1 Compound Interest

INTEREST

It is the additional money besides the original money paid by the borrower to the moneylender (bank, financial agency or individual) in lieu of the money used by him.

Principal. *The money borrowed (or the money lent) is called* ***principal.***

Amount. *The sum of the principal and the interest is called* ***amount.***

Thus, **amount = principal + interest.**

Rate. *It is the interest paid on* ₹ *100 for a specified period.*

Time. *It is the time for which the money is borrowed.*

Simple Interest. *It is the interest calculated on the original money (principal) for any given time and rate.*

Formula : Simple Interest $= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

1.1 COMPOUND INTEREST

At the end of the first year (or any other fixed period), if the interest accrued is not paid to the moneylender but is added to the principal, then this amount becomes the principal for the next year (or any other fixed period) and so on. This process is repeated until the amount for the whole time is found.

The difference between the final amount and the (original) principal is called ***compound interest.***

Remark

In the case of simple interest, the principal remains constant for the whole time but in the case of compound interest, the principal keeps on changing every year (or any other fixed period).

If the interest is *compounded annually,* the principal changes after every year and if the interest is *compounded half-yearly* (or *any other fixed period*), the principal changes after every six months (or any other fixed period).

Baron Rothschild

"I don't know what the seven wonders of the world are, but I know the eighth—**Compound Interest**".

ILLUSTRATIVE EXAMPLES

Example 1. *Find the amount and compound interest on ₹14000 for 2 years at 5%.*

Solution. Principal for the first year = ₹14000.

$$\text{Interest for the first year} = ₹\frac{14000 \times 5 \times 1}{100} = ₹700.$$

Amount after the first year = ₹14000 + ₹700 = ₹14700.

Principal for the 2nd year = ₹14700.

$$\text{Interest for the 2nd year} = ₹\frac{14700 \times 5 \times 1}{100} = ₹735.$$

Amount after 2 years = ₹14700 + ₹735 = ₹15435.

Compound interest for 2 years = final amount – (original) principal

= ₹15435 – ₹14000 = ₹1435.

Note

The compound interest may also be obtained by adding together the interest of consecutive years.

Thus, in the above example,

compound interest = interest of first year + interest of 2nd year

= ₹700 + ₹735 = ₹1435.

Example 2. *Find the compound interest to the nearest rupee on ₹1200 for 2 years 4 months at 10% per annum reckoned annually.*

Solution. Principal for the first year = ₹1200.

$$\text{Interest for the first year} = ₹\frac{1200 \times 10 \times 1}{100} = ₹120.$$

Amount after the first year = ₹1200 + ₹120 = ₹1320.

Principal for the 2nd year = ₹1320.

$$\text{Interest for the 2nd year} = ₹\frac{1320 \times 10 \times 1}{100} = ₹132.$$

Amount after 2 years = ₹1320 + ₹132 = ₹1452.

Principal for the third year = ₹1452.

$$\text{Remaining time} = 4 \text{ months} = \frac{4}{12} \text{ year} = \frac{1}{3} \text{ year}.$$

$$\text{Interest for the next } \frac{1}{3} \text{ year} = ₹\frac{1452 \times 10 \times \frac{1}{3}}{100} = ₹48{\cdot}40.$$

$$\text{Amount after } 2\frac{1}{3} \text{ years} = ₹1452 + ₹48{\cdot}40 = ₹1500{\cdot}40.$$

$\therefore$ Compound interest for $2\frac{1}{3}$ years = amount – (original) principal

= ₹1500·40 – ₹1200 = ₹300·40

= ₹300 (*to the nearest rupee*)

Example 3. *Find the amount and the compound interest on ₹8000 for $1\frac{1}{2}$ years at 10% per annum, the interest being compounded half-yearly.*

Solution. Since the rate of interest is 10% per annum, therefore, the rate of interest half-yearly = 5%.

Principal for the first half-year = ₹ 8000.

Interest for the first half-year = ₹ $\frac{8000 \times 5 \times 1}{100}$ = ₹ 400.

∴ Amount after the first half-year = ₹ 8000 + ₹ 400 = ₹ 8400.

Principal for the 2nd half-year = ₹ 8400.

Interest for the 2nd half-year = ₹ $\frac{8400 \times 5 \times 1}{100}$ = ₹ 420.

∴ Amount after one year = ₹ 8400 + ₹ 420 = ₹ 8820.

Principal for the 3rd half-year = ₹ 8820.

Interest for the 3rd half-year = ₹ $\frac{8820 \times 5 \times 1}{100}$ = ₹ 441.

∴ Amount after $1\frac{1}{2}$ years = ₹ 8820 + ₹ 441 = ₹ 9261.

Compound interest for $1\frac{1}{2}$ years = final amount – (original) principal

= ₹ 9261 – ₹ 8000 = ₹ 1261.

Example 4. *Nikita invests ₹ 6000 for two years at a certain rate of interest compounded annually. At the end of first year it amounts to ₹ 6720. Calculate :*

(i) the rate of interest.

(ii) the amount at the end of the second year. *(2010)*

Solution. Given, principal = ₹ 6000, amount after one year = ₹ 6720.

(*i*) Interest for the first year = ₹ 6720 – ₹ 6000 = ₹ 720.

Let the rate of interest be R% p.a., then

S.I. = $\frac{P \times R \times T}{100}$ ⇒ 720 = $\frac{6000 \times R \times 1}{100}$ ⇒ R = 12.

Hence, the rate of interest = 12% p.a.

(*ii*) Principal for the second year = ₹ 6720.

Interest for the second year = ₹ $\frac{6720 \times 12 \times 1}{100}$ = ₹ 806·40

∴ The amount at the end of second year = ₹ 6720 + ₹ 806·40

= ₹ 7526·40.

Example 5. *Calculate the amount due and the compound interest on ₹ 7500 in 2 years when the rate of interest on successive years is 8% and 10% respectively.*

Solution. Principal for the first year = ₹ 7500, rate = 8%.

Interest for the first year = ₹ $\frac{7500 \times 8 \times 1}{100}$ = ₹ 600.

Amount after the first year = ₹ 7500 + ₹ 600 = ₹ 8100.

Principal for the second year = ₹ 8100, rate = 10%.

Interest for the second year = ₹ $\frac{8100 \times 10 \times 1}{100}$ = ₹ 810.

∴ Amount due after 2 years = ₹ 8100 + ₹ 810 = ₹ 8910.

Compound interest for 2 years = amount – principal

= ₹ 8910 – ₹ 7500 = ₹ 1410.

Example 6. *Calculate the difference between the compound interest and the simple interest on ₹ 12000 at 9% per annum in 2 years.*

Solution. Given principal = ₹ 12000, rate = 9% p.a. and time = 2 years

∴ S.I. = ₹ $\frac{12000 \times 9 \times 2}{100}$ = ₹ 2160.

For C.I.

Principal for the first year = ₹12000.

Interest for the first year = $₹\frac{12000 \times 9 \times 1}{100}$ = ₹1080.

Amount after one year = ₹12000 + ₹1080 = ₹13080.

Principal for the second year = ₹13080.

Interest for the 2nd year = $₹\frac{13080 \times 9 \times 1}{100}$

= ₹1177·20

∴ C.I. of 2 years = ₹1080 + ₹1177·20 = ₹2257·20.

∴ Difference between compound interest and simple interest in 2 years

= ₹2257·20 – ₹2160

= ₹97·20.

Example 7. *The simple interest on a sum of money for 2 years at 4% per annum is ₹340. Find*

(i) the sum of money

(ii) the compound interest on this sum for one year payable half-yearly at the same rate.

(2008)

Solution. Given, S.I. = ₹340, rate = 4% p.a. and time = 2 years

(*i*) Let the sum of money be P, then

$$\text{S.I.} = \frac{P \times R \times T}{100} \Rightarrow ₹340 = \frac{P \times 4 \times 2}{100}$$

$$\Rightarrow \quad P = ₹\frac{340 \times 100}{4 \times 2} = ₹4250.$$

(*ii*) Since the rate of interest is 4% per annum, therefore, the rate of interest half-yearly = 2%.

Principal for the first half-year = ₹4250.

Interest for the first half-year = $₹\frac{4250 \times 2 \times 1}{100}$ = ₹85.

∴ Amount after the first half-year = ₹4250 + ₹85 = ₹4335.

