21 Heights and Distances

21.1 PRACTICAL USE OF TRIGONOMETRY

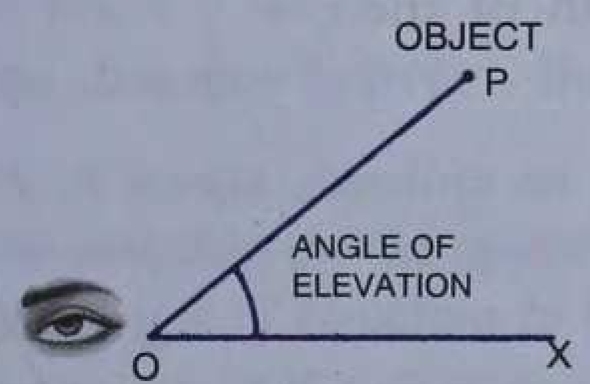
Trigonometry helps us in finding the heights of objects and the distances between points, the actual measurement of these heights and distances is very difficult. For example, it enables us in obtaining the height of a mountain (pillar or minar) or the breadth of a river. For this, we require the following two definitions :

Angle of Elevation and Angle of Depression

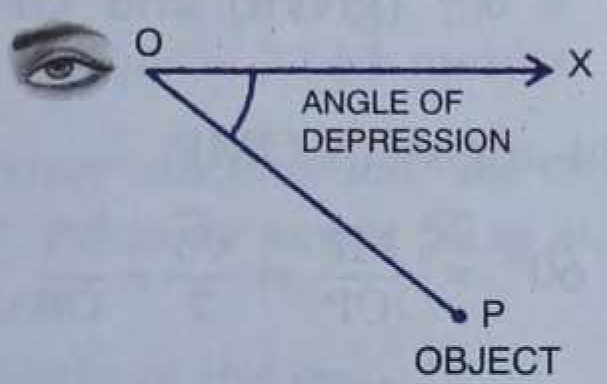
Let O be an eye of an observer and OX be the horizontal line drawn through O. Let P be an object, then

(i) if P is above OX as in figure (i), $\angle XOP$ is called the ***angle of elevation*** *of P (as observed from O).*

(ii) if P is below OX as in figure (ii), $\angle XOP$ is called the ***angle of depression*** *of P (as observed from O).*



(i)



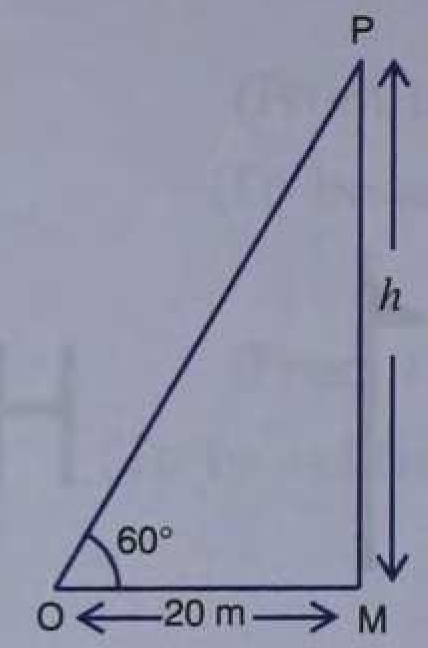
(ii)

Remarks

- The line OP, joining the eye to the object, is called the *line of sight*.
- While dealing with problems on heights and distances, usually, we find $\frac{\text{required side}}{\text{known side}}$ = a certain *t*-ratio of a known angle.
- Remember : $\sqrt{2} = 1{\cdot}414$, $\sqrt{3} = 1{\cdot}732$.

ILLUSTRATIVE EXAMPLES

Example 1. *From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 60°. Find the height of the tower.*



Solution. Let MP be the tower of height h metres and O be the point 20 m away from the foot of tower, then

$\angle MOP = 60°$ (given), the angle of elevation.

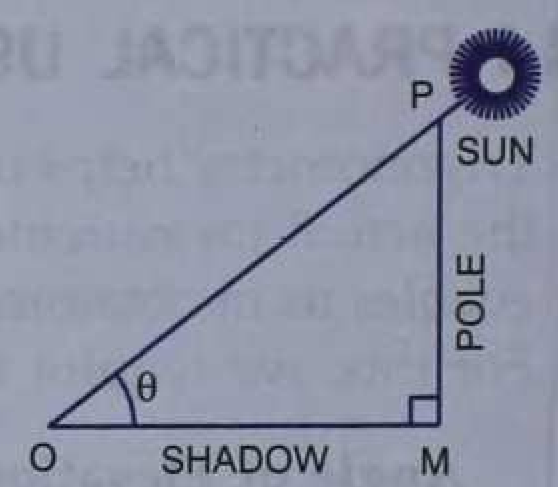
From right angled ΔOMP, we get

$$\tan 60° = \frac{MP}{OM} \Rightarrow \sqrt{3} = \frac{h}{20}$$

$\Rightarrow \quad h = 20\sqrt{3} = 20 \times 1{\cdot}732 = 34{\cdot}64.$

$\therefore$ The height of the tower = 34·64 metres.

Example 2. *If the length of a shadow cast by a pole be $\sqrt{3}$ times the length of the pole, find the angle of elevation of the sun.*



Solution. Let MP be the pole, then its shadow

$= OM = \sqrt{3}\ MP$ (according to given).

Let $\angle MOP = \theta$, the angle of elevation of the sun.

From right-angled Δ OMP, we get

$$\tan \theta = \frac{MP}{OM} \text{ but } OM = \sqrt{3}\ MP$$

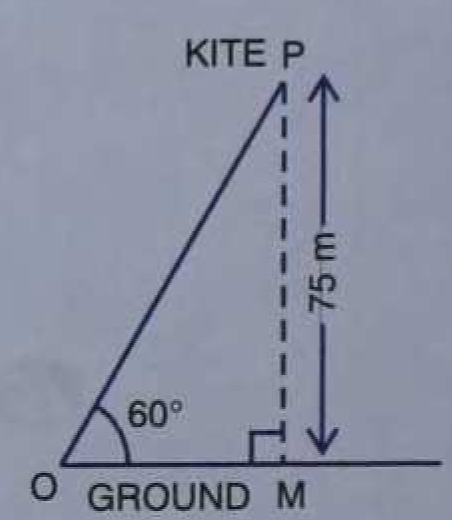
$$\Rightarrow \quad \tan \theta = \frac{MP}{\sqrt{3}\ MP} = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\Rightarrow \quad \theta = 30°.$$

Hence, the angle of elevation of the sun = 30°.

Example 3. *A kite is flying at a height of 75 metres from the level ground, attached to a string inclined at 60° to the horizontal. Find the length of the string to the nearest metre.*

Solution. Let P be the kite and MP = 75 metres, the height of the kite. Let the string be held at the point O, then $\angle MOP = 60°$ (given) and OP is the length of the string.



From right-angled Δ OMP,

$$\sin 60° = \frac{MP}{OP} \Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{OP}$$

$$\Rightarrow \quad OP = \frac{150}{\sqrt{3}} = 50 . \sqrt{3} = 50 \times 1{\cdot}732 = 86{\cdot}6.$$

$\therefore$ The length of the string (to the nearest metre) = 87 metres.

Example 4. *A pole being broken by the wind, the top struck the ground at an angle of 30° and at a distance of 8 m from the foot of the pole. Find the whole height of the pole.*

Solution. Let ABC be the pole. When broken at B by the wind, let its top A strike the ground so that $\angle CAB = 30°$ and AC = 8 m.

