UNIT – 3
ALGEBRA

CHAPTER 13

FUNDAMENTAL CONCEPTS

(INCLUDING FUNDAMENTAL OPERATIONS)

13.1 ELEMENTARY TREATMENT

1. *Constants and Variables :*

In Arithmetic, we use digits (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) each of which has a fixed value and so these digits and the numbers formed by these digits are called ***constants,*** whereas *in Algebra,* we use letters of English alphabet which can be assigned any value according to the requirement. So the letters used in Algebra are called ***variables.***

(i) A combination of two or more than two constants is always a constant.

e.g. 3 is a constant and 8 is also a constant, therefore each of 3 + 8, 3 – 8, 8 – 3, 8 ÷ 3, 8 ÷ 3, 3 × 8, 38, 83, etc., is also a constant.

Similarly, 5 is a constant, 3 is a constant and 2 is also a constant; so each of 5 × 3 ÷ 2, 3 ÷ 5 × 2, 3 + 5 × 2, 352, 5 × 3 – 2, 532, etc., is also a constant.

(ii) A combination of two or more variables is always a variable.

e.g. x, y and z are variables and so each of $x + y - z$, $x - yz$, $x - y + z$, $x \div y \times z$, etc., is also a variable.

(iii) A combination of one or more constants and one or more variables is also a variable.

e.g. Each of $6 + x$, $x - 8$, $10z$, $8x \div 3y$, $2x - 3y + 4z$, etc., is a variable.

2. *Term :*

A term is *a number* (constant), *a variable* or *a combination* (product or quotient) *of numbers and variables.*

e.g. 5, $3a$, x, ax, xy, $-4xy$, $\frac{4x}{3y}$, $\frac{8pq}{7a}$, $\frac{8}{z}$, $\frac{5ax}{12}$, etc.

3. *Algebraic Expression :*

An algebraic expression is *a collection of one or more terms*, which are *separated* from each other *by addition* (+) *or subtraction* (–) sign*(s).*

e.g. $4x$, $3xy - 7$, $a + 2b$, $5x - 7y$, $2x^2 + 5xy + 3$, $ax - by - cz$, etc.

Only plus (+) and minus (–) signs separate the terms, whereas the product (×) and division (÷) do not separate the terms.

For example :

The expression $3x + 4y$ has two terms, the expression $3x - 4y$ also has two terms but each of $3x \times 4y$ and $3x \div 4y$ has only one term.

4. *Types of algebraic expressions :*

Name	Condition	Examples
1. Monomial	has **only one term**	$x, 5xy, \frac{-7x}{4}, \frac{ax^2}{7}$, etc.
2. Binomial	has **two terms**	$2a + x, \frac{7x}{4} - 8, 2x^2 - y^2$, etc.
3. Trinomial	has **three terms**	$ax^2 + bx + c, a^2 - 4x + 8x^2, xy^2 - xy + \frac{x}{4}$, etc.
4. Multinomial	has **more than three terms**	$4 - a + ax + by, x^2 - 4x - xy + 8y + a$, etc.
5. Polynomial	has **two or more than two terms**	every binomial, every trinomial, every multinomial, etc.

An expression of the type $\frac{4}{x}$ does not form a monomial unless x is not equal to zero (0).

Reason : *Since x is a variable, it can take any value. If it takes the value zero i.e., if x = 0, the expression $\frac{4}{x}$ is equal to $\frac{4}{0}$, which is not defined.*

Similarly, $\frac{7}{y}$ *is not a monomial unless* $y \neq 0$,

$\frac{15}{xy}$ *is not a monomial unless* $x \neq 0$ *and* $y \neq 0$.

5. *Product :*

When two or more quantities (constants, variables or both) *are multiplied together, the result is called their* **product.**

For example :

(i) 4ay is the product of 4, a and y.

(iii) 8b is the product of 8 and b and so on.

6. *Factor :*

Each of the quantities (constants or variables) *multiplied together to form a term is called a factor of the term.*

For example :

(i) 3, a and y are the factors of the term 3ay.

(ii) 2, a and b are the factors of the term 2ab and so on.

Infact, factor of a quantity is each and every constant, variable, combination of constant and variable, etc., by which the given quantity is completely divisible.

For example :

Quantity 3*ay* is completely divisible by each of 1, 3, *a*, *y*, 3*a*, 3*y*, *ay* and 3*ay*, so the factors of 3*ay* are : 1, 3, *a*, *y*, 3*a*, 3*y*, *ay* and 3*ay*.

Similarly, the factors of 2*ab* are : 1, 2, *a*, *b*, 2*a*, 2*b*, *ab* and 2*ab*.

7. Co-efficient :

In a monomial, any factor or group of factors of a term is called the coefficient of the remaining part of the monomial.

For example :

In 5*xyz*, 5 is the co-efficient of *xyz*, *x* is the co-efficient of 5*yz*, *y* is the coefficient of 5*xz*, 5*x* is the co-efficient of *yz*, *xy* is the co-effcient of 5*z* and so on.

> *If a factor is a numerical quantity (i.e., constant), it is called* **numerical co-efficient,** *while the factor involving letter(s) is called the* **literal co-efficient.**
>
> *Thus in 5xyz, 5 is numerical co-efficient and each of x, y, z, 5x, 5y, 5z, xy, yz, xz, 5xy, 5yz, 5xz, xyz and 5xyz are the literal co-efficients.*

8. Degree of a monomial :

The degree of a monomial is the exponent of its variable or the sum of the exponents of its variables.

