

17

Constructions

17.1 CONSTRUCTION OF TANGENTS

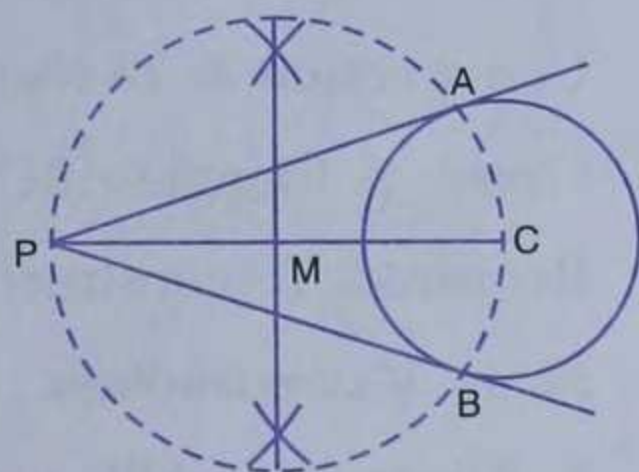
Construction 1. To construct tangents to a given circle from an exterior point.

Given. A circle with centre C and an exterior point P .

Required. To construct tangents from P to the given circle.

Steps of construction.

1. Join CP .
2. Draw a circle with CP as diameter to cut the given circle at the points A and B (to construct a circle with CP as diameter; draw perpendicular bisector of CP , let M be the mid-point of CP , with M as centre and CM (or PM) as radius, draw a circle).
3. Join PA and PB , then PA and PB are the required tangents from the point P to the given circle.



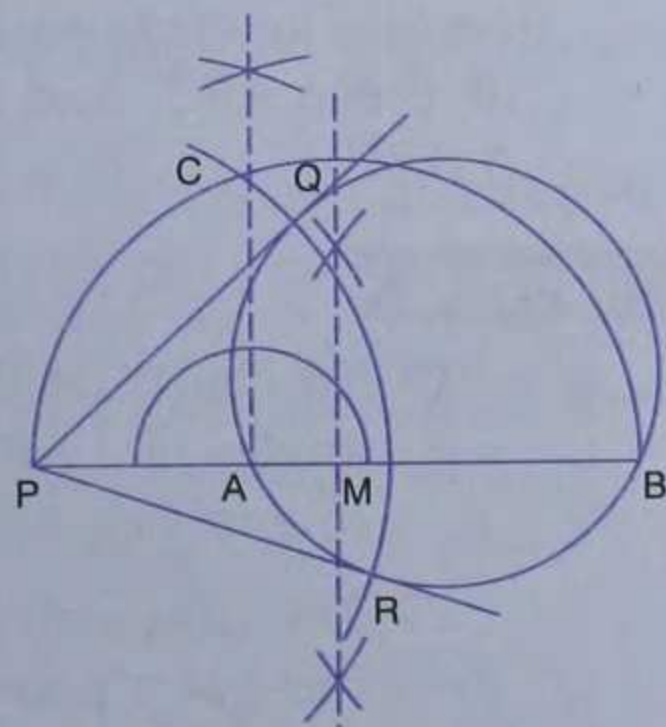
Construction 2. To construct tangents to a given circle from an exterior point when the centre of the circle is not known.

Given. A circle and an exterior point P .

Required. To construct tangents from P to the given circle.

Steps of construction.

1. Draw any secant PAB to the circle.
2. Draw right bisector of PB . Let M be mid-point of PB .
3. Taking M as centre and MP as radius, draw a semicircle.
4. At A , draw a perpendicular to PB . Let this perpendicular meet the semicircle at C .
5. Taking P as centre and CP as radius, draw an arc to meet the given circle at two points, say Q and R .
6. Join PQ and PR . Then PQ and PR are the required tangents from P to the given circle.



