times$ AN

and area of ΔADC = $\frac{1}{2} \times$ DC $\times$ AN

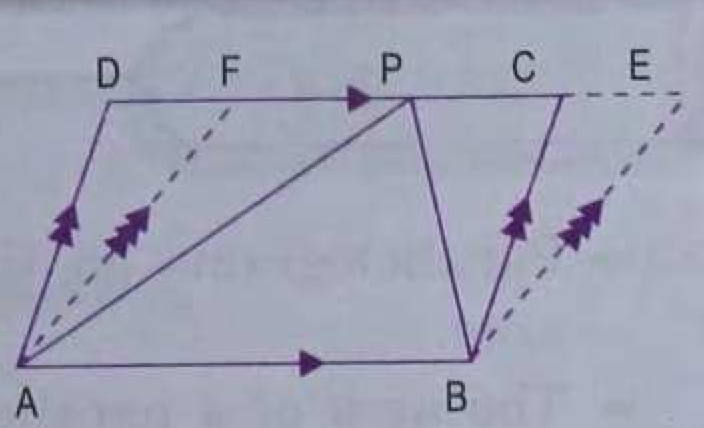
= $\frac{1}{2} \times$ BD $\times$ AN (∵ BD = DC)

$\Rightarrow$ area of ΔADC = area of Δ ABD

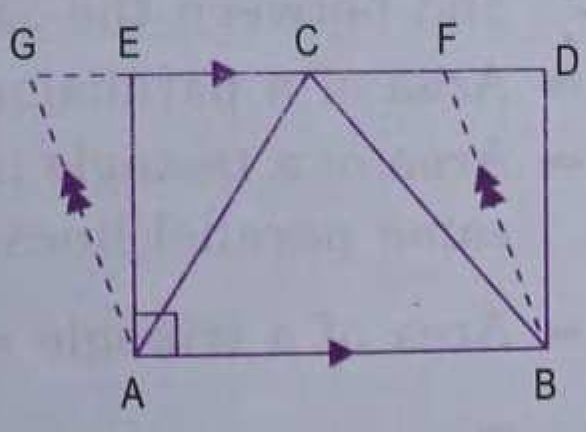
Hence, a median of a triangle divides it into two triangles of equal area.

Exercise 25

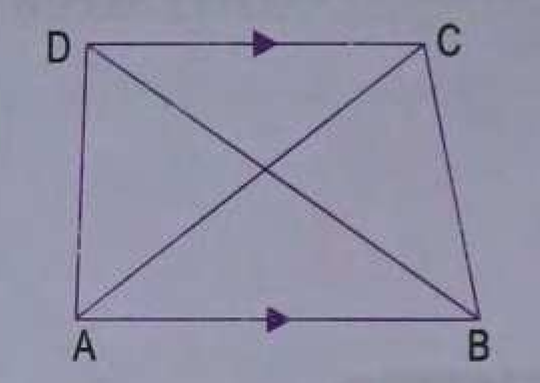
1. In the adjoining figure, area of parallelogram ABCD = 37 cm². Find
   (*i*) area of parallelogram ABEF
   (*ii*) area of ΔABP.



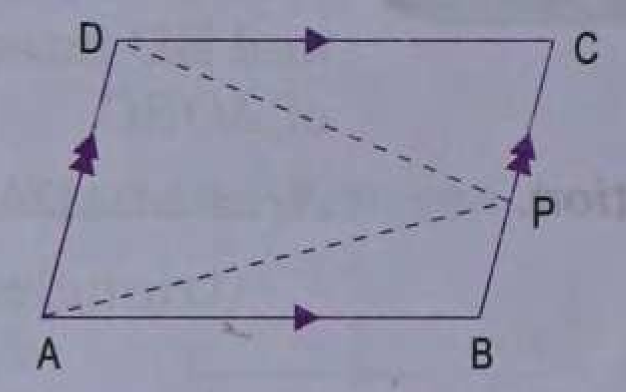
2. In the adjoining figure, area of ΔABC = 21 square units. Find
   (*i*) area of parallelogram ABFG
   (*ii*) area of rectangle ABDE.



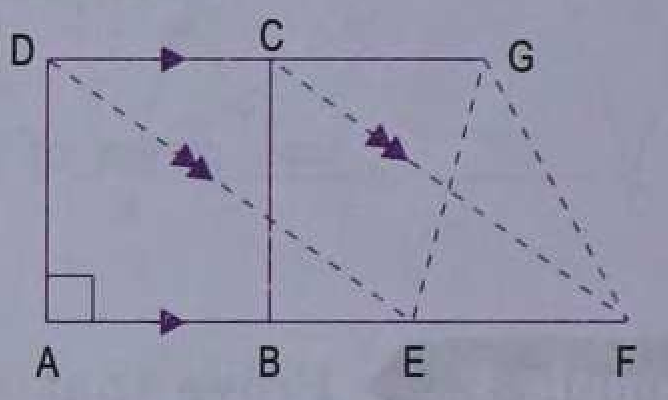
3. In the adjoining figure, AB || DC. If area of ΔABC = 19 square units, find the area of ΔABD.



