Mensuration

Exercise 18.1

1. Given

Ratio of length and breadth of rectangular field = 9:5

Area of field = 14580 m^2

Cost of fence = ₹ 3.25 /m

Let length, breadth = $9x, 5x$

Area = 14580

$l \times b = 14580$

$9x \times 5x = 14580$

$45x^2 = 14580$

$x^2 = \frac{14580}{45}$

$x^2 = 324$

$x = \sqrt{324}$

$x = 18m.$

$\therefore$ length $= 9x = 9 \times 18 = 162m$

breadth $= 5x = 5 \times 18 = 90m$

Length of fence = perimeter of rectangle section

$= 2(l+b)$

$= 2(162+90)$

$= 2(252)$

Length of fence $= 504m$

Cost of fence = 504×3.25

= ₹ 1638

2. Given

Dimensions of the rectangle = $16m \times 9m$

Let side of square = x m.

perimeter of rectangle = $2(16+9) = 50m$

Area of rectangle = Area of square

$l \times b = x^2$

$16 \times 9 = x^2$

$x = \sqrt{16 \times 9}$

$x = 4 \times 3 = 12m$

$x = 12m$

∴ Side of square = 12m

∴ perimeter of square = $4x$

$= 4 \times 12$

$= 48m$

∴ perimeter of rectangle exceeds perimeter of square by $50 - 48 = 2m$.

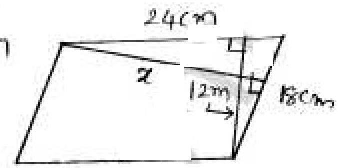
3. Given

lengths of adjacent sides = 24cm and 18cm

⊥er Distance b/w longer sides = 12cm.

Let ⊥er Distance b/w Shorter sides = x cm

Area of Parallelogram =



Side × ⊥er distance b/w the opposite sides

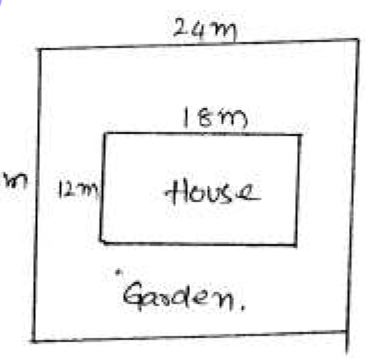
$\therefore$ $24 \times 12 = 18 \times x$

$$x = \frac{24 \times 12}{18}$$

$$x = 16\text{cm}$$

$\therefore$ ⊥er distance b/w Shorter Sides = 16cm.

4. Given



plot dimension = 24m × 24m

House dimensions = 18m × 12m.

$\therefore$ Garden Area =

Plot Area − House Area.

$= 24 \times 24 - 18 \times 12$

Garden Area = 360m^2

Given cost of developing garden = ₹ 50/m²

$\therefore$ Total cost of developing garden around House

$= 360 \times 50$

$=$ ₹ 18000

5. Dimension of tiles (Parallelogram) = 18cm × 6cm
↳ Height

Floor Area = $540 m^2$

Area of one tile = 18cm × 6cm (b×h)

= $108 cm^2$

Area of one tile = $108 \times 10^{-4} m^2$ ($\because$ $1cm = 10^{-2} m$)

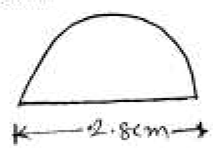
No. of tiles required = $\frac{\text{Total Area}}{\text{Area of one tile}}$

$= \frac{540}{108 \times 10^{-4}}$

No. of tiles required = 50000

6.

(a) diameter of semi circle = 2.8cm



Perimeter of semi circle

$= \frac{\pi d}{2}$

$= \frac{\pi \times 2.8}{2}$

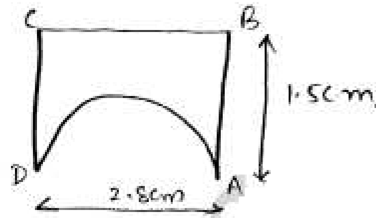
$= \frac{3.14 \times 2.8}{2}$

$= 3.14 \times 1.4$

Perimeter of semi circle = 4.398 c.m

(b)

Perimeter of given shape



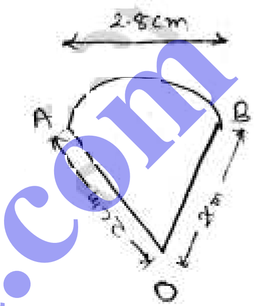
$= \overline{AB} + \overline{BC} + \overline{CD} +$ Semi circle perimeter

$= 1.5 + 2.8 + 1.5 + 4.398$

$= 10.198$ cm.

(c)

Perimeter of given shape



$= \overline{OA} +$ Semi circle AB $+ \overline{OB}$

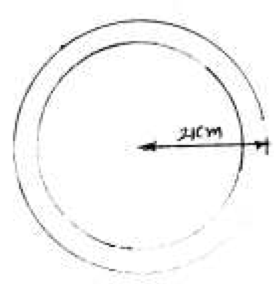
$= 2 + 4.398 + 2$

$= 8.398$ cm

∴ Comparing ~~three~~ p. figures perimeter values, we can say in case of figure 'b' Ant has covered more distance.

7. Given

Area b/w enclosed concentric circle $= 770\ cm^2$



Outer circle radius $= 21$ cm.

Let inner circle radius $= r$ cm

Outer Circle Area − Inner Circle Area = $770 cm^2$

$\pi(21)^2 - \pi r^2 = 770$

$\pi(21^2 - r^2) = 770$

$21^2 - r^2 = 245.098$

$441 - r^2 = 245.098$

$r^2 = 441 - 245.098$

$r^2 = 195.90$

$r = \sqrt{195.9}$

$r = 13.996 \approx 14 cm.$

Radius of Inner Circle = 14cm.

8. Given

Area of square = $121 cm^2$

$s^2 = 121$

$s = \sqrt{121}$

$s = 11 cm.$

∴ Side of Square = 11cm

∴ length of ~~Circle~~ wire = Perimeter of square = $4 \times 11 cm$

$= 44 cm.$

Now wire is bent into a form of Circle

∴ length of wire = perimeter of circle

$44 = 2\pi r$ $\quad r$ = radius of Circle

$\pi r = \frac{44}{2}$

$$\pi r = 22$$

$$r = \frac{22}{\pi}$$

$$r = \frac{22}{3.14}$$

$$r = 7\,cm$$

radius of Circle = 7cm.

