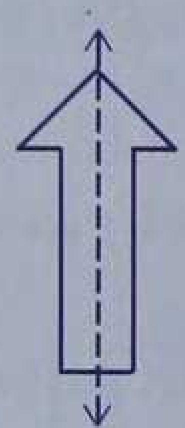
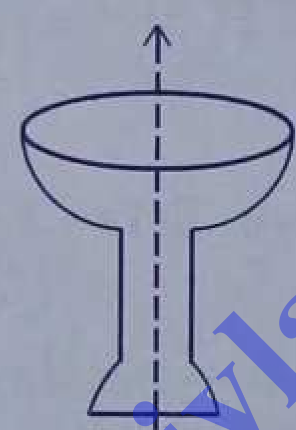
13 Symmetry

13.1 SYMMETRY

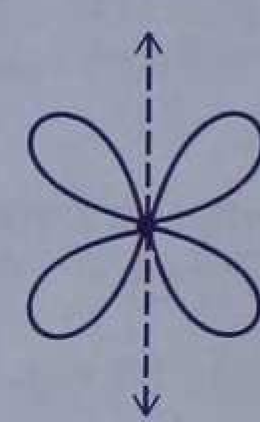
Look at the following plane figures :



(*i*)



(*ii*)



(*iii*)

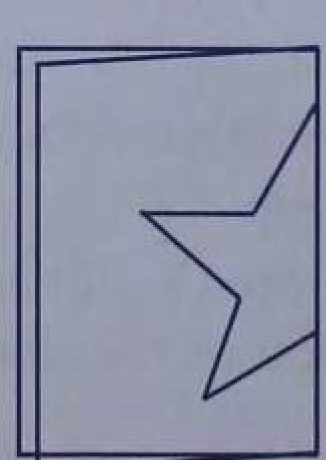
We observe that if these figures are folded along a specific line (shown dotted), each figure on the left hand of the dotted line fits exactly on the top of the figure on the right hand side of the dotted line *i.e.* each figure is divided into identical parts about the dotted line. This leads to :

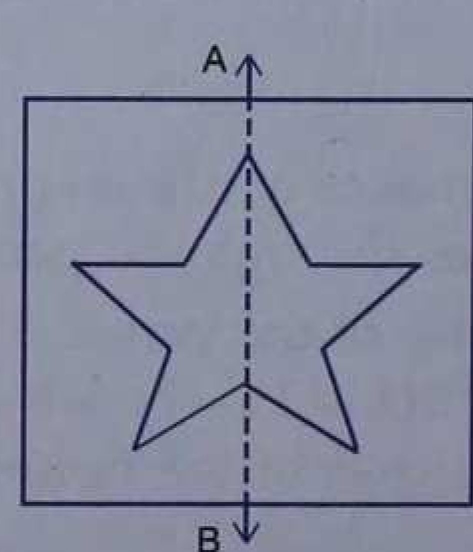
Line symmetry

If a figure is divided into two identical parts by a line then the figure is called ***symmetrical*** *about that line, and the line is called its* ***line*** *(or* ***axis****)* ***of symmetry*** *or* ***mirror line.*** *We also say that the figure has a* ***line symmetry.***

Each one of the figures ((*i*), (*ii*) and (*iii*)) shown above is symmetrical about the dotted line, and the dotted line is its axis (or line) of symmetry.

Plane symmetrical figures can be obtained by folding and cutting paper. Fold a piece of paper and cut out a shape based on the folded line. On unfolding the paper,





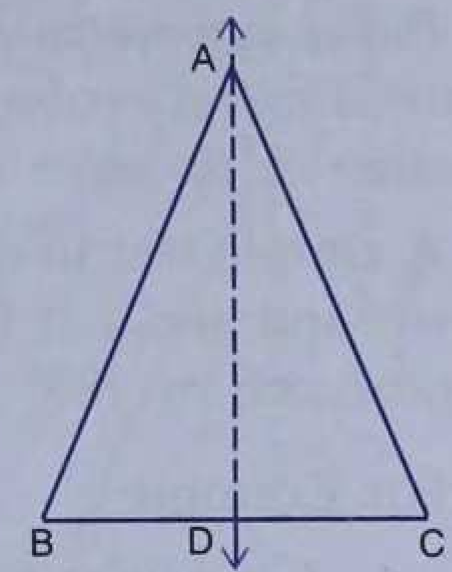
a design is obtained. Observe that the design obtained above is identical on both sides of the crease of the paper (shown by dotted line AB). So, the above design (decagon) is symmetrical about the line AB.

A simple test to determine whether a figure has line symmetry is to fold the figure along the supposed line of symmetry and to see if the two halves of the figure coincide.

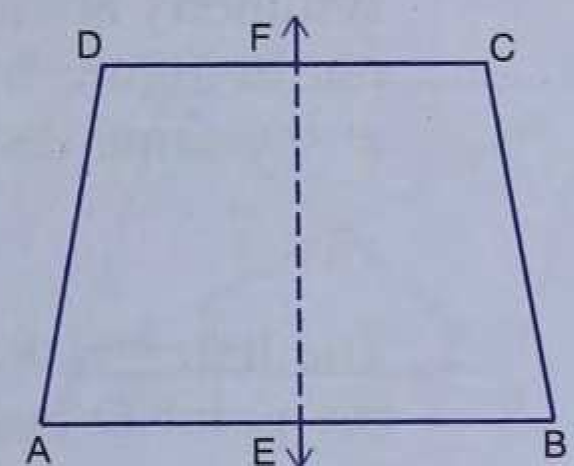
A two dimensional (plane) figure may or may not have a line of symmetry. Moreover, a plane figure may have more than one line of symmetry.

For Example :

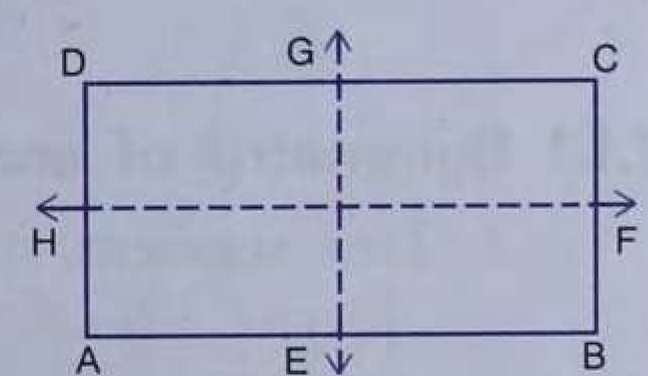
1. Let ABC be an isosceles triangle with AB = AC and D be mid-point of BC, then Δs ABD and ACD are identical (because AB = AC, BD = DC and AD is common), therefore, Δ ABC is symmetrical about the line AD and the line AD is its axis of symmetry.



2. Let ABCD be an isosceles trapezium with AD = BC and AB || DC, and E, F be mid-points of AB, CD respectively, then quad. AEFD is identical to quad. BEFC (check). Therefore, the trapezium ABCD is symmetrical about the line EF and the line EF is its axis of symmetry.

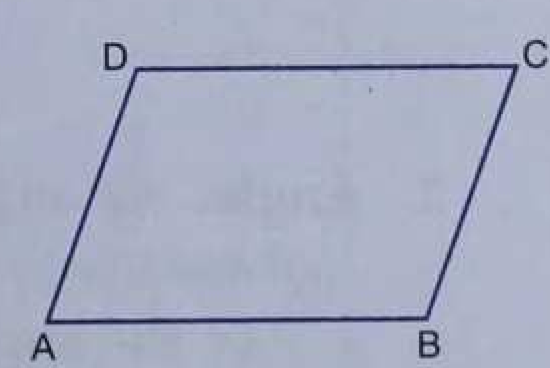


3. Let ABCD be a rectangle and E, F, G, H be mid-points of sides AB, BC, CD, DA respectively, then the rectangle is symmetrical about the lines EG and HF. Note that the rectangle ABCD has two axes of symmetry EG and HF.

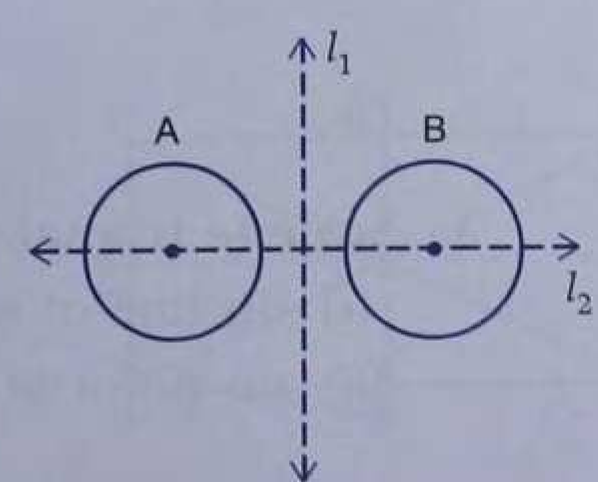


4. Let ABCD be a parallelogram. Observe that there is no line in the plane of parallelogram about which it can be folded so that the two parts may coincide.

   Therefore, parallelogram has no line of symmetry.



5. Let A, B be two equal circles, then l_1 and l_2 are two lines of symmetry. Note that l_1 does not pass through the figure.



Remarks

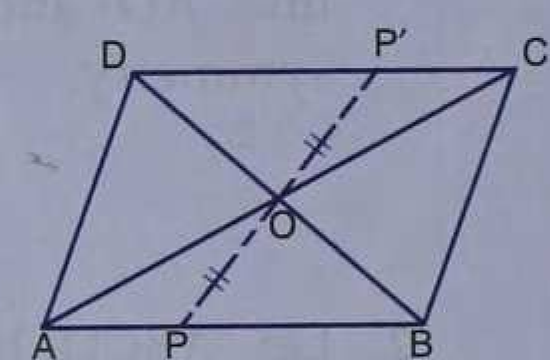
- If a figure is folded about its line of symmetry, then the two parts coincide *i.e.* it is divided into two congruent parts.
- A figure may have more than one line of symmetry.
- A figure may not have a line of symmetry.
- A figure may not pass through line of symmetry.

Point symmetry

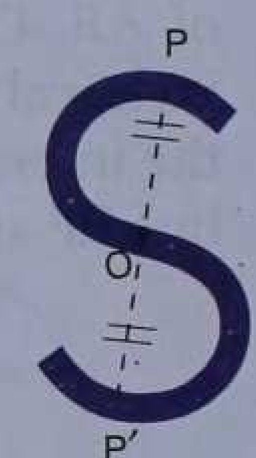
Point symmetry *exists when a figure is built around a single point called the* ***centre*** *of the figure. For every point of the figure, there is another point found directly opposite to it at the same distance on the other side of the centre.*

A simple test to determine whether a figure has point symmetry is to turn it upside-down and see if it looks the same. A figure that has point symmetry is unchanged in appearance by 180° rotation.