17.2 CONSTRUCTION OF CIRCUMSCRIBED AND INSCRIBED CIRCLES

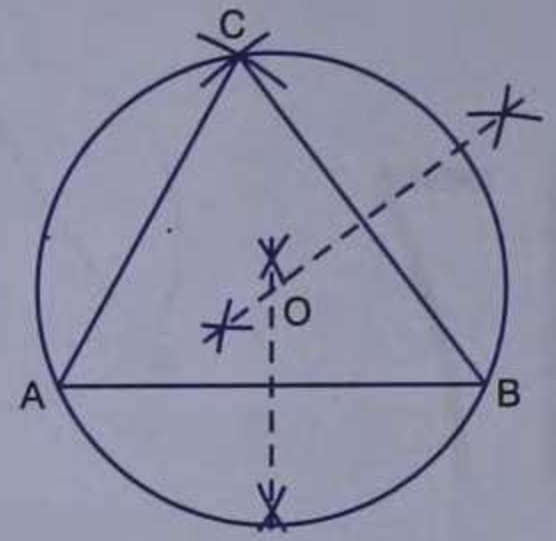
Construction 3. To construct the circumscribed circle of a given triangle.

Given. A triangle with sides $AB = 4.5$ cm, $BC = 4$ cm and $CA = 3.5$ cm.

Required. To construct the circumcircle of ΔABC .

Steps of construction.

1. Construct ΔABC with the given data.
2. Draw the perpendicular bisectors of AB and BC . Let these bisectors meet at the point O .
3. With O as centre and radius equal to OA , draw a circle. The circle so drawn passes through the points A , B and C , and is the required circumcircle of ΔABC .



Remarks

- To construct the circumcircle of ΔABC , draw the perpendicular bisectors of any two sides of ΔABC .
- The perpendicular bisectors of the sides of a triangle are concurrent *i.e.* they meet at a point (the point O of the above construction).
The point O is called the circumcentre of the triangle.
- The circumcentre of the triangle is equidistant from the vertices of the triangle.

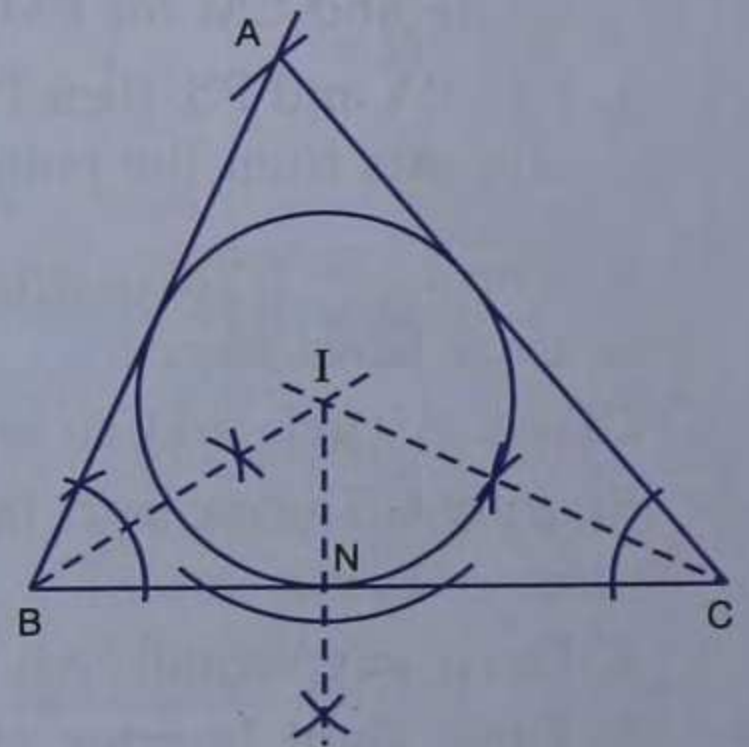
Construction 4. To construct the inscribed circle of a given triangle.

Given. A triangle ABC with $BC = 6.4$ cm, $CA = 5.8$ cm and $\angle B = 60^\circ$.

Required. To construct the incircle of ΔABC .

Steps of construction.

1. Construct ΔABC with the given data.
2. Draw the (internal) bisectors of $\angle B$ and $\angle C$. Let these bisectors meet at the point I .
3. From I , draw IN perpendicular to the side BC .
4. With I as centre and radius equal to IN , draw a circle. The circle so drawn touches all the sides of the ΔABC , and is the required incircle of ΔABC .