For example :

(i) The degree of $4x^2 = 2$ [Since, exponent of x^2 is 2]

(ii) The degree of $7x = 1$ [Since, $x = x^1$]

(iii) The degree of $8x^2y^3 = 2 + 3 = 5$ [Sum of the exponents of the variables x and y]

(iv) The degree of $\frac{2}{7}xy^4 = 1 + 4 = 5$

(v) The degree of $2 = 0$ [Since, it has no variable]

9. Degree of a polynomial :

The degree of a polynomial is the degree of its highest degree term.

e.g. (i) In expression $5x^4 + 7x^3y^2 + 2xy^2$, the degree of term $5x^4 = 4$, the degree of term $7x^3y^2 = 3 + 2 = 5$ and the degree of term $2xy^2 = 1 + 2 = 3$.

Since, the highest degree term is $7x^3y^2$ and its degree is 5, therefore, degree of the given polynomial is 5.

(ii) The degree of polynomial $x^5 - x^2y^4 + x^3y$ is $2 + 4 = 6$.

10. Like and unlike terms :

Terms having the *same literal coefficients* or alphabetic letters are called ***like terms,*** whereas the terms with *different literal co-efficients* are called ***unlike terms***.

For example :

(i) $5x$ and $8x$ are *like terms,* whereas $5x$ and $8y$ are *unlike terms.*

(ii) $7x^2$ and $2x^2$ are *like terms,* whereas $7x^2$ and $2x$ are *unlike terms.*

(iii) $3xy^2$ and $4xy^2$ are *like terms,* whereas $3xy^2$ and $4x^2y$ are *unlike terms.*

Example 1 :

For algebraic expression $5 - 8xy + 6x^2y^3 + 5xy^2 - 8x^3y^4$, find :

(i) number of terms

(ii) degree of the expression

(iii) coefficient of x^3 in $-8x^3y^4$

(iv) coefficient of x in $6x^2y^3$

(v) constant term

(vi) literal coefficient of $-8xy$

(vii) coefficient of x^2 in $-8x^3y^4$

(viii) all the factors of $5xy^2$.

Solution :

(i) No. of terms = 5 **(Ans.)**

(ii) Degree of the expression = 3 + 4 = 7 **(Ans.)**

(iii) Coefficient of x^3 in $-8x^3y^4 = -8y^4$ **(Ans.)**

(iv) Coefficient of x in $6x^2y^3 = 6xy^3$ **(Ans.)**

(v) Constant term = 5 **(Ans.)**

(vi) Literal coefficient of $-8xy = -8$ **(Ans.)**

(vii) Coefficient of x^2 in $-8x^3y^4 = -8xy^4$ **(Ans.)**

(viii) All the factors of $5xy^2$ = 1, 5, x, y, xy, y^2, xy^2, $5x$, $5xy$, $5y$, $5y^2$ and $5xy^2$ **(Ans.)**

EXERCISE 13(A)

1. Fill in the blanks :

(i) 8 is a ...., x is a .... and $8x$ is a ....

(ii) y is a ...., 15 is a .... and $y + 15$ is a ....

(iii) $7x$ is a ...., y is a .... and $7xy$ is a ....

(iv) $6x + 2y + 7xy$ is an .... with .... terms.

(v) $6x \div 2y - 7xy$ is an .... with .... terms.

(vi) $6x - 2y \times 7xy$ is an .... with .... terms.

(vii) Every binomial is a ....

(viii) Every trinomial is a ....

(ix) A trinomial has .... terms, a polynomial has .... terms and a multinomial has .... terms.

(x) $7xyz$ is product of ....

(xi) All possible factors of $7xyz$ are ....

(xii) The degree of $x + y^2$ is .... and the degree of xy^2 is ....

(xiii) The degree of $8x^5 + 7xy^3 - 6xy^8$ is ....

(xiv) $5x$ and $5y$ are .... terms, whereas $5x^2y^3$ and $28x^2y^3$ are .... terms.

(xv) The degrees of two like terms are always ....

(xvi) The degrees of two unlike terms can be .... as well as ....

2. Separate *constant terms* and *variable terms* from the following :

8, x, $6xy$, $6 + x$, $-5xy^2$, $15az^2$, $\dfrac{32z}{xy}$, $\dfrac{y^2}{3x}$

3. For each expression, given below, state whether it is a *monomial, binomial* or *trinomial* :

(i) $2x \div 15$	(ii) $ax + 9$	(iii) $3x^2 \times 5x$
(iv) $5 + 2a - 3b$	(v) $2y - \dfrac{7}{3}z \div x$	(vi) $3p \times q \div z$
(vii) $12z \div 5x + 4$	(viii) $12 - 5z - 4$	(ix) $a^3 - 3ab^2 \times c$

4. Write the coefficient of :

(i) xy in $-3axy$ (ii) z^2 in p^2yz^2

(iii) mn in $-mn$ (iv) 15 in $-15p^2$

5. For each of the following monomials, write its *degree* :

(i) $7y$ (ii) $-x^2y$ (iii) xy^2z

(iv) $-9y^2z^3$ (v) $3m^3n^4$ (vi) $-2p^2q^3r^4$

6. Write the *degree of* each of the following *polynomials* :

(i) $3y^3 - x^2y^2 + 4x$ (ii) $p^3q^2 - 6p^2q^5 + p^4q^4$ (iii) $-8mn^6 + 5m^3n$

(iv) $7 - 3x^2y + y^2$ (v) $3x - 15$ (vi) $2y^2z + 9yz^3$

7. Group the *like terms* together :

(i) $9x^2$, xy, $-3x^2$, x^2 and $-2xy$ (ii) ab, $-a^2b$, $-3ab$, $5a^2b$ and $-8a^2b$.

