24 Probability

24.1 PROBABILITY

In everyday life, we come across statements such as :

(i) It **may** rain today.

(ii) **Probably** Rajesh will top the council examination this year.

(iii) I **doubt** she will pass the test.

(iv) It is **unlikely** that India will win the world cup.

(v) **Chances** are high that the prices of petrol will go up.

The words, 'may', 'probably', 'doubt', 'unlikely', 'chances' used in the above statements indicate *uncertainties*. For example, in the statement (i), 'may rain today' means it may rain or may not rain today. We are predicting rain today based on our past experience when it rained under similar conditions. Similar predictions are made in other cases listed (ii) to (v).

Probability is a measure of uncertainty.

The theory of probability developed as a result of studies of game of chance or gambling. Suppose you pay ₹ 5 to throw a die – if it comes up with number 1, you get ₹ 20, otherwise you get nothing. Should you play such a game? Does it give a *fair chance* of winning? Search for mathematical answers of these types of questions led to the development of modern theory of probability.

24.1.1 Some terms related to probabiltiy

1. **Experiment.** *An action which results in some (well defined) outcomes is called an experiment.*
2. **Random experiment.** *An experiment is called* ***random*** *if it has more than one possible outcome and it is not possible to tell (predict) the outcome in advance.*

Pierre Simon Laplace

'It is remarkable that a science, which began with the consideration of games of chance, should be elevated to the rank of the most important subject of human knowledge.'

For example :

(*i*) tossing a coin

(*ii*) tossing two coins simultaneously

(*iii*) throwing a die

(*iv*) drawing a card from a pack of 52 (playing) cards.

All these are random experiments.

From now onwards, whenever the word experiment is used it will mean random experiment.

3. **Sample space.** *The collection of all possible outcomes of a random experiment is called* ***sample space.***

4. **Event.** *A subset of the sample space associated with a random experiment is called an event.*

For example :

(*i*) When a die is thrown, we can get any number 1, 2, 3, 4, 5, 6. So the sample space S = {1, 2, 3, 4, 5, 6}. A few events of this experiment could be —

'Getting a six' : {6}

'Getting an even number' : {2, 4, 6}

'Getting a prime number' : {2, 3, 5}

'Getting a number less than 5' : {1, 2, 3, 4}, etc.

(*ii*) When a pair of coins is tossed, the sample space of the experiment

S = {HH, HT, TH, TT}. A few events of this experiment could be —

'Getting exactly one head' : {HT, TH}

'Getting exactly two heads' : {HH}

'Getting atleast one head' : {HH, HT, TH}, etc.

5. *Occurrence of an event. When the outcome of an experiment satisfies the condition mentioned in the event, then we say that* ***event has occurred.***

For example :

(*i*) In the experiment of tossing a coin, an event E may be getting a head. If the coin comes up with head, then we say that event E has occurred, otherwise, if the coin comes up with tail, we say that event E has not occurred.

(*ii*) In the experiment of tossing a pair of coins simultaneously, an event E may be getting two heads. If the pair of coins come up with two heads, then we say that event E has occurred, otherwise, we say that event E has not occured.

(*iii*) In the experiment of throwing a die, an event E may be taken as 'getting an even number'. If the die comes up with any of the numbers 2, 4 or 6, we say that event E has occurred, otherwise, if the die comes up with 1, 3 or 5, we say that event E has not occurred.

6. **Favourable outcomes.** *The outcomes which ensure the occurrence of an event are called* ***favourable outcomes*** *to that event.*

7. **Equally likely outcomes.** *If there is no reason for any one outcome to occur in preference to any other outcome, we say that the outcomes are* ***equally likely.***

For example :

(*i*) In tossing a coin, it is equally likely that the coin lands either head up or tail up.

(*ii*) In throwing a die, each of the six numbers 1, 2, 3, 4, 5, 6 is equally likely to show up.

Note

From now onwards, we will **assume that all the outcomes are equally likely.**

24.1.2 Definition of probability

The assumption that all the outcomes are equally likely leads to the following definition of probability :

The probability of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{number of outcomes favourable to E}}{\text{total number of possible outcomes of the experiment}}.$$

Sure event. *An event which always happens is called a* ***sure event*** *or a* ***certain event.***

For example, when we throw a die, then the event 'getting a number less than 7' is a sure event.

The probability of a sure event is 1.

Impossible event. *An event which never happens is called an* ***impossible event.*** For example, when we throw a die, then the event 'getting a number greater than 6' is an impossible event.

The probability of an impossible event is 0.

Elementary event. *An event which has one (favourable) outcome from the sample space is called an* ***elementary event.***

An event which has more than one (favourable) outcome from the sample space is called a ***compound event.***

For example, when we throw a die, then the event getting number 5 is an elementary event whereas the event getting an even number (2, 4, or 6) is a compound event.