Remarks

- To construct the incircle of ΔABC , draw (internal) bisectors of any two angles of ΔABC . Let these bisectors meet at I . From I , draw perpendicular to any side.
- The (internal) bisectors of the angles of a triangle are concurrent *i.e.* they meet at a point (the point I of the above construction).
The point I is called the incentre of the triangle.
- The incentre of the triangle is equidistant from the sides of the triangle.

17.2.1 Construction of circles in and about a regular polygon

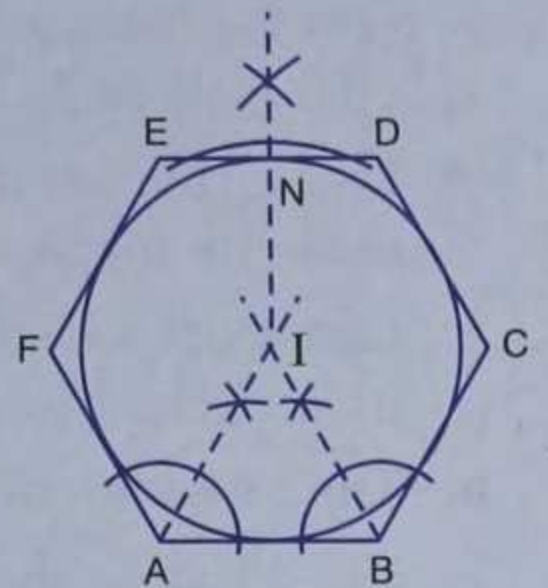
Construction 5. To construct a circle in a given regular hexagon.

Given. A regular hexagon of side 3 cm.

Required. To construct a circle touching the sides of the given regular hexagon.

Steps of construction.

1. Construct regular hexagon ABCDEF with side = 3 cm.
2. Draw bisectors of $\angle A$ and $\angle B$. Let these bisectors meet at the point I.
3. From I, draw IN perpendicular to ED.
4. With I as centre and radius equal to IN, draw a circle. This circle touches all the sides of the hexagon ABCDEF, and is the required circle *in* the regular hexagon.



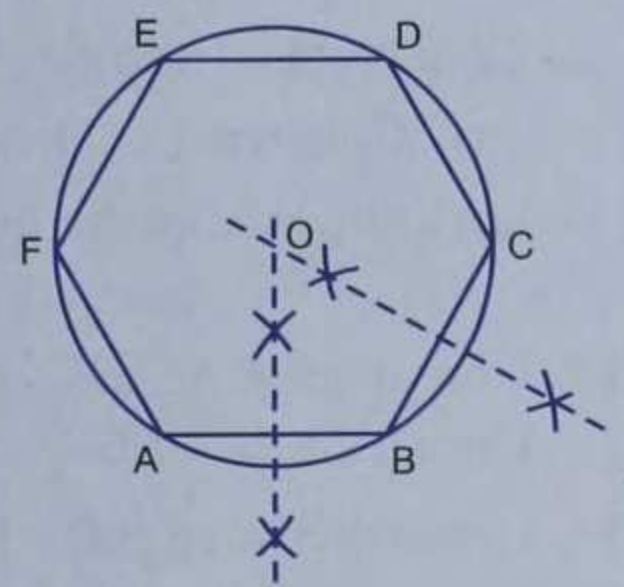
Construction 6. To construct a circle about a given regular hexagon.

Given. A regular hexagon of side 2.5 cm.

Required. To construct a circle passing through the vertices of the given regular hexagon.

Steps of construction.

1. Construct a regular hexagon ABCDEF with side = 2.5 cm.
2. Draw the perpendicular bisectors of the sides AB and BC. Let these bisectors meet at the point O.
3. With O as centre and radius equal to OA, draw a circle. This circle passes through all the vertices of the regular hexagon ABCDEF, and is the required circle *about* the regular hexagon.



