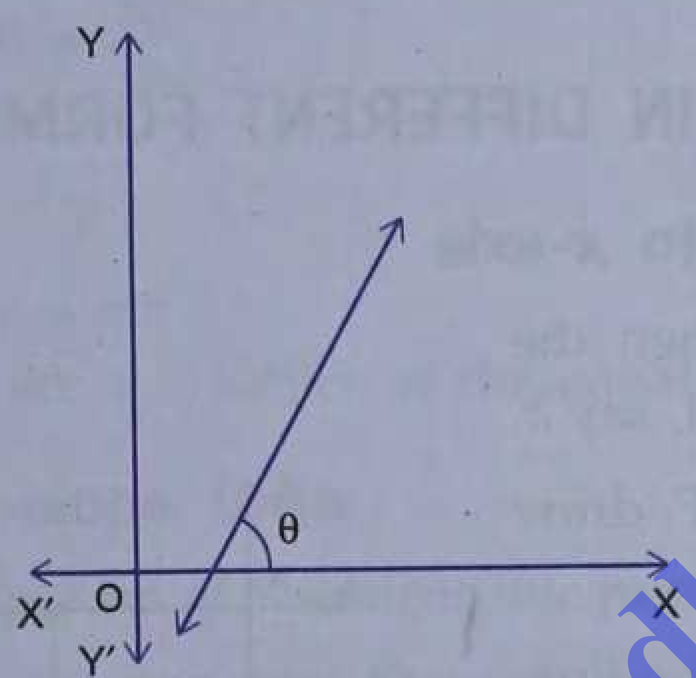
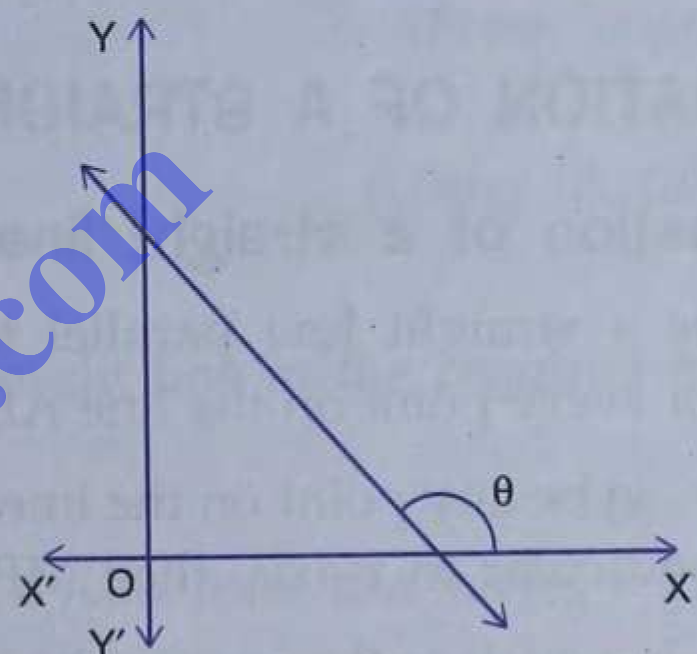
12 Equation of a Straight Line

12.1 DEFINITIONS

1. Inclination of a straight line.





The angle which a straight line makes with the positive direction of x-axis measured in the anticlockwise direction is called the ***inclination*** *(or* ***angle of inclination****) of the line. The inclination is usually denoted by θ.*

In particular :

(*i*) Inclination of a line parallel to *y*-axis or the *y*-axis itself is 90°.

(*ii*) Inclination of a line parallel to *x*-axis or the *x*-axis itself is 0°.

2. Horizontal, Vertical and Oblique lines.

(*i*) Any line parallel to *x*-axis is called a *horizontal line.*

(*ii*) Any line parallel to *y*-axis is called a *vertical line.*

(*iii*) A line which is neither parallel to *x*-axis nor parallel to *y*-axis is called an *oblique line.*

3. Slope (or gradient) of a straight line.

If $\theta(\neq 90°)$ *is the inclination of a line, then tan θ is called its* ***slope*** *(or* ***gradient****).*

The slope of a line is usually denoted by m.

Thus, if θ (≠ 90°) is the inclination of a line then $m = \tan\theta$.

Remark

Since tan θ is not defined when θ = 90°, slope of a vertical line is not defined.

4. Intercepts made by a line on the axes.

If a straight line meets x-axis in A and y-axis in B (shown in the figure given below), then

(*i*) OA is called *x-intercept* or the *intercept* made by the line on x-axis.

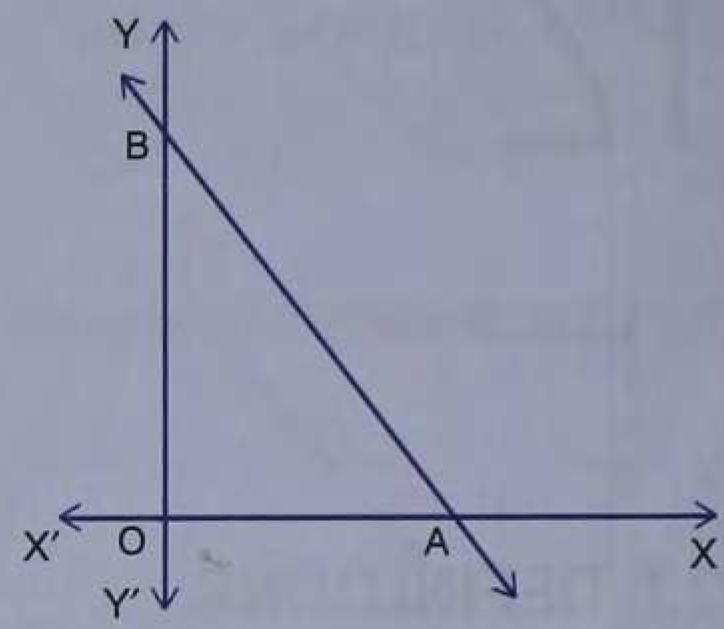
(*ii*) OB is called *y-intercept* or the *intercept* made by the line on y-axis.
The y-intercept is usually denoted by c.

(*iii*) OA and OB taken together in this very order are called the *intercepts* made by the line on axes.

Convention for the signs of intercepts.

(*i*) x-intercept is considered positive if it is measured to the right of origin and negative if it is measured to the left of origin.

(*ii*) y-intercept is considered positive if it is measured above the origin and negative if it is measured below the origin.



Remark

A horizontal line has no x-intercept and a vertical line has no y-intercept.

12.2 EQUATION OF A STRAIGHT LINE IN DIFFERENT FORMS

12.2.1 Equation of a straight line parallel to x-axis

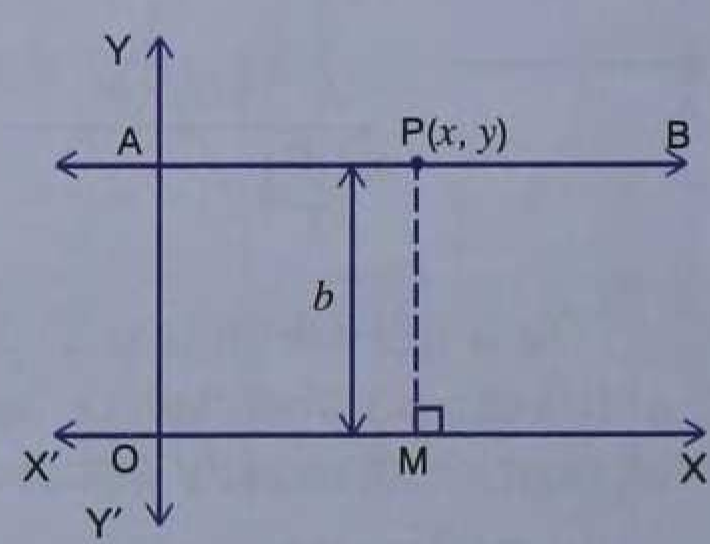
Let AB be a straight line parallel to x-axis, then the ordinate of every point on the line AB is constant, say b.

Let P(x , y) be any point on the line AB. From P, draw PM perpendicular to x-axis, then MP = y.

$\therefore$ $y = b$, which is the required equation of the line.

Corollary. *The equation of x-axis is $y = 0$.*

(For, if $b = 0$ then the line AB coincides with x-axis.)



Remark

The line $y = b$ lies above or below the x-axis according as b is positive or negative.

12.2.2 Equation of a straight line parallel to y-axis

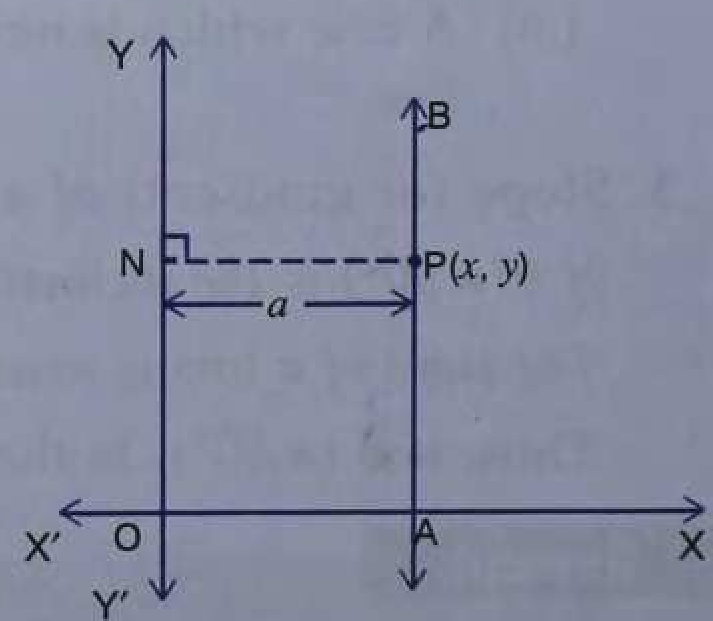
Let AB be a straight line parallel to y-axis, then the abscissa of every point on the line AB is constant, say a.

Let P(x , y) be any point on the line AB. From P, draw PN perpendicular to y-axis, then NP = x.

$\therefore$ $x = a$, which is the required equation of the line.

Corollary. *The equation of y-axis is $x = 0$.*

(For, if $a = 0$ then the line AB coincides with y-axis.)



Remark

The line $x = a$ lies to the right or left of y-axis according as a is positive or negative.

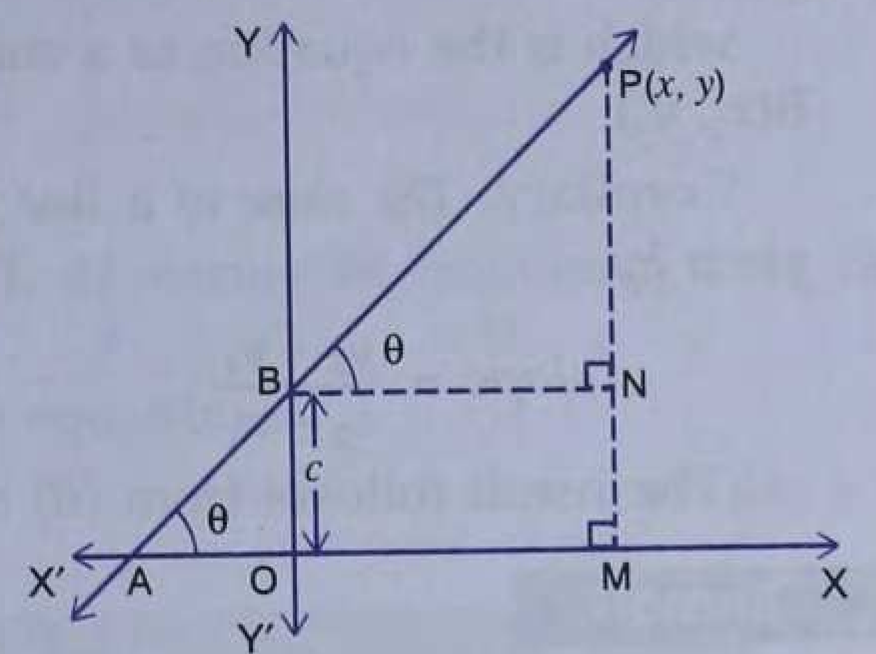
12.2.3 Slope-intercept form

To find the equation of a straight line in the form $y = mx + c$.