(iii) $7p$, $8pq$, $-5pq$, $-2p$ and $3p$

8. Write *numerical coefficient of each of the followings :*

(i) y (ii) $-y$ (iii) $2x^2y$

(iv) $-8xy^3$ (v) $3py^2$ (vi) $-9a^2b^3$

9. In $-5x^3y^2z^4$; write the coefficient of :

(i) z^2 (ii) y^2 (iii) yz^2

(iv) x^3y (v) $-xy^2$ (vi) $-5xy^2z$

Also, write the degree of the given algebraic expression.

13.2 ADDITION AND SUBTRACTION

Method : *Add or subtract (as required) the numerical coefficients of like terms.*

For example :

(i) Addition of $8xy$, $15xy$ and $3xy$ $= 8xy + 15xy + 3xy$
$= (8 + 15 + 3)xy = \mathbf{26xy}$

(ii) Subtraction of $8xy$ from $15xy$ $= 15xy - 8xy$
$= (15 - 8)xy = \mathbf{7xy}$

Similarly :

(iii) $12x - 3x + 4x = (12 - 3 + 4)x = (16 - 3)x = \mathbf{13x}$

(iv) $23m^2n - 15m^2n - 20m^2n = (23 - 15 - 20)\ m^2n = (23 - 35)\ m^2n = \mathbf{-12m^2n}$

(v) $6a + 8b - 3a - 2b = 6a - 3a + 8b - 2b$ [Grouping like terms]
$= (6 - 3)\ a + (8 - 2)\ b = \mathbf{3a + 6b}$

(vi) $-3x^2y + 15xy^2 - 4x^2y - 6xy^2 = -3x^2y - 4x^2y + 15xy^2 - 6xy^2$
$= \mathbf{-7x^2y + 9xy^2}$

Example 2 :

Add : (i) $3a + 2b - 4c$ and $2c - 5b + 8a$.

(ii) $2ax + 3by + 4cy$, $5by - 3cy - ax$ and $6cy + 4ax - 9by$.

Solution :

Column method : Re-write the given expressions in such a way that their like terms are one below the other, then operate (add or subtract, as the case may be) like terms column-wise. Thus :

(i)
$$\begin{array}{r} 3a + 2b - 4c \\ 8a - 5b + 2c \\ \hline 11a - 3b - 2c \\ \hline \end{array} \quad \text{(Ans.)}$$

(ii)
$$\begin{array}{r} 2ax + 3by + 4cy \\ -ax + 5by - 3cy \\ 4ax - 9by + 6cy \\ \hline 5ax - by + 7cy \\ \hline \end{array} \quad \text{(Ans.)}$$

Row method :

Steps :

1. In a single row, write each of the given polynomials (expressions) in a bracket with plus sign between the consecutive brackets.
2. Remove the brackets without changing the sign of any term.
3. Group the like terms and add.

Thus for Example 2, given above, we have :

(i) $(3a + 2b - 4c) + (2c - 5b + 8a)$ [Step 1]

$= 3a + 2b - 4c + 2c - 5b + 8a$ [Step 2]

$= 3a + 8a + 2b - 5b - 4c + 2c$ [Step 3]

$= 11a - 3b - 2c$ (Ans.)

(ii) $(2ax + 3by + 4cy) + (5by - 3cy - ax) + (6cy + 4ax - 9by)$

$= 2ax + 3by + 4cy + 5by - 3cy - ax + 6cy + 4ax - 9by$

$= 2ax - ax + 4ax + 3by + 5by - 9by + 4cy - 3cy + 6cy$

$= 5ax - by + 7cy$ (Ans.)

Example 3 :

Add : $4x^2 + 3xy - 8y^2$, $2xy - 6x^2 + 7y^2$ and $y^2 - xy + 5x^2$

Solution :

Row Method :

$(4x^2 + 3xy - 8y^2) + (2xy - 6x^2 + 7y^2) + (y^2 - xy + 5x^2)$

$= 4x^2 + 3xy - 8y^2 + 2xy - 6x^2 + 7y^2 + y^2 - xy + 5x^2$

$= 4x^2 - 6x^2 + 5x^2 + 3xy + 2xy - xy - 8y^2 + 7y^2 + y^2$

$= 3x^2 + 4xy$ (Ans.)

Column method :

$$\begin{array}{r} 4x^2 + 3xy - 8y^2 \\ -6x^2 + 2xy + 7y^2 \\ +5x^2 - xy + y^2 \\ \hline 3x^2 + 4xy \qquad \\ \hline \end{array} \quad \text{(Ans.)}$$

Rules of addition of like terms :

1. If the signs of both the terms are same, add their numerical coefficients and affix the same sign. e.g.

(i)
$$\begin{array}{r} 8xy^2 \\ 5xy^2 \\ \hline 13xy^2 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} +\ 8xy^2 \\ +\ 5xy^2 \\ \hline +13xy^2 \\ \hline \end{array} \quad \text{and} \quad \text{(ii)} \quad \begin{array}{r} -\ 8xy^2 \\ -\ 5xy^2 \\ \hline -13xy^2 \\ \hline \end{array}$$

2. If the signs of both the terms are different, subtract the smaller numerical coefficient from the greater and affix the sign of the greater coefficient. e.g.

