10 Reflection

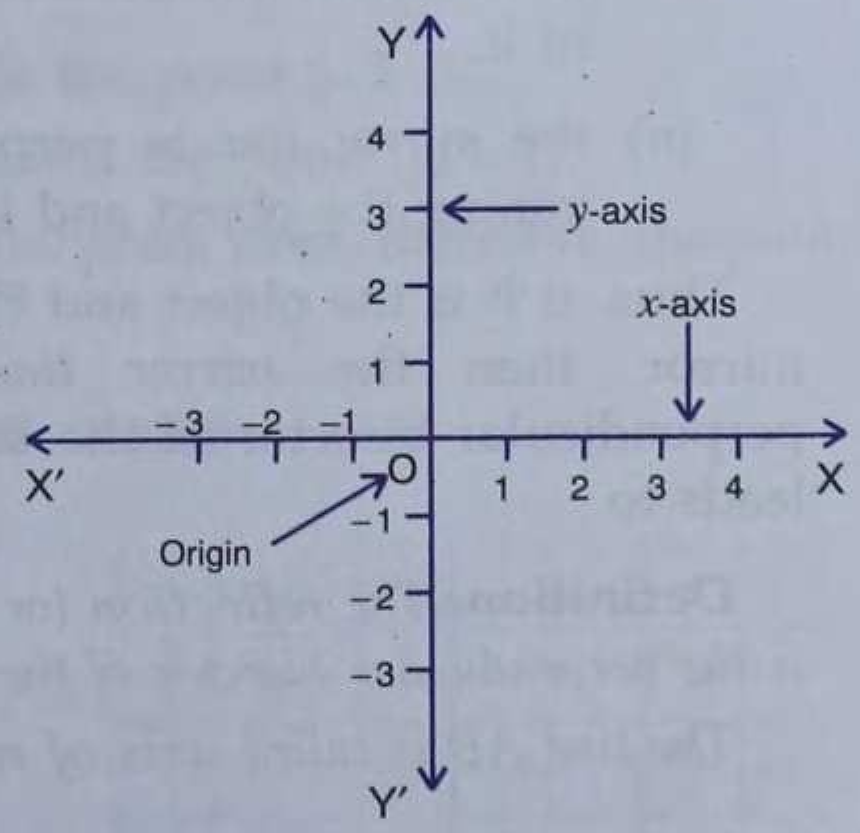
10.1 CO-ORDINATE GEOMETRY

Co-ordinate geometry is that branch of Mathematics which deals with the study of geometry by means of algebra. In co-ordinate geometry, we represent a point in a plane by an ordered pair of real numbers, called co-ordinates of the point; and a straight line or a curve by an algebraic equation with real coefficients. Thus, we use algebra advantageously to the study of straight lines and geometric curves.

Recall that there is one and only one point on a number line associated with each real number. A similar situation exists for points in a plane and **ordered pairs** of real numbers.

10.1.1 The cartesian system of co-ordinates

When two numbered lines perpendicular to each other (usually horizontal and vertical) are placed together so that the two origins (the points corresponding to zero) coincide and the lines are perpendicular, then the resulting configuration is called a **cartesian coordinate system** or a **coordinate plane.**



Let X′OX and Y′OY, two number lines perpendicular to each other, meet at the point O (shown in the adjoining figure), then

(*i*) X′OX is called ***x*-axis.**

(*ii*) Y′OY is called ***y*-axis.**

(*iii*) X′OX and Y′OY taken together are called **coordinate axes.**

(*iv*) the point O is called the **origin.**

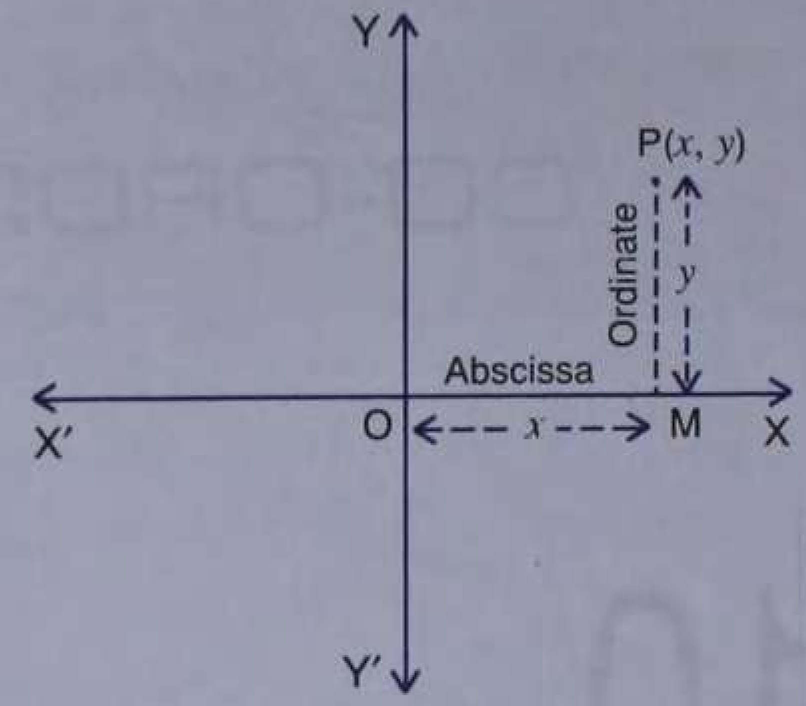
10.1.2 Co-ordinates of a point

Let P be any point in the co-ordinate plane. From P draw PM perpendicular to X′OX, then

(*i*) OM is called ***x*-co-ordinate** or **abscissa** of P and is usually denoted by x.

(*ii*) MP is called ***y*-co-ordinate** or **ordinate** of P and is usually denoted by y.

(*iii*) x and y taken together are called **cartesian co-ordinates** or simply co-ordinates of P and are denoted by (x, y).



Remarks

- The co-ordinates of a point indicate its position with reference to coordinate axes.
- In stating the co-ordinates of a point, the ***abscissa*** precedes the ***ordinate***. The two are separated by a comma and are enclosed in the bracket ( ). Thus, a point P whose abscissa is 'x' and ordinate is 'y' is written as (x, y) or P (x, y).
- The co-ordinates of the origin O are (0, 0).

Convention for signs of coordinates

(*i*) The x-coordinate (abscissa) of a point is ***positive*** if it is measured to the right of O *i.e.* along OX and is ***negative*** if it is measured to the left of O *i.e.* along OX'.

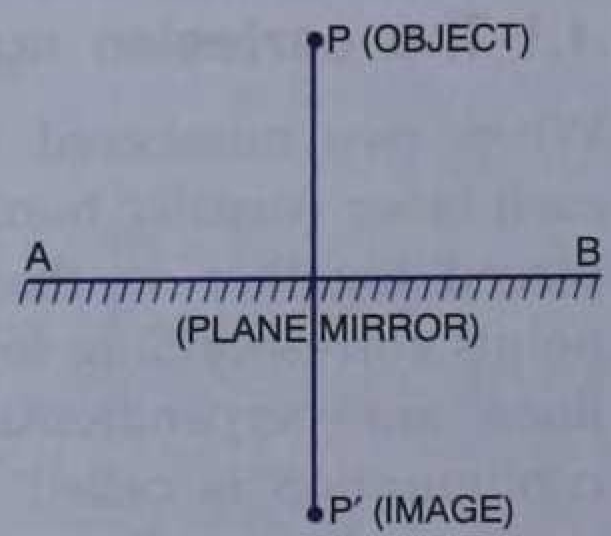
(*ii*) The y-coordinate (ordinate) of a point is ***positive*** if it is measured upwards *i.e.* along OY and is ***negative*** if it is measured downwards *i.e.* along OY'.

10.2 REFLECTION OF A POINT IN A LINE

When we see the image of an object in a plane mirror, we notice that :

(*i*) the distance of the *image* behind the mirror is the same as the distance of the *object* in front of it.

(*ii*) the *mirror line* is perpendicular to the line joining the object and its image.



Thus, if P is the object and P' its image in a plane mirror, then the *mirror line* (say AB) is the perpendicular bisector of the line segment PP'. This leads to :

Definition. *The* ***reflection*** (*or* ***image***) *of a point P in a line AB is a point P' such that AB is the perpendicular bisector of the line segment PP'.*

The line AB is called ***axis of reflection*** (or ***mediator***).

Invariant point

In particular, if the point P lies on the line AB, then the image of P is P itself. Such a point is called an *invariant point* with respect to the line AB. This leads to :

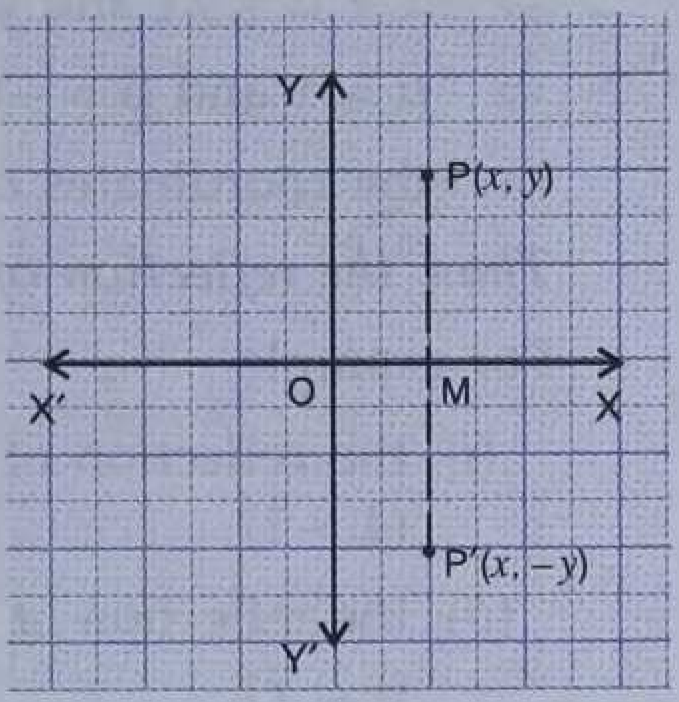
Definition. *A point is called an* ***invariant point*** *with respect to a given line if and only if it lies on the line.*

10.2.1 Reflection of a point in the *x*-axis

Let $P(x, y)$ be any point in the co-ordinate plane. From P, draw PM perpendicular to x-axis and produce it to a point P' such that MP' = MP. Then the point P' is the *reflection* of the point P in the x-axis.