Principal for the 2nd half-year = ₹4335.

Interest for the 2nd half-year = $₹\frac{4335 \times 2 \times 1}{100}$ = ₹86·70

∴ Compound interest on the above sum for one year payable half-yearly

= ₹85 + ₹86·70 = ₹171·70.

Example 8. *The simple interest on a certain sum of money for 3 years at 5% per annum is ₹1200. Find the amount due and the compound interest on this sum of money at the same rate after 3 years, interest is reckoned annually.*

Solution. Given simple interest for 3 years = ₹1200.

∴ Simple interest for one year = $\frac{1}{3}$ of ₹1200 = ₹400

$$\text{S.I.} = \frac{P \times R \times T}{100} \Rightarrow ₹400 = \frac{P \times 5 \times 1}{100}$$

$$\Rightarrow \quad P = ₹\frac{400 \times 100}{5 \times 1} = ₹8000.$$

∴ Amount after one year = ₹8000 + ₹400 = ₹8400.

Principal for the second year = ₹8400.

Interest for the second year = ₹ $\frac{8400 \times 5 \times 1}{100}$ = ₹ 420.

Amount after 2 years = ₹ 8400 + ₹ 420 = ₹ 8820.

Interest for the third year = ₹ $\frac{8820 \times 5 \times 1}{100}$ = ₹ 441.

∴ Amount due after 3 years = ₹ 8820 + ₹ 441 = ₹ 9261.

Compound interest for 3 years = ₹ 9261 – ₹ 8000 = ₹ 1261.

Example 9. *A man borrows ₹ 10000 at 12% per annum compound interest. If he repays ₹ 3200 at the end of the first year and ₹ 3960 at the end of the second year, find the amount of the loan outstanding at the beginning of the third year.*

Solution. Principal for the first year = ₹ 10000, rate = 12%.

Interest for the first year = ₹ $\frac{10000 \times 12 \times 1}{100}$ = ₹ 1200.

Amount after the first year = ₹ 10000 + ₹ 1200 = ₹ 11200.

Money refunded at the end of first year = ₹ 3200.

∴ Principal for the second year = ₹ 11200 – ₹ 3200 = ₹ 8000.

Interest for the second year = ₹ $\frac{8000 \times 12 \times 1}{100}$ = ₹ 960.

Amount after the second year = ₹ 8000 + ₹ 960 = ₹ 8960.

Money refunded at the end of 2nd year = ₹ 3960.

∴ The loan outstanding at the beginning of the third year

= ₹ 8960 – ₹ 3960 = ₹ 5000.

Example 10. *Mr. Kumar borrowed ₹ 15000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays ₹ 6200 at the end of first year, find the outstanding amount at the end of the second year.* (2011)

Solution. Principal for the first year = ₹ 15000, rate = 8% p.a.

Interest for the first year = ₹ $\frac{15000 \times 8 \times 1}{100}$ = ₹ 1200.

Amount after one year = ₹ 15000 + ₹ 1200 = ₹ 16200.

Money repaid at the end of first year = ₹ 6200.

∴ Principal for the second year = ₹ 16200 – ₹ 6200 = ₹ 10000;

rate of interest for second year = 10% p.a.

Interest for the second year = ₹ $\frac{10000 \times 10 \times 1}{100}$ = ₹ 1000.

Amount after second year = ₹ 10000 + ₹ 1000 = ₹ 11000.

∴ The amount outstanding at the end of second year = ₹ 11000.

Example 11. *Sulekha deposits ₹ 8000 in a bank every year in the beginning of the year, at 10% per annum compound interest. Calculate the amount due to her at the end of three years. Also find her gain in three years.*

Solution. Principal for the first year = ₹ 8000, rate = 10% p.a.

Interest for the first year = ₹ $\frac{8000 \times 10 \times 1}{100}$ = ₹ 800.

Amount after one year = ₹ 8000 + ₹ 800 = ₹ 8800.

Money deposited at the beginning of 2nd year = ₹ 8000.

Principal for the 2nd year = ₹8800 + ₹8000 = ₹16800.

Interest for the 2nd year = ₹ $\frac{16800 \times 10 \times 1}{100}$ = ₹1680.

Amount after 2 years = ₹16800 + ₹1680 = ₹18480.

Money deposited at the beginning of 3rd year = ₹8000.

Principal for the 3rd year = ₹18480 + ₹8000 = ₹26480.

Interest for the 3rd year = ₹ $\frac{26480 \times 10 \times 1}{100}$ = ₹2648.

Amount after 3 years = ₹26480 + ₹2648 = ₹29128.

∴ The amount due to Sulekha at the end of 3 years = ₹29128.

Money deposited by Sulekha in 3 years = ₹8000 + ₹8000 + ₹8000
= ₹24000.

∴ Gain of Sulekha in 3 years = ₹29128 – ₹24000
= ₹5128.

Example 12. *A man borrows ₹5000 at 12% compound interest per annum, interest payable after six months. He pays back ₹1800 at the end of every six months. Calculate the third payment he has to make at the end of 18 months in order to clear the entire loan.* (2001)

Solution. Since the rate of interst is 12% per annum, therefore, rate of interest half-yearly = 6%.

Principal for the first six months = ₹5000.

Interest for the first six months = ₹ $\frac{5000 \times 6 \times 1}{100}$ = ₹300.

Amount after first six months = ₹5000 + ₹300 = ₹5300.

Money refunded at the end of first six months = ₹1800.

∴ Principal for the second six months = ₹5300 – ₹1800
= ₹3500.

Interest for the second six months = ₹ $\frac{3500 \times 6 \times 1}{100}$ = ₹210.

Amount after second six months = ₹3500 + ₹210
= ₹3710.

Money refunded at the end of second six months = ₹1800.

∴ Principal for the third six months = ₹3710 – ₹1800
= ₹1910.

Interest for the third six months = ₹ $\frac{1910 \times 6 \times 1}{100}$ = ₹114·60.

∴ The payment to be made at the end of 18 months to clear the entire loan
= ₹1910 + ₹114·60
= ₹2024·60.

Example 13. *Asha invests ₹240000 for 2 years at 10% per annum compounded annually. If the income tax at 20% is deducted at the end of each year on interest accrued, find the amount she received at the end of 2 years.*

Solution. Principal for the first year = ₹240000, rate = 10%.

Interest for the first year = ₹ $\frac{240000 \times 10 \times 1}{100}$ = ₹24000.

Income tax on ₹24000 = ₹ $\left(\frac{20}{100} \times 24000\right)$ = ₹4800.

∴ Net amount after the first year = ₹(240000 + 24000 – 4800)
= ₹259200.

$\therefore$ Principal for the second year = ₹ 259200.

Interest for the second year = ₹ $\frac{259200 \times 10 \times 1}{100}$ = ₹ 25920.

Income tax on ₹ 25920 = ₹ $\left(\frac{20}{100} \times 25920\right)$ = ₹ 5184.

$\therefore$ Net amount after 2 years = ₹ (259200 + 25920 – 5184)

= ₹ 279936.

Exercise 1.1

1. Find the amount and the compound interest on ₹ 8000 at 5% per annum for 2 years.

2. A person invests ₹ 5600 at 14% p.a. compound interest for 2 years. Calculate :

(*i*) the interest for the first year.

(*ii*) the amount at the end of the first year.

(*iii*) the interest for the 2nd year, correct to the nearest Re.

3. A man invests ₹ 46875 at 4% per annum compound interest for 3 years. Calculate :

(*i*) the interest for the first year.

(*ii*) the amount standing to his credit at the end of the second year.

(*iii*) the interest for the third year.

4. Calculate the compound interest for the second year on ₹ 8000 invested for 3 years at 10% p.a.

Also find the sum due at the end of third year.

5. Ramesh invests ₹ 12800 for three years at the rate of 10% per annum compound interest. Find :

(*i*) the sum due to Ramesh at the end of the first year.

(*ii*) the interest he earns for the second year.

(*iii*) the total amount due to him at the end of three years. (2007)

6. The simple interest on a sum of money for 2 years at 12% per annum is ₹ 1380. Find

(*i*) the sum of money.

(*ii*) the compound interest on this sum for one year payable half-yearly at the same rate.

7. A person invests ₹ 10000 for two years at a certain rate of interest, compounded annually. At the end of one year this sum amounts to ₹ 11200. Calculate :

(*i*) the rate of interest per annum.