(i)
$$\begin{array}{r} +\ 8xy^2 \\ -\ 5xy^2 \\ \hline +\ 3xy^2 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 8xy^2 \\ -\ 5xy^2 \\ \hline 3xy^2 \\ \hline \end{array} \quad \text{and} \quad \text{(ii)} \quad \begin{array}{r} -\ 8xy^2 \\ 5xy^2 \\ \hline -\ 3xy^2 \\ \hline \end{array}$$

Example 4 :

Subtract : (i) $2a + 3b - c$ from $4a + 5b + 6c$.

(ii) $2x^2 + 5x - 6$ from $5x^2 - 3x + 1$.

Solution :

Column method :

1. Write the given expressions in two rows in such a way that the like terms are written one below the other, taking care that the expression to be subtracted is written in the second row.
2. Change the sign of each term in the second row (lower row).
3. With these new signs of the terms in lower row, add the like terms columnwise.

Thus :

(i)

$$\begin{array}{lrrr} \textbf{Step 1 :} & 4a & +\ 5b & +\ 6c \\ & 2a & +\ 3b & -\ c \\ \textbf{Step 2 :} & - & - & + \\ \hline \textbf{Step 3 :} & \mathbf{2a} & \mathbf{+\ 2b} & \mathbf{+\ 7c} \\ \hline \end{array} \quad \textbf{(Ans.)}$$

(ii)

$$\begin{array}{rrr} 5x^2 & -\ 3x & +\ 1 \\ 2x^2 & +\ 5x & -\ 6 \\ - & - & + \\ \hline \mathbf{3x^2} & \mathbf{-\ 8x} & \mathbf{+\ 7} \\ \hline \end{array} \quad \textbf{(Ans.)}$$

Row method :

1. Write both the expressions in a single row with the expression to be subtracted in a bracket and put a *minus* sign before (outside) this bracket (minus sign is for subtraction).
2. Open the bracket by changing the sign of each term inside the bracket.
3. Add the like terms.

Thus, for Example 4, given above, we have :

(i) $\mathbf{4a + 5b + 6c - (2a + 3b - c)}$ **[Step 1]**

$= 4a + 5b + 6c - 2a - 3b + c$ **[Step 2]**

$= 4a - 2a + 5b - 3b + 6c + c$

$= \mathbf{2a + 2b + 7c}$ **[Step 3]** **(Ans.)**

(ii) $\mathbf{5x^2 - 3x + 1 - (2x^2 + 5x - 6)}$

$= 5x^2 - 3x + 1 - 2x^2 - 5x + 6$

$= 5x^2 - 2x^2 - 3x - 5x + 1 + 6 = \mathbf{3x^2 - 8x + 7}$ **(Ans.)**

Example 5 :

Subtract : $a^2 + 2ab$ from $b^2 + 4ab$.

Solution :

Column method :

$$\begin{array}{rrr} b^2 & +\ 4ab & \\ & +\ 2ab & +\ a^2 \\ & - & - \\ \hline \mathbf{b^2} & \mathbf{+\ 2ab} & \mathbf{-\ a^2} \\ \hline \end{array} \quad \textbf{(Ans.)}$$

Row method :

$b^2 + 4ab - (a^2 + 2ab)$

$= b^2 + 4ab - a^2 - 2ab$

$= \mathbf{b^2 + 2ab - a^2}$ **(Ans.)**

The statement for subtracting $x + 2y$ from $2x + 5y$ can be given in many ways, such as :

1. Take away $x + 2y$ from $2x + 5y$.
2. What is the excess of $2x + 5y$ over $x + 2y$?
3. By how much does $2x + 5y$ exceed $x + 2y$?
4. How much is $x + 2y$ less than $2x + 5y$?
5. What must be subtracted from $2x + 5y$ to get $x + 2y$?
6. What should be added to $x + 2y$ to get $2x + 5y$?

13.3 ADDITION OR SUBTRACTION OF UNLIKE TERMS

As seen above, the two like terms can be added or subtracted to get a single like term, but two unlike terms cannot be added or subtracted together to get a single term. All that can be done is to connect them by the sign as required.

For example :

(i) Addition of $5x$ and $7y = 5x + 7y$

(ii) Addition of $8xy^2$ and $5x^2y = 8xy^2 + 5x^2y$

(iii) Subtraction of $9y$ from $3xy = 3xy - 9y$ and so on.

Example 6 :

Subtract : $5x^2 + 9y^2 - 3z^2$ from the sum of $6x^2 - 5y^2$ and $y^2 + 5z^2$.

Solution :

Column method :

Sum of $6x^2 - 5y^2$ and $y^2 + 5z^2$.

$$\begin{array}{rrrr} = & 6x^2 & -5y^2 & \\ & & y^2 & +5z^2 \\ \hline = & 6x^2 & -4y^2 & +5z^2 \\ \hline \end{array}$$

Required :

$$\begin{array}{rrrr} = & 6x^2 & -4y^2 & +5z^2 \\ & 5x^2 & +9y^2 & -3z^2 \\ & - & - & + \\ \hline = & x^2 & -13y^2 & +8z^2 \\ \hline \end{array}$$ **(Ans.)**

Row method :

Required : $= (6x^2 - 5y^2) + (y^2 + 5z^2) - (5x^2 + 9y^2 - 3z^2)$

$= 6x^2 - 5y^2 + y^2 + 5z^2 - 5x^2 - 9y^2 + 3z^2$

$= 6x^2 - 5x^2 - 5y^2 + y^2 - 9y^2 + 5z^2 + 3z^2 = x^2 - 13y^2 + 8z^2$ **(Ans.)**

EXERCISE 13 (B)

1. Fill in the blanks :

(i) $8x + 5x =$ ....

