UNIT – 2
ARITHMETIC PROBLEMS

CHAPTER 8
RATIO
(Including Proportions)

8.1 INTRODUCTION

Most of the time, we compare things, numbers, etc. (say, ***x*** and ***y***) by saying :

(i) ***x*** is *greater* than ***y*** (ii) ***x*** is *less* than ***y***

(iii) ***x*** is *double* of ***y*** (iv) ***x*** is *one-third* of ***y***

(v) $\frac{x}{y} = \frac{4}{5}$ (vi) $\frac{y}{x} = \frac{3}{2}$, etc.

The method of comparing two quantities (numbers, things, etc.) *by dividing one quantity by the other, is called* **ratio.**

Thus : $\frac{x}{y} = \frac{4}{5}$ represents the ratio of ***x*** to ***y***.

and, $\frac{y}{x} = \frac{3}{2}$ represents the ratio of ***y*** to ***x***.

8.2 RATIO

The relation of two quantities (both of the same kind and in the same unit) *obtained on dividing one quantity by the other is called their* ***ratio***.

∴ The **ratio** of two quantities ***x*** **to** ***y***, both of the same kind and in the same unit, is $\frac{x}{y}$, and is often written as ***x*** **:** ***y*** (read as ***x*** **to** ***y*** or ***x*** **is to** ***y***).

Meaning of the two quantities of the same kind and in the same unit :

1. **Both the quantities must of the same kind, means :** If one quantity is length, the other quantity must also be length; if quantity represents mass the other quantity must also be representing mass and so on.

   The ratio between unlike quantities has no meaning.

   For example, the ratio of length to mass has no meaning.

2. **Both the quantities must be in the same unit, means :** The two quantities must have the same unit of measurement.

   For example, if the lengths of two objects are given to be 60 cm and 1·5 m; then before finding the ratio of one length to that of other, both of these lengths must either be converted into cm or into m.

Examples :

(i) The ratio of 5 kg to 15 kg $= \frac{5\text{ kg}}{15\text{ kg}} = \frac{1}{3} =$ **1 : 3**

(ii) The ratio of 800 gm to 1.2 kg

$= \frac{800\text{ gm}}{1200\text{ gm}} = \frac{2}{3} =$ **2 : 3**

1.2 kg = 1.2 × 1000 gm = 1200 gm

(iii) The ratio of 2 m to 80 cm

$$= \frac{2\,\text{m}}{80\,\text{cm}} = \frac{200\,\text{cm}}{80\,\text{cm}} = \frac{5}{2} = \mathbf{5 : 2}$$

2 m = 2 × 100 cm = 200 cm

(iv) The ratio of $1\frac{1}{2}$ years to 10 months

$$= \frac{18\,\text{months}}{10\,\text{months}} = \frac{9}{5} = \mathbf{9 : 5}$$

$1\frac{1}{2}$ years = $\frac{3}{2}$ × 12 months = 18 months

A ratio is a pure number and so has no unit.

1. *The ratio of two numbers or quantities is denoted by the colon mark " : ".*
   Thus, the ratio of two quantities p and q = **p : q**
2. *The ratio of two quantities of same kind and in the same unit is obtained on dividing one quantity by the other.*

   Thus, *the ratio of* 20 kg *to* 80 kg $= \frac{20\,\text{kg}}{80\,\text{kg}} = \frac{1}{4} =$ **1 : 4**
3. *The* ***first term*** *of a ratio is called the* ***antecedent*** *and the* ***second term*** *is called the* ***consequent.***

   In the ratio 1 : 4, *antecedent* = 1 *and consequent* = 4.
4. *A ratio must always by expressed in its lowest terms.*
5. *Whatever be the units of the terms of a ratio, the ratio has no unit. The ratio of 15 km and 20 km* $= \frac{15\,\text{km}}{20\,\text{km}} = \frac{3}{4} = 3{:}4$. *Here, the two quantities 15 km and 20 km have unit km, but their ratio 3:4 has no unit. On dividing, units cancel out.*
6. *The terms of a ratio are written in a definite order :*

