CHAPTER 17

FORMULA: CHANGE OF SUBJECT OF A FORMULA
(INCLUDING SUBSTITUTION)

17.1 FORMULA

A formula is an equation, which shows the relationship between two or more quantities (variables).

Infact, a formula is a translation from words to symbols.

For example :

1. Formula showing the relationship between the area of rectangle (**A**), its length (***l***) and its breadth (***b***) is :

$$A = l \times b$$

2. Formula showing the relationship between the distance (***d***) covered by a body in time (***t***) and with velocity (***v***) is :

$$d = v \times t$$

Example 1 :

Frame a formula for each of the following :

(i) Seven more than a certain number is twenty.

(ii) After seven years, the age of father will be three times the age of his son.

Solution :

(i) Let the number be x.

Then the formula for the given statement is :

$x + 7 = 20$ **(Ans.)**

(ii) Let the present age of father = x years and the present age of son = y years.

After 7 years :

Father's age will be $x + 7$ years and son's age will be $y + 7$ years.

$\therefore$ Formula for the given statement is : $x + 7 = 3(y + 7)$ **(Ans.)**

Example 2 :

On monday, Rohit worked at a shop for 11 hours, out of which 8 hours was normal work and remaining hours was overtime. If he gets ₹ x for each hour of normal work and ₹ $2x$ for each hour of overtime, find (in terms of x) the total amount of money he gets on monday.

Solution :

$\because$ Number of hours of normal work = 8 hours

and, for each hour of normal work, he gets ₹ x

$\therefore$ Money, he gets for 8 hours of normal work = $8 \times$ ₹ x = ₹ $8x$

$\because$ Number of hours of overtime = (11 – 8) hours = 3 hours

and, for each hour of overtime, he gets ₹ $2x$

$\therefore$ Money, he gets for 3 hours of overtime = $3 \times$ ₹ $2x$ = ₹ $6x$

$\Rightarrow$ **The total money he gets on monday** = ₹ $8x$ + ₹ $6x$ = **₹ $14x$** **(Ans.)**

EXERCISE 17(A)

Frame a formula for each of the following statements :

1. D is the number of days in w weeks and p days.
2. Twelve less than thrice a certain number is twenty-four.
3. Half of a number added to $\frac{1}{3}$ of the same number is 10.
4. When two is subtracted from twice of a certain number, the result is twenty-two.
5. If five is subtracted from a certain number and the difference is divided by fifteen, the result is three.
6. If a number is multiplied by nine and then two is subtracted from it, the result is 88.
7. The sum of three consecutive integers is seventy-eight.
8. The sum of three consecutive odd integers is fifty-seven.
9. Ajay went to a market with ₹ 500. He buys a tennis ball for ₹ 10 and spends ₹ 75 on a racket plus ₹ 5 on conveyance. He still has ₹ x left.
10. A worker is paid ₹ 3 per hour for normal work and double this rate for overtime. Form a formula to find his earnings in a week (6 days) of 8 hours per day of normal work plus total overtime during this week being ten hours.
11. The final velocity *(v)* of a body is the sum of its initial velocity *(u)* and the product of acceleration produced *(a)* and time *(t)*.
12. A taxi, in Delhi, charges ₹ 23 for the first kilometre and then ₹ 12 per kilometre for the remaining distance. Form an equation, if the taxi-driver charges ₹ 203 for a distance of x kilometre.
13. Eight years hence, Geeta will be twice as old as her age 5 years ago. Taking Geeta's present age as x years, form an equation in terms of x.
14. Mr. Verma is an officer in a Central Government office, which works for 5 days in a week. Mrs. Verma is also an officer in a State Government office which works for 6 days in a week. If per day earning of Mr. Verma is ₹ 280 more than that of Mrs. Verma; form an equation to find one week's earnings of Mrs. and Mr. Verma. Assume that one day earning of Mr. Verma is ₹ x.

17.2 CHANGING THE SUBJECT OF A FORMULA

The subject of a formula is the variable which is expressed in terms of other variables.

For example :

1. In formula, $A = l \times b$. A is expressed in terms of l and b, so, A is the subject of the formula.
2. In $I = \frac{P \times R \times T}{100}$, I is expressed in terms of P, R and T, so, I is the subject of the formula.

To change the subject of a given formula means to obtain a formula for a particular (required) quantity.

For example :

1. The formula $A = l \times b$ can be re-written as :
   (i) $l = \frac{A}{b}$; here l is the subject of the formula.
   (ii) $b = \frac{A}{l}$; here b is the subject of the formula.