Area of Circle = πr^2

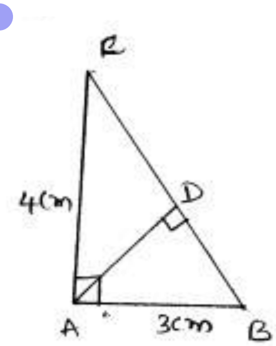
$= 3.14 \times 7^2$

Area of Circle $= 153.938\ cm^2$

9.

(i)

Area of $\triangle$le ABC



$= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \times 4$ ($\because$ right angle triangle)

$= \frac{1}{2} \times 3 \times 4$

$= \frac{1}{2} \times 12$

$= 6\ cm^2$

(ii)

$BC^2 = AB^2 + AC^2$ ($\because$ Pythogorus Theorem)

$BC^2 = 3^2 + 4^2$

$BC^2 = 9 + 16$

$BC^2 = 25$

$BC = \sqrt{25}$

$BC = 5\,cm$

(ii) Area of triangle ABC = $6\,cm^2$

By taking $\overline{BC}$ as base

Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times BC \times AD$

$6 = \frac{1}{2} \times 6 \times AD$

$AD = \frac{6 \times 2}{6}$

$AD = 2\,cm$

10. Dimension of rectangular garden = $80m \times 40m$

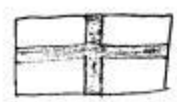
Width of path (w) = 2.5 m

i) Area of cross path $= l \times w + b \times w - (w \times w)$

$= 80 \times 2.5 + 40 \times 2.5 - (2.5 \times 2.5)$

$= 293.75\,m^2$

ii) Area of unshaded portion =



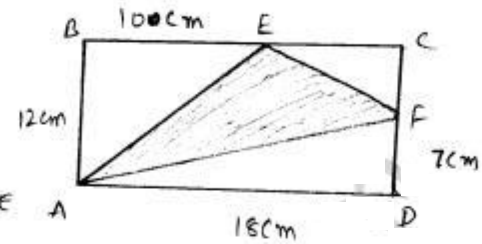
= Area of garden − Area of cross path

$= 80 \times 40 - (293.75)$

$= 2906.25\,m^2$

11.

Area of shaded portion:



Area of □ ABCD − [Area of $\triangle^{le}$ ABE + Area of $\triangle^{le}$ AFD + $\triangle^{le}$ EFC]

$18 \times 12 - \left[\frac{1}{2} \times 7 \times 18 + \frac{1}{2} \times 12 \times 10 + \frac{1}{2} \times 5 \times 8\right]$

$216 - [7 \times 9 + 6 \times 10 + 5 \times 4]$

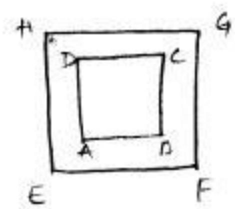
$216 - [63 + 60 + 20]$

$216 - 143$

$73\ cm^2$

∴ Area of shaded portion $73 cm^2$.

12. Given



Area of square ~~ABCD~~ EFGH = $729 m^2$

∴ Side of ~~ABCD~~ lawn = $\sqrt{729}$

= 27 m

Area of square ~~EFGH~~ ABCD = $295 m^2$

Side of ABCD = $\sqrt{295}$

Side of ABCD = 17.175 m.

i) ∴ length of square field enclosing lawn & path

= 27m

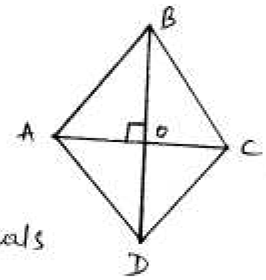
(ii) width of the path = Side of EFGH − Side of ABCD

$= 27 - 17.175$

width of the path $= 9.825$ m.

Exercise 18.2

1. Let ABCD is a Rhombus



$AB = BC = CD = AD = 13cm.$

$AC = 10cm.$

'O' intersection point of diagnals

$\overline{OA} = \overline{OC} = 5cm.$

In $\triangle^{le}$ AOB

(i) $\overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2$ ($\because$ Pythogorus theorem)

$13^2 = 5^2 + \overline{OB}^2$

$169 = 25 + \overline{OB}^2$

$\overline{OB}^2 = 169 - 25$

$\overline{OB}^2 = 144$

$\overline{OB} = \sqrt{144}$

$\overline{OB} = 12\,cm$

$\overline{BD} = 2 \times \overline{OB}$

$= 2 \times 12$

$\overline{BD} = 24\,cm$

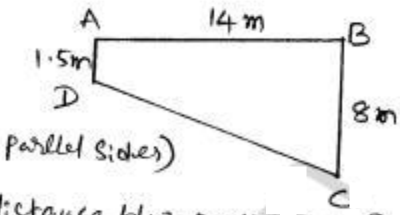
(ii) length of diagnel = 24 cm

Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 10 \times 24$

Area of rhombus $= 120\,cm^2$

2. Given ABCD is a trapezium



Area of trapezium = $\frac{1}{2} \times$ (Sum of parllel sides) $\times$ (distance b/w parllel sides)

$= \frac{1}{2} \times (1.5+8) \times 14$

Area of trapezium $= 66.5\ m^2$

3. Given

Area of a trapezium $= 360 m^2$

distance b/w two parallel side = 20m

length of one parallel side = 25m

Let unknown parallel sides = x

Area of a trapezium = $\frac{1}{2}$ (Sum of parallel sides) $\times$ (distance b/w parallel sides)

$360 = \frac{1}{2} (25+x) \times 20$

$(25+x) = \frac{360 \times 2}{20}$

$25+x = 36$

$x = 36-25$

$x = 11m$

Unknown length of

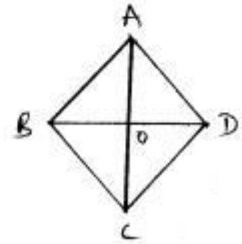
∴ another parlle side length = 11m.

4. Given ABCD is a rhombus

$\overline{BD} = 13cm$

$\overline{AB} = \overline{BC} = \overline{CD} = \overline{AD} = 6.5cm$

Altitude $\overline{AC} = 5cm.$



(i) Area of rhombus $= \frac{1}{2} \times$ (product of diognals)

$= \frac{1}{2} \times (13 \times 5)$

$= 6.5 \times 5$

Area of rhombus $= 32.5\ cm^2$

(ii) Another diagnal $\overline{AC} = 5cm.$

5.

