

QUADRILATERALS

A closed plane figure bounded by four line segments is called a **quadrilateral**.

In the adjoining diagram, ABCD is a quadrilateral.

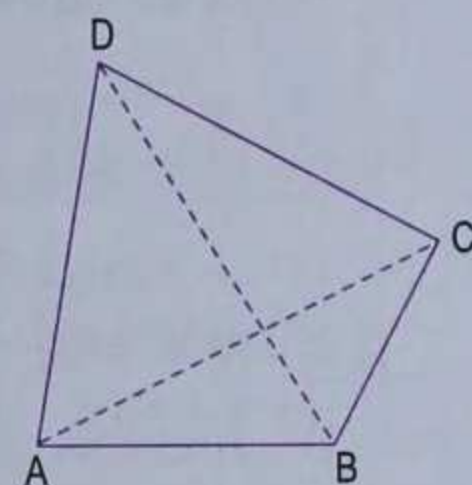
It has

four sides — AB, BC, CD and DA

four (interior) angles — $\angle A$, $\angle B$, $\angle C$ and $\angle D$

four vertices — A, B, C and D

two diagonals — AC and BD.



Sum of (interior) angles of a quadrilateral is 360°

In the adjoining figure, ABCD is *any* quadrilateral. Diagonal AC divides it into two triangles. We know that the sum of angles of a triangle is 180° ,

in $\triangle ABC$, $\angle 1 + \angle B + \angle 2 = 180^\circ$...*(i)*

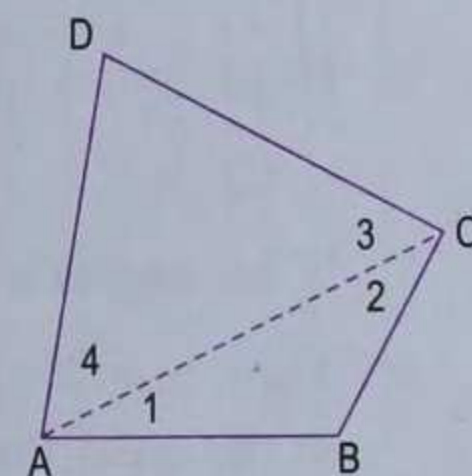
in $\triangle ACD$, $\angle 4 + \angle D + \angle 3 = 180^\circ$...*(ii)*

On adding *(i)* and *(ii)*, we get

$$\angle 1 + \angle 4 + \angle B + \angle D + \angle 2 + \angle 3 = 360^\circ$$

$$\Rightarrow \angle A + \angle B + \angle D + \angle C = 360^\circ \quad (\text{from figure})$$

Hence, the sum of (interior) angles of a quadrilateral is 360° .



Example 1. From the adjoining diagram, calculate the value of x .

Solution.

As the sum of (interior) angles of a quadrilateral is 360° ,

$$90^\circ + 110^\circ + 83^\circ + \angle ABC = 360^\circ$$

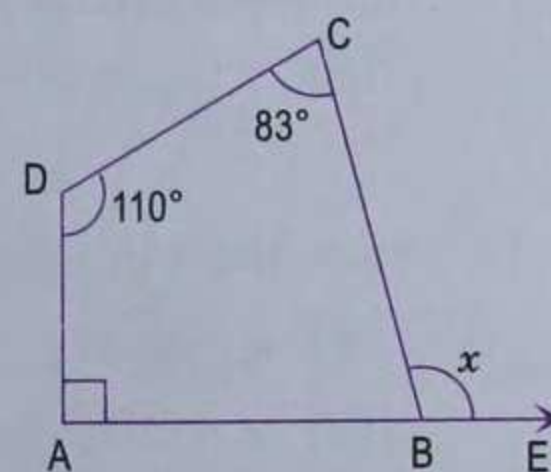
$$\Rightarrow \angle ABC = 360^\circ - 90^\circ - 110^\circ - 83^\circ = 77^\circ.$$

As ABE is a straight line,

$$x + \angle ABC = 180^\circ$$

$$\Rightarrow x = 180^\circ - 77^\circ$$

$$\Rightarrow x = 103^\circ.$$



($\because \angle ABC = 77^\circ$)

Example 2. If the angles of a quadrilateral are in the ratio 5 : 8 : 11 : 12, find the angles.

Solution.

Since the angles of the quadrilateral are in the ratio 5 : 8 : 11 : 12, let these angles be $5x$, $8x$, $11x$ and $12x$.

As the sum of angles of a quadrilateral is 360° ,

$$5x + 8x + 11x + 12x = 360^\circ$$

$$\Rightarrow 36x = 360^\circ$$

$$\Rightarrow x = 10^\circ$$

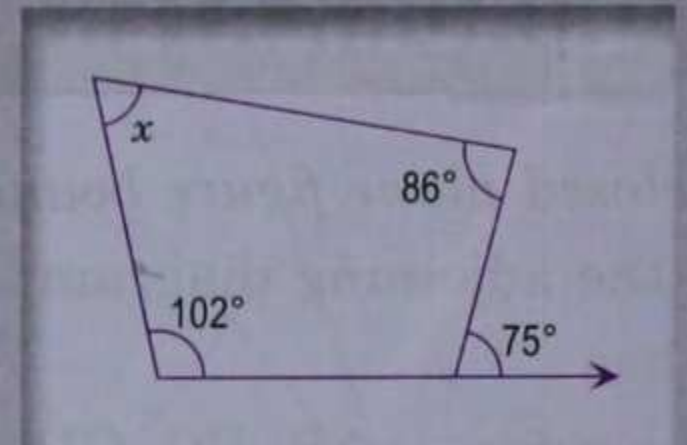
\therefore The angles of the quadrilateral are

$5 \times 10^\circ, 8 \times 10^\circ, 11 \times 10^\circ$ and $12 \times 10^\circ$ i.e. $50^\circ, 80^\circ, 110^\circ$ and 120° .

Exercise 23.1

1. If three angles of a quadrilateral are $70^\circ, 83^\circ$ and 112° , find the fourth angle.

2. From the adjoining diagram, find the value of x .



3. If two angles of a quadrilateral are 76° and 138° and the other two angles are equal, find the measure of equal angles.
4. A quadrilateral has three interior angles each equal to 95° . Find the size of the fourth interior angle.

5. If one of the angles of a quadrilateral is 210° and the remaining three angles are equal, find the measure of the equal angles.

Note. It is a re-entrant quadrilateral.

6. If the angles of a quadrilateral are $x^\circ, (x - 20)^\circ, (x - 30)^\circ$ and $(x + 10)^\circ$, find

(i) x (ii) the angles of the quadrilateral.

7. If the angles of a quadrilateral are in the ratio $2 : 3 : 4 : 6$, find the angles.

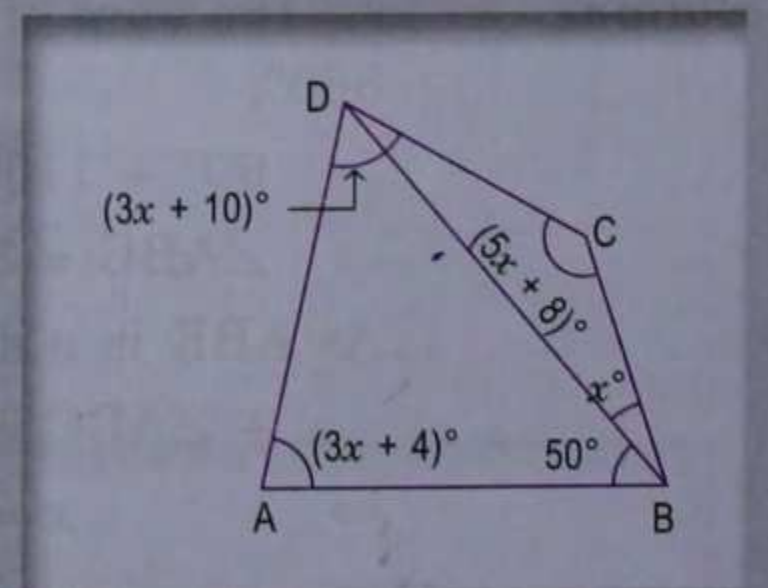
8. Three angles of a quadrilateral are in the ratio $3 : 5 : 6$. If the fourth angle is 80° , find the other angles of the quadrilateral.