For Example :

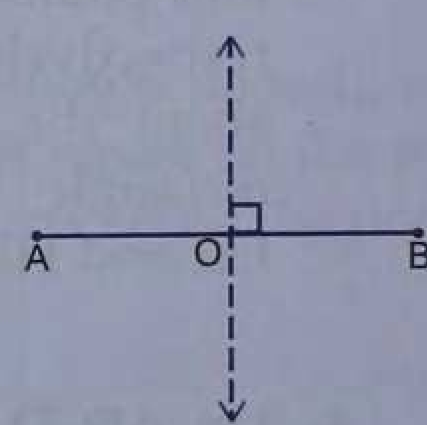


1. Let ABCD be a parallelogram and O be the point of intersection of its diagonals. It has a point symmetry about the point O because every point P on the figure has a point P′ directly opposite to it at the same distance on the other side of O.

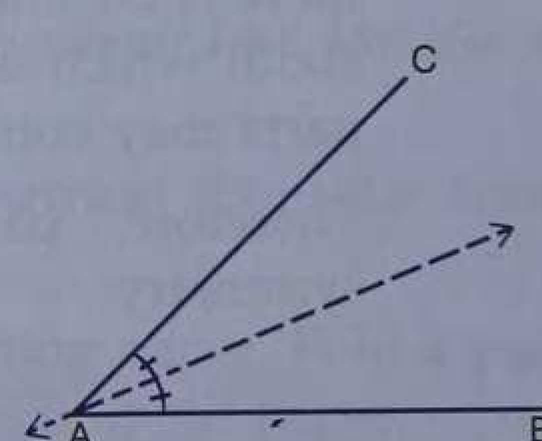


2. The letter 'S' has a point symmetry about the point O because every point P on the figure has a point P′ directly opposite to it at the same distance on the other side of O.

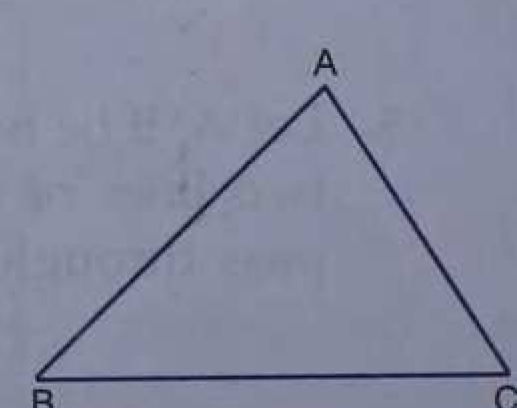
13.1.1 Symmetry of some figures



1. **Line segment.** A line segment has
   (*i*) *one line of symmetry* — the perpendicular bisector of the segment.
   (*ii*) *one point of symmetry* — the mid-point of the segment.



2. **Angle.** An angle with equal arms has
   (*i*) *one line of symmetry* — the internal bisector of the angle.
   (*ii*) no point of symmetry.

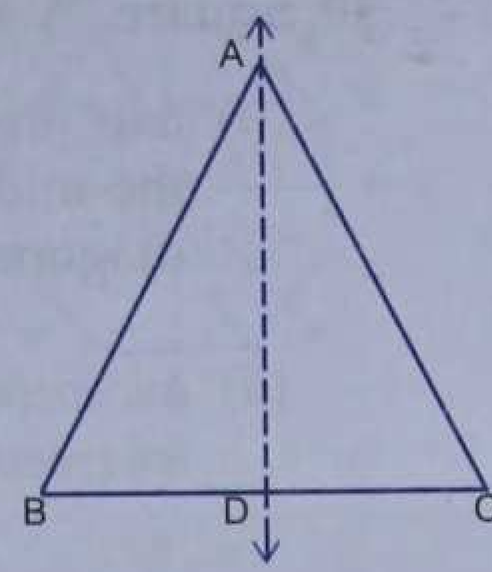


3. **Scalene triangle.** A scalene triangle has
   (*i*) no line of symmetry.
   (*ii*) no point of symmetry.

4. **Isosceles triangle.** An isosceles triangle has

(*i*) *one line of symmetry* — the median through the vertex A.

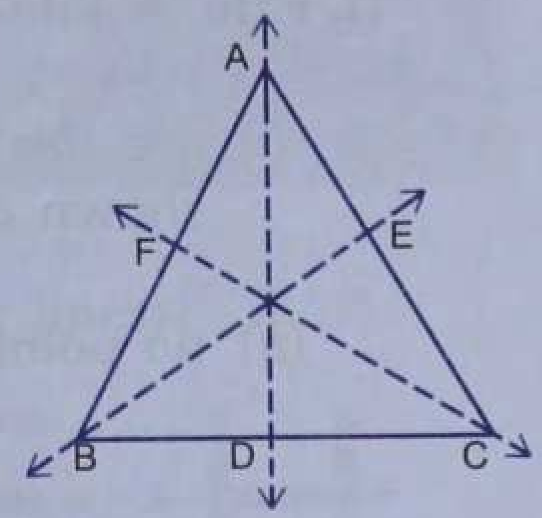
(*ii*) no point of symmetry.



5. **Equilateral triangle.** An equilateral triangle has

(*i*) *three lines of symmetry* — the three medians of the triangle.

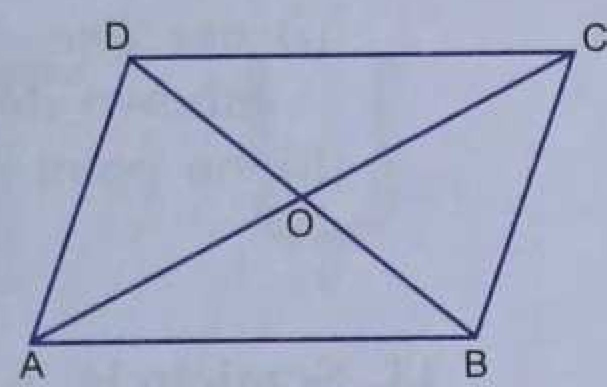
(*ii*) no point of symmetry.



6. **Parallelogram.** A parallelogram has

(*i*) no line of symmetry.

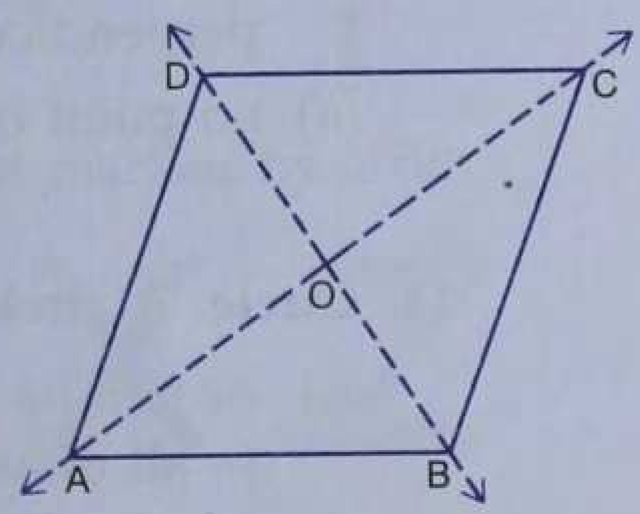
(*ii*) *one point of symmetry* — the point of intersection of the diagonals.



7. **Rhombus.** A rhombus has

(*i*) *two lines of symmetry* — the diagonals of the rhombus.

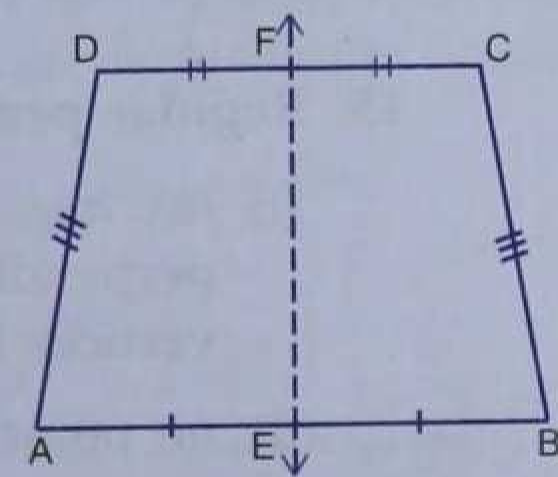
(*ii*) *one point of symmetry* — the point of intersection of the diagonals.



8. **Isosceles trapezium.** An isosceles trapezium has

(*i*) *one line of symmetry* — the line joining the mid-points of the bases of the trapezium.

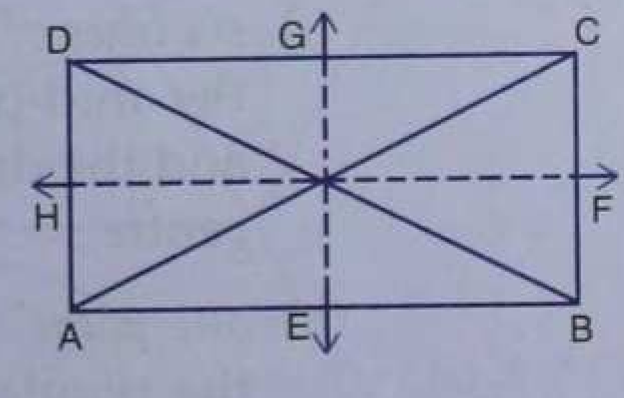
(*ii*) no point of symmetry.



9. **Rectangle.** A rectangle has

(*i*) *two lines of symmetry* — the lines joining the mid-points of the opposite sides.

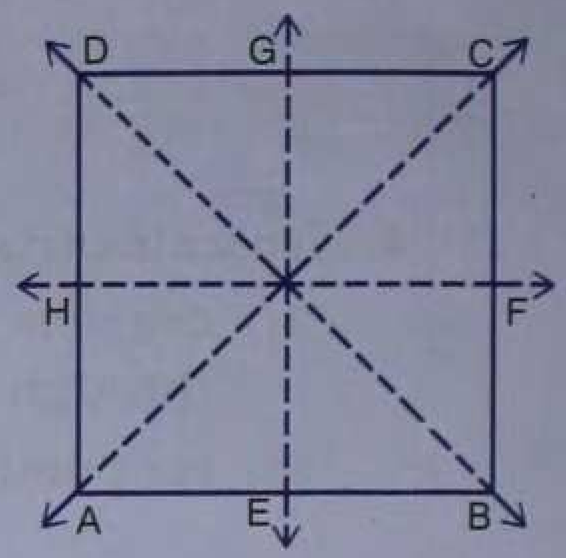
(*ii*) *one point of symmetry* — the point of intersection of the diagonals.