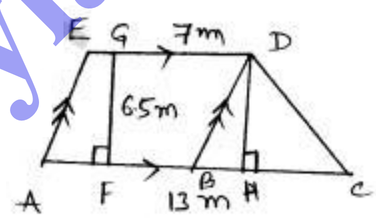
(i) Area of trapezium ACDE

$= \frac{1}{2}(ED + AC) \times FG$

$= \frac{1}{2}(7+13) \times 6.5$

$= \frac{1}{2} \times 20 \times 6.5$

$= 65m^2$



(ii) Area of parallelogram ABDE = base × perdistance b/w parallel sides

$= 7 \times 6.5$

$= 45.5\ m^2$

(iii) The area of triangle BCD $= \frac{1}{2} \times BC \times DH$

$$AC = AB + BC$$
$$13 = 7 + BC$$
$$BC = 13 - 7$$
$$BC = 6m$$
$$DH = GF = 6.5m$$

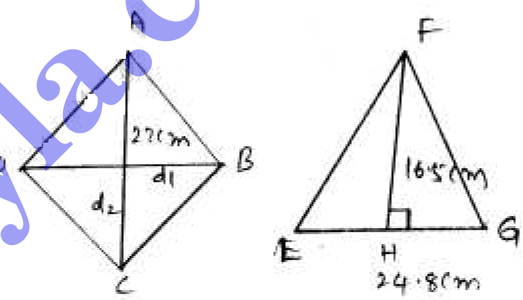
$\therefore$ The area of triangle BCD $= \frac{1}{2} \times 6 \times 6.5$

$$= 3 \times 6.5$$
$$= 19.5\ m^2$$

6.

ABCD is a rhombus and

EFG is a triangle



Given

Area of rhombus = Area of a triangle

$$\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times b \times h$$

$$\frac{1}{2} \times 22 \times d_1 = \frac{1}{2} \times 24.8 \times 16.5$$

$$22 \times d_1 = 24.8 \times 16.5$$

$$d_1 = \frac{24.8 \times 16.5}{22}$$

$$d_1 = 18.6cm.$$

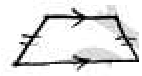
length of diagonal = 18.6cm.

7. Given

Perimeter of trapezium = 52cm

length of non-parallel side = 10cm.

Altude = 8cm.



length of parllel sides = perimeter − 2(parallel side)

= 52 − 2×10

= 52 − 20

Sum of parallel sides = 32 cm

Area of trapezium = $\frac{1}{2}$ × (sum of parallel side) × Altitude

= $\frac{1}{2}$ × 32 × 8

= 32×4

Area of trapezium = 128 cm^2

8. Given

Area of trapeziom = 540 cm^2

Altitude = 18cm.

Ratio of lengths of parallel sides = 7:5

Let lengths of parallel sides = 7x, 5x

∴ Area of trapezium = $\frac{1}{2}$ × (sum of parallel sides) × Altitude.

540 = $\frac{1}{2}$ × (7x+5x) × 18

$540 = \frac{1}{2}(12x) \times 18$

$540 = \cancel{\frac{1}{2}}\ 6 \times 18 \times x$

$x = \frac{540}{6 \times 18}$

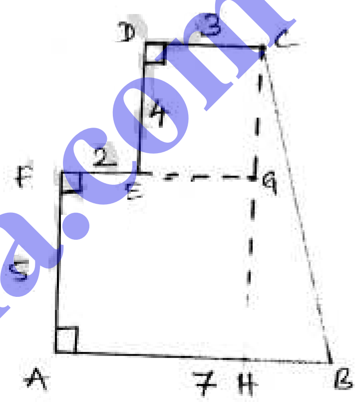
$x = 5$ cm

length of parallel sides $= 7x = 7 \times 5 = 35$ cm

$5x = 5 \times 5 = 25$ cm.

9.

(i)



Area enclosed by shape

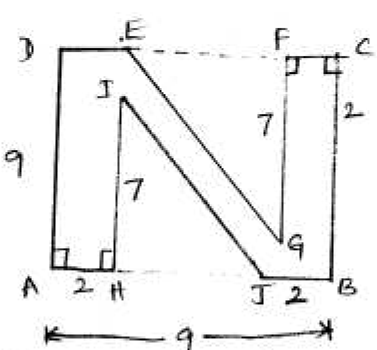
$=$ Area of $\square$ AHGF + Area of $\triangle$ BCH + Area of $\square$ DCGE

$= 5 \times 5 + \frac{1}{2} \times 2 \times 9 + 4 \times 3$

$= 25 + 9 + 12$

$= 46$ cm^2

(ii)



Area enclosed by shape

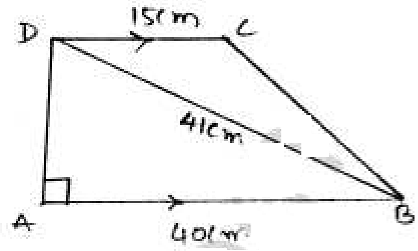
$=$ Area of $\square$ ABCD $-$ [Area of $\triangle$le EFG + Area of $\triangle$le HIJ]

$= 9 \times 9 - \left[\frac{1}{2} \times 5 \times 7 + \frac{1}{2} \times 5 \times 7\right]$

$= 81 - [5 \times 7] \Rightarrow 81 - 35 = 46$ cm^2.

10.

(i) In $\triangle^{le}$ ABD



$AB^2 + AD^2 = DB^2$ ($\because$ Pythogorus theorem)

$40^2 + AD^2 = 41^2$

$AD^2 = 41^2 - 40^2$

$= 1681 - 1600$

$AD^2 = 81$

$AD = \sqrt{81}$

$AD = 9$ cm.

(ii) Area of trapezium $= \frac{1}{2}$ (Sum of parallel sides) $\times$ Altitude

$= \frac{1}{2}(15+40) \times 9$ ($\because$ AD = 9 cm)

$= \frac{1}{2} \times 55 \times 9$

Area of trapezium $= 247.5 \text{ cm}^2$

iii) Area of $\triangle^{le}$ BCD $=$ Area of $\square$ABCD $-$ [Area of $\triangle^{le}$ ADB]

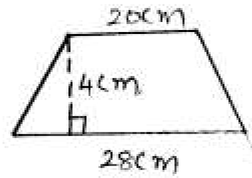
$= 247.5 - \left[\frac{1}{2} \times AB \times AD\right]$