Let a straight line, say AB, make an intercept c on y-axis then OB = c.

Let m be the slope of the line and θ be its inclination, then $m = \tan\theta$ ...(*i*)

Let P(x, y) be any point on the line AB. From P, draw PM perpendicular to x-axis, and from B, draw BN perpendicular on MP.



From the figure,

$$BN = OM = x \quad ...(ii)$$

and $NP = MP - MN = MP - OB = y - c$ ...(*iii*)

Also $\angle PBN = \angle BAO = \theta$ (corresp. $\angle$s)

From right-angled Δ BNP,

$$\tan\theta = \frac{NP}{BN} \quad \text{(From Trigonometry)}$$

$$\Rightarrow \quad m = \frac{y-c}{x} \quad \text{(Using (i), (ii) and (iii))}$$

$$\Rightarrow \quad y - c = mx$$

$\Rightarrow$ $y = mx + c$, which is the equation of a straight line in the required form.

12.2.4 Point-slope form

To find the equation of a straight line passing through a fixed point and having a given slope.

Let a straight line pass through the fixed point $A(x_1, y_1)$ and have slope m.

We know that the equation of a straight line having slope m is

$$y = mx + c \quad ...(i) \quad \text{(Art. 12.2.3)}$$

where c is unknown constant.

Since the line (*i*) passes through the point $A(x_1, y_1)$, we get

$$y_1 = mx_1 + c \quad ...(ii)$$

To eliminate c, subtracting (*ii*) from (*i*), we get

$$y - y_1 = m(x - x_1)$$

which is the required equation of a straight line passing through the fixed point $A(x_1, y_1)$ and having slope m.

This is also known as **one-point form.**

12.2.5 Two-point form

To find the equation of a straight line passing through two fixed points.

Let a straight line pass through two fixed points $A(x_1, y_1)$ and $B(x_2, y_2)$.

We know that the equation of a straight line passing through the fixed point $A(x_1, y_1)$ is

$$y - y_1 = m(x - x_1) \quad ...(i) \quad \text{(Art. 12.2.4)}$$

where m is unknown constant.

Since the line (i) passes through the point $B(x_2, y_2)$, we get

$$y_2 - y_1 = m\,(x_2 - x_1)$$

$$\Rightarrow \quad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \ldots(ii)$$

To eliminate m, substituting the value of m from (ii) in (i), we get

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$$

which is the equation of a straight line passing through two fixed points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Corollary. *The slope of a line passing through two fixed points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by*

$$\textit{slope} = \frac{y_2 - y_1}{x_2 - x_1}.$$

(The result follows from (ii) of the above article.)

Remark

We observe that the equation of a straight line in any of the above forms is a linear equation in x and y. Thus, the equation of a straight line can be written in the form $ax + by + c = 0$. In fact, the converse is also true *i.e.* every linear equation in x and y represents a straight line.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the slope of a line whose inclination is 60°.*

Solution. Let m be the slope of the line, then

$m = \tan 60° = \sqrt{3}$. (From Trigonometry)

Example 2. *Find the equation of a straight line parallel to x-axis and passing through the point (3, –5).*

Solution. We know that the equation of a st. line parallel to x-axis is

$y = b$ ...(i)

Since (i) passes through the point (3, –5), we get

$-5 = b$ *i.e.* $b = -5$.

Substituting this value of b in (i), we get

$y = -5$ *i.e.* $y + 5 = 0$, which is the required equation.

Example 3. *Find the equation of a straight line whose inclination is 45° and whose y-intercept is –3.*

Solution. Let m be the slope of the line, then

$m = \tan 45° = 1$.

Also y-intercept is – 3 *i.e.* $c = -3$.

∴ The equation of the line is $y = 1 \cdot x + (-3)$ | $y = mx + c$

i.e. $x - y - 3 = 0$.

Example 4. *The equation of a straight line is 3x – 3y – 7 = 0. Find :*

(*i*) *the gradient of the line.*

(*ii*) *the inclination of the line.*

(*iii*) *the y-intercept of the line.*

Solution. The equation of the line is $3x - 3y - 7 = 0$.

It can be written as $-3y = -3x + 7$

or $y = 1 . x - \frac{7}{3}$.

Comparing it with $y = mx + c$, we get $m = 1$ and $c = -\frac{7}{3}$.

(*i*) The gradient of the line $= m = 1$.

(*ii*) Let θ be the inclination of the line, then

$\tan\theta = m = 1$ (Using (*i*))

$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$.

(*iii*) y-intercept $= c = -\frac{7}{3}$.

Example 5. *Find the equation of the line through (1, 3) making an intercept of 5 on the y-axis.*

Solution. Since the y-intercept of the line is 5, its equation is

$y = mx + 5$ ...(*i*) $|\ y = mx + c$

where m is unknown constant.

As the line (*i*) passes through the point (1, 3), we get

$3 = m . 1 + 5 \Rightarrow m = -2$.

Substituting this value of m in (*i*), we get

$y = -2x + 5$, which is the required equation.

Example 6. *Find the equation of a straight with slope −2 and which intersects x-axis at a distance of 3 units to the left of origin.*

Solution. Here slope of the line $= m = -2$.

Since the line intersects x-axis at a distance of 3 units to the left of origin, it passes through the point (−3, 0).

The equation of the line passing through the point (−3, 0) and with slope −2 is

$y - 0 = (-2)(x - (-3))$ $|\ y - y_1 = m(x - x_1)$

$\Rightarrow y = -2(x + 3)$

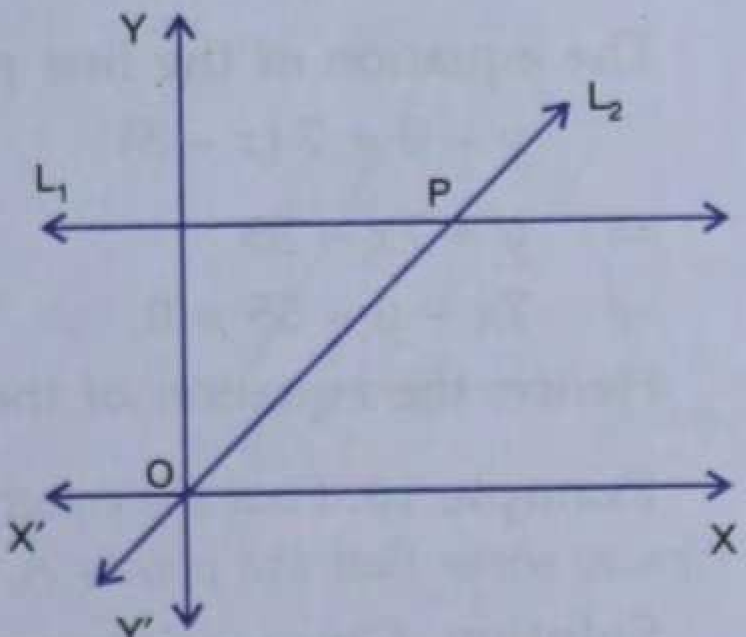
$\Rightarrow 2x + y + 6 = 0$.

Example 7. *Given equation of line L_1 is $y = 4$.*

(*i*) *Write the slope of line L_2 if L_2 is the bisector of angle O.*

(*ii*) *Write the co-ordinates of the point P.*

(*iii*) *Find the equation of L_2.* (2011)



Solution. From P, draw MP ⊥ OX. As the equation of line L_1 is $y = 4$, MP = 4.

(*i*) Since line L_2 is the bisector of ∠O, therefore,

$\angle MOP = \frac{1}{2}\angle O = \frac{1}{2} \times 90° = 45°$

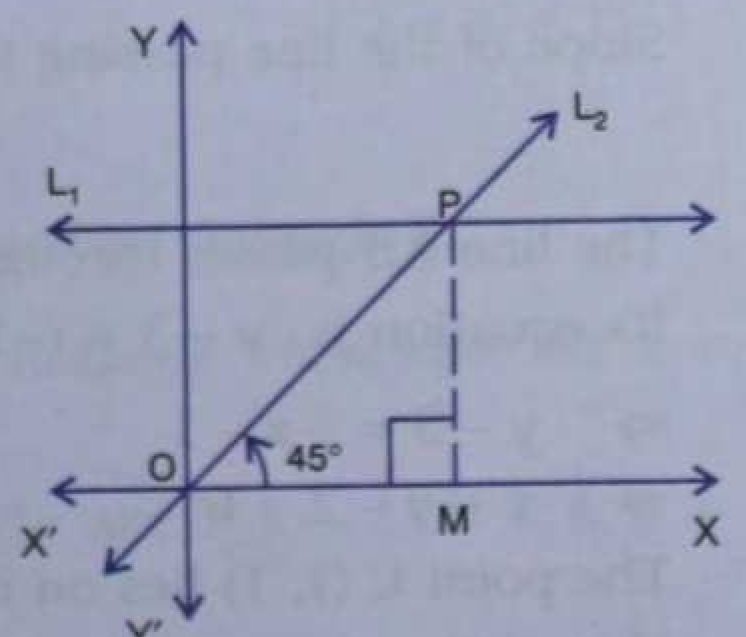
$\Rightarrow$ the inclination of line L_2 is 45°.

$\therefore$ Slope of line $L_2 = \tan 45° = 1$.

(*ii*) Note that ΔOMP is an isosceles right-angled triangle.

So, OM = MP = 4.

$\therefore$ The co-ordinates of point P are (4, 4).



(*iii*) The line L_2 has slope 1 and passes through origin (0, 0), therefore, the equation of line L_2 is

$y - 0 = 1\,(x - 0)$ | using $y - y_1 = m\,(x - x_1)$

i.e. $x - y = 0$.

Example 8. *A straight line passes through the points P(2, – 5) and Q(4, 3). Find :*

(i) the slope of the line PQ.

(ii) the equation of the line PQ.

(iii) the value of p if PQ passes through the point (p – 1, p + 4).

Solution. Given points are P(2, –5) and Q(4, 3).

(*i*) The slope of the line PQ $= \frac{3-(-5)}{4-2}$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8}{2} = 4.$

(*ii*) The line PQ passes through the point P(2, –5) and has slope 4, therefore, its equation is

$y - (-5) = 4\,(x - 2)$ | $y - y_1 = m\,(x - x_1)$

$\Rightarrow \quad y + 5 = 4x - 8$

$\Rightarrow \quad 4x - y - 13 = 0.$

(*iii*) As the line PQ passes through the point $(p - 1, p + 4)$, we get

$4(p - 1) - (p + 4) - 13 = 0$

$\Rightarrow \quad 4p - 4 - p - 4 - 13 = 0$

$\Rightarrow \quad 3p - 21 = 0 \quad \Rightarrow \quad p = 7.$

Example 9. *Find the equation of a line with x-intercept = 5 and passing through the point (4, –7).* (2009)

Solution. Since x-intercept is 5, the line passes through the point (5, 0).