10. Square. A square has

(*i*) *four lines of symmetry* — the lines joining the mid-points of opposite sides, and the diagonals.

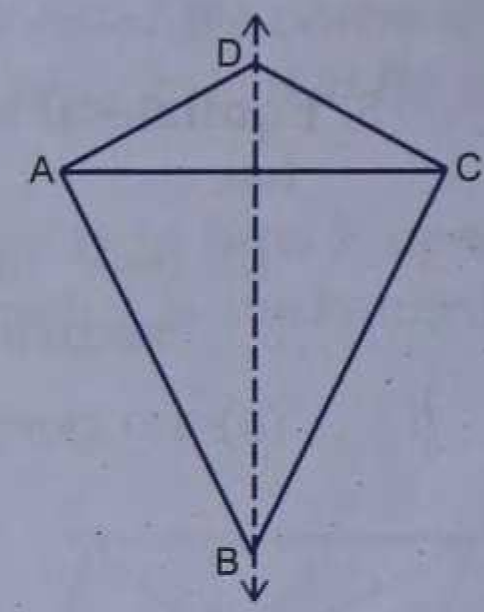
(*ii*) *one point of symmetry* — the point of intersection of the diagonals.



11. Kite. A kite (or diamond) has

(*i*) *one line of symmetry* — the diagonal shown dotted in the adjoining figure.

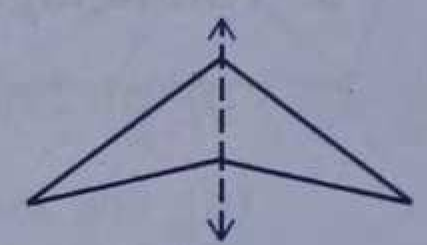
(*ii*) no point of symmetry.



12. Arrow head. An arrow head has

(*i*) *one line of symmetry* — the diagonal shown dotted in the adjoining figure.

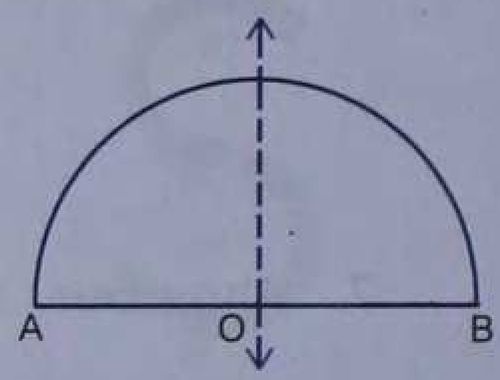
(*ii*) no point of symmetry.



13. Semicircle. A semicircle has

(*i*) *one line of symmetry* — the perpendicular bisector of AB.

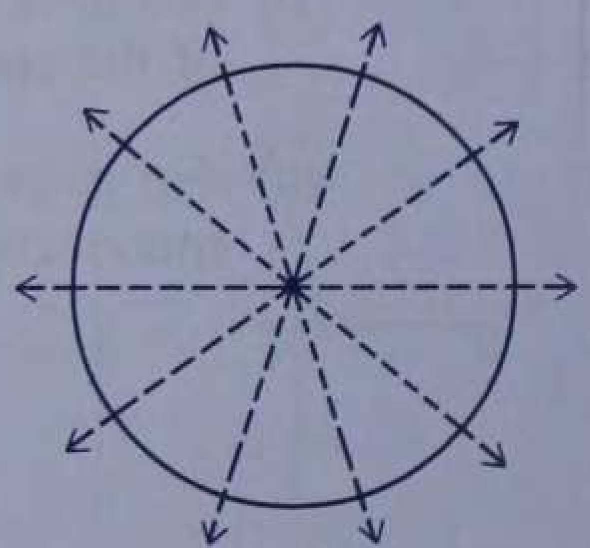
(*ii*) no point of symmetry.



14. Circle. A circle has

(*i*) *an infinite number of lines of symmetry* — all diameters.

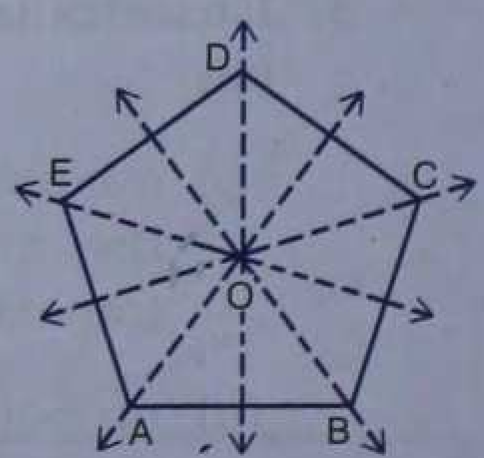
(*ii*) *one point of symmetry* — the centre of the circle.



15. Regular pentagon. A regular pentagon has

(*i*) *five lines of symmetry* — the perpendiculars drawn from the vertices to the opposite sides.

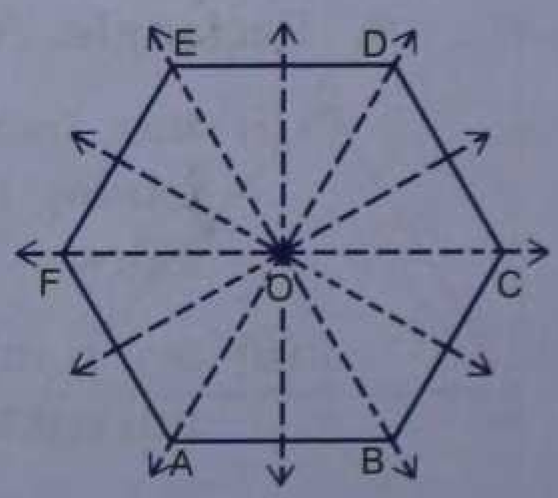
(*ii*) no point of symmetry.



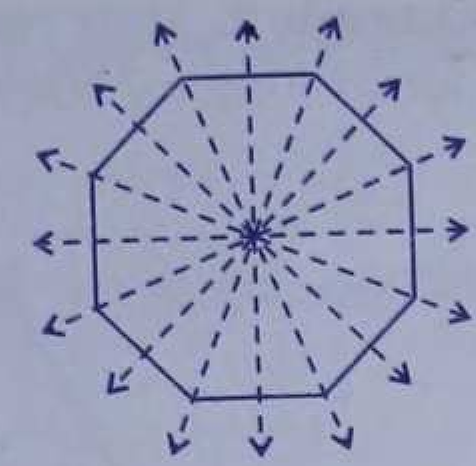
16. Regular hexagon. A regular hexagon has

(*i*) *six lines of symmetry* — the lines joining the mid-points of the opposite sides, and the diagonals passing through the centre.

(*ii*) *one point of symmetry* — the centre of the regular hexagon.



17. Regular octagon. A regular octagon has



(*i*) *eight lines of symmetry* — the lines joining the mid-points of the opposite sides, and the diagonals passing through the centre.

(*ii*) *one point of symmetry* — the centre of the regular octagon.

Remark

A regular polygon of n sides has

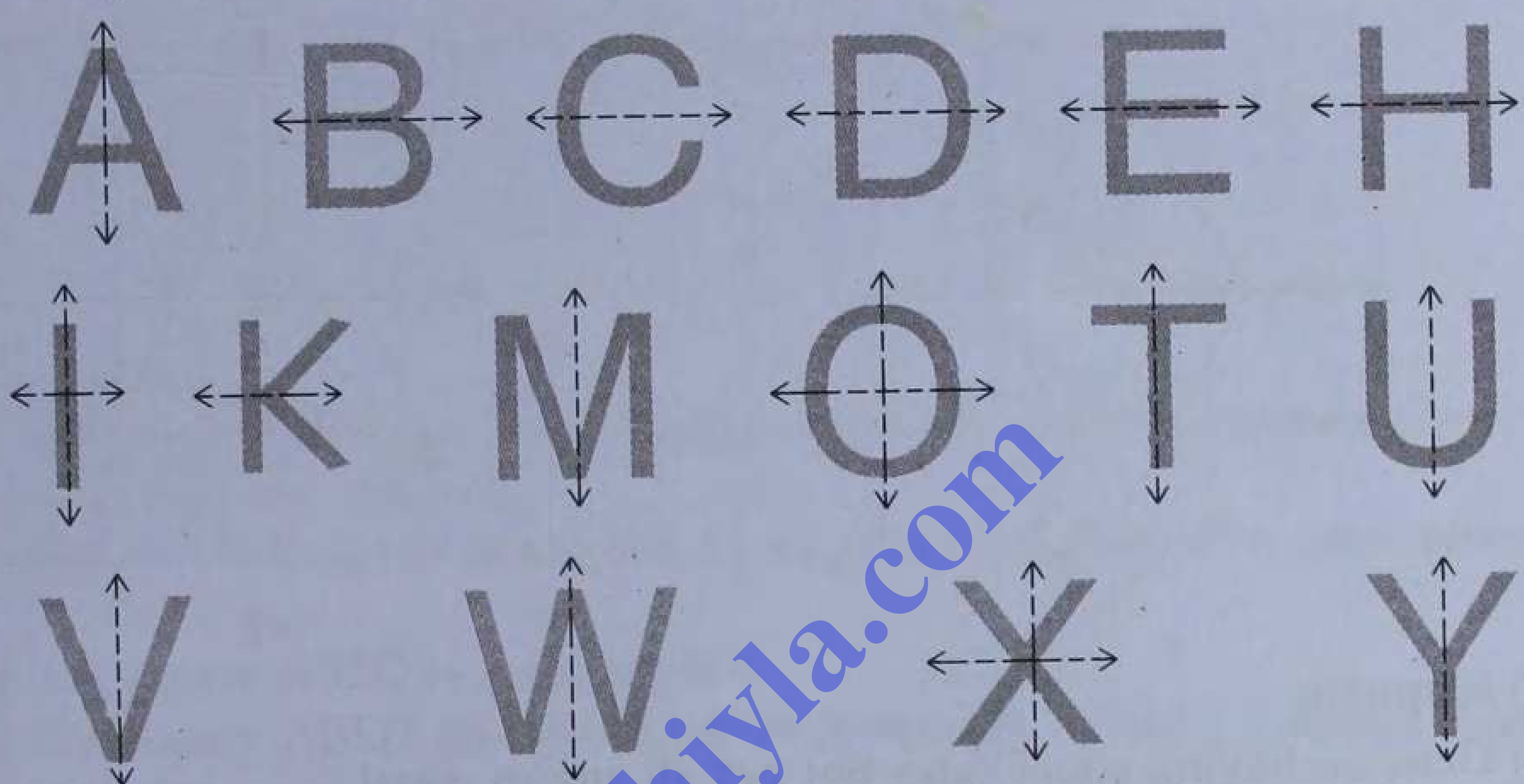
(*i*) n *lines of symmetry.*

(*ii*) If n is *even, one point of symmetry* — the centre of the regular polygon.

