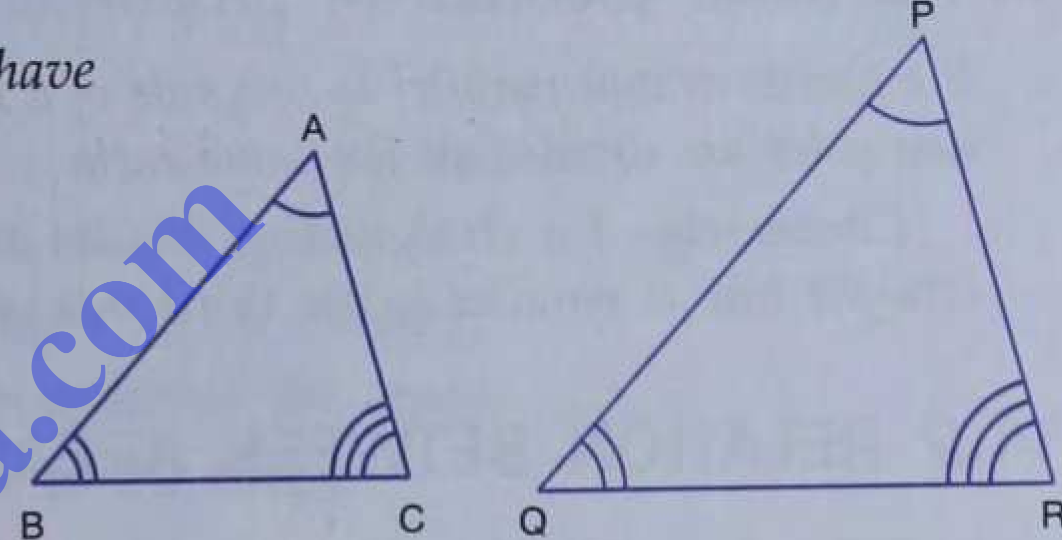
14 Similarity

14.1 SIMILARITY OF TRIANGLES

*Two triangles are called **similar** if and only if they have the same shape, but not necessarily the same size.*



If two triangles ABC and PQR are similar, written as $\Delta ABC \sim \Delta PQR$, then we shall find that

(*i*) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

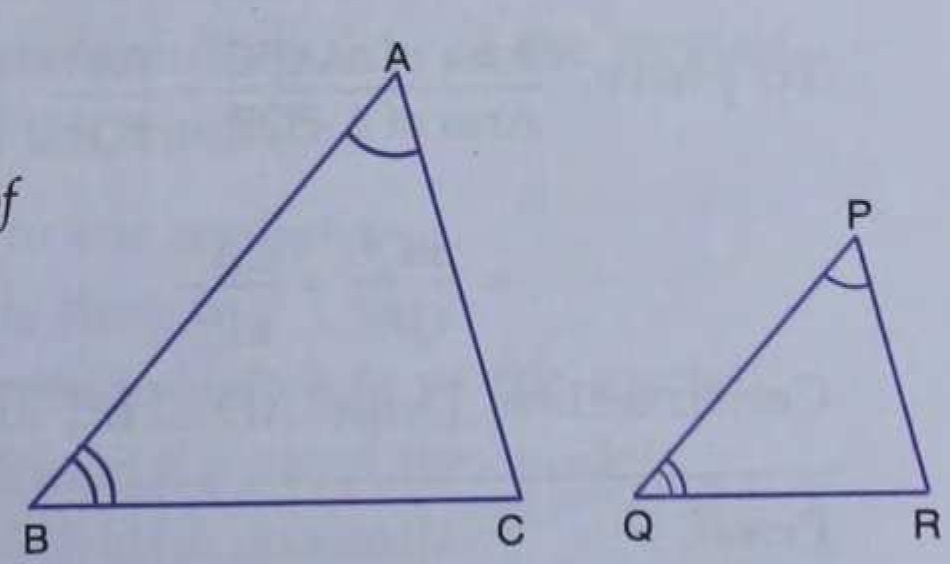
(*ii*) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$.

Thus, when two triangles are similar, then their corresponding angles are equal and corresponding sides are proportional.

14.1.1 Axioms of similarity of triangles

1. A.A. (Angle-Angle) axiom of similarity

If two angles of a triangle are equal to two angles of another triangle, then the two triangles are similar.



In the adjoining diagram,

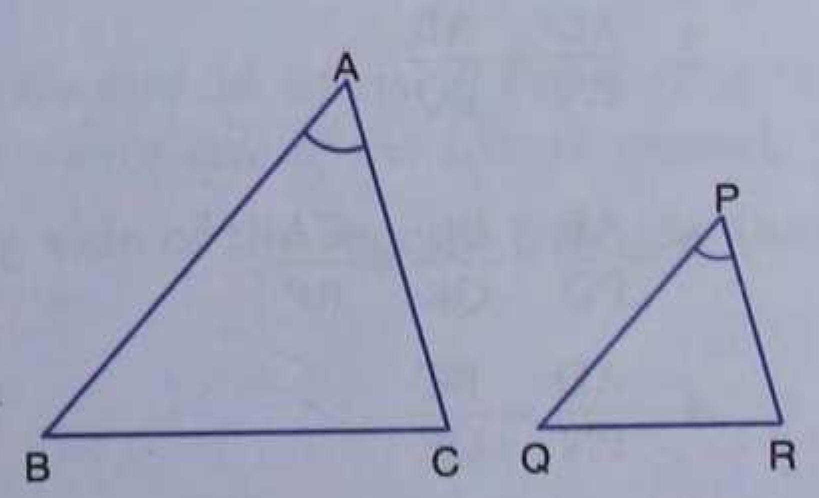
Δs ABC and PQR are such that

$\angle A = \angle P$ and $\angle B = \angle Q$,

$\therefore \ \Delta ABC \sim \Delta PQR$.

2. S.A.S. (Side-Angle-Side) axiom of similarity

If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles are similar.



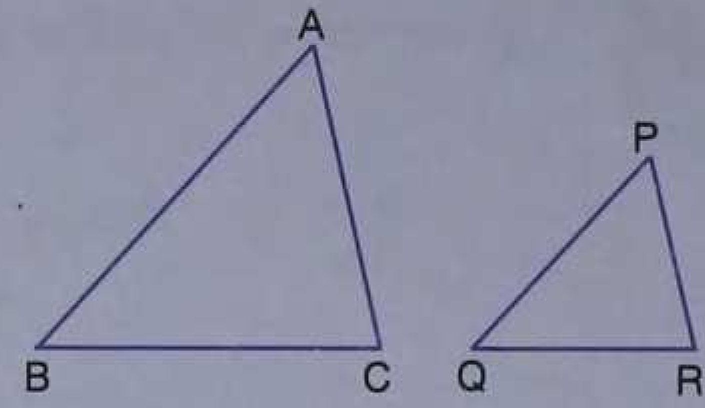
In the adjoining diagram, Δs ABC and PQR are such that

$\angle A = \angle P$ and $\frac{AB}{PQ} = \frac{AC}{PR}$,

$\therefore \ \Delta ABC \sim \Delta PQR$.

3. S.S.S. (Side-Side-Side) axiom of similarity

If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.



In the adjoining diagram, Δs ABC and PQR are such that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP},$$

$\therefore\ \Delta ABC \sim \Delta PQR$.

Remark

- If two angles of one triangle are respectively equal to the two angles of another triangle, then their third angles are necessarily equal, because the sum of three angles of a triangle is always 180°.
- Congruent triangles are necessarily similar but the similar triangles may not be congruent.
- If two triangles are similar to a third triangle, then they are similar to each other.

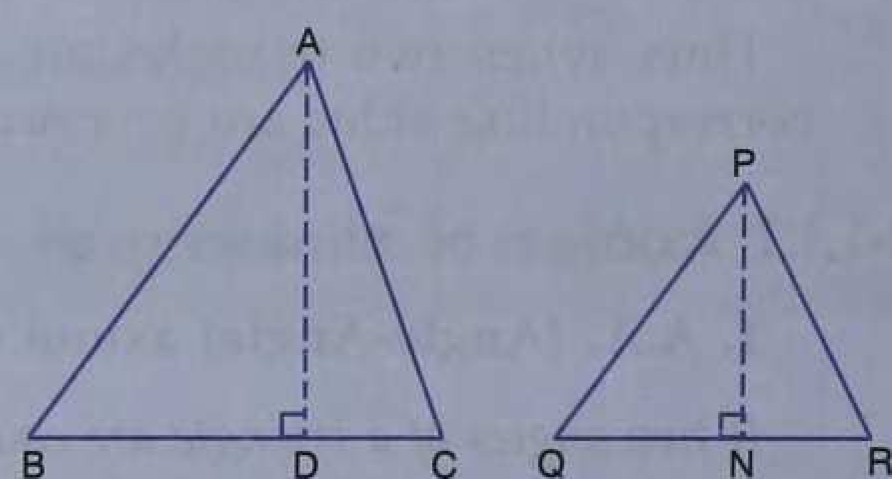
14.1.2 Basic theorem of proportionality

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

Conversely, if a straight line divides any two sides of a triangle in the same ratio, then the straight line is parallel to the third side of the triangle.

14.2 RELATION BETWEEN AREAS OF SIMILAR TRIANGLES

†Theorem 23. *The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.*



Given. $\Delta ABC \sim \Delta PQR$.

To prove. $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \dfrac{AB^2}{PQ^2}$

$$= \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}.$$

Construction. Draw $AD \perp BC$ and $PN \perp QR$.

Proof. *Statements*	*Reasons*
1. $\Delta ABD \sim \Delta PQN$	1. $\angle B = \angle Q$ (since $\Delta ABC \sim \Delta PQR$) and $\angle ADB = \angle PNQ$ ($\because AD \perp BC, PN \perp QR$)
2. $\dfrac{AD}{PN} = \dfrac{AB}{PQ}$	2. Corres. sides of similar Δs are proportional.
3. $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$	3. $\Delta ABC \sim \Delta PQR$.
4. $\dfrac{AD}{PN} = \dfrac{BC}{QR}$	4. From 2 and 3.