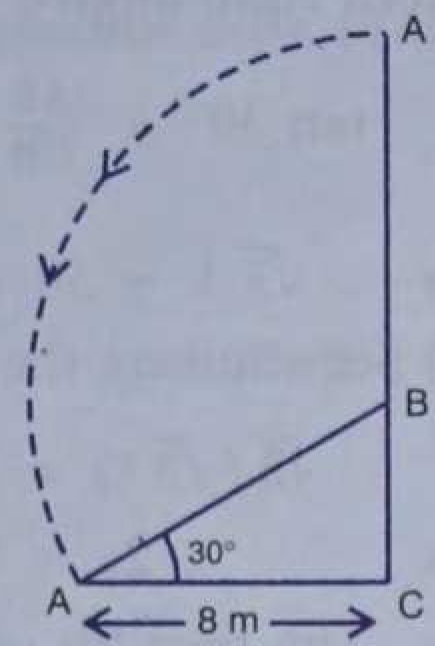
From right-angled Δ ACB,

$$\tan 30° = \frac{BC}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$\Rightarrow \quad BC = \frac{8}{\sqrt{3}} \text{ m.}$$

Also $\cos 30° = \frac{AC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AB}$

$\Rightarrow \quad AB = \frac{16}{\sqrt{3}}$ m.



$\therefore$ The height of the pole $= AB + BC = \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right)$ m

$= \frac{24}{\sqrt{3}}$ m $= 8 \times \sqrt{3}$ m

$= 8 \times 1{\cdot}732$ m $= 13{\cdot}856$ m

$= 13{\cdot}86$ metres nearly.

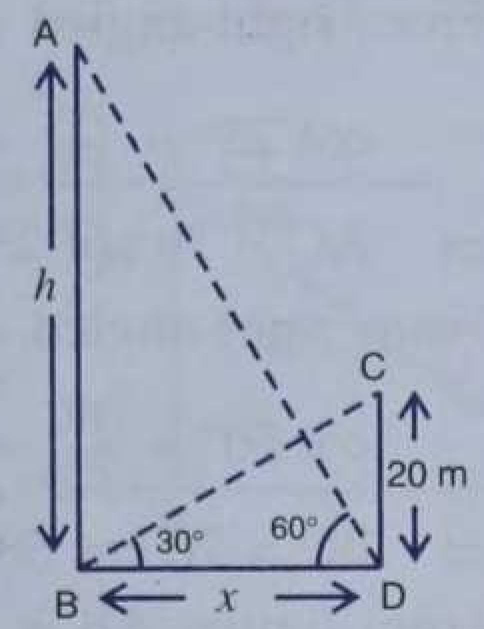
Example 5. *The angle of elevation of the top of a hill at the foot of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 20 m high, find*

(i) the height of the hill *(ii) the distance between the hill and the tower.*

Solution. Let $AB = h$ metres be the height of the hill and CD be the tower, then $CD = 20$ metres (given). Let $BD = x$ metres be the distance between the foot of the hill and the foot of the tower. Then

$\angle ADB = 60°$ and $\angle CBD = 30°$ (given).

From right angled Δ ABD, we get



$\tan 60° = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}\,x$ ...(1)

From right angled ΔCBD, we get

$\tan 30° = \frac{CD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{x}$

$\Rightarrow \quad x = 20\sqrt{3}$ ...(2)

(*i*) Substituting the value of x from (2) in (1), we get

$h = \sqrt{3}\,(20\sqrt{3}) = 60.$

$\therefore$ The height of the hill = 60 metres.

(*ii*) From (2), $x = 20\sqrt{3} = 20 \times 1{\cdot}732 = 34{\cdot}64$.

$\therefore$ The distance between the hill and the tower = 34·64 metres.

Example 6. *A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 50 m away from the bank he finds that the angle of elevation to be 30°. Calculate :*

(i) the width of the river and *(ii) the height of the tree.* (2003)

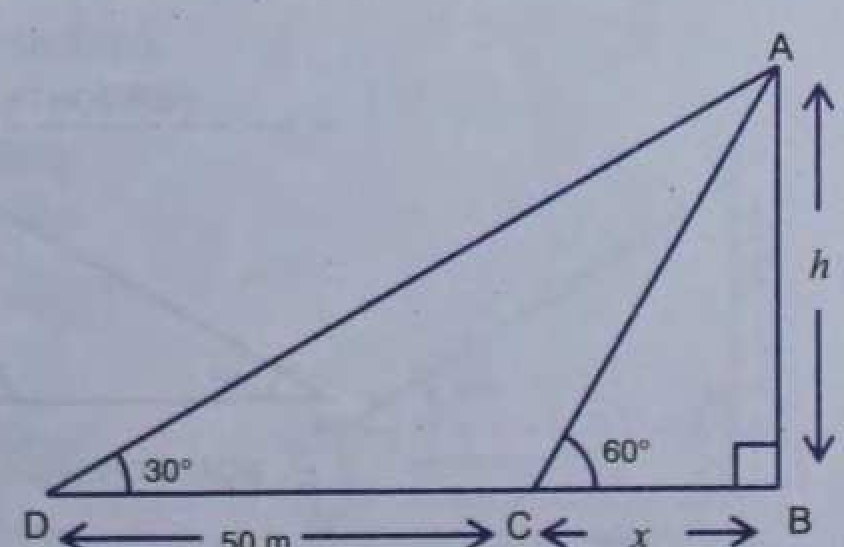
Solution. Let $AB = h$ metres be the height of the tree and $CB = x$ metres be the breadth of the river.

Let C be the first position of the man, and D be the position after moving 50 metres away from the bank, then

$DC = 50$ m,

$\angle ACB = 60°$ and $\angle ADB = 30°$.

From right-angled Δ ACB, we get



$\tan 60° = \frac{AB}{CB} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}\,x$ ...(1)

From right-angled Δ ADB, we get

$$\tan 30° = \frac{AB}{DB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{50 + x}$$

$\Rightarrow \quad \sqrt{3}\, h = 50 + x$ ...(2)

(*i*) Substituting the value of h from (1) in (2), we get

$$\sqrt{3}\,(\sqrt{3}\,x) = 50 + x \Rightarrow 3x = 50 + x$$

$\Rightarrow \quad 2x = 50 \Rightarrow x = 25.$

$\therefore$ The breadth of the river = 25 metres.

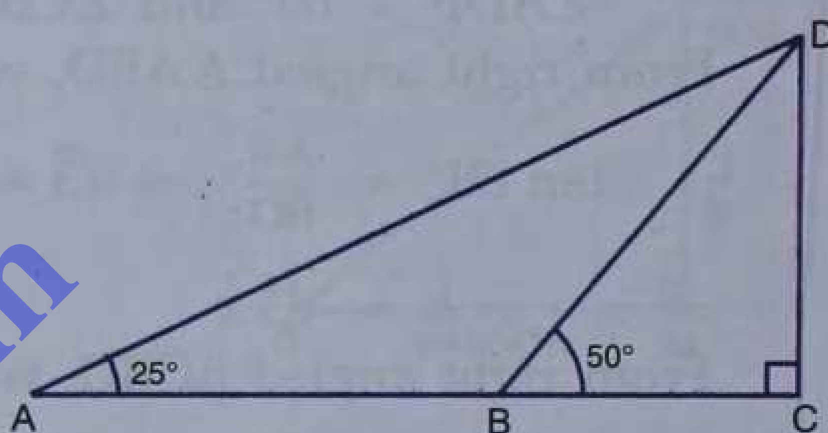
(*ii*) From (1), we get $h = \sqrt{3} \times 25$ m $= 1{\cdot}732 \times 25$ m $= 43{\cdot}3$ m.