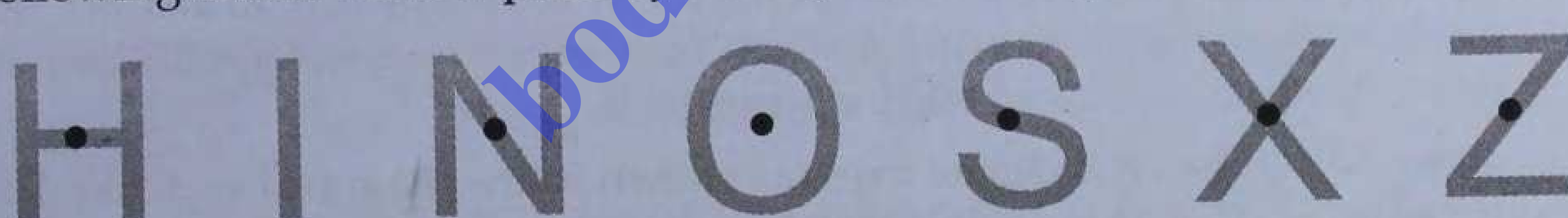
(*iii*) If n is *odd*, then no point of symmetry.

Letters

The following letters are symmetrical about the dotted line (or lines) :



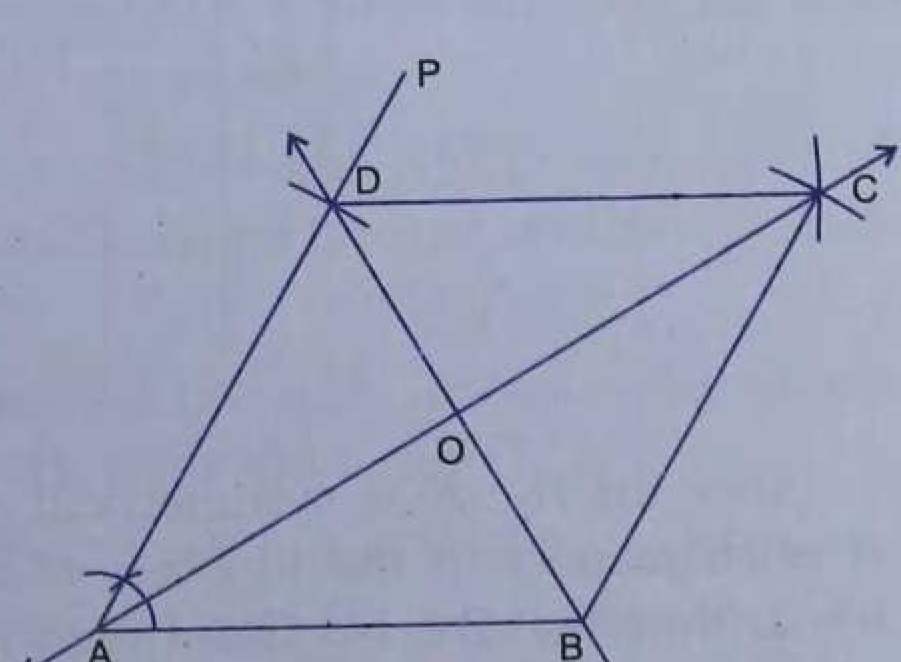
The following letters have a point symmetry about the point marked by a dot :



ILLUSTRATIVE EXAMPLES

Example 1. *Construct a rhombus* **ABCD** *of side 4·5 cm and* $\angle BAD = 60°$*, by using ruler and compasses only. Draw its lines of* **symmetry**. *Hence, prove that diagonals are perpendicular to each other.*

Solution. Draw AB = 4·5 cm. At A, construct $\angle BAP = 60°$. Construct rhombus ABCD. Join AC and BD to meet at O. The diagonals AC and BD are lines of symmetry of rhombus ABCD.



To prove $AC \perp BD$.

Since BD is line of symmetry,

$\Delta AOD \cong \Delta COD$

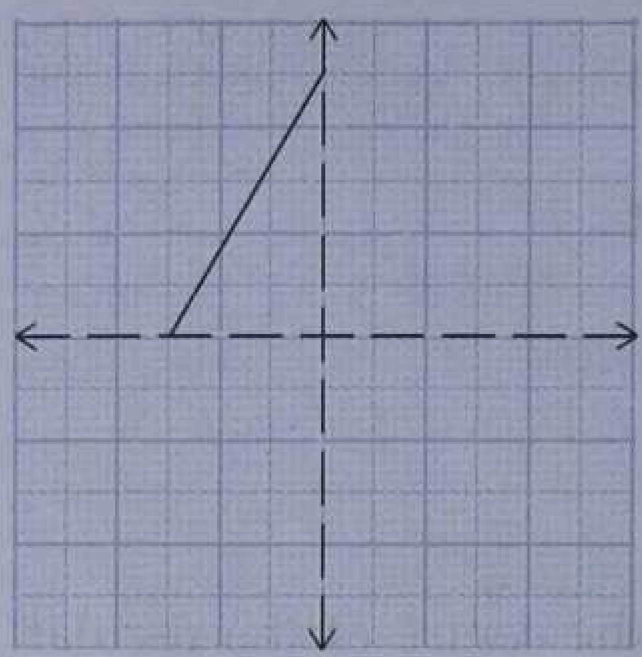
$\Rightarrow \angle AOD = \angle DOC.$

But $\angle AOD + \angle DOC = 180°$

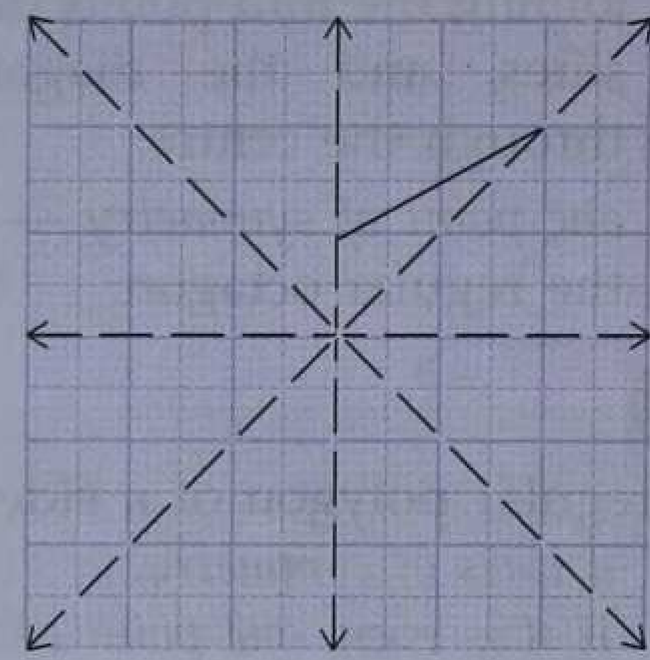
($\because$ AOC is a straight line)

$\Rightarrow \angle AOD = 90° \Rightarrow AC \perp BD.$

Example 2. *Make copies of the following shapes on the graph paper and complete them so that the dotted lines are lines of symmetry :*



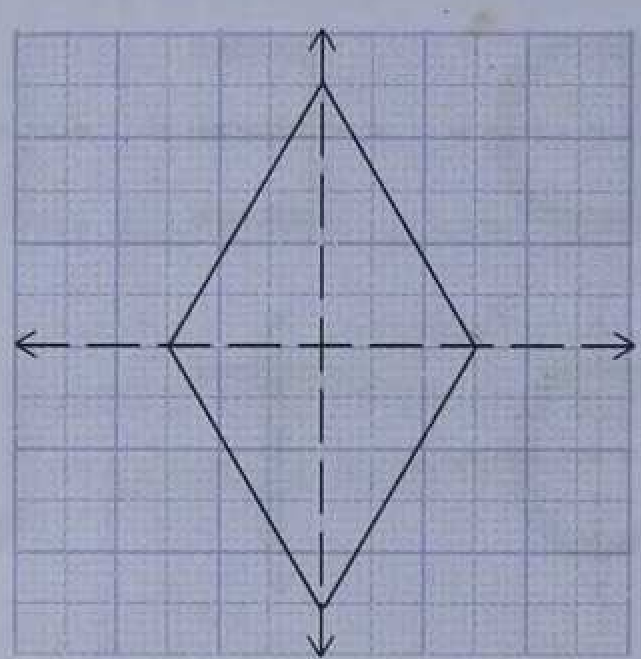
(*i*)



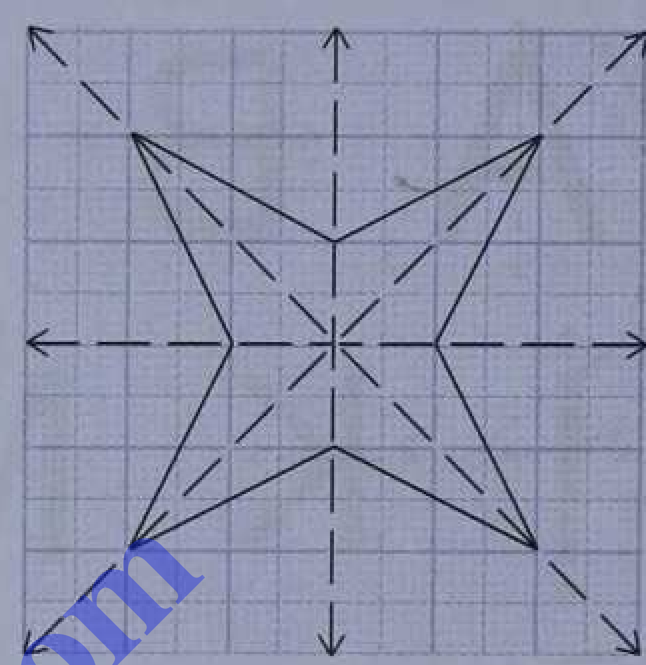
(*ii*)

Also give the name of the geometrical shape formed.