† We have continued the number of theorems from Understanding I.C.S.E. Mathematics Class IX.

5. $\dfrac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ PQR}} = \dfrac{\frac{1}{2}.\text{BC} \times \text{AD}}{\frac{1}{2}.\text{QR} \times \text{PN}}$	5. Area of $\Delta = \frac{1}{2}$ base $\times$ height.
$= \dfrac{\text{BC}}{\text{QR}} \cdot \dfrac{\text{AD}}{\text{PN}} = \dfrac{\text{BC}}{\text{QR}} \cdot \dfrac{\text{BC}}{\text{QR}} = \dfrac{\text{BC}^2}{\text{QR}^2}$	$\dfrac{\text{AD}}{\text{PN}} = \dfrac{\text{BC}}{\text{QR}}$ (proved above)
6. $\dfrac{\text{BC}^2}{\text{QR}^2} = \dfrac{\text{AB}^2}{\text{PQ}^2} = \dfrac{\text{CA}^2}{\text{RP}^2}$	6. Using 3
$\therefore \dfrac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ PQR}} = \dfrac{\text{AB}^2}{\text{PQ}^2} = \dfrac{\text{BC}^2}{\text{QR}^2} = \dfrac{\text{CA}^2}{\text{RP}^2}$.	**Q.E.D.**

Corollary. *The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding altitudes.*

Proof. By the above theorem,

$$\frac{\text{area of } \Delta \text{ ABC}}{\text{area of } \Delta \text{ PQR}} = \frac{\text{AB}^2}{\text{PQ}^2} \quad \ldots(i) \qquad \text{(See figure of theorem 23)}$$

Also $\dfrac{\text{AD}}{\text{PN}} = \dfrac{\text{AB}}{\text{PQ}}$ ...(*ii*) (From step 2 of above theorem)

From (*i*) and (*ii*), we get $\dfrac{\text{area of } \Delta \text{ ABC}}{\text{area of } \Delta \text{ PQR}} = \dfrac{\text{AD}^2}{\text{PN}^2}$.

MAPS

The map of a plane figure and the actual figure are similar to one another.

If the map of a plane figure is drawn to the scale 1 : k, then

(*i*) the length of the actual figure = $k \times$ (length of the map).

(*ii*) the breadth of the actual figure = $k \times$ (breadth of the map).

(*iii*) the area of the actual figure = $k^2 \times$ (area of the map).

Scale factor. *The number k is called scale factor.*

MODELS

The model of a plane figure and the actual figure are similar to one another.

If the model of a plane figure is drawn to the scale 1 : k, then

(*i*) each side of the actual figure = $k \times$ (the corresponding side of the model).

(*ii*) the area of the actual figure = $k^2 \times$ (area of the model).

The model of a solid and the actual solid are similar to one another.

If the model of a solid is drawn to the scale 1 : k, then

(*i*) each side of the actual solid = $k \times$ (the corresponding side of the model).

(*ii*) the surface area of the actual solid = $k^2 \times$ (surface area of the model).

(*iii*) the volume of the actual solid = $k^3 \times$ (volume of the model).

ILLUSTRATIVE EXAMPLES

Example 1. *The areas of two similar triangles are 25 sq cm and 36 sq cm. If one side of the first triangle is 3 cm long, what is the length of the corresponding side of the second triangle ?*

Solution. Let x cm be the length of the corresponding side of the second triangle, then by theorem 23,

$$\frac{x^2}{3^2} = \frac{36}{25} \Rightarrow x^2 = \frac{36}{25} \times 9 \Rightarrow x = \frac{18}{5} = 3\frac{3}{5}.$$

$\therefore$ The length of the corresponding side of the second triangle = $3\frac{3}{5}$ cm.

Example 2. *The corresponding altitudes of two similar triangles are 5 cm and 7 cm respectively. Find the ratio of their areas.*

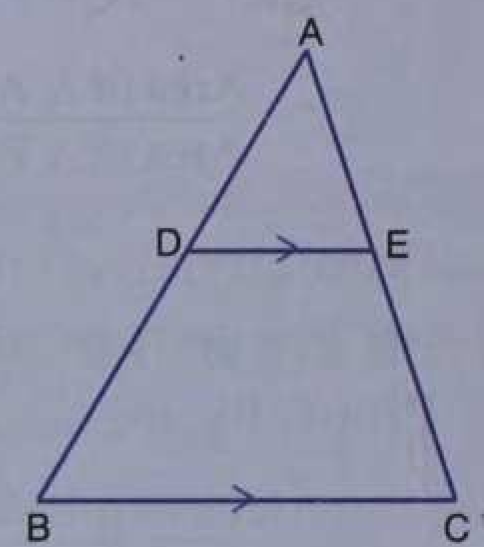
Solution. By corollary to theorem 23, the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding altitudes,

$\therefore$ the ratio of their areas $= \frac{5^2}{7^2} = \frac{25}{49}$ *i.e.* 25 : 49.

Example 3. *In the adjoining figure, DE ∥ BC and* $\frac{AD}{DB} = \frac{2}{3}$. *Calculate the value of :*

(i) $\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC}$.

(ii) $\frac{\text{area of trapezium DBCE}}{\text{area of } \Delta ABC}$.



Solution. Given $\frac{AD}{DB} = \frac{2}{3} \Rightarrow \frac{AD}{AD + DB} = \frac{2}{2+3}$

$\Rightarrow \quad \frac{AD}{AB} = \frac{2}{5}$ (See figure)

In Δs ADE and ABC,

$\angle ADE = \angle ABC$ ($\because$ DE ∥ BC, corres. ∠s are equal)

and $\angle A = \angle A$

$\Rightarrow \quad \Delta ADE \sim \Delta ABC$. (A.A. axiom of similarity)

(i) As the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$\therefore \quad \frac{\text{area of } \Delta \text{ ADE}}{\text{area of } \Delta \text{ ABC}} = \frac{AD^2}{AB^2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$.

(ii) From (i), we get

$4 \,.\, \text{area of } \Delta \text{ ABC} = 25 \,.\, \text{area of } \Delta \text{ ADE}$

$= 25.(\text{area of } \Delta \text{ ABC} - \text{area of trapezium DBCE})$

$\Rightarrow \quad 25 \,.\, \text{area of trapezium DBCE} = 21 \,.\, \text{area of } \Delta \text{ ABC}$

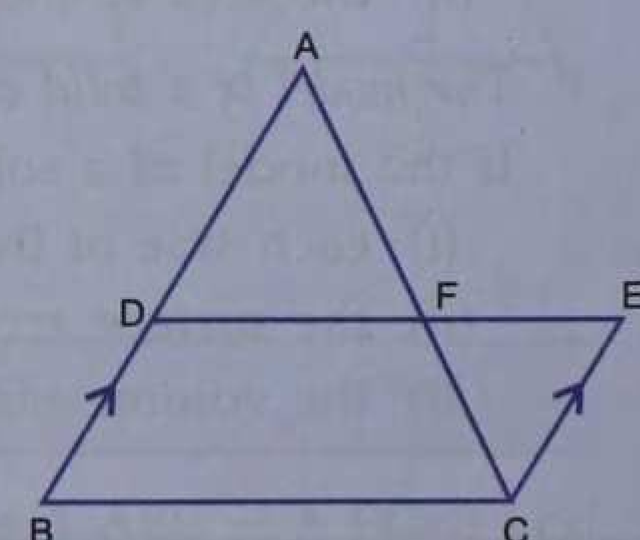
$\Rightarrow \quad \frac{\text{area of trapezium DBCE}}{\text{area of } \Delta \text{ ABC}} = \frac{21}{25}$.

Example 4. *In the adjoining figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.*

(i) Prove that ΔADF ~ ΔCEF.

(ii) Find AD if CE = 6 cm.

(iii) If DF is parallel to BC, find area of ΔADF : area of ΔABC. (2009)



Solution.

(i) In ΔADF and ΔCEF

$\angle AFD = \angle CFE$ (Vert. opp. ∠s)

and $\angle DAF = \angle ECF$ ($\because$ BA ∥ CE, alt. ∠s)

$\therefore \quad \Delta ADF \sim \Delta CEF$ (A.A. axiom of similarity)

(ii) Given AF : AC = 5 : 8 $\Rightarrow \frac{AF}{AC} = \frac{5}{8}$

$\Rightarrow \quad 8\,AF = 5\,AC \Rightarrow 8AF = 5\,(AF + FC)$

$\Rightarrow \quad 3\,AF = 5\,FC \Rightarrow \frac{AF}{FC} = \frac{5}{3}$.

As $\Delta ADF \sim \Delta CEF$ (From part (*i*))

$$\frac{AD}{CE} = \frac{AF}{FC} \Rightarrow \frac{AD}{6\text{ cm}} = \frac{5}{3} \Rightarrow AD = \frac{5}{3} \times 6\text{ cm} = 10\text{ cm}.$$

(*iii*) Given DF is parallel to BC,

$\therefore \angle ADF = \angle ABC$ (corres. $\angle$s are equal)

In ΔADF and ΔABC

$\angle A = \angle A$ (common)

and $\angle ADF = \angle ADC$ (proved above)

$\therefore \Delta ADF \sim \Delta ABC$ (A.A. axiom of similarity)

$$\therefore \frac{\text{area of } \Delta ADF}{\text{area of } \Delta ABC} = \frac{AF^2}{AC^2} \quad \text{(theorem 23)}$$

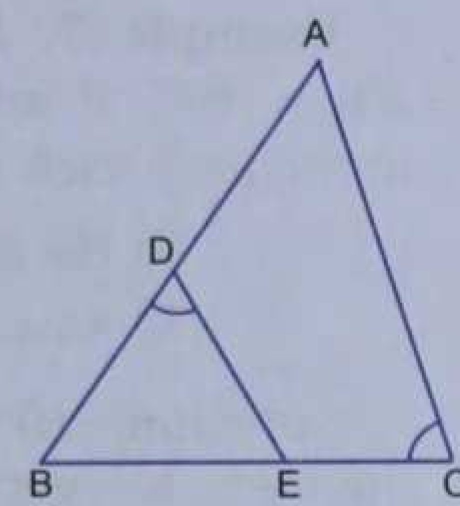
$$= \left(\frac{AF}{AC}\right)^2 = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

$\Rightarrow$ area of ΔADF : area of ΔABC = 25 : 64.