9. Two angles of a quadrilateral are 78° and 87° . If the other two angles are in the ratio $5 : 8$, find the size of each of them.

10. In a quadrilateral ABCD, $AB \parallel DC$. If $\angle A : \angle D = 2 : 3$ and $\angle B : \angle C = 7 : 8$, find the measure of each angle.

11. From the adjoining figure, find

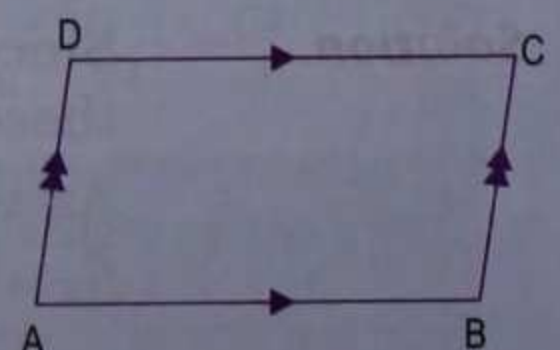
(i) x (ii) $\angle DAB$
(iii) $\angle ADB$



PARALLELOGRAM

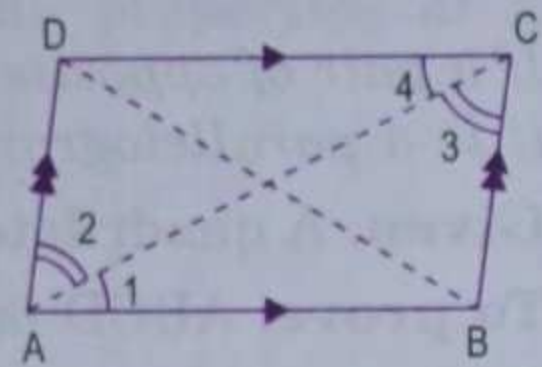
A quadrilateral in which both pairs of opposite sides are parallel is called a **parallelogram**.

In the adjoining quadrilateral, $AB \parallel DC$ and $AD \parallel BC$, so ABCD is a parallelogram.



Theorem 1

- (i) The opposite sides of a parallelogram are equal.
- (ii) The opposite angles of a parallelogram are equal.
- (iii) Each diagonal bisects the parallelogram.



Given. A parallelogram ABCD.

- To prove.**
- (i) $AB = DC$ and $AD = BC$
 - (ii) $\angle B = \angle D$ and $\angle A = \angle C$
 - (iii) Area of $\triangle ABC =$ area of $\triangle ACD$
and area of $\triangle ABD =$ area of $\triangle BCD$

Construction. Join AC and BD.

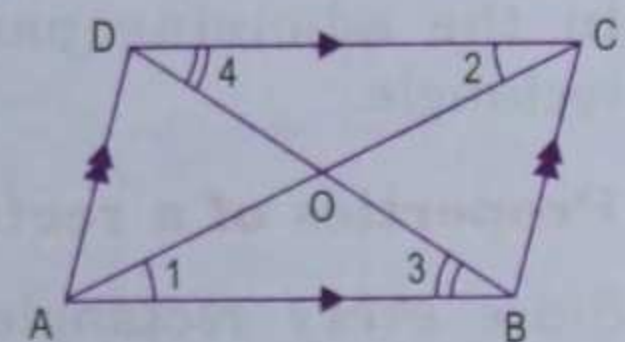
Proof.

Statements	Reasons
In $\triangle ABC$ and $\triangle CDA$	
1. $\angle 1 = \angle 4$	1. Alt. \angle s, as $AB \parallel DC$ and AC cuts them
2. $\angle 3 = \angle 2$	2. Alt. \angle s, as $AD \parallel BC$ and AC cuts them
3. $AC = AC$	3. Common
4. $\triangle ABC \cong \triangle CDA$	4. A.S.A. axiom of congruency
(i) $AB = DC$ and $AD = BC$	'c.p.c.t.'
(ii) $\angle B = \angle D$ and $\angle A = \angle C$	'c.p.c.t.'
(iii) area of $\triangle ABC =$ area of $\triangle CDA$	Adding 1 and 2, $\angle 1 + \angle 2 = \angle 3 + \angle 4$ $\triangle ABC \cong \triangle CDA$ and congruent triangles have equal area.
$= \frac{1}{2}$ (area of parallelogram ABCD)	
\Rightarrow AC bisects parallelogram ABCD	
Similarly, $\triangle ABD \cong \triangle CDB$	
\Rightarrow area of $\triangle ABD =$ area of $\triangle CDB$	
\Rightarrow BD bisects parallelogram ABCD	
Q.E.D.	

Theorem 2

The diagonals of a parallelogram bisect each other.

Given. A parallelogram ABCD whose diagonals AC and BD intersect at O.



To prove. $OA = OC$ and $OB = OD$.

Proof.

Statements	Reasons
In $\triangle OAB$ and $\triangle OCD$	
1. $\angle 1 = \angle 2$	1. Alt. \angle s, as $AB \parallel DC$ and AC cuts them
2. $\angle 3 = \angle 4$	2. Alt. \angle s, as $AB \parallel DC$ and BD cuts them
3. $AB = DC$	3. Opp. sides of a \parallel gm are equal, Theorem 1
4. $\triangle OAB \cong \triangle OCD$	4. A.S.A. axiom of congruency
$\therefore OA = OC$ and $OB = OD$	'c.p.c.t.'
Q.E.D.	

Theorem 3

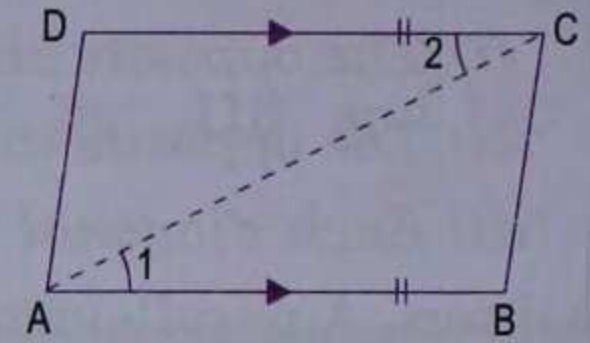
If a pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.

Given. A quadrilateral ABCD in which $AB \parallel DC$ and $AB = DC$.

To prove. ABCD is a parallelogram.

Construction. Join AC.

Proof.



Statements	Reasons
In $\triangle ABC$ and $\triangle CDA$	
1. $\angle 1 = \angle 2$	1. Alt. \angle s, as $AB \parallel DC$ and AC cuts them
2. $AB = DC$	2. Given
3. $AC = AC$	3. Common
4. $\triangle ABC \cong \triangle CDA$	4. S.A.S. (axiom of congruency)
5. $\angle ACB = \angle CAD$	5. 'c.p.c.t.'
6. $AD \parallel BC$	6. AC cuts AD and BC, and alt. \angle s are equal
Hence, ABCD is a parallelogram	By definition
Q.E.D.	

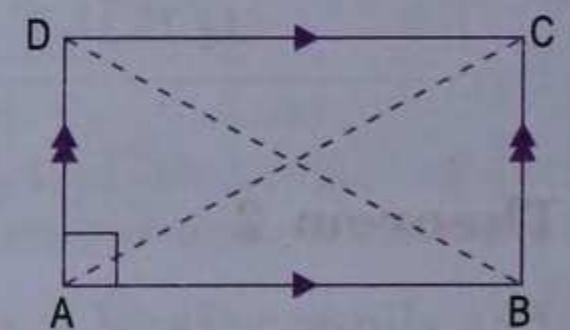
Properties of a parallelogram

- Both pairs of opposite sides are parallel (by definition).
- Both pairs of opposite sides are equal.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other.
- Each diagonal bisects the parallelogram.