Complementary events. *If E is an event, then the event 'not E' is complementary event of E.*

For example, when we throw a die, let E be the event getting a number less than or equal to 2, then the event 'not E' *i.e.* getting a number greater than 2 is complementary event of E.

Complement of E is denoted by $\bar{E}$ or E^c.

Let E be an event, then the number of outcomes favourable to E is greater than or equal to zero and is less than or equal to total number of outcomes. If follows that $0 \le P(E) \le 1$.

Remarks

- Let E be an event, then we have :
  - (*i*) $0 \le P(E) \le 1$
  - (*ii*) $P(\bar{E}) = 1 - P(E)$
  - (*iii*) $P(E) = 1 - P(\bar{E})$
  - (*iv*) $P(E) + P(\bar{E}) = 1$.
- The sum of the probabilities of all the elementary events of an experiment is 1.

ILLUSTRATIVE EXAMPLES

Example 1. *A coin is tossed once. Find the probability of getting*

(*i*) *a head* (*ii*) *a tail.*

Solution. When a coin is tossed once, the possible outcomes are Head (H) and Tail (T). So, the total number of possible outcomes = 2.

(*i*) Let E_1 be the event of getting a head, then the number of favourable outcomes to E_1 (*i.e.* of getting a head) = 1

$$\therefore P(E_1) = \frac{\text{number of favourable outcomes to } E_1}{\text{total number of possible outcomes}} = \frac{1}{2}.$$

(*ii*) Let E_2 be the event of getting a tail, then the number of favourable outcomes to E_2 (*i.e.* of getting a tail) = 1.

$$\therefore P(E_2) = \frac{\text{number of favourable outcomes to } E_2}{\text{total number of possible outcomes}} = \frac{1}{2}.$$

Note that E_1 and E_2 are elementary events and $P(E_1) + P(E_2) = \frac{1}{2} + \frac{1}{2} = 1$.

Thus, the sum of the probabilities of all the elementary events of an experiment = 1.

Example 2. *Malini buys a fish from a shop for her aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?*

Solution. As a fish is taken out at random from the tank, all the outcomes are equally likely.

Total number of fish in the tank = 5 + 8 = 13.

∴ Total number of possible outcomes = 13.

Let E be the event 'taking out a male fish'. As there are 5 male fish in the tank, the number of favourable outcomes to the event E = 5.

$$\therefore P(E) = \frac{\text{number of favourable outcomes to E}}{\text{total number of possible outcomes}} = \frac{5}{13}.$$

Example 3. *A die is tossed once. Find the probability of getting*

(*i*) *number 4* (*ii*) *a number greater than 4*

(*ii*) *a number less than 4* (*iv*) *an even number*

(*v*) *a number greater than 6* (*vi*) *a number less than 7.*

Solution. When a die is tossed once, the possible outcomes are the numbers 1, 2, 3, 4, 5, 6. So, the total number of possible outcomes = 6.

(*i*) The event is getting the number 4.

The number of favourable outcomes to the event getting number 4 = 1.

$$\therefore P\text{ (getting number 4)} = \frac{1}{6}.$$

(*ii*) The event is getting a number greater than 4 and the outcomes greater than 4 are 5 and 6. So, the number of favourable outcomes to the event getting a number greater than 4 = 2

$$\therefore P\text{ (getting a number greater than 4)} = \frac{2}{6} = \frac{1}{3}.$$

(*iii*) The event is getting a number less than 4 and the outcomes less than 4 are 1, 2, 3. So, the number of favourable outcomes to the event getting a number less than 4 = 3.

$$\therefore P\text{ (getting a number less than 4)} = \frac{3}{6} = \frac{1}{2}.$$

(*iv*) The event is getting an even number and the even numbers (outcomes) are 2, 4, 6. So, the number of favourable outcomes to the event getting an even number = 3.

$$\therefore P\text{ (getting an even number)} = \frac{3}{6} = \frac{1}{2}.$$

(*v*) The event is getting a number greater than 6 and there is no outcome greater than 6. So, the number of favourable outcomes to the event getting a number greater than 6 = 0.

$\therefore$ P (getting a number greater than 6) = $\frac{0}{6} = 0$.

Note that in this experiment, getting a number greater than 6 is an ***impossible event***.

(*vi*) The event is getting a number less than 7 and all the outcomes 1, 2, 3, 4, 5, 6 are less than 7. So, the number of favourable outcomes to the event getting a number less than 7 = 6.

$\therefore$ P (getting a number less than 7) = $\frac{6}{6} = 1$.

Note that in this experiment, getting a number less than 7 is a ***sure event***.

Example 4. *A bag contains 5 red, 4 green and 6 white balls. If a ball is drawn at random from the bag, find the probability that it will be*

(i) white *(ii) red* *(iii) green.*

Solution. Saying that a ball is drawn at random from the bag means that all the balls are equally likely to be drawn.