   The ratio of 5 kg and 8 kg $= \frac{5\,\text{kg}}{8\,\text{kg}} = \frac{5}{8} = 5 : 8$ *and*

   the ratio of 8 kg and 5 kg $= \frac{8\,\text{kg}}{5\,\text{kg}} = \frac{8}{5} = 8 : 5$

   Remember : *5 : 8 and 8 : 5 are not equal to each other.*

Example 1 :

Find the ratio of : (i) 60 to 48 (ii) 3.75 kg to 750 gm

Solution :

(i) **Required ratio** $= \frac{60}{48} = \frac{5}{4}$

$= \mathbf{5 : 4}$ **(Ans.)**

(ii) Since 3.75 kg = 3.75 × 1000 gm = 3750 gm

Required ratio $= \frac{3.75\ \text{kg}}{750\ \text{gm}}$

$= \frac{3750\ \text{kg}}{750\ \text{gm}} = \frac{5}{1} = \mathbf{5 : 1}$ **(Ans.)**

8.3 CONVERTING INTO SIMPLE RATIO

Example 2 :

Express as simple ratio : (i) $3\frac{1}{2} : 2\frac{1}{3}$ (ii) $\frac{2}{3} : \frac{4}{5} : \frac{1}{2}$.

Solution :

(i) Divide the first term of the ratio by its second term and then simplify.

Given ratio $= 3\frac{1}{2} : 2\frac{1}{3} = \frac{7}{2} : \frac{7}{3}$

$= \frac{7}{2} \times \frac{3}{7} = \frac{3}{2} =$ **3 : 2** **(Ans.)**

Alternative method :

Multiply each terms of the ratio by the L.C.M. of their denominators and then simplify.

$\therefore$ **Given ratio** $= \frac{7}{2} : \frac{7}{3} = \frac{7}{2} \times 6 : \frac{7}{3} \times 6$ L.C.M. of 2 and 3 = 6

$= 21 : 14 = \frac{21}{14} = \frac{3}{2} =$ **3 : 2** **(Ans.)**

(ii) **Given ratio** $= \frac{2}{3} : \frac{4}{5} : \frac{1}{2} = \frac{2}{3} \times 30 : \frac{4}{5} \times 30 : \frac{1}{2} \times 30$ L.C.M. of 3,5 and 2 = 30

$=$ **20 : 24 : 15** **(Ans.)**

Example 3 :

The strength of a class is 50 with 30 boys and the remaining girls. Find the ratio of the number of boys to the number of girls in the class.

Solution :

Since the strength of the class = 50

and the number of boys in the class = 30

$\Rightarrow$ The number of girls in the class = 50 – 30 = 20

$\therefore$ **Required ratio** $= \frac{\text{No. of boys in the class}}{\text{No. of girls in the class}}$

$= \frac{30}{20} = \frac{3}{2} =$ **3 : 2** **(Ans.)**

Example 4 :

A man's monthly income is ₹ 15,000, out of which he spends ₹ 12,500 every month. Find the ratio of his :

(i) savings to expenditure (ii) expenditure to income

(iii) income to savings

Solution :

Since the monthly income of the man = ₹ 15,000

And his monthly expenditure = ₹ 12,500

$\Rightarrow$ His savings per month = ₹ 15,000 – ₹ 12,500 = ₹ 2,500

(i) **Ratio of savings to expenditure** $= \frac{₹\,2,500}{₹\,12,500} = \frac{1}{5} =$ **1 : 5** **(Ans.)**

(ii) **Ratio of expenditure to income** $= \frac{₹\,12,500}{₹\,15,000} = \frac{5}{6} =$ **5 : 6** **(Ans.)**

(iii) **Ratio of income to savings** $= \frac{₹\,15,000}{₹\,2,500} = \frac{6}{1} =$ **6 : 1** **(Ans.)**