Some special parallelograms**Rectangle**

If one angle of a parallelogram is a right angle then it is called a **rectangle**.

In the adjoining parallelogram ABCD, $\angle A = 90^\circ$, so it is a rectangle.

**Properties of a rectangle**

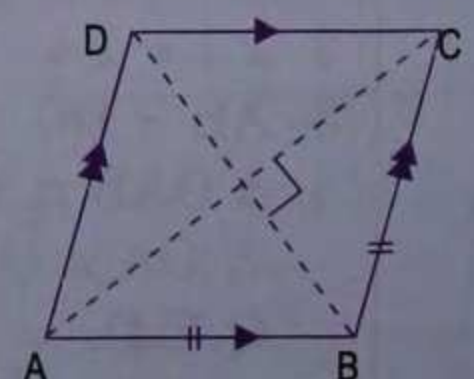
Since every rectangle is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are:

- All the (interior) angles of a rectangle are right angles.
In the above diagram, $\angle A = \angle B = \angle C = \angle D = 90^\circ$.
- The diagonals of a rectangle are equal.
In the above diagram, $AC = BD$.

Rhombus

If two adjacent sides of a parallelogram are equal, then it is called a **rhombus**.

In the adjoining parallelogram, $AB = BC$, so ABCD is a rhombus.



Properties of rhombus

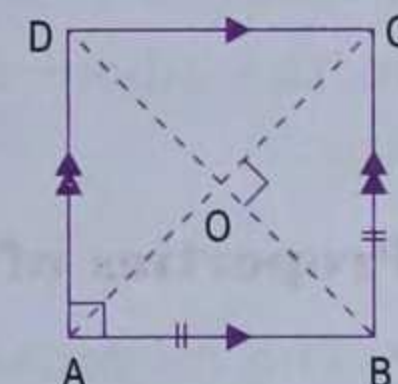
Since every rhombus is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are :

- All the sides of a rhombus are equal.
In the above diagram, $AB = BC = CD = DA$.
- The diagonals of a rhombus intersect at right angles.
In the above diagram, $AC \perp BD$.
- The diagonals bisect the angles of a rhombus.
In the above diagram, diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Square

If two adjacent sides of a rectangle are equal, then it is called a **square**. Alternatively, if one angle of a rhombus is a right angle, then it is called a **square**.

In the adjoining rectangle, $AB = BC$, so $ABCD$ is a square.



Properties of a square

Since every square is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a square are :

- All the interior angles of a square are right angles.
In the above diagram, $\angle A = \angle B = \angle C = \angle D = 90^\circ$.
- All the sides of a square are equal.
In the above diagram, $AB = BC = CD = DA$.
- The diagonals of a square are equal.
In the above diagram, $AC = BD$.
- The diagonals of a square intersect at right angles.
In the above diagram, $AC \perp BD$.
- The diagonals bisect the angles of a square.
In the above diagram, diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

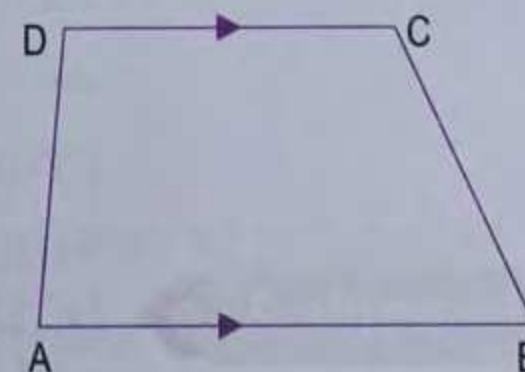
In fact, a square is a rectangle as well as rhombus, so it has all the properties of a rectangle as well as that of a rhombus.

Trapezium

A quadrilateral in which one pair of opposite sides is parallel is called a **trapezium**.

The parallel sides are called the **bases** of the trapezium.

In the adjoining quadrilateral, $AB \parallel DC$, so $ABCD$ is a trapezium.



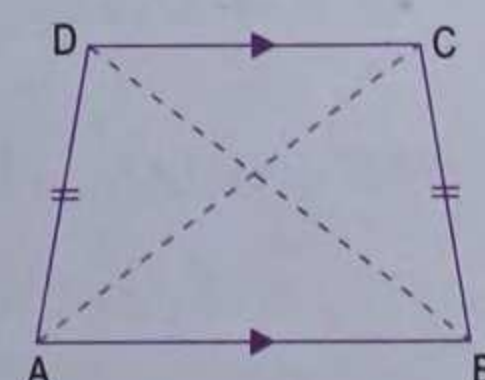
Property of a trapezium

- Co-interior angles of a trapezium are supplementary angles.
In the above diagram, $\angle A + \angle D = 180^\circ$ and $\angle B + \angle C = 180^\circ$.

Isosceles trapezium

If two non-parallel sides of a trapezium are equal then it is called an **isosceles trapezium**.

In the adjoining quadrilateral, $AB \parallel DC$ and $AD = BC$, so $ABCD$ is an isosceles trapezium.



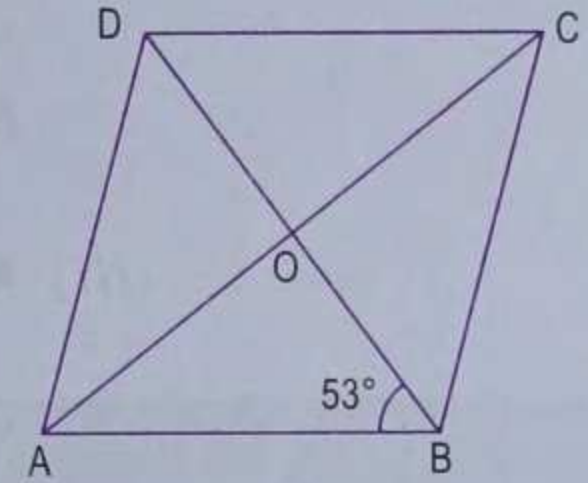
$$\begin{aligned} \Rightarrow & \quad AO = OB \\ \Rightarrow & \quad \angle ABO = \angle OAB \quad (\text{angles opp. equal sides in } \triangle OAB) \\ \Rightarrow & \quad \angle ABO = 30^\circ. \quad (\because \angle OAB = 30^\circ \text{ given}) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \angle AOB + 30^\circ + 30^\circ &= 180^\circ \\ & \text{(sum of angles in } \triangle AOB) \\ \Rightarrow \quad \angle AOB &= 180^\circ - 30^\circ - 30^\circ = 120^\circ \\ \quad \angle COD &= \angle AOB = 120^\circ. \quad (\text{vert. opp. } \angle s) \\ \text{(iv)} \quad \angle BOC + 120^\circ &= 180^\circ \quad (\text{angles on a straight line}) \\ \Rightarrow \quad \angle BOC &= 180^\circ - 120^\circ = 60^\circ. \end{aligned}$$

Example 3.

In the adjoining rhombus ABCD, diagonals intersect at O. If $\angle ABO = 53^\circ$, find

- (i) $\angle OAB$
- (ii) $\angle ADC$
- (iii) $\angle BCD$.



Solution.