$\therefore$ The height of the tree = 43·3 metres.

Example 7. *Two persons standing on the same side of a tower in a straight line with it, measure the angles of elevation of the top of the tower as 25° and 50° respectively. If the height of the tower is 70 m, find the distance between the two persons.* (2004)

Solution. Let A, B be the positions of the two persons and CD be the tower, then CD = 70 m (given)

$$\angle DAC = 25° \text{ and } \angle DBC = 50°.$$



From right-angled ΔDAC, we get

$$\cot 25° = \frac{AC}{CD} = \frac{AC}{70}$$

$\Rightarrow \quad AC = 70 \cot 25°$ ...(1)

From right-angled ΔDBC, we get

$$\cot 50° = \frac{BC}{CD} = \frac{BC}{70}$$

$\Rightarrow \quad BC = 70 \cot 50°$ ...(2)

From figure, AB = AC – BC

$\Rightarrow \quad AB = 70 \cot 25° - 70 \cot 50°$ (Using (1) and (2))

$= 70 (\cot 25° - \cot 50°)$

$= 70 [\cot (90° - 65°) - \cot (90° - 40°)]$

$= 70 (\tan 65° - \tan 40°)$

$= 70 (2{\cdot}1445 - 0{\cdot}8391)$ (From tables)

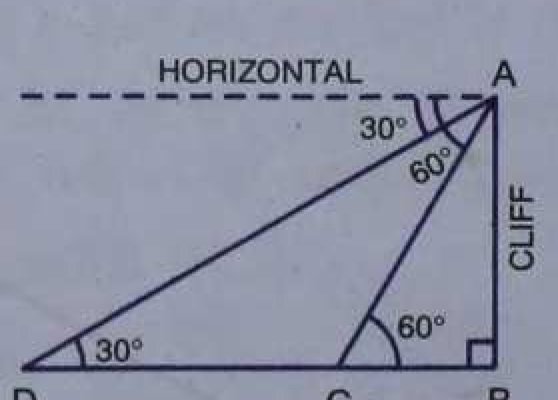
$= 70 \times 1{\cdot}3054 = 91{\cdot}378.$

$\therefore$ The distance between the two persons = 91·4 metres (approximately).

Example 8. *From the top of a cliff 150 m high, the angles of depression of two boats are 60° and 30°. Find the distance between the boats, if the boats are*

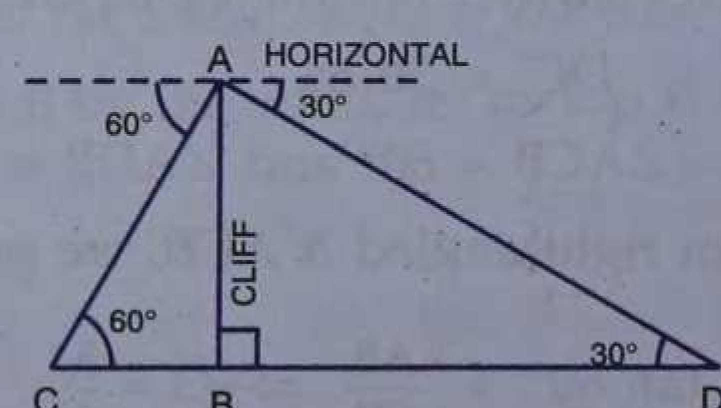
(*i*) *on the same side of the cliff* (*ii*) *on the opposite sides of the cliff.*

Solution. Let AB = 150 m be the height of the cliff and let C, D be the two boats, then ∠ACB = 60° and ∠ADB = 30°.



BOATS ON SAME SIDE

(*i*)



BOATS ON OPPOSITE SIDES

(*ii*)

In both cases (boats on same side of the cliff or boats on opposite sides of the cliff), from right-angled ΔACB, we get

$$\tan 60° = \frac{AB}{CB} \Rightarrow \sqrt{3} = \frac{150}{CB}$$

$$\Rightarrow \quad CB = \frac{150}{\sqrt{3}} = 50\sqrt{3} \quad \ldots(1)$$

From right-angled ΔABD, we get

$$\tan 30° = \frac{AB}{DB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{DB}$$

$$\Rightarrow \quad DB = 150\sqrt{3} \quad \ldots(2)$$

(*i*) *Boats on the same side of the cliff.*

Distance between the boats = DB – CB (From figure (*i*))

$$= (150\sqrt{3} - 50\sqrt{3}) \text{ m} = 100 \times \sqrt{3} \text{ m}$$

$$= 100 \times 1{\cdot}732 \text{ m} = 173{\cdot}2 \text{ m}.$$

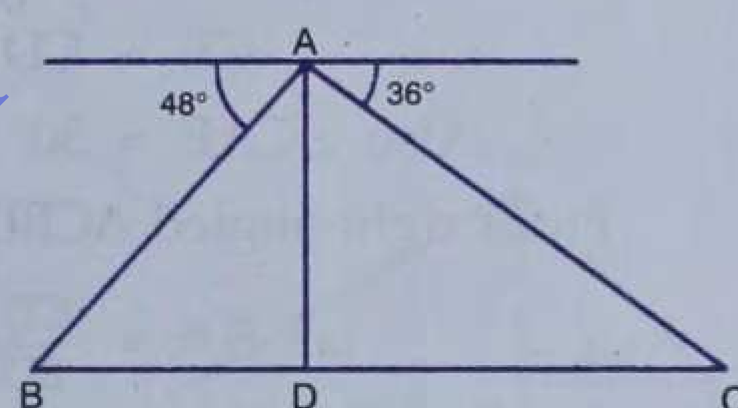
(*ii*) *Boats on the opposite sides of the cliff.*

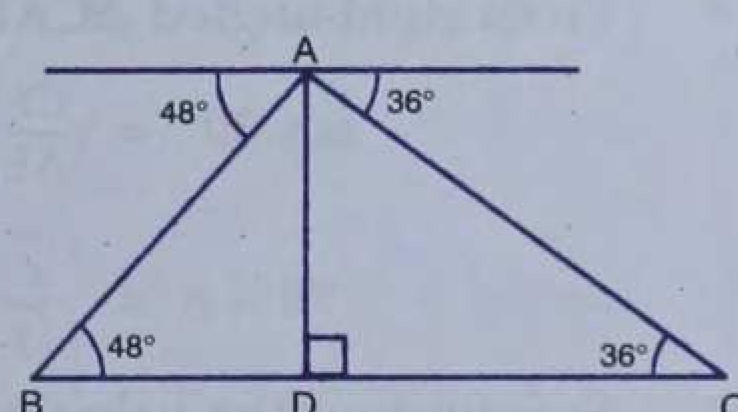
Distance between the boats = CB + BD (From figure (*ii*))

$$= (50\sqrt{3} + 150\sqrt{3}) \text{ m} = 200 \times \sqrt{3} \text{ m}$$

$$= 200 \times 1{\cdot}732 \text{ m} = 346{\cdot}4 \text{ m}.$$

Example 9. *From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.* (2010)





Solution. $\angle ABD = 48°$ and $\angle ACD = 36°$.

