Chapter 11

FORMULAE

In previous classes, you have learnt how to express a mathematical statement as a formula, using literals and symbols in place of words. You also learnt changing the subject of a formula and its evaluation by substitution—replacing the literals with particular values. In this chapter, we will strengthen these concepts and solve a few tougher problems.

FRAMING A FORMULA

An equation expressing the relationship between two or more quantities is called a ***formula.***

To express a statement in the form of an equation using literals and symbols in place of words is called ***framing the formula.***

We illustrate the concept with a few examples.

Example 1. Raman's present age is x years and his mother is 26 years older than him. After 14 years, Raman will be half as old as his mother. Write the formula for this statement.

Solution. Given Raman's present age = x years

His mother's present age = $(x + 26)$ years

After 14 years,

age of Raman will be $(x + 14)$ years

and age of his mother will be $(x + 26 + 14)$ years *i.e.* $(x + 40)$ years

According to given information,

$$x + 14 = \frac{1}{2}(x + 40), \text{ which is the required formula.}$$

Example 2. A shopkeeper buys a kg of tea at the rate of ₹x per kg and b kg of tea at the rate of ₹y per kg. He mixes the two brands of tea and sells this tea at the rate of ₹z per kg. Find :

(i) his profit (ii) profit percentage.

Solution. (i) Cost of a kg of tea = ₹ax

Cost of b kg of tea = ₹by

$\therefore$ Total cost of $(a + b)$ kg of tea = ₹$(ax + by)$

Selling price of $(a + b)$ kg of tea = ₹$(a + b)z$

$\therefore$ Profit = S.P. − C.P. = ₹$[(a + b)z - (ax + by)]$.

(ii) Profit percentage $= \left(\frac{\text{profit}}{\text{C.P.}} \times 100\right)\%$

$= \left(\frac{(a + b)z - (ax + by)}{ax + by} \times 100\right)\%$.

Example 3. In a shooting competition, a marksman receives ₹ 2 if he hits the mark and pays 50 paise if he misses it. If he hits the mark x times and tried 60 shots, find the money earned by him.

Solution. As the marksman tried 60 shots and hits x times, so he misses the mark $(60 - x)$ times.

Since he receives ₹ 2 on hitting the mark,

$\therefore$ for x hits, he receives ₹ $2x$.

As he pays 50 paise *i.e.* ₹ $\frac{1}{2}$ for missing the mark,

$\therefore$ for $(60 - x)$ misses, he pays ₹ $\frac{1}{2}(60 - x)$.

$\therefore$ The money earned by him = ₹ $\left(2x - \frac{1}{2}(60 - x)\right)$ = ₹ $\left(\frac{5}{2}x - 30\right)$.

Example 4. In a two digit number, the unit's digit is x. If the ten's digit exceed its unit's digit by 3, find the number.

Solution. Given, in a two digit number, the unit's digit is x.

As its ten's digit exceed its unit's digit by 3,

its ten's digit is $x + 3$

$\therefore$ The number = $(x + 3) \times 10 + x$.

[For example, if the unit's digit is 2, then ten's digit = 2 + 3 = 5. So the number is $5 \times 10 + 2$ *i.e.* 52.]

Exercise 11.1

Frame a formula for each of the following (1 to 7) statements :

1. If you multiply a number x by 5 and take away 23, you get 7 more than thrice the number.
2. The length of a rectangle is 10 units more than its breadth x units, and the perimeter is 7 times the breadth.
3. Anu is presently y years old. In 4 years time, she will be three times old as she was 2 years ago.
4. The area A sq. units of a circle is π times the square of its radius r units.
5. The area A of a circular ring (track) is π times the difference between the squares of its outer radius R and the inner radius r.
6. A two digit number having x at ten's digit and y at unit's digit is 4 times the sum of its digits.
7. The number d of diagonals that can be drawn from one vertex of an n-sided polygon to all other vertices is 3 less than n.
8. A donation box contains x coins of ₹ 2 each, y coins of rupee 1 each, z coins of 50 paise each and t coins of paise 25 each. Find the total money M (in rupees).
9. A man earns ₹ 260 per day and a woman earns ₹ 235 per day. Find the monthly earning of x men and y women, assuming that there are 26 working days in a month.
10. In a class of 45 students, the average marks of x students are 8 and the average marks of the remaining students are 7. Find a formula for the average marks of all the students of the class.
11. A cyclist rides for t hours at x km/hr and for s hours at y km/hr. Find the formula for his average speed.
12. In a class of x students, each one of y students pays ₹ 50 and each of the remaining students of the class pays ₹ 10 for a charity show. Find the total collection C (in rupees).