Example 5. *In the given figure, ABC is a triangle with $\angle EDB = \angle ACB$. If BE = 6 cm, EC = 4 cm, BD = 5 cm and area of $\Delta BED = 9\text{ cm}^2$, calculate*

(i) the length of AB

(ii) the area of ΔABC. (2010)



Solution. From figure, BC = BE + EC = 6 cm + 4 cm = 10 cm.

In Δs ABC and EBD

$\angle ACB = \angle EDB$ (given)

and $\angle B$ is common

$\Rightarrow \Delta ABC \sim \Delta EBD$ (A.A. axiom of similarity)

(*i*) $\frac{AB}{BE} = \frac{BC}{BD}$ ($\because \Delta ABC \sim \Delta EBD$)

$$\Rightarrow \frac{AB}{6\text{ cm}} = \frac{10\text{ cm}}{5\text{ cm}} \Rightarrow AB = (2 \times 6)\text{ cm} = 12\text{ cm}.$$

Hence, the length of AB = 12 cm.

(*ii*) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = \frac{BC^2}{BD^2}$ (theorem 23)

$$\Rightarrow \frac{\text{Area of } \Delta ABC}{9\text{ cm}^2} = \left(\frac{BC}{BD}\right)^2 = \left(\frac{10\text{ cm}}{5\text{ cm}}\right)^2 = 4$$

$\Rightarrow$ Area of $\Delta ABC = (4 \times 9)\text{ cm}^2 = 36\text{ cm}^2$.

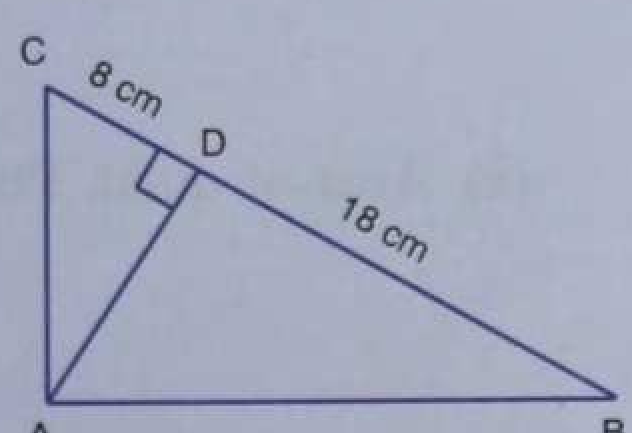
Hence, the area of $\Delta ABC = 36\text{ cm}^2$.

Example 6. *In the adjoining figure, ABC is a right-angled triangle with $\angle BAC = 90°$.*

(i) Prove that $\Delta ADB \sim \Delta CDA$.

(ii) If BD = 18 cm and CD = 8 cm, find AD.

(iii) Find the ratio of the area of ΔADB is to the area of ΔCDA. (2011)



Solution. (*i*) Since $AD \perp BC$, $\angle ADB = 90° = \angle ADC$.

$\therefore \angle C + \angle CAD = 90°$ ($\because$ In ΔACD, $\angle D = 90°$)

Also $\angle CAD + \angle DAB = \angle A = 90°$ (given)

$\Rightarrow \angle CAD + \angle DAB = \angle C + \angle CAD$

$\Rightarrow \angle DAB = \angle C$.

In ΔADB and ΔCDA (each = 90°)

$\angle ADB = \angle ADC$ (proved above)

and $\angle DAB = \angle C$

$\therefore$ $\Delta ADB \sim \Delta CDA$ (A.A. axiom of similarity)

(*ii*) Since $\Delta ADB \sim \Delta CDA$,

$$\frac{BD}{AD} = \frac{AD}{CD} \Rightarrow AD^2 = BD \times CD$$

$\Rightarrow$ $AD^2 = 18 \times 8$ ($\because$ BD = 18 cm, CD = 8 cm given)

$\Rightarrow$ $AD^2 = 144 \Rightarrow AD = 12$ cm.

(*iii*) As the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides and $\Delta ADB \sim \Delta CDA$,

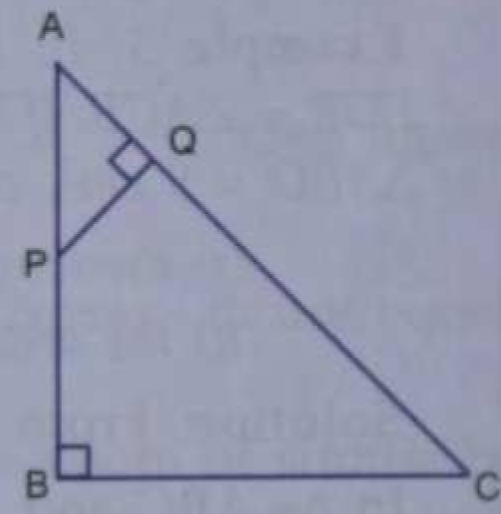
$$\therefore \quad \frac{\text{area of } \Delta ADB}{\text{area of } \Delta CDA} = \left(\frac{BD}{AD}\right)^2 = \left(\frac{18}{12}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$\Rightarrow$ area of ΔADB : area of ΔCDA = 9 : 4.

Example 7. *In ΔABC, AB = 8 cm, AC = 10 cm and $\angle B = 90°$. P and Q are points on the sides AB and AC respectively such that PQ = 2 cm and $\angle PQA = 90°$, find :*

(i) the area of ΔAQP.

(ii) area of quad. PBCQ : area of ΔABC.



Solution. (*i*) In ΔABC, $\angle B = 90°$, by Pythagoras theorem, we get

$BC^2 = AC^2 - AB^2 = 10^2 - 8^2 = 100 - 64 = 36$

$\Rightarrow$ BC = 6 cm.

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AB$

$= \left(\frac{1}{2} \times 6 \times 8\right) \text{cm}^2 = 24 \text{ cm}^2$.

In Δs AQP and ABC,

$\angle PQA = 90° = \angle ABC$ and $\angle A$ is common

$\Rightarrow$ $\Delta AQP \sim \Delta ABC$ (A.A. axiom of similarity)

$\therefore$ $\frac{\text{area of } \Delta AQP}{\text{area of } \Delta ABC} = \frac{PQ^2}{BC^2}$ (theorem 23)

$\Rightarrow$ $\frac{\text{area of } \Delta AQP}{24 \text{ cm}^2} = \left(\frac{PQ}{BC}\right)^2 = \left(\frac{2 \text{ cm}}{6 \text{ cm}}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\Rightarrow$ area of $\Delta AQP = \left(\frac{1}{9} \times 24\right) \text{cm}^2$

$= \frac{8}{3} \text{ cm}^2$.

(*ii*) Area of quad. PBCQ = area of ΔABC – area of ΔAQP (from figure)

$= 24 \text{ cm}^2 - \frac{8}{3} \text{ cm}^2$

$= \left(24 - \frac{8}{3}\right) \text{cm}^2 = \frac{64}{3} \text{ cm}^2$.

$\therefore$ $\frac{\text{area of quad. PBCQ}}{\text{area of } \Delta ABC} = \frac{\frac{64}{3} \text{cm}^2}{24 \text{ cm}^2} = \frac{64}{72} = \frac{8}{9}$

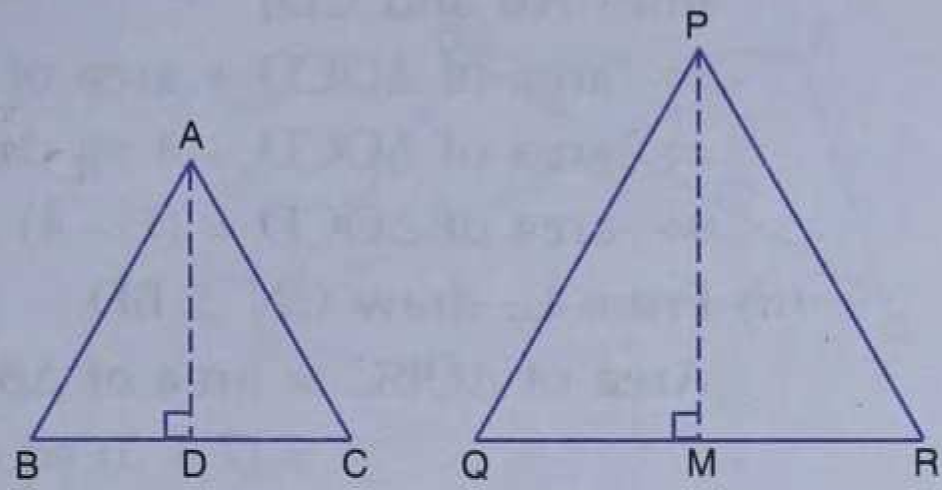
$\Rightarrow$ area of quad PBCQ : area of ΔABC = 8 : 9.

Example 8. *Two isosceles triangles have equal vertical angles and their areas are in the ratio 4 : 9. Find the ratio of their corresponding heights.*

Solution. Let ABC and PQR be two isosceles triangles such that

$AB = AC,\ PQ = PR,\ \angle A = \angle P$

and $\dfrac{\text{area of } \Delta\ ABC}{\text{area of } \Delta\ PQR} = \dfrac{4}{9}$.



Draw $AD \perp BC$ and $PM \perp QR$.