From right angled ΔABC,

$$\cot 48° = \frac{BD}{AD}$$

$$\Rightarrow \quad BD = AD \cot 48°.$$

From right angled ΔACD,

$$\cot 36° = \frac{DC}{AD}$$

$$\Rightarrow \quad DC = AD \cot 36°$$

$\therefore$ The distance between the two ships = BD + DC

$$= AD \cot 48° + AD \cot 36°$$

$$= AD (\cot (90° - 42°) + \cot (90° - 54°)$$

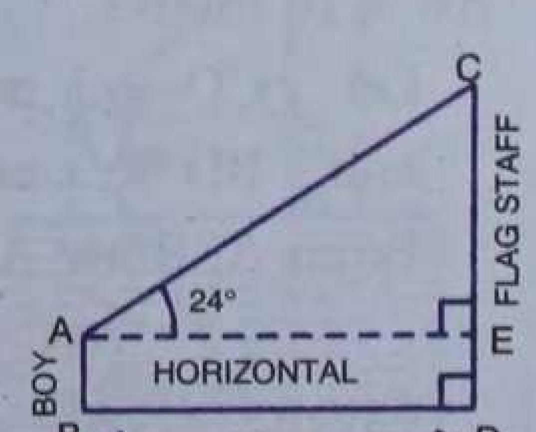
$$= AD (\tan 42° + \tan 54°)$$

$$= 100 \times (0{\cdot}9004 + 1{\cdot}3764) \text{ metres}$$

$$= 100 \times 2{\cdot}2768 \text{ metres} = 227{\cdot}68 \text{ metres}$$

$$= 228 \text{ metres (to the nearest metre).}$$

Example 10. *A boy of height 1·7 m is standing 20 m away from a flagstaff on the same level ground. He observes that the angle of elevation of the top of the flagstaff is 24°. Calculate the height of the flagstaff.*



Solution. Let AB be the boy of height 1·7 m and CD be the flagstaff of height *h* metres, then

$$BD = 20 \text{ m (given).}$$

Through A (eye of the boy) draw horizontal line AE to meet CD at E, then $\angle CAE = 24°$ (given)

and AE = BD = 20 m. (∵ ABDE is a rectangle)

Also CE = CD – ED = CD – AB = $(h - 1{\cdot}7)$ metres.

From right-angled ΔCAE, we get

$$\tan 24° = \frac{CE}{AE} \Rightarrow \tan 24° = \frac{h - 1{\cdot}7}{20}$$

$$\Rightarrow \quad h - 1{\cdot}7 = 20 \times \tan 24°$$

$$\Rightarrow \quad h - 1{\cdot}7 = 20 \times {\cdot}4452 \qquad \text{(Using tables of natural tangents)}$$

$$\Rightarrow \quad h - 1{\cdot}7 = 8{\cdot}904$$

$$\Rightarrow \quad h = 8{\cdot}904 + 1{\cdot}7 = 10{\cdot}604.$$

∴ The height of the flagstaff = 10·604 metres.

Example 11. *From the top of a cliff 90 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.*

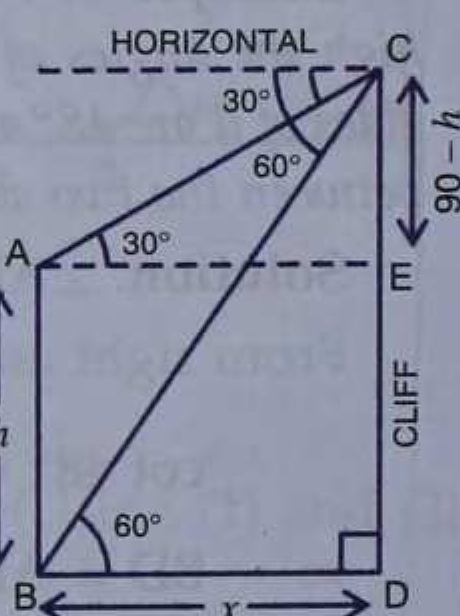
Solution. Let AB be the tower of height h metres and CD be the cliff. Let BD = x metres be the distance between the tower and the cliff.

Through A (top of the tower) draw a horizontal line to meet CD at E, then

AE = BD = x metres and

CE = CD – ED = $(90 - h)$ metres.

Also $\angle CAE = 30°$ and $\angle CBD = 60°$.



From right-angled ΔCBD, we get

$$\tan 60° = \frac{CD}{BD} \Rightarrow \sqrt{3} = \frac{90}{x} \Rightarrow x = \frac{90}{\sqrt{3}}$$

$$\Rightarrow \quad x = 30\sqrt{3} \qquad \ldots(1)$$

From right-angled ΔCAE, we get

$$\tan 30° = \frac{CE}{AE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{90 - h}{x}$$

$$\Rightarrow \quad 90 - h = \frac{x}{\sqrt{3}} \qquad \ldots(2)$$

Substituting the value of x from (1) in (2), we get

$$90 - h = \frac{30\sqrt{3}}{\sqrt{3}} \Rightarrow 90 - h = 30 \Rightarrow h = 60.$$

∴ The height of the tower = 60 metres.

Example 12. *A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.*

Solution. Let AB be the deck of the ship and the man be standing at A, then AB = 10 metres.

Let CD = h metres be the height of the hill

and BD = x metres be the distance between the hill and the ship.

From A, draw AN ⊥ CD, then

AN = DB = x metres, ND = AB = 10 metres and CN = $(h - 10)$ metres.

Given $\angle NAD = 30°$ and $\angle CAN = 60°$.

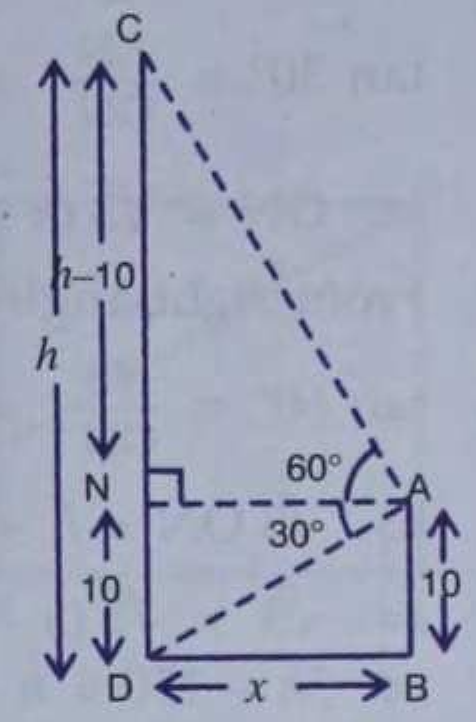
From right-angled ΔAND, we get

$$\tan 30^\circ = \frac{ND}{AN} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow \quad x = 10\sqrt{3} = 10 \times 1{\cdot}732 = 17{\cdot}32.$$

∴ The distance between the hill and the ship

= 17·32 metres.



From right-angled ΔCNA, we get

$$\tan 60^\circ = \frac{h-10}{AN} \Rightarrow \sqrt{3} = \frac{h-10}{x}$$

$$\Rightarrow \quad h - 10 = \sqrt{3}\,x \quad \Rightarrow h - 10 = \sqrt{3}\,(10\sqrt{3})$$

$$\Rightarrow \quad h = 10 + 30 = 40.$$

∴ The height of the hill = 40 metres.

Example 13. *A boy standing on the ground finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 m high building finds the angle of elevation of the same bird to be 45°. The boy and the girl are on opposite sides of the bird. Find the distance of the bird from the girl, correct to nearest cm.*

Solution. Let the bird be at the point P. The boy is at the point B on the ground and the girl is at the point G on the roof, 20 m above the ground.