From figure, the co-ordinates of the point P′ are $(x, -y)$.

Thus, the reflection of the point $P(x, y)$ in the x-axis is the point $P'(x, -y)$.



Rule to find the reflection of a point in the x-axis :

(*i*) *Retain the abscissa i.e. x-coordinate.*

(*ii*) *Change the sign of ordinate i.e. y-coordinate.*

For example :

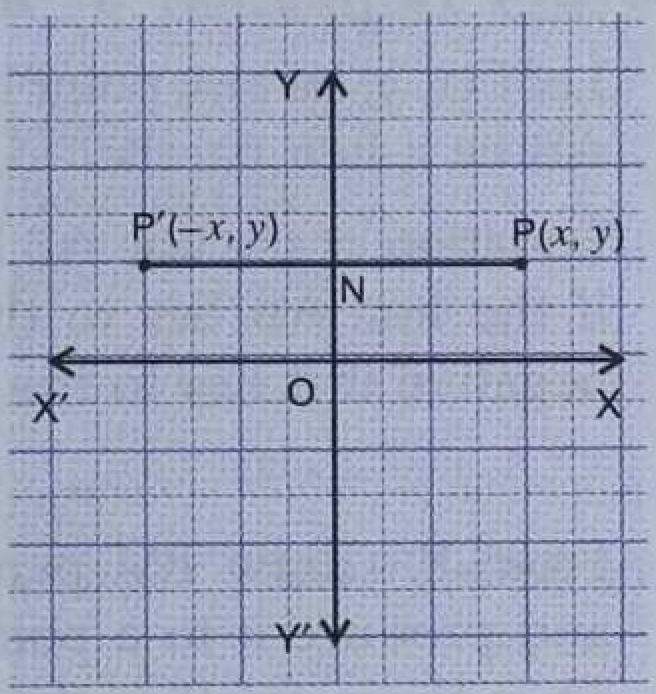
(*i*) the reflection of the point (2, 3) in the x-axis is the point (2, – 3).

(*ii*) the reflection of the point (– 4, – 1) in the x-axis is the point (– 4, 1).

(*iii*) the reflection of the point (5, 0) in the x-axis is the point itself, therefore, the point (5, 0) is *invariant* with respect to x-axis.

10.2.2 Reflection of a point in the y-axis

Let $P(x, y)$ be any point in the co-ordinate plane. From P, draw PN perpendicular to y-axis and produce it to a point P′ such that NP′ = NP. Then the point P′ is the *reflection* of the point P in the y-axis.



From figure, the co-ordinates of the point P′ are $(-x, y)$.

Thus, the reflection of the point $P(x, y)$ in the y-axis is the point $P'(-x, y)$.

Rule to find the reflection of a point in the y-axis :

(*i*) *Change the sign of abscissa i.e. x-coordinate.*

(*ii*) *Retain the ordinate i.e. y-coordinate.*

For example :

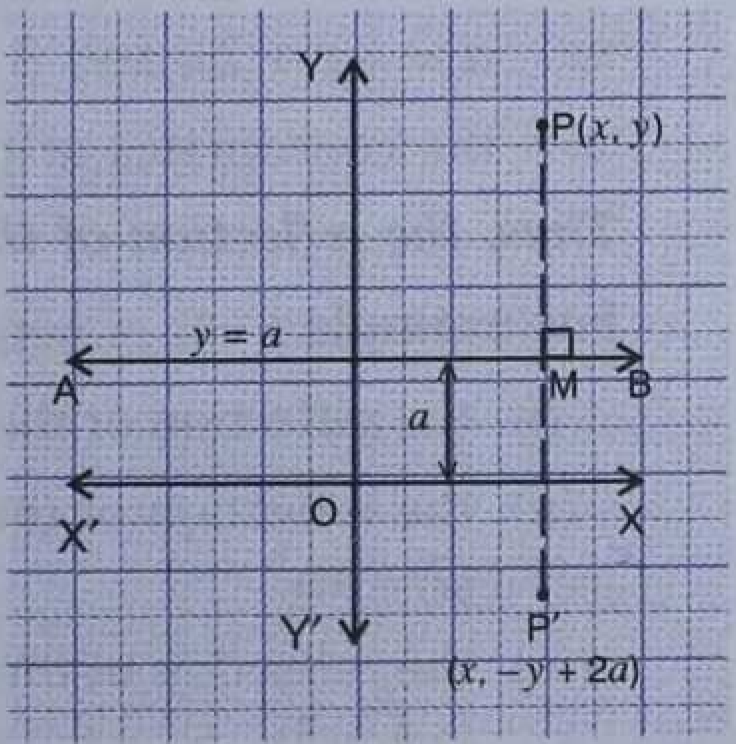
(*i*) the reflection of the point (2, 3) in the y-axis is the point (– 2, 3).

(*ii*) the reflection of the point (– 4, – 1) in the y-axis is the point (4, – 1).

(*iii*) the reflection of the point (0, 5) in the y-axis is the point itself, therefore, the point (0, 5) is *invariant* with respect to y-axis.

10.2.3 Reflection of a point in a line parallel to x-axis

Let $P(x, y)$ be any point in the coordinate plane and AB be a line parallel to x-axis.



Equation of the line AB is $y = a$, where a is positive if the line AB lies above the x-axis and a is negative if it lies below the x-axis. (See article 12.2.1)

From P, draw PM perpendicular to the line AB and produce it to a point P′ such that MP′ = MP. Then the point P′ is the *reflection* of the point P in the line AB *i.e.* in the line $y = a$.

From figure, the coordinates of the point M are (x, a). Let the coordinates of the point P′ be (α, β).

Since MP′ = MP *i.e.* $M(x, a)$ is the mid-point of the line segment PP′, we have

$$\frac{x+\alpha}{2} = x \text{ and } \frac{y+\beta}{2} = a \qquad \text{(See article 11.2.1)}$$

$\Rightarrow \quad x + \alpha = 2x$ and $y + \beta = 2a$

$\Rightarrow \quad \alpha = x$ and $\beta = -y + 2a$

$\Rightarrow$ the coordinates of the point P′ are $(x, -y + 2a)$.

Thus, the reflection of the point P (x, y) in the line y = a is the point P (x, −y + 2a).

For example :

(i) the reflection of the point (4, 5) in the line $y = 2$ is the point $(4, -5 + 2.2)$ *i.e.* $(4, -1)$

(ii) the reflection of the point $(-2, -3)$ in the line $y = 2$ is the point $(-2, 3 + 2.2)$ *i.e.* $(-2, 7)$.

(iii) the reflection of the point (3, 4) in the line $y = -1$ is the point $(3, -4 + 2.(-1))$ *i.e.* $(3, -6)$.

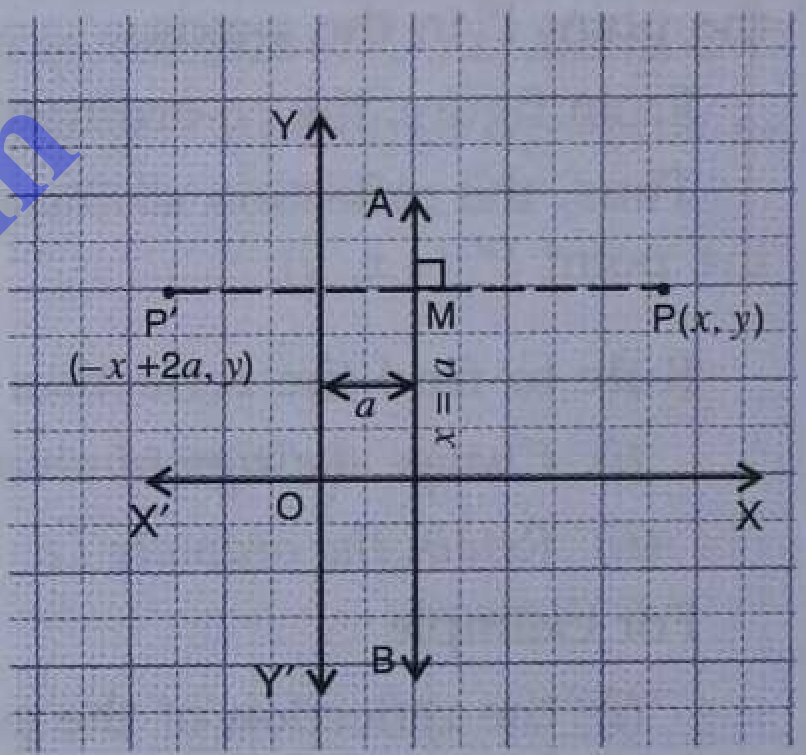
(iv) the reflection of the point $(-3, 2)$ in the line $y = 2$ is the point $(-3, -2 + 2.2)$ *i.e.* $(-3, 2)$ *i.e.* the point itself, therefore, the point $(-3, 2)$ is *invariant* with respect to the line $y = 2$.