Solution.



(*i*)



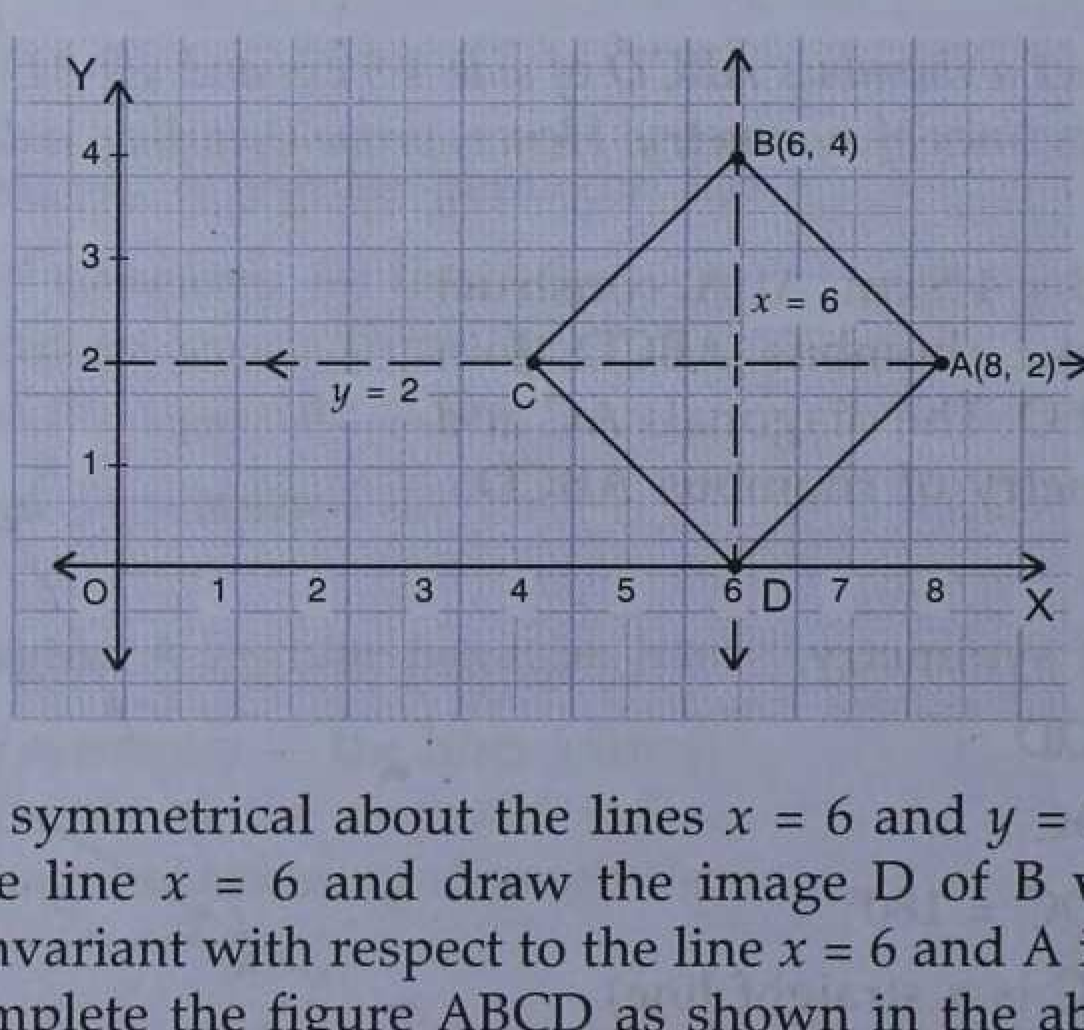
(*ii*)

(*i*) Rhombus

(*ii*) Octagon having equal sides but not all angles equal.

Example 3. *Use graph paper for this question. Plot the points A(8, 2) and B(6, 4). These two points are the vertices of a figure which is symmetrical about x = 6 and y = 2. Complete the figure on the graph. Write down the geometrical name of the figure.* (2000)

Solution. Select the co-ordinate axes (as shown in the diagram) and take 1 cm = 1 unit on both the axes. Plot the points A(8, 2) and B(6, 4) on the graph paper (co-ordinate plane). Draw the lines of symmetry $x = 6$ (vertical) and $y = 2$ (horizontal).



Since the figure is symmetrical about the lines $x = 6$ and $y = 2$, draw the image C of A with respect to the line $x = 6$ and draw the image D of B with respect to the line $y = 2$. Note that B is invariant with respect to the line $x = 6$ and A is invariant with respect to the line $y = 2$. Complete the figure ABCD as shown in the above diagram.

The above figure ABCD is a **square**.

Example 4. *Use a graph paper for this question (Take 1 cm = 1 unit on both axes). Plot the points A(–2, 0), B(4, 0), C(1, 4) and D(–2, 4).*

(i) Draw the line of symmetry of ΔABC. Name it L_1.

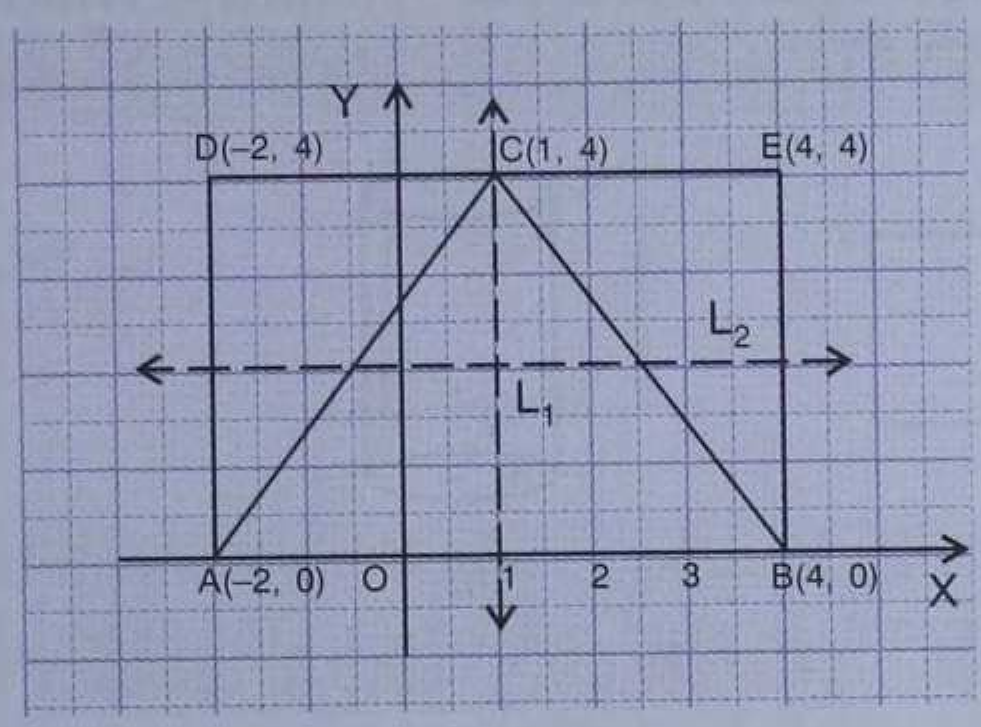
(ii) Point D is reflected about the line L_1 to get the image E. Write the coordinates of E.

(iii) Name the figure ABED.

(iv) Draw all the lines of symmetry of the figure ABED. *(2008)*

Solution. Select the coordinate axes as shown in the diagram and take 1 cm = 1 unit on both axes.

Plot the points A(–2, 0), B(4, 0), C(1, 4) and D(–2, 4) on the graph paper as shown in the diagram.



(*i*) We observe that ABC is an isosceles triangle, its line of symmetry L_1 is shown dotted in the diagram.

(*ii*) Reflect the point D in the line L_1 to get its image E. The coordinates of E are (4, 4).

(*iii*) The figure ABED is a rectangle.

(*iv*) The figure ABED has two lines of symmetry L_1 and L_2 shown dotted in the diagram.

Exercise 13

1. Construct an equilateral triangle each of whose side is 4 cm. Draw all its lines of symmetry.
2. Construct an angle of 60° and draw its axis of symmetry.
3. Construct an isosceles triangle with base = 3·8 cm and one base angle = 45°. Draw its line of symmetry.
4. Using ruler and compasses only, construct a parallelogram ABCD such that BC = 4 cm, the diagonal AC = 8·6 cm and diagonal BD = 4· 4 cm. Measure the side AB. Mark the point of symmetry of the parallelogram as O.
5. Without using set square or protractor, construct a rhombus ABCD with sides of length 4 cm and one diagonal AC of length 5 cm. Draw its lines of symmetry. Also mark its point of symmetry.
6. Construct a rhombus ABCD of side 4·6 cm and ∠BCD = 135°, by using ruler and compasses only. Indicate the point of symmetry with the letter O.
7. (*i*) Construct a rectangle ABCD in which AD = 2·5 cm and BD = 5·5 cm. Measure CD. (Steps of construction need not be written. Lines of construction must be shown clearly).

   (*ii*) Draw all lines of symmetry of the rectangle ABCD.

8. Use a ruler and compass only in this question.

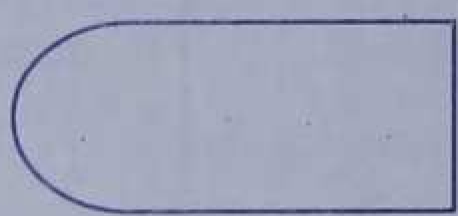
(*i*) Construct the quadrilateral ABCD in which AB = 5 cm, BC = 7 cm and $\angle ABC = 120°$, given that AC is its only line of symmetry.

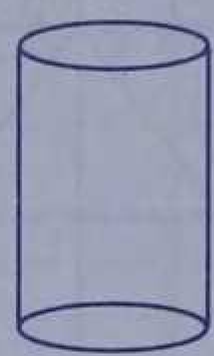
(*ii*) Write down the geometrical name of the quadrilateral.

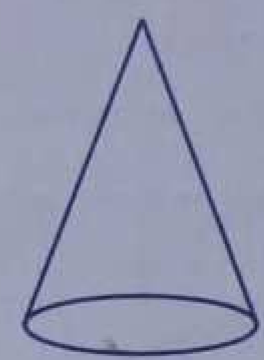
(*iii*) Measure and record the length of BD in cm.