Exercise 17

1. Use a ruler and compass only in this question.
 - (i) Draw a circle, centre O and radius 4 cm.
 - (ii) Mark a point P such that $OP = 7$ cm.Construct two tangents to the circle from P. Measure and record the length of one of the tangents.
2. Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent. (2011)
3. Draw a line $AB = 6$ cm. Construct a circle with AB as diameter. Mark a point P at a distance of 5 cm from the mid-point of AB. Construct two tangents from P to the circle with AB as diameter. Measure the length of each tangent.
4. Draw an equilateral triangle of side 4 cm. Draw its circumcircle.
5. Using a ruler and a pair of compasses only, construct :
 - (i) a triangle ABC, given $AB = 4$ cm, $BC = 6$ cm and $\angle ABC = 90^\circ$.
 - (ii) a circle which passes through the points A, B and C and mark its centre as O. (2008)

6. Construct a triangle with sides 3 cm, 4 cm and 5 cm. Draw its circumcircle and measure its radius.
7. Using ruler and compasses only :
- Construct a triangle ABC with the following data :
Base AB = 6 cm, AC = 5.2 cm and $\angle CAB = 60^\circ$.
 - In the same diagram, draw a circle which passes through the points A, B and C, and mark its centre O.
8. Using ruler and compasses only, draw an equilateral triangle of side 5 cm and draw its inscribed circle. Measure the radius of the circle.
9. Construct a triangle ABC with BC = 6.4 cm, CA = 5.8 cm and $\angle ABC = 60^\circ$. Draw its incircle. Measure and record the radius of incircle. (2007)
10. The bisectors of angles A and B of a scalene triangle ABC meet at O.
- What is the point O called ?
 - OR and OQ are drawn perpendicular to AB and CA respectively. What is the relation between OR and OQ ?
 - What is the relation between $\angle ACO$ and $\angle BCO$?
11. Using ruler and compasses only, construct a triangle ABC in which BC = 4 cm, $\angle ACB = 45^\circ$ and the perpendicular from A on BC is 2.5 cm. Draw the circumcircle of triangle ABC and measure its radius.
12. Construct a triangle ABC in which base BC = 6 cm, AB = 5.5 cm and $\angle ABC = 120^\circ$.
- Construct a circle circumscribing the triangle ABC.
 - Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C. (2012)
13. Construct a ΔABC , given that AB = 4.5 cm, BC = 7 cm and median AD = 4 cm. Construct inscribed circle of ΔABC and measure its radius.
14. Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon. (2010)
15. Draw a regular hexagon of side 3.5 cm. Construct its circumcircle. Measure and record its radius.
16. Draw a regular hexagon of side 4 cm and construct its incircle.

CHAPTER TEST

1. Draw a circle of radius 3 cm. Mark its centre as C and mark a point P such that $CP = 7$ cm. Using ruler and compasses only, construct two tangents from P to the circle.
2. Using ruler and compasses only, construct a triangle ABC having given $c = 6$ cm, $b = 7$ cm and $\angle A = 30^\circ$. Measure side a . Draw the circumcircle of the triangle.
3. Using ruler and compasses only, construct an equilateral triangle of height 4 cm and draw its circumcircle.
4. Using ruler and compasses only :
 - (i) Construct a triangle ABC with the following data :
 $BC = 7$ cm, $AB = 5$ cm and $\angle ABC = 45^\circ$.
 - (ii) Draw the inscribed circle to ΔABC drawn in part (i).
5. Draw a triangle ABC, given that $BC = 4$ cm, $\angle C = 75^\circ$ and that radius of circumcircle of ΔABC is 3 cm.

Hint

Draw perpendicular bisector of BC. With B as centre and radius 3 cm, draw an arc to meet the bisector at O. With O as centre and radius 3 cm, draw a circle. At C, construct $\angle BCD = 75^\circ$ to meet the circle at A. Then ABC is the required triangle.

6. Draw a regular hexagon of side 2.8 cm. Construct its circumcircle and measure its radius.
7. Draw a line $AQ = 7$ cm. Mark a point P on AQ such that $AP = 4$ cm. Using ruler and compasses only, construct :
 - (i) a circle with AP as diameter
 - (ii) two tangents to the above circle from the point Q.
8. Construct a triangle ABC with the following data:
 $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = 90^\circ$.
 - (i) Find a point P which is equidistant from B and C and is 5 cm from A. How many such points are there ?
 - (ii) Construct the inscribed circle of ΔABC drawn above.