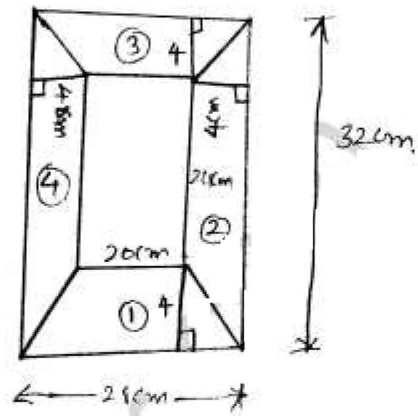
$= 247.5 - \left[\frac{1}{2} \times 40 \times 9\right]$

$= 247.5 - [180]$

Area of $\triangle^{le}$ BCD $= 67.5 \text{ cm}^2$

11. Area of section (1)





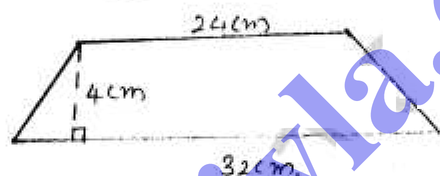
Area of trapezium =

$\frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Altitude)

$= \frac{1}{2} \times (28+20) \times 4$

$= 96 \text{ cm}^2$ $\therefore$ Area of section (1) $= 96 \text{ cm}^2$

Area of section (2)



Area of trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Altitude)

$= \frac{1}{2} \times (24+32) \times 4$

Area of section (2) $= 112 \text{ cm}^2$

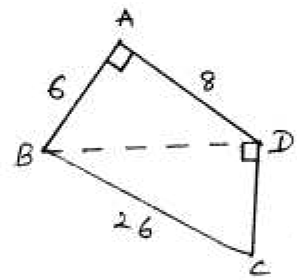
Section (3) dimension are same as section (1)

$\therefore$ Area of section (3) $= 96 \text{ cm}^2$

Section (4) dimensions are same as section (2)

$\therefore$ Area of section (4) $= 112 \text{ cm}^2$

12.



From $\triangle^{le}$ ABD

$$BD^2 = AB^2 + AD^2$$
$$BD^2 = 6^2 + 8^2$$
$$BD^2 = 36 + 64$$
$$BD^2 = 100$$
$$BD = 10\text{ cm.}$$

From $\triangle^{le}$ BDC

$$BC^2 = BD^2 + DC^2$$
$$26^2 = 10^2 + DC^2$$
$$676 = 100 + DC^2$$
$$DC^2 = 676 - 100$$
$$DC^2 = 576$$
$$DC = \sqrt{576}$$
$$DC = 24\text{ cm.}$$

Area of quadrilateral ABCD = Area of $\triangle^{le}$ BAD + Area of $\triangle^{le}$ BDC

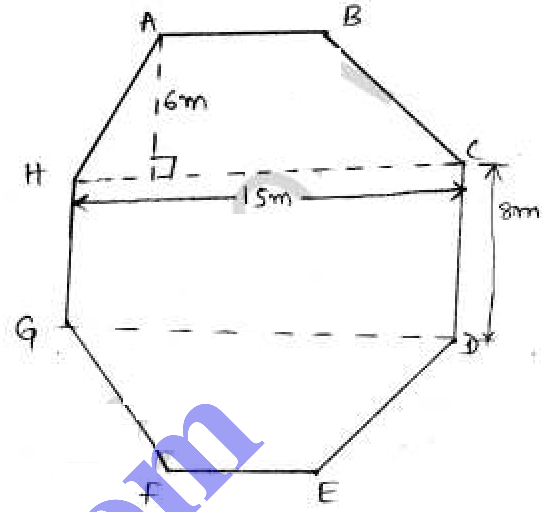
$$= \frac{1}{2}(AB \times AD) + \frac{1}{2}(BD \times DC)$$
$$= \frac{1}{2}(6 \times 8) + \frac{1}{2}(10 \times 24)$$
$$= \frac{1}{2}(48) + \frac{1}{2}(240)$$
$$= 24 + 120$$

Area of Quadrilateral ABCD $= 144\text{ cm}^2$

13.

Given ABCDEFGH a Regular octagon

Area of octagon ABCDEFGH



$=$ Area of $\triangle$ ABCH $+$

Area of $\square$ HCDG $+$

Area of $\triangle$ GDEF

$=$ $2\times$ Area of $\triangle$ ABCH $+$

Area of $\square$ HCDG

$= 2\times\left(\frac{1}{2}\times(8+15)\times 6\right)+(8\times 15)$

$= \left(2\times\frac{1}{2}\times 23\times 6\right)+(8\times 15)$

$= 23\times 6 + 8\times 15$

$= 138+120$

$= 258\ m^2$

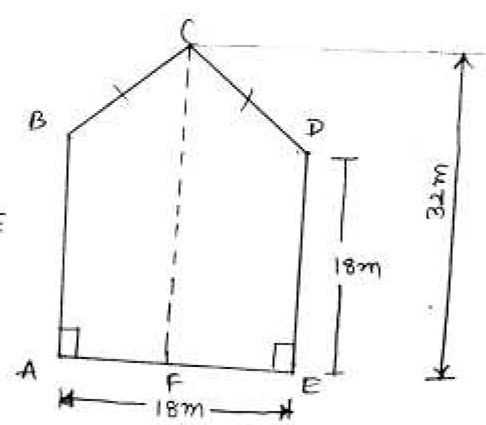
14. Jaspreet's diagram

Area of ABCDE $=$

Area of $\triangle$ ABCF $+$ Area of $\triangle$ FCDE

$= 2\times($Area of $\triangle$ ABCF$)$

($\because$ both are Symmetric)



Jaspreet's diagram

$= 2\times\left(\frac{1}{2}\times(AB+CF)\times AF\right)$

$2\times\frac{1}{2}\times(18+32)\times\frac{18}{2}$

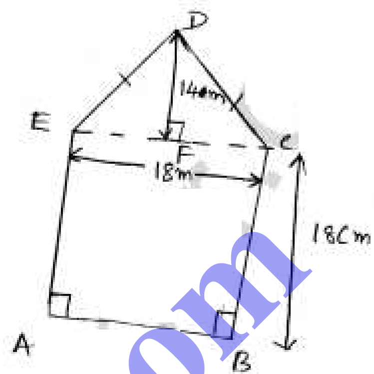
$= 50 \times 9$

Area of ABCDE $= 450 \text{ cm}^2$

Rahul's diagram.