EXERCISE 8(A)

1. Express each of the following ratios in its simplest form :

(a) (i) 4 : 6 (ii) 48 : 54 (iii) 200 : 250

(b) (i) 5 kg : 800 gm (ii) 30 cm : 2 m (iii) 3 m : 90 cm

(iv) 2 years : 9 months (v) 1 hour : 45 min (vi) 4 min : 45 sec

(c) (i) $1\frac{1}{2} : 2\frac{1}{2}$ (ii) $3\frac{1}{2} : 7$ (iii) $2\frac{1}{3} : 3\frac{1}{2} : 1\frac{1}{4}$

(iv) $x^2 : 4x$ (v) 2.5 : 1.5 (vi) 2.5 : 5

2. A field is 80 m long and 60 m wide. Find the ratio of its width to its length.

3. The monthly salary of a person is ₹ 12,000 and his monthly expenditure is ₹ 8,500. Find the ratio of his :

(i) salary to expenditure (ii) expenditure to savings

(iii) savings to salary

4. The strength of a class is 65, including 30 girls. Find the ratio of the number of :

(i) girls to boys (ii) boys to the whole class

(iii) the whole class to girls.

5. The weekly expenses of a boy have increased from ₹ 150 to ₹ 225. Find the ratio of :

(i) increase in expenses to original expenses.

(ii) original expenses to increased expenses.

(iii) increased expenses to increase in expenses.

6. Reduce each of the following ratios to their lowest terms :

(i) 1 hour 20 min : 2 hours (ii) 4 weeks : 49 days

(iii) 3 years 4 months : 5 years 5 months (iv) 2 m 40 cm : 1 m 44 cm

(v) 5 kg 500 gm : 2 kg 750 gm

8.4 TO DIVIDE A GIVEN QUANTITY AS A GIVEN RATIO

Example 5 :

12 sweets are to be divided between A and B in the ratio 1 : 3. Find how many sweets each gets ?

Solution :

Here, A and B get sweets in the ratio 1 : 3.

This means, if all the sweets are divided in 1 + 3 = 4 equal parts,

Then, **A gets** = One part out of the 4 equal parts made

$= \frac{1}{4}$ of the total number of sweets

$= \frac{1}{4} \times 12$ sweets = **3 sweets** **(Ans.)**

And **B gets** = 3 parts out of the 4 equal parts made

$= \frac{3}{4}$ of the total number of sweets

$= \frac{3}{4} \times 12$ sweets = **9 sweets** **(Ans.)**

Thus, if a whole quantity is divided into two parts in the ratio 3 : 4m

$\therefore$ The first part $= \frac{3}{7} \times$ the whole quantity [As, 3 + 4 = 7]

and the second part $= \frac{4}{7} \times$ the whole quantity

Example 6 :

A pole of length 165 cm is divided into two parts such that their lengths are in the ratio 7 : 8. Find the length of each part of the pole.

Solution :

Here, 165 cm is to be divided into two lengths in the ratio 7 : 8 and 7 + 8 = 15.

$\therefore$ **Length of one** (shorter) **part** $= \frac{7}{15} \times 165$ cm = **77 cm**

and **length of the other** (longer) **part** $= \frac{8}{15} \times 165$ cm = **88 cm** **(Ans.)**

Example 7 :

Divide 99 into three parts in the ratio 2 : 4 : 5.

Solution :

Since 2 + 4 + 5 = 11

$\therefore$ **1st part** $= \frac{2}{11} \times 99 =$ **18**; **2nd part** $= \frac{4}{11} \times 99 =$ **36**

and **3rd part** $= \frac{5}{11} \times 99 =$ **45** **(Ans.)**

Example 8 :

Divide 268 into two parts in the ratio $2\frac{1}{3} : 3\frac{1}{4}$.