Now, AB = AC and PQ = PR

$\Rightarrow \quad \dfrac{AB}{PQ} = \dfrac{AC}{PR}$

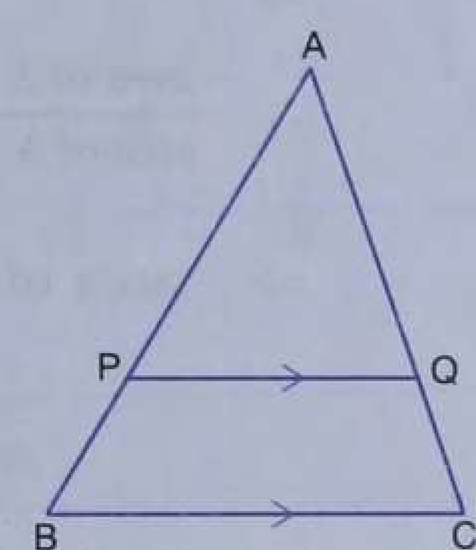
$\Rightarrow \quad \Delta\ ABC \sim \Delta\ PQR$ (S.A.S. axiom of similarity)

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding heights,

$\therefore \quad \dfrac{AD^2}{PM^2} = \dfrac{\text{area of } \Delta\ ABC}{\text{area of } \Delta\ PQR} = \dfrac{4}{9} = \left(\dfrac{2}{3}\right)^2$

$\Rightarrow \quad \dfrac{AD}{PM} = \dfrac{2}{3} \quad \Rightarrow \quad AD : PM = 2 : 3.$

Examples 9. *P, Q are points on the sides AB, AC respectively of a Δ ABC such that PQ ∥ BC and divides Δ ABC into two parts, equal in area. Find PB : AB.*



Solution. According to given,

area of Δ APQ = area of trap. PBCQ

= area of Δ ABC – area of Δ APQ

$\Rightarrow$ 2 × area of Δ APQ = area of Δ ABC ...(*i*)

In Δs APQ and ABC,

$\angle APQ = \angle ABC$ ($\because$ PQ ∥ BC, corres. ∠s are equal)

and $\angle A = \angle A$

$\Rightarrow \quad \Delta\ APQ \sim \Delta\ ABC$ (A.A. axiom of similarity)

As the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$\therefore \quad \dfrac{\text{area of } \Delta\ APQ}{\text{area of } \Delta\ ABC} = \dfrac{AP^2}{AB^2}$

$\Rightarrow \quad \dfrac{\text{area of } \Delta\ APQ}{2 \,.\, \text{area of } \Delta\ APQ} = \dfrac{AP^2}{AB^2}$ (Using (*i*))

$\Rightarrow \quad 2.AP^2 = AB^2 \Rightarrow \sqrt{2}\ .\ AP = AB$

$\Rightarrow \quad \sqrt{2}\,(AB - PB) = AB \Rightarrow (\sqrt{2} - 1)\ .\ AB = \sqrt{2}\ .\ PB$

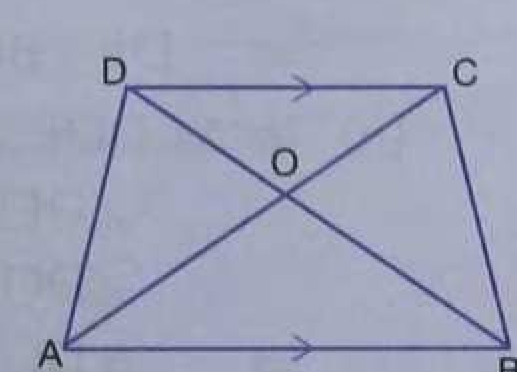
$\Rightarrow \quad \dfrac{PB}{AB} = \dfrac{\sqrt{2} - 1}{\sqrt{2}} \Rightarrow PB : AB = (\sqrt{2} - 1) : \sqrt{2}.$

Example 10. *In the adjoining figure, AB ∥ DC, area of Δ AOD = 4 sq cm and area of Δ BCD = 7 sq cm. Calculate :*

(i) area of Δ OCD.

(ii) the ratio BO : OD.

(iii) area of Δ OAB.



Solution.

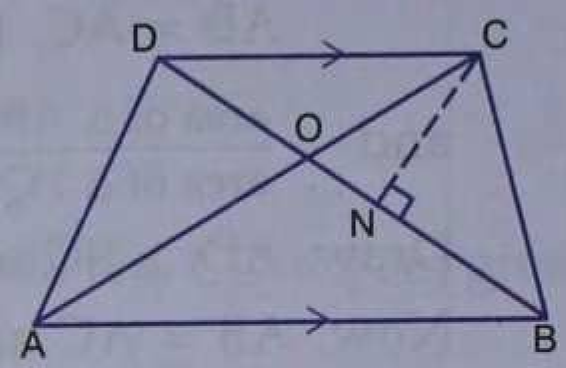
(*i*) Area of ΔACD = area of ΔBCD

($\because$ Δs ACD and BCD have the same base CD and lie between the same parallel lines AB and CD)

$\Rightarrow$ area of ΔOCD + area of ΔAOD = 7 sq. cm

$\Rightarrow$ area of ΔOCD + 4 sq cm = 7 sq. cm

$\Rightarrow$ area of ΔOCD = (7 – 4) sq cm = 3 sq. cm.



(*ii*) From C, draw CN $\perp$ BD.

Area of ΔOBC = area of ΔBCD – area of ΔOCD

= (7 – 3) sq. cm = 4 sq. cm.

$$\text{Now } \frac{\text{area of } \Delta\ OBC}{\text{area of } \Delta\ OCD} = \frac{\frac{1}{2}\ BO \times CN}{\frac{1}{2}\ OD \times CN}$$

$$\Rightarrow \frac{4}{3} = \frac{BO}{OD} \quad \Rightarrow \quad BO : OD = 4 : 3.$$

(*iii*) In Δs OAB and OCD,

$\angle OAB = \angle OCD$ (alt. $\angle$s, since AB ∥ DC)

and $\angle AOB = \angle COD$ (vert. opp. $\angle$s)

$\Rightarrow$ $\Delta OAB \sim \Delta OCD$ (A.A. axiom of similarity)

$\therefore$ $\frac{\text{area of } \Delta\ OAB}{\text{area of } \Delta\ OCD} = \frac{OB^2}{OD^2}$ (theorem 23)

$\Rightarrow$ area of Δ OAB = $\left(\frac{4}{3}\right)^2 \times 3$ sq. cm (from parts (*i*) and (*ii*))

$= \frac{16}{9} \times 3$ sq. cm $= 5\frac{1}{3}$ sq. cm.

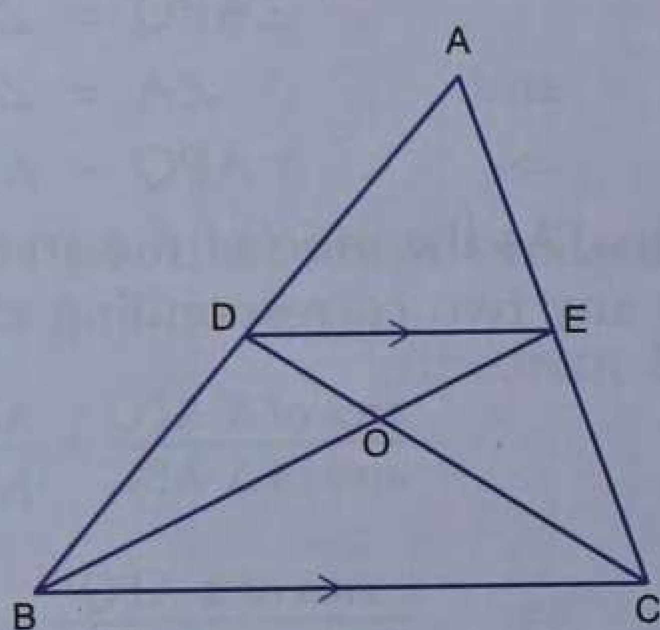
Example 11. *In the adjoining figure, DE ∥ BC and AD : DB = 5 : 4. Find :*

(i) DE : BC

(ii) DO : DC

(iii) area of ΔDOE : area of ΔDCE.

(iv) area of ΔDOE : area of ΔCOB.



Solution.

(*i*) Given $\frac{AD}{DB} = \frac{5}{4} \Rightarrow \frac{AD}{AD + DB} = \frac{5}{5+4}$

$\Rightarrow \frac{AD}{AB} = \frac{5}{9}$.

In Δs ADE and ABC,

$\angle ADE = \angle ABC$ (DE ∥ BC, corres. $\angle$s are equal)

and $\angle A = \angle A$

$\Rightarrow$ $\Delta ADE \sim \Delta ABC$

$\Rightarrow \frac{DE}{BC} = \frac{AD}{AB} = \frac{5}{9}$

$\Rightarrow$ DE : BC = 5 : 9.

(*ii*) In Δs DOE and COB

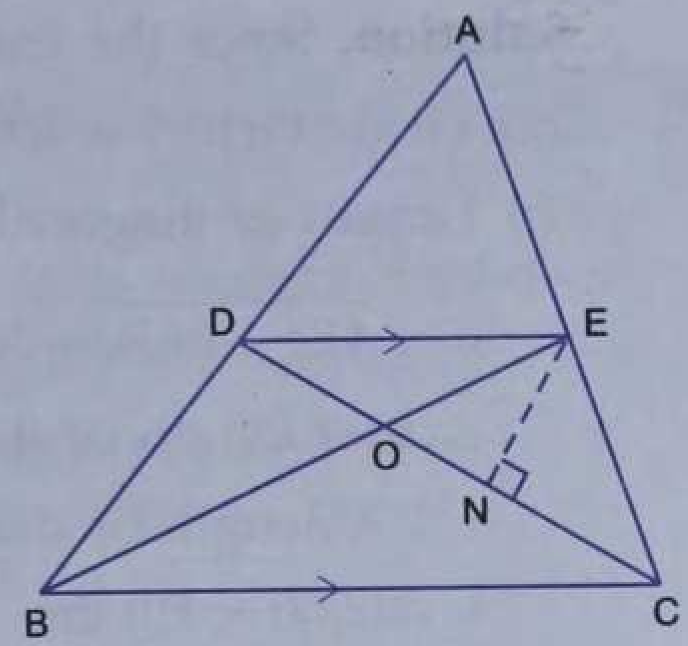
$\angle OED = \angle OBC$ (DE ∥ BC, alt. $\angle$s are equal)

$\angle DOE = \angle BOC$ (vert. opp. $\angle$s)

$\Rightarrow$ $\Delta DOE \sim \Delta COB$ (A.A. axiom of similarity)

$$\Rightarrow \quad \frac{DO}{OC} = \frac{DE}{BC} = \frac{5}{9} \Rightarrow \frac{DO}{DO + OC} = \frac{5}{5+9}$$

$$\Rightarrow \quad \frac{DO}{DC} = \frac{5}{14} \Rightarrow DO : DC = 5 : 14.$$



(*iii*) From E, draw EN ⊥ DC.