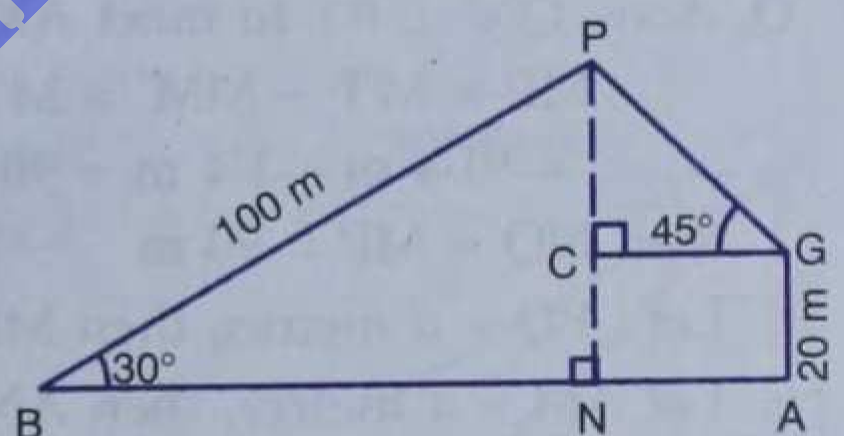
Given BP = 100 m and

AG = 20 m.

From right-angled ΔPBN,

$$\sin 30^\circ = \frac{NP}{BP}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{NP}{100} \Rightarrow NP = 50 \text{ m}$$



$$\therefore \quad CP = NP - NC = NP - AG$$

$$= 50 \text{ m} - 20 \text{ m}$$

$$= 30 \text{ m}.$$

From right-angled ΔPCG,

$$\sin 45^\circ = \frac{CP}{GP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{30}{GP}$$

$$\Rightarrow \quad GP = 30\sqrt{2} \text{ m} = (30 \times 1{\cdot}4142) \text{ m} = 42{\cdot}426 \text{ m} = 42{\cdot}43 \text{ m (nearly)}$$

Hence, the bird is 42·43 m away from the girl.

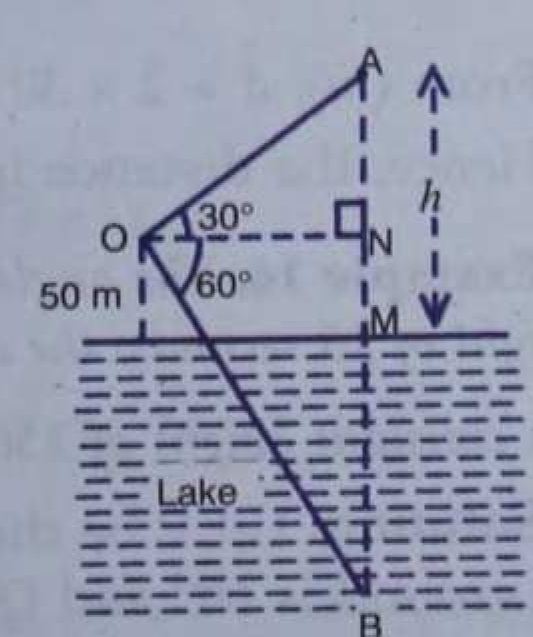
Example 14. *The angle of elevation of a bird from a point 50 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the bird.*

Solution. Let O be the point of observation 50 metres above lake level and let the bird be at the point A. If B is the reflection of the bird in the lake, then MB = MA. Let the height of the bird above the lake level be h metres, then

AN = (h − 50) metres and

NB = (h + 50) metres.

∠AON = 30° and ∠BON = 60°.



From the right-angled ΔOAN,

$\tan 30° = \frac{AN}{ON} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-50}{ON}$

$\Rightarrow ON = \sqrt{3}(h-50)$ ...(1)

From right-angled ΔOBN,

$\tan 60° = \frac{NB}{ON} \Rightarrow \sqrt{3} = \frac{h+50}{ON}$

$\Rightarrow \sqrt{3}\,ON = h + 50$

$\Rightarrow \sqrt{3} \times \sqrt{3}(h-50) = h + 50$ [Using (1)]

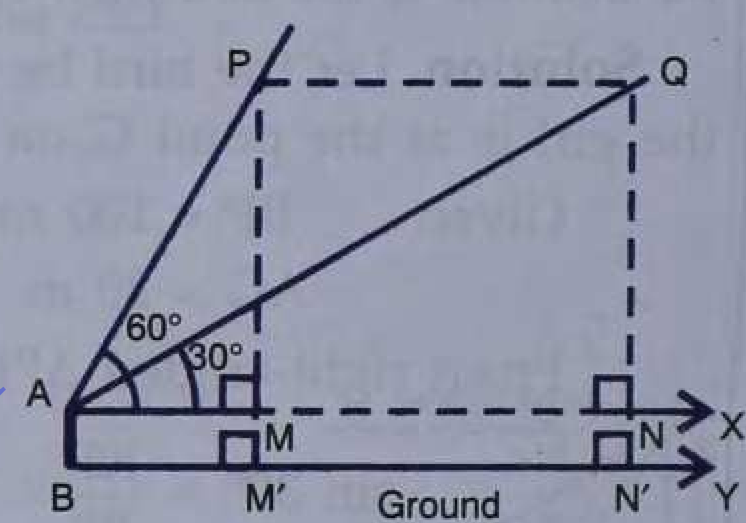
$\Rightarrow 3h - 150 = h + 50$

$\Rightarrow 2h = 200 \Rightarrow h = 100.$

∴ The height of the bird (above lake level) = 100 metres.

Example 15. *A 1·4 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 91·4 m from the ground. The angle of elevation of the balloon from the eyes of the girl at that instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during that interval. Take $\sqrt{3}$ = 1·732.*

Solution. Let AB represent the girl 1·4 m tall and P, Q be the two positions of the balloon when the angles of elevation observed from the point A are 60° and 30° respectively. Through A, draw a horizontal line AX. BY represents ground.



From P, draw PM′ ⊥ BY to meet AX at M; and from Q, draw QN′ ⊥ BY to meet AX at N.

MP = M′P – MM′ = M′P – AB

= 91·4 m – 1·4 m = 90 m,

then NQ = MP = 90 m (∵ PMNQ is a rectangle)

Let PQ = d metres, then MN = PQ = d metres

Let AM = x metres, then AN = AM + MN = $(x + d)$ metres

In right angled ΔAMP, ∠PAM = 60°

∴ $\tan 60° = \frac{MP}{AM} \Rightarrow \sqrt{3} = \frac{90}{x}$ ...(*i*)

In right angled ΔANQ, ∠QAN = 30°

∴ $\tan 30° = \frac{NQ}{AN} \Rightarrow \frac{1}{\sqrt{3}} = \frac{90°}{x+d}$ ...(*ii*)

Dividing (*i*) by (*ii*), we get

$\sqrt{3} \times \frac{\sqrt{3}}{1} = \frac{90}{x} \times \frac{x+d}{90} \Rightarrow 3 = \frac{x+d}{x}$

$\Rightarrow 3x = x + d \Rightarrow d = 2x$ ...(*iii*)

From (*i*), $x = \frac{90}{\sqrt{3}} = 30\sqrt{3}$.

From (*iii*), $d = 2 \times 30\sqrt{3} = 60\sqrt{3} = 60 \times 1·732 = 103·92$.