CHANGE OF SUBJECT OF A FORMULA

Subject of a formula *is the variable which is expressed in terms of the other variables involved in the formula.*

For example :

The perimeter P of a rectangle is given by the formula

$P = 2(l + b)$, where l and b are the length and breadth of a rectangle.

Here P is the subject of this formula.

The same formula can be re-written as

(*i*) $l = \frac{P}{2} - b$, l is the subject of the formula.

(*ii*) $b = \frac{P}{2} - l$, b is the subject of the formula.

Evaluation of the subject of the formula

To evaluate the subject of the formula, proceed as under :

Substitute the values of all the variables (except the subject) in the formula and simplify it to find the value of the subject.

Sometimes, we may have to change the subject of the formula to find the value of the unknown variable.

Example 1. Nine-fifth of the temperature (C) in centigrade of body increased by 32° is equal to its temperature in Fahrenheit (F).

(*i*) Frame a formula for the above statement, making F as the subject.

(*ii*) Find F when C = 35°.

(*iii*) Make C as the subject of the formula.

(*iv*) Find C when F = 68°.

Solution. (*i*) From the given statement, we can write the formula as

$$F = \frac{9}{5}C + 32°, \text{ F is the subject of the formula.}$$

(*ii*) Substituting C = 35° in the above formula, we get

$$F = \frac{9}{5} \times 35° + 32° = 9 \times 7° + 32° = 63° + 32° = 95°.$$

(*iii*) To make C as the subject of the formula :

From (*i*), we get $F - 32° = \frac{9}{5}C$

$$\Rightarrow \quad C = \frac{5}{9}(F - 32°), \text{ C is the subject of the formula.}$$

(*iv*) Substituting F = 68° in the above formula, we get

$$C = \frac{5}{9}(68° - 32°) = \frac{5}{9} \times 36° = 5 \times 4° = 20°.$$

Example 2. In the formula $S = \frac{n}{2}[2a + (n - 1)\,d]$, make a as the subject. Find a when S = 60, n = 10 and d = 2

Solution. To make a as the subject of the formula :

Given $S = \frac{n}{2}[2a + (n - 1)\,d]$

$$\Rightarrow \quad \frac{2S}{n} = 2a + (n-1)\,d \quad \Rightarrow \quad 2a + (n-1)\,d = \frac{2S}{n}$$

$$\Rightarrow \quad 2a = \frac{2S}{n} - (n-1)\,d \quad \Rightarrow \quad a = \frac{1}{2}\left[\frac{2S}{n} - (n-1)\,d\right]$$

$$\Rightarrow \quad a = \frac{S}{n} - \frac{1}{2}(n-1)\,d\,,\ a \text{ is the subject of the formula.}$$

Substituting S = 60, $n = 10$ and $d = 2$ in the above formula, we get

$$a = \frac{60}{10} - \frac{1}{2}(10-1) \times 2 = 6 - 9 = -3.$$

Example 3. The area A of a circle is given by the formula $A = \pi r^2$ where $\pi = \frac{22}{7}$ and r is the radius of the circle.

(*i*) Find A when $r = 3{\cdot}5$ cm.

(*ii*) Make r as the subject of the formula.

Hence find r when $A = 154$ cm^2.

Solution. (*i*) Given $A = \pi r^2$

Substituting $\pi = \frac{22}{7}$ and $r = 3{\cdot}5$ cm $= \frac{7}{2}$ cm, we get

$$A = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2 = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2 = 38{\cdot}5 \text{ cm}^2.$$

(*ii*) To make r as the subject of the formula, we have

$A = \pi r^2 \Rightarrow r^2 = \frac{A}{\pi} \Rightarrow r = \sqrt{\frac{A}{\pi}}$, r is the subject of the formula.