Now $\frac{\text{area of } \Delta DOE}{\text{area of } \Delta DCE} = \frac{\frac{1}{2} . DO \times EN}{\frac{1}{2} . DC \times EN} = \frac{DO}{DC} = \frac{5}{14}$

$\Rightarrow$ area of Δ DOE : area of Δ DCE = 5 : 14.

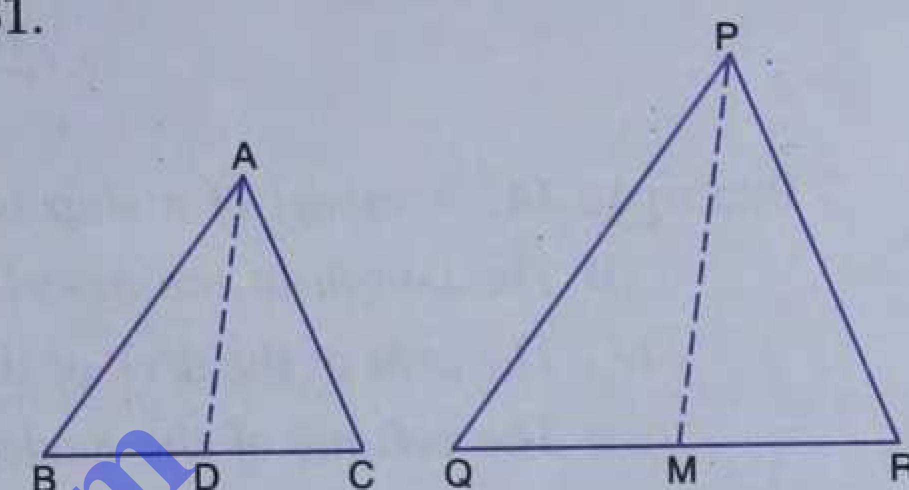
(*iv*) From (*ii*) part, we have $\Delta DOE \sim \Delta COB$,

$\therefore \frac{\text{area of } \Delta DOE}{\text{area of } \Delta COB} = \frac{DE^2}{BC^2}$ (theorem 23)

$$= \left(\frac{DE}{BC}\right)^2 = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$$

$\Rightarrow$ area of Δ DOE : area of Δ COB = 25 : 81.

Example 12. *The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding medians.*



Given. Δ ABC ~ Δ PQR; AD is median of Δ ABC and PM is median of Δ PQR.

To prove. $\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ PQR}} = \frac{AD^2}{PM^2}$.

Proof. *Statements*	*Reasons*
1. $\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ PQR}} = \frac{AB^2}{PQ^2}$	1. ΔABC ~ ΔPQR
2. $\frac{AB}{PQ} = \frac{BC}{QR}$	2. Δ ABC ~ ΔPQR
3. $\frac{AB}{PQ} = \frac{2 . BD}{2 . QM}$	3. D is mid-point of BC and M is mid-point of QR
In Δs ABD and PQM	
1′. $\frac{AB}{PQ} = \frac{BD}{QM}$	1′. From 3
2′. ∠B = ∠Q	2′. Δ ABC ~ Δ PQR
3′. Δ ABD ~ Δ PQM	3′. S.A.S. axiom of similarity
4′. $\frac{AB}{PQ} = \frac{AD}{PM}$	4′. Using 3′
$\therefore \frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ PQR}} = \frac{AD^2}{PM^2}$.	Using 1 and 4′
Q.E.D.	

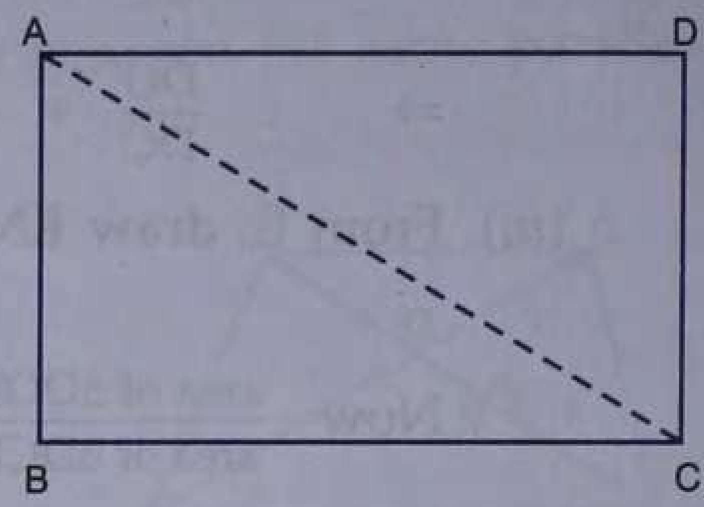
Example 13. *On a map drawn to a scale of 1 : 40000, a rectangular plot of land, ABCD has the following measurements : AB = 6 cm and BC = 8 cm. Calculate :*

(i) the diagonal distance of the plot in km.

(ii) the area of the plot in sq. km.

Solution. Since the map is drawn to the scale 1 : 40000

$\therefore$ k (scale factor) = 40000.



(*i*) Length of diagonal AC (shown on map)

$= \sqrt{AB^2 + BC^2} = \sqrt{6^2 + 8^2}$ cm = 10 cm.

Actual length of the diagonal of the plot

$= k \times$ (length of diagonal on the map)

$= (40000 \times 10)$ cm

$= \frac{40000 \times 10}{100 \times 1000}$ km = 4 km.

(*ii*) Area of the plot on the map = (8×6) cm^2 = 48 cm^2.

Actual area of the plot = $k^2 \times$ (area of plot on the map)

$= (40000)^2 \times 48$ cm^2

$= \frac{40000 \times 40000 \times 48}{100 \times 100 \times 1000 \times 1000}$ km^2

$= \frac{16 \times 48}{100}$ km^2 = 7·68 km^2.

Example 14. *A model of a ship is made to a scale of 1 : 200.*

(*i*) *The length of the model is 4 m. Calculate the length of the ship.*

(*ii*) *The area of the deck of the ship is 160000 m^2. Find the area of the deck of the model.*

(*iii*) *The volume of the model is 200 litres. Calculate the volume of the ship in m^3.*

Solution.

(*i*) Since the model of the ship is made to scale 1 : 200,

$\therefore$ k (scale factor) = 200.

Actual length of the ship = $k \times$ (the length of the model)

$= (200 \times 4)$ m = 800 m.

(*ii*) The area of the deck of the ship = $k^2 \times$ (the area of the deck of the model)

$\Rightarrow$ 160000 m^2 = $(200)^2 \times$ (the area of the deck of the model)

$\Rightarrow$ the area of the deck of the model = $\frac{160000}{200 \times 200}$ m^2 = 4 m^2

(*iii*) Volume of the ship = $k^3 \times$ (the volume of the model)

$= (200)^3 \times 200$ litres

$= \frac{8000000 \times 200}{1000}$ m^3 ($\because$ 1000 litres = 1 m^3)

$= 1600000$ m^3.

Exercise 14

1. Given that Δs ABC and PQR are similar. Find :

 (*i*) the ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.

 (*ii*) the ratio of their corresponding sides if area of ΔABC : area of ΔPQR = 25 : 36.

2. ΔABC ~ ΔDEF. If area of ΔABC = 9 sq cm, area of ΔDEF = 16 sq. cm and BC = 2·1 cm, find the length of EF.

3. Δ ABC ~ Δ DEF. If BC = 3 cm, EF = 4 cm and area of Δ ABC = 54 sq. cm, determine the area of Δ DEF.

4. The areas of two similar triangles are 36 cm^2 and 25 cm^2. If an altitude of the first triangle is 2·4 cm, find the corresponding altitude of the other triangle.

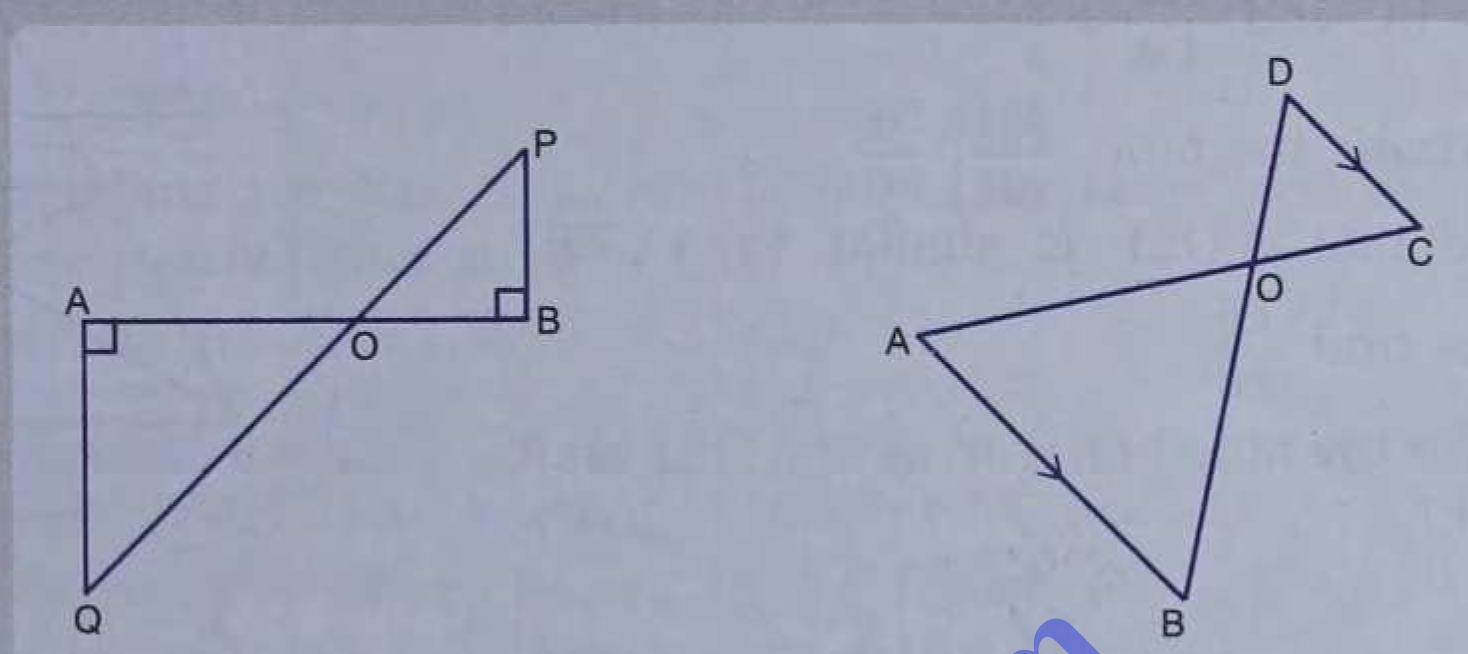
5. (*a*) In the figure (*i*) given below, PB and QA are perpendiculars to the line segment AB. If PO = 6 cm, QO = 9 cm and the area of Δ POB = 120 cm^2, find the area of Δ QOA. (*2006*)