10.2.4 Reflection of a point in a line parallel to *y*-axis

Let $P(x, y)$ be any point in the coordinate plane and AB be a line parallel to *y*-axis.

Equation of the line AB is $x = a$, where a is positive if the line AB lies to the right of *y*-axis and a is negative if it lies to the left of *y*-axis.

(See article 12.2.2)



From P, draw PM perpendicular to the line AB and produce it to a point P′ such that MP′ = MP. Then the point P′ is the *reflection* of the point P in the line AB *i.e.* in the line $x = a$.

From figure, the coordinates of the point M are (a, y). Let the coordinates of the point P′ be (α, β).

Since MP′ = MP *i.e.* $M(a, y)$ is mid-point of the line segment PP′, we have

$$\frac{x+\alpha}{2} = a \text{ and } \frac{y+\beta}{2} = y$$ (See article 11.2.1)

$\Rightarrow \quad x + \alpha = 2a$ and $y + \beta = 2y$

$\Rightarrow \quad \alpha = -x + 2a$ and $\beta = y$

$\Rightarrow$ the coordinates of the point P′ are $(-x + 2a, y)$.

Thus, the reflection of the point P (x, y) in the line x = a is the point P′(−x + 2a, y).

For example :

(i) the reflection of the point (3, 2) in the line $x = 2$ is the point $(-3 + 2 . 2, 2)$ *i.e.* $(1, 2)$.

(ii) the reflection of the point $(-3, -4)$ in the line $x = 2$ is the point $(3 + 2 . 2, -4)$ *i.e.* $(7, -4)$.

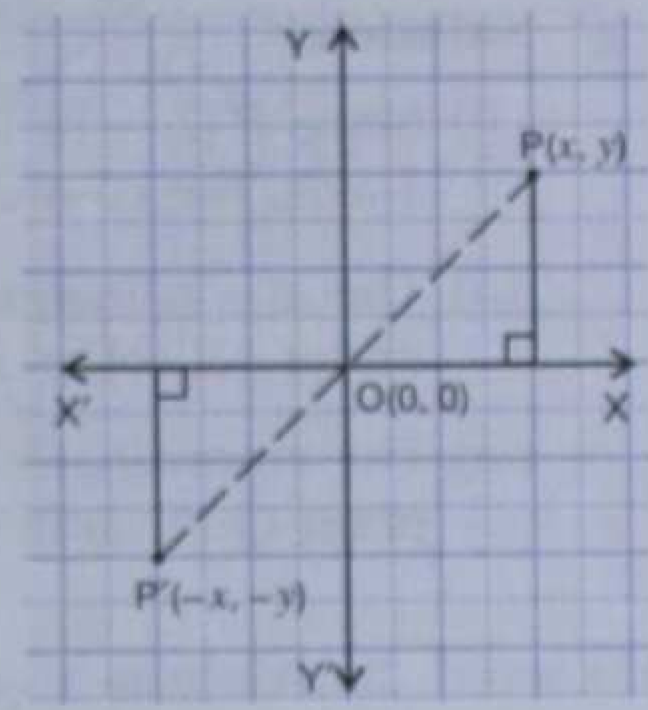
(iii) the reflection of the point (2, 5) in the line $x = -1$ is the point $(-2 + 2 . (-1), -5)$ *i.e.* $(-4, -5)$.

(iv) the reflection of the point $(2, -3)$ in the line $x = 2$ is the point $(-2 + 2 . 2, -3)$ *i.e.* $(2, -3)$ *i.e.* the point itself, therefore, the point $(2, -3)$ is *invariant* with respect to the line $x = 2$.

10.3 REFLECTION OF A POINT IN A POINT

Definition. *The reflection (or image) of a point P in a given point M is a point P′ such that M is the mid-point of the line segment PP′.*

10.3.1 Reflection of a point in the origin



Let P (x, y) be any point in the co-ordinate plane. Join PO, and produce it to a point P′ such that OP′ = OP. Then the point P′ is the reflection of the point P in the origin.

From figure, the co-ordinates of the point P′ are $(-x, -y)$.

Thus, the reflection of the point $P(x, y)$ in the origin is the point $P'(-x, -y)$.

Rule to find the reflection of a point in the origin :

(i) *Change the sign of abscissa i.e. x-coordinate.*

(ii) *Change the sign of ordinate i.e. y-coordinate.*

For example :

(i) the reflection of the point (2, 3) in the origin is the point (– 2, – 3).

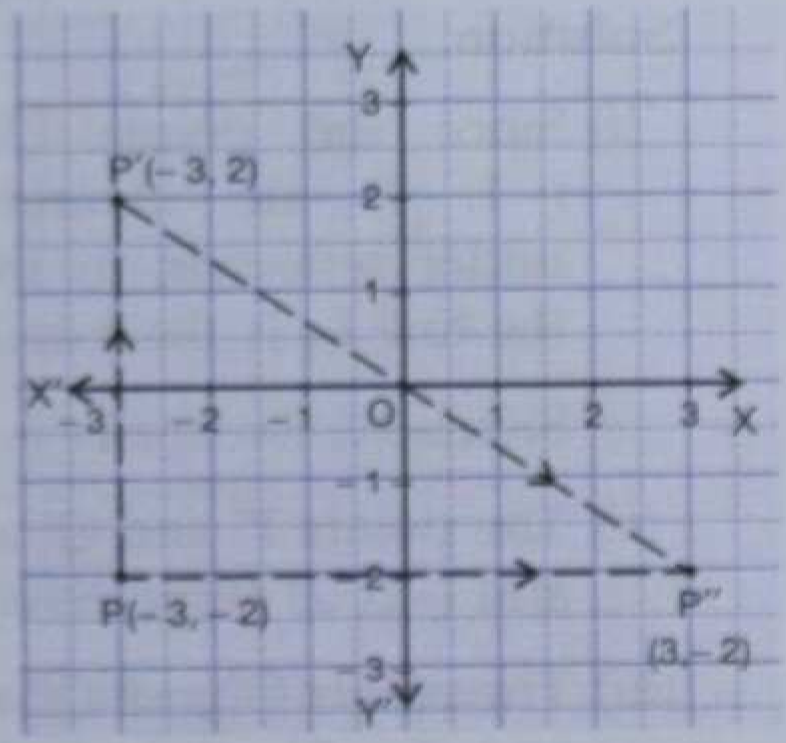
(ii) the reflection of the point (– 4, – 1) in the origin is the point (4, 1).

(iii) the reflection of the point (5, 0) in the origin is the point (– 5, 0).

ILLUSTRATIVE EXAMPLES

Example 1. *The point P (– 3, – 2) on reflection in x-axis is mapped on P′. Then P′ on reflection in the origin is mapped as P″. Find the co-ordinates of P′ and P″.*

Write down a single transformation that maps P onto P″.



Solution. Since the point P′ is the reflection of the point P (– 3, – 2) in x-axis, the co-ordinates of P′ are (– 3, 2).

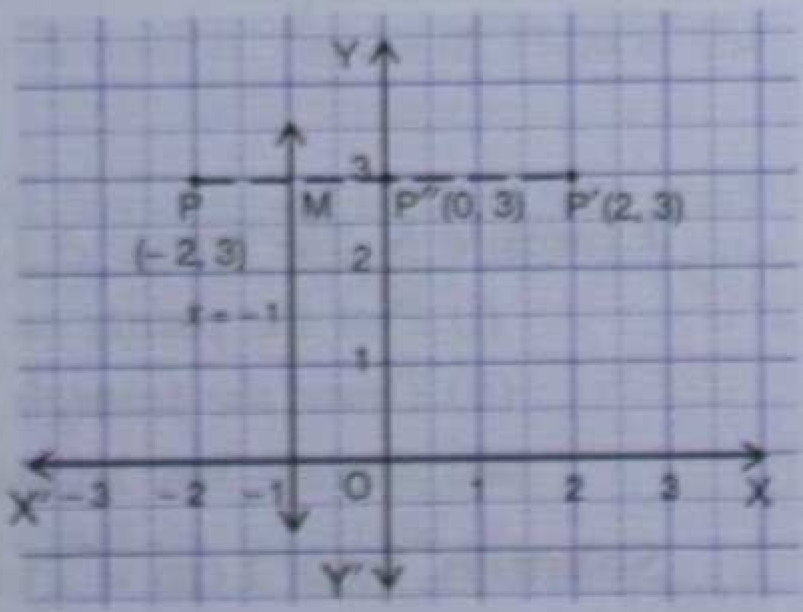
Further, as the point P″ is the reflection of the point P′ (– 3, 2) in the origin, the co-ordinates of P″ are (3, – 2).

The single transformation — the reflection of the point P (– 3, – 2) in y-axis maps it onto P″ (3, – 2).

Example 2. *A point P under reflection in y-axis is mapped onto P′(2, 3).*

(i) *Find the coordinates of P.*

(ii) *Find the coordinates of the image of P under reflection in the line $x = -1$.*



Solution.

(i) Since P′(2, 3) is the image of P under reflection in the y-axis, the coordinates of P are (–2, 3).