(ii) $8x - 5x =$ ....

(iii) $6xy^2 + 9xy^2 =$ ....

(iv) $6xy^2 - 9xy^2 =$ ....

(v) The sum of $8a$, $6a$ and $5b =$ ....

(vi) The addition of 5, $7xy$, 6 and $3xy =$ ....

(vii) $4a + 3b - 7a + 4b =$ ....

(viii) $-15x + 13x + 8 =$ ....

(ix) $6x^2y + 13xy^2 - 4x^2y + 2xy^2 =$ ....

(x) $16x^2 - 9x^2 =$ .... and $25xy^2 - 17xy^2 =$ ....

2. Add :

(i) $-9x$, $3x$ and $4x$ (ii) $23y^2$, $8y^2$ and $-12y^2$

(iii) $18pq$, $-15pq$ and $3pq$

3. Simplify :

(i) $3m + 12m - 5m$ (ii) $7n^2 - 9n^2 + 3n^2$

(iii) $25zy - 8zy - 6zy$ (iv) $-5ax^2 + 7ax^2 - 12ax^2$

(v) $-16am + 4mx + 4am - 15mx + 5am$

4. Add :

(i) $a + b$ and $2a + 3b$ (ii) $2x + y$ and $3x - 4y$

(iii) $-3a + 2b$ and $3a + b$ (iv) $4 + x$, $5 - 2x$ and $6x$

5. Find the sum of :

(i) $3x + 8y + 7z$, $6y + 4z - 2x$ and $3y - 4x + 6z$

(ii) $3a + 5b + 2c$, $2a + 3b - c$ and $a + b + c$

(iii) $4x^2 + 8xy - 2y^2$ and $8xy - 5y^2 + x^2$

(iv) $9x^2 - 6x + 7$, $5 - 4x$ and $6 - 3x^2$

(v) $5x^2 - 2xy + 3y^2$, $-2x^2 + 5xy + 9y^2$ and $3x^2 - xy - 4y^2$

(vi) $a^2 + b^2 + 2ab$, $2b^2 + c^2 + 2bc$ and $4c^2 - a^2 + 2ac$

(vii) $9ax - 6bx + 8$, $4ax + 8bx - 7$ and $-6ax - 4bx - 3$

(viii) $abc + 2ba + 3ac$, $4ca - 4ab + 2bca$ and $2ab - 3abc - 6ac$

(ix) $4a^2 + 5b^2 - 6ab$, $3ab$, $6a^2 - 2b^2$ and $4b^2 - 5ab$

(x) $x^2 + x - 2$, $2x - 3x^2 + 5$ and $2x^2 - 5x + 7$

(xi) $4x^3 + 2x^2 - x + 1$, $2x^3 - 5x^2 - 3x + 6$, $x^2 + 8$ and $5x^3 - 7x$

6. Find the sum of :

(i) x and $3y$ (ii) $-2a$ and $+5$ (iii) $-4x^2$ and $+7x$

(iv) $+4a$ and $-7b$ (v) x^3, $3x^2y$ and $2y^2$ (vi) 11 and $-by$

7. The sides of a triangle are $2x + 3y$, $x + 5y$ and $7x - 2y$. Find its perimeter.

8. The two adjacent sides of a rectangle are $6a + 9b$ and $8a - 4b$. Find its perimeter.

9. Subtract the second expression from the first :

(i) $2a + b$, $a + b$ (ii) $-2b + 2c$, $b + 3c$

(iii) $5a + b$, $-6b + 2a$ (iv) $a^3 - 1 + a$, $3a - 2a^2$

(v) $p + 2$, 1 (vi) $x + 2y + z$, $-x - y - 3z$

(vii) $3a^2 - 8ab - 2b^2$, $3a^2 - 4ab + 6b^2$ (viii) $4pq - 6p^2 - 2q^2$, $9p^2$

(ix) $10abc$, $2a^2 + 2abc - 4b^2$ (x) $a^2 + ab + c^2$, $a^2 - d^2$

10. Subtract :

(i) $4x$ from $8 - x$

(ii) $-8c$ from $c + 3d$

(iii) $-5a - 2b$ from $b + 6c$

(iv) $4p + p^2$ from $3p^2 - 8p$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

11. Subtract $-5a^2 - 3a + 1$ from the sum of $4a^2 + 3 - 8a$ and $9a - 7$.

12. By how much does $8x^3 - 6x^2 + 9x - 10$ exceed $4x^3 + 2x^2 + 7x - 3$?

13. What must be added to $2a^3 + 5a - a^2 - 6$ to get $a^2 - a - a^3 + 1$?

14. What must be subtracted from $a^2 + b^2 + 2ab$ to get $-4ab + 2b^2$?

15. Find the excess of $4m^2 + 4n^2 + 4p^2$ over $m^2 + 3n^2 - 5p^2$.

16. By how much is $3x^3 - 2x^2y + xy^2 - y^3$ less than $4x^3 - 3x^2y - 7xy^2 + 2y^3$?

17. Subtract the sum of $3a^2 - 2a + 5$ and $a^2 - 5a - 7$ from the sum of $5a^2 - 9a + 3$ and $2a - a^2 - 1$.

18. The perimeter of a rectangle is $28x^3 + 16x^2 + 8x + 4$. One of its sides is $8x^2 + 4x$. Find the other side.

19. The perimeter of a triangle is $14a^2 + 20a + 13$. Two of its sides are $3a^2 + 5a + 1$ and $a^2 + 10a - 6$. Find its third side.