(*ii*) the amount at the end of second year. (2006)

8. A man invests ₹ 5000 for three years at a certain rate of interest, compounded annually. At the end of one year it amounts to ₹ 5600. Calculate :

(*i*) the rate of interest per annum.

(*ii*) the interest accrued in the second year.

(*iii*) the amount at the end of the third year.

9. Find the amount and the compound interest on ₹ 2000 at 10% p.a. for $2\frac{1}{2}$ years.

10. Find the amount and the compound interest on ₹ 50000 for $1\frac{1}{2}$ years at 8% per annum, the interest being compounded semi-annually.

11. Calculate the amount and the compound interest on ₹ 5000 in 2 years when the rate of interest for successive years is 6% and 8% respectively.

12. Calculate the amount and the compound interest on ₹ 17000 in 3 years when the rate of interest for successive years is, 10%, 10% and 14% respectively.

13. A sum of ₹ 9600 is invested for 3 years at 10% per annum at compound interest.

(*i*) What is the sum due at the end of the first year ?

(*ii*) What is the sum due at the end of the second year ?

(*iii*) Find the compound interest earned in 2 years.

(*iv*) Find the difference between the answers in (*ii*) and (*i*) and find the interest on this sum for one year.

(*v*) Hence, write down the compound interest for the third year.

14. The simple interest on a certain sum of money for 2 years at 10% per annum is ₹ 1600. Find the amount due and the compound interest on this sum of money at the same rate after 3 years, interest being reckoned annually.

15. A man invests ₹ 4000 for three years at compound interest. After one year the money amounts to ₹ 4320. Find the amount (to the nearest rupee) due at the end of 3 years.

16. A man borrows ₹ 6000 at 5% compound interest. If he repays ₹ 1200 at the end of each year, find the amount outstanding at the beginning of the third year.

17. Mr. Dubey borrows ₹ 100000 from State Bank of India at 11% per annum compound interest. He repays ₹ 41000 at the end of first year and ₹ 47700 at the end of second year. Find the amount outstanding at the beginning of the third year. *(2009)*

18. Vikram borrowed ₹ 20000 from a bank at 10% per annum simple interest. He lent it to his friend Venkat at the same rate but compounded annually. Find his gain after $2\frac{1}{2}$ years.

19. Sachin invests ₹ 200000 for 2 years at 12% per annum compounded annually. If the interest accrued is subject to income tax at 25% at the end of each year, find the amount he received at the end of 2 years.

1.2 FORMULA FOR COMPOUND INTEREST

Compound interest (*abbreviated* **C.I.**) *can be easily calculated by the following formula :*

Formula : $\mathbf{A = P\left(1+\frac{r}{100}\right)^n}$

where A is the final amount, P is the principal, r is the rate of interest compounded yearly and n is the number of years.

$$\text{C.I.} = \text{A} - \text{P} = \text{P}\left(1+\frac{r}{100}\right)^n - \text{P} = \text{P}\left[\left(1+\frac{r}{100}\right)^n - 1\right].$$

Remark

If the interest is calculated for any other fixed period (like 6 months), then the principal keeps on changing every term of the fixed period (like 6 months).

The time from one specified interest period to the next period is called a ***conversion period.*** If this specified period is one year (*i.e.* the interest is compounded annually), then there is *one conversion period* in a year; if this period is six months (*i.e.* the interest is compounded semi-annually), then there are *two conversion periods* in a year ; if this period is three months (*i.e.* the interest is compounded quarterly), then

there are *four conversion periods* in a year. In view of this discussion, we can restate the formula as :

Formula : $$A = P\left(1+\frac{r}{100}\right)^n$$

where A is the final amount, P is the principal, r is the rate of interest per conversion period and n is the number of conversion periods.

Note

Obviously, if the rate of interest per annum is 10% and if the interest is compounded semi-annually, then the rate of interest per conversion period is $\frac{1}{2}$ of 10% *i.e.* 5%. If the interest is compounded quarterly, then the rate of interest per conversion period is $\frac{1}{4}$ of 10% *i.e.* 2·5%.

1.2.1 In solving problems on compound interest, remember the following :

1. $A = P\left(1+\frac{r}{100}\right)^n$ and C.I. $= P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

   where A is the amount, P is the principal, r is the rate of interest per conversion period and n is the number of conversions periods.

2. When the rates of interest for the successive fixed periods are $r_1\%$, $r_2\%$, $r_3\%$, ..., then amount A is given by

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\ldots.$$

3. S.I. (simple interest) and C.I. are equal for the first conversion period on the same sum and at the same rate.

4. C.I. of 2nd conversion period is more than the C.I. of 1st conversion period and C.I. of 2nd conversion period – C.I. of 1st conversion period = S.I. on the interest of the 1st conversion period.

5. C.I. for the nth conversion period = amount after n conversion periods – amount after $(n-1)$ conversion periods.

6. When the total time is not a complete number of conversion periods, we consider simple interest for the last partial period. For example, if time is 2 years 5 months and the interest is r % per annum compounded annually, then

$$A = P\left(1+\frac{r}{100}\right)^2 \times \left(1+\frac{\frac{5}{12}r}{100}\right).$$

ILLUSTRATIVE EXAMPLES

Example 1. *Find the amount and compound interest on ₹ 16000 for 2 years at 15%, interest being payable annually.*

Solution. Using the formula, we get

$$A = ₹16000\left(1+\frac{15}{100}\right)^2 \qquad \left| A = P\left(1+\frac{r}{100}\right)^n \right.$$

$$= ₹16000 \times \left(\frac{115}{100}\right)^2 = ₹16000 \times \left(\frac{23}{20}\right)^2$$

$$= ₹16000 \times \frac{23}{20} \times \frac{23}{20} = ₹21160.$$

$$\text{C.I.} = \text{A} - \text{P} = ₹21160 - ₹16000 = ₹5160.$$

Example 2. *Rohit borrows ₹ 86000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of two years.* (2010)

Solution. Principal = ₹ 8600, rate = 5% and time = 2 years.

Simple interest paid by Rohit in 2 years :

$$\text{S.I.} = \frac{\text{P} \times \text{R} \times \text{T}}{100} = ₹ \frac{86000 \times 5 \times 2}{100} = ₹8600.$$

Compound interest earned by Rohit in 2 years

If A is the amount after 2 years, then

$$\text{A} = \text{P}\left(1+\frac{\text{R}}{100}\right)^n = ₹86000\left(1+\frac{5}{100}\right)^2 = ₹86000 \times \left(\frac{21}{20}\right)^2$$

$$= ₹86000 \times \frac{441}{400} = ₹94815.$$

∴ C.I. earned by Rohit in 2 years

= amount after 2 years – principal

= ₹ 94815 – ₹ 86000 = ₹ 8815

∴ Rohit's profit at the end of 2 years

= C.I. earned – S.I. paid

= ₹ 8815 – ₹ 8600 = ₹ 215.

Example 3. *Calculate the interest earned and the amount due, if a sum of ₹ 15000 is invested for $1\frac{1}{2}$ years at 8% per annum compound interest, interest being compounded semi-annually.*

Solution. Since rate of interest is 8% per annum, therefore, rate of interest per conversion period (half-yearly) = 4%.

As the money is invested for $1\frac{1}{2}$ years, therefore,

n (the number of conversion periods) = 3.

$$\therefore \quad \text{A} = \text{P}\left(1+\frac{r}{100}\right)^n = ₹15000\left(1+\frac{4}{100}\right)^3$$

$$= ₹15000 \times \left(\frac{26}{25}\right)^3 = ₹15000 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25}$$

$$= ₹ \frac{421824}{25} = ₹16872{\cdot}96.$$

Interest earned = A – P = ₹ 16872·96 – ₹ 15000 = ₹ 1872·96.

Example 4. *How much will ₹ 25000 amount to in 2 years, at compound interest, if the rates for the successive years are 4% and 5% per year ?*

Solution. $\text{A} = \text{P}\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$

$$= ₹25000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right) = ₹25000 \times \frac{26}{25} \times \frac{21}{20} = ₹27300.$$

Example 5. *Compute the amount and the compound interest on ₹10000 compounded annually for $2\frac{1}{2}$ years at 4% per annum.*

Solution. First, we find the amount after 2 years

$$A = P\left(1+\frac{r}{100}\right)^n = ₹10000\left(1+\frac{4}{100}\right)^2$$

$$= ₹10000 \times \left(\frac{26}{25}\right)^2 = ₹10000 \times \frac{26}{25} \times \frac{26}{25} = ₹10816.$$

Principal for the third year = ₹10816.