(ii) We know that the reflection of the point (x, y) in the line $x = a$ is the point $(-x + 2a, y)$, therefore, the image of the point P(–2, 3) under reflection in the line $x = -1$ is the point P″(– (–2) + 2 . (–1), 3) *i.e.* the point P″(0, 3).

Example 3. *A point P is reflected to P′ in the x-axis. The co-ordinates of its image are (2, – 3). Find :*

(i) the co-ordinates of P.

(ii) the co-ordinates of the image P″ of P under reflection in the y-axis.

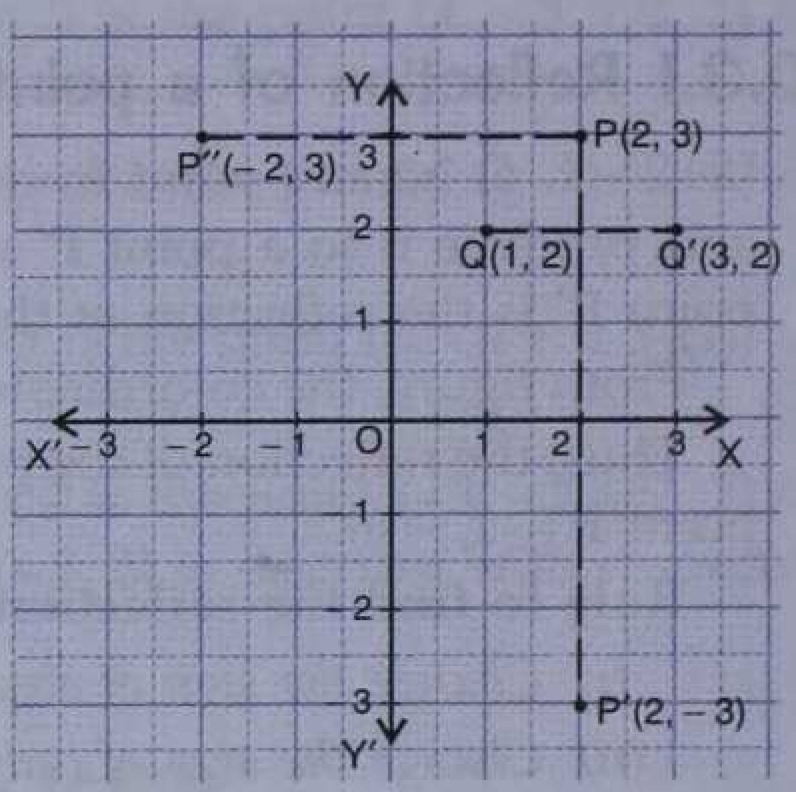
(iii) the co-ordinates of the image Q′ of the point Q (1, 2) in the line PP′.

Solution.

(*i*) Since P′(2, – 3) is the image of P in the *x*-axis, the co-ordinates of P are (2, 3).

(*ii*) The co-ordinates of the image P″ of P under reflection in the *y*-axis are (– 2, 3).

(*iii*) The co-ordinates of the image Q′ of the point Q (1, 2) in the line PP′ are (3, 2).



Example 4. *Points (3, 0) and (– 1, 0) are invariant points under reflection in the line L_1; points (0, – 3) and (0, 1) are invariant points on reflection in line L_2.*

(i) Name the lines L_1 and L_2.

(ii) Write down the images of points P(3, 4) and Q(– 5, – 2) on reflection in L_1. Name the images as P′ and Q′ respectively.

(iii) Write down the images of P and Q on reflection in L_2. Name the images as P″ and Q″ respectively.

(iv) State or describe a single transformation that maps P′ onto P″.

Solution.

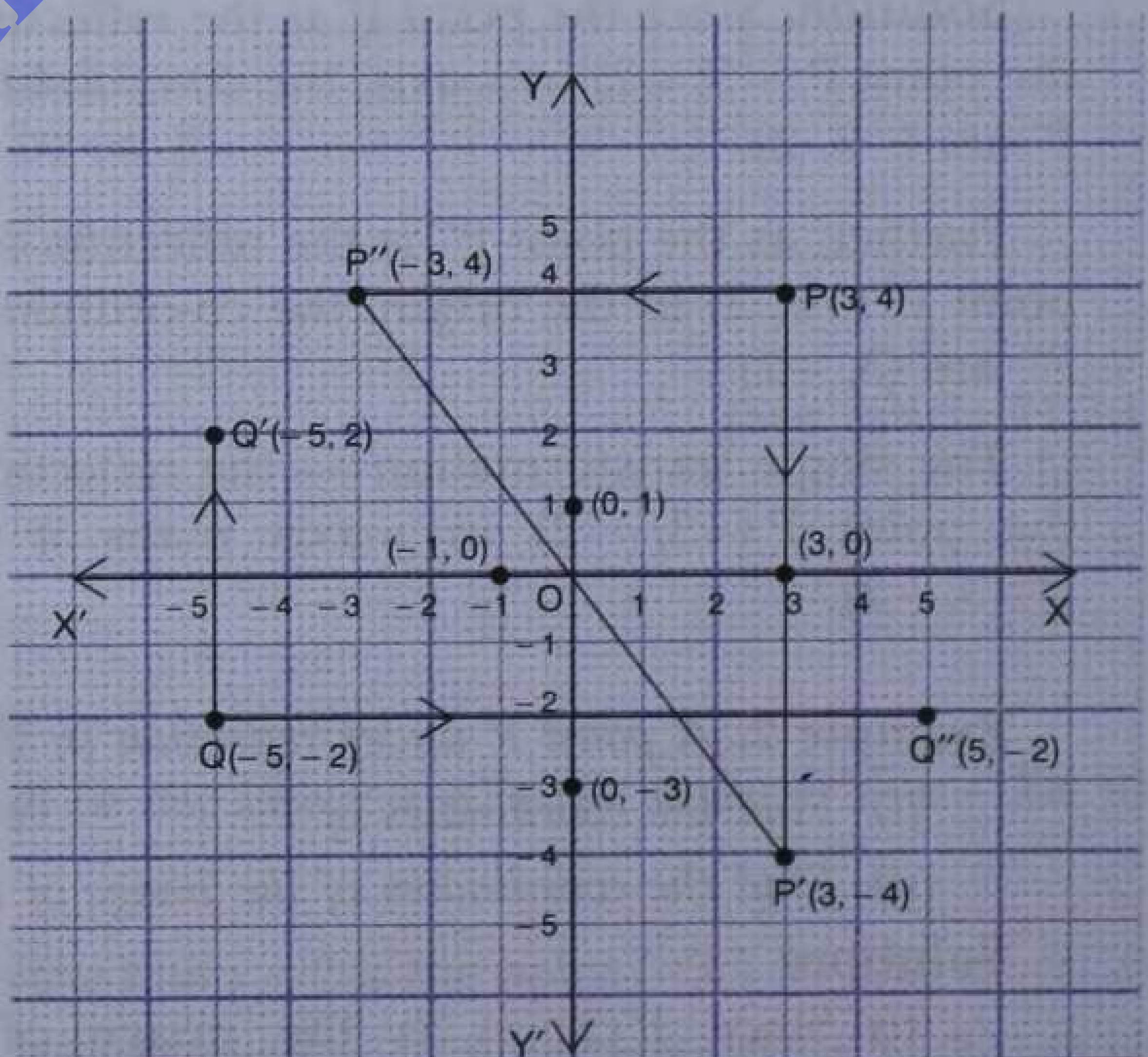
(*i*) Since the points (3, 0) and (– 1, 0) are invariant points under reflection in the line L_1, the line L_1 is the *x-axis*.

Similarly, the line L_2 is the *y-axis*.

(*ii*) The points P′ and Q′ are (3, – 4) and (– 5, 2) respectively.

(*iii*) The points P″ and Q″ are (– 3, 4) and (5, – 2) respectively.

(*iv*) The single transformation that maps P′ onto P″ is the reflection in the origin.



Example 5. *The points A (2, 1), B (0, 3) and C (– 2, – 2) are the vertices of a triangle.*

(i) Plot the points on the graph paper.

(ii) Draw the triangle formed by reflecting these points in the x-axis.