20. If $x = 4a^2 + b^2 - 6ab$, $y = 3b^2 - 2a^2 + 8ab$ and $z = 6a^2 + 8b^2 - 6ab$, find :

(i) $x + y + z$

(ii) $x - y - z$

21. If $m = 9x^2 - 4xy + 5y^2$ and $n = -3x^2 + 2xy - y^2$, find :

(i) $2m - n$

(ii) $m + 2n$

(iii) $m - 3n$.

13.4 MULTIPLICATION

1. *Multiplication of monomials :*

Steps : 1. Multiply the numerical coefficients together.

2. Multiply the literal coefficients separately together.

For example :

(i) $\mathbf{8x \times 3y} = (8 \times 3) \times (x \times y)$

$= 24 \times xy = \mathbf{24xy}$

(ii) $\mathbf{5a \times 3b \times 6c} = (5 \times 3 \times 6) \times (a \times b \times c) = \mathbf{90\ abc}$

1. *If both the terms (monomials) have same signs, the sign of the product is always positive.*
   ∴ $(5a) \times (4b) = 20ab$ *and* $(-5a) \times (-4b) = 20ab$
2. *If both the terms (monomials) have opposite signs, the sign of the product is always negative.*
   ∴ $(-5a) \times 4b = -20ab$ *and* $(5a) \times (-4b) = -20ab$

2. *Multiplication of a polynomial by a monomial :*

Steps : 1. Write the given polynomial inside a bracket and the monomial outside it.

2. Multiply the monomial with each term of the polynomial and simplify.

For example :

(i) Multiplication of $2x + y - 8$ and $4x$ $= 4x(2x + y - 8)$

$= 4x \times 2x + 4x \times y - 4x \times 8$

$= \mathbf{8x^2 + 4xy - 32x}$

(ii) Multiplication of $-2a^2$ with $6a + 2b - 3c$ $= -2a^2(6a + 2b - 3c)$

$= -2a^2 \times 6a - 2a^2 \times 2b + (-2a^2) \times (-3c)$

$= \mathbf{-12a^3 - 4a^2b + 6a^2c}$

Alternative method :

Instead of multiplying the polynomial by the monomial horizontally, as done above, these may be multiplied vertically as shown below :

(i)
$$\begin{array}{r} 2x + y - 8 \\ 4x \\ \hline 8x^2 + 4xy - 32x \\ \hline \end{array}$$

∵ $4x \times 2x = 8x^2$

$4x \times y = 4xy$

and $4x \times -8 = -32x$

(ii)
$$\begin{array}{r} 6a + 2b - 3c \\ -2a^2 \\ \hline -12a^3 - 4a^2b + 6a^2c \\ \hline \end{array}$$

∵ $-2a^2 \times 6a = -12a^3$

$-2a^2 \times 2b = -4a^2b$

and $-2a^2 \times -3c = 6a^2c$

3. *Multiplication of a polynomial by a polynomial :*

Steps : 1. Multiply each term of one polynomial by each term of other polynomial.

2. Combine (add or subtract) the like terms.

Thus for the multiplication of $a + b$ and $2a + 3b$, we have :

Column method :

$$\begin{array}{r} a + b \\ 2a + 3b \\ \hline 2a^2 + 2ab \\ 3ab + 3b^2 \\ \hline 2a^2 + 5ab + 3b^2 \\ \hline \end{array}$$

$[2a(a + b) = 2a^2 + 2ab]$

$[3b(a + b) = 3ab + 3b^2]$

[Adding like terms]

Row method :

Multiplication of $a + b$ and $2a + 3b$ $= (a + b)(2a + 3b)$

$= a(2a + 3b) + b(2a + 3b)$

$= a(2a) + a(3b) + b(2a) + b(3b)$

$= 2a^2 + 3ab + 2ab + 3b^2$

$= \mathbf{2a^2 + 5ab + 3b^2}$

Similarly;

(i) $(x - 2y)(3x - 5y)$

$= x(3x - 5y) - 2y(3x - 5y)$

$= 3x^2 - 5xy - 6xy + 10y^2$

$= \mathbf{3x^2 - 11xy + 10y^2}$

(ii) $(4x + 8)(3x^3 - x + 1)$

$= 4x(3x^3 - x + 1) + 8(3x^3 - x + 1)$

$= 12x^4 - 4x^2 + 4x + 24x^3 - 8x + 8$

$= 12x^4 + 24x^3 - 4x^2 + 4x - 8x + 8$

$= \mathbf{12x^4 + 24x^3 - 4x^2 - 4x + 8}$

Example 7 :

Evaluate : $(2x - 5y)(3x + 7y)(8x - 9y)$.

Solution :

$(2x - 5y)(3x + 7y)(8x - 9y)$

$= [2x(3x + 7y) - 5y(3x + 7y)](8x - 9y)$

$= [6x^2 + 14xy - 15xy - 35y^2](8x - 9y)$

$= [6x^2 - xy - 35y^2](8x - 9y)$

$= 6x^2(8x - 9y) - xy(8x - 9y) - 35y^2(8x - 9y)$

$= 48x^3 - 54x^2y - 8x^2y + 9xy^2 - 280xy^2 + 315y^3$

$= \mathbf{48x^3 - 62x^2y - 271xy^2 + 315y^3}$ **(Ans.)**

EXERCISE 13 (C)

1. Fill in the blanks :

(i) $3 \times 6x =$ ....