Interest for the next $\frac{1}{2}$ year = $₹\frac{10816 \times 4 \times \frac{1}{2}}{100}$

= ₹216·32.

Amount after $2\frac{1}{2}$ years = ₹10816 + ₹216·32

= ₹11032·32.

∴ Compound interest for $2\frac{1}{2}$ years = A – P

= ₹11032·32 – ₹10000

= ₹1032·32.

Alternative method (by using formula)

Amount at the end of $2\frac{1}{2}$ years = $₹10000\left(1+\frac{4}{100}\right)^2\left(1+\frac{\frac{6}{12}\times 4}{100}\right)$

= $₹\left(10000 \times \frac{26}{25} \times \frac{26}{25} \times \frac{51}{50}\right)$ = ₹11032·32.

∴ Compound interest for $2\frac{1}{2}$ years = ₹11032·32 – ₹10000

= ₹1032·32.

Example 6. *Rohan borrowed ₹40000 at 10% p.a. simple interest. He immediately invested this money at 10% p.a., the interest compounded half-yearly. Calculate Rohan's gain in 18 months.*

Solution. Principal = ₹40000, rate = 10% p.a., time = 18 months = $\frac{18}{12}$ years = $\frac{3}{2}$ years.

Simple interest paid by Rohan in 18 months :

$$S.I. = \frac{P \times R \times T}{100} = ₹\frac{40000 \times 10 \times \frac{3}{2}}{100} = ₹6000.$$

Compound interest earned by Rohan in 18 months :

Since the rate of interest is 10% p.a., therefore, the rate of interest per conversion period (half-yearly) = 5%.

As the money is invested for 18 months, therefore, n(the number of conversion periods) = 3.

$$A = P\left(1+\frac{r}{100}\right)^n = ₹40000\left(1+\frac{5}{100}\right)^3$$

$$= ₹40000 \times \left(\frac{21}{20}\right)^3 = ₹40000 \times \frac{9261}{8000} = ₹46305.$$

$\therefore$ C.I. earned by Rohan in 18 months

= amount after 18 months – principal

= ₹46305 – ₹40000 = ₹6305.

$\therefore$ Rohan's gain in 18 months = C.I. earned – S.I. paid

= ₹6305 – ₹6000 = ₹305.

Example 7. *What sum of money will amount to ₹3630 in two years at 10% per annum compound interest ?* (2003)

Solution. $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow ₹3630 = P\left(1+\frac{10}{100}\right)^2$

$\Rightarrow \quad ₹3630 = P \times \frac{11}{10} \times \frac{11}{10}$

$\Rightarrow \quad P = ₹\left(3630 \times \frac{10}{11} \times \frac{10}{11}\right) = ₹3000.$

Example 8. *What sum of money will amount to ₹3704·40 in 3 years at 5% compound interest ?*

Solution. $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow ₹3704{\cdot}40 = P\left(1+\frac{5}{100}\right)^3$

$\Rightarrow \quad ₹\frac{370440}{100} = P \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

$\Rightarrow \quad P = ₹\frac{37044}{10} \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21} = ₹3200.$

Example 9. *On a certain sum, the compound interest for 2 years is ₹2172. If the rates of interest for successive years are 6% and 8% per year, then find the sum.*

Solution. $A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$

$= P\left(1+\frac{6}{100}\right)\left(1+\frac{8}{100}\right) = P \times \frac{53}{50} \times \frac{27}{25}$

$= \frac{1431}{1250}P.$

$\therefore \quad C.I. = A - P = \frac{1431}{1250}P - P = \frac{181}{1250}P.$

Given, C.I. = ₹2172

$\Rightarrow \quad \frac{181}{1250}P = ₹2172 \Rightarrow P = ₹2172 \times \frac{1250}{181} = ₹15000.$

Hence, the sum = ₹15000.

Example 10. *The compound interest on a certain sum of money at 5% per annum for two years is ₹246. Calculate the simple interest on the same sum for three years at 6% per annum.* (2004)

Solution. $C.I. = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

$\Rightarrow \quad ₹246 = P\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$

$$\Rightarrow \quad ₹246 = P\left[\frac{21}{20}\times\frac{21}{20}-1\right] = P\times\frac{41}{400}$$

$$\Rightarrow \quad P = ₹\left(246\times\frac{400}{41}\right) = ₹2400.$$

$\therefore$ Simple interest for the same sum for 3 years at 6% per annum

$$= ₹\frac{2400\times6\times3}{100} = ₹432.$$

Example 11. *At what rate percent per annum compound interest will ₹5000 amount to ₹ 5832 in 2 years ?*

Solution. $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow ₹5832 = ₹5000\left(1+\frac{r}{100}\right)^2$

$$\Rightarrow \quad \left(1+\frac{r}{100}\right)^2 = \frac{5832}{5000} = \frac{729}{625} = \left(\frac{27}{25}\right)^2$$

$$\Rightarrow \quad 1+\frac{r}{100} = \frac{27}{25} \Rightarrow \frac{r}{100} = \frac{27}{25}-1 = \frac{2}{25}$$

$$\Rightarrow \quad r = \frac{2}{25}\times100 = 8.$$

Hence, rate = 8% per annum.

Example 12. *At what rate % p.a. will a sum of ₹4000 yield ₹1324 as compound interest in 3 years?* *(2013)*

Solution. Given P = ₹4000, C.I. = ₹1324 and T = 3 years.

A = P + C.I. = ₹4000 + ₹1324 = ₹5324.

Let the rate of interest be r% p.a.

Here, time = 3 years, so $n = 3$.

Using the formula :

$$A = P\left(1+\frac{r}{100}\right)^n, \text{ we get}$$

$$₹5324 = ₹4000\left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \quad \left(1+\frac{r}{100}\right)^3 = \frac{5324}{4000} = \frac{1331}{1000}$$

$$\Rightarrow \quad \left(1+\frac{r}{100}\right)^3 = \left(\frac{11}{10}\right)^3 \Rightarrow 1+\frac{r}{100} = \frac{11}{10}$$

$$\Rightarrow \quad \frac{r}{100} = \frac{11}{10}-1 \Rightarrow \frac{r}{100} = \frac{1}{10} \Rightarrow r = 10.$$

Hence, the rate of interest = 10% p.a.

Example 13. *A sum of ₹16000 earns a compound interest of ₹2522 in 18 months, when the interest is compounded half-yearly. Find the rate of interest.*

Solution. Given P = ₹16000, C.I. = ₹2522 and T = 18 months.

Let the rate of interest per conversion period (half-year) be r%.

As the interest is compounded half-yearly *i.e.* after 6 months and the time is 18 months,

so n (number of conversion periods) $= \frac{18}{6} = 3.$

$$A = P + C.I. = ₹16000 + ₹2522 = ₹18522.$$

$$A = P\left(1+\frac{r}{100}\right)^n \Rightarrow 18522 = 16000\left(1+\frac{r}{100}\right)^3$$

$$\Rightarrow \quad \left(1+\frac{r}{100}\right)^3 = \frac{18522}{16000} = \frac{9261}{8000}$$

$$\Rightarrow \quad \left(1+\frac{r}{100}\right)^3 = \left(\frac{21}{20}\right)^3 \Rightarrow 1 + \frac{r}{100} = \frac{21}{20}$$

$$\Rightarrow \quad \frac{r}{100} = \frac{21}{20} - 1 \Rightarrow \frac{r}{100} = \frac{1}{20} \Rightarrow r = 5.$$

Thus, rate of interest per conversion period (6 months) = 5%.

$\therefore$ Rate of interest per annum = 10%.

Example 14. *Determine the rate of interest per annum for a sum that becomes* $\frac{729}{625}$ *of itself in one year, compounded half-yearly.*

Solution. Let the principal be P and the rate of interest be r % per conversion period (half-year).

Here, interest is compounded half-yearly.

Total time is one year, so n(the number of conversion periods) = 2.