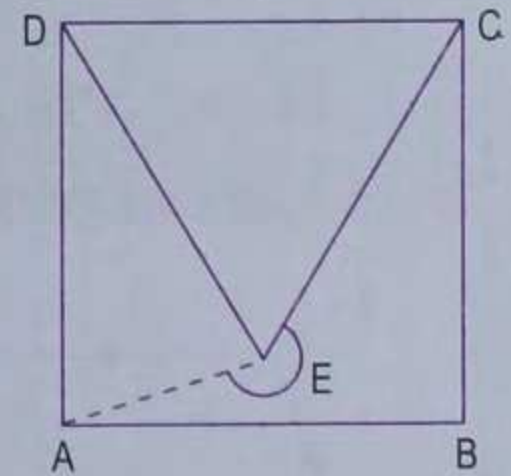
Given, ABCD is a rhombus.

$$\begin{aligned} \text{(i)} \quad \angle AOB &= 90^\circ \quad (\text{diagonals intersect at right angles}) \\ \quad \angle OAB + 53^\circ + 90^\circ &= 180^\circ \quad (\text{sum of angles in } \triangle OAB) \\ \Rightarrow \quad \angle OAB &= 180^\circ - 53^\circ - 90^\circ = 37^\circ. \\ \text{(ii)} \quad \text{As diagonal BD bisects } \angle ABC, \\ \quad \angle ABC &= 2 \angle ABO = 2 \times 53^\circ = 106^\circ \\ \therefore \quad \angle ADC &= \angle ABC = 106^\circ. \quad (\text{opp. } \angle s \text{ are equal}) \\ \text{(iii)} \quad \angle BCD + 106^\circ &= 180^\circ \quad (\text{AD} \parallel \text{BC, sum of co-int. } \angle s = 180^\circ) \\ \Rightarrow \quad \angle BCD &= 180^\circ - 106^\circ = 74^\circ. \end{aligned}$$

Example 4.

In the adjoining figure, ABCD is a square and CDE is an equilateral triangle. Find

- (i) $\angle AED$
- (ii) $\angle EAB$
- (iii) reflex $\angle AEC$.



Solution.

Given, ABCD is a square and CDE is an equilateral triangle. We know that each angle in a square = 90° and each angle in an equilateral triangle is 60° .

$$\begin{aligned} \text{(i)} \quad \text{From figure, } \angle ADE &= 90^\circ - 60^\circ = 30^\circ \\ \quad ED &= DC \quad (\text{sides of an equilateral triangle}) \\ \quad AD &= DC \quad (\text{sides of a square}) \\ \Rightarrow \quad ED &= AD \\ \Rightarrow \quad \angle DAE &= \angle AED \quad (\text{angles opp. equal sides in } \triangle AED) \\ \quad \angle DAE + \angle AED + \angle ADE &= 180^\circ \quad (\text{sum of angles in } \triangle AED) \\ \Rightarrow \quad 2 \angle AED &= 180^\circ - 30^\circ = 150^\circ \quad (\because \angle ADE = 30^\circ) \\ \Rightarrow \quad \angle AED &= 75^\circ. \\ \text{(ii)} \quad \angle EAB &= 90^\circ - 75^\circ = 15^\circ. \quad (\because \angle DAE = \angle AED = 75^\circ) \\ \text{(iii)} \quad \angle AEC &= \angle AED + \angle DEC = 75^\circ + 60^\circ = 135^\circ \\ \therefore \quad \text{Reflex } \angle AEC &= 360^\circ - 135^\circ = 225^\circ. \end{aligned}$$

Also $\angle CEB = \angle A$ ($\because CE \parallel DA$. corres. \angle s are equal)
 $\Rightarrow \angle CEB = 60^\circ$ ($\because \angle A = 60^\circ$ given)
 $\angle B = \angle A$ (\because In an isosceles trap., base angles are equal)
 $\Rightarrow \angle B = 60^\circ$ ($\because \angle A = 60^\circ$ given)
 $\angle ECB + \angle CEB + \angle B = 180^\circ$ (sum of angles of a $\Delta = 180^\circ$)
 $\Rightarrow \angle ECB + 60^\circ + 60^\circ = 180^\circ$
 $\Rightarrow \angle ECB = 60^\circ$
 $\Rightarrow \angle CEB$ is an equilateral triangle.
 $\therefore EB = BC$
 $\Rightarrow EB = 15 \text{ cm}$ ($\because BC = AD = 15 \text{ cm}$ given)

From figure,

$$AB = AE + EB = 20 \text{ cm} + 15 \text{ cm} = 35 \text{ cm.}$$

Hence, length of $AB = 35 \text{ cm.}$

Exercise 23.2

1. State whether the following statements are true or false :

- | | |
|--|--|
| (i) Every rectangle is a rhombus. | (ii) Every square is a rhombus. |
| (iii) Every square is a rectangle. | (iv) Every square is a parallelogram. |
| (v) Every rectangle is a square. | (vi) Every rectangle is a parallelogram. |
| (vii) Every rhombus is a square. | (viii) Every rhombus is a parallelogram. |
| (ix) Every parallelogram is a rhombus. | |

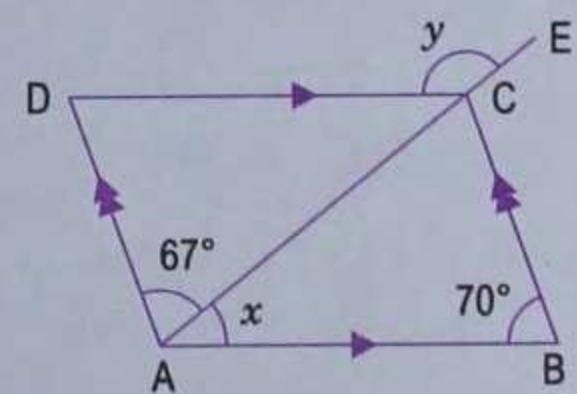
2. In a parallelogram $ABCD$, $\angle A = (4x - 5)^\circ$ and $\angle C = (3x + 10)^\circ$. Find $\angle A$ and $\angle B$.

3. If in a square $ABCD$, $AB = (2x + 3) \text{ cm}$ and $BC = (3x - 5) \text{ cm}$, find BD .

[Hint. $BD^2 = AB^2 + AD^2$ by Pythagoras theorem, $AD = BC$.]

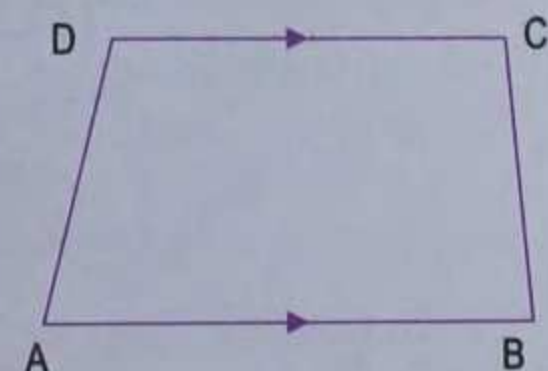
4. If the ratio of two conjoined angles of a parallelogram is $5 : 7$, find the angles of the parallelogram.

5. In the adjoining figure, $ABCD$ is a parallelogram. Find the values of x and y .



6. In the adjoining figure, $ABCD$ is a trapezium.

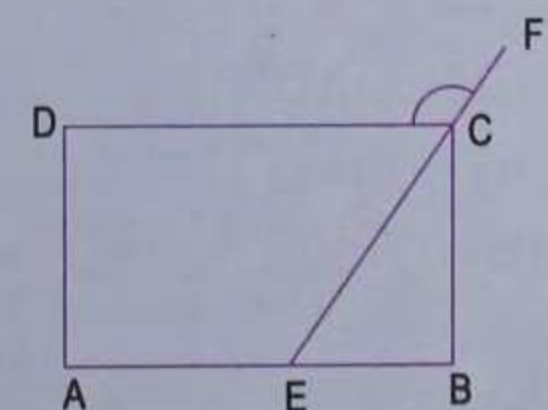
If $\angle A : \angle D = 5 : 7$, $\angle B = (3x + 11)^\circ$ and $\angle C = (5x - 31)^\circ$, then find all the angles of the trapezium.



7. In the adjoining figure, $ABCD$ is a rectangle.

If $\angle CEB : \angle ECB = 3 : 2$, find

- (i) $\angle CEB$ (ii) $\angle DCF$.



8. In the adjoining figure, ABCD is a rectangle and diagonals intersect at O. If $\angle AOB = 118^\circ$, find

(i) $\angle ABO$ (ii) $\angle ADO$
 (ii) $\angle OCB$.

9. In the adjoining figure, ABCD is a rhombus and $\angle ABD = 50^\circ$. Find

(i) $\angle CAB$ (ii) $\angle BCD$
 (iii) $\angle ADC$.

10. In the adjoining figure, ABCD is a parallelogram and diagonals intersect at O. Find

(i) $\angle CAD$ (ii) $\angle ACD$
 (iii) $\angle ADC$.