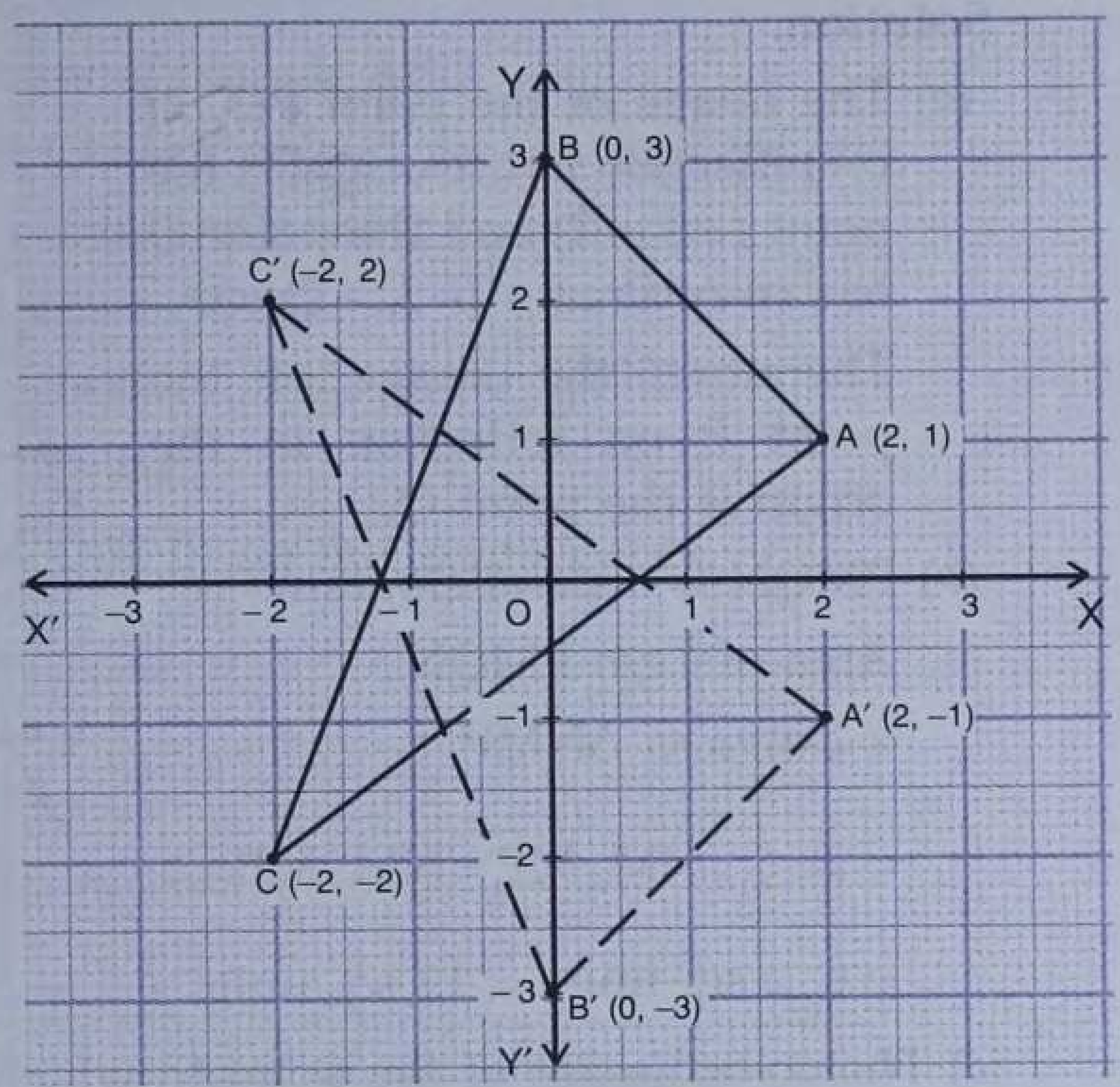
(iii) Are the two triangles congruent ?

Solution. Take 1 cm = 1 unit on both axes.

(*i*) Plot the points A (2, 1), B (0, 3) and C (– 2, – 2) as shown in the adjoining diagram.

(*ii*) The points A′ (2, – 1), B′ (0, – 3) and C′ (– 2, 2) are the reflections of the points A, B and C respectively in the *x*-axis. The triangle formed by the points A′, B′ and C′ has been shown in the diagram by dotted lines.

(*iii*) The two triangles ABC and A′B′C′ are congruent (measure the distances and check it).

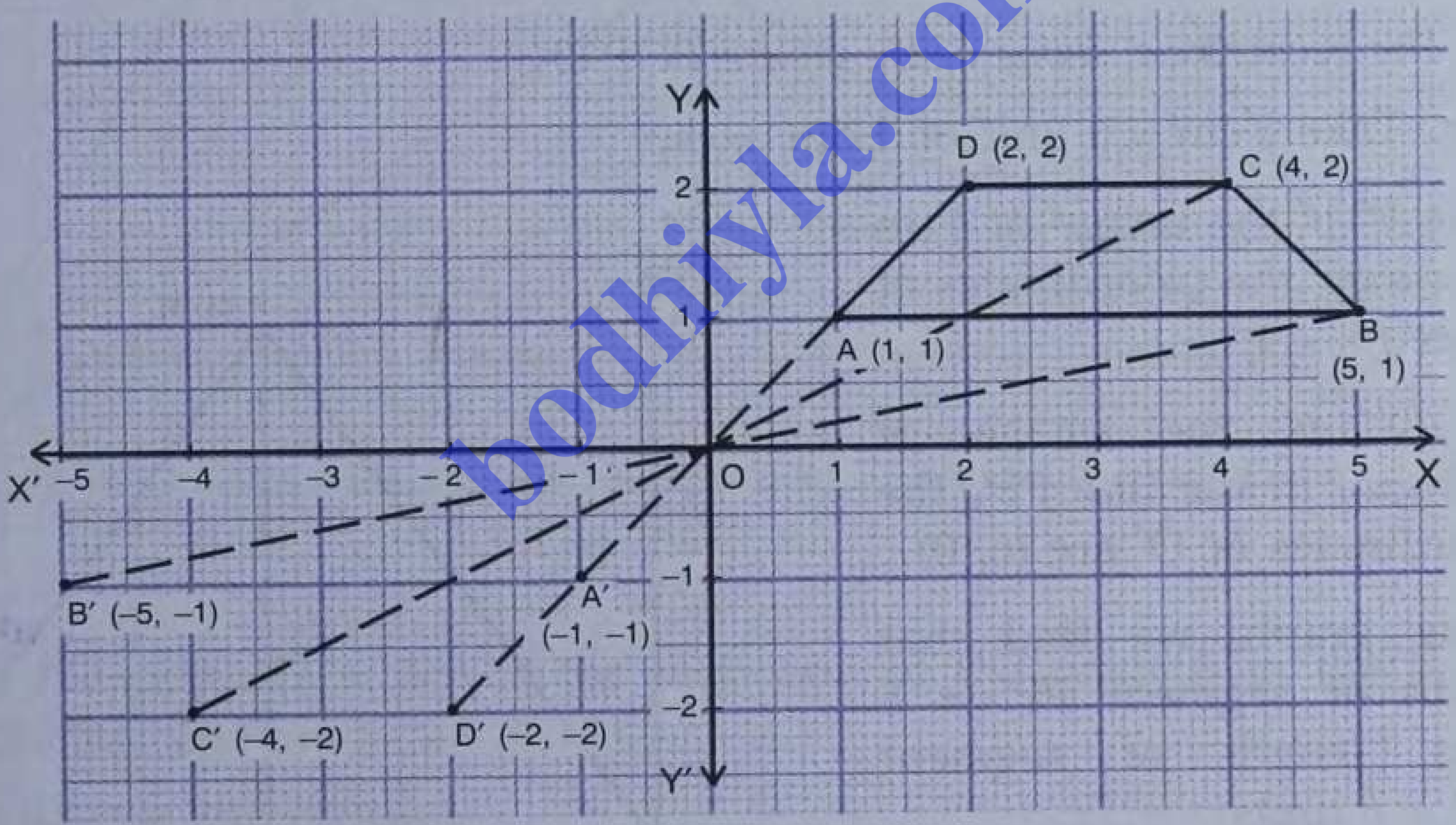


Example 6. *Use graph paper for this question.*

A (1, 1), B (5, 1), C (4, 2) and D (2, 2) are the vertices of a quadrilateral. Name the quadrilateral ABCD. A, B, C and D are reflected in the origin on to A′, B′, C′ and D′ respectively. Locate A′, B′, C′ and D′ on the graph sheet and write their co-ordinates. Are D, A, A′ and D′ collinear ? (2004)

Solution. Choose the co-ordinate axes as shown in the graph paper.

Take 1 cm = 1 unit on both axes.



Plot the points A (1, 1), B (5, 1), C (4, 2) and D (2, 2) on the graph paper.

The quadrilateral ABCD is an isosceles trapezium.

Reflect the points A, B, C and D in the origin onto the points A′, B′, C′ and D′ respectively as shown in the graph paper. The co-ordinates of these points are :

A′ (– 1, – 1), B′ (– 5, – 1), C′ (– 4, – 2) and D′ (– 2, – 2).

On joining the points D and D′, we find that the points A and A′ lie on it. Hence, the points D, A, A′ and D′ are collinear.

Example 7. *Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axes).*

Plot the points P(3, 2) and Q(– 3, – 2). From P and Q, draw perpendiculars PM and QN on the x-axis.

(a) Name the image of P on reflection in the origin.

(b) Assign the special name to the geometrical figure PMQN and find its area.