Area of pentagon ABCDE =

Area of $\triangle^{le}$ DEC + Area of $\square$ ECBA



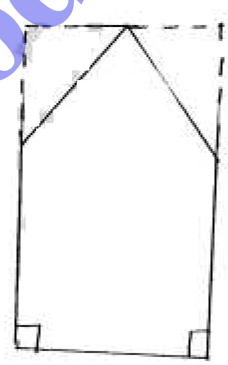
$= \frac{1}{2}(EC \times DF) + BC \times AB$

$= \frac{1}{2} \times 18 \times 14 + 18 \times 18$

$= 126 + 324$

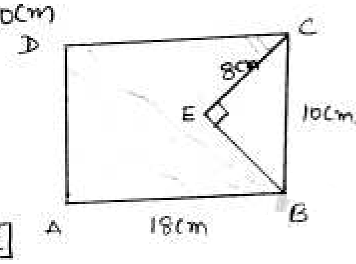
$= 450 \text{ cm}^2$

We can find area of pentagon often in this way



Mahesh's diagram.

15. Given ABCD is a rectangle of 18cm x 10cm



Area of shaded pentagon ABECD

$= \text{Area of} \square ABCD - [\text{Area of} \triangle^{le} BEC]$

$= 18 \times 10 - [\frac{1}{2} \times 8 \times EB] \rightarrow (1)$

From $\triangle^{le}$ BEC

$BC^2 = EC^2 + EB^2$

$10^2 = 8^2 + EB^2$

$EB^2 = 100 - 64$

$EB^2 = 36$

$EB = \sqrt{36}$

$EB = 6$ cm.

Sub EB value in eq(1)

$\therefore$ Area of Shaded pentagon ABECD $= 180 - [\frac{1}{2} \times 8 \times 6]$

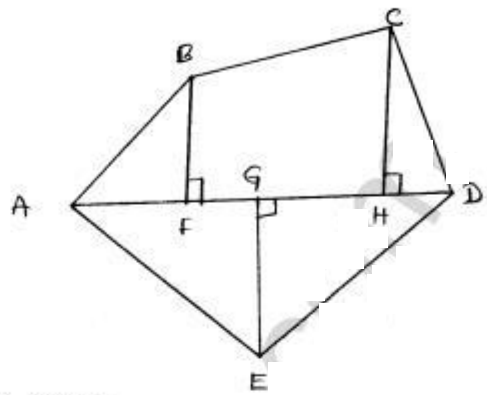
$= 180 - [4 \times 6]$

$= 180 - 24$

$\therefore$ Area of Shaded pentagon ABECD $= 156$ cm^2

16. Given

ABCDE is a polygon.



AD = 8 cm

AH = 6 cm

AG = 4 cm

AF = 3 cm.

BF = 2 cm, CH = 3 cm, EG = 2.5 cm.

Area of polygon ABCDE = Area of Δ^{le} ABF + Area of ▭ BCHF + Area of Δ^{le} CHD

+ ~~Δ^{le}~~ Area of Δ^{le} ADE + ~~Area of Δ^{le}~~ DGE

$= \frac{1}{2}(AF \times BF) + \frac{1}{2}(BF + CH) \times FH + \frac{1}{2}(DH \times CH) +$

$\frac{1}{2}(AD \times EG)$

AD = AH + HD

8 = 6 + HD

HD = 8 − 6

$\boxed{HD = 2\text{ cm}}$

AH = AF + FH

6 = 3 + FH

FH = 6 − 3

$\boxed{FH = 3\text{ cm}}$

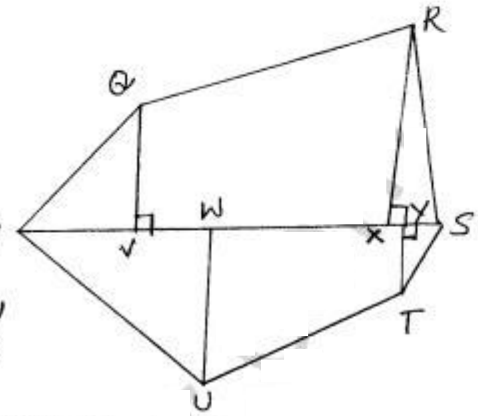
∴ Area of Polygon ABCDE $= \frac{1}{2}(3 \times 2) + \frac{1}{2}(5) \times 3 + \frac{1}{2} \times 2 \times 3 + \frac{1}{2}(8 \times 2.5)$

$= 3 + 7.5 + 3 + 10$

Area of Polygon ABCDE $= 23.5\text{ cm}^2$

17.

Given PQRSTU is a polygon



$PS = 11cm$

$PY = 9cm$

$PX = 8cm$

$PW = 5cm$

$PV = 3cm$

$QV = 5cm$

$UW = 4cm$

$RX = 6cm$

$TY = 2cm$

$VX = PX - PY$
$= 8 - 3$
$VX = 5cm$

$WY = PY - PW \Rightarrow 9 - 5 = 4cm.$

$XS = PS - PX$
$= 11 - 8$
$XS = 3cm$

$YS = PS - PY$
$= 11 - 9$
$YS = 2cm$

Area of polygon PQRSTU = Area of Δ^{le} PQV + Area of ⏢ QRXV + Area of Δ^{le} XRS + Area of Δ^{le} PUW + Area of ⏢ UWYT + Area of Δ^{le} YST

$= \left(\frac{1}{2} \times PV \times QV\right) + \frac{1}{2}(QV + RX) \times (VX) + \frac{1}{2}(RX)(XS) + \frac{1}{2} \times PW \times UW + \frac{1}{2}(UW + YT) \times (YW) + \frac{1}{2}(YS) \times (YT)$

$= \frac{1}{2} \times 3 \times 5 + \frac{1}{2}(5+6) \times 5 + \frac{1}{2}(6 \times 3) + \frac{1}{2}(5 \times 4) + \frac{1}{2}(4+2) \times 4 + \frac{1}{2}(2 \times 2)$

$= \frac{1}{2}(15 + 55 + 18 + 20 + 24 + 4)$

$= \frac{1}{2}(136)$

$= 68 \text{ cm}^2$

Exercise 18.3:

1. Given volume of cube $= 343\,cm^3$

Let 's' be edge of cube

$\therefore$ volume of cube $= s^3$

$$s^3 = 343$$

$$s = \sqrt[3]{343}$$

$$s = 7\,cm$$

$\therefore$ length of an edge of cube $= 7\,cm$.

2.

	Volume of Cuboid	Length	Breadth	Height
i.	$90\,cm^3$	6cm	5cm	3cm
ii.	$840\,cm^3$	15cm	8cm	7cm.
iii.	$62.5\,m^3$	10m	5m	1.25 m

3. Given

Volume of Cuboid $= 312\,cm^3$.