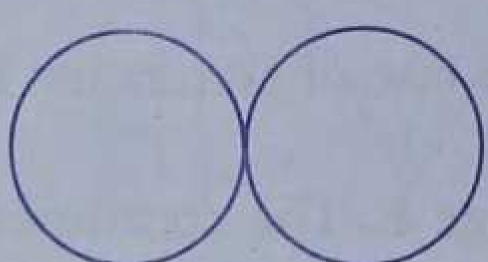
9. Using ruler and compasses only, construct a regular hexagon of side 3 cm. Draw all its lines of symmetry.

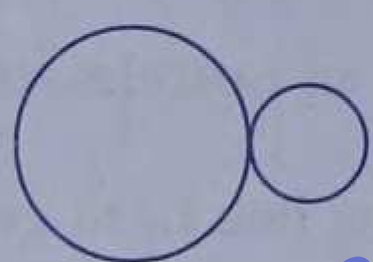
10. How many lines of symmetry do the following figures have ? Copy these figures and in each case, draw line (or lines) of symmetry. Which of these figures have the point of symmetry ?

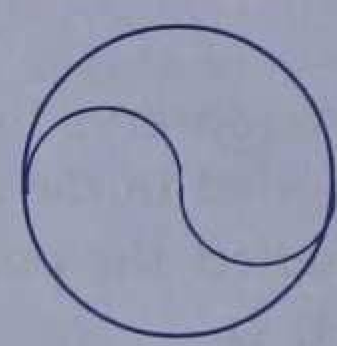

(*i*)

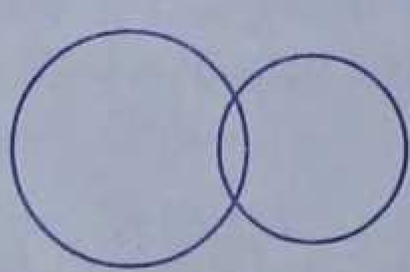

(*ii*)


(*iii*)

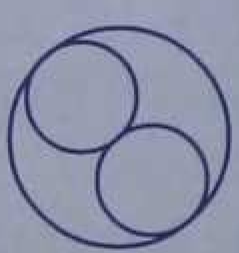

(*iv*)

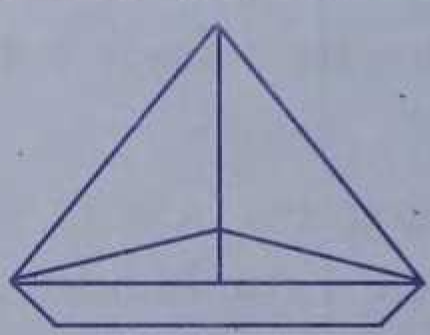

(*v*)

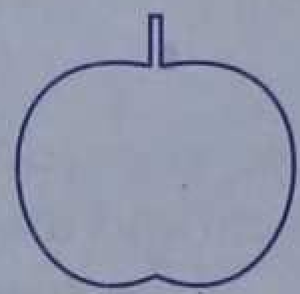

(*vi*)

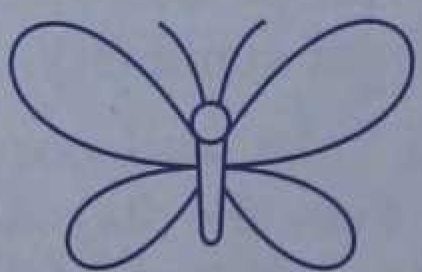

(*vii*)


(*viii*)


(*ix*)


(*x*)


(*xi*)


(*xii*)

11. Draw neat diagrams showing the line (or lines) of symmetry and give the specific name to the quadrilateral :

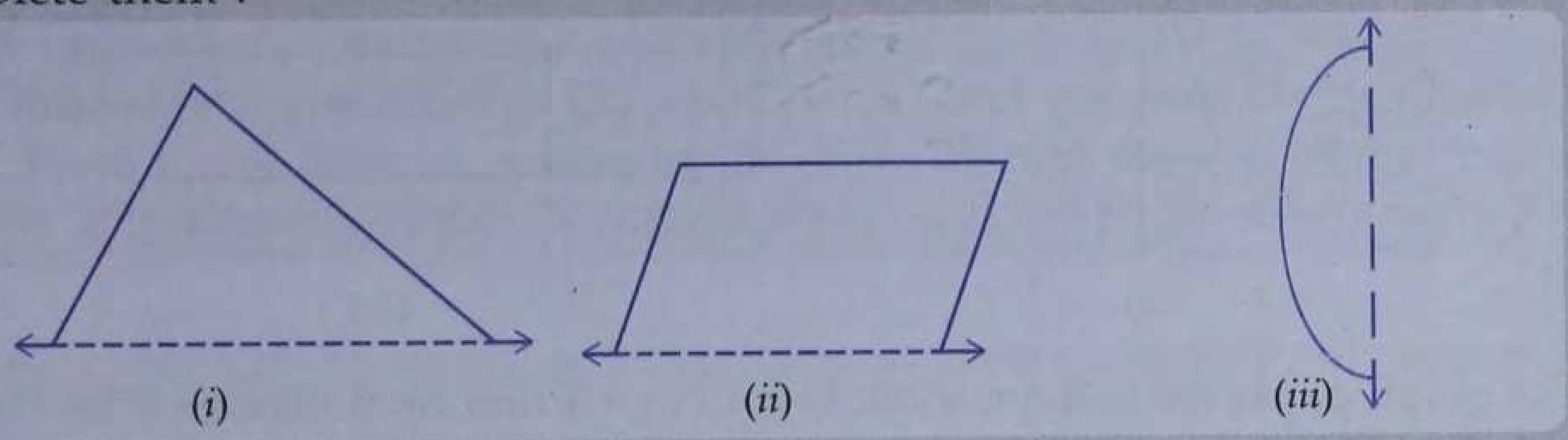
(*i*) Quadrilateral having only one line of symmetry. How many such quadrilaterals are there ?

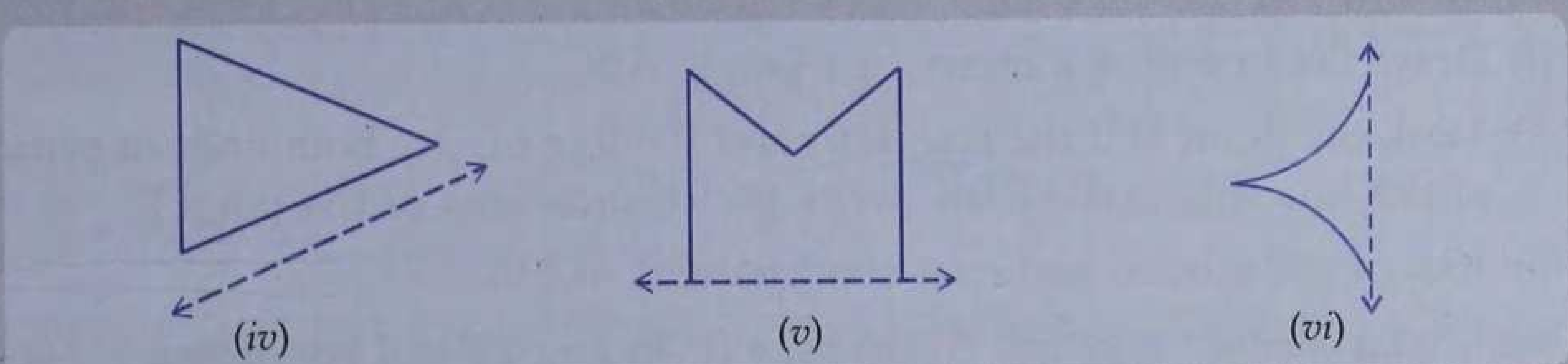
(*ii*) Quadrilateral having its diagonals as the only lines of symmetry.

(*iii*) Quadrilateral having two lines of symmetry other than diagonals.

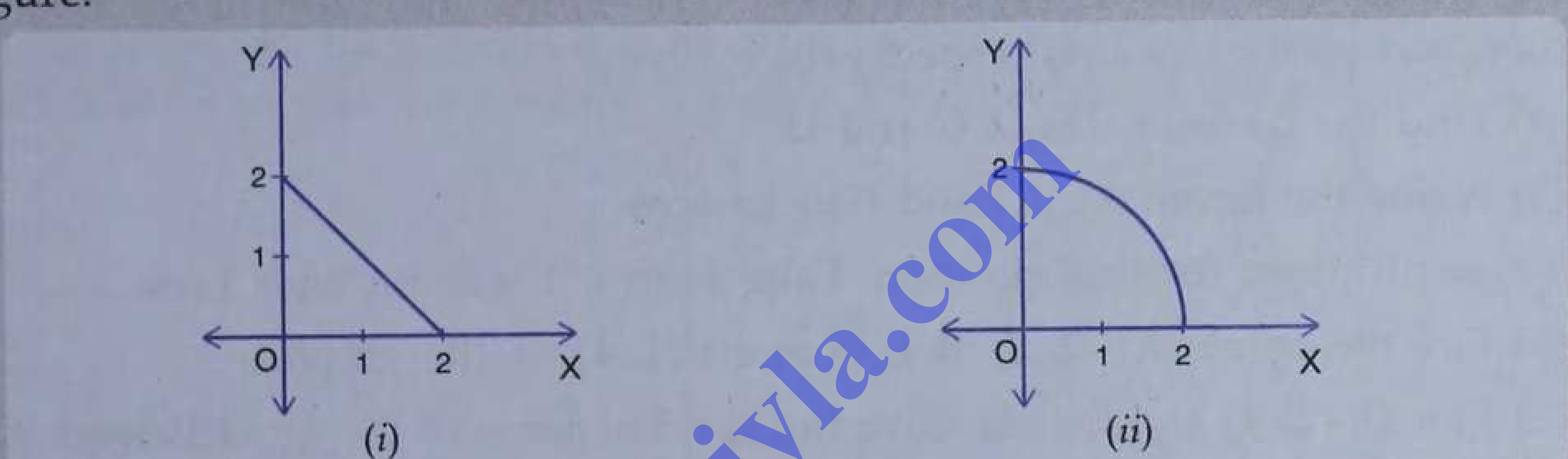
(*iv*) Quadrilateral having more than two lines of symmetry.