Solution :

Given ratio $= 2\frac{1}{3} : 3\frac{1}{4} = \frac{7}{3} : \frac{13}{4} = \frac{7}{3} \times \frac{4}{13} = \frac{28}{39} = 28 : 39$

Since 28 + 39 = 67

$\therefore$ **1st part** $= \frac{28}{67} \times 268 = 28 \times 4 =$ **112** **(Ans.)**

And **2nd part** $= \frac{39}{67} \times 268 = 39 \times 4 =$ **156** **(Ans.)**

Example 9 :

The total weight of the mixture of two things *A* and *B* is 50 kg. If *A* and *B* are mixed in the ratio 3 : 7, find the quantity of B in the mixture.

Solution :

Since *A* and *B* are mixed in the ratio 3 : 7 and 3 + 7 = 10,

$\therefore$ **Quantity of B in the mixture** $= \frac{7}{10} \times 50 \text{ kg} =$ **35 kg** **(Ans.)**

Example 10 :

420 articles are divided among *A*, *B* and *C*, such that *A* gets three times that of *B* and *B* gets five times that of *C*. Find the number of articles received by *B*.

Solution :

Let the number of articles *C* gets = 1

$\Rightarrow$ The number of articles that B gets = five times that of C = 5 × 1 = 5

and the number of articles that A gets = three times that of B = 3 × 5 = 15

$\therefore$ A : B : C = 15 : 5 : 1 and 15 + 5 + 1 = 21

$\Rightarrow$ **The no. of articles received by B** $= \frac{5}{21} \times 420 =$ **100** **(Ans.)**

EXERCISE 8(B)

1. ₹ 120 is to be divided between Hari and Gopi in the ratio 5 : 3. How much does each get ?
2. Divide 72 in the ratio $2\frac{1}{2} : 1\frac{1}{2}$.
3. Divide 81 into three parts in the ratio 2 : 3 : 4.
4. Divide ₹ 10,400 among *A*, *B* and *C* in the ratio 1/2 : 1/3 : 1/4.
5. A profit of ₹ 2,500 is to be shared among three persons in the ratio 6 : 9 : 10. How much does each person get ?
6. The angles of a triangle are in the ratio 3 : 7 : 8. Find the greatest and the smallest angles.

> The sum of the angles of a triangle is 180°.

7. The sides of a triangle are in the ratio 3 : 2 : 4. If the perimeter of the triangle is 27 cm, find the length of each side.
8. An alloy of zinc and copper weighs $12\frac{1}{2}$ kg. If, in the alloy, the ratio of zinc and copper is 1 : 4, find the weight of copper in it.
9. How will ₹ 31,500 be shared between *A*, *B* and *C*, if *A* gets the double of what *B* gets, and *B* gets the double of what *C* gets ?
10. Mr. Gupta divides ₹ 81,000 among his three children, Ashok, Mohit and Geeta, in such a way that Ashok gets four times what Mohit gets and Mohit gets 2·5 times what Geeta gets. Find the share of each of them.

8.5 PROPORTION

Consider the following examples :

1. What is the ratio of the number of boys to the number of girls in a group of 8 boys and 12 girls ?

The required ratio $= \frac{\text{Number of boys}}{\text{Number of girls}} = \frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \frac{2}{3}$

2. What is the ratio of ₹ 18 to ₹ 27 ?

The required ratio $= \frac{18}{27} = \frac{2 \times 9}{3 \times 9} = \frac{2}{3}$

It is observed in the examples given above that the ratios $\frac{8}{12}$ and $\frac{18}{27}$ are equal.

i.e. $\frac{8}{12} = \frac{18}{27}$ or **8 : 12 = 18 : 27**

Such an equality of two ratios is called a **proportion** and read as :

"8 is to 12 as 18 is to 27".

Thus, a proportion is an expression which states that the two given ratios are equal.

The numbers 8, 12, 18 and 27 that are used in the proportion are called its **terms,** *i.e.* **8** is the **first term, 12** is the **second term, 18** is the **third term** and **27** is the **fourth term** of the proportion **8 : 12 = 18 : 27.**

In general, the symbol for representing a proportion is " : : ".