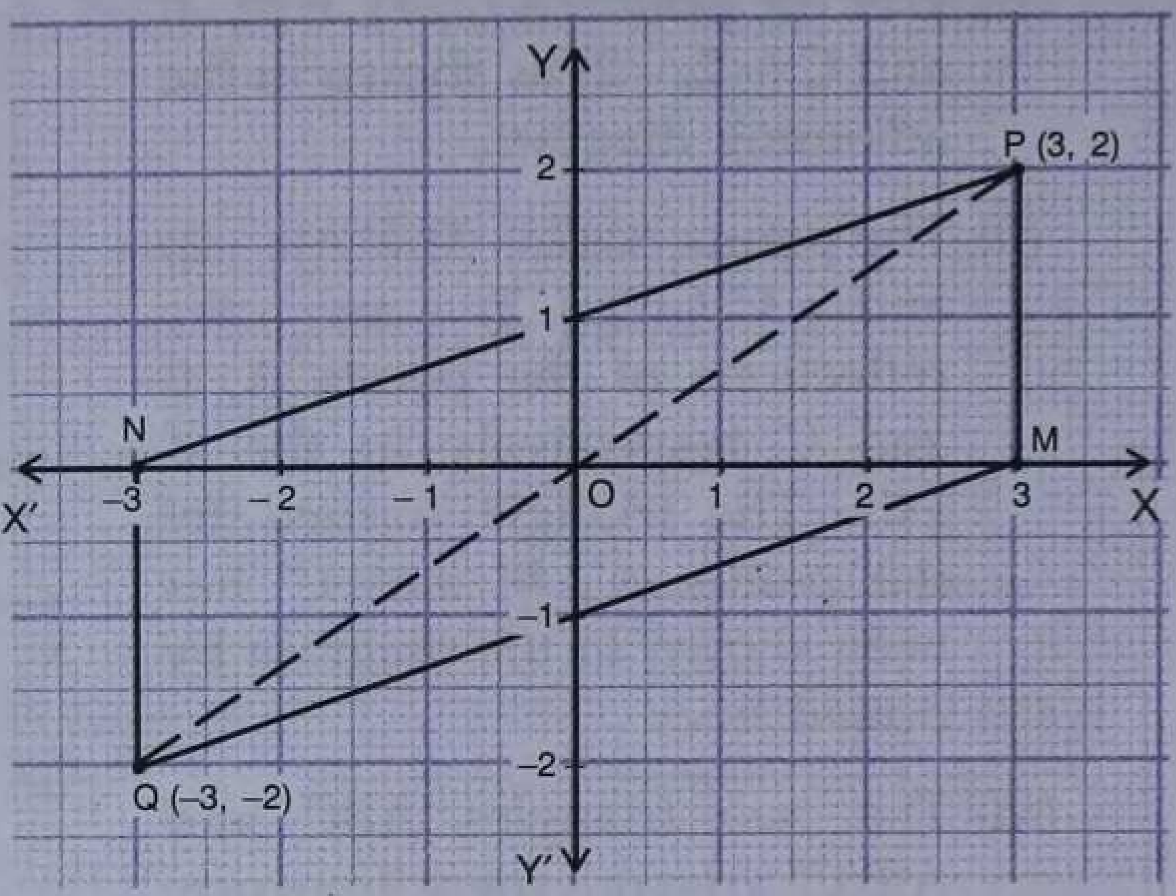
(c) Write the co-ordinates of the point to which M is mapped on reflection in

(i) x-axis *(ii) y-axis* *(iii) origin.* (2003)

Solution.

(*a*) The image of the point P(3, 2) on reflection in the origin is the point (– 3, – 2) and the point Q has co-ordinates (– 3, – 2) :

Therefore, the image of the point P on reflection in the origin is the point Q.



(*b*) PMQN is a parallelogram.

Area of || gm PMQN

= 2.area of ΔNMP

$= 2 . \frac{1}{2} . NM . MP = NM . MP$

= 6 . 2 sq. units = 12 sq. units.

(*c*) The co-ordinates of the point to which M is mapped on reflection in

(*i*) *x*-axis are (3, 0) (*ii*) *y*-axis are (– 3, 0) (*iii*) origin are (– 3, 0).

Example 8. *The point P(3, 4) is reflected to P′ in the x-axis; and O′ is the image of O (the origin) when reflected in the line PP′. Using graph paper, give :*

(i) The co-ordinates of P′ and O′.

(ii) The lengths of the segments PP′ and OO′.

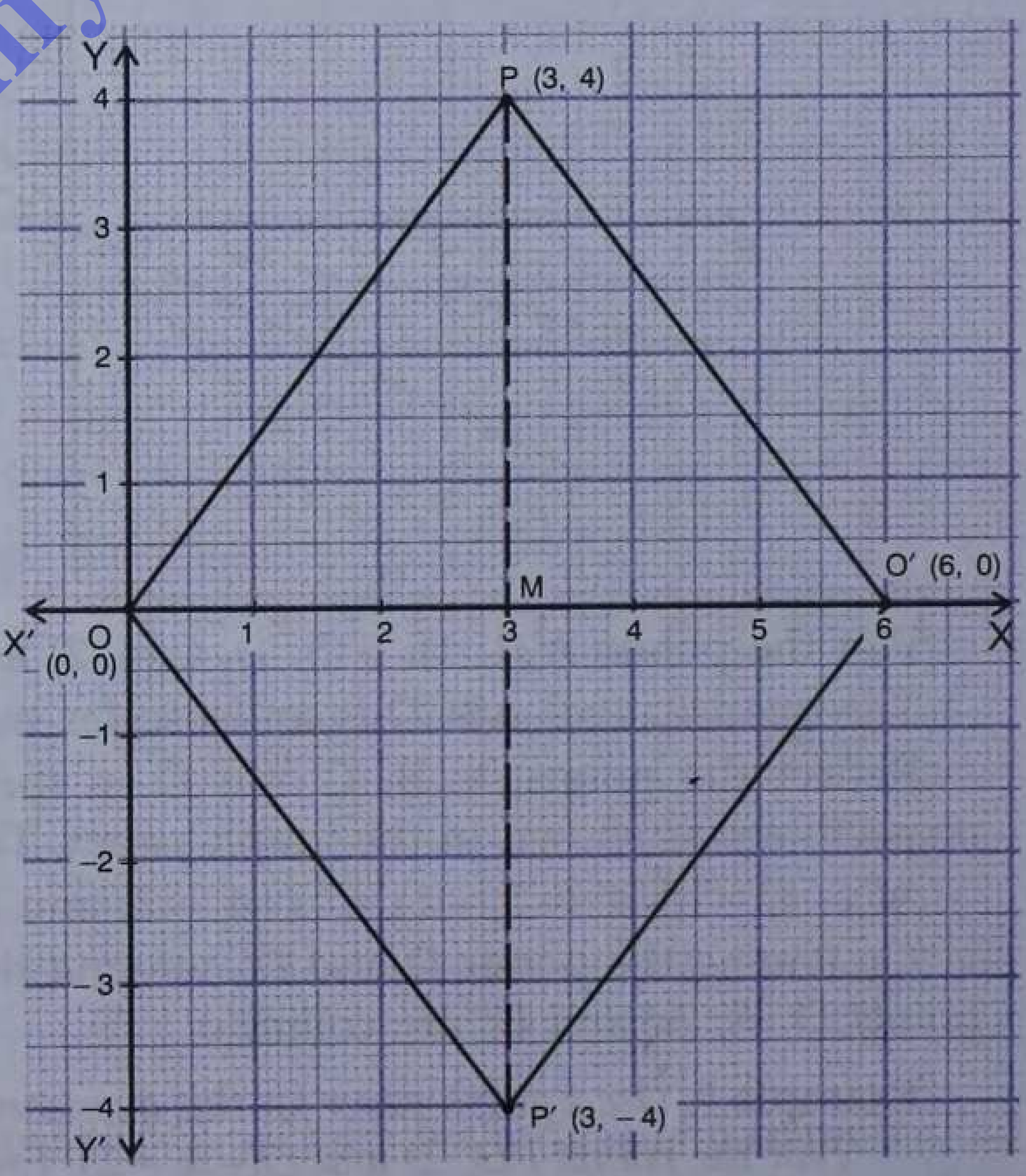
(iii) The perimeter of the quadrilateral POP′O′.

(iv) The geometrical name of the figure POP′O′. (2002)

Solution. Take 1 cm = 1 unit on both axes.

(*i*) Since P′ is the image of P in the *x*-axis, the co-ordinates of P′ are (3, – 4).

As O′ is the image of O(origin) in the line PP′, the co-ordinates of O′ are (6, 0).



(*ii*) Using graph, we find that length of segment PP′ = 8 units, length of segment OO′ = 6 units.

(*iii*) OM = 3 units, MP = 4 units.

From right angled ΔOMP, by Pythagoras theorem, we get

$OP^2 = OM^2 + MP^2$

$= 3^2 + 4^2 = 25$

$\Rightarrow$ OP = 5 units.

Similarly,

OP′ = O′P′ = O′P = 5 units.

∴ Perimeter of the quadrilateral POP′O′ = (5 + 5 + 5 + 5) units = 20 units.

(*iv*) POP′O′ is a rhombus.

Example 9. *Use graph paper for this question.*

A(0, 3), B(3, −2) and O(0, 0) are the vertices of triangle ABO.

(i) Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.