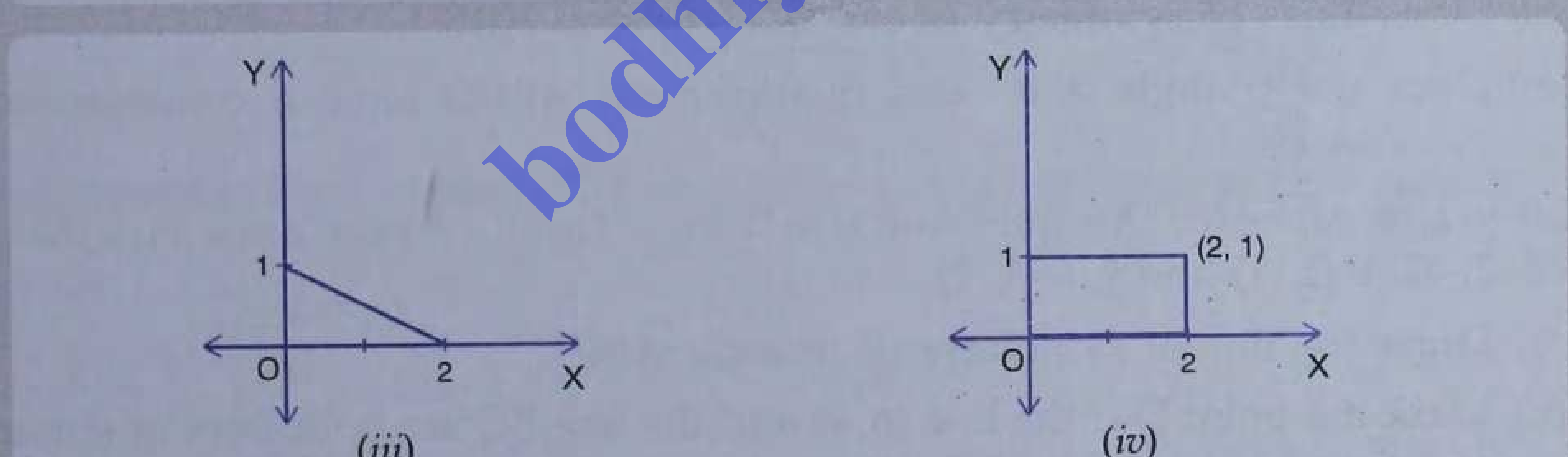
12. Each of the following figures shows half of a symmetrical figure about a line of symmetry indicated as a dotted line. Copy these figures in your note book and complete them :





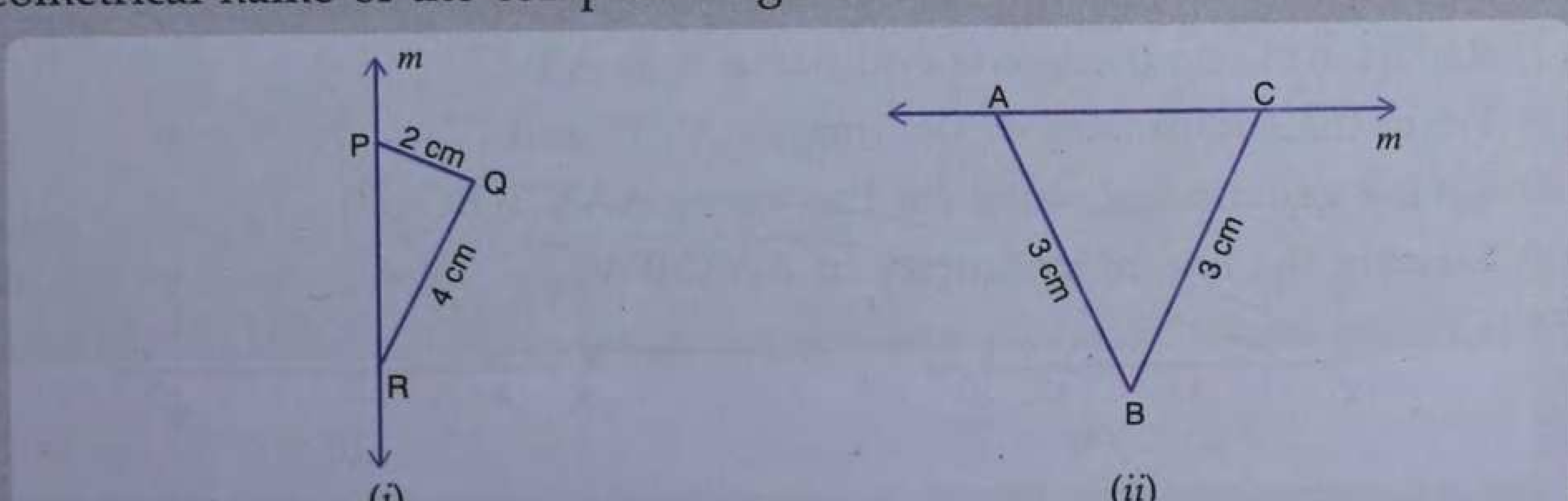
13. Part of a geometrical figure is given in each of the diagram below. Complete the figures so that both x-axis and y-axis are lines of symmetry of the completed figure.

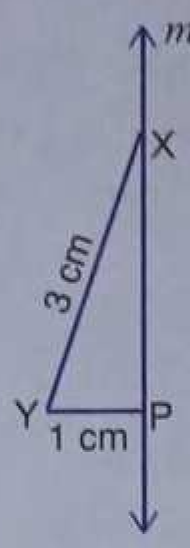




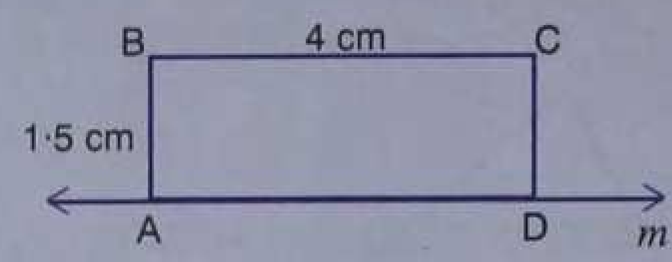
Give the geometrical name of the completed figure. (You may use graph paper if required). Freehand sketches would be sufficient.

14. Part of a geometrical figure is given in each of the diagrams below. Complete the figures, so that the line *'m'*, in each case, is the line of symmetry, of the completed figure. Recognizable freehand sketches, would be awarded full marks. Give the geometrical name of the completed figure.





(iii)



(iv)

15. Use graph paper for this question. Use 1 cm = 1 unit on both axes. Plot the points A(5, 3), B(2, –1) and C(2, 7).

(*i*) Draw the line of symmetry of triangle ABC.

(*ii*) Mark the point D if the line in (*i*) and the line BC are both lines of symmetry of the quadrilateral ACDB; write the co-ordinates of the point D.

(*iii*) Assign the special name to quadrilateral ACDB.

16. Construct a ΔABC, in which AB = AC = 3 cm and BC = 2 cm. Using a ruler and compasses only, draw the reflection A′BC of ΔABC, in BC. Draw lines of symmetry of the figure ABA′C. (2002)

17. The *y*-axis is a line of symmetry for the figure ACBD where A, B have co-ordinates (3, 6), (–3, 4) respectively.

(*i*) Find the co-ordinates of C and D.

(*ii*) Name the figure ACBD and find its area.

18. Use graph paper for this question. Take 1 cm = 1 unit on both axes.

(*i*) Plot the points A(–2, 0), B(4, 0) and C(1, 4) on the graph.

(*ii*) Plot D(–2, 4) and E(4, 4). Give the specific name of the quadrilateral ABED.

(*iii*) Draw lines of symmetry of the quadrilateral ABED.

(*iv*) Does the triangle ABC and quadrilateral ABED have a common line of symmetry ?

19. Use graph paper for this question. Use 2 cm = 1 unit on both axes. Plot the point A (–2, 4), B (2, 1) and C (–6, 1).

(*i*) Draw the line of symmetry of triangle ABC.

(*ii*) Mark the point D if the line in (*i*) and the line BC are both lines of symmetry of the quadrilateral ABDC. Write down the co-ordinates of the point D.

(*iii*) What kind of quadrilateral is figure ABDC?

(*iv*) Write down the equations of BC and the line symmetry named in (*i*).

20. Use a graph paper to answer the following question.

(Take 1 cm = 1 unit on both axes)

(*i*) Plot A (4, 4), B (4, –6) and C (8, 0), the vertices of a triangle ABC.

(*ii*) Reflect ABC on the *y*-axis and name it as A′B′C′.

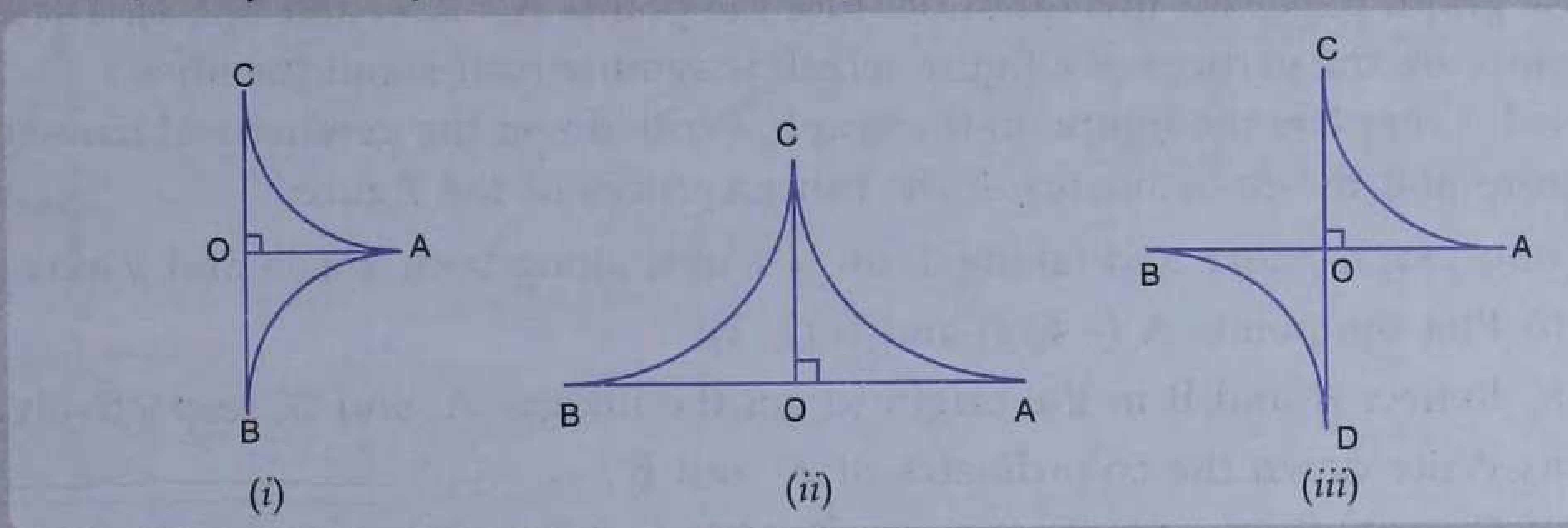
(*iii*) Write the coordinates of the images A′, B′ and C′.