11. In the adjoining figure, ABCD is a parallelogram and diagonals intersect at O. Prove that O is mid-point of PQ.

[Hint. Show that $\triangle AOQ \cong \triangle COP$.]

12. In the adjoining figure, ABCD is a rhombus and its diagonals intersect at O. Prove that

(i) the diagonals bisect each other.
 (ii) the diagonals are at right angles.

[Hint. (i) Show that $\triangle AOB \cong \triangle COD$

(ii) Show that $\triangle AOB \cong \triangle COB$.]

13. In the adjoining figure, ABCD is a parallelogram and $AP \parallel CQ$. Prove that

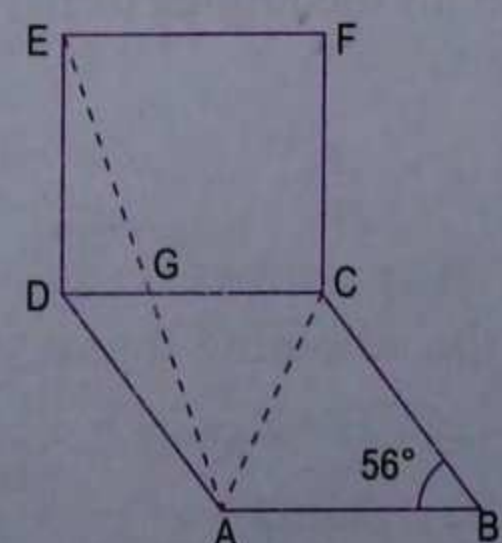
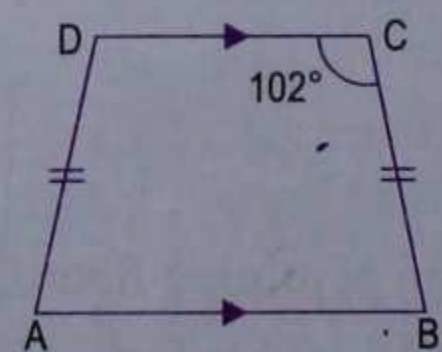
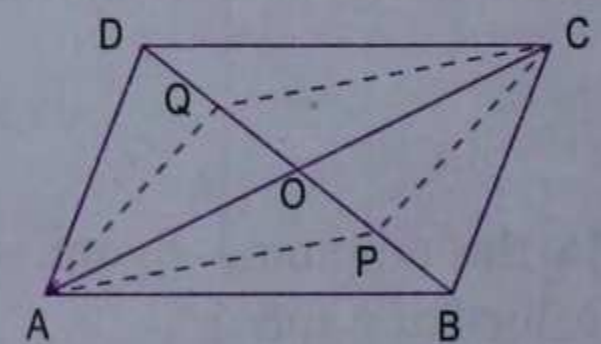
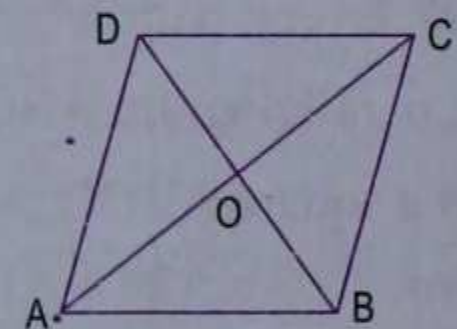
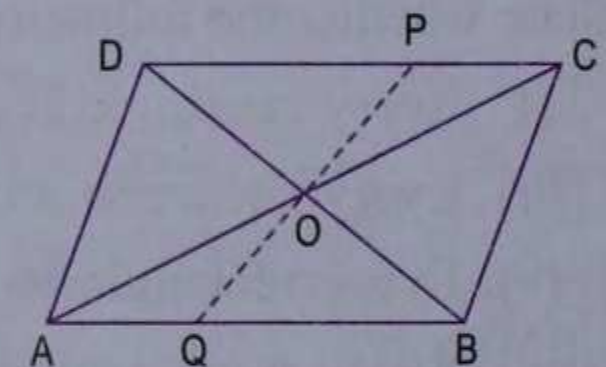
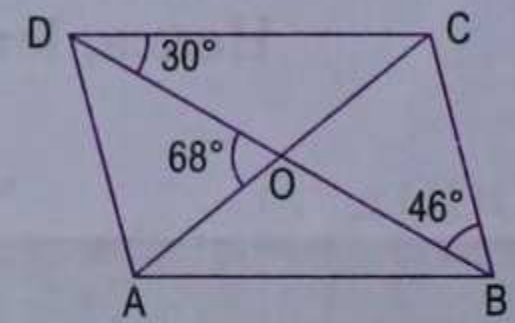
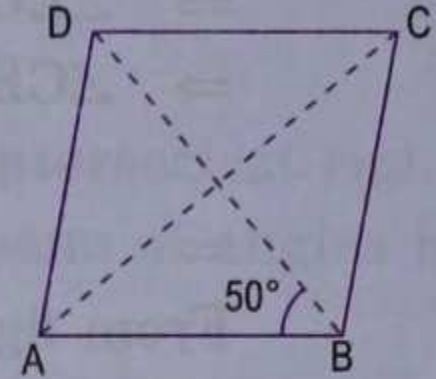
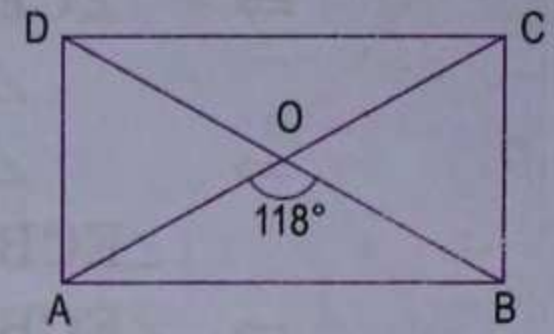
(i) $\triangle OAP \cong \triangle OCQ$ (ii) $AP = CQ$
 (iii) APCQ is a parallelogram.

14. In the adjoining isosceles trapezium ABCD, $\angle C = 102^\circ$. Find all the remaining angles of the trapezium.

15. In the adjoining figure, ABCD is a rhombus and DCFE is a square. If $\angle ABC = 56^\circ$, find

(i) $\angle DAG$ (ii) $\angle FEG$
 (iii) $\angle GAC$ (iv) $\angle AGC$.

[Hint. (i) $\angle EDA = 90^\circ + 56^\circ = 146^\circ$, $ED = AD$.]



POLYGONS

A closed plane figure bounded by line segments is called a **polygon**.

The line segments are called its **sides** and the points of intersection of consecutive sides are called its **vertices**. An angle formed by two consecutive sides of a polygon inside the polygon is called an **interior angle** or simply an **angle** of the polygon.

A polygon has the same number of angles as it has sides. A polygon is named according to the number of sides/angles it has :

Number of sides / angles	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon/Septagon
8	Octagon
9	Nonagon
10	Decagon

In general, a polygon having n sides is called **n -sided polygon** or **n -gon**. Thus, a polygon having 20 sides is called 20-gon.

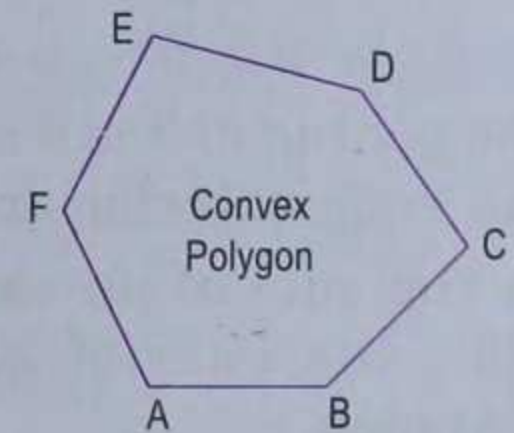
Diagonal of a polygon

Line segment joining any two non-consecutive vertices of a polygon is called its **diagonal**.