   (*b*) In the figure (*ii*) given below, AB ∥ DC. AO = 10 cm, OC = 5 cm, AB = 6·5 cm and OD = 2·8 cm.

   (*i*) Prove that Δ OAB ~ Δ OCD.

   (*ii*) Find CD and OB.

   (*iii*) Find the ratio of areas of Δ OAB and Δ OCD.

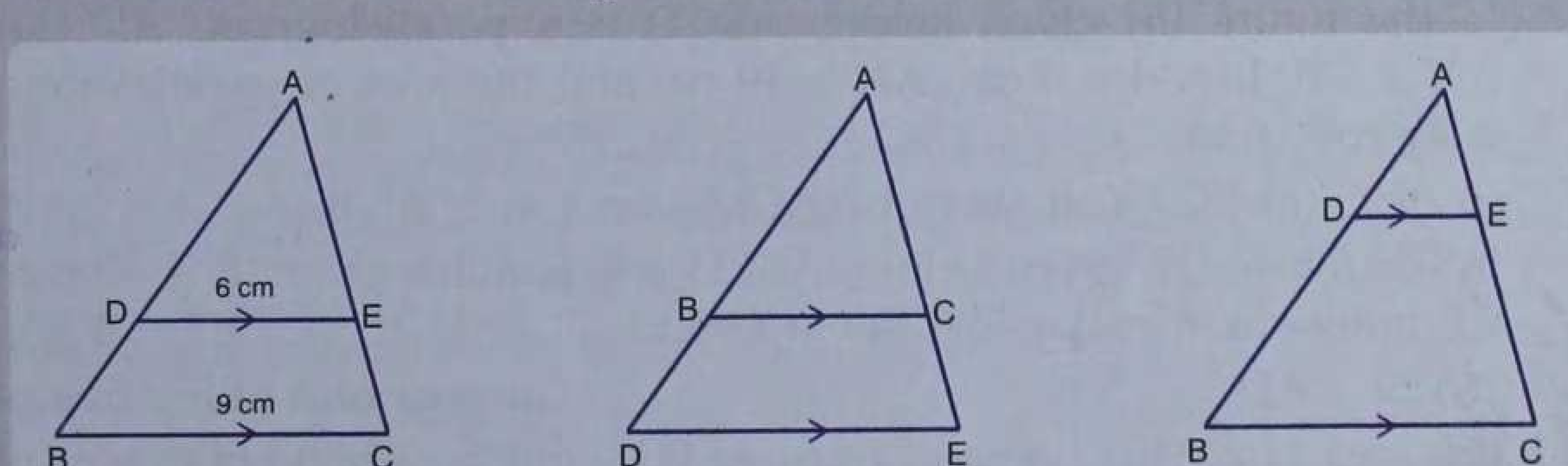


(*i*) (*ii*)

6. (*a*) In the figure (*i*) given below, DE ∥ BC. If DE = 6 cm, BC = 9 cm and area of Δ ADE = 28 sq. cm, find the area of Δ ABC.

   (*b*) In the figure (*ii*) given below, BC is parallel to DE. Area of triangle ABC = 25 cm^2, area of trapezium BCED = 24 cm^2, DE = 14 cm. Calculate the length of BC. (*2000*)

   (*c*) In the figure (*iii*) given below, DE ∥ BC and AD : DB = 1 : 2, find the ratio of the areas of Δ ADE and trapezium DBCE.



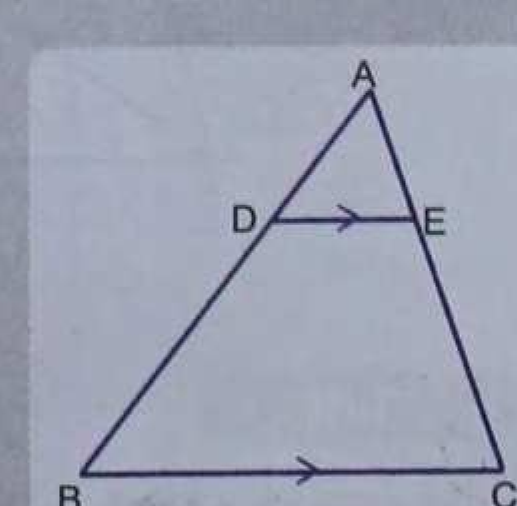
(*i*) (*ii*) (*iii*)

7. In the given figure, DE ∥ BC.

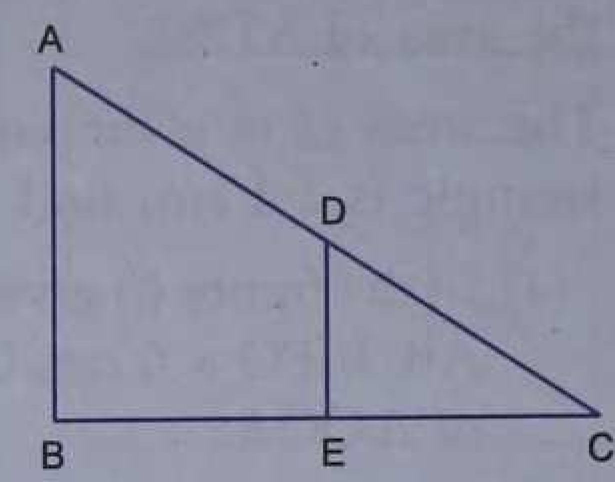
   (*i*) Prove that Δ ADE and Δ ABC are similar.

   (*ii*) Given that AD = $\frac{1}{2}$ BD, calculate DE, if BC = 4·5 cm.

   (*iii*) If area of Δ ABC = 18 cm^2, find area of trapezium DBCE. (*2004*)



8. In the given figure, AB and DE are perpendiculars to BC.



(i) Prove that $\Delta ABC \sim \Delta DEC$.

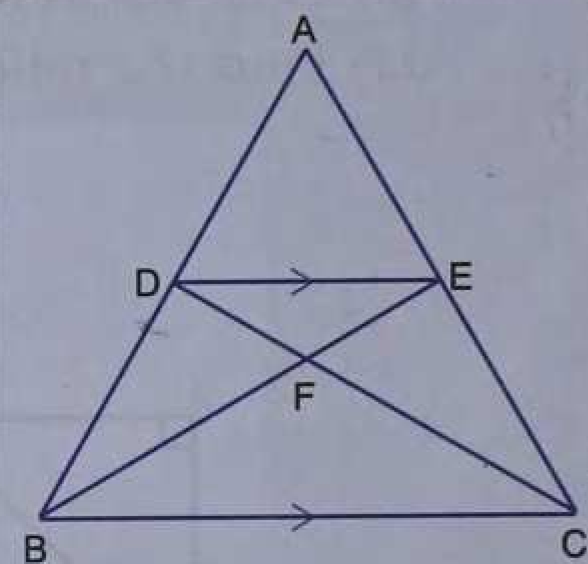
(ii) If AB = 6 cm, DE = 4 cm and AC = 15 cm, calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC. *(2013)*

Hint

(ii) As $\Delta ABC \sim \Delta DEC$, $\frac{AB}{DE} = \frac{AC}{CD}$.

9. In the adjoining figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

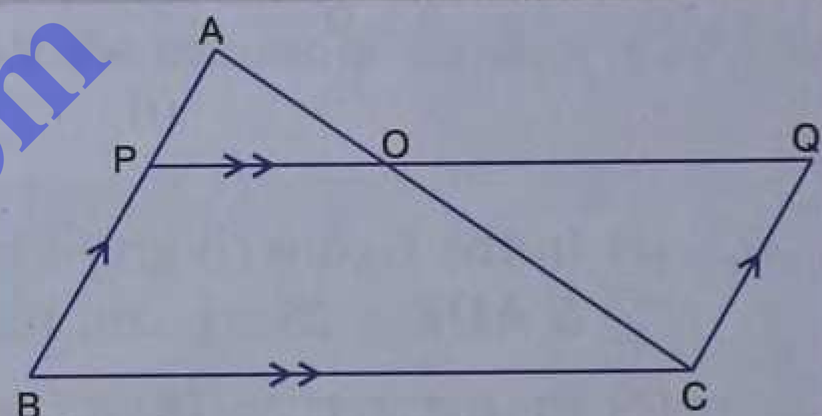


(i) Determine the ratio $\frac{AD}{AB}, \frac{DE}{BC}$.

(ii) Prove that ΔDEF is similar to ΔCBF. Hence find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of ΔDEF and ΔCBF? *(2007)*

10. In ΔABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA. Find :



(i) area ΔAPO : area ΔABC

(ii) area ΔAPO : area ΔCQO. *(2008)*

11. (a) In the figure (i) given below, ABCD is a trapezium in which AB ∥ DC and AB = 2 CD. Determine the ratio of the areas of Δ AOB and Δ COD.