(*iv*) Give a geometrical name for the figure AA′C′B′BC.

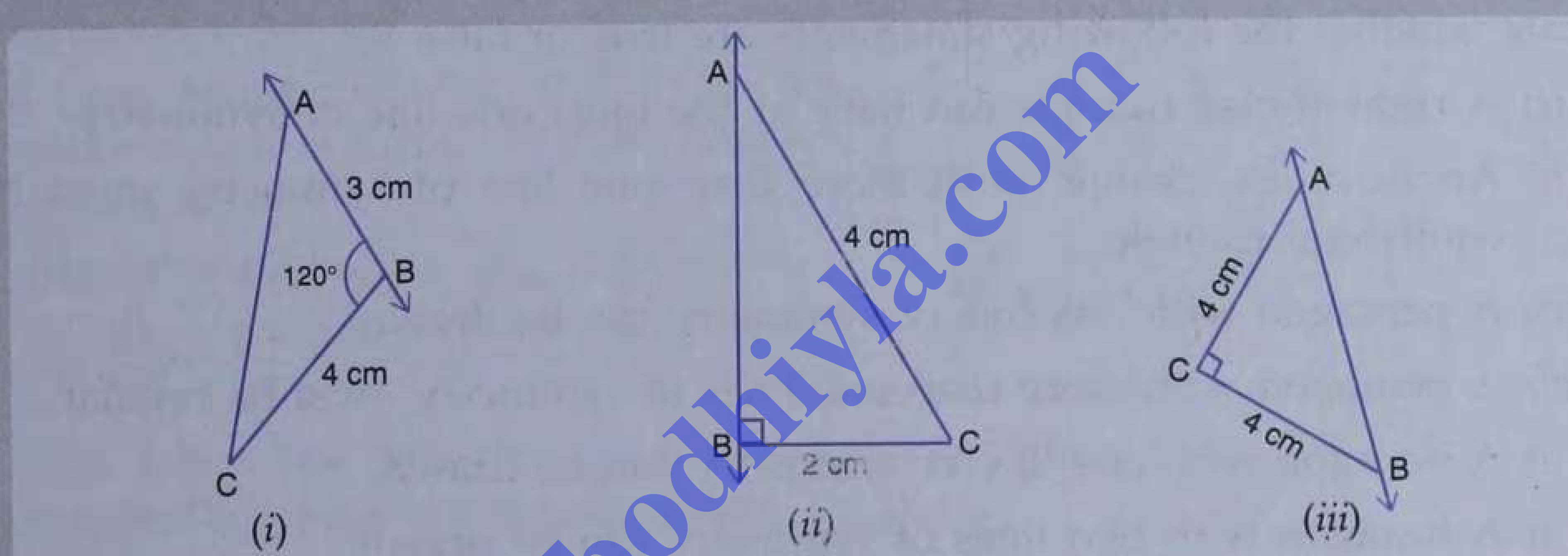
(*v*) Identify the line of symmetry of AA′C′B′BC. (2011)

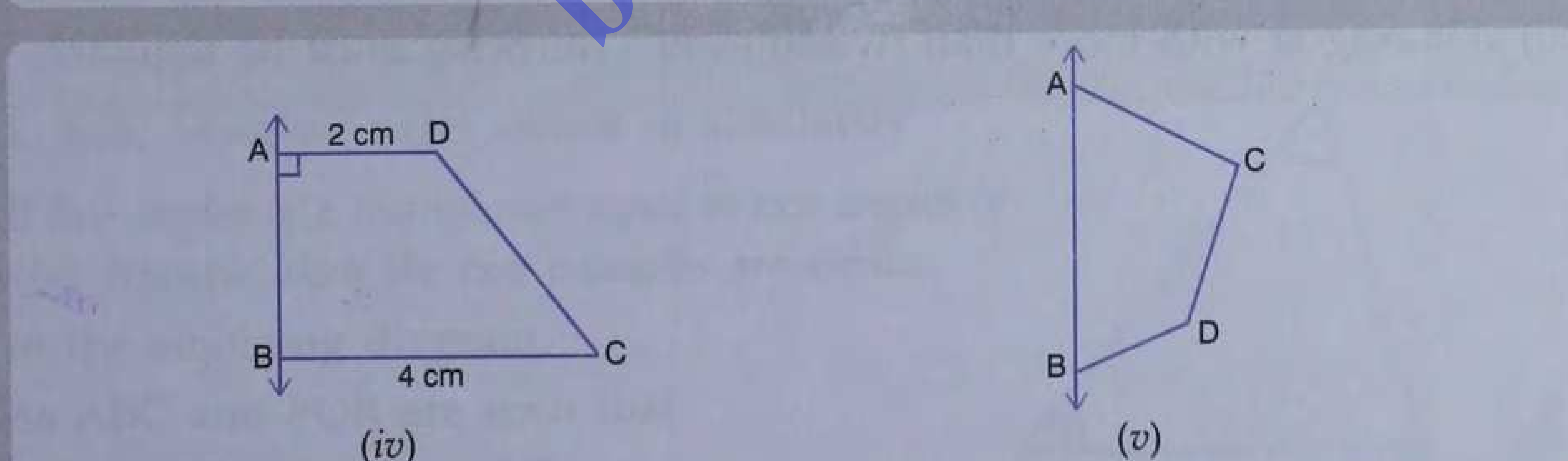
CHAPTER TEST

1. Using ruler and compasses only, construct a rectangle ABCD with AB = 5 cm and AD = 3 cm, and construct its lines of symmetry.
2. In the following figures, OA = OB = OC = OD and the arcs are quadrants of circles. How many lines of symmetry do these figures have ? Name the line (or lines) of symmetry. Which of these figures have the point of symmetry ?

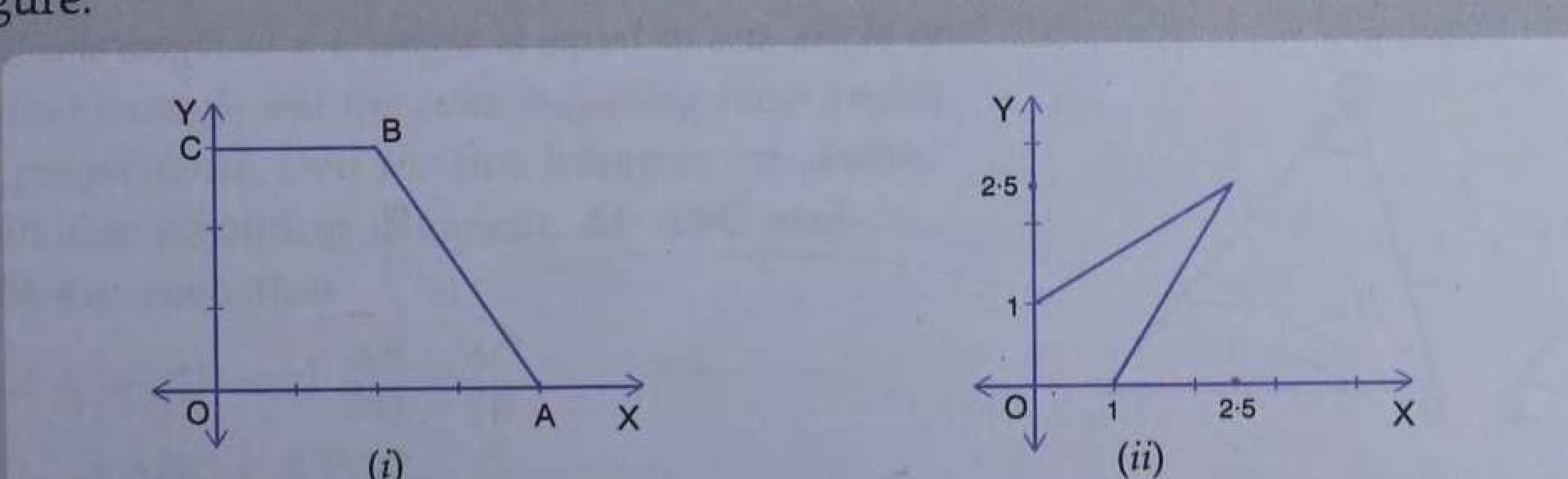


3. Part of a geometrical figure is given in each of the diagram below. Complete the figures so that the line AB in each case is a line of symmetry of the completed figure.
   Give also the geometrical name for the completed figure. Recognizable freehand sketches would be awarded full marks.





4. Part of a geometrical figure is given in each of the diagram below. Complete the figures so that both *x*-axis and *y*-axis are lines of symmetry of the completed figure.



5. In a circle, prove that every line through the centre is a line of symmetry.

6. ABCD is a quadrilateral such that

(*i*) the diagonal AC and BD of the quadrilateral are equal.

(*ii*) the quadrilateral has only two lines of symmetry.

What kind of quadrilateral is ABCD ? Justify your answer.

7. Use graph paper for this question. Plot the points A(– 2, 4) and B(2, 1). These two points are the vertices of a figure which is symmetrical about the lines $x = -2$ and $y = 1$. Complete the figure on the graph. Write down the geometrical name of the figure and the co-ordinates of the other vertices of the figure.

8. Using graph paper and taking 1 cm = 1 unit along both x-axis and y-axis :

(*i*) Plot the points A (– 4, 4) and B (2, 2).

(*ii*) Reflect A and B in the origin to get the images A′ and B′ respectively.

(*iii*) Write down the co-ordinates of A′ and B′.

(*iv*) Give the geometrical name for the figure AB A′ B′.

(*v*) Draw and name its lines of symmetry. *(2012)*

9. Construct ΔABC in which AB = AC = 3 cm and BC = 4 cm. Draw the reflection A_1 of A in the line BC. Draw the lines of symmetry of quadrilateral ABA_1C. Give the specific name of quadrilateral ABA_1C.

10. State whether the following statements are true or false :

(*i*) A right-angled triangle can have at the most one line of symmetry.

(*ii*) An isosceles triangle with more than one line of symmetry must be an equilateral triangle.

(*iii*) A pentagon with one line of symmetry can be drawn.

(*iv*) A pentagon with more than one lines of symmetry must be regular.

(*v*) A hexagon with one line of symmetry can be drawn.

(*vi*) A hexagon with two lines of symmetry can be drawn.

(*vii*) A hexagon with more than two lines of symmetry must be regular.