The slope of the line passing through the points (4, – 7) and (5, 0)

$= \frac{0-(-7)}{5-4}$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= 7.$

The equation of the line passing through (5, 0) and with slope 7 is

$y - 0 = 7\,(x - 5)$ | $y - y_1 = m\,(x - x_1)$

$\Rightarrow \quad y = 7x - 35$

$\Rightarrow \quad 7x - y - 35 = 0.$

Hence, the equation of the required line is $7x - y - 35 = 0$.

Example 10. *Find the equation of the line passing through the points A (–1, 3) and B (0, 2). Hence, show that the points A, B and C (1, 1) are collinear.*

Solution. Given points are A (–1, 3), B (0, 2) and C (1, 1).

Slope of the line passing through A and B $= \frac{2-3}{0-(-1)}$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= -1.$

The line AB passes through the point A (–1, 3) and has slope = –1.

Its equation is $y - 3 = (-1)\,(x - (-1))$ | $y - y_1 = m\,(x - x_1)$

$\Rightarrow \quad y - 3 = -x - 1$

$\Rightarrow \quad x + y - 2 = 0$

The point C (1, 1) lies on it if $1 + 1 - 2 = 0$ *i.e.* if $0 = 0$, which is true.

Hence, the given points A, B and C (1, 1) are collinear.

Example 11. *A straight line passes through the points P(– 1, 4) and Q(5, – 2). It intersects the co-ordinate axes at points A and B. M is mid-point of the segment AB. Find :*

(i) the equation of the line.

(ii) the co-ordinates of A and B.

(iii) the co-ordinates of M. (2003)

Solution. Given points are P(–1, 4) and Q(5, –2).

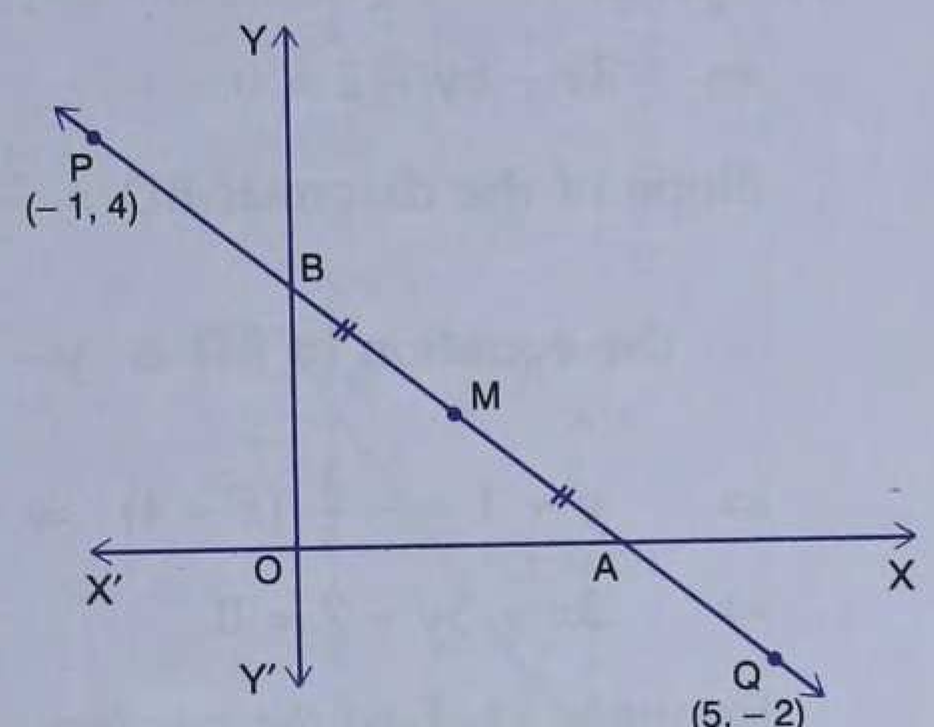
(*i*) The slope of the line PQ $= \frac{-2-4}{5-(-1)} = -1.$

The line passes through P(–1, 4) and has slope –1.

Its equation is $y - 4 = -1(x - (-1))$

$\quad | \; y - y_1 = m(x - x_1)$

$\Rightarrow \quad x + y - 3 = 0.$



(*ii*) The line PQ meets *x*-axis *i.e.* $y = 0$ where $x + 0 - 3 = 0 \;\Rightarrow x = 3.$

∴ The co-ordinates of A are (3, 0).

The line PQ meets *y*-axis *i.e.* $x = 0$ where $0 + y - 3 = 0 \;\Rightarrow y = 3.$

∴ The co-ordinates of B are (0, 3).

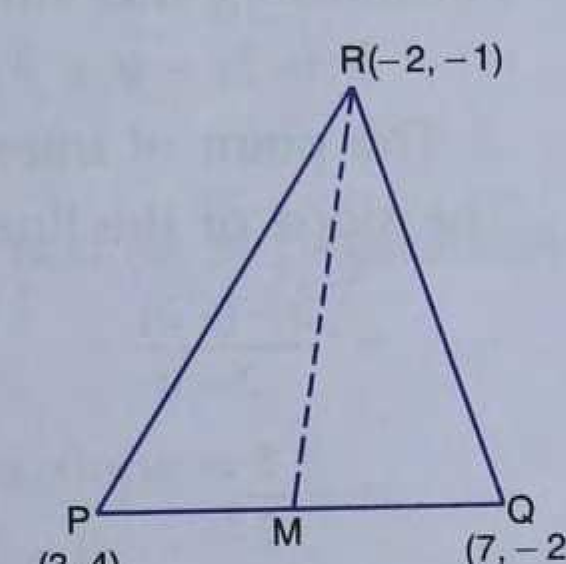
(*iii*) Since M is mid-point of the segment AB, its co-ordinates are

$$\left(\frac{3+0}{2}, \frac{0+3}{2}\right) \; i.e. \left(\frac{3}{2}, \frac{3}{2}\right).$$

Example 12. *If P(3, 4), Q(7, – 2) and R(– 2, – 1) are the vertices of a triangle PQR. Write down the equation of the median of the triangle through R.* (2004)

Solution. The vertices of ΔPQR are P(3, 4), Q(7, – 2) and R(–2, – 1).

Let M be the mid-point of PQ, then RM is the median through R.



Co-ordinates of M are $\left(\frac{3+7}{2}, \frac{4+(-2)}{2}\right)$ *i.e.* (5, 1).

∴ Slope of RM $= \frac{1-(-1)}{5-(-2)} \qquad \left| \; m = \frac{y_2 - y_1}{x_2 - x_1}\right.$

$= \frac{2}{7}.$

The equation of the line RM is

$y - (-1) = \frac{2}{7}(x - (-2)) \qquad | \; y - y_1 = m(x - x_1)$

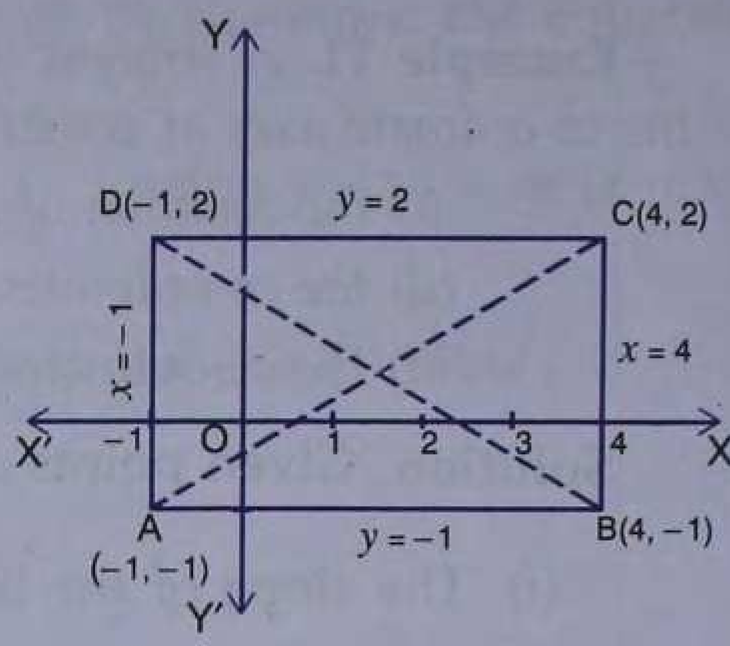
$\Rightarrow \quad 7y + 7 = 2x + 4$

$\Rightarrow \quad 2x - 7y - 3 = 0$, which is the equation of the median through R.

Example 13. *Find the equations of the diagonals of a rectangle whose sides are $x = -1$, $x = 4$, $y = -1$ and $y = 2$.*

Solution. The equations of the sides of a rectangle are $x = -1$, $x = 4$, $y = -1$ and $y = 2$.

Let ABCD be the rectangle formed by these lines as shown in the adjoining figure. Clearly, its vertices are A(– 1, – 1), B(4, – 1), C(4, 2) and D(– 1, 2).



Slope of the diagonal AC $= \frac{2-(-1)}{4-(-1)} = \frac{3}{5}$,

$\therefore$ the equation of AC is $y - (-1) = \frac{3}{5}(x - (-1))$

$\Rightarrow \quad y + 1 = \frac{3}{5}(x + 1) \Rightarrow 5y + 5 = 3x + 3$

$\Rightarrow \quad 3x - 5y - 2 = 0.$

Slope of the diagonal BD $= \frac{2-(-1)}{-1-4} = -\frac{3}{5}$,

$\therefore$ the equation of BD is $y - (-1) = -\frac{3}{5}(x - 4)$

$\Rightarrow \quad y + 1 = -\frac{3}{5}(x - 4) \Rightarrow 5y + 5 = -3x + 12$

$\Rightarrow \quad 3x + 5y - 7 = 0.$

Example 14. *Find the equation of the line passing through the point (0, –2) and the point of intersection of the lines 4x + 3y = 1 and 3x – y + 9 = 0.*

Solution. The given lines are

$4x + 3y - 1 = 0$ ...(*i*)

and $3x - y + 9 = 0$ ...(*ii*)

To find the point of intersection of the lines (*i*) and (*ii*), we solve these equations simultaneously.

Multiplying (*ii*) by 3 and adding it to (*i*), we get

$13x + 26 = 0 \Rightarrow x = -2.$

Substituting this value of x in (*ii*), we get

$3.(-2) - y + 9 = 0 \Rightarrow -6 - y + 9 = 0 \Rightarrow y = 3.$

$\therefore$ The point of intersection of the given lines is (–2, 3).

The slope of the line passing the points (0, – 2) and (– 2, 3)

$= \frac{3-(-2)}{-2-0}$ $\quad \left| \; m = \frac{y_2 - y_1}{x_2 - x_1} \right.$

$= -\frac{5}{2}.$

The equation of the line passing through the point (0, –2) and with slope $-\frac{5}{2}$ is

$y - (-2) = -\frac{5}{2}(x - 0)$

$\Rightarrow \quad 2y + 4 = -5x \Rightarrow 5x + 2y + 4 = 0.$

Hence, the equation of the required line is $5x + 2y + 4 = 0$.