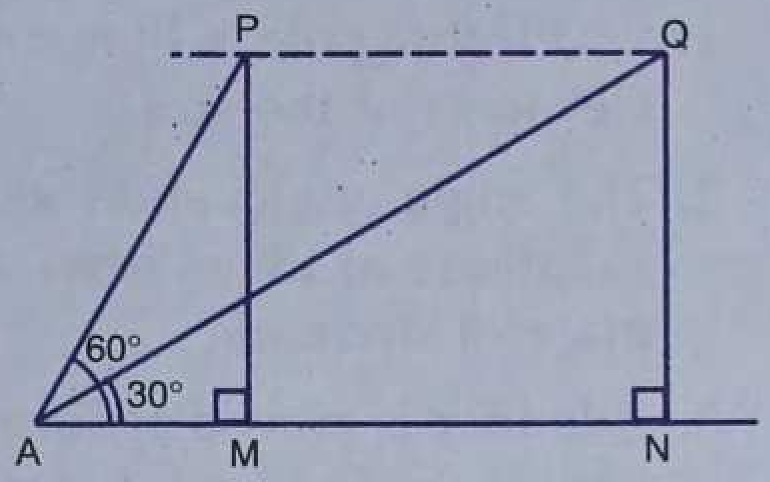
Hence, the distance travelled by the balloon during the interval = 103·92 metres.

Example 16. *The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet plane is flying horizontally at a constant height of $1500\sqrt{3}$ metres, find the speed of the jet plane.*

Solution. Let P be the position of the jet plane when its elevation from a point A on the ground is 60° and Q be its position when the angle of elevation is 30°.

Given MP = NQ = $1500\sqrt{3}$ m.

From right angled ΔAMP,



$$\tan 60° = \frac{MP}{AM} \Rightarrow \sqrt{3} = \frac{1500\sqrt{3}\text{ m}}{AM}$$

$\Rightarrow$ AM = 1500 m.

From right angled ΔANQ,

$$\tan 30° = \frac{NQ}{AN} \Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}\text{ m}}{AN}$$

$\Rightarrow$ AN = 4500 m

$\therefore$ PQ = MN = AN – AM = (4500 – 1500) m = 3000 m.

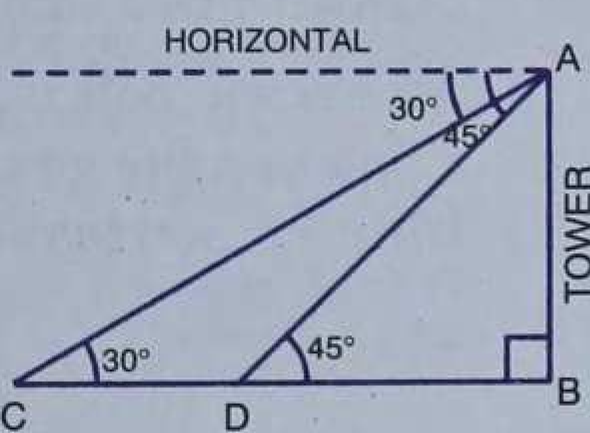
$\therefore$ The speed of the plane $= \frac{3000}{15}$ m/s = 200 m/s

$$= \left(200 \times \frac{18}{5}\right)\text{km/h} = 720 \text{ km/hr.}$$

Example 17. *A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower ? Give your answer correct to the nearest second.*

Solution. Let AB be the tower of height h metres, C be the initial position of the car and D be its position after 12 minutes, then

$\angle$ACB = 30° and $\angle$ADB = 45°.



Let x metres per minute be the speed of the car and let the car reach the observation tower *i.e.* the point B after t minutes, then

CD = 12 x metres and DB = $t\,x$ metres.

[$\because$ distance covered by the car = speed × time]

From right-angled ΔADB, we get

$$\tan 45° = \frac{AB}{DB} \Rightarrow 1 = \frac{h}{t\,x}$$

$\Rightarrow$ $h = t\,x$ ...(1)

From right-angled ΔACB, we get

$$\tan 30° = \frac{AB}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{12x + t\,x}$$

$\Rightarrow$ $12\,x + t\,x = \sqrt{3}\,h$ ...(2)

Substituting the value of h from (1) in (2), we get

$$12\,x + t\,x = \sqrt{3}\,t\,x \Rightarrow 12 + t = \sqrt{3}\,t$$

$\Rightarrow$ $12 = (\sqrt{3} - 1)\,t$

$$\Rightarrow t = \frac{12}{\sqrt{3}-1} = \frac{12}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{12(\sqrt{3}+1)}{3-1} = 6(\sqrt{3}+1)$$

$= 6\,(1{\cdot}732 + 1) = 6 \times 2{\cdot}732 = 16{\cdot}392.$

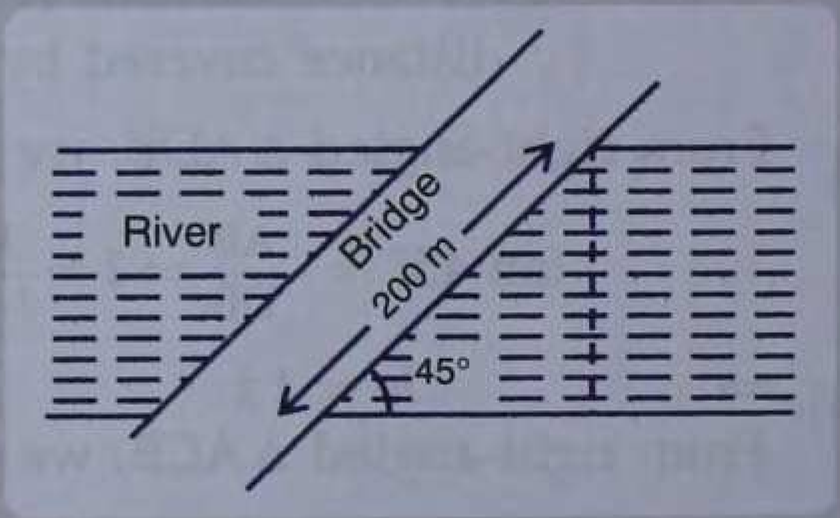
$\therefore$ The time to reach the observation tower = 16·392 minutes

= 16 minutes 24 seconds.

($\because$ ·392 minutes $= \frac{392}{1000} \times 60$ sec = 23·52 sec = 24 sec nearly.)

Exercise 21

1. An electric pole is 10 metres high. If its shadow is $10\sqrt{3}$ metres in length, find the elevation of the sun.
2. The angle of elevation of the top of a tower, from a point on the ground and at a distance of 150 m from its foot, is 30°. Find the height of the tower correct to one place of decimal.
3. A ladder is placed against a wall such that it just reaches the top of the wall. The foot of the ladder is 1·5 metres away from the wall and the ladder is inclined at an angle of 60° with the ground. Find the height of the wall.
4. What is the angle of elevation of the sun when the length of shadow of a vertical pole is equal to its height ?
5. A river is 60 m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to the foot of the tree, on the other bank, is 30°. Find the height of the tree.
6. From a point P on level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, how far is P from the foot of the tower ?
7. From the top of a cliff 92 m high, the angle of depression of a buoy is 20°. Calculate to the nearest metre, the distance of the buoy from the foot of the cliff. *(2005)*
8. A boy is flying a kite with a string of length 100 m. If the string is tight and the angle of elevation of the kite is 26° 32′, find the height of the kite correct to one decimal place (ignore the height of the boy).
9. An electric pole is 10 m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.
10. A bridge across a river makes an angle of 45° with the river bank. If the length of the bridge across the river is 200 metres, what is the breadth of the river ?