Convex polygon

If all the (interior) angles of a polygon are less than 180° , it is called a **convex polygon**.

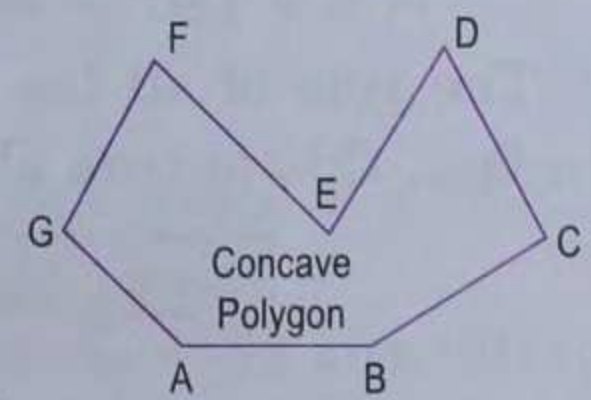
In the adjoining figure, ABCDEF is a convex polygon. In fact, it is a convex hexagon.



Concave polygon

If one or more of the (interior) angles of a polygon is greater than 180° i.e. reflex, it is called a **concave (or re-entrant) polygon**.

In the adjoining figure, ABCDEFG is a concave (or re-entrant) polygon. In fact, it is a concave heptagon.



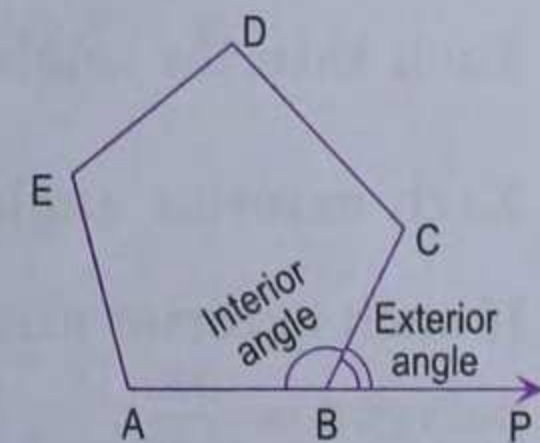
Exterior angle of a convex polygon

If we produce a side of a convex polygon, the angle it makes with the next side is called an **exterior angle**.

In the adjoining figure, ABCDE is a pentagon. Its side AB has been produced to P, then $\angle CBP$ is an exterior angle.

Notice that corresponding to each interior angle, there is an exterior angle.

Also, as an exterior angle and its adjacent interior angle make a straight line, we have :



An exterior angle + adjacent interior angle = 180°

Regular polygon

A polygon is called **regular polygon** if all its sides have equal length and all its angles have equal size.

Thus, in a regular polygon :

- all sides are equal in length
- all interior angles are equal in size
- all exterior angles are equal in size.

All regular polygons are convex.

Angle Property of a Polygon

Sum of interior angles of a polygon

In the adjoining figure, ABCDE is a pentagon. It has 5 sides and 5 (interior) angles. Take any point O inside the pentagon and join it with vertices. We notice that 5 triangles are formed.

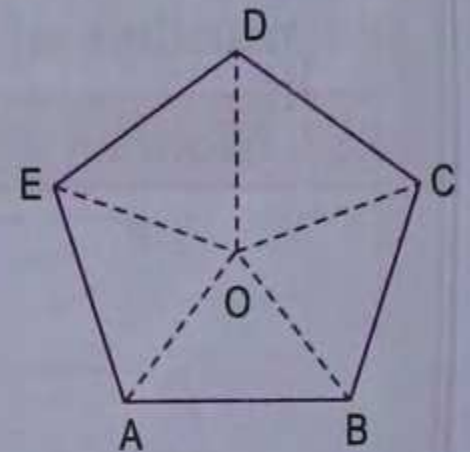
As the sum of angles of a triangle is 2 right angles, therefore, the sum of all the angles of the 5 triangles

$$= (2 \times 5) \text{ right angles.}$$

Also the sum of angles at the point O = 4 right angles. It follows that the sum of all the (interior) angles of the pentagon ABCDE = $(2 \times 5 - 4)$ right angles.

In fact, this is true about every polygon of n sides. So, we have an important result :

The sum of interior angles of a polygon of n sides = $(2n - 4)$ right angles



Sum of exterior angles of a convex polygon

In the adjoining figure, ABCDE is a convex pentagon. It has 5 sides and 5 interior angles. On putting $n = 5$ in the above formula, sum of interior angles of a pentagon

$$= (2 \times 5 - 4) \text{ right angles} = 6 \times 90^\circ = 540^\circ.$$

The pentagon has 5 exterior angles (the sides are produced in order) and each exterior angle has an adjacent interior angle.

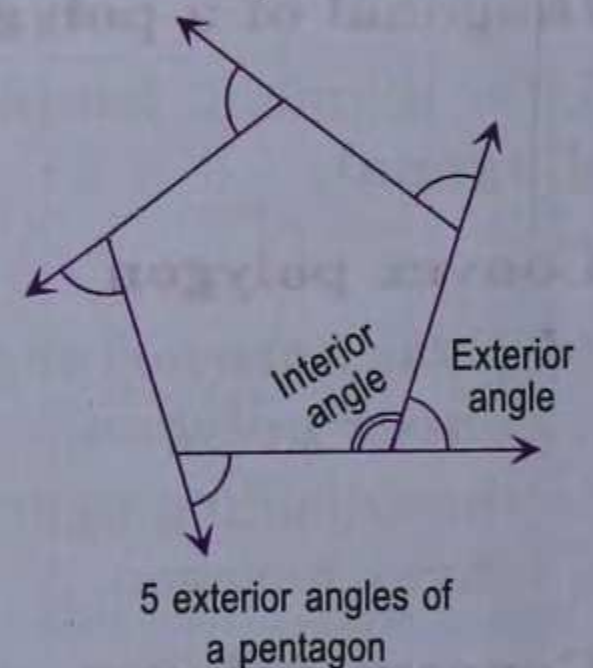
As the sum of an exterior angle and its adjacent interior angle is 180° , the sum of all the exterior and the interior angles of a pentagon

$$= 5 \times 180^\circ = 900^\circ.$$

\therefore The sum of all the exterior angles = $900^\circ - 540^\circ = 360^\circ$.

In fact, this is true about every convex polygon. So, we have another important result :

The sum of exterior angles of a convex polygon = 360°



From the above two results, it follows that :

- Each interior angle of a regular polygon of n sides = $\frac{2n - 4}{n}$ right angles
- Each exterior angle of a regular polygon of n sides = $\frac{360^\circ}{n}$
- If each exterior angle of a regular polygon is x° , then the number of sides in the regular polygon = $\frac{360}{x}$.

Example 1.

- Find the sum of interior angles of nonagon.
- Find the measure of each interior angle of a regular 16-gon.

Solution.

- A nonagon has 9 sides.

$$\begin{aligned} \text{Sum of its interior angles} &= (2 \times 9 - 4) \text{ right angles} = 14 \times 90^\circ \\ &= 1260^\circ \end{aligned}$$

(ii) Each exterior angle of a regular 16-sided polygon

$$= \frac{360^\circ}{16} = \frac{45^\circ}{2} = 22.5^\circ = 22^\circ 30'$$

\therefore Each interior angle of regular 16-gon $= 180^\circ - 22^\circ 30' = 157^\circ 30'$.

Example 2.

A heptagon has four equal angles each of 132° and three equal angles. Find the size of equal angles.

Solution.

A heptagon has 7 sides.

$$\begin{aligned} \text{Sum of its interior angles} &= (2 \times 7 - 4) \text{ right angles} \\ &= 10 \times 90^\circ = 900^\circ. \end{aligned}$$

Let the size of each equal angle be x° , so we have

$$4 \times 132^\circ + 3x^\circ = 900^\circ$$

$$\Rightarrow 3x^\circ = 900^\circ - 528^\circ = 372^\circ \quad \Rightarrow x = 124$$

Hence, the size of each equal angle $= 124^\circ$.

Example 3.