Example 15. *A line passes through the point P (3, 2) and cuts off positive intercepts, on the x-axis and the y-axis in the ratio 3 : 4. Find the equation of the line.*

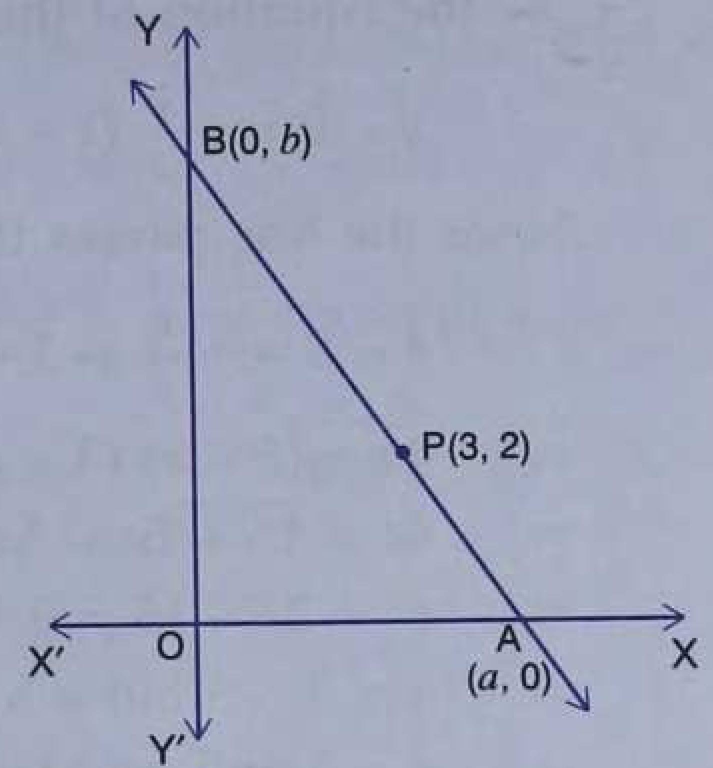
Solution. Let the line make positive intercepts a, b on the co-ordinates axes, then the line passes through the points A (a, 0) and B (0 , b), shown in the given diagram.

According to given $a : b = 3 : 4$

$\Rightarrow \quad \frac{a}{b} = \frac{3}{4}$ ...(*i*)

Slope of the line AB $= \frac{b-0}{0-a}$ $\quad \left| m = \frac{y_2 - y_1}{x_2 - x_1} \right.$

$= -\frac{b}{a} = -\frac{4}{3}$. (Using (*i*))



Thus, the line passes through P (3, 2) and has slope $-\frac{4}{3}$.

The equation of the line is

$$y - 2 = -\frac{4}{3}(x - 3)$$

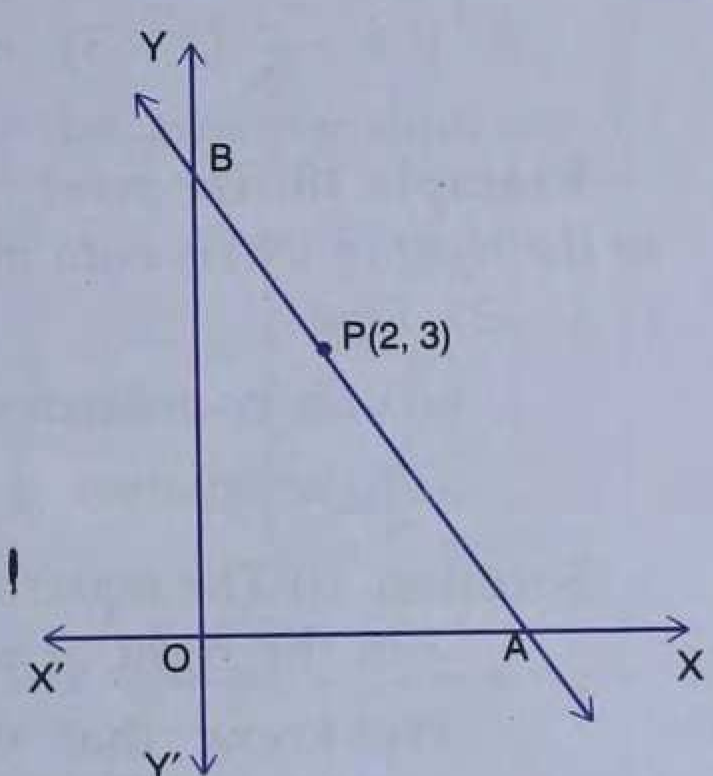
$\Rightarrow \quad 3y - 6 = -4x + 12$

$\Rightarrow \quad 4x + 3y - 18 = 0.$

Example 16. *A line passes through the point P(2, 3) and meets the coordinates axes at the points A and B (as shown in the adjoining figure). If 2PA = 3PB, find*

(i) the coordinates of A and B.

(ii) the equation of the line AB.

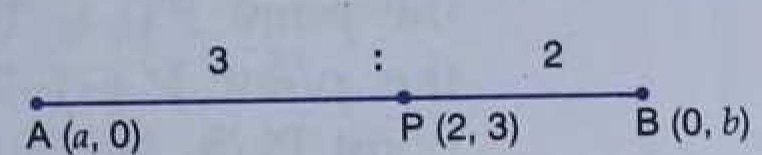


Solution. (*i*) Let OA = a and OB = b, then the coordinates of points A and B are $(a, 0)$, $(0, b)$ respectively.

Given 2PA = 3PB $\Rightarrow \frac{PA}{PB} = \frac{3}{2}$

$\Rightarrow$ PA : PB = 3 : 2

$\Rightarrow$ the point P divides the line segment AB in the ratio 3 : 2.



$\therefore$ The coordinates of the point P are $\left(\frac{2.a + 3.0}{2+3}, \frac{2.0 + 3.b}{2+3}\right)$ *i.e.* $\left(\frac{2a}{5}, \frac{3b}{5}\right)$.

But the coordinates of the point P are (2, 3)

$\Rightarrow \quad \frac{2a}{5} = 2$ and $\frac{3b}{5} = 3$

$\Rightarrow \quad a = 5$ and $b = 5$.

$\therefore$ The coordinates of the points A and B are (5, 0) and (0, 5) respectively.

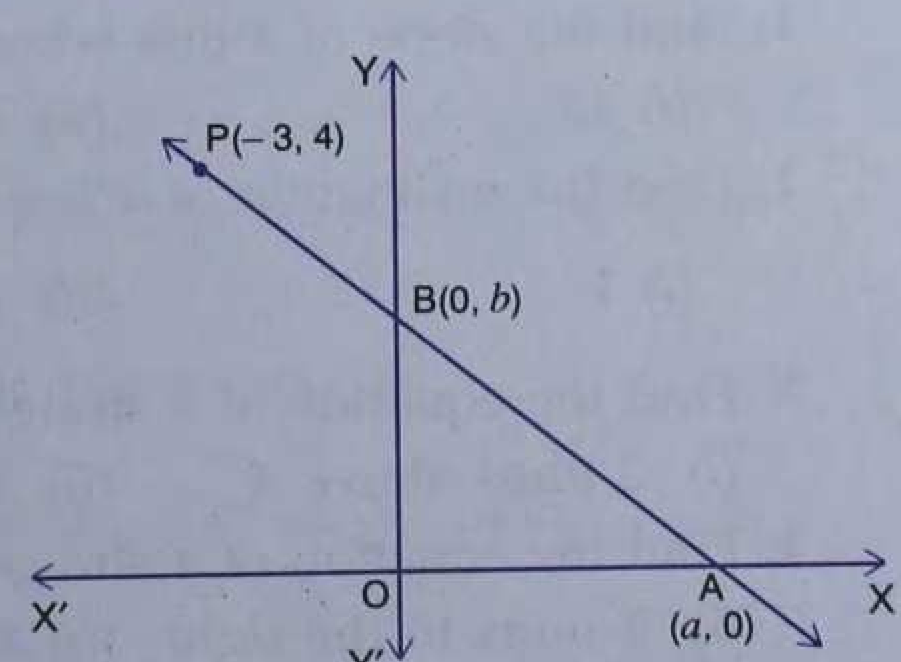
(*ii*) Slope of line AB $= \frac{5-0}{0-5} = -1$.

The line AB passes through the point A(5, 0) and has slope = −1.

$\therefore$ Its equation is $y - 0 = -1(x - 5) \quad \Rightarrow \quad x + y - 5 = 0$.

Example 17. *A straight line makes on the co-ordinate axes positive intercepts whose sum is 5. If the line passes through the point P(−3, 4), find its equation.*

Solution. Let the line make positive intercepts a, b on the co-ordinate axes, then the line passes through the points A$(a, 0)$, B$(0, b)$, shown in the adjoining diagram.



According to given $a + b = 5$

$\Rightarrow \quad b = 5 - a$ ...(*i*)

Slope of the line AB $= \frac{b-0}{0-a} = -\frac{b}{a}$,

∴ the equation of the line AB is

$$y - 0 = -\frac{b}{a}(x - a) \qquad \ldots(ii) \quad |y - y_1 = m(x - x_1)$$

Since the line passes through the point P(–3, 4), we get

$$4 - 0 = -\frac{b}{a}(-3 - a) \Rightarrow 4a = b(3 + a)$$

$\Rightarrow \quad 4a = (5 - a)(3 + a)$ (Using (*i*))

$\Rightarrow \quad 4a = 15 + 5a - 3a - a^2$

$\Rightarrow \quad a^2 + 2a - 15 = 0 \Rightarrow (a - 3)(a + 5) = 0$

$\Rightarrow \quad a = 3, -5$ but $a > 0$

$\therefore \quad a = 3$ and on using (*i*), $b = 5 - 3 = 2$.

Substituting these values of a and b in (*ii*), the equation of the required line is

$$y = -\frac{2}{3}(x - 3) \Rightarrow 2x + 3y - 6 = 0.$$

Example 18. *The point P(–1, 3) is reflected in the line parallel to y-axis at a distance 2 unit to the right of y-axis onto the point P′. The point Q is reflected in the origin onto the point Q′ (–3, –2). Find :*

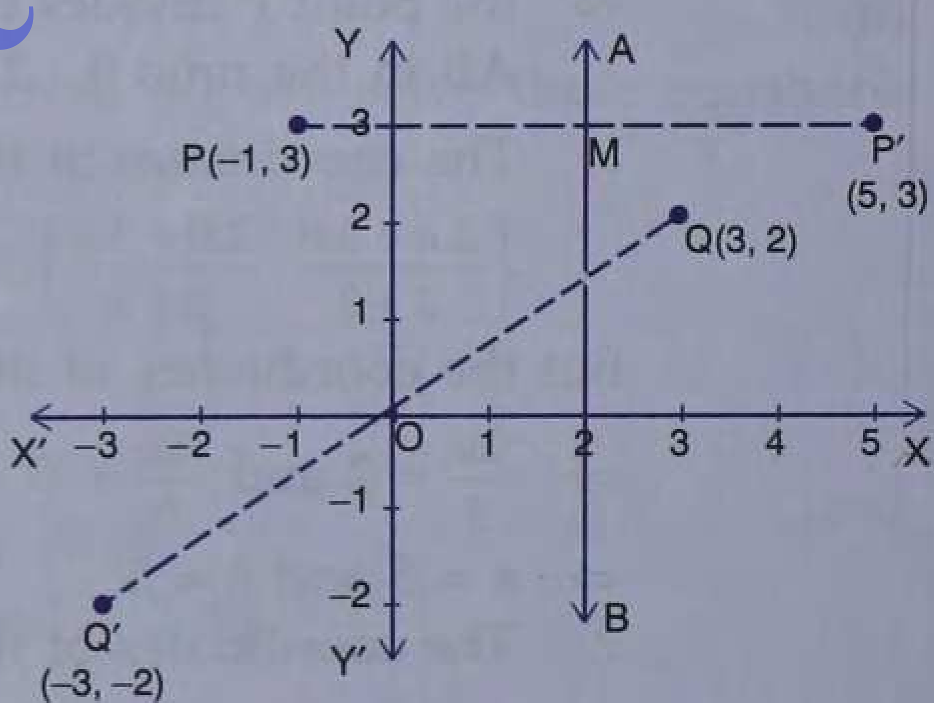
(i) the co-ordinates of P′ and Q.