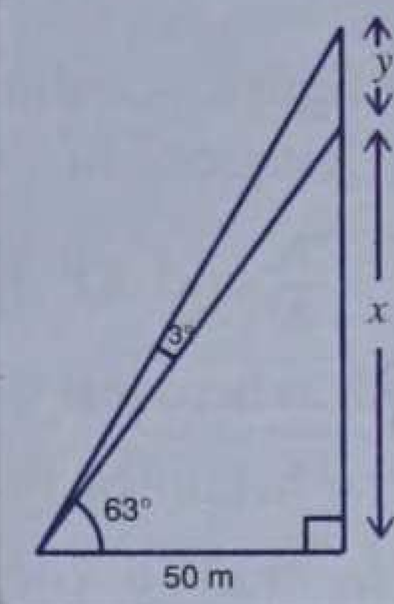
11. An aeroplane is 675 m directly above one end of a bridge. The angle of depression of the other end of the bridge is 56° 40′. How long is the bridge ?
12. A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0·53. How far is he standing from the foot of the tower ? *(2001)*

Hint

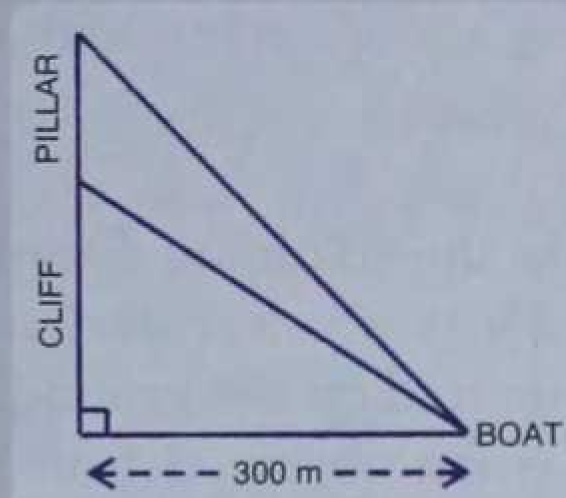
If θ is the angle of elevation, then $\cos\theta = \cdot 53 \Rightarrow \theta = 58°$.

13. The upper part of a tree broken by wind, falls to the ground without being detached. The top of the broken part touches the ground at an angle of 38° 30′ at a point 6 m from the foot of the tree. Calculate :
    (*i*) the height at which the tree is broken.
    (*ii*) the original height of the tree correct to two decimal places.

14. Some students wished to find the height x of a building and the height y of the flag-pole on the building. They made the measurements as shown in the diagram. Find x and y. Give your answer to the nearest metre.



15. From a boat 300 metres away from a vertical cliff, the angles of elevation of the top and the foot of a vertical concrete pillar at the edge of the cliff are 55° 40′ and 54° 20′ respectively. Find the height of the pillar correct to the nearest metre.



16. From a point P on the ground, the angle of elevation of the top of a 10 m tall building and a helicopter, hovering over the top of the building, are 30° and 60° respectively. Find the height of the helicopter above the ground.

17. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°; when he retires 20 m from the bank, he finds the angle to be 30°. Find the height of the tree and the breadth of the river.

18. The shadow of a vertical tower on a level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower, correct to two decimal places. *(2006)*

Remark

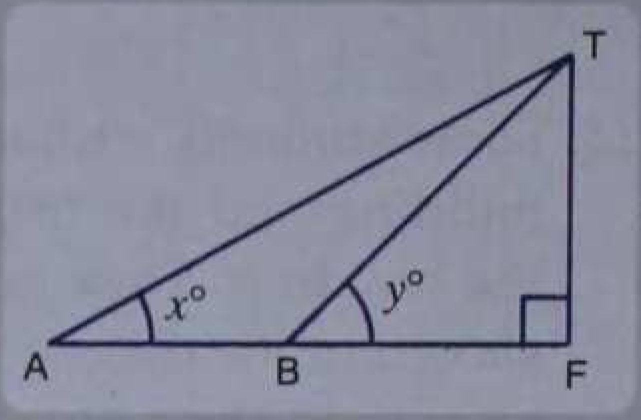
Altitude of the sun means angle of elevation of the sun.

19. From the top of a hill, the angles of depression of two consecutive kilometre stones, due east are found to be 30° and 45° respectively. Find the distance of two stones from the foot of the hill. *(2007)*

20. A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m, the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre. *(2011)*

21. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 m towards the tower, the tangent of the angle is found to be $\frac{3}{4}$. Find the height of the tower.

22. In the figure, not drawn to scale, TF is a tower. The elevation of T from A is $x°$ where $\tan x = \frac{2}{5}$ and AF = 200 m. The elevation of T from B, where AB = 80 m, is $y°$. Calculate :

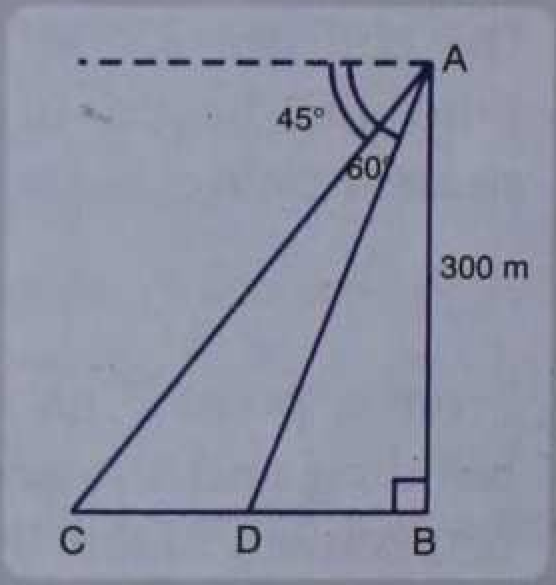


(*i*) the height of the tower TF.

(*ii*) the angle y, correct to the nearest degree.

23. From the top of a church spire 96 m high, the angles of depression of two vehicles on a road, at the same level as the base of the spire and on the same side of it are $x°$ and $y°$, where $\tan x° = \frac{1}{4}$ and $\tan y° = \frac{1}{7}$. Calculate the distance between the vehicles.

24. In the adjoining figure, not drawn to the scale, AB is a tower and two objects C and D are located on the ground, on the same side of AB. When observed from the top A of the tower, their angles of depression are 45° and 60°. Find the distance between the two objects, if the height of the tower is 300 m. Give your answer to the nearest metre.



25. The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower, when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, find the height of the first tower.

26. As observed from the top of a 80 m tall light house, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre. *(2012)*

27. The angle of elevation of a pillar from a point A on the ground is 45° and from a point B diametrically opposite to A and on the other side of the pillar is 60°. Find the height of the pillar, given that the distance between A and B is 15 m.

28. From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10 m, find the distance between A and B correct to two decimal places. *(2009)*

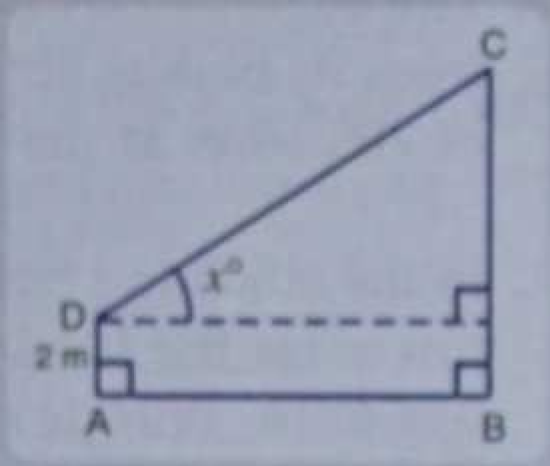
29. The angles of depression of two boats on a river from the top of a tower on the bank of the river are 30° and 50°. If the height of the tower is 30 m and the boats are in line with the tower on the same side of it, find the distance between the boats.