Base Area $= 26\,cm^2$

Volume $= 312\,cm^3$

Area $\times$ height $= 312$

$$26 \times h = 312$$

$$h = \frac{312}{26}$$

$$\boxed{h = 12\,cm}$$

4. Given
godown dimensions $(l \times b \times h) = 55m \times 45m \times 30m$

Cuboidal box volume $= 1.25 m^3$

godown volume $= l \times b \times h$

$= 55 \times 45 \times 30$

$= 74250 m^3$

No. of Cuboidal boxes $= \frac{\text{godown Volume}}{\text{box volume}}$

$= \frac{74250}{1.25}$

No. of Cuboidal boxes $= 59400$

5.
Given Dimensions of rectangular pit $= 1.4m \times 90cm \times 70cm$

Volume of pit $= l \times b \times h$

$= 140 \times 90 \times 70 \ cm^3$

Volume of pit $= 882000 cm^3$

Given
Brick dimension $(l \times b) = 21cm \times 10.5cm$

Let 'h' height of brick

~~Ara~~ $1000 \times$ Brick volume $=$ Pit volume

$1000 \times 21 \times 10.5 \times h = 882000$

$h = \frac{882000}{21 \times 10.5 \times 1000}$

$h = 4 cm$

$\therefore$ Height of brick $= 4cm$

6. ~~Et~~ Let 'a' be edge of cube.

Volume of ~~a~~ cube $= a^3$

If each edge of cube is tripled $= a' = 3a$

Volume of new cube $= a'^3$

$= (3a)^3$

$= 27a^3$

The Volume becomes 27 times the original volume of cube.

7. Given

milk tank is in the form of cylinder

diameter of ~~diameter~~ tank $= 1.4m \times 2$

Height of tank $= 8m$.

Volume of tank $= \frac{\pi}{4} d^2 \times h$

$= \frac{\pi}{4} \times (1.4)^2 \times 8$

Volume of tank $=$ ~~12.315 m³~~ $49.260\ m^3$

~~8.~~

$\therefore$ Volume of tank $= 49260$ lit

8. Given

External dimensions of box $= 84\ cm \times 75\ cm \times 64\ cm$

Thickness of box $= 2\ cm$.

$\therefore$ Internal dimensions of box $= (84 - 2\times2)\ cm, (75 - 2\times2)\ cm,$

$(64 - 2\times2)\ cm$

$= 80\ cm \times 71\ cm \times 60\ cm$

Volume of wood = External Volume − Internal Volume

$= (84 \times 75 \times 64) - (80 \times 71 \times 60)$

$= 403200 - 340800$

Volume of wood $= 62400\ cm^3$.

9. Given

Two cylinder Jar has Same Volume

Let d_1, d_2 are diameter of Jar

h_1, h_2 are heights of Jar

given $d_1 : d_2 = 3:4$

Volume of cylinder equal

$\therefore \frac{\pi}{4} d_1^2 \times h_1 = \frac{\pi}{4} d_2^2 \times h_2$

$d_1^2 \times h_1 = d_2^2 \times h_2$

$\left(\frac{d_1}{d_2}\right)^2 = \frac{h_2}{h_1}$

$\left(\frac{3}{4}\right)^2 = \frac{h_2}{h_1}$

$\frac{9}{16} = \frac{h_2}{h_1}$

$\frac{h_1}{h_2} = \frac{16}{9}$

$h_1 : h_2 = 16:9$

$\therefore$ heights of cylinders are in the ratio $= 16:9$

10. Let 'r' be the radius of cylinder

h be the height of cylinder

Volume $V = \pi r^2 \times h$

Now radius is halved $= r' = \frac{r}{2}$

Height is doubled $= h' = 2h$

new volume $V' = \pi r'^2 \times h'$

$= \pi \left(\frac{r}{2}\right)^2 \times (2h)$

$= \frac{\pi r^2}{4} \times 2h$

$V' = \frac{\pi r^2 \times h}{2}$

$V' = \frac{V}{2}$

$\therefore$ New volume is half of original volume.

11.

Dimensions of tin sheet = 30cm × 18cm

<u>When rolled along it's length (30cm)</u>

$2\pi r = 30$, $h = 18cm$

$r = \frac{30}{2\pi}$

$r = 4.77cm$

Volume $= \pi r^2 \times h$

$= \pi \times 4.77^2 \times 18$

Volume $= 1289.155\ cm^3$

When rolled along breadth (18cm)

$2\pi r = 18$, $h = 30cm$

$r = \frac{18}{2\pi}$

$r = 2.86cm$

Volume $= \pi r^2 \times h$

$= \pi \times 2.86^2 \times 30$

$= 773.493 \ cm^3$

(b) 12.

(i) Given dia of pipe = 7cm = 0.07m

Velocity = 5m/sec

Discharge = Area × Velocity

$= \frac{\pi d^2}{4} \times V$

$= \frac{\pi \times (0.07)^2}{4} \times 5$

Discharge $= 0.0192 \ m^3/sec$

∴ Discharge = 19.2 lits/sec

$= 19.2 \times 60$ lits/min

Discharge = 1154.53 lits/min.

∴ Discharge ≈ 1155 lit/min.

(ii) Dimension of tank = $4m \times 3m \times 2.31m$

Discharge = $0.0192\ m^3/sec$

$= 1.154\ m^3/min$

Time taken to fill the tank = $\frac{\text{Volume of tank}}{\text{Discharge}}$

$= \frac{4 \times 3 \times 2.31}{1.154}$

Time taken to fill the tank = 24 min

13. Given

	vessel 1	vessel 2
radius	15cm	20cm
height	40cm	45cm
Volume	$\pi \times r^2 \times h$	$\pi \times r^2 \times h$
	$\pi \times (15)^2 \times 40$	$\pi \times (20)^2 \times 45$
Volume	$28274.33\ cm^3$	$56548.667\ cm^3$

given another vessel with capacity equal to sum of vessel 1 and vessel 2.

Let radius of vessel '3' = R

Height of vessel 3 = 30cm.

$(\pi R^2) \times 30 = 28274.33 + 56548.667$

$30 \times (\pi R^2) = 84823$

$R^2 = \frac{84823}{\pi \times 30}$

$R^2 = 900$

$R = \sqrt{900}$

$\boxed{R = 30\,cm}$

$\therefore$ Radius of Vessel $= 30cm$.