(ii) the equation of the line P′Q.

Solution. (*i*) The equation of the line, say AB, parallel to y-axis at a distance of 2 units to the right of y-axis is $x = 2$.

We know that the reflection of the point (x, y) in the line $x = a$ is the point $(-x + 2a, y)$, therefore, the reflection of the point P (–1, 3) in the line $x = 2$ is the point P′ (–(–1) + 2.2, 3) *i.e.* the point P′ (5, 3).

Since Q′ (–3, –2) is the image of the point Q in the origin, the co-ordinates of the point Q are (3, 2).



(*ii*) Slope of the line P′Q $= \frac{2 - 3}{3 - 5} = \frac{1}{2}$.

The equation of the line P′Q is

$$y - 3 = \frac{1}{2}(x - 5)$$

$\Rightarrow \quad 2y - 6 = x - 5$

$\Rightarrow \quad x - 2y + 1 = 0.$

Exercise 12.1

1. Find the slope of a line whose inclination is

   (*i*) 45° (*ii*) 30°.

2. Find the inclination of a line whose gradient is

   (*i*) 1 (*ii*) $\sqrt{3}$ (*iii*) $\frac{1}{\sqrt{3}}$.

3. Find the equation of a straight line parallel to x-axis which is at a distance

   (*i*) 2 units above it (*ii*) 3 units below it.

4. Find the equation of a straight line parallel to y-axis which is at a distance

   (*i*) 3 units to the right (*ii*) 2 units to the left.

5. Find the equation of a straight line parallel to y-axis and passing through the point $(-3, 5)$.

6. Find the equation of a line whose

   (*i*) slope = 3, y-intercept = -5. (*ii*) slope = $-\frac{2}{7}$, y-intercept = 3.

   (*iii*) gradient = $\sqrt{3}$, y-intercept = $-\frac{4}{3}$. (*iv*) inclination = $30°$, y-intercept = 2.

7. Find the slope and y-intercept of the following lines :

   (*i*) $x - 2y - 1 = 0$ (*ii*) $4x - 5y - 9 = 0$ (*iii*) $3x + 5y + 7 = 0$

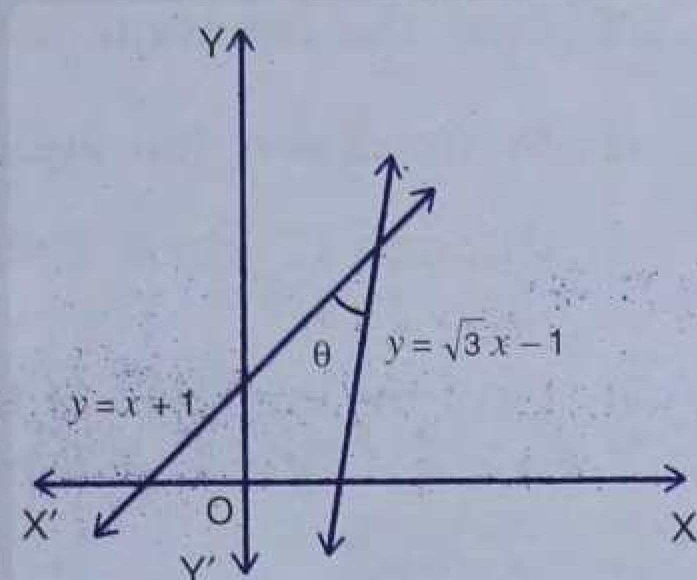
   (*iv*) $\frac{x}{3} + \frac{y}{4} = 1$ (*v*) $y - 3 = 0$ (*vi*) $x - 3 = 0$.

8. The equation of the line PQ is $3y - 3x + 7 = 0$.

   (*i*) Write down the slope of the line PQ.

   (*ii*) Calculate the angle that the line PQ makes with the positive direction of x-axis.

9. The given figure represents the lines $y = x + 1$ and $y = \sqrt{3}x - 1$. Write down the angles which the lines make with the positive direction of the x-axis. Hence, determine θ.



Hint

Ext. ∠ = sum of two opp. int. ∠s ; $60° = \theta + 45°$.

10. Find the value of p, given that the line $\frac{y}{2} = x - p$ passes through the point $(-4, 4)$.

11. Given that $(a, 2a)$ lies on the line $\frac{y}{2} = 3x - 6$, find the value of a.

12. The graph of the equation $y = mx + c$ passes through the points $(1, 4)$ and $(-2, -5)$. Determine the values of m and c.

13. Find the equation of the line passing through the point $(2, -5)$ and making an intercept of -3 on the y-axis.

14. Find the equation of a straight line passing through $(-1, 2)$ and whose slope is $\frac{2}{5}$.

15. Find the equation of a straight line whose inclination is $60°$ and which passes through the point $(0, -3)$.

16. Find the gradient of a line passing through the following pairs of points :

    (*i*) $(0, -2)$, $(3, 4)$ (*ii*) $(3, -7)$, $(-1, 8)$.

17. The co-ordinates of two points E and F are $(0, 4)$ and $(3, 7)$ respectively. Find :

    (*i*) the gradient of EF.

    (*ii*) the equation of EF.

    (*iii*) the co-ordinates of the point where the line EF intersects the x-axis.

18. Find the intercepts made by the line $2x - 3y + 12 = 0$ on the co-ordinate axes.

Hint

To find x-intercept, put $y = 0$; to find y-intercept, put $x = 0$.

19. Find the equation of the line passing through the points P(5, 1) and Q(1, −1). Hence, show that the points P, Q and R (11, 4) are collinear.

20. The graph of a linear equation in x and y passes through (4, 0) and (0, 3). Find the value of k, if the graph passes through (k, 1·5).

21. Use a graph paper for this question.
The graph of a linear equation in x and y, passes through A(–1, –1) and B(2, 5). From your graph, find the values of h and k, if the line passes through (h, 4) and $\left(\frac{1}{2}, k\right)$. (2005)

22. ABCD is a parallelogram where A (x, y), B (5, 8), C (4, 7) and D (2, –4). Find
(*i*) the co-ordinates of A.
(*ii*) the equation of the diagonal BD. (2011)

23. In ΔABC, A(3, 5), B(7, 8) and C(1, –10). Find the equation of the median through A. (2013)

24. Find the equation of a line passing through the point (–2, 3) and having x-intercept 4 units. (2002)

25. Find the equation of the line whose x-intercept is 6 and y-intercept is –4.

26. Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A(–2, 6) and B(3, –4) in the ratio 2 : 3. (2001)

27. Find the equation of the line passing through the point (1, 4) and intersecting the line $x - 2y - 11 = 0$ on the y-axis.

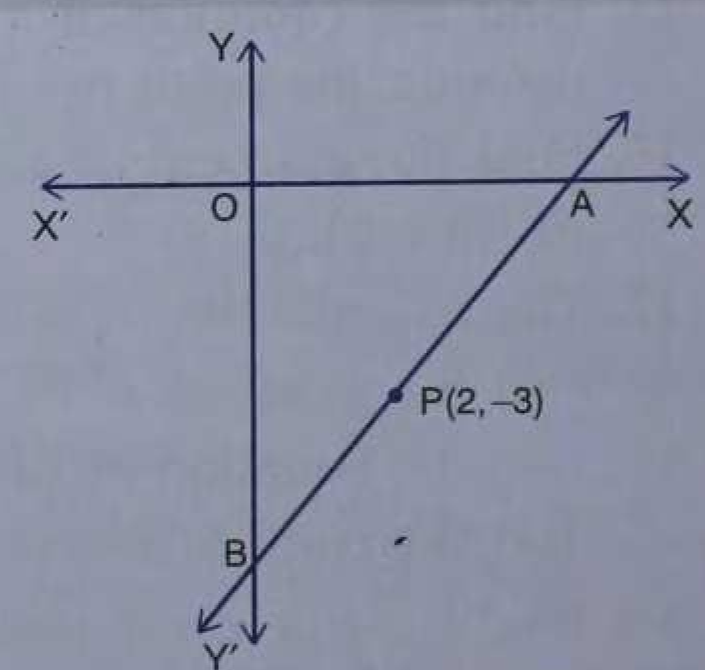
Hint

The line $x - 2y - 11 = 0$ intersects y-axis at the point $\left(0, -\frac{11}{2}\right)$.

28. Find the equation of the straight line containing the point (3, 2) and making positive equal intercepts on axes.

29. The intercepts made by a straight line on the axes are –3 and 2 units. Find :
(*i*) the gradient of the line.
(*ii*) the equation of the line.
(*iii*) the area of the triangle enclosed between the line and the co-ordinate axes.

30. A and B are two points on the x-axis and y-axis respectively. P(2, –3) is the mid-point of AB.
Find :
(*i*) the co-ordinates of A and B.
(*ii*) the slope of the line AB.
(*iii*) the equation of the line AB. (2010)



31. Find the equations of the diagonals of a rectangle whose sides are $x = -1$, $x = 2$, $y = -2$ and $y = 6$.

32. Find the equation of a straight line passing through the origin and through the point of intersection of the lines $5x + 7y = 3$ and $2x - 3y = 7$.

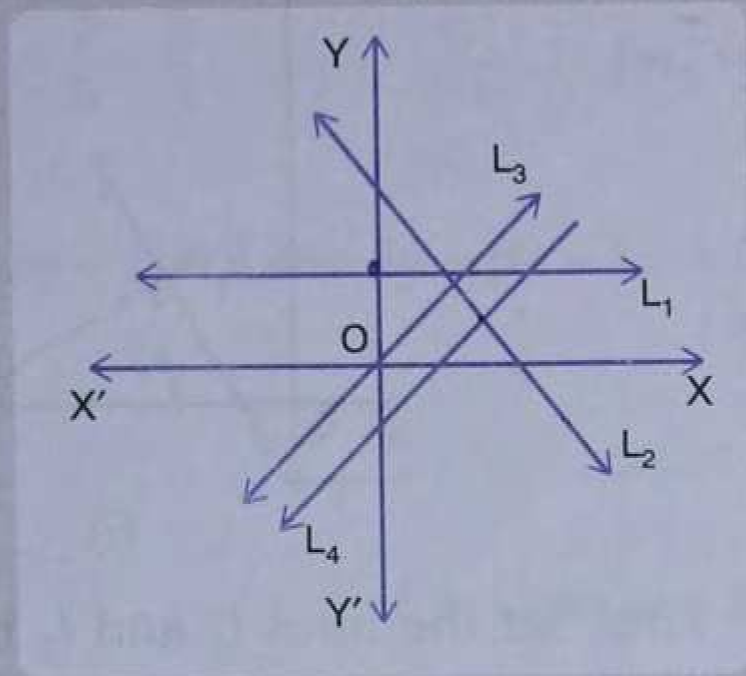
33. Match the equations A, B, C, D with the lines L_1, L_2, L_3, L_4 whose graphs are roughly drawn in the adjoining diagram.

$A \equiv y = 2x$

$B \equiv y - 2x + 2 = 0$

$C \equiv 3x + 2y = 6$

$D \equiv y = 2.$



34. Point A(3, −2) on reflection in the x-axis is mapped as A′ and point B on reflection in the y-axis is mapped onto B′(−4, 3).

(*i*) Write down the co-ordinates of A′ and B.

(*ii*) Find the slope of the line A′B, hence find its inclination.