(ii) $6 \times 3y =$ ....

(iii) $3x \times 6y =$ ....

(iv) $-5x \times 6x =$ ....

(v) $2x^2 \times 8xy =$ ....

(vi) $4x^2y \times y^5 =$ ....

(vii) $8a \times 7y =$ ....

(viii) $5ab^2 \times 4ab =$ ....

(ix) $-2m \times mn =$ ....

(x) $-4m \times -3mn^2 =$ ....

(xi) $-a^2 \times -b^2 =$ ....

(xii) $4pq \times -8q =$ ....

(xiii) $-3 \times -2ab \times -4xy =$ ....

(xiv) $-15x \times 9xy^2 \times -2 =$ ....

2. Multiply :

(i) $3x$, $5x^2y$ and $2y$

(ii) 5, $3a$ and $2ab^2$

(iii) $5x + 2y$ and $3xy$

(iv) $6a - 5b$ and $-2a$

(v) $4a + 5b$ and $4a - 5b$

(vi) $9xy + 2y^2$ and $2x - 3y$

(vii) $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

(viii) $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

3. Copy and complete the following multiplications :

(i) $\begin{array}{r} 3a + 2b \\ \times \quad -3xy \\ \hline \end{array}$ (ii) $\begin{array}{r} 9x - 5y \\ \times \quad -3xy \\ \hline \end{array}$ (iii) $\begin{array}{r} 3xy - 2x^2 - 6x \\ \times \quad -5x^2y \\ \hline \end{array}$

(iv) $\begin{array}{r} a + b \\ \times \quad a + b \\ \hline \end{array}$ (v) $\begin{array}{r} ax - b \\ \times \quad 2ax + 2b^2 \\ \hline \end{array}$ (vi) $\begin{array}{r} 2a - b + 3c \\ \times \quad 2a - 4b \\ \hline \end{array}$

(vii) $\begin{array}{r} 3m^2 + 6m - 2n \\ \times \quad 5n - 3m \\ \hline \end{array}$ (viii) $\begin{array}{r} 6 - 3x + 2x^2 \\ \times \; 1 + 5x - x^2 \\ \hline \end{array}$ (ix) $\begin{array}{r} 4x^3 - 10x^2 + 6x - 8 \\ \times \quad 3 + 2x - x^2 \\ \hline \end{array}$

4. Evaluate :

(i) $(c + 5)(c - 3)$ (ii) $(3c - 5d)(4c - 6d)$

(iii) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$ (iv) $(a^2 + 2ab + b^2)(a + b)$

(v) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$ (vi) $(4m - 2)(m^2 + 5m - 6)$

(vii) $(8 - 12x + 7x^2 - 6x^3)(5 - 2x)$ (viii) $(4x^2 - 4x + 1)(2x^3 - 3x^2 + 2)$

(ix) $(6p^2 - 8pq + 2q^2)(-5p)$ (x) $-4y(15x + 12y - 8z)(x - 2y)$

(xi) $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

5. Evaluate :

(i) $(a + b)(a - b)$.

(ii) $(a^2 + b^2)(a + b)(a - b)$, using the result of (i).

(iii) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$, using the result of (ii).

6. Evaluate :

(i) $(3x - 2y)(4x + 3y)$.

(ii) $(3x - 2y)(4x + 3y)(8x - 5y)$.

(iii) $(a + 5)(3a - 2)(5a + 1)$

(iv) $(a + 1)(a^2 - a + 1)$ and $(a - 1)(a^2 + a + 1)$;
and then : $(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$.

(v) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$.

13.5 DIVISION

1. *Dividing a monomial by a monomial :*

Steps : 1. Write the dividend in numerator and divisor in denominator.

2. Simplify the fraction, obtained in Step (1).

For example :

(i) Division of $15xy$ by $5x = \frac{15xy}{5x} = \mathbf{3y}$

(ii) $(6x^2y) \div (2xy) = \frac{6x^2y}{2xy} = \mathbf{3x}$

(iii) $(-12a^2bc) \div (9ab^2c^2) = \frac{-12a^2bc}{9ab^2c^2} = \frac{-3 \times 2 \times 2 \times a \times a \times b \times c}{3 \times 3 \times a \times b \times b \times c \times c} = -\frac{\mathbf{4a}}{\mathbf{3bc}}$

(iv) $\frac{3ab}{5} \div \frac{-4a}{5} = \frac{3ab}{5} \times \frac{5}{-4a} = -\frac{3b}{4}$

(v) $(72x^5y^2z^3) \div 54x^2y^5z^7$

$$= \frac{72x^5y^2z^3}{54x^2y^5z^7} = \frac{4\times18x^{5-2}}{3\times18y^{5-2}z^{7-3}} = \frac{4x^3}{3y^3z^4}$$

2. *Dividing a polynomial by a monomial :*

Divide each term of the polynomial by the monomial :

For example :

(i) Division of $4x^2 + 5x$ by $2x$

$$= (4x^2 + 5x) \div 2x = \frac{4x^2}{2x} + \frac{5x}{2x} = 2x + \frac{5}{2}$$

(ii) $(8a^2b^3c + 4ab^2c^2 - 6abc) \div (-2a^2bc)$

$$= \frac{8a^2b^3c}{-2a^2bc} + \frac{4ab^2c^2}{-2a^2bc} - \frac{6abc}{-2a^2bc} = -4b^2 - \frac{2bc}{a} + \frac{3}{a}$$

3. *Dividing a polynomial by a polynomial :*

Example 8 :

Divide : $6x^2 + 19x + 10$ by $3x + 2$.