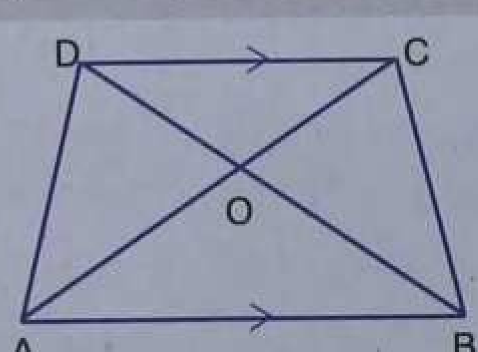
(b) In the figure (ii) given below, ABCD is a parallelogram. AM ⊥ DC and AN ⊥ CB. If AM = 6 cm, AN = 10 cm and the area of parallelogram ABCD is 45 cm^2, find

(i) AB (ii) BC (iii) area of Δ ADM : area of Δ ANB.

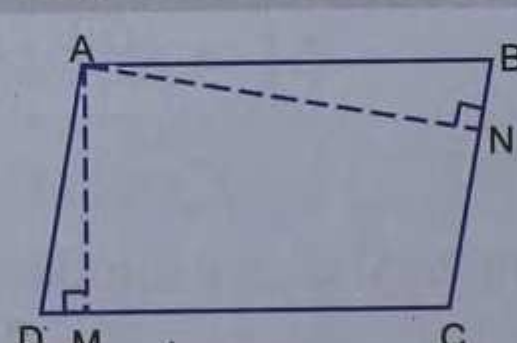
(c) In the figure (iii) given below, ABCD is a parallelogram. E is a point on AB, CE intersects the diagonal BD at O and EF ∥ BC. If AE : EB = 2 : 3, find

(i) EF : AD (ii) area of Δ BEF : area of Δ ABD

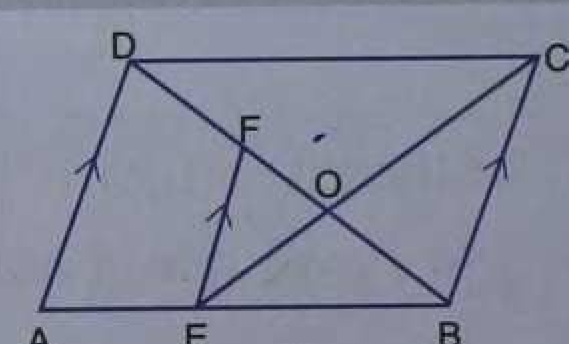
(iii) area of Δ ABD : area of trap. AEFD (iv) area of Δ FEO : area of Δ OBC.



(i)



(ii)



(iii)

Hint

(b) $\Delta ADM \sim \Delta ANB$.

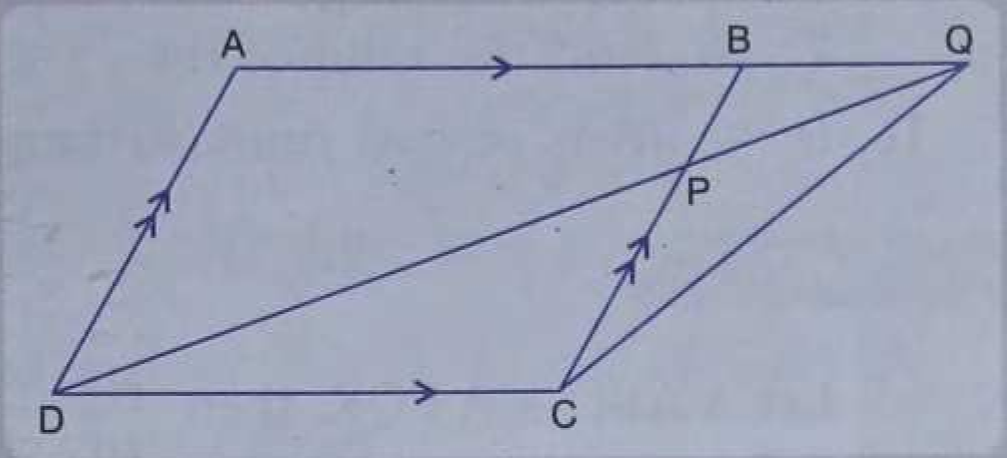
12. In the adjoining figure, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2 and DP produced meets AB produced at Q.

If area of $\Delta CPQ = 20 \text{ cm}^2$, find

(*i*) area of ΔBPQ.

(*ii*) area ΔCDP.

(*iii*) area of || gm ABCD.



Hint

(*i*) From Q, draw QN ⊥ CB (produced)

$$\frac{\text{area of } \Delta BPQ}{\text{area of } \Delta CPQ} = \frac{\frac{1}{2} \times BP \times QN}{\frac{1}{2} \times PC \times QN} = \frac{BP}{PC} = \frac{1}{2} \text{(given)}$$

(*ii*) ΔCDP ~ ΔBQP

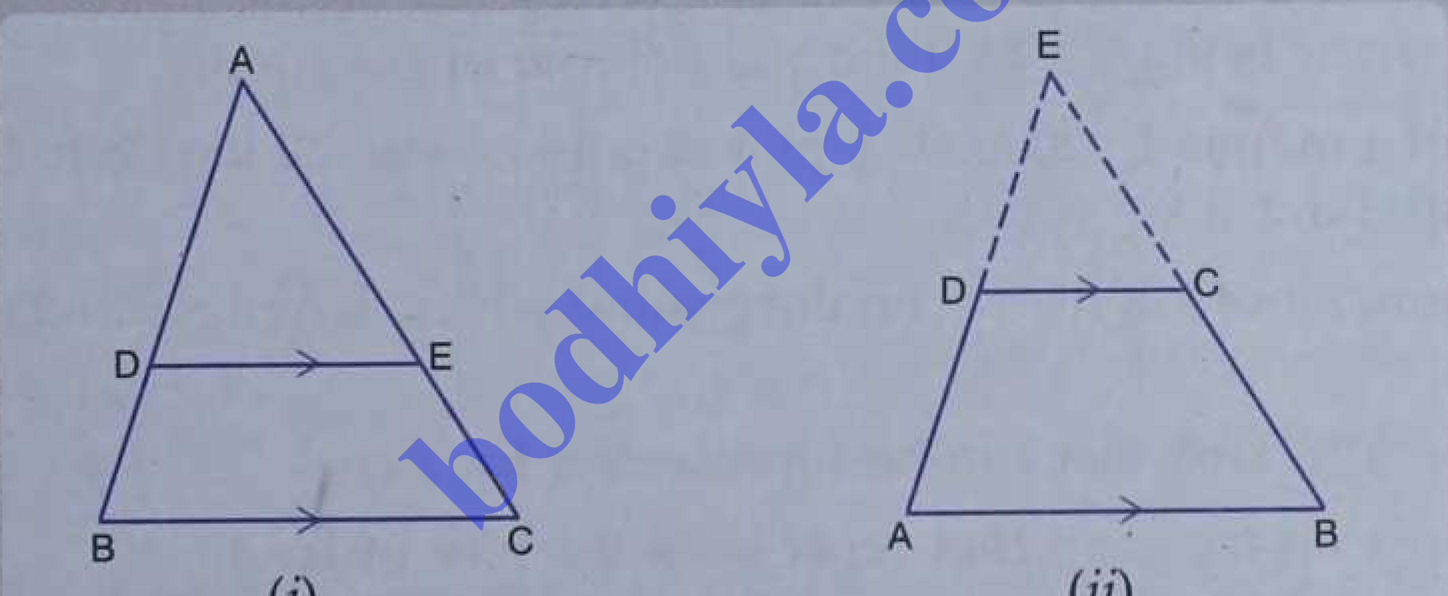
(*iii*) As ΔDCQ and parallelogram ABCD are on the same base DC and between the same parallel lines AB and DC,

area of || gm ABCD = 2 area of ΔDCQ.

13. (*a*) In the figure (*i*) given below, DE || BC and the ratio of the areas of Δ ADE and trapezium DBCE is 4 : 5. Find the ratio of DE : BC.

(*b*) In the figure (*ii*) given below, AB || DC and AB = 2DC. If AD = 3 cm, BC = 4 cm and AD, BC produced meet at E, find

(*i*) ED (*ii*) BE (*iii*) area of Δ EDC : area of trapezium ABCD.

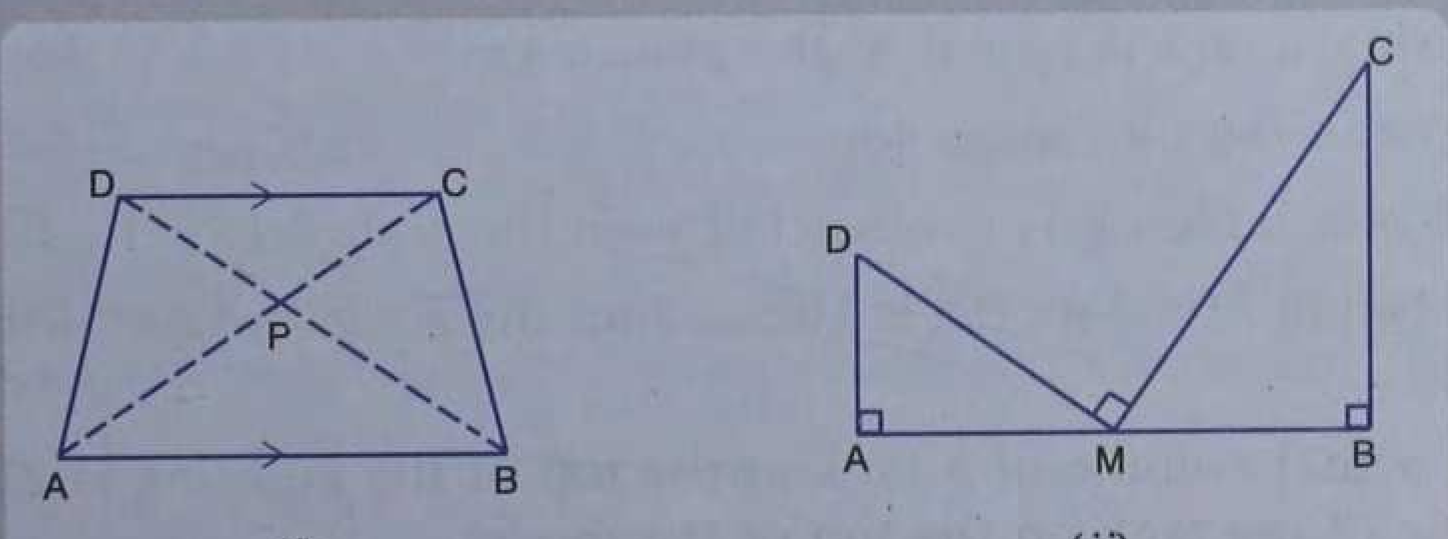


14. (*a*) In the figure (*i*) given below, ABCD is a trapezium in which DC is parallel to AB. If AB = 9 cm, DC = 6 cm and BD = 12 cm, find

(*i*) BP (*ii*) the ratio of areas of Δ APB and Δ DPC.