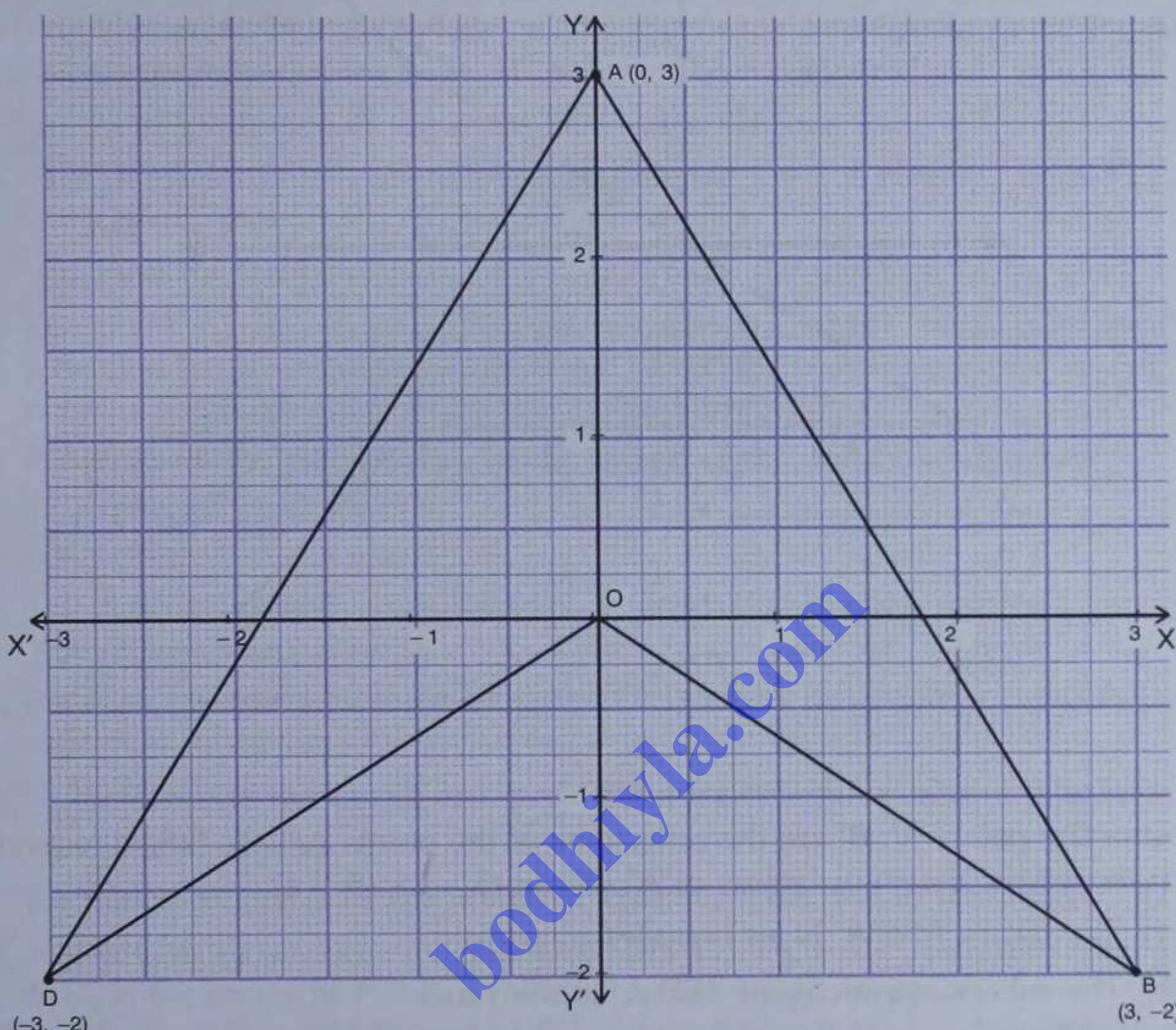
(ii) Plot D the reflection of B in the y-axis, and write it co-ordinates.

(iii) Give the geometrical name of the figure ABOD.

(iv) Write the equation of the line of symmetry of the figure ABOD. (2010)

Solution. Take 2 cm = 1 unit on both the axes

(*i*) Plot the point A(0, 3), B(3, −2) and O(0, 0) on the graph paper.



(*ii*) Plot the point D which is the reflection of the point B in the *y*-axis. The co-ordinates of D are (−3, −2).

(*iii*) ABOD is a kite (arrow).

(*iv*) The axis of symmetry of the kite ABOD is the *y*-axis, its equation is $x = 0$.

Example 10. *Attempt this question on graph paper.*

(i) Plot A(3, 2) and B(5, 4) on the graph paper.

(ii) Reflect A and B in the x-axis to A′, B′. Plot these on the same graph paper.

(iii) Write down

(a) the geometrical name of the figure ABB′A′.

(b) the axis of symmetry of ABB′A′.

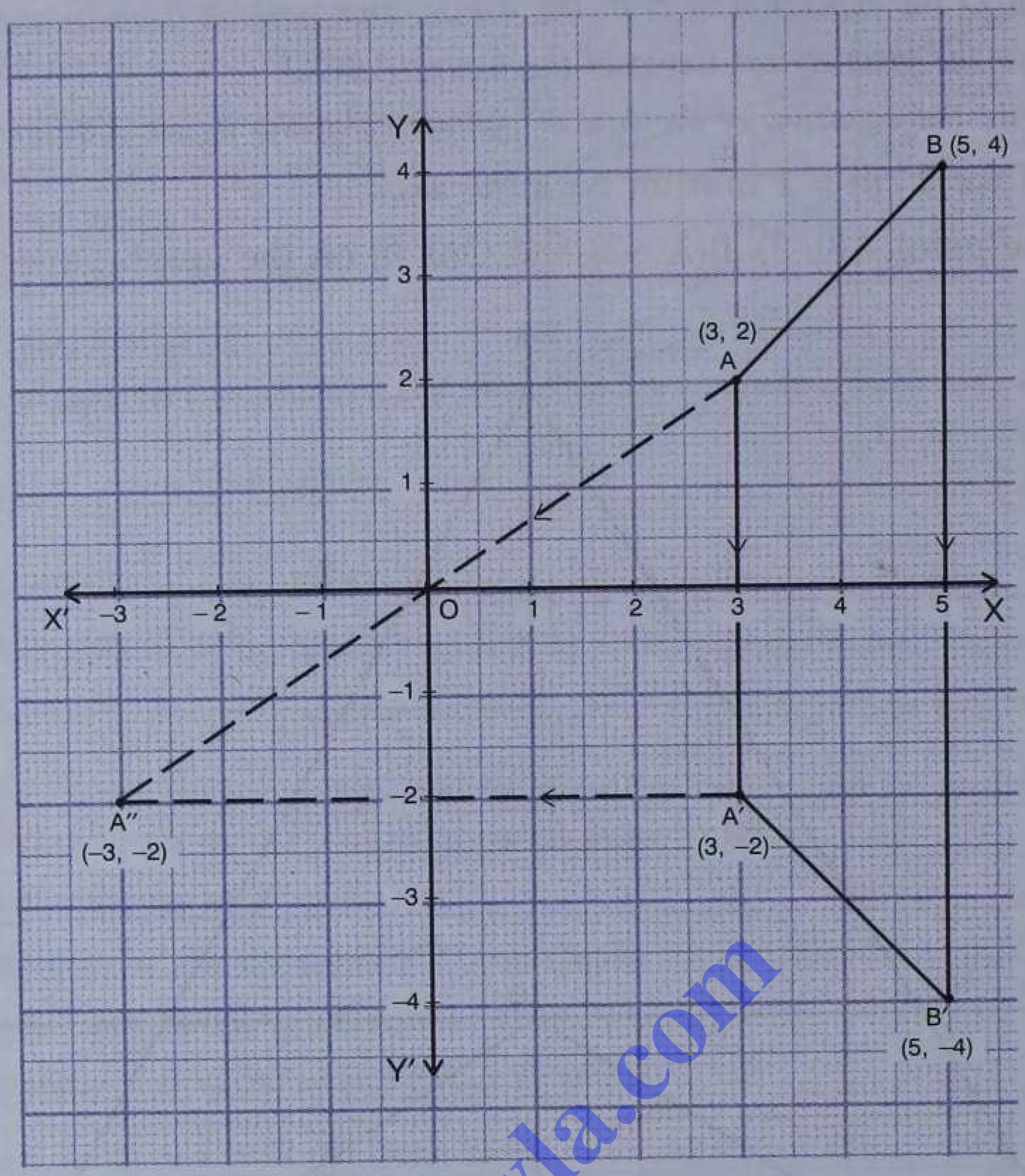
(c) the measure of the angle ABB′.

(d) the image A″ of A, when A is reflected in the origin.

(e) the single transformation that maps A′ to A″.

Solution. Take 1 cm = 1 unit on both axes.

(*i*) See the following diagram :



(*ii*) Since the points A′, B′ are the reflections of the points A(3, 2), B(5, 4) respectively in the x-axis, the co-ordinates of A′, B′ are (3, – 2), (5, – 4).

(*iii*) (*a*) The figure ABB′A′ is an isosceles trapezium.

(*b*) The axis of symmetry of ABB′A′ is the x-axis.

(*c*) ∠ABB′ = 45°.

(*d*) The co-ordinates of A″ are (–3, –2).

(*e*) The single transformation that maps A′ to A″ is the reflection in the y-axis.

Exercise 10

1. Find the co-ordinates of the images of the following points under reflection in the x-axis :

(*i*) (2, – 5) (*ii*) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (*iii*) (– 7, 0).

2. Find the co-ordinates of the images of the following points under reflection in the y-axis :

(*i*) (2, – 5) (*ii*) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ (*iii*) (0, – 7).

3. Find the co-ordinates of the images of the following points under reflection in the origin :

(*i*) (2, – 5) (*ii*) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (*iii*) (0, 0).

4. The image of a point P under reflection in the x-axis is $(5, -2)$. Write down the co-ordinates of P.

5. A point P is reflected in the x-axis. Co-ordinates of its image are $(8, -6)$.
   (*i*) Find the co-ordinates of P.
   (*ii*) Find the co-ordinates of the image of P under reflection in the y-axis.

6. A point P is reflected in the origin. Co-ordinates of its image are $(2, -5)$. Find
   (*i*) the co-ordinates of P.
   (*ii*) the co-ordinates of the image of P in the x-axis.

7. (*i*) The point P(2, 3) is reflected in the line $x = 4$ to the point P′. Find the co-ordinates of the point P′.
   (*ii*) Find the image of the point P(1, −2) in the line $x = -1$.

8. (*i*) The point P(2, 4) on reflection in the line $y = 1$ is mapped onto P′. Find the co-ordinates of P′.
   (*ii*) Find the image of the point P(−3, −5) in the line $y = -2$.

9. The point P (−4, −5) on reflection in y-axis is mapped on P′. The point P′ on reflection in the origin is mapped on P″. Find the co-ordinates of P′ and P″. Write down a single transformation that maps P onto P″.