Step 1 : Set the two expressions as : $3x + 2 \overline{)\,6x^2 + 19x + 10}$

Step 2 : Divide first term of the dividend by the first term of divisor to get the first term of quotient. Here, $\frac{6x^2}{3x} = 2x$, which is the first term of the quotient.

$$\begin{array}{r} 2x \\ 3x+2 \overline{)\,6x^2 + 19x + 10} \end{array}$$

Step 3 : Multiply quotient $(2x)$ with each term of divisor $(3x + 2)$ to get :

$$2x(3x + 2) = 6x^2 + 4x$$

$$\begin{array}{r} 2x \\ 3x+2 \overline{)\,6x^2 + 19x + 10} \\ 6x^2 + 4x \quad\quad \end{array}$$

Step 4 : Subtract the result of Step 3 and take the next term/ terms of the dividend down :

$$\begin{array}{r} 2x \\ 3x+2 \overline{)\,6x^2 + 19x + 10} \\ 6x^2 + 4x \quad\quad \\ - \quad - \quad\quad\quad \\ \hline 15x + 10 \end{array}$$

Step 5 : Repeat the process from Step 2 to Step 4 taking remainder $15x + 10$ as new dividend :

$$\because \; 5(3x + 2) = 15x + 10$$

$$\begin{array}{r} 2x + 5 \\ 3x+2 \overline{)\,6x^2 + 19x + 10} \\ 6x^2 + 4x \quad\quad \\ - \quad - \quad\quad\quad \\ \hline 15x + 10 \\ 15x + 10 \\ - \quad - \\ \hline 0 \\ \hline \end{array}$$

Thus, $(6x^2 + 19x + 10) \div (3x + 2) = 2x + 5$ **(Ans.)**

While dividing one polynomial by another polynomial, arrange the terms of both the dividend and the divisor in descending or in ascending order of their powers.

In Example 8, given above, both dividend $6x^2 + 19x + 10$ and divisor $3x + 2$ are in descending order of their powers.

Example 9 :

Divide : $6x^2 + 7xy - 3y^2$ by $2x + 3y$.

Solution :

$$\begin{array}{r|l} & 3x - y \\ \hline 2x + 3y & 6x^2 + 7xy - 3y^2 \\ & 6x^2 + 9xy \\ & - \quad - \\ \hline & \quad - 2xy - 3y^2 \\ & \quad - 2xy - 3y^2 \\ & \quad + \quad + \\ \hline & \qquad 0 \end{array}$$

$\because$ $6x^2$ divided by $2x$ gives $3x$

and, $3x(2x + 3y) = 6x^2 + 9xy$

$\because$ $-2xy$ divided by $2x$ gives $-y$

and, $-y(2x + 3y) = -2xy - 3y^2$

$\therefore$ $(6x^2 + 7xy - 3y^2) \div (2x + 3y) = 3x - y$ **(Ans.)**

EXERCISE 13 (D)

1. Fill in the blanks :

(i) $6x \div 3 =$ .... (ii) $8m \div 4m =$ ....

(iii) $4x^2y^3 \div 2xy^2 =$ .... (iv) $12abc \div 4bc =$ ....

(v) $-18xyz \div (-3zx) =$ .... (vi) $-18xyz \div (-9yz) =$ ....

(vii) $18xyz \div (-3zx) =$ .... (viii) $35a^2bc \div 7ab =$ ....

(ix) $-35a^2bc \div (-5ac) =$ .... (x) $-35a^2bc \div a^2c =$ ....

(xi) $35a^2bc \div (-7abc) =$ .... (xii) $(6a + 3) \div 3 =$ ....

(xiii) $(12x^2 - y^2) \div x =$ .... (xiv) $(-16m^2 + 24n^2) \div (-12) =$ ....

(xv) $(6a^2 - 5ab) \div (-a) =$ .... (xvi) $(45x^2y - 36xy^2) \div (-9xy) =$ ....

2. Divide :

(i) $-16ab^2c$ by $6abc$ (ii) $25x^2y$ by $-5y^2$ (iii) $8x + 24$ by 4

(iv) $4a^2 - a$ by $-a$ (v) $8m - 16$ by -8 (vi) $-50 + 40p$ by $10p$

(vii) $4x^3 - 2x^2$ by $-x$ (viii) $10a^3 - 15a^2b$ by $-5a^2$

(ix) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$ (x) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

3. Divide :

(i) $n^2 - 2n + 1$ by $n - 1$ (ii) $m^2 - 2mn + n^2$ by $m - n$

(iii) $4a^2 + 4a + 1$ by $2a + 1$ (iv) $p^2 + 4p + 4$ by $p + 2$

(v) $x^2 + 4xy + 4y^2$ by $x + 2y$ (vi) $2a^2 - 11a + 12$ by $a - 4$

(vii) $6x^2 + 5x - 6$ by $2x + 3$ (viii) $8a^2 + 4a - 60$ by $2a - 5$

(ix) $9x^2 - 24xy + 16y^2$ by $3x - 4y$ (x) $15x^2 + 31xy + 14y^2$ by $5x + 7y$

(xi) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$ (xii) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

4. The area of a rectangle is $6x^2 - 4xy - 10y^2$ square unit and its length is $2x + 2y$ unit. Find its breadth.

5. The area of a rectangular field is $25x^2 + 20xy + 3y^2$ square unit. If its length is $5x + 3y$ unit, find its breadth. Hence, find its perimeter.