(*b*) In the figure (*ii*) given below, M is mid point of AB, ∠A = ∠B = 90° = ∠CMD, prove that

(*i*) Δ DAM is similar to Δ MBC (*ii*) $\frac{\text{area of } \Delta DAM}{\text{area of } \Delta MBC} = \frac{AD}{BC}$ (*iii*) $\frac{AD}{BC} = \frac{MD^2}{MC^2}$.



15. Two isosceles triangles have equal vertical angles and their areas are in the ratio 7 : 16. Find the ratio of their corresponding heights.

16. If the areas of two similar triangles are equal, prove that they are congruent.

Hint

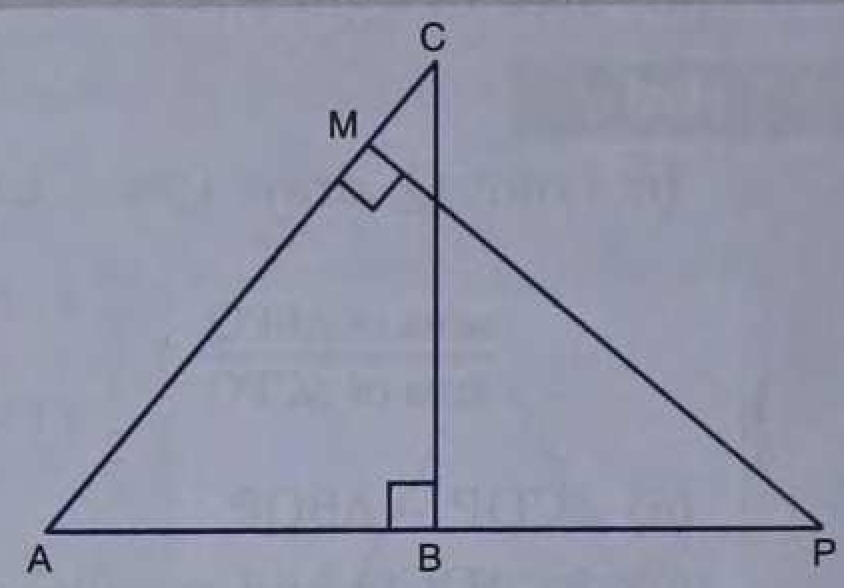
Let Δ ABC ~ Δ PQR, then $\dfrac{\text{area of } \Delta \text{ ABC}}{\text{area of } \Delta \text{ PQR}} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$.

17. In the given figure, ΔABC and ΔAMP are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(*i*) Prove that ΔABC ~ ΔAMP.

(*ii*) Find AB and BC. *(2012)*



Hint

(*i*) ΔABC ~ ΔAMP (by A.A. axiom of similarity)

(*ii*) $\dfrac{BC}{MP} = \dfrac{AC}{AP} \Rightarrow \dfrac{BC}{12\text{ cm}} = \dfrac{10\text{ cm}}{15\text{ cm}} \Rightarrow$ BC = 8 cm.

By Pythagoras theorem, $AB^2 = AC^2 - BC^2 \Rightarrow AB = \sqrt{10^2 - 8^2}$ cm.

18. The volume of a machine is 27000 cm^3. A model of the machine is made, the reduction factor being 2 : 15. Find the volume of the model.

19. The scale of a map is 1 : 200000. A plot of land of area 20 km^2 is to be represented on the map. Find :

(*i*) The number of kilometres on the ground which is represented by 1 cm on the map.

(*ii*) The area in km^2 that can be represented by 1 cm^2.

(*iii*) The area on the map that represents the plot of land.

20. On a map drawn to a scale of 1 : 250000, a triangular plot of land has the following measurements :

AB = 3 cm, BC = 4 cm, angle ABC = 90°. Calculate :

(*i*) the actual length of AB in km.

(*ii*) the area of the plot in sq. km.

21. On a map drawn to a scale of 1 : 25000, a rectangular plot of land, ABCD has the following measurements AB = 12 cm and BC = 16 cm. Angles A, B, C and D are 90° each. Calculate :

(*i*) the distance of a diagonal of the plot in km.

(*ii*) the area of the plot in sq. km.

22. The model of a building is constructed with the scale factor 1 : 30.

(*i*) If the height of the model is 80 cm, find the actual height of the building in metres.

(*ii*) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model. *(2009)*

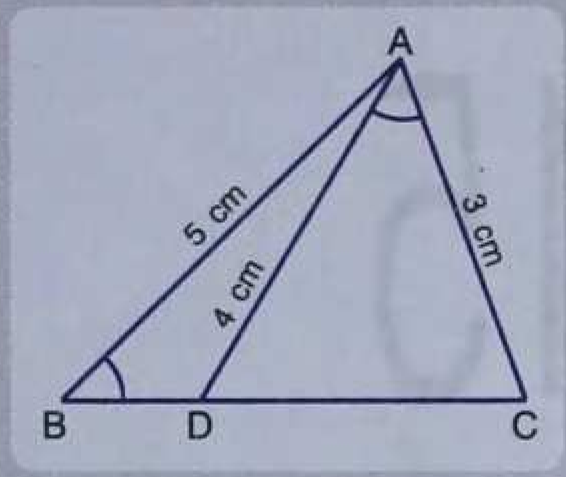
CHAPTER TEST

1. If the areas of two similar triangles are 360 cm^2 and 250 cm^2 and if one side of the first triangle is 8 cm, find the length of the corresponding side of the second triangle.

2. In the adjoining figure, D is a point on BC such that $\angle ABD = \angle CAD$. If AB = 5 cm, AC = 3 cm and AD = 4 cm, find

   (*i*) BC (*ii*) DC

   (*iii*) area of ΔACD : area of ΔBCA.

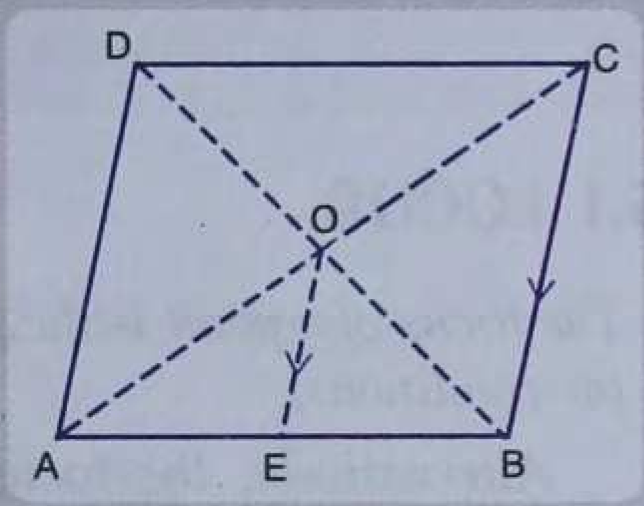


Hint

$\Delta ACD \sim \Delta BCA$.

3. In the adjoining figure, the diagonals of a parallelogram intersect at O. OE is drawn parallel to CB to meet AB at E, find

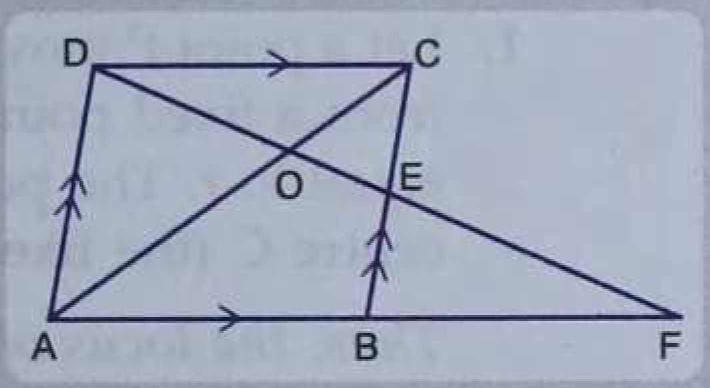
   area of Δ AOE : area of || gm ABCD.



4. In the adjoining figure, ABCD is a parallelogram. E is mid-point of BC. DE meets the diagonal AC at O and meet AB (produced) at F. Prove that

   (*i*) DO : OE = 2 : 1

   (*ii*) area of ΔOEC : area of ΔOAD = 1 : 4.



5. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding angle bisector segments.

6. D, E, F are mid points of the sides BC, CA and AB respectively of a Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.

Hint

FD || AC, DE || AB $\Rightarrow$ AFDE is a parallelogram

$\Rightarrow \angle A = \angle FDE$. Similarly, $\angle B = \angle DEF \Rightarrow \Delta DEF \sim \Delta ABC$.

7. A model of a ship is made to a scale of 1 : 250. Calculate :

   (*i*) the length of the ship, if the length of model is 1·6 m.

   (*ii*) the area of the deck of the ship, if the area of the deck of model is 2·4 m^2.

   (*iii*) the volume of the model, if the volume of the ship is 1 km^3.