10. Write down the co-ordinates of the image of the point $(3, -2)$ when :
    (*i*) reflected in the x-axis.
    (*ii*) reflected in the y-axis.
    (*iii*) reflected in the x-axis followed by reflection in the y-axis.
    (*iv*) reflected in the origin. *(2000)*

11. Find the co-ordinates of the image of (3, 1) under reflection in x-axis followed by reflection in the line $x = 1$.

12. If P′(−4, −3) is the image of a point P under reflection in the origin, find
    (*i*) the co-ordinates of P.
    (*ii*) the co-ordinates of the image of P under reflection in the line $y = -2$.

13. A point P(a, b) is reflected in the x-axis to P′(2, −3), write down the values of a and b. P″ is the image of P, when reflected in the y-axis. Write down the co-ordinates of P″. Find the co-ordinates of P‴, when P is reflected in the line, parallel to y-axis such that $x = 4$.

14. (*i*) Point P(a, b) is reflected in the x-axis to P′(5, −2). Write down the values of a and b.
    (*ii*) P″ is the image of P when reflected in the y-axis. Write down the co-ordinates of P″.
    (*iii*) Name a single transformation that maps P′ to P″.

15. Points A and B have co-ordinates (2, 5) and (0, 3). Find :
    (*i*) the image A′ of A under reflection in the x-axis.
    (*ii*) the image B′ of B under reflection in the line AA′.

16. Plot the points A (2, −3), B(−1, 2) and C (0, −2) on the graph paper. Draw the triangle formed by reflecting these points in the x-axis. Are the two triangles congruent ?

17. The points (6, 2), (3, −1) and (−2, 4) are the vertices of a right angled triangle. Check whether it remains a right angled triangle after reflection in the y-axis.

18. The triangle ABC where A (1, 2), B (4, 8), C (6, 8) is reflected in the x-axis to triangle A′ B′ C′. The triangle A′ B′ C′ is then reflected in the origin to triangle

A″B″C″. Write down the co-ordinates of A″, B″, C″. Write down a single transformation that maps ABC onto A″B″C″.

19. The image of a point P on reflection in a line l is point P′. Describe the location of the line l.

20. Given two points P and Q, and that (1) the image of P on reflection in y-axis is the point Q and (2) the mid point of PQ is invariant on reflection in x-axis. Locate (*i*) the x-axis (*ii*) the y-axis and (*iii*) the origin.

21. The point (−3, 0) on reflection in a line is mapped as (3, 0) and the point (2, −3) on reflection in the same line is mapped as (−2, −3).

(*i*) Name the mirror line.

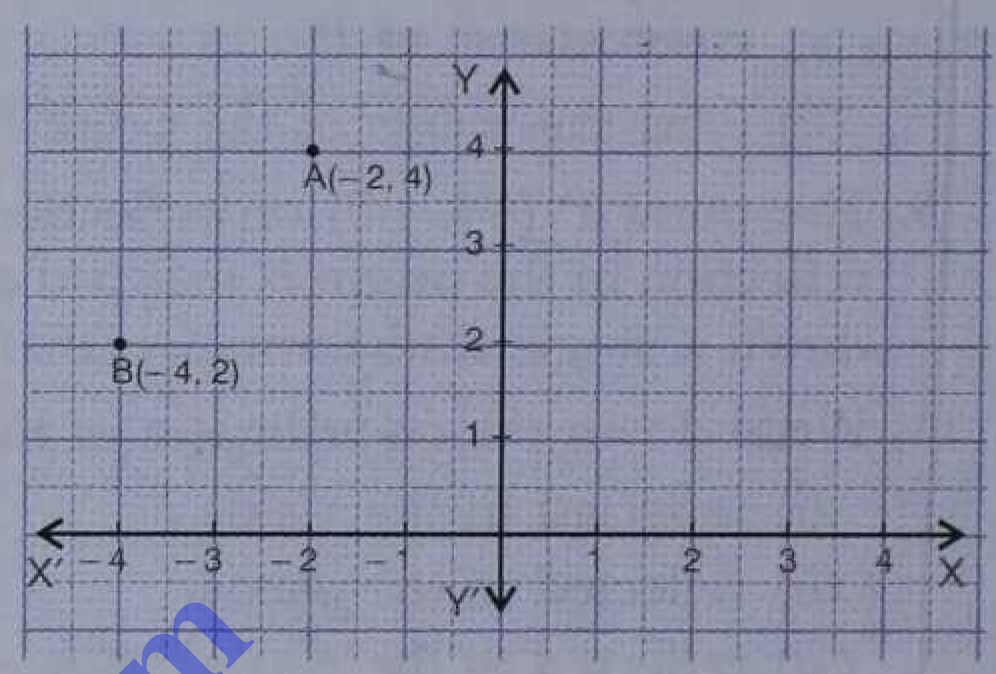
(*ii*) Write the co-ordinates of the image of (−3, −4) in the mirror line.

22. A (−2, 4) and B (−4, 2) are reflected in the y-axis. If A′ and B′ are images of A and B respectively, find

(*i*) the co-ordinates of A′ and B′.

(*ii*) Assign special name to quad. AA′B′B.

(*iii*) State whether AB′ = BA′.



23. Use graph paper for this question.

(*i*) The point P(2, −4) is reflected about the line $x = 0$ to get the image Q. Find the co-ordinates of Q.

(*ii*) Point Q is reflected about the line $y = 0$ to get the image R. Find the co-ordinates of R.

(*iii*) Name the figure PQR.

(*iv*) Find the area of the figure PQR. *(2007)*

24. Use graph paper for this question. The point P(5, 3) was reflected in the origin to get the image P′.

(*i*) Write down the co-ordinates of P′.

(*ii*) If M is the foot of perpendicular from P to the x-axis, find the co-ordinates of M.

(*iii*) If N is the foot of the perpendicular from P′ to the x-axis, find the co-ordinates of N.

(*iv*) Name the figure PMP′N.

(*v*) Find the area of the figure PMP′N. *(2001)*

25. Using a graph paper, plot the points A(6, 4) and B(0, 4).

(*i*) Reflect A and B in the origin to get images A′ and B′.

(*ii*) Write the co-ordinates of A′ and B′.

(*iii*) State the geometrical name for the figure ABA′B′.

(*iv*) Find its perimeter. *(2013)*

26. Use graph paper to answer this question.

(*i*) Plot the points A (4, 6) and B (1, 2).

(*ii*) If A′ is the image of A when reflected in the *x*-axis, write the co-ordinates of A′.

(*iii*) If B′ is the image of B when reflected in the line AA′, write the co-ordinates of B′.

(*iv*) Give the geometrical name for the figure ABA′B′. (*2009*)

27. The points A(2, 3), B(4, 5) and C(7, 2) are the vertices of Δ ABC.

(*i*) Write down the co-ordinates of A_1, B_1, C_1 if Δ $A_1B_1C_1$ is the image of ΔABC when reflected in the origin.

(*ii*) Write down the co-ordinates of A_2, B_2, C_2 if Δ $A_2B_2C_2$ is the image of ΔABC when reflected in the *x*-axis.

(*iii*) Assign the special name to the quadrilateral BCC_2B_2 and find its area. (*2006*)

28. The point P(3, 4) is reflected to P′ in the *x*-axis and O′ is the image of O (origin) in the line PP′. Find :

(*i*) the co-ordinates of P′ and O′.

(*ii*) the length of segments PP′ and OO′.

(*iii*) the perimeter of the quadrilateral POP′O′.

29. Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axes).

P and Q have co-ordinates (0, 5) and (–2, 4).

(*i*) P is invariant when reflected in an axis. Name the axis.

(*ii*) Find the image of Q on reflection in the axis found in (*i*).

(*iii*) (0, *k*) on reflection in the origin is invariant. Write the value of *k*.

(*iv*) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin followed by reflection in *x*-axis. (*2005*)

CHAPTER TEST

1. The point P(4, –7) on reflection in x-axis is mapped onto P′. Then P′ on reflection in the y-axis is mapped onto P″. Find the co-ordinates of P′ and P″. Write down a single transformation that maps P onto P″.
2. The point P(a, b) is first reflected in the origin and then reflected in the y-axis to P′. If P′ has co-ordinates (3, –4), evaluate a, b.
3. A point P (a, b) becomes (–2, c) after reflection in the x-axis, and P becomes (d, 5) after reflection in the origin. Find the values of a, b, c and d.
4. A (4, –1), B(0, 7) and C (–2, 5) are the vertices of a triangle. ΔABC is reflected in the y-axis and then reflected in the origin. Find the co-ordinates of the final images of the vertices.
5. The points A (4, –11), B(5, 3), C (2, 15) and D (1, 1) are the vertices of a parallelogram. If the parallelogram is reflected in the y-axis and then in the origin, find the co-ordinates of the final images. Check whether it remains a parallelogram. Write down a single transformation that brings the above change.
6. The triangle OAB is reflected in the origin O to triangle OA′B′. A′ and B′ have co-ordinates (–3, –4) and (0, –5) respectively.
   (*i*) Find the co-ordinates of A and B.
   (*ii*) Draw a diagram to represent the given information.
   (*iii*) What kind of figure is the quadrilateral ABA′B′?
   (*iv*) Find the co-ordinates of A″, the reflection of A in the origin followed by reflection in the y-axis.
   (*v*) Find the co-ordinates of B″, the reflection of B in the x-axis followed by reflection in the origin.