12.3 PARALLELISM AND PERPENDICULARITY

12.3.1 Slopes of parallel lines

Two (non-vertical) lines are ***parallel*** *if and only if their slopes are equal.*

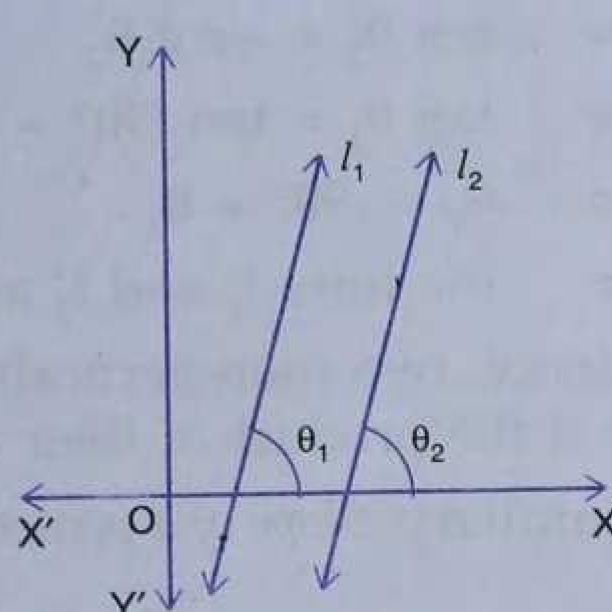
Proof. Let l_1, l_2 be two (non-vertical) lines and m_1, m_2 be their slopes. Let θ_1, θ_2 be the inclinations of these lines.

First, let the lines l_1 and l_2 be parallel

$\Rightarrow \quad \theta_1 = \theta_2$

$\Rightarrow \quad \tan\theta_1 = \tan\theta_2$

$\Rightarrow \quad m_1 = m_2.$



Thus, if two lines are parallel then their slopes are equal.

Conversely, let the lines l_1 and l_2 have equal slopes *i.e.*

$m_1 = m_2$

$\Rightarrow \quad \tan\theta_1 = \tan\theta_2$

$\Rightarrow \quad \theta_1 = \theta_2$

$\Rightarrow$ the lines l_1 and l_2 are parallel.

Hence, two (non-vertical) lines are parallel if and only if $m_1 = m_2$ *i.e.* if and only if their slopes are equal.

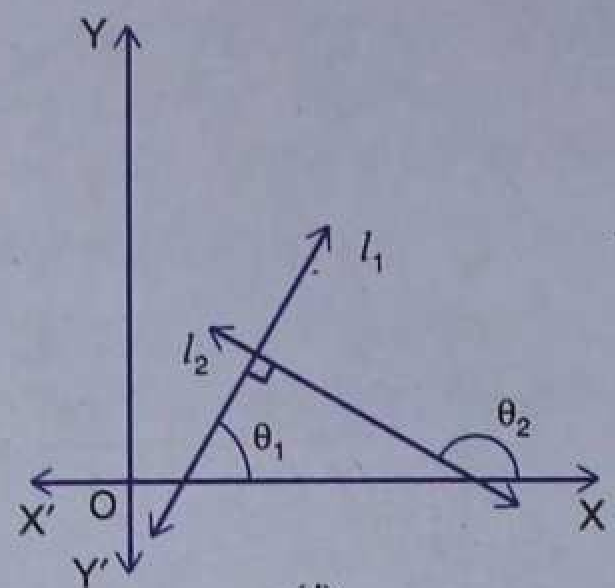
Remark

Since inclination of every line parallel to x-axis is 0°, its slope = tan 0° = 0. Therefore, *slope of every horizontal line is zero.*

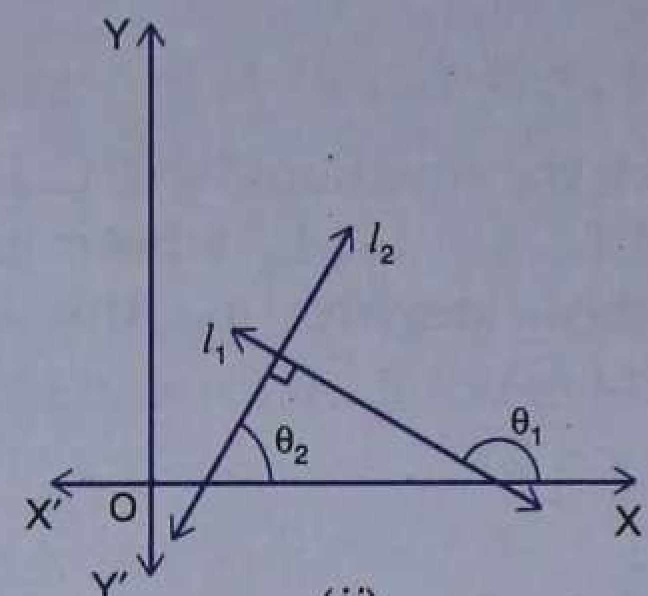
12.3.2 Slopes of perpendicular lines

Two (non-vertical) lines are ***perpendicular*** *if and only if the product of their slopes is −1.*

Proof. Let l_1, l_2 be two (non-vertical) lines and m_1, m_2 be their slopes. Let θ_1, θ_2 be the inclinations of these lines.



(*i*)



(*ii*)

First, let the lines l_1 and l_2 be perpendicular

$\Rightarrow \quad \theta_2 = 90° + \theta_1 \quad$ or $\quad \theta_1 = 90° + \theta_2$

$\Rightarrow \quad \tan\theta_2 = \tan(90° + \theta_1) \quad$ or $\quad \tan\theta_1 = \tan(90° + \theta_2)$

$\Rightarrow \quad \tan\theta_2 = -\cot\theta_1 \quad$ or $\quad \tan\theta_1 = -\cot\theta_2 \qquad$ (From Trigonometry)

$\Rightarrow \quad \tan\theta_2 = -\frac{1}{\tan\theta_1} \quad$ or $\quad \tan\theta_1 = -\frac{1}{\tan\theta_2}$

$\Rightarrow \quad m_2 = -\frac{1}{m_1} \quad$ or $\quad m_1 = -\frac{1}{m_2}$.

Therefore, in either case $m_1m_2 = -1$.

Thus, if two lines are perpendicular then the product of their slopes is -1.

Conversely, let the lines l_1 and l_2 be such that the product of their slopes is -1 *i.e.* $m_1m_2 = -1$

$\Rightarrow \quad \tan\theta_1 \tan\theta_2 = -1$

$\Rightarrow \quad \tan\theta_1 = -\frac{1}{\tan\theta_2}$

$\Rightarrow \quad \tan\theta_1 = -\cot\theta_2$

$\Rightarrow \quad \tan\theta_1 = \tan(90° + \theta_2)$

$\Rightarrow \quad \theta_1 = 90° + \theta_2$

$\Rightarrow$ the lines l_1 and l_2 are perpendicular.

Hence, two (non-vertical) lines are perpendicular if and only if $m_1m_2 = -1$ *i.e.* if and only if the product of their slopes is -1.

Corollary. *Slope of a perpendicular line is the negative reciprocal of the slope of the given line.*

$$\left(\because m_1 m_2 = -1 \Rightarrow m_2 = -\frac{1}{m_1}\right)$$

ILLUSTRATIVE EXAMPLES

Example 1. *If $2x - 3y + 5 = 0$ and $px + 6y + 7 = 0$ are parallel lines, find the value of p.*

Solution. Given $2x - 3y + 5 = 0$ $\quad \ldots(i)$

$\Rightarrow \quad -3y = -2x - 5$

$\Rightarrow \quad y = \frac{2}{3}x + \frac{5}{3}$. $\qquad$ (Converting into the form $y = mx + c$)

$\therefore \quad$ The slope of the line (*i*) $= \frac{2}{3}$.

Given $px + 6y + 7 = 0$ $\quad \ldots(ii)$

$\Rightarrow \quad 6y = -px - 7$

$\Rightarrow \quad y = -\frac{p}{6}x - \frac{7}{6}$ $\qquad$ (Converting into the form $y = mx + c$)

$\therefore \quad$ The slope of the line (*ii*) $= -\frac{p}{6}$.

Since the given lines (*i*) and (*ii*) are parallel, we get $-\frac{p}{6} = \frac{2}{3}$ | $m_1 = m_2$

$\Rightarrow \quad p = -4.$

Example 2. *Find the value of p for which the lines $2x + 3y - 7 = 0$ and $4y - px - 12 = 0$ are perpendicular to each other.* (2009)

Solution. Given $2x + 3y - 7 = 0$ ...(*i*)

$\Rightarrow \quad 3y = -2x + 7$

$\Rightarrow \quad y = -\frac{2}{3}x + \frac{7}{3}$ | $y = mx + c$ form

$\therefore$ The slope of the line (*i*) $= -\frac{2}{3}$.

Given $4y - px - 12 = 0$ ...(*ii*)

$\Rightarrow \quad 4y = px + 12$

$\Rightarrow \quad y = \frac{p}{4}x + 3$ | $y = mx + c$ form

$\therefore$ The slope of the line (*ii*) $= \frac{p}{4}$.

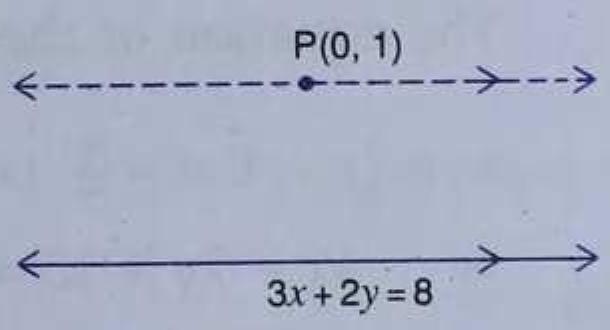
Since the given lines are perpendicular to each other,

$\left(-\frac{2}{3}\right)\cdot\left(\frac{p}{4}\right) = -1$ | $m_1 m_2 = -1$

$\Rightarrow \quad -\frac{p}{6} = -1 \quad \Rightarrow \quad p = 6.$

Example 3. *Find the equation of the line parallel to the line $3x + 2y = 8$ and passing through the point (0, 1).* (2007)

Solution. Given $3x + 2y = 8$ ...(*i*)



$\Rightarrow \quad 2y = -3x + 8$

$\Rightarrow \quad y = -\frac{3}{2}x + 4.$ | $y = mx + c$ form

$\therefore$ The slope of the line (*i*) $= -\frac{3}{2}$.

$\therefore$ The slope of a line parallel to (*i*) $= -\frac{3}{2}$. | $m_1 = m_2$

The equation of the line through (0, 1) and having slope $-\frac{3}{2}$ is

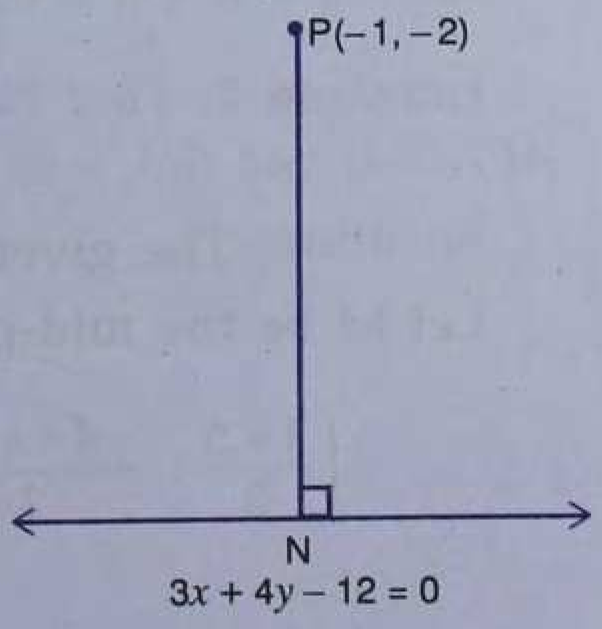
$y - 1 = -\frac{3}{2}(x - 0)$ | $y - y_1 = m(x - x_1)$

$\Rightarrow \quad 2y - 2 = -3x$

$\Rightarrow \quad 3x + 2y - 2 = 0$, which is the required equation.