Is it possible to have a regular polygon whose each interior angle is 105° ?

Solution.

Given each interior angle $= 105^\circ$,

so each exterior angle $= 180^\circ - 105^\circ = 75^\circ$.

\therefore The number of sides of the polygon $= \frac{360}{75} = \frac{24}{5} = 4\frac{4}{5}$, which is not a natural number.

Therefore, no regular polygon is possible whose each interior angle is 105° .

Example 4.

The sum of interior angles of a polygon is 2700° . How many sides this polygon has?

Solution.

Let the polygon have n sides, then the sum of its interior angles

$$= (2n - 4) \text{ right angles} = (2n - 4) \times 90^\circ$$

$$\text{By the question, } (2n - 4) \times 90^\circ = 2700^\circ$$

$$\Rightarrow 2n - 4 = 30 \quad \Rightarrow 2n = 34 \quad \Rightarrow n = 17.$$

Hence, the polygon has 17 sides.

Example 5.

The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 8. Find the number of sides in the polygon.

Solution.

In a regular polygon, all exterior angles are equal in size and also interior angles are equal in size.

Let an exterior angle be x , then interior angle is $180^\circ - x$.

$$\text{According to given information, } \frac{x}{180^\circ - x} = \frac{1}{8}$$

$$\Rightarrow 8x = 180^\circ - x \quad \Rightarrow 9x = 180^\circ \quad \Rightarrow x = 20^\circ.$$

$$\therefore \text{ The number of sides in the polygon} = \frac{360}{20} = 18.$$

Example 6.

Each interior angle of a regular polygon is 144° . Find the interior angle of a regular polygon which has double the number of sides as the first polygon.

Solution.

Each interior angle of the first polygon $= 144^\circ$ (given),

\therefore each exterior angle of the first polygon $= 180^\circ - 144^\circ = 36^\circ$

- \therefore The number of sides in the first polygon = $\frac{360}{36} = 10$
 \therefore The number of sides in the second polygon = $2 \times 10 = 20$
 \therefore Each exterior angle in the second polygon = $\frac{360^\circ}{20} = 18^\circ$
 \therefore Each interior angle in the second polygon = $180^\circ - 18^\circ = 162^\circ$.

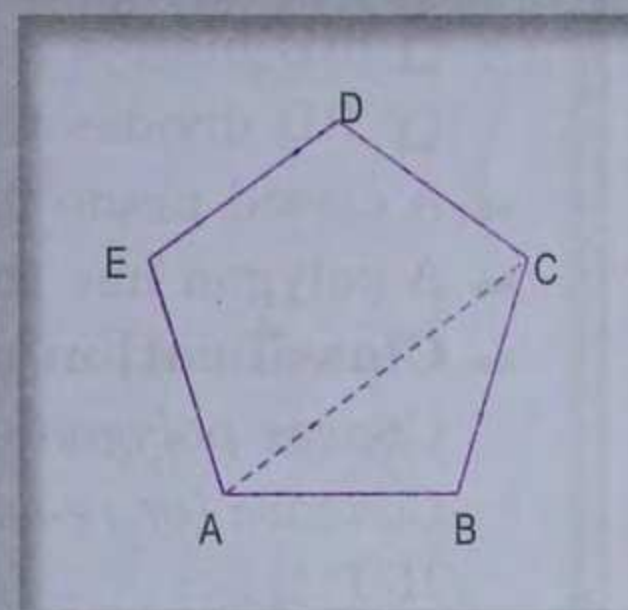
Exercise 23.3

- Find the sum of interior angles of a :
 - hexagon
 - octagon
 - decagon.
- Find the sum of interior angles of a polygon with
 - 11 sides
 - 19 sides
 - 25 sides.
- Find the measure of each interior angle of a regular
 - hexagon
 - heptagon
 - octagon
 - decagon
 - 18-gon
 - 24-gon.
- Find the number of sides of a regular polygon if each of its exterior angles is
 - 72°
 - 45°
 - 24°
 - $\left(51\frac{3}{7}\right)^\circ$.
- Find the number of sides of a regular polygon if each of its interior angles is
 - 162°
 - 108°
 - 120°
 - 140°
 - $\left(147\frac{3}{11}\right)^\circ$.
- Find the number of sides in a polygon if the sum of its interior angles is :
 - 1260°
 - 1980°
 - 3420° .
- Is it possible to have a polygon the sum of whose interior angles is
 - 1800°
 - 450°
 - 1120°
 - 31 right angles?
- Is it possible to have a regular polygon each of whose interior angle is
 - 130°
 - 165°
 - $1\frac{3}{4}$ right angles?
- The angles of a pentagon are x° , $(x - 10)^\circ$, $(x + 20)^\circ$, $(2x - 44)^\circ$ and $(2x - 70)^\circ$. Calculate x .
- The exterior angles of a pentagon are in the ratio 1 : 2 : 3 : 4 : 5. Find all the interior angles of the pentagon.
[Hint. Let exterior angles be x , $2x$, $3x$, $4x$, $5x$, then $x + 2x + 3x + 4x + 5x = 360^\circ \Rightarrow x = 24^\circ$.]
- Five angles of a hexagon are each 116° , calculate the size of the sixth angle.
- A heptagon has three equal angles each of 120° and four equal angles. Find the size of equal angles.
- The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 5. Find
 - the measure of each exterior angle
 - the measure of each interior angle
 - the number of sides in the polygon.
- Each interior angle of a regular polygon is double of its exterior angle. Find the number of sides in the polygon.

15. Each interior angle of a regular polygon is 150° . Find the interior angle of a regular polygon which has double the number of sides as the given polygon.

16. In the adjoining figure, ABCDE is a regular pentagon. Find

- (i) $\angle ABC$
- (ii) $\angle CAB$
- (iii) $\angle ACD$.



Summary

➔ A closed plane figure bounded by four line segments is called a quadrilateral. It has four sides, four (interior) angles, four vertices and two diagonals.

➔ Sum of interior angles of a quadrilateral is 360° .

➔ Properties of a parallelogram

- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are equal.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other.
- Each diagonal bisects the parallelogram.

➔ If two opposite sides of a quadrilateral are equal and parallel, then it is a parallelogram.

➔ Properties of a rectangle

It has all the properties of a parallelogram. Its additional properties are:

- Each (interior) angle = 90° .
- The diagonals are equal (in length).

➔ Properties of a rhombus

It has all the properties of a parallelogram. Its additional properties are:

- All the sides are equal (in length).
- The diagonals intersect at right angles.
- The diagonals bisect the angles of a rhombus.

➔ Properties of a square

It has all the properties of a parallelogram. Its additional properties are:

- Each (interior) angle = 90° .
- All the sides are equal (in length).
- The diagonals are equal (in length).
- The diagonals intersect at right angles.
- The diagonals bisect the angles of a square.

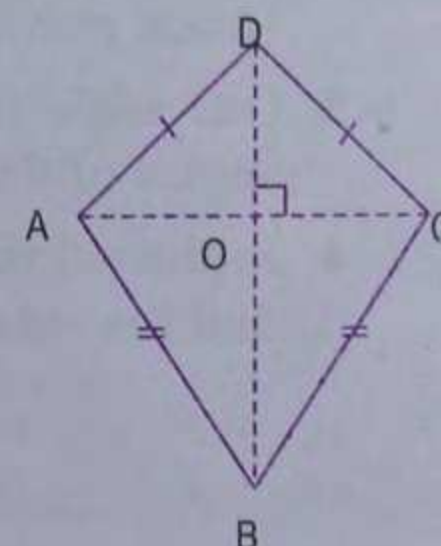
➔ Properties of an isosceles trapezium

- Co-interior angles are supplementary.
- Angles on the same base are equal.
- Diagonals are equal (in length).

➔ Properties of a kite

In the adjoining diagram, ABCD is a kite.

- The diagonals intersect at right angles.
- $\angle A = \angle C$.