Example 11 :

Check whether or not the two ratios form a proportion.

(i) ₹ 6 : ₹ 8 and 12 kg : 16 kg (ii) 6 kg : 9 kg and 10 m : 16 m

Solution :

(i) Since ₹ 6 : ₹ 8 $= \frac{6}{8} = \frac{3}{4}$ and 12 kg : 16 kg $= \frac{12}{16} = \frac{3}{4}$

∴ **Ratios ₹6 : ₹8 and 12 kg : 16 kg** are **equal**, they **form a proportion.**

(Ans.)

(ii) Since, 6 kg : 9 kg $= \frac{6}{9} = \frac{2}{3}$ and 10 m : 16 m $= \frac{10}{16} = \frac{5}{8}$

∴ **Ratio 6 kg : 9 kg ≠ ratio 10 m : 16 m, they do not form a proportion.**

(Ans.)

1. *In a proportion, the* **first two terms** *(quantities) must be of the same kind and of the same unit, whereas the last* **two terms** *(quantities) must also be of the same kind and of the same unit.*

   [All the* four *quantities in a proportion may be of the same kind and the same unit.]

2. *In a proportion, the* **first** *and the* **fourth terms** *are called* **extremes** *whereas the* **second** *and the* **third terms** *are called* **means.**

   Thus, in 8 : 12 = 18 : 27; the terms **12** *and* **18** *are the* **means** *and* **8** *and* **27** *are the* **extremes.**

   Also **Product of extremes = Product of means**

Example 12 :

Check whether or not the given ratios form a proportion :

(i) 15 : 24 and 35 : 56 (ii) $2\frac{1}{4} : 5\frac{2}{5}$ and $3\frac{1}{3} : 4\frac{1}{6}$.

Solution :

(i) Product of extremes = $15 \times 56 = 840$

and product of means = $24 \times 35 = 840$

Since, product of extremes = product of means

$\Rightarrow$ **The given two ratios form a proportion** **(Ans.)**

(ii) $\because$ Product of extremes $= 2\frac{1}{4} \times 4\frac{1}{6} = \frac{9}{4} \times \frac{25}{6} = \frac{75}{8}$

and product of means $= 5\frac{2}{5} \times 3\frac{1}{3} = \frac{27}{5} \times \frac{10}{3} = \frac{18}{1}$

and since product of extremes $\neq$ product of means

$\Rightarrow$ **The given two ratios do not form a proportion** **(Ans.)**

Example 13 :

(i) The numbers 8, x, 9 and 36 are in proportion. Find x.

(ii) If x : 15 = 8 : 12, find x.

Solution :

(i) The numbers 8, x, 9 and 36 are in proportion

$\Rightarrow$ $8 : x = 9 : 36$

$\Rightarrow$ $x \times 9 = 8 \times 36$

Product of means = Product of extremes

$\Rightarrow$ $x = \frac{8 \times 36}{9} = 32$ **(Ans.)**

(ii) $x : 15 = 8 : 12 \Rightarrow x \times 12 = 15 \times 8$

$\Rightarrow$ $x = \frac{15 \times 8}{12} = 10$ **(Ans.)**

Example 14 :

The first, third and fourth terms of a proportion are 12, 8 and 14, respectively. Find the second term.

Solution :

Let the second term be x.

$\therefore$ 12, x, 8 and 14 are in proportion, *i.e.* $12 : x = 8 : 14$

$\Rightarrow$ $x \times 8 = 12 \times 14$

$\Rightarrow$ $x = \frac{12 \times 14}{8} = 21$

$\therefore$ **The second term** of the proportion **is 21**. **(Ans.)**

Example 15 :

The ratio of the length and the width of a sheet of paper is 3 : 2. If the length is 12 cm, find the width.