Example 4. *Find the equation of the perpendicular from the point P(−1, −2) on the line $3x + 4y - 12 = 0$. Also find the co-ordinates of the foot of perpendicular.*

Solution. The given line is $3x + 4y - 12 = 0$ ...(*i*)



$\Rightarrow \quad 4y = -3x + 12$

$\Rightarrow \quad y = -\frac{3}{4}x + 3,$

$\therefore$ the slope of the line (*i*) $= -\frac{3}{4}$.

From P(−1, −2), draw PN perpendicular to the given line.

$\therefore$ The slope of the line PN $= \frac{4}{3}$ | $m_2 = -\frac{1}{m_1}$

The equation of the line through P(–1, – 2) and having slope $\frac{4}{3}$ is

$y - (-2) = \frac{4}{3}(x - (-1)) \Rightarrow 3y + 6 = 4x + 4$

$\Rightarrow \quad 4x - 3y - 2 = 0$ ...(*ii*)

which is the required equation of the perpendicular from P to the given line.

To find the co-ordinates of N (the foot of perpendicular), solve (*i*) and (*ii*) simultaneously.

Multiplying (*i*) by 3 and (*ii*) by 4, and on adding, we get

$$25x - 44 = 0 \Rightarrow x = \frac{44}{25}.$$

Multiplying (*i*) by 4 and (*ii*) by 3, and on subtracting, we get

$$25y - 42 = 0 \Rightarrow \frac{42}{25}.$$

Hence, the co-ordinates of the foot of perpendicular are $\left(\frac{44}{25}, \frac{42}{25}\right)$.

Example 5. *Find the equation of the line through the point P(– 5, 1) and parallel to the line joining the points A(7, – 1) and B(0, 3).*

Solution. Slope of the line joining the points A(7, – 1) and B(0, 3)

$= \frac{3-(-1)}{0-7}$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= -\frac{4}{7}$.

∴ The slope of a line parallel to line AB $= -\frac{4}{7}$. | $m_1 = m_2$

The equation of the line through P(– 5, 1) and having slope $-\frac{4}{7}$ is

$y - 1 = -\frac{4}{7}(x - (-5)) \Rightarrow 7y - 7 = -4x - 20$

$\Rightarrow \quad 4x + 7y + 13 = 0$, which is the required equation.

Example 6. *Find the equation of the perpendicular dropped from the point (– 1, 2) onto the line joining (1, 4) and (2, 3).*

Solution. Slope of the line joining the points A(1, 4) and B(2, 3)

$= \frac{3-4}{2-1}$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= -1.$

∴ The slope of a line perpendicular to line AB = 1. | $m_2 = -\frac{1}{m_1}$

The equation of the line through (–1, 2) and having slope 1 is

$y - 2 = 1(x - (-1)) \Rightarrow y - 2 = x + 1$

$\Rightarrow \quad x - y + 3 = 0$, which is the required equation.

Example 7. *Find the equation of the right bisector of the line segment joining the points A(3, – 4) and B(5, – 6).*

Solution. The given points are A(3, – 4) and B(5, – 6).

Let M be the mid-point of the segment AB, then the co-ordinates of M are

$\left(\frac{3+5}{2}, \frac{-4+(-6)}{2}\right)$ *i.e.* (4, –5).

Slope of line AB = $\frac{-6-(-4)}{5-3} = \frac{-2}{2} = -1$.

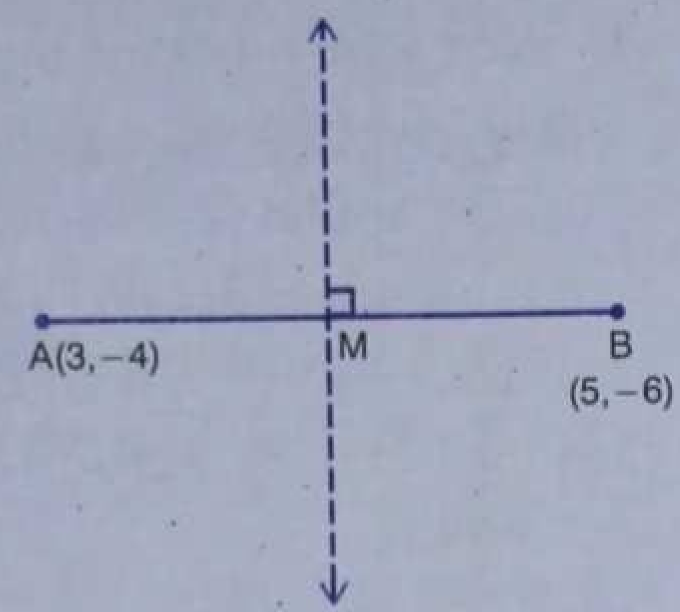
$\therefore$ The slope of a line perpendicular to line AB = 1.

The equation of the line through M(4, – 5) and having slope 1 is

$$y - (-5) = 1\,(x - 4) \Rightarrow y + 5 = x - 4$$

$$\Rightarrow \quad x - y - 9 = 0,$$

which is the required equation of the right bisector of the line segment joining the given points.



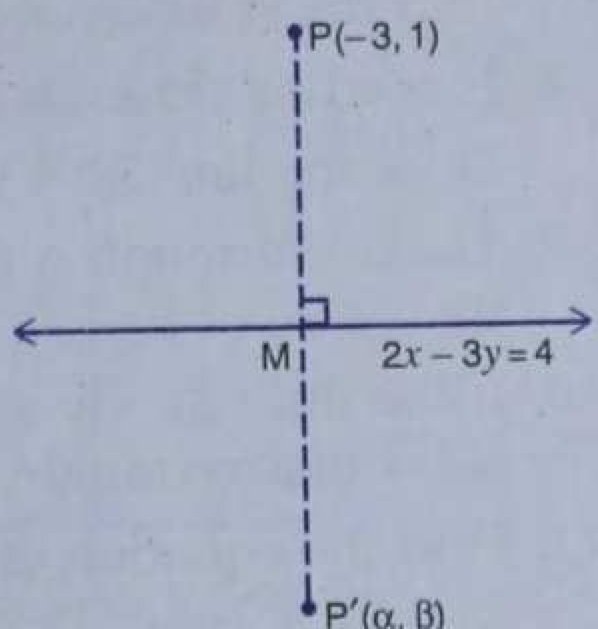
Example 8. *Find the image of the point P(–3, 1) in the line 2x – 3y = 4.*

Solution. The given line is $2x - 3y = 4$ …(*i*)

$$\Rightarrow \quad -3y = -2x + 4 \Rightarrow y = \frac{2}{3}x - \frac{4}{3},$$

$\therefore$ the slope of the line (*i*) = $\frac{2}{3}$.

From P, draw PM perpendicular to the line (*i*) and produce it to a point P′ such that P′M = MP, then P′ is the image of P in the line (*i*) and the line (*i*) is the right bisector of the segment PP′.



Let P′ be (α, β).

Then slope of PP′ = $\frac{\beta - 1}{\alpha + 3}$.

As (*i*) is perpendicular to PP′, we get

$$\frac{\beta - 1}{\alpha + 3} \cdot \frac{2}{3} = -1 \qquad | \; m_1 m_2 = -1$$

$$\Rightarrow \quad 2\beta - 2 = -3\alpha - 9 \Rightarrow 3\alpha + 2\beta + 7 = 0 \qquad \ldots(ii)$$

Also the mid-point of PP′ is M $\left(\frac{\alpha - 3}{2}, \frac{\beta + 1}{2}\right)$.

Since (*i*) is the right bisector of the segment PP′, M lies on (*i*)

$$\Rightarrow \quad 2 \cdot \frac{\alpha - 3}{2} - 3 \cdot \frac{\beta + 1}{2} = 4 \Rightarrow 2\alpha - 6 - 3\beta - 3 = 8$$

$$\Rightarrow \quad 2\alpha - 3\beta - 17 = 0 \qquad \ldots(iii)$$

To find the values of α and β, solve (*ii*) and (*iii*) simultaneously.

Multiplying (*ii*) by 3 and (*iii*) by 2, and on adding, we get

$$13\alpha - 13 = 0 \quad \Rightarrow \quad \alpha = 1.$$

Substituting this value of α in (*ii*), we get

$$3.1 + 2\beta + 7 = 0 \quad \Rightarrow \quad \beta = -5.$$

Hence, the image of P in the given line is P′(1, –5).

Exercise 12.2

1. State which one of the following is true :
   The straight lines $y = 3x - 5$ and $2y = 4x + 7$ are
   (*i*) parallel (*ii*) perpendicular
   (*iii*) neither parallel nor perpendicular.
2. If $6x + 5y - 7 = 0$ and $2px + 5y + 1 = 0$ are parallel lines, find the value of p.
3. Lines $2x - by + 5 = 0$ and $ax + 3y = 2$ are parallel. Find the relation connecting a and b.
4. Given that the line $\frac{y}{2} = x - p$ and the line $ax + 5 = 3y$ are parallel, find the value of a.
5. If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p. *(2006)*
6. Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are perpendicular to each other. *(2003)*
7. If the lines $3x + by + 5 = 0$ and $ax - 5y + 7 = 0$ are perpendicular to each other, find the relation connecting a and b.
8. Is the line through $(-2, 3)$ and $(4, 1)$ perpendicular to the line $3x = y + 1$?
   Does the line $3x = y + 1$ bisect the join of $(-2, 3)$ and $(4, 1)$?
9. The line through A $(-2, 3)$ and B $(4, b)$ is perpendicular to the line $2x - 4y = 5$. Find the value of b. *(2012)*
10. If the lines $3x + y = 4$, $x - ay + 7 = 0$ and $bx + 2y + 5 = 0$ form three consecutive sides of a rectangle, find the values of a and b.
11. Find the equation of a line, which has the y-intercept 4, and is parallel to the line $2x - 3y - 7 = 0$. Find the co-ordinates of the point where it cuts the x-axis.
12. Find the equation of a st. line perpendicular to the line $2x + 5y + 7 = 0$ and with y-intercept -3 units.
13. Find the equation of a st. line perpendicular to the line $3x - 4y + 12 = 0$ and having same y-intercept as $2x - y + 5 = 0$.
14. Find the equation of the line which is parallel to $3x - 2y = -4$ and passes through the point $(0, 3)$.
15. Find the equation of the line passing through $(0, 4)$ and parallel to the line $3x + 5y + 15 = 0$.
16. The equation of a line is $y = 3x - 5$. Write down the slope of this line and the intercept made by it on the y-axis. Hence, or otherwise, write down the equation of a line which is parallel to the line and which passes through the point $(0, 5)$.
17. Write down the equation of the line perpendicular to $3x + 8y = 12$ and passing through the point $(-1, -2)$.
18. (*i*) The line $4x - 3y + 12 = 0$ meets the x-axis at A. Write down the co-ordinates of A.
    (*ii*) Determine the equation of the line passing through A and perpendicular to $4x - 3y + 12 = 0$.
19. Find the equation of the line that is parallel to $2x + 5y - 7 = 0$ and passes through the mid-point of the line segment joining the points $(2, 7)$ and $(-4, 1)$.
20. Find the equation of the line that is perpendicular to $3x + 2y - 8 = 0$ and passes through the mid-point of the line segment joining the points $(5, -2)$ and $(2, 2)$.
21. Find the equation of a straight line passing through the intersection of $2x + 5y - 4 = 0$ with x-axis and parallel to the line $3x - 7y + 8 = 0$.