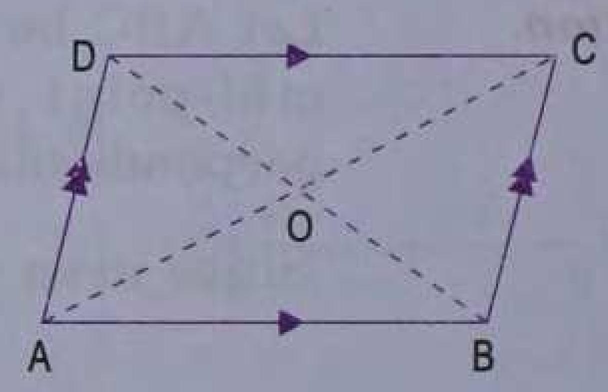
4. In the adjoining figure, ABCD is a parallelogram and P is a point on BC.
If area of Δ APD = 17·6 cm², find
   (*i*) the area of parallelogram ABCD
   (*ii*) the sum of the areas of triangles ABP and DPC.



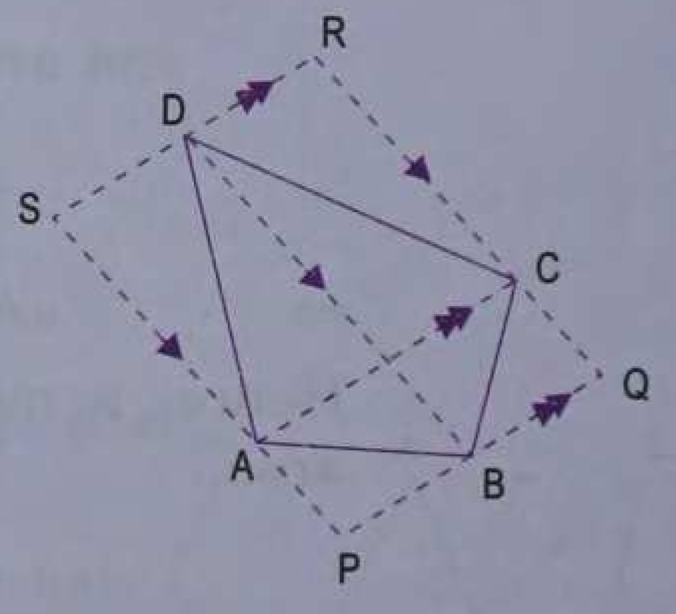
5. In the adjoining figure, ABCD is a rectangle with sides AB = 4 cm and AD = 6 cm. Find
   (*i*) the area of parallelogram DEFC.
   (*ii*) the area of ΔEFG.



6. In the adjoining figure, ABCD is a parallelogram and its diagonals AC and BD intersect at O. Prove that area of Δ OAB = area of ΔOBC= area of ΔOCD = area of ΔOAD.
[Hint. O is mid-point of AC, by example 3,
area of $\Delta OAB = \frac{1}{2}$ area of ΔABC.]



7. In the adjoining figure, PQRS is a parallelogram formed by drawing lines parallel to the diagonals of a quadrilateral ABCD through its corners. Prove that area of parallelogram PQRS = 2 . area of quadrilateral ABCD
[Hint. Area of $\Delta ABC = \frac{1}{2}$ area of parallelogram APQC.]