Total number of balls in the bag = 5 + 4 + 6 = 15.

$\therefore$ The number of possible outcomes = 15.

(*i*) Let W be the event 'the ball drawn is white'. As there are 6 white balls in the bag, the number of favourable outcomes to the event W = 6.

$\therefore P(W) = \frac{6}{15} = \frac{2}{5}$.

(*ii*) Let R be the event 'the ball draw is red'. As there are 5 red balls in the bag, the number of favourable outcomes to the event R = 5.

$\therefore P(R) = \frac{5}{15} = \frac{1}{3}$.

(*iii*) Let G be the event 'the ball drawn is green'. As there are 4 green balls in the bag, the number of favourable outcomes to the event G = 4.

$\therefore P(G) = \frac{4}{15}$.

Note that $P(W) + P(R) + P(G) = \frac{2}{5} + \frac{1}{3} + \frac{4}{15} = \frac{6+5+4}{15} = \frac{15}{15} = 1$.

Example 5. *If the probability of Sania winning a tennis match is 0·63, what is the probability of her losing the match?*

Solution. Let E be the event 'Sania winning the match', then P(E) = 0·63 (given).

$\therefore$ P (Sania losing the match) = P (not E) = $P(\bar{E})$
= 1 – P(E) = 1 – 0·63
= 0·37.

Note that winning the match and losing the match are complementary events.

Example 6. *Ankita and Nagma are friends. They were both born in 1990. What is the probability that they have*

(i) same birthday? *(ii) different birthdays?*

Solution. Out of two friends, say, Ankita's birthday can be any day of the 365 days in a (non-leap) year. Also Nagma's birthday can be any day of the 365 days of the same year. So, the total number of outcomes = 365.

We assume that all these 365 outcomes are equally likely.

(*i*) Let E be the event 'Ankita and Nagma have same birthday', then the number of favourable outcomes to the event E = 1

$\therefore P(E) = \frac{1}{365}$.

(*ii*) P (Ankita and Nagma have different birthdays) = P (not E)

$= P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{1}{365} = \frac{364}{365}$.

Example 7. *A box contains 7 blue, 8 white and 5 black marbles. If a marble is drawn at random from the box, what is the probability that it will be*

(i) black? *(ii) blue or black?* *(iii) not black?* *(iv) green?*

Solution. Saying that a marble is drawn at random from the box means that all the marbles are equally likely to be drawn.

Total number of marbles in the box = 7 + 8 + 5 = 20.

∴ The number of possible outcomes = 20.

(*i*) Let the event be 'marble drawn is black'. As there are 5 black marbles in the box, the number of favourable outcomes to the event = 5.

$\therefore P \text{ (black)} = \frac{5}{20} = \frac{1}{4}$.

(*ii*) Let the event be 'marble drawn is blue or black', so the favourable outcomes to the event are any blue marble or any black marble.

∴ The number of favourable outcomes to the event

= number of blue marbles + number of black marbles

= 7 + 5 = 12.

$\therefore P \text{ (blue or black)} = \frac{12}{20} = \frac{3}{5}$.

(*iii*) $P \text{ (not black)} = 1 - P \text{ (black)} = 1 - \frac{1}{4} = \frac{3}{4}$.

Alternatively

The event 'marble drawn is not black' means that the marble drawn is either blue or white.

∴ The number of favourable outcomes to the event 'marble drawn is not black'

= number of blue marbles + number of white marbles

= 7 + 8 = 15.

$\therefore P \text{ (not black)} = \frac{15}{20} = \frac{3}{4}$.

(*iv*) Let the event be 'marble drawn is green'. As there is no green marble in the box, the number of favourable outcomes to the event = 0.

$\therefore P \text{ (green)} = \frac{0}{20} = 0$.

Example 8. *A box contains 17 cards numbered 1, 2, 3, ..., 17 and are mixed thoroughly. A card is drawn at random from the box. Find the probability that the number on the card is*

(i) odd *(ii) even* *(iii) prime* *(iv) divisible by 3*

(v) divisible by 3 and 2 both *(vi) divisible by 3 or 2.*

Solution. The cards are mixed thoroughly and a card is drawn at random from the box means that all the outcomes are equally likely.

As the box contains 17 cards, total number of possible outcomes = 17.

(*i*) Let the event be 'the number on the card drawn is odd'. The outcomes favourable to the event are 1, 3, 5, 7, 9, 11, 13, 15, 17.

∴ The number of favourable outcomes to the event = 9.

∴ P (odd) = $\frac{9}{17}$.

(*ii*) Let the event be 'the number on the card drawn is even'. The outcomes favourable to the event are 2, 4, 6, 8, 10, 12, 14, 16.

∴ The number of favourable outcomes to the event = 8.