22. The equation of a line is $3x + 4y - 7 = 0$. Find

(*i*) the slope of the line.

(*ii*) the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$ (2010)

23. Find the equation of the perpendicular from the point $(1, -2)$ on the line $4x - 3y - 5 = 0$. Also find the co-ordinates of the foot of perpendicular.

24. Prove that the line through (0, 0) and (2, 3) is parallel to the line through (2, –2) and (6, 4).

25. Prove that the line through (–2, 6) and (4, 8) is perpendicular to the line through (8, 12) and (4, 24).

26. Show that the triangle formed by the points A(1, 3), B(3, –1) and C(–5, –5) is a right angled triangle (by using slopes).

27. Find the equation of the line through the point (–1, 3) and parallel to the line joining the points (0, –2) and (4, 5).

28. A(1, 4), B(3, 2) and C(7, 5) are the vertices of a Δ ABC. Find :

(*i*) the co-ordinates of the centroid G of ΔABC.

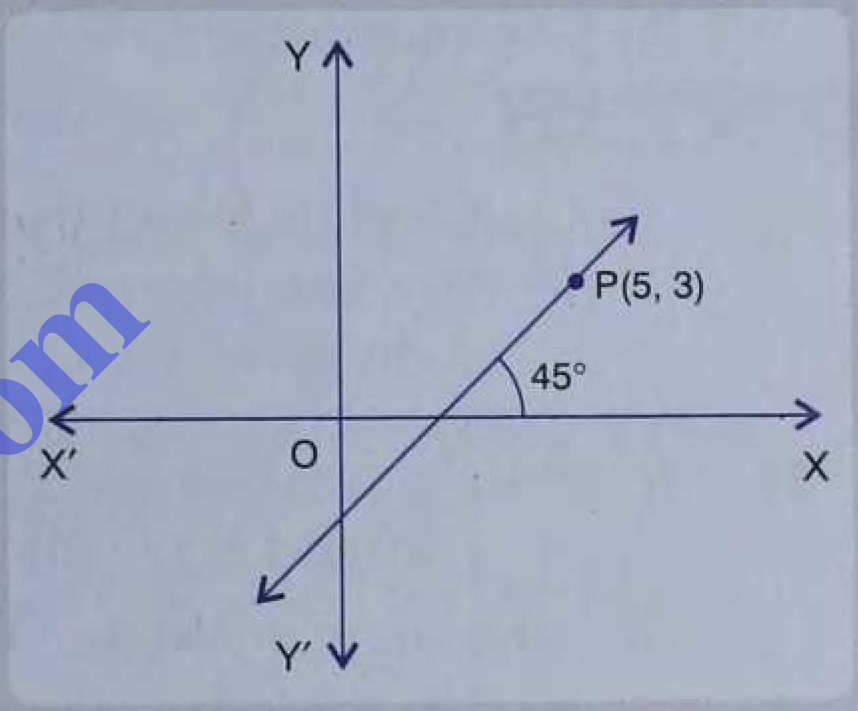
(*ii*) the equation of a line through G and parallel to AB. (2002)

29. The line through P (5, 3) intersects *y*-axis at Q.

(*i*) Write the slope of the line.

(*ii*) Write the equation of the line.

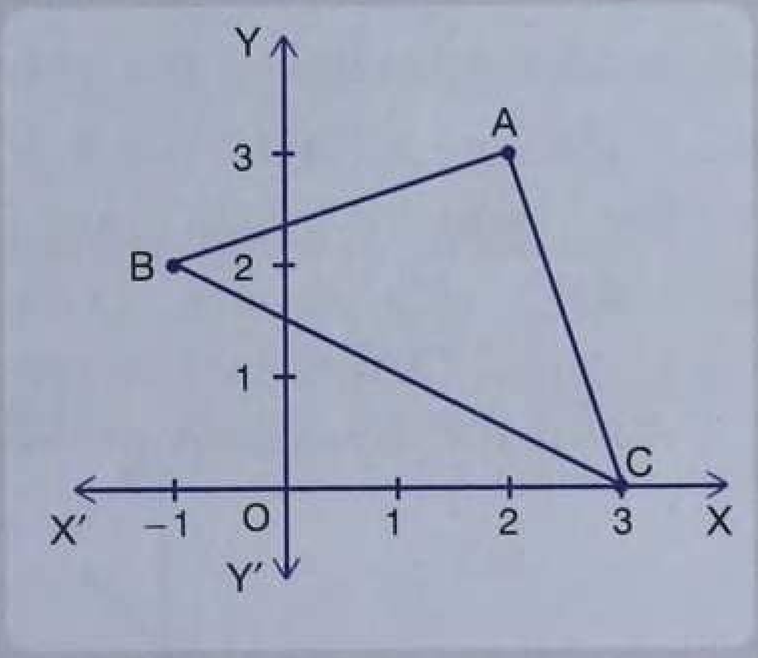
(*iii*) Find the coordinates of Q. (2012)



30. In the adjoining diagram, write down

(*i*) the co-ordinates of the points A, B and C.

(*ii*) the equation of the line through A, parallel to BC. (2005)



31. Find the equation of the line through (0, –3) and perpendicular to the line joining the points (–3, 2) and (9, 1).

32. The vertices of a triangle are A(10, 4), B(4, –9) and C(–2, –1). Find the equation of the altitude through A.

[*The perpendicular drawn from a vertex of a triangle to the opposite side is called* ***altitude***.]

33. A(2, – 4), B(3, 3) and C(–1, 5) are the vertices of triangle ABC. Find the equation of :

(*i*) the median of the triangle through A.

(*ii*) the altitude of the triangle through B.

34. Find the equation of the right bisector of the line segment joining the points (1, 2) and (5, –6).

35. Points A and B have coordinates (7, −3) and (1, 9) respectively. Find

(i) the slope of AB.

(ii) the equation of the perpendicular bisector of the line segment AB.

(iii) the value of p if $(-2, p)$ lies on it. *(2008)*

36. The points B(1, 3) and D(6, 8) are two opposite vertices of a square ABCD. Find the equation of the diagonal AC.

Hint

AC is the right bisector of BD.

37. ABCD is a rhombus. The co-ordinates of A and C are (3, 6) and (−1, 2) respectively. Write down the equation of BD. *(2000)*

Hint

BD is the right bisector of AC.

38. Find the equation of the line passing through the intersection of the lines $4x + 3y = 1$ and $5x + 4y = 2$ and

(i) parallel to the line $x + 2y - 5 = 0$

(ii) perpendicular to the x-axis.

Hint

(i) It will be found that the point of intersection of the given lines is (−2, 3).

(ii) Any line perpendicular to x-axis is parallel to y-axis and its equation is of the form $x = a$.

39. (i) Write down the co-ordinates of the point P that divides the line joining A(−4, 1) and B(17, 10) in the ratio 1 : 2.

(ii) Calculate the distance OP, where O is the origin.

(iii) In what ratio does the y-axis divide the line AB ?

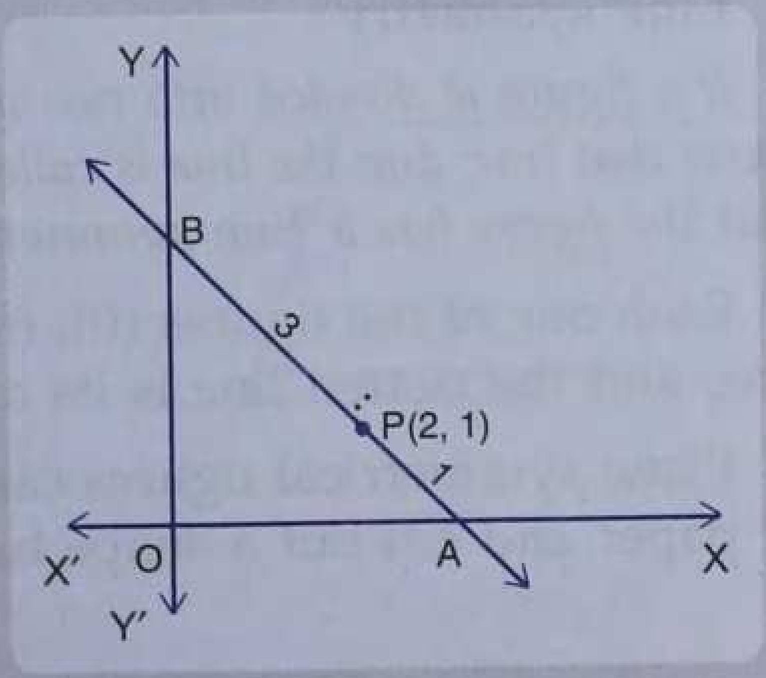
40. Find the image of the point (1, 2) in the line $x - 2y - 7 = 0$.

41. If the line $x - 4y - 6 = 0$ is the perpendicular bisector of the line segment PQ and the co-ordinates of P are (1, 3), find the co-ordinates of Q.

42. OABC is a square, O is the origin and the points A and B are (3, 0) and (p, q). If OABC lies in the first quadrant, find the values of p and q. Also write down the equations of AB and BC.

CHAPTER TEST

1. Find the equation of a line whose inclination is $60°$ and y-intercept is -4.
2. Write down the gradient and the intercept on the y-axis of the line $3y + 2x = 12$.
3. If the equation of a line is $y = \sqrt{3}\,x + 1$, find its inclination.
4. If the line $y = mx + c$ passes through the points $(2, -4)$ and $(-3, 1)$, determine the values of m and c.
5. If the points $(1, 4)$, $(3, -2)$ and $(p, -5)$ lie on a line, find the value of p.
6. Find the inclination of the line joining the points P(4, 0) and Q(7, 3).
7. Find the equation of the line passing through the point of intersection of the lines $2x + y = 5$ and $x - 2y = 5$ and having y-intercept equal to $-\frac{3}{7}$.
8. If the point A is reflected in the y-axis, the co-ordinates of its image A_1 are $(4, -3)$.
   (*i*) Find the co-ordinates of A.
   (*ii*) Find the co-ordinates of A_2, A_3, the images of the points A, A_1 respectively under reflection in the line $x = -2$.
9. If the lines $\frac{x}{3} + \frac{y}{4} = 7$ and $3x + ky = 11$ are perpendicular to each other, find the value of k.
10. Write down the equation of a line parallel to $x - 2y + 8 = 0$ and passing through the point $(1, 2)$.
11. Write down the equation of the line passing through $(-3, 2)$ and perpendicular to the line $3y = 5 - x$.
12. Find the equation of the line perpendicular to the line joining the points A(1, 2) and B(6, 7), and passing through the point which divides the line segment AB in the ratio 3 : 2.
13. The points A(7, 3) and C(0, −4) are two opposite vertices of a rhombus ABCD. Find the equation of the diagonal BD.
14. A straight line passes through P(2, 1) and cuts the axes in points A, B. If BP : PA = 3 : 1, find :
   (*i*) the co-ordinates of A and B.
   (*ii*) the equation of the line AB.



15. A straight line makes on the co-ordinate axes positive intercepts whose sum is 7. If the line passes through the point $(-3, 8)$, find its equation.
16. If the co-ordinates of the vertex A of a square ABCD are $(3, -2)$ and the equation of the diagonal BD is $3x - 7y + 6 = 0$, find the equation of the diagonal AC. Also find the co-ordinates of the centre of the square.