According to given, $A = \frac{729}{625} P$

$$\Rightarrow \quad \frac{729}{625} P = P\left(1+\frac{r}{100}\right)^2 \qquad \left| A = P\left(1+\frac{r}{100}\right)^n \right.$$

$$\Rightarrow \quad \left(1+\frac{r}{100}\right)^2 = \frac{729}{625} = \left(\frac{27}{25}\right)^2$$

$$\Rightarrow \quad 1 + \frac{r}{100} = \frac{27}{25} \Rightarrow \frac{r}{100} = \frac{27}{25} - 1 = \frac{2}{25}$$

$$\Rightarrow \quad r = 8$$

i.e. rate of interest per conversion period (half-year) = 8%.

$\therefore$ Rate of interest per annum = 16%.

Example 15. *The compound interest on a sum of money for 2 years is ₹ 410 and the simple interest on the same sum for the same period and at the same rate is ₹ 400. Find the sum and the rate of interest.*

Solution. S.I. for 2 years = ₹ 400

$\therefore$ S.I. for 1 year = $\frac{₹\ 400}{2}$ = ₹ 200

$\therefore$ C.I. for first year = ₹ 200 ($\because$ for first year, C.I. = S.I.)

Given, C.I. for 2 years = ₹ 410

$\therefore$ C.I. for the second year = ₹ 410 – ₹ 200 = ₹ 210.

$\therefore$ Difference of interests = ₹ 210 – ₹ 200 = ₹ 10

$\Rightarrow$ ₹ 10 is the interest on ₹ 200 for one year.

$$\therefore \quad \text{Rate of interest} = \frac{10 \times 100}{200 \times 1}\% = 5\% \qquad \left| R = \frac{\text{S.I.} \times 100}{P \times T} \right.$$

$$\text{Principal} = ₹\ \frac{200 \times 100}{5 \times 1} = ₹\ 4000. \qquad \left| P = \frac{\text{S.I.} \times 100}{R \times T} \right.$$

Example 16. *A certain sum amounts to ₹ 4840 in 2 years and to ₹ 5324 in 3 years at compound interest. Find the rate and the sum.*

Solution. Let the rate of compound interest be r % per annum.

The amount after 2 years will be the principal for the third year,

$$₹\ 5324 = ₹\ 4840\left(1+\frac{r}{100}\right) \qquad \left| A = P\left(1+\frac{r}{100}\right) \right.$$

$$\Rightarrow \quad 1 + \frac{r}{100} = \frac{5324}{4840}$$

$$\Rightarrow \quad \frac{r}{100} = \frac{5324}{4840} - 1 = \frac{5324 - 4840}{4840} = \frac{484}{4840} = \frac{1}{10}$$

$$\Rightarrow \quad r = \frac{1}{10} \times 100 = 10.$$

$\therefore$ The rate of compound interest = 10% per annum.

Let the original sum be P.

$$₹4840 = P\left(1+\frac{10}{100}\right)^2 \qquad \left| A = P\left(1+\frac{r}{100}\right)^n \right.$$

$$\Rightarrow \quad ₹4840 = P \times \frac{11}{10} \times \frac{11}{10}$$

$$\Rightarrow \quad P = ₹4840 \times \frac{10}{11} \times \frac{10}{11} = ₹4000.$$

$\therefore$ The original sum is ₹4000.

Example 17. *The compound interest, calculated yearly, on a certain sum of money for the second year is ₹880 and for the third year it is ₹968. Calculate the rate of interest and the original money.*

Solution. C.I. for the third year = ₹968 and

C.I. for the second year = ₹880.

$\therefore$ S.I. on Rs 880 for one year = ₹968 − ₹880 = ₹88.

$$\therefore \quad \text{Rate of interest} = \frac{88 \times 100}{880 \times 1}\% = 10\% \qquad \left| R = \frac{\text{S.I.} \times 100}{P \times T} \right.$$

Let the original money be ₹P.

Amount after 2 years − amount after one year = C.I. for second year

$$\Rightarrow \quad P\left(1+\frac{10}{100}\right)^2 - P\left(1+\frac{10}{100}\right) = 880$$

$$\Rightarrow \quad P\left[\left(\frac{11}{10}\right)^2 - \frac{11}{10}\right] = 880 \Rightarrow P\left(\frac{121}{100} - \frac{11}{10}\right) = 880$$

$$\Rightarrow \quad P \times \frac{11}{100} = 880 \qquad \Rightarrow \quad P = 8000.$$

$\therefore$ The rate of interest = 10%, and original money = ₹8000.

Example 18. *In how many years will ₹4000 amount to ₹5324 at 10% compound interest ?*

Solution. $A = P\left(1+\frac{r}{100}\right)^n \Rightarrow ₹5324 = ₹4000\left(1+\frac{10}{100}\right)^n$

$$\Rightarrow \quad \frac{5324}{4000} = \left(\frac{11}{10}\right)^n \qquad \Rightarrow \left(\frac{11}{10}\right)^n = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3$$

$\Rightarrow \quad n = 3.$

$\therefore$ The required time = 3 years.

Example 19. *In what period of time will ₹12000 yield ₹3972 as compound interest at 10% per annum, if compounded on yearly basis.* *(2011)*

Solution. P = ₹12000, C.I. = ₹3972, rate = 10% p.a.

A = P + C.I. = ₹12000 + ₹3972 = ₹15972.

Using $A = P\left(1+\frac{r}{100}\right)^n$, we get

$$₹15972 = ₹12000 \times \left(1+\frac{10}{100}\right)^n$$

$$\Rightarrow \quad \frac{15972}{12000} = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \quad \left(\frac{11}{10}\right)^n = \frac{1331}{1000} \Rightarrow \left(\frac{11}{10}\right)^n = \left(\frac{11}{10}\right)^3$$

$$\Rightarrow \quad n = 3.$$

$\therefore$ The required time = 3 years.

Example 20. *A sum of ₹25000 invested at 8% p.a. compounded semi-annually amounts to ₹28121·60. Compute the time period of investment.*

Solution. Here, P = ₹25000, A = ₹28121·60.

As the interest is compounded semi-annually, r (rate of interest per conversion period)

$$= \frac{1}{2} \text{ of } 8\% = 4\%.$$

Let the number of conversion periods be n, then

$$₹28121·60 = ₹25000\left(1+\frac{4}{100}\right)^n \qquad \left| A = P\left(1+\frac{r}{100}\right)^n \right.$$

$$\Rightarrow \quad \frac{2812160}{100} = 25000\left(\frac{26}{25}\right)^n$$

$$\Rightarrow \quad \left(\frac{26}{25}\right)^n = \frac{2812160}{100\times 25000} = \frac{17576}{15625} = \left(\frac{26}{25}\right)^3$$

$$\Rightarrow \quad n = 3.$$

$\therefore$ Time period of investment is 3 half-years *i.e.* $1\frac{1}{2}$ years.

Example 21. *A sum of money invested at compound interest doubles itself in 4 years, interest being payable annually. In how much time will it be eight times?*

Solution. Let the principal be ₹P and rate of interest be r% p.a.

As the money doubles itself in 4 years, we have

$$\Rightarrow \quad 2P = P\left(1+\frac{r}{100}\right)^4 \Rightarrow \left(1+\frac{r}{100}\right)^4 = \frac{2P}{P} = 2$$

$$\Rightarrow \quad 1 + \frac{r}{100} = 2^{1/4} \qquad \ldots(i)$$

Let the money become eight times in n years, then

$$8P = P\left(1+\frac{r}{100}\right)^n \Rightarrow \left(1+\frac{r}{100}\right)^n = \frac{8P}{P} = 8$$

$$\Rightarrow \quad (2^{1/4})^n = 2^3 \qquad \text{(Using } (i)\text{)}$$

$$\Rightarrow \quad 2^{n/4} = 2^3 \Rightarrow \frac{n}{4} = 3 \Rightarrow n = 12.$$

Hence, the required time = 12 years.