30. From a tower 126 m high, the angles of depression of two rocks which are in a horizontal line through the base of the tower are 16° and 12° 20′. Find the distance between the rocks if they are on

(*i*) the same side of the tower (*ii*) the opposite sides of the tower.

31. Two pillars of equal height stand on either side of a roadway which is 120 m wide. At a point in the road between pillars, the elevations of the pillars are 60° and 30°. Find the height of the pillars and the position of the point.

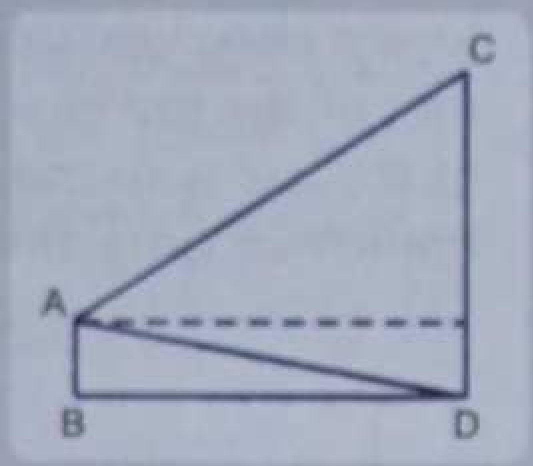
32. With reference to the figure given alongside, a man stands on the ground at point A, which is on the same horizontal plane as B, the foot of the vertical pole BC. The height of the pole is 10 m. The man's eye is 2 m above the ground. He observes the angle of elevation at C, the top of the pole as x°, where $\tan x^\circ = \frac{2}{5}$. Calculate :



(*i*) the distance AB in metres.

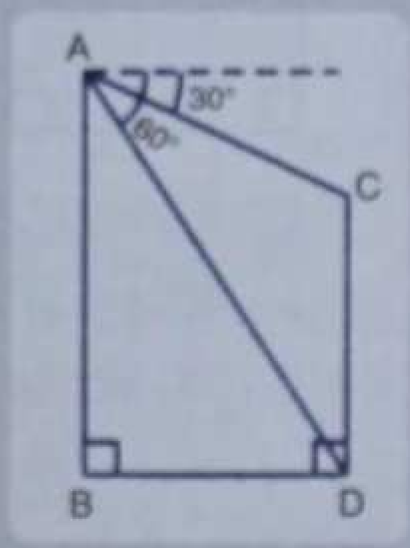
(*ii*) the angle of elevation of the top of the pole when he is standing 15 m from the pole. Give your answer to the nearest degree.

33. From a window A, 10 m above ground the angle of elevation of the top C of a tower is x°, where $\tan x^\circ = \frac{5}{2}$ and the angle of depression of the foot D of the tower is y°, where $\tan y^\circ = \frac{1}{4}$ (see the adjoining figure). Calculate the height CD of the tower in metres. *(2000)*



34. A man 1·8 m high stands at a distance of 3·6 m from a lamp post and casts a shadow of 5·4 m on the ground. Find the height of the lamp post.

35. In the figure given alongside, from the top of a building AB, 60 metres high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :



(*i*) the horizontal distance between AB and CD.

(*ii*) the height of the lamp post. *(2013)*

36. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings, correct to two decimal places.

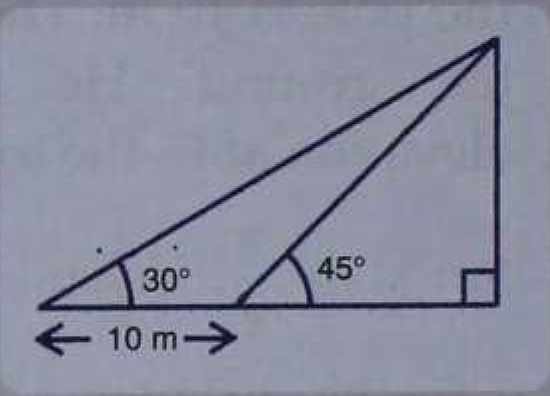
37. A pole of height 5 m is fixed on the top of a tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of the tower. (Take $\sqrt{3} = 1{\cdot}732$).

38. A vertical pole and a vertical tower are on the same level ground. From the top of the pole, the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m. *(2008)*

39. From the top of a building 20 m high, the angle of elevation of the top of a monument is 45° and the angle of depression of its foot is 15°. Find the height of the monument.

40. The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same point may be 60° ?

41. In the adjoining figure, the shadow of a vertical tower on the level ground increases by 10 m, when the altitude of the sun changes from 45° to 30°. Find the height of the tower and give your answer, correct to $\frac{1}{10}$ of a metre. (2002)



42. From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 30° and 45°. Find the height of the hill in km correct to two places of decimal.

43. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

CHAPTER TEST

1. The angle of elevation of the top of a tower from a point A (on the ground) is 30°. On walking 50 m towards the tower, the angle of elevation is found to be 60°. Calculate :

   (*i*) the height of the tower (correct to one decimal place).

   (*ii*) the distance of the tower from A.

2. An aeroplane 3000 m high, passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes.

3. A 7 m long flagstaff is fixed on the top of a tower. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 45° and 36° respectively. Find the height of the tower correct to one place of decimal.

4. A boy, 1·6 m tall, is 20 m away from a tower and observes that the angle of elevation of the top of the tower is 60°. Find the height of the tower.

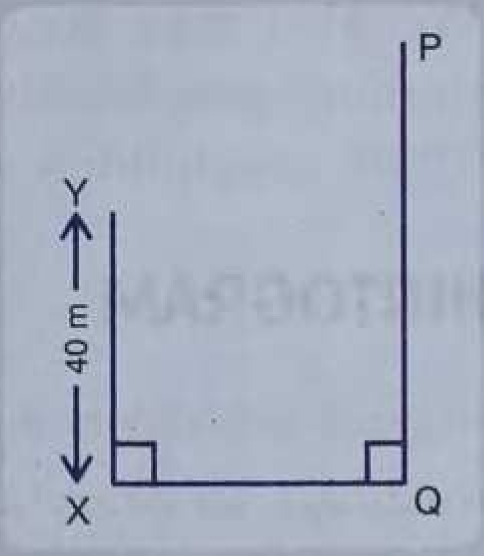
5. A boy 1·54 m tall can just see the sun over a wall 3·64 m high which is 2·1 m away from him. Find the angle of elevation of the sun.

6. In the adjoining figure, the angle of elevation of the top P of a vertical tower from a point X is 60°; at a point Y, 40 m vertically above X, the angle of elevation is 45°. Find :

   (*i*) the height of the tower PQ

   (*ii*) the distance XQ.

   (Give your answer to the nearest metre)



7. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/hr.

8. A man on the deck of a ship is 16 m above the water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

9. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point A on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of the tower.

10. There is a small island in between a river 100 metres wide. A tall tree stands on the island. P and Q are points directly opposite each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are 30° and 45° respectively, find the height of the tree.

11. The angles of elevation of the top of a tower from two points P and Q at distances a and b respectively, from the base and in the same straight line with it, are complementary. Prove that the height of the tower is $\sqrt{ab}$.