∴ P (even) = $\frac{8}{17}$.

(*iii*) Let the event be 'the number on the card drawn is prime'. The outcomes favourable to the event are 2, 3, 5, 7, 11, 13, 17.

∴ The number of favourable outcomes to the event = 7.

∴ P (prime) = $\frac{7}{17}$.

(*iv*) Let the event be 'the number on the card is divisible by 3'. The outcomes favourable to the event are 3, 6, 9, 12, 15.

∴ The number of favourable outcomes to the event = 5.

∴ P (divisible by 3) = $\frac{5}{17}$.

(*v*) Let the event be 'the number on the card drawn is divisible by 3 and 2 both'. The outcomes favourable to the event are 6 and 12.

∴ The number of favourable outcomes to the event = 2.

∴ P (divisible by 3 and 2 both) = $\frac{2}{17}$.

(*vi*) Let the event be 'the number on the card drawn is divisible by 3 or 2'. The outcomes favourable to the event are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16.

∴ The number of favourable outcomes to the event = 11.

∴ P (divisible by 3 or 2) = $\frac{11}{17}$.

Example 9. *Find the probability of having 53 Sundays in*

(i) a non-leap year *(ii) a leap year.*

Solution.

(*i*) In non-leap year, there are 365 days and 364 days make 52 weeks. Therefore, we have to find the probability of having a Sunday out of the remaining 1 day.

Hence, probability (having 53 Sundays) = $\frac{1}{7}$.

(*ii*) In a leap year, there are 366 days and 364 days make 52 weeks and in each week there is one Sunday. Therefore, we have to find the probability of having a Sunday out of the remaining 2 days.

Now the 2 days can be (Sunday, Monday) or (Monday, Tuesday) or (Tuesday, Wednesday) or (Wednesday, Thursday) or (Thursday, Friday) or (Friday, Saturday) or (Saturday, Sunday). Note that Sunday occurs 2 times in these 7 pairs.

Let the event be 'having a Sunday', then the number of favourable outcomes to the event = 2.

∴ Probability (having 53 Sundays) = $\frac{2}{7}$.

Example 10. *A bag contains 12 balls out of which x are black.*

(i) If a ball is drawn at random, what is the probability that it will be a black ball?

(ii) If 6 more black balls are put in the bag, the probability of drawing a black ball will be double than that of (i). Find the value of x.

Solution. All outcomes are equally likely.

(*i*) Total number of outcomes = 12.

Let the event be 'drawing a black ball', then the number of favourable outcome to the event = x.

$\therefore$ P (drawing a black ball) = $\frac{x}{12}$.

(*ii*) Now, 6 more black balls are put in the bag.

$\therefore$ Total number of outcomes = 12 + 6 = 18.

Number of black balls in the bag become $x + 6$.

Let the event be 'drawing a black ball', then the number of favourable outcomes to the event = $x + 6$.

$\therefore$ P (drawing a black ball) = $\frac{x+6}{18}$.

According to the question, $\frac{x+6}{18} = 2 \times \frac{x}{12}$

$\Rightarrow x + 6 = 3x \quad \Rightarrow 2x = 6 \quad \Rightarrow x = 3$

Hence, the value of x is 3.

A Note on a Pack of 52 Cards

You must have seen a pack or (deck) of playing cards. It consists of 52 cards which are divided into 4 suits of 13 cards each – spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Spades and clubs are of black colour, while hearts and diamonds are of red colour. The cards in each suit are ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, queens and jacks are called *face* (or *picture* or *court*) *cards*.

The cards bearing numbers 10, 9, 8, 7, 6, 5, 4, 3 and 2 are called *numbered cards*. Thus a pack of playing cards has 4 aces, 12 face cards and 36 numbered cards.

The aces together with face cards *i.e.* aces, kings, queens and jacks are called *cards of honour*; so there are 16 cards of honour.

Example 11. *A card is drawn from a well-shuffled deck of 52 cards. Find the probability that the card drawn is :*

(i) an ace | *(ii) a red card*

(iii) neither a king nor a queen | *(iv) a face card*

(v) a card of spade or an ace | *(vi) non-face card of red colour.*

Solution. Well-shuffling ensure equally likely outcomes.

Total number of outcomes = 52.

(*i*) There are 4 aces, one of each suit.

$\therefore$ The number of favourable outcomes to the event 'an ace' = 4.

$\therefore$ P (an ace) = $\frac{4}{52} = \frac{1}{13}$.

(*ii*) There are 26 red cards, 13 of hearts and 13 of diamonds.

$\therefore$ The number of favourable outcomes to the event 'a red card' = 26.

$\therefore$ P (a red card) = $\frac{26}{52} = \frac{1}{2}$.

(*iii*) There are 4 kings and 4 queens.