14. Given

Pole height $=$ ~~700cm.~~ $7m$

Pole diameter $=$ $20cm$ $= 0.2\,m$

density $=$ $225\,Kg/m^3$

Volume of wood $= \frac{\pi d^2}{4} \times h$

$= \frac{\pi}{4} \frac{(20)^2}{10^4} \times 7$

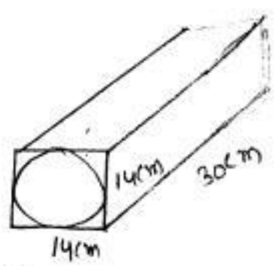
Volume of wood $= 0.219\,m^3$

weight of wood $=$ Volume $\times$ density

$= 0.219 \times 225$

weight of wood $= 49.48\,Kg$.

15.



A cylinder with diameter of 14cm and height of 30cm is the maximum volume that can be cut from ~~the~~ given cuboid.

Volume of cylinder = $\frac{\pi}{4} d^2 \times h$

$= \frac{\pi}{4} (14)^2 \times 30$

Volume of cylinder = 4618.14 cm^3

Volume of wood wasted = Volume of cuboid − Volume of cylinder

$= 14 \times 14 \times 30 - (4618.14)$

Volume of wood wasted. = 1261.85 cm^3

Exercise 18.4

1. Given Surface area = $384\ cm^2$

(i) Let length of side of cube = a

Surface area of cube = $6a^2$

$$6a^2 = 384$$

$$a^2 = \frac{384}{6}$$

$$a^2 = 64$$

$$a = \sqrt{64}$$

$$\boxed{a = 8cm}$$

$\therefore$ length of an edge = 8cm

(ii) Volume of the cube.

Volume of the cube = a^3

$= 8^3$

$= \cancel{256}$

Volume of the cube $= 512\ cm^3$

2. Given

radius of Cylinder = 5cm

height of Cylinder = 10cm

Surface area of Cylinder = $2\pi r h$

$= 2\pi \times 5 \times 10$

Surface area of Cylinder $= 100\pi$

3. Given

Aquarium dimensions = 70cm × 28cm × 35cm

To cover base, side and back faces total area of

paper needed = $2(lb + bh + lh)$

$= 2(70 \times 28 + 28 \times 35 \times 70 \times 35)$

$= 2(5390)$

$= 10780\ cm^2$

4.

Given

Internal dimensions of hall = 15m × 12m × 4m

Area of four walls = $2(lb+$

$= lh + bh + lh + bh$

$= 2(lh + bh)$

$= 2(15 \times 4 + 12 \times 4)$

$= 2(60 + 48)$

Area of four walls $= 2 \times 108$

$= 216\ m^2$

Given

4 windows of dimension = 2m × 1.5m

2 doors of dimension = $1.5 \times 2.5\ m^2$

∴ Remaining walls area = Area of four walls − [4 × Area of window + 2 × area of door]

$= 216 - [4 \times 2 \times 1.5 + 2 \times 1.5 \times 2.5]$

$= 216 - [12 + 7.5]$

$= 216 - 19.5$

∴ Remaining walls area $= 196.5\ m^2$

Given

Cost for white washing walls = ₹5/m^2

Total cost of white washing walls = 5×196.5

$= ₹982.5$

Area of ceiling = $lb = 15 \times 12$

$= 180 m^2$

Total Area = Area of walls + Area of Ceiling

$= 196.5 + 180$

Total Area $= 376.5 m^2$

Total cost of white washing walls including ceiling

$= 5 \times 376.5$

$= ₹1882.5$

5. Swimming pool length = 50m

breadth = 30m

height = 2.5m

Area of walls and base = $lb + lh + bh + lh + bh$

$= 2(lh + bh) + lb$

$= 2(50 \times 2.5 + 30 \times 2.5) + 50 \times 30$

$= 400 + 1500$

Area of walls and base $= 1900 m^2$

Given Cementing rate = ₹ 27/m^2

∴ Total cost for cementing = 27×1900

= ₹ 51300

6. Given rectangular hall perimeter = 236 m

hall height = 4.5 m

Surface area of walls = $2h(l+b)$

= 4.5×236

Surface area of walls ≈ 1062 m^2

Painting of walls cost = ₹ 8.4/m^2

Total cost of painting = 8.4×1062

Total cost of painting = ₹ 8920.8

7. Dimension of fish tank = 30cm × 20cm × 20cm

Given only $\frac{3}{4}$th of tank contains water

∴ Volume of water = 30cm × 20cm × $20 \times \frac{3}{4}$ cm

Volume of water = 30cm × 20cm × 15cm.

Area of tank in contact with water =

walls area upto water level + base area

= $2h(l+b) + lb$

= $2(30+20) \times 15 + 30 \times 20$

= $1500 + 600$

Area of tank in contact with water. = 2100 cm^2

8. Given

(i)

Volume of cuboid = $448\ cm^3$

Let side of square = a cm

height = 7 cm.

$a^2 \times 7 = 448$

$a^2 = \frac{448}{7}$

$a^2 = 64$

$a = \sqrt{64}$

$\boxed{a = 8cm}$

Side of square base = 8cm

(ii)

Surface area cuboid = $2(a^2 + 2ah)$

$= 2(8^2 + 2\times8\times7)$

$= 2(176)$

Surface area of cuboid. $= 352\ cm^2$

9. Given

total Surface area of rectangular solid = $1216\ cm^3$

Ratio of length, breadth and height = 5:4:2

Let length, breadth and height = $5x, 4x, 2x$

total Surface area = 1216

$2(lb + bh + hl) = 1216$

$2(5x\times4x + 4x\times2x + 2x\times5x) = 1216$

$2(20x^2 + 8x^2 + 10x^2) = 1216$

$2\times38x^2 = 1216$

$$76x^2 = 1216$$

$$x^2 = \frac{1216}{76}$$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = 4\,cm$$

$\therefore$ length $5x = 5 \times 4 = 20$ cm

breadth $4x = 4 \times 4 = 16$ cm

height $2x = 2 \times 4 = 8$ cm.

Volume of rectangular solid $= l \times b \times h$

$= 20 \times 16 \times 8$

$= \cancel{2880}$

Volume of rectangular solid $= 2560\ cm^3$

10. Dimension of room $= 6 \times 5 \times 3.5\ m^3$

Dimensions of window $=$ ~~1.4m × 2m~~ $1.5m \times 1.4m$

Dimension of door $= 1.1m \times 2m$

Area of walls $= 2h(l+b) - [3 \times 1.5 \times 1.4 + 2 \times 1.1 \times 2]$

$= 2 \times 3.5(6+5) - [6.3 + 4.4]$

$= 77 - 10.7$

Area of walls $= 66.3\ m^2$

Area of ceiling $= lb = 6 \times 5 = 30\ m^2$

Total area = $66.3 + 30 = 96.3 m^2$

cost of white washing = ₹ $5.3/m^2$

Total cost = area × cost/m^2

= 96.3×5.3

Total cost = ₹ 510.39

11. Given

dimensions of Cuboidal block = $36cm \times 32cm \times 25cm$.