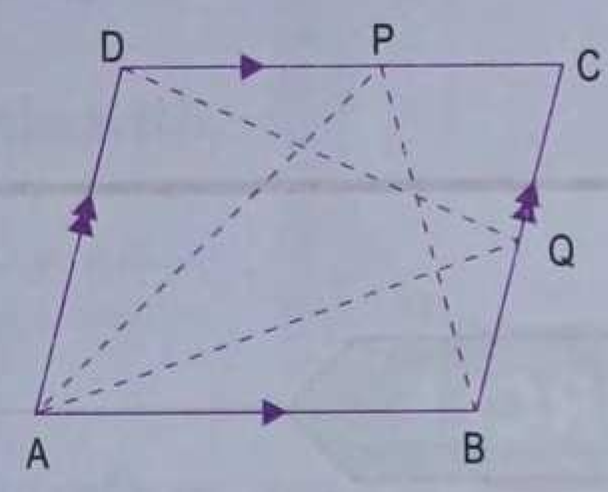


Summary

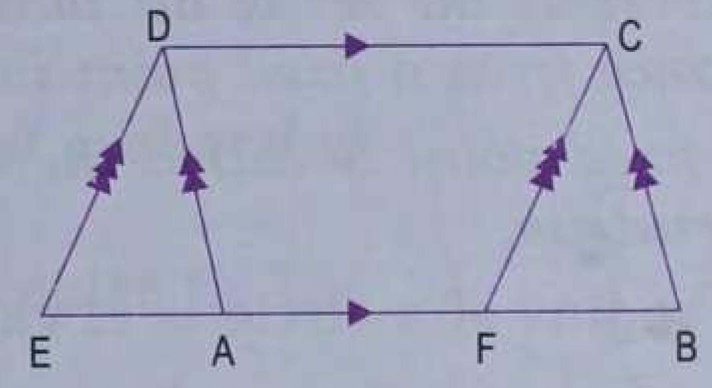
- Parallelograms on the same base and between the same parallels are equal in area.
- The area of a parallelogram is equal to the area of a rectangle on the same base and between the same parallel lines.
- Area of a parallelogram = base × height.
- Area of a triangle is half that of a parallelogram on the same base and between the same parallel lines.
- Area of a triangle = $\frac{1}{2}$ base × height.
- Triangles on the same base and between the same parallel lines are equal in area.

Check Your Progress

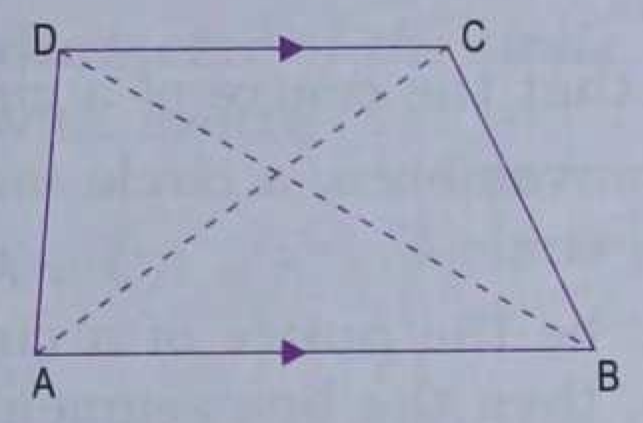
1. In the adjoining diagram, ABCD is a parallelogram. P and Q are points on the sides DC and BC respectively. Prove that
   (*i*) area of ΔAPB = area of ΔAQD.
   (*ii*) area of ΔAPB = area of ΔABQ + area of ΔDQC.



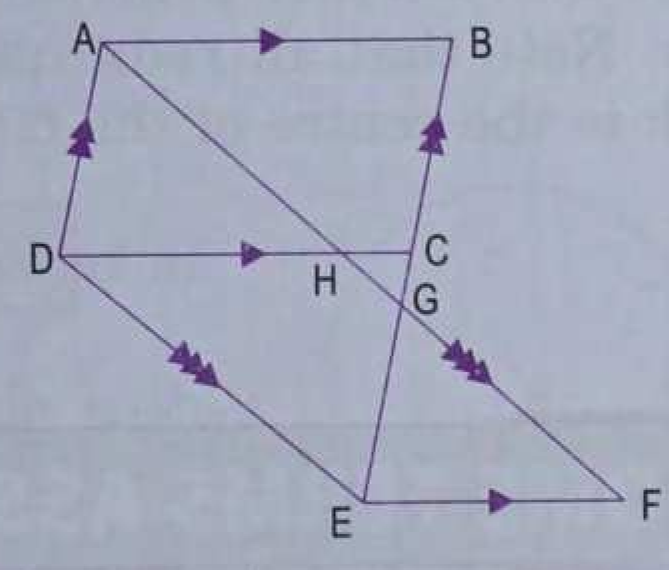
2. In the adjoining figure, area of the parallelogram ABCD is 29 cm^2. If AB = 5·8 cm, find the height of the parallelogram EFCD.



3. In the adjoining figure, AB || DC and area of ΔABD is 24 sq. units. If AB = 8 units, find the height of ΔABC.



4. In the adjoining figure, AB || DC || EF, DA || EB and DE || AF. Prove that area of parallelogram DEFH = area of parallelogram ABCD.
   [Hint. Area of parallelogram ABCD
   = area of parallelogram ADEG
   = area of parallelogram DEFH.]



5. In the adjoining figure, DE is parallel to the side BC of ΔABC. BE and CD intersect at O. Prove that
   (*i*) area of ΔBED = area of ΔCED.
   (*ii*) area of ΔBOD = area of ΔCOE.
   (*iii*) area of ΔABE = area of ΔADC.