Substituting $A = 154$ cm^2 and $\pi = \frac{22}{7}$, we get

$$r = \sqrt{\frac{154}{\frac{22}{7}}} \text{ cm} = \sqrt{\frac{154}{1} \times \frac{7}{22}} \text{ cm} = \sqrt{49} \text{ cm} = 7 \text{ cm}.$$

Example 4. Make u as the subject of the formula : $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$. Hence find u when $f = 6$ cm and $v = 10$ cm.

Solution. To make u as the subject of the formula, we have

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{v-f}{fv} \Rightarrow u = \frac{fv}{v-f}\,,\ u \text{ is the subject of the formula.}$$

Substituting $f = 6$ cm and $v = 10$ cm, we get

$$u = \frac{6 \times 10}{10 - 6} \text{ cm} = \frac{60}{4} \text{ cm} = 15 \text{ cm}.$$

Exercise 11.2

1. Change the subject of each of the following formulae to the letter given against them :

(*i*) $A = P\left(1 + \frac{RT}{100}\right)$; R

(*ii*) $a = \frac{A - 2b}{2}$; b

(*iii*) $ax + 2x = b$; x

(*iv*) $s = ut - \frac{1}{2}gt^2$; g

(*v*) $s = \frac{u+v}{2}t$; v (*vi*) $m = \sqrt{\frac{l+n}{l-n}}$; n

(*vii*) $A = 2\pi r(r + h)$; h (*viii*) $V = \pi r^2 h$; r

2. If $A = P + \frac{PRT}{100}$, make T the subject of the formula. Hence find T when $A = 575$, $P = 500$ and $R = 6$.

3. Make b the subject of the formula $P = 2(l + b)$. Find b when $P = 72$ and $l = 22$.

4. In the formula $c = \frac{ab}{a+b}$, make b the subject of the formula. Hence find b when $c = 12{\cdot}5$ and $a = 15$.

5. In the formula $T = 2\pi\sqrt{\frac{l}{g}}$, make g as the subject of the formula. Find g when $\pi = \frac{22}{7}$, $l = 20$ and $T = \frac{44}{49}$.

6. In the formula $v = u + at$, find :

(*i*) v when $u = 10$, $a = 2$ and $t = 4$ (*ii*) u when $v = 18$, $a = 2{\cdot}5$ and $t = 2$

(*iii*) a when $v = 25$, $u = 18$ and $t = 5$ (*iv*) t when $v = 37$, $u = 23$ and $a = 1{\cdot}75$.

7. In the formula $S = \frac{n}{2}[2a + (n - 1)d]$, make d as the subject of the formula. Find d when $S = 255$, $a = -4$ and $n = 15$.

8. In the formula $V = \pi (R^2 - r^2) h$, make h as the subject of the formula. Find h when $V = 3234$, $R = 14$, $r = 10{\cdot}5$ and $\pi = \frac{22}{7}$.

Summary

- An equation expressing the relationship between two or more quantities is called a formula.
- To express a mathematical statement in the form of an equation using literals and symbols in place of words is called framing the formula.
- Subject of a formula is the variable which is expressed in terms of the other variables involved in the formula.
- To find the value of the subject—substitute the values of all the variables (except the subject) in the formula and simplify it.

Check Your Progress

1. Write a formula for the statement 'A two digit number has digit x at its unit place and the sum of both digits is 9. On subtracting 9 from the number, it becomes 45'.
2. Make n as the subject of the formula $l = a + (n - 1)d$. Find n when $l = -29$, $a = -5$ and $d = -2$.
3. Make s as the subject of the formula $v = \sqrt{u^2 + 2as}$. Find s when $v = 15$, $u = 20$ and $a = -2$.
4. Make x as the subject of the formula $\frac{1}{x} + \frac{1}{y} = \frac{1}{y-1}$. Find x when $y = 13$.