Example 22. *The simple interest on a sum of money for 2 years at 4% is ₹450. Find the compound interest on this sum of money at the same rate for 1 year if the interest is reckoned half-yearly.*

Solution. $\text{S.I.} = \dfrac{\text{P} \times \text{R} \times \text{T}}{100} \Rightarrow ₹450 = \dfrac{\text{P} \times 4 \times 2}{100}$

$\Rightarrow \qquad \text{P} = ₹ \dfrac{450 \times 100}{4 \times 2} = ₹5625.$

Rate of interest per conversion period (half-year) = 2% and n(the number of conversion periods) = 2.

$$\text{C.I.} = \text{P}\left[\left(1+\frac{r}{100}\right)^n - 1\right] = ₹5625\left[\left(1+\frac{2}{100}\right)^2 - 1\right]$$

$$= ₹5625\left[\frac{51}{50} \times \frac{51}{50} - 1\right] = ₹5625 \times \left(\frac{2601}{2500} - 1\right)$$

$$= ₹\left(5625 \times \frac{101}{2500}\right) = ₹\frac{225 \times 101}{100} = ₹227{\cdot}25.$$

Example 23. *The simple interest in 3 years and the compound interest in 2 years on a certain sum at the same rate are ₹1200 and ₹832 respectively. Find :*

(i) the rate of interest *(ii) the principal*

(iii) the difference between the C.I. and S.I. for 3 years.

Solution. (*i*) Let the principal be ₹P and rate of interest be R% p.a.

According to the first condition of the question,

$$\frac{\text{P} \times \text{R} \times 3}{100} = 1200 \Rightarrow \text{P} \times \text{R} = 40000 \qquad \ldots(1)$$

According to the second condition of the question,

$$\text{P}\left[\left(1+\frac{\text{R}}{100}\right)^2 - 1\right] = 832$$

$\Rightarrow \qquad \dfrac{40000}{\text{R}}\left[\dfrac{(100+\text{R})^2}{(100)^2} - 1\right] = 832$ (Using (1))

$\Rightarrow \qquad 40000 \times \dfrac{(100+\text{R})^2 - (100)^2}{100 \times 100} = 832\ \text{R}$

$\Rightarrow \quad 4[(100)^2 + \text{R}^2 + 2 \times 100 \times \text{R} - (100)^2] = 832\ \text{R}$

$\Rightarrow \qquad \text{R}^2 + 200\ \text{R} = 208\ \text{R} \Rightarrow \text{R}^2 + 200\ \text{R} - 208\ \text{R} = 0$

$\Rightarrow \qquad \text{R}^2 - 8\text{R} = 0 \Rightarrow \text{R}(\text{R} - 8) = 0$

$\Rightarrow \qquad$ either R = 0 or R − 8 = 0

$\Rightarrow \qquad$ either R = 0 or R = 8, but R cannot be zero.

$\therefore \qquad \text{R} = 8 \Rightarrow$ rate of interest = 8% p.a.

(*ii*) On using (1), we get

$$\text{P} \times 8 = 40000 \Rightarrow \text{P} = 5000.$$

$\therefore \qquad$ Principal = ₹5000.

(*iii*) Rate of compound interest = 8% p.a. and principal = ₹5000.

$$\text{Amount due after 3 years} = ₹5000\left(1+\frac{8}{100}\right)^3$$

$$= ₹5000 \times \left(\frac{27}{25}\right)^3 = ₹5000 \times \frac{27}{25} \times \frac{27}{25} \times \frac{27}{25}$$

$$= ₹\frac{157464}{25} = ₹6298{\cdot}56.$$

$\therefore$ C.I. for 3 years = A − P = ₹6298·56 − ₹5000

= ₹1298·56

$\therefore$ The difference between the C.I. and S.I. for 3 years

= ₹1298·56 − ₹1200 = ₹98·56.

Example 24. *A certain sum of money is invested at the rate of 5% per annum compound interest, the interest compounded annually. If the difference between the interests of third year and first year is ₹102·50, find the sum.*

Solution. Let the sum of money invested be ₹x.

Rate of interest = 5% p.a.

$\therefore$ Interest for the first year = ₹ $\frac{x \times 5 \times 1}{100}$ = ₹ $\frac{x}{20}$.

Interest for the 3rd year *i.e.* the compound interest for the 3rd year

= amount after 3 years − amount after 2 years

$= ₹x\left(1+\frac{5}{100}\right)^3 - ₹x\left(1+\frac{5}{100}\right)^2$

$= ₹x\left[\left(\frac{21}{20}\right)^3 - \left(\frac{21}{20}\right)^2\right] = ₹\left(x \times \left(\frac{21}{20}\right)^2\left(\frac{21}{20}-1\right)\right)$

$= ₹\left(x \times \frac{441}{400} \times \frac{1}{20}\right) = ₹\frac{441x}{8000}$.

Given, difference between interests of third year and first year is ₹102·50

$\Rightarrow \quad \frac{441x}{8000} - \frac{x}{20} = \frac{205}{2}$

$\Rightarrow \quad \frac{(441-400)x}{8000} = \frac{205}{2} \Rightarrow \frac{41}{8000}x = \frac{205}{2}$

$\Rightarrow \quad x = \frac{205}{2} \times \frac{8000}{41} = 20000.$

$\therefore$ The sum of money invested = ₹20000.

Example 25. *On a certain sum of money, the difference between the compound interest for a year, payable half-yearly, and the simple interest for a year is ₹180. Find the sum lent out, if the rate of interest in both the cases is 10%.* (2002)

Solution. Let the principal be ₹x.

S.I. for 1 year at 10% per annum = ₹ $\frac{x \times 10 \times 1}{100}$ = ₹ $\frac{x}{10}$.

Rate of interest per conversion period (half-year) = $\frac{1}{2}$ of 10% = 5%

and n(the number of conversion periods) = 2.

C.I. = ₹$x\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$ | C.I. = P $\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

$= ₹x\left(\frac{21}{20} \times \frac{21}{20} - 1\right) = ₹x\left(\frac{441}{400} - 1\right) = ₹\frac{41}{400}x.$

$\therefore$ C.I. − S.I. = ₹ $\frac{41}{400}x$ − ₹ $\frac{x}{10}$ = ₹ $\left(\frac{41}{400} - \frac{1}{10}\right)x$ = ₹ $\frac{x}{400}$.

According to the given information, $\frac{x}{400} = 180 \Rightarrow x = 72000.$

$\therefore$ The sum lent out = ₹72000.

Example 26. *The difference between the compound interest and the simple interest on ₹ 42000 for two years is ₹ 105 at the same rate of interest per annum. Find :*

(i) the rate of interest (ii) the compound interest earned in the second year.

Solution. (*i*) Let the rate of interest be $r\%$ per annum.

Here P (principal) = ₹ 42000 and T (time) = 2 years.

$$\text{S.I.} = ₹\ \frac{42000 \times r \times 2}{100}.$$

$$\text{C.I.} = ₹\,42000\left[\left(1+\frac{r}{100}\right)^2 - 1\right] = ₹\,42000\left[\left(\frac{r}{100}\right)^2 + \frac{2r}{100}\right]$$

$$\therefore \quad \text{C.I.} - \text{S.I.} = ₹\,42000\left[\left(\frac{r}{100}\right)^2 + \frac{2r}{100}\right] - ₹\,42000 \times \frac{2r}{100}$$

$$= ₹\,42000 \times \left(\frac{r}{100}\right)^2 = ₹\ \frac{21}{5}r^2.$$

According to given, $\frac{21}{5}r^2 = 105$

$\Rightarrow \quad r^2 = 25 \Rightarrow r = 5, -5$, but r cannot be negative

$\Rightarrow \quad r = 5.$

Hence the rate of interest = 5% per annum.

(*ii*) C.I. earned in two years $= ₹\,42000\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$

$$= ₹\,42000\left[\left(\frac{21}{20}\right)^2 - 1\right] = ₹\,42000 \times \frac{41}{400}$$

$$= ₹\,4305.$$

Interest earned in one year $= ₹\ \frac{42000 \times 5 \times 1}{100} = ₹\,2100.$

$\therefore$ C.I. earned in the second year = ₹ 4305 − ₹ 2100 = ₹ 2205.

Example 27. *A man borrowed a sum of money and agrees to pay off by paying ₹ 3150 at the end of the first year and ₹ 4410 at the end of the second year. If the rate of compound interest is 5% per annum, find the sum borrowed.*

Solution. Let the money borrowed be ₹ x.

$$\text{Amount after one year} = ₹\,x\left(1+\frac{5}{100}\right) = ₹\ \frac{21}{20}x.$$

Money refunded at the end of first year = ₹ 3150.