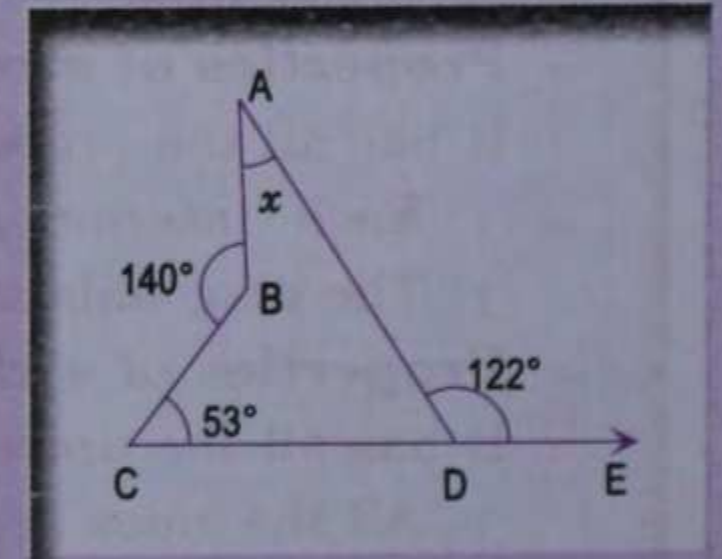


- BD bisects $\angle B$ as well as $\angle D$.
- BD divides the kite into two congruent triangles.
- ➔ A closed plane figure bounded by line segments is called a polygon.
- ➔ A polygon has the same number of (interior) angles as it has sides.
- ➔ **Classification of polygons**
 - Convex polygon* — all interior angles are less than 180° .
 - Concave (or re-entrant) polygon* — one or more of the interior angles is greater than 180° .
 - Regular polygon* — all sides have equal length and all interior angles have equal size. Of course, all exterior angles will also have equal size.
- ➔ All regular polygons are convex.
- ➔ **Angle property of a polygon**
 - The sum of interior angles of a polygon of n sides = $(2n - 4)$ right angles.
 - The sum of exterior angles of a convex polygon is 360° .
- ➔ Each interior angle of a regular polygon of n sides = $\frac{2n - 4}{n}$ right angles.
- ➔ Each exterior angle of a regular polygon of n sides = $\frac{360^\circ}{n}$.
- ➔ If each exterior angle of a regular polygon is x° , then the number of sides in the polygon = $\frac{360}{x}$.

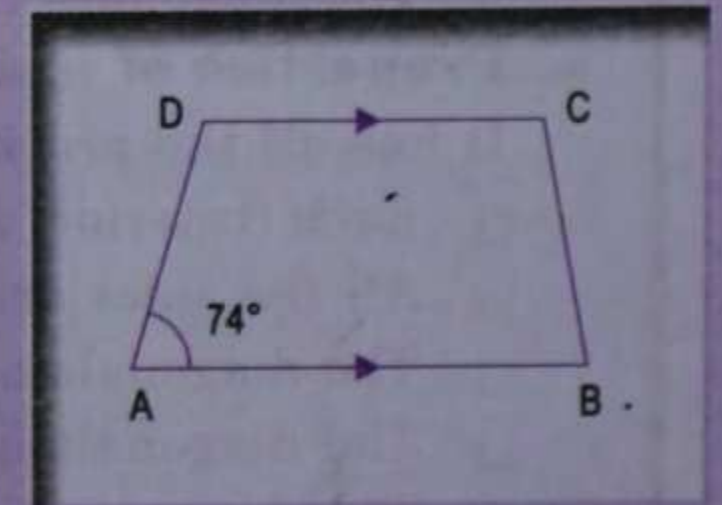
Check Your Progress

1. From the adjoining diagram, find the value of x .

[Hint. Reflex angle B = 220° , $\angle ADC = 58^\circ$
Sum of interior angles is 360° .]



2. If two angles of a quadrilateral are $76^\circ 37'$ and $57^\circ 23'$, and out of the remaining two angles, one angle is 10° smaller than the other, find these angles.

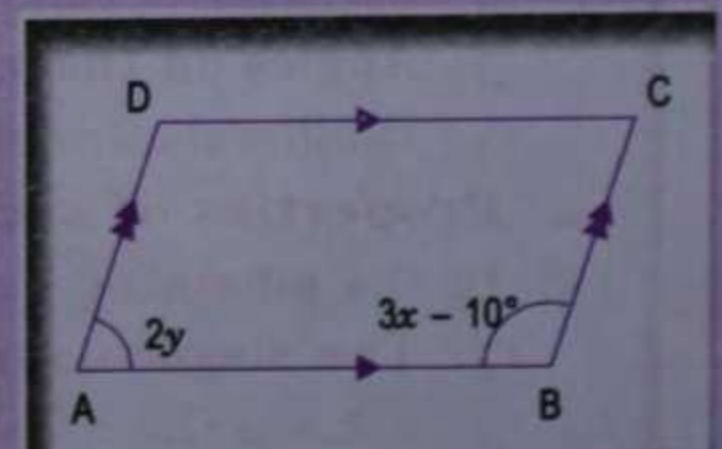


3. In the adjoining figure, $AB \parallel DC$, $\angle A = 74^\circ$ and $\angle B : \angle C = 4 : 5$. Find
- (i) $\angle D$
 - (ii) $\angle B$
 - (iii) $\angle C$.

4. In quadrilateral ABCD, $\angle A : \angle B : \angle C : \angle D = 3 : 4 : 6 : 7$. Find all the angles of the quadrilateral. Hence prove that AB and DC are parallel. Is BC also parallel to AD?

5. One angle of a parallelogram is two-third of the other. Find the angles of the parallelogram.

6. In the adjoining figure, ABCD is a parallelogram. If $x - y = 5^\circ$, find the values of x and y .



7. In the adjoining figure, ABCD is a parallelogram. If $AB = 2x + 5$, $CD = y + 1$, $AD = y + 5$ and $BC = 3x - 4$, then find the ratio of $AB : BC$.

8. In the adjoining figure, ABCD is a rhombus and EDC is an equilateral triangle. If $\angle DAB = 48^\circ$, find

- (i) $\angle BEC$
- (ii) $\angle DEB$

(iii) $\angle BFC$.

[Hint. $\angle BCE = 48^\circ + 60^\circ = 108^\circ$, $BC = EC$.]

9. In the adjoining figure, ABCD is a kite. If $\angle BCD = 52^\circ$ and $\angle ADB = 42^\circ$, find the values of x , y and z .

[Hint. Join AC.]

10. In the adjoining figure, ABCD is a rectangle. Prove that $AC = BD$.

[Hint. $\triangle ABC \cong \triangle BAD$.]

11. In the adjoining figure, ABCD is a parallelogram. AM and CN are drawn perpendiculars from A and C respectively on the diagonal BD. Prove that $AM = CN$.

[Hint. Prove that $\triangle ADM \cong \triangle CBN$.]

12. Find the measure (in degrees) of each interior angle of a regular 40-gon.

13. Find the number of sides of a regular polygon if each of its interior angle is $157^\circ 30'$.

14. If the sum of interior angles of a polygon is 3780° , find the number of sides.

15. Find the number of sides in a regular polygon if its interior and exterior angles are equal.

16. Two angles of a polygon are right angles and every other angle is 120° . Find the number of sides of the polygon.

[Hint. Let the number of sides be n , then $2 \times 90^\circ + (n - 2) \times 120^\circ = (2n - 4) \times 90^\circ$.]

17. The sum of interior angles of a regular polygon is twice the sum of its exterior angles. Find the number of sides of the polygon.

18. The angles of a hexagon are $(2x + 5)^\circ$, $(3x - 5)^\circ$, $(x + 40)^\circ$, $(2x + 20)^\circ$, $(2x + 25)^\circ$ and $(2x + 35)^\circ$. Find the value of x .

19. An exterior angle of a regular polygon is one-fourth of its interior angle. Find the number of sides in the polygon.

20. The adjoining figure represents a part of the regular octagon ABCD... with the diagonal AC drawn. Find

- (i) $\angle ABC$
- (ii) $\angle CAB$
- (iii) $\angle ACD$.