∴ The number of cards which are neither a king nor a queen

$= 52 - 4 - 4 = 44.$

∴ The number of favourable outcomes to the event 'neither a king nor a queen' = 44.

∴ P (neither a king nor a queen) = $\frac{44}{52} = \frac{11}{13}$.

(*iv*) Each suit has one king, one queen and one jack, therefore, in total we have 4 kings, 4 queens and 4 jacks.

∴ The number of face cards = 12.

∴ The number of favourable outcomes to the event 'a face card' = 12.

∴ P (a face card) = $\frac{12}{52} = \frac{3}{13}$.

(*v*) The event is 'a card of spade or an ace'.

The number of outcomes favourable to the event 'a card of spade or an ace' = number of cards of spade + number of aces other than spade

$= 13 + 3 = 16.$

∴ P (a card of spade or an ace) = $\frac{16}{52} = \frac{4}{13}$.

(*vi*) There are 26 cards of red colour which contain 3 face cards of hearts and 3 face cards of diamonds.

∴ The number of non-face cards of red colour = 26 – 3 – 3 = 20.

∴ The number of favourable outcomes to the event 'non-face card of red colour' = 20.

∴ P (non-face card of red colour) = $\frac{20}{52} = \frac{5}{13}$.

Example 12. *From a pack of 52 cards, a black jack, a red queen and two black kings fell down. A card was then drawn from the remaining pack at random. Find the probability that the card drawn is*

(i) a black card *(ii) a king* *(iii) a red queen.*

Solution. As a black jack, a red queen and two black kings fell down, the number of cards left in the pack = 52 – 4 = 48.

∴ The total number of outcomes = 48.

(*i*) Number of black cards left in the pack = 26 – 3 = 23.

∴ The number of favourable outcomes to the event 'a black cards' = 23.

∴ P (a black card) = $\frac{23}{48}$.

(*ii*) Number of kings left in the pack = 4 – 2 = 2.

∴ The number of favourable outcomes to the event 'a king'

∴ P (a king) = $\frac{2}{48} = \frac{1}{24}$.

(*iii*) Number of red queens left in the pack = 2 – 1 = 1.

∴ P (a red queen) = $\frac{1}{48}$.

Example 13. *From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random. What is the probability that the card drawn is*

(i) a face card (King, Jack or Queen)

(ii) an even numbered red card? (2011)

Solution. Cards bearing numbers 2, 3, 4, 5, 6, 7, 8, 9 and 10 are numbered cards in the pack of 52 playing cards.

The cards whose numbers are multiples of 3 are 3, 6 and 9.

As each suit has one '3', one '6' and one '9', therefore, in total we have four 3's, four 6's and four 9's.

$\therefore$ Total number of cards whose numbers are multiples of 3 = 12.

On removing these cards from the pack, the number of cards left in the pack

$$= 52 - 12 = 40.$$

(*i*) As each suit has one king, one queen and one jack, therefore, in total we have 4 kings, 4 queens and 4 jacks.

$\therefore$ Total number of face cards in the pack = 12.

$\therefore$ The number of outcomes favourable to the event 'a face card' = 12.

$\therefore$ Probability (a face card) = $\frac{12}{40} = \frac{3}{10}$.

(*ii*) Even numbered *red* cards left in the pack are 2, 4, 8, 10 of hearts and 2, 4, 8, 10 of diamonds.

$\therefore$ Total number of even numbered red cards in the remaing pack = 8.

$\therefore$ The number of outcomes favourable to the event 'an even numbered red card' = 8.

$\therefore$ P (an even numbered red card) = $\frac{8}{40} = \frac{1}{5}$.

A Note on Tossing Two Different Coins Simultaneously

Let two different coins (say, one is of ₹1 and other of ₹2) be tossed simultaneously. When ₹1 coin shows head (H) up, then ₹2 coin may show head (H) up or tail (T) up. When ₹1 coin shows tail (T) up, then ₹2 coin may show head (H) up or tail (T) up. Thus, the outcomes are :

(H, H), (H, T), (T, H), (T, T)

These out comes can also be written as : HH, HT, TH, TT.

Also one head means one head and one tail. The outcomes favourable to the event 'one head' are (H, T), (T, H) *i.e.* HT, TH. Similarly, one tail means one tail and one head. The outcomes favourable to the event 'one tail' are (H, T), (T, H) *i.e.* HT, TH.

Atleast one head means either one head or two heads. The outcomes favourable to the event 'atleast one head' are (H, T), (T, H), (H, H) *i.e.* HT, TH, HH.

Similarly, atmost one head means either one head or no head. The outcomes favourable to the event 'atmost one head' are (H,T), (T, H), (T, T) *i.e.* HT, TH, TT.