Solution :

Let width = x cm

The ratio of length to width = 12 : x [Given length = 12 cm]

According to the given statement, 12 : x = 3 : 2

$\Rightarrow$ $x \times 3 = 12 \times 2$

$\Rightarrow$ $x = \frac{12 \times 2}{3} = 8$

$\therefore$ **Width = 8 cm** **(Ans.)**

EXERCISE 8(C)

1. In each of the following, check whether or not the given ratios form a proportion :
   (i) 8 : 16 and 12 : 15 (ii) 16 : 28 and 24 : 42 (iii) 12 ÷ 3 and 8 ÷ 2
   (iv) 25 : 40 and 20 : 32 (v) $\frac{15}{18}$ and $\frac{10}{12}$ (vi) $\frac{7}{8}$ and 14 : 16

2. Find the value of x in each of the following proportions :
   (i) x : 4 = 6 : 8 (ii) 14 : x = 7 : 9 (iii) 4 : 6 = x : 18
   (iv) 8 : 10 = x : 25 (v) 5 : 15 = 4 : x (vi) 16 : 24 = 6 : x

3. Find the value of x so that the given four numbers are in proportion :
   (i) x, 6, 10 and 15 (ii) x, 4, 15 and 30 (iii) 2, x, 10 and 25
   (iv) 4, x, 6 and 18 (v) 9, 12, x and 8 (vi) 4, 10, 36 and x.

4. The first, second and the fourth terms of a proportion are 6, 18 and 75, respectively. Find its third term.

5. Find the second term of the proportion whose first, third and fourth terms are 9, 8 and 24, respectively.

6. Find the fourth term of the proportion whose first, second and third terms are 18, 27 and 32, respectively.

7. The ratio of the length and the width of a school ground is 5 : 2. Find the length, if the width is 40 metres.

8. The ratio of the sale of eggs on a Sunday and that of the whole week at a grocery shop was 2 : 9. If the total value of the sale of eggs in the same week was ₹ 360, find the value of the sale of eggs that Sunday.

9. The ratio of copper and zinc in an alloy is 9 : 8. If the weight of zinc in the alloy is 9·6 kg, find the weight of copper in the alloy.

10. The ratio of the number of girls to the number of boys in a school is 2 : 5. If the number of boys is 225, find :
    (i) the number of girls in the school.
    (ii) the number of students in the school.

Revision Exercise (Chapter 8)

1. A school has 1625 students, out of which 750 are girls. Find the ratio of the number of boys to the number of girls in the school.

2. Rohit earns ₹ 1,040 in 8 days and John earns ₹ 960 in 6 days. Find :
   (i) one day's earnings of Rohit (ii) one day's earnings of John.
   (iii) the ratio of one day's earnings of Rohit and John.

3. A sample of an alloy consists of $8\frac{3}{4}$ gm copper and $1\frac{1}{2}$ gm silver. Find the ratio of the weights of :
   (i) alloy to copper (ii) copper to silver (iii) silver to alloy.

4. Divide ₹ 1,450 between Richa and Sonu in the ratio 12 : 17.

5. Are the following numbers in proportion ?
   (i) 32, 40, 48 and 60 (ii) 12, 15, 18 and 20

6. Find the value of x in each of the following such that the given numbers are in proportion.
   (i) 14, 42, x and 75 (ii) 45, 135, 90 and x

7. The costs of two articles are in the ratio 7 : 4. If the cost of the first article is ₹ 2,800, find the cost of the second article.

   If the cost of the second article is ₹ x, $\Rightarrow$ 7 : 4 = 2800 : x

8. The ratio of the length and the width of a rectangular sheet of paper is 8 : 5. If the width of the sheet is 17·5 cm, find the length.

   Let length = x cm $\Rightarrow$ 8 : 5 = x : 17·5

9. The ages of A and B are in the ratio 6 : 5. If A's age is 18 years, find the age of B.

10. A sum of ₹ 10,500 is divided among A, B and C in the ratio 5 : 6 : 4. Find the share of each.