$\therefore$ Principal for the 2nd year $= ₹\left(\frac{21}{20}x - 3150\right).$

$$\text{Amount at the end of 2nd year} = ₹\left(\frac{21}{20}x - 3150\right) \times \left(1+\frac{5}{100}\right)$$

$$= ₹\left(\frac{21}{20}x - 3150\right) \times \frac{21}{20}.$$

But the money paid at the end of the 2nd year = ₹ 4410,

$\therefore \quad \left(\frac{21}{20}x - 3150\right) \times \frac{21}{20} = 4410$

$\Rightarrow \quad \frac{21}{20}x - 3150 = 4410 \times \frac{20}{21} = 4200$

$$\Rightarrow \quad \frac{21}{20}x = 4200 + 3150 = 7350$$

$$\Rightarrow \quad x = 7350 \times \frac{20}{21} = 7000.$$

$\therefore$ The sum borrowed = ₹ 7000.

Exercise 1.2

1. Find the amount and the compound interest on ₹ 5000 for 2 years at 6% per annum, interest payable yearly.
2. Find the amount and the compound interest on ₹ 8000 for 4 years at 10% per annum, interest reckoned yearly.
3. If the interest is compounded half yearly, calculate the amount when the principal is ₹ 7400, the rate of interest is 5% and the duration is one year. *(2005)*
4. Find the amount and the compound interest on ₹ 5000 at 10% p.a. for $1\frac{1}{2}$ years, compound interest reckoned semi-annually.
5. Find the amount and the compound interest on ₹ 100000 compounded quarterly for 9 months at the rate of 4% p.a.

Hint

$r = \frac{1}{4}$ of 4% = 1% and $n = \frac{9}{3} = 3$.

6. Find the difference between C.I. and S.I. on sum of ₹ 4800 for 2 years at 5% per annum payable yearly.
7. Find the difference between the simple interest and compound interest on ₹ 2500 for 2 years at 4% per annum, compound interest being reckoned semi-annually.
8. Find the amount and the compound interest on ₹ 2000 in 2 years if the rate is 4% for the first year and 3% for the second year.
9. Find the compound interest on ₹ 3125 for 3 years if the rates of interest for the first, second and third year are respectively 4%, 5% and 6% per annum.
10. What sum of money will amount to ₹ 9261 in 3 years at 5% per annum compound interest? *(2009)*
11. What sum invested at 4% per annum compounded semi-annually amounts to ₹ 7803 at the end of one year ?
12. What sum invested for $1\frac{1}{2}$ years compounded half-yearly at the rate of 4% p.a. will amount to ₹ 132651 ?
13. On what sum will the compound interest for 2 years at 4% per annum be ₹ 5712 ?
14. A man invests ₹ 1200 for two years at compound interest. After one year the money amounts to ₹ 1275. Find the interest for the second year correct to the nearest rupee.
15. At what rate percent per annum compound interest will ₹ 2304 amount to ₹ 2500 in 2 years ?
16. A sum compounded annually becomes $\frac{25}{16}$ times of itself in two years. Determine the rate of interest per annum.

17. At what rate percent will ₹2000 amount to ₹2315·25 in 3 years at compound interest ?

18. If ₹40000 amounts to ₹48620·25 in 2 years, compound interest payable half-yearly, find the rate of interest per annum.

19. Determine the rate of interest for a sum that becomes $\frac{216}{125}$ times of itself in $1\frac{1}{2}$ years, compounded semi-annually.

20. At what rate percent p.a. compound interest would ₹80000 amount to ₹88200 in two years, interest being compounded yearly. Also find the amount after 3 years at the above rate of compound interest.

21. A certain sum amounts to ₹5292 in 2 years and to ₹5556·60 in 3 years at compound interest. Find the rate and the sum.

22. A certain sum amounts to ₹798·60 after 3 years and ₹878·46 after 4 years. Find the interest rate and the sum.

23. In what time will ₹15625 amount to ₹17576 at 4% per annum compound interest ?

24. Find the time (in years) in which ₹12500 will produce ₹3246.40 as compound interest at 8% per annum, interest compounded annually.

25. ₹16000 invested at 10% p.a., compounded semi-annually, amounts to ₹18522. Find the time period of investment.

26. What sum will amount to ₹2782·50 in 2 years at compound interest, if the rates are 5% and 6% for the successive years ?

27. A sum of money is invested at compound interest payable annually. The interest in two successive years is ₹225 and ₹240. Find :
    (*i*) the rate of interest.
    (*ii*) the original sum.
    (*iii*) the interest earned in the third year.

28. On what sum of money will the difference between the compound interest and simple interest for 2 years be equal to ₹ 25 if the rate of interest charged for both is 5% p.a.? *(2012)*

29. The difference between the compound interest for a year payable half-yearly and the simple interest on a certain sum of money lent out at 10% for a year is ₹15. Find the sum of money lent out.

30. The amount at compound interest which is calculated yearly on a certain sum of money is ₹1250 in one year and ₹1375 in two years. Calculate the rate of interest.

31. The simple interest on a certain sum for 3 years is ₹225 and the compound interest on the same sum at the same rate for 2 years is ₹153. Find the rate of interest and the principal.

32. Find the difference between compound interest on ₹8000 for $1\frac{1}{2}$ years at 10% p.a. when compounded annually and semi-annually.

33. A sum of money is lent out at compound interest for two years at 20% p.a., C.I. being reckoned yearly. If the same sum of money is lent out at compound interest at the same rate percent per annum, C.I. being reckoned half-yearly, it would have fetched ₹482 more by way of interest. Calculate the sum of money lent out.

34. A sum of money amounts to ₹13230 in one year and to ₹13891·50 in $1\frac{1}{2}$ years at compound interest, compounded semi-annually. Find the sum and the rate of interest per annum.

1.3 GROWTH AND DEPRECIATION

Growth. We know certain things grow (increase, appreciate). For example, India's population, Delhi's pollution level, height of a plant, weight of a child, cost of goods etc.

The growth per year (or unit of time) is called ***rate of growth.***

If the rate of growth is constant, then

$$V = V_0\left(1+\frac{r}{100}\right)^n$$

where r % is the rate of growth per year, n is the number of years, V_0 is the present measure of the quantity and V is the measure of the quantity after n years.

Similarly, if V_0 is the measure of the quantity n years ago and V is the present measure of the quantity, then

$$V = V_0\left(1+\frac{r}{100}\right)^n$$

Remark

If the rate of growth is r_1% during the first year (or unit of time) and r_2% during the second year (or unit of time), then

$$V = V_0\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$$

where V_0 is the present measure of the quantity and V is the measure of the quantity after two years (or units of time).

This formula can be extended for more than 2 years (or units of time).

Depreciation. The value of a machine or vehicle etc. decreases with time due to wear and tear. *The decrease of the value per year (or unit of time) is called* ***rate of depreciation.***

If the rate of depreciation is constant, then

$$V = V_0\left(1-\frac{r}{100}\right)^n$$

where r% is the rate of depreciation per year, n is the number of years, V_0 is the present value and V is the value after n years.

Similarly, if V_0 is the value n years ago and V is the present value, then

$$V = V_0\left(1-\frac{r}{100}\right)^n$$

The number of years a machine can be effectively used is called its ***life span****, after which it is sold as* ***waste*** *or* ***scrap****.*

ILLUSTRATIVE EXAMPLES

Example 1. *The present population of a town is 48000. It is increasing at the rate of 5% every year. What will be the increase in population in next 3 years ?*

Solution. Population after 3 years = (present population) $\times \left(1+\frac{5}{100}\right)^3$

$\left| V = V_0\left(1+\frac{r}{100}\right)^n \right.$

$$= 48000 \times \frac{21}{20}\times\frac{21}{20}\times\frac{21}{20} = 55566.$$

$\therefore$ Increase in population = 55566 – 48000 = 7566.

Example 2. *The present population of a village is 5408. If it has increased at the rate of 4% every year, what was its population two years ago ?*

Solution. Present population = (population 2 years ago) $\times \left(1+\frac{4}{100}\right)^2$

$\Rightarrow$ $5408 = (\text{population 2 years ago}) \times \frac{26}{25} \times \frac{26}{25}$

$\Rightarrow$ population 2 years ago $= 5408 \times \frac{25}{26} \times \frac{25}{26} = 5000.$

Example 3. *The value of a machine, purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97200, find :*

(i) its value after 3 years (ii) its value when it was purchased.