2. The formula $I = \frac{P \times R \times T}{100}$ can be re-written as :

(i) $P = \frac{I \times 100}{R \times T}$; here P is the subject of the formula.

(ii) $R = \frac{I \times 100}{P \times T}$; here R is the subject of the formula and so on.

For changing the subject of a given formula, we use the same steps as are used in solving the equations.

Example 3 :

Given : $p = 2l + 2b$. Make b the subject.

Solution :

$p = 2l + 2b \Rightarrow p - 2l = 2l + 2b - 2l$ [Subtracting $2l$ from both the sides]

$\Rightarrow p - 2l = 2b$

$\Rightarrow \frac{p - 2l}{2} = \frac{2b}{2}$ [Dividing each side by 2]

$\Rightarrow \frac{p - 2l}{2} = b$ or, $b = \frac{p - 2l}{2}$ **(Ans.)**

Example 4 :

Given : $a = \frac{b + c}{m}$. Make c the subject.

Solution :

$a = \frac{b + c}{m} \Rightarrow am = b + c$

or $b + c = am$

$\Rightarrow c = am - b$ **(Ans.)**

EXERCISE 17(B)

Change the subject for the following formulae to the indicated letter (variable) :

1. $x + 2y = m$; for y
2. $v^2 = u^2 + 2as$; for s
3. $A = \frac{1}{2}(a + b)h$; for h
4. $s = \frac{n}{2}(a + l)$; for l
5. $C = \frac{5}{9}(F - 32)$; for F
6. $F = \frac{9}{5}C + 32$; for C
7. $A = p(a + rt)$; for p
8. $s = ut + \frac{1}{2}at^2$; a
9. $s = \frac{n}{2}\{2a + (n - 1)d\}$; d
10. $a = \frac{x - y}{x + y}$; x
11. $\frac{m - a}{m + b} = \frac{2c}{3d}$; m
12. $\frac{5x + 8y}{3y - x} = 2a$; y

17.3 TO EVALUATE THE UNKNOWN, USING CHANGE OF SUBJECT OF FORMULA AND SUBSTITUTION METHODS

Steps : 1. Change, if required, the formula to the required subject.

2. In the new formula, substitute the values of the given quantities and simplify.

Example 5 :

Given : $\frac{m + c}{m} = x$, find c, if $x = 5$ and $m = 10$.

Solution :

Step 1 : $\frac{m + c}{m} = x \Rightarrow m + c = mx$

$\Rightarrow \boldsymbol{c = mx - m}$

Step 2 : Substituting $x = 5$ and $m = 10$;

we get : $\boldsymbol{c} = 10 \times 5 - 10 = \boldsymbol{40}$ **(Ans.)**

| Since, we are not asked to form a formula for c, it can be done directly. So,

$\frac{m + c}{m} = x \Rightarrow \frac{10 + c}{10} = 5$

$\Rightarrow 10 + c = 50$

$\Rightarrow c = 50 - 10$

$= \boldsymbol{40}$ **(Ans.)**

EXERCISE 17(C)

1. $C = \frac{5}{9}(F - 32)$. Find F, if C = 40.

2. $V = \frac{1}{3}\pi r^2 h$. Find h, if $V = 110\ cm^3$ and $r = 4$ cm.

3. $A = \frac{1}{2}(l + b)h$. Find b, if $A = 60$, $l = 6$ and $h = 10$.

4. $t = 4\sqrt{\frac{h}{32}}$; express h in terms of t. Then calculate h, if $t = 12$.

5. In the formula $p = \pi r + 2r$, make r the subject. Hence, find r, if $p = 40$ and $\pi = 3{\cdot}142$.

6. If $2y = \frac{x + 3}{x - 1}$ and $y = 3$. Find x.

7. $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$. Find v when $u = 15$ and $f = 5$.

8. If $x = 3$ and $y = -1$; find z, if :

   (i) $z = (x + y)^2 - 5(x - y)$

   (ii) $z = 8xy + x^2 - y^2$

   (iii) $z = x^3 - y^3 - 3x^2y + 3xy^2$

9. Given : $A = 2\pi r(r + h)$, find h, if $A = 2816\ cm^2$, $\pi = 3\frac{1}{7}$ and $r = 14$ cm.

10. Given : $a = 5$, $b = -3$ and $c = 2$. Find m, if :

    (i) $m = abc + a^2 - b^2 + c^2$

    (ii) $m = a^3 + b^3 + c^3 - 5ab - 6bc$