Example 14. *Kiran tossed two different coins simultaneously. Find the probability of getting :*

(i) two heads *(ii) one head* *(iii) no head*

(iv) atleast one head *(v) atmost one head.*

Solution. When two different coins are tossed simultaneously, the possible outcomes are (H, H), (H, T), (T, H), (T, T). All the outcomes are equally like.

Total number of possible outcomes = 4.

(*i*) The outcome favourable to event 'two heads' is (H, H).

$\therefore$ P (two heads) = $\frac{1}{4}$.

(*ii*) The outcomes favourable to the even 'one head' are (H, T) and (T, H).

$\therefore$ P (one head) = $\frac{2}{4} = \frac{1}{2}$.

(*iii*) The outcome favourable to the event 'no head' is (T, T)

$\therefore$ P (no head) = $\frac{1}{4}$.

(*iv*) The outcomes favourable to the event 'atleast one head' are (H, T), (T, H) and (H, H).

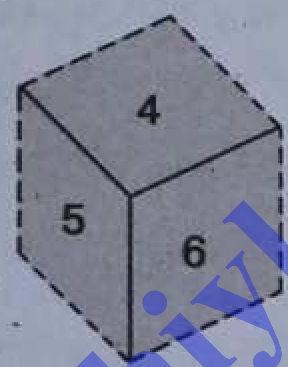
$\therefore$ P (atleast one head) = $\frac{3}{4}$.

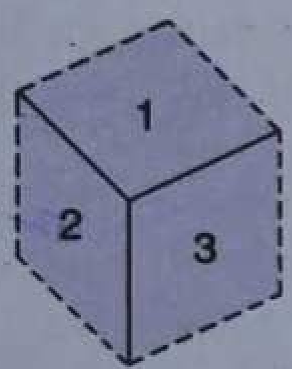
(*v*) The outcomes favourbale to the event 'atmost one head' are (H, T), (T, H) and (T, T).

$\therefore$ P (atmost one head) = $\frac{3}{4}$.

A Note on Throwing Two Different Dice Simultaneously

Let two different dice (say, one red and other grey) be thrown simultaneously. When the red die shows 1, then the grey die could show any of the number 1, 2, 3, 4, 5, 6. The same is true when the red die shows '2', '3', '4', '5' or '6'. The possible oucomes of the experiment are listed in the table below ; the first number in each ordered pair is the number appearing on the red die and the second number is that of the grey die.





	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Note that the pair (1, 2) is different from (2, 1).
The total number of possible outcomes = 6 × 6 = 36.
All the 36 outcomes are equally likely.
The outcomes (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6,) are called **doublets**.

Example 15. *Two different dice are thrown simultaneously. What is the probability that the sum of two numbers appearing on the top of dice is*

(*i*) *9* (*ii*) *10* (*iii*) *atleast 10* (*iv*) *13* (*v*) *less than or equal to 12*
(*vi*) *a multiple of 2 on one die and a multiple of 3 on the other die.*

Solution. When two different dice are thrown, the total number of outcomes is 36 and all the outcomes are equally likely.

(*i*) The outcomes favourable to the event 'sum of two number is 9' are (6, 3), (5, 4), (4, 5) and (3, 6). these are 4 in number.

$\therefore$ P (sum of 9) = $\frac{4}{36} = \frac{1}{9}$.

(*ii*) The outcomes favourable to the event 'sum of two numbers is 10' are (6, 4), (5, 5) and (4, 6). These are 3 in number.

$\therefore$ P (sum of 10) = $\frac{3}{36} = \frac{1}{12}$.

(*iii*) The sum of atleast 10 means that sum is 10, 11 or 12. Therefore, the outcomes favourable to the event 'sum of atleast 10' are (6, 4), (5, 5), (4, 6), (6, 5), (5, 6) and (6, 6). These are 6 in number.

$\therefore$ P (sum of atleast 10) = $\frac{6}{36} = \frac{1}{6}$.

(*iv*) As the sum of two numbers appearing on the top of two dice can never be 13, there is no outcome favourable to the event 'sum of two numbers is 13'.

$\therefore$ P (sum of 13) = $\frac{0}{36}$ = 0.

(*v*) As the sum of two numbers appearing on the top of two dice is always less than or equal to 12, all the 36 outcomes are favourable to the event 'sum is less than or equal to 12'.

$\therefore$ P (sum is less than or equal to 12) = $\frac{36}{36}$ = 1.

(*vi*) The outcomes favourable to the event 'a multiple of 2 on one die and a multiple of 3 on the other die' are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2) and (6, 4).

$\therefore$ The number of outcomes favourable to the given event = 11.

$\therefore$ Required probability = $\frac{11}{36}$.

Exercise 24

1. A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Anjali takes out a ball from the bag without looking into it. What is the probability that she takes out

   (*i*) yellow ball ? (*ii*) red ball ? (*iii*) blue ball ?