Solution. (*i*) Value after 3 years = (present value) $\times \left(1-\frac{10}{100}\right)^3$ $\quad \left| V = V_0\left(1-\frac{r}{100}\right)^n \right.$

$= ₹\left(97200 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right) = ₹\ \frac{708588}{10}$

$= ₹\ 70858{\cdot}80.$

(*ii*) Present value = (value 2 years ago) $\times \left(1-\frac{10}{100}\right)^2$

$\Rightarrow$ $₹\ 97200 = (\text{value 2 years ago}) \times \frac{9}{10} \times \frac{9}{10}$

$\Rightarrow$ value 2 years ago $= ₹\left(97200 \times \frac{10}{9} \times \frac{10}{9}\right) = ₹\ 120000.$

Example 4. *The population of a village 2 years ago was 6250. Due to migration to cities, it decreases at the rate of 8% every year. Find the decrease in its population in the last 2 years.*

Solution. Present population = (population 2 years ago) $\times \left(1-\frac{8}{100}\right)^2$

$= 6250 \times \frac{23}{25} \times \frac{23}{25} = 5290.$

$\therefore$ The decrease in the population = 6250 – 5290 = 960.

Example 5. *The value of a car depreciates by 12·5 % every year. By what percent will the value of the car decrease after 3 years ?*

Solution. Let the present value of the car be $₹\ V_0$.

Value of the car after 3 years = (present value) $\times \left(1-\frac{12{\cdot}5}{100}\right)^3$

$= ₹\ V_0 \times \left(1-\frac{1}{8}\right)^3 = ₹\ V_0 \times \left(\frac{7}{8}\right)^3$

$\therefore$ Decrease in the value of car $= ₹\ V_0 - ₹\ V_0 \times \left(\frac{7}{8}\right)^3$

$= ₹\ V_0\left[1-\left(\frac{7}{8}\right)^3\right] = ₹\ V_0 \times \frac{512-343}{512}$

$= ₹\left(V_0 \times \frac{169}{512}\right).$

$\therefore$ Percentage decrease in the value of the car after 3 years

$$= \left(\frac{\text{decrease}}{\text{present value}} \times 100\right)\% = \left(\frac{V_0 \times \frac{169}{512}}{V_0} \times 100\right)\%$$

$$= \left(\frac{169}{512} \times 100\right)\% = \frac{169 \times 25}{128}\% = \frac{4225}{128}\%$$

$$= 33\frac{1}{128}\%.$$

Example 6. *6000 workers were employed to construct a river bridge in four years. At the end of first year, 20% workers were retrenched. At the end of second year, 5% of the workers at that time were retrenched. However, to complete the project in time, the number of workers was increased by 15% at the end of third year. How many workers were working during the fourth year ?*

Solution. The number of workers who were working during the fourth year

$$= 6000\left(1-\frac{20}{100}\right)\left(1-\frac{5}{100}\right)\left(1+\frac{15}{100}\right) \qquad \left| V = V_0\left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\right.$$

$$= 6000 \times \frac{4}{5} \times \frac{19}{20} \times \frac{23}{20} = 5244.$$

Example 7. *24000 blood donors were registered with 'red cross' at Kolkata. The number of donors increased at the rate of 5% every six months. Find the time period at the end of which the total number of blood donors becomes 27783.*

Solution. Here rate of increase of the number of blood donors per unit time (six months) = 5%. Let the number of units of time (six months) be n.

Using $V = V_0\left(1+\frac{r}{100}\right)^n$, we get

$$27783 = 24000\left(1+\frac{5}{100}\right)^n$$

$$\Rightarrow \left(\frac{21}{20}\right)^n = \frac{27783}{24000} = \frac{9261}{8000} = \left(\frac{21}{20}\right)^3$$

$$\Rightarrow n = 3.$$

$\therefore$ The time period = 3 × (six months) = $1\frac{1}{2}$ years.

Exercise 1.3

1. The present population of a town is 15625. If the population increases at the rate of 4% every year, what will be the increase in the population in next 3 years ?
2. A town has 64000 inhabitants. If the population increases at the rate of $2\frac{1}{2}$ % every year, find the number of inhabitants in the town after 3 years.
3. The population of a city increases each year by 4% of what it had been at the beginning of each year. If its present population is 6760000, find :

   (*i*) its population 2 years hence (*ii*) its population 2 years ago.

4. The cost of a refrigerator is ₹9000. Its value depreciates at the rate of 5% every year. Find the total depreciation in its value at the end of 2 years.

5. Dinesh purchased a scooter for ₹24000. The value of the scooter is depreciating at the rate of 5% per annum. Calculate its value after 3 years.

6. A farmer increases his output of wheat in his farm every year by 8%. This year he produced 2187 quintals of wheat. What was the yearly produce of wheat two years ago ?

7. The value of a property decreases every year at the rate of 5%. If its present value is ₹411540, what was its value three years ago ?

8. Ahmed purchased an old scooter for ₹16000. If the cost of the scooter after 2 years depreciates to ₹14440, find the rate of depreciation.

9. A factory increased its production of cars from 80000 in the year 2006-2007 to 92610 in 2009-2010. Find the annual rate of growth of production of cars.

10. The value of a machine worth ₹500000 is depreciating at the rate of 10% every year. In how many years will its value be reduced to ₹364500 ?

11. Mahindra set up a factory by investing ₹2500000. During the first two years, his profits were 5% and 10% respectively. If each year the profit was on previous year's capital, calculate his total profit.

12. The value of a property is increasing at the rate of 25% every year. By what percent will the value of the property increase after 3 years?

13. Mr. Durani bought a plot of land for ₹180000 and a car for ₹320000 at the same time. The value of the plot of land grows uniformly at the rate of 30% p.a., while the value of the car depreciates by 20% in the first year and by 15% p.a. thereafter. If he sells the plot of land as well as the car after 3 years, what will be his profit or loss?

CHAPTER TEST

1. ₹10000 was lent for one year at 10% per annum. By how much more will the interest be, if the sum was lent at 10% per annum, interest being compounded half-yearly ?

2. A man invests ₹3072 for two years at compound interest. After one year the money amounts to ₹3264. Find the rate of interest and the amount due at the end of 2nd year.

3. What sum will amount to ₹28090 in two years at 6% per annum compound interest? Also find the compound interest.

4. Two equal sums were lent at 5% and 6% per annum compound interest for 2 years. If the difference in the compound interest was ₹422, find :
   (*i*) the equal sums (*ii*) compound interest for each sum.

5. The compound interest on a sum of money for 2 years is ₹1331·20 and the simple interest on the same sum for the same period and at the same rate is ₹1280. Find the sum and the rate of interest per annum.

6. On what sum will the difference between the simple and compound interest for 3 years at 10% p.a. is ₹232·50 ?

7. The simple interest on a certain sum for 3 years is ₹1080 and the compound interest on the same sum at the same rate for 2 years is ₹741·60. Find :
   (*i*) the rate of interest (*ii*) the principal.

8. In what time will ₹2400 amount to ₹2646 at 10% p.a. compounded semi-annually ?

9. Sudarshan invested ₹60000 in a finance company and received ₹79860 after $1\frac{1}{2}$ years. Find the rate of interest per annum compounded half-yearly.

10. A man invests ₹65000 for 3 years at 4·5% p.a. compound interest reckoned yearly. Income tax at the rate of 20% is deducted at the end of each year from interest accrued. Find the amount at the end of third year.

11. The population of a city is 320000. If the annual birth rate is 9·2% and the annual death rate is 1·7%, calculate the population of the town after 3 years.

Hint

Net growth rate = 9·2% − 1·7% = 7·5%.

12. The cost of a car, purchased 2 years ago, depreciates at the rate of 20% every year. If its present worth is ₹315600, find :
   (*i*) its purchase price (*ii*) its value after 3 years.

13. Amar Singh started a business with an initial investment of ₹400000. In the first year, he incurred a loss of 4%. However, during the second year, he earned a profit of 5% which in the third year rose to 10%. Calculate his net profit for the entire period of 3 years.

14. The cost of a washing machine depreciates by ₹720 during the second year and by ₹648 during the third year. Calculate :
   (*i*) the rate of depreciation per annum (*ii*) the original cost of the machine
   (*iii*) the value of the machine at the end of third year.