(i) Volume of cuboidal block = $36 \times 32 \times 25$

= $28800\ cm^3$

Cube of edge = 4cm.

Volume of cube = 4^3

= $64\ cm^3$

no. of cubes = $\frac{\text{Volume of cuboidal block}}{\text{Volume of cube}}$

= $\frac{28800}{64}$

no. of cubes = 450

∴ From given Cuboid 450 cubes of edge 4cm can be casted.

(ii) Cost of silver coating = ₹ 0.75 /cm²

Surface area of cube = $6a^2$

$= 6 \times 4^2$

$= 6 \times 16$

Surface area of cube $= 96\ cm^2$

Total surface area of cubes $= 450 \times 96$

$= 43200\ cm^2$

Total surface area of cubes $= 4.32\ m^2$

Cost of silver coating of all cubes $= 4.32 \times 0.75 \times 10^4$

$= ₹ 32400$

∴ Total cost for silver coating of cubes is ₹ 32400

12. Given three cubes of edge lengths = 3cm, 4cm, 5cm.

New cube edge length = a cm

∴ $a^3 = 3^3 + 4^3 + 5^3$

$a^3 = 216$

$a = (216)^{1/3}$

$\boxed{a = 6cm}$

Surface area of cube $= 6a^2$

$= 6 \times 6^2$

Surface area of cube $= 216\ cm^2$

Cost of gold coating = ₹ 3.5 / cm²

Total cost ~~for~~ of gold coating of cube

$= \text{Area} \times \text{cost}/cm^2$

$= 216 \times 3.5$

$= ₹756$

$\therefore$ Total cost of gold coating of cube $= ₹756$

13. Given

Surface area of cylinder $= 4375\ cm^2$

rectangular sheet width $= 35\ cm$.

Perimeter of circle (base) $= 35\ cm$

$2\pi r = 35$

$r = \frac{35}{2\pi}$

radius of base $(r) = 5.57\ cm$

Surface area $= 2\pi r h = 4375$

$= 2\pi \times r \times h = 4375$

$35 \times h = 4375$

$h = \frac{4375}{35}$

$h = 125\ cm$

Height of cylinder $= 125\ cm$

$\therefore$ ~~Height of Sheet~~ =

length of ~~cylinder~~ sheet $= 125\ cm$

Perimeter of sheet $= 2(l+w)$

$= 2(125+35) \Rightarrow 2(160)$

$= 320\ cm$

14. Road roller diameter = 0.7m

Road roller width = 1.2m

Play ground size = 120m × 44m

Area of ground = 5280 m^2

Surface area of roller = $\pi d w$

$= \pi \times 0.7 \times 1.2$

Surface area of roller = $2.638 m^2$

No. of revolutions to cover ground = $\frac{\text{Area of ground}}{\text{Surface area of roller}}$

$= \frac{5280}{2.638}$

$= 2000.8 \approx \cancel{2001}$ 2000

∴ ~~No.~~ Minimum no. of revolutions to cover ground is 2000.

15. Given Diameter of cylindrical container = 14cm

Height of cylindrical container = 20cm

Label height = 20 − (2+2)

= 16cm.

∴ Area of label = Surface area of cylinder of height 16cm

$\cancel{= \frac{\pi}{4} d^2 \times h}$

$\cancel{= \frac{\pi}{4} \times 14^2 \times 16}$

$\cancel{= 2463}$

$= \pi d h$

$= \pi \times 14 \times 16$

$= \frac{22}{7} \times 14 \times 16$

$\therefore$ Area of label $= 704\ cm^2$

16. given

Sum of radius and height of cylinder $= 37\ cm$

$r + h = 37 \rightarrow (1)$

total Surface area $= 1628\ cm^2$

$2\pi r h = 1628$

$rh = \frac{1628}{2 \times \pi}$

$rh = \frac{1628 \times 7}{2 \times 22}$

$rh = 259$

$(37 - h) h = 259$

$37h - h^2 = 259$

$h^2 - 37h + 259 = 0$

$h_1 = 27.63$

$r_1 = 37 - 27.63$

$r_1 = 9.37\ cm$

Volume of Cylinder

$= \pi r_1^2 h_1$

$= \pi \times 9.37^2 \times 27.63$

$V_1 = 7620.96\ cm^3$

$h_2 = 9.37$

$r_2 = 37 - 9.37$

$r_2 = 27.63\ cm$

Volume of Cylinder

$= \pi r_2^2 h_2$

$= \pi \times 27.63^2 \times 9.37$

$V_2 = 22472.5\ cm^3$

17. Given

Ratio b/w Curved Surface area and total Surface area $= 1:2$

total Surface area $= 616\ cm^2$

$$2\pi rh : 2\pi r(h+r) = 1:2$$

$$\frac{h}{h+r} = \frac{1}{2}$$

$$2h = h+r$$

$$\boxed{h = r}$$

$\therefore$ Height = radius

total Surface area $= 616\ cm^2$

$$2\pi r(h+r) = 616\ cm^2$$

$$2\pi r(r+r) = 616\ cm^2$$

$$2(2\pi r^2) = 616\ cm^2$$

$$2\pi r^2 = 308\ cm^2$$

$$r^2 = \frac{308}{2\pi}$$

$$r^2 = 49$$

$$r = \sqrt{49}$$

$$r = 7\ cm$$

$$r = h = 7\ c.m$$

Volume of cylinder $= \pi r^2 h$

$= \pi (7)^2 \times 7$

Volume of cylinder $= 1077.56\ cm^3$

18. length of cylinder $= 77cm = h$

Inner diameter$(d_1) = 4\ cm$

Outer diameter$(d_2) = 4.4cm$

(i) Inner curved surface area

$$= \pi d_1 h$$

$$= \pi \times 4 \times 77$$

$$= 967.61\ cm^2$$

(ii) Outer curved surface area

$$= \pi d_2 h$$

$$= \pi \times 4.4 \times 77$$

$$= 1064.37\ cm^2$$

(iii) total surface area

$$\pi d_1 h + \pi d_2 h + 2 \times \pi (r_2^2 - r_1^2)$$

$$967.63 + 1064.37 + 2\pi (2.2^2 - 2^2)$$

$$= 967 + 1064.37 + 5.27$$

$$= 2038.08\ cm^2$$