2. A box contains 600 screws, one-tenth are rusted. One screw is taken out at random from this box. Find the probability that it is a good screw.
3. In a lottery, there are 5 prized tickets and 995 blank tickets. A person buys a lottery ticket. Find the probability of his winning a prize.
4. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
5. If the probability of winning a game is $\frac{5}{11}$, what is the probability of losing?
6. Two players, Sania and Sonali, play a tennis match. It is known that the probability of Sania winning the match is 0·69. What is the probability of Sonali winning ?
7. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is

   (*i*) red ? (*ii*) not red ?

8. There are 40 students in Class X of a school of which 25 are girls and the others are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of

(*i*) a girl ? (*ii*) a boy ?

9. A letter is chosen from the word 'TRIANGLE'. What is the probability that it is a vowel?

10. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be

(*i*) red ? (*ii*) white ? (*iii*) not green ?

11 A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is

(*i*) red (*ii*) black or white (*iii*) not black.

12. A bag contains 5 red, 6 black and 4 white balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is

(*i*) white (*ii*) not black

(*iii*) red or black (*iv*) neither red nor black.

13. If 65% of the population have black eyes, 25% have brown eyes and the remaining have blue eyes, what is the probability that a person selected at random has

(*i*) blue eyes (*ii*) brown or black eyes

(*iii*) blue or black eyes (*iv*) neither blue nor brown eyes?

14. A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the ball is :

(*i*) white (*ii*) red or black

(*iii*) not green (*iv*) neither white nor black.

15. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Peter, a trader, will only accept the shirts which are good, but Salim, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that

(*i*) it is acceptable to Peter ?(*ii*)it is acceptable to Salim ?

16. A die is thrown once. What is the probability that the

(*i*) number is even

(*ii*) number is greater than 2 ? (*2009*)

17. In a single throw of a die, find the probability of getting :

(*i*) an odd number (*ii*) a number less than 5

(*iii*) a number greater than 5 (*iv*) a prime number

(*v*) a number less than 8 (*vi*) a number divisible by 3

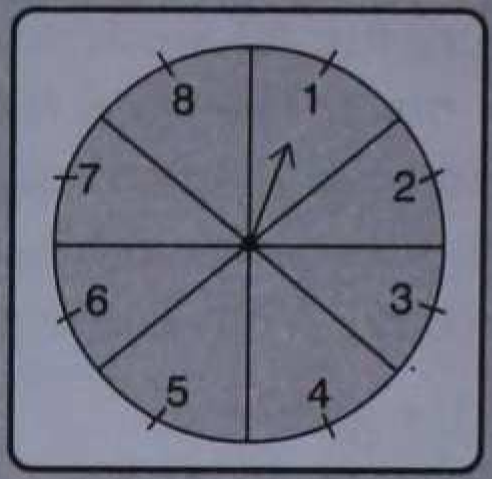
(*vii*) a number between 3 and 6 (*viii*) a number divisible by 2 or 3.

18. A child has a die whose six faces show the letters as given below :

A B C D E A

The die is thrown once. What is the probability of getting (*i*) A ? (*ii*) D ?

19. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (shown in the adjoining figure) and these are equally likely outcomes. What is the probability that it will point at



(*i*) 8 ?

(*ii*) an odd number ?

(*iii*) a number greater than 2 ?

(*iv*) a number less than 9 ?

20. Find the probability that the month of January may have 5 Mondays in

(*i*) a leap year (*ii*) a non-leap year.

21. Find the probability that the month of February may have 5 Wednesdays in

(*i*) a leap year (*ii*) a non-leap year.

22. Cards marked with numbers 1, 2, 3, 4, ..., 20 are well-shuffled and a card is drawn at random. What is the probability that the number on the card is

(*i*) a prime number (*ii*) divisible by 3 (*iii*) a perfect square? *(2010)*

23. A box contains 25 cards, numbered from 1 to 25. A card is drawn from the box at random. Find the probability that the number on the card is :

(*i*) even (*ii*) prime (*iii*) multiple of 6.

24. A box contains 15 cards numbered 1, 2, 3, ..., 15 which are mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the card is :

(*i*) odd (*ii*) prime

(*iii*) divisible by 3 (*iv*) divisible by 3 and 2 both

(*v*) divisible by 3 or 2 (*vi*) a perfect square number.

25. A box contains 19 balls bearing numbers 1, 2, 3, ..., 19. A ball is drawn at random from the box. Find the probability that the number on the ball is :

(*i*) a prime number (*ii*) divisible by 3 or 5

(*iii*) neither divisible by 5 nor by 10 (*iv*) an even number.

26. Tickets numbered 3, 5, 7, 9, ..., 29 are placed in a box and mixed thoroughly. One ticket is drawn at random from the box. Find the probability that the number on the ticket is

(*i*) a prime number (*ii*) a number less than 16

(*iii*) a number divisible by 3.

27. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(*i*) a two-digit number (*ii*) a perfect square number

(*iii*) a number divisible by 5.

28. Cards marked with numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn at random from this box. Find the probability that the number on the card is

(*i*) an even number

(*ii*) a number less than 14

(*iii*) a number which is a perfect square

(*iv*) a prime number less than 30.

29. A bag contains 15 balls of which some are white and others are red. If the probability of drawing a red ball is twice that of a white ball, find the number of white balls in the bag.

30. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball is twice that of a red ball, find the number of blue balls in the bag.

31. A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. *(2013)*

32. A jar contains 24 marbles; some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

33. A card is drawn from a well-shuffled pack of 52 cards. Find the probability of getting :

(*i*) '2' of spades (*ii*) a jack
(*iii*) a king of red colour (*iv*) a card of diamond
(*v*) a king or a queen (*vi*) a non-face card
(*vii*) a black face card (*viii*) a black card
(*ix*) a non-ace (*x*) non-face card of black colour
(*xi*) neither a spade nor a jack (*xii*) neither a heart nor a red king.

34. The king, queen and jack of clubs are removed from a deck of 52 playing cards and then shuffled. A card is drawn from the remaining deck. Find the probability of getting :

(*i*) a heart (*ii*) a queen
(*iii*) a club (*iv*) '9' of red colour.

35. Two coins are tossed once. Find the probability of getting :

(*i*) 2 heads (*ii*) atleast one tail. *(2012)*

36. Two different coins are tossed simultaneously. Find the probability of getting :

(*i*) two tails (*ii*) one tail
(*iii*) no tail (*iv*) atmost one tail.

37. Two different dice are thrown at the same time. Find the probability of getting :

(*i*) a doublet (*ii*) a sum of 8
(*iii*) sum divisble by 5 (*iv*) sum of atleast 11.

CHAPTER TEST

1. A lot consists of 144 ball pens of which 20 are defective and the others are good. Rohana will buy a pen if it is good, but will not buy it if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
   (*i*) She will buy it ? (*ii*) She will not buy it ?
2. A letter is chosen at random from the letter of English alphabet. Find the probability that the letter chosen is a
   (*i*) vowel (*ii*) consonant
   (*iii*) a letter of the word SHUNTED.
3. A bag contains 6 red, 5 black and 4 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is
   (*i*) white (*ii*) red
   (*iii*) not black (*iv*) red or white.
4. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn from the bag at random. Find the probability that the drawn ball is
   (*i*) red or white (*ii*) not black (*iii*) neither white nor black.
5. A bag contains 5 white balls, 7 red balls, 4 black balls and 2 blue balls. One ball is drawn at random from the bag. What is the probability that the ball drawn is :
   (*i*) white or blue (*ii*) red or black
   (*iii*) not white (*iv*) neither white nor black?
6. A box contains 20 balls bearing numbers 1, 2, 3, 4, ..., 20. A ball is drawn at random from the box. What is the probability that the number on the ball is
   (*i*) an odd number (*ii*) divisible by 2 or 3
   (*iii*) prime number (*iv*) not divisible by 10?
7. Find the probability that a number selected at random from the numbers 1, 2, 3, ..., 35 is a
   (*i*) prime number (*ii*) multiple of 7
   (*iii*) multiple of 3 or 5.
8. Cards marked with numbers 13, 14, 15, ..., 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the card is
   (*i*) divisible by 5 (*ii*) a number which is a perfect square.
9. A box has cards numbered 14 to 99. Cards are mixed thoroughly and a card is drawn at random from the box. Find the probability that the card drawn from the box has
   (*i*) an odd number (*ii*) a perfect square number.
10. Find the probability of getting 53 Fridays in a leap year.
11. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is four times that of a red ball, find the number of balls in the bags.
12. A card is drawn from a well-shuffled pack of 52 cards. Find the probability that the card drawn is :
    (*i*) a red face card
    (*ii*) neither a club nor a spade

(*iii*) neither an ace nor a king of red colour

(*iv*) neither a red card nor a queen

(*v*) neither a red card nor a black king.

13. All kings, jacks and diamonds have been removed from a pack of 52 playing cards and the remaining cards are well shuffled. A card is drawn from the remaining pack. Find the probability that the card drawn is :

(*i*) a red queen (*ii*) a face card

(*iii*) a black card (*iv*) a heart.

14. Two different dice are thrown simultaneously. Find the probability of getting :

(*i*) six as a product (*ii*) sum ≤ 3 (*iii*) sum ≤ 10.

15. Three coins are tossed simultaneously. Find the probability of getting :

(*i*) three heads (*ii*) two heads

(*iii*) one head (*iv*) atleast one head

(*v*) atleast two heads (*vi*) atmost two heads.

Hint

Possible outcomes are :

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.