14

MATHEMATICAL REASONING

INTRODUCTION

In ancient Greece, a group of travelling teachers called Sophists had the reputation of being able to argue for any point. They would confuse people with arguments like :

Laika is my cat.

Laika is a mother.

Therefore, Laika is my mother.

To help expose the logical tricks used by Sophists, Aristotle wrote a book titled 'Refutation of the Sophists'. This book mentions many illogical patterns of reasoning (fallacies), which are quite effective for detecting bad argumentation. In general, the goal of study of Logic is to construct good or sound arguments, and to recognise bad or unsound arguments. *Thus, Logic is the science of reasoning.*

Though Aristotle was one of the earliest writers on Logic, the first one to employ mathematical methods in the study of Logic was English mathematician George Boole (1815 – 1864). Hence sometimes, the study of logic in mathematics is called Boolean logic.

14.1 PROPOSITIONS (STATEMENTS)

To start the study of Geometry, you start with concepts of points and lines. Study of logic is based on the concept of a proposition.

It is important to distinguish between three related concepts :

Utterance : Any verbal expression includes nonsense expressions such as "gubbledook pinaca", as well as sentences (see next).

Sentence : Any utterance which conveys meaning. Sentences include questions ("Where are you going?"), commands ("Eat your lunch!"), expressions of feelings ("Hip, hip, hurray !"), request ("Give me a glass of water"), wish ("Wish you best of luck"), and propositions (see next).

Proposition : (*Statement or Assertion*) Any sentence which is either true or false. Some examples are :

(*i*) Two plus two equals four.

(*ii*) π is a rational number.

(*iii*) Indira Gandhi was first woman President of India.

(*iv*) An elephant weighs more than a human being.

(*v*) It was raining in Delhi during new year's eve in 2004.

We can immediately see that first and fourth sentences are correct (true) while second and third sentences are incorrect (false). There is no confusion. However, though we may not recall or know immediately whether the fifth sentence above is true or false, it is obvious that it will have only one value — true or false. It cannot be both true and false at the same moment. Hence it is also a statement.

Notice that a statement is either *true* or *false*, or equivalently, either *valid* or *invalid*. It cannot be both true or false at the same time. This fact is known as the *law of the excluded middle*.

Thus a sentence is called a mathematical acceptable statement (or proposition) if it is either true or false but not both.

Now consider the following sentences :

(*i*) Girls are more intelligent than boys.

(*ii*) Sum of two integers x and y is greater than 0.

(*iii*) Product of two integers x and y is greater than 0.

(*iv*) Open the door.

(*v*) Where were you born?

(*vi*) What a lovely day!

(*vii*) Tomorrow is sunday.

(*viii*) It was raining yesterday.

(*ix*) He is a Mathematics teacher.

(*x*) Srinagar is far from here.

None of these sentences is a statement (proposition). Regarding first sentence, some people may think that it is true while some people may think that it is false. This means that it is an ambiguous sentence and hence it is not a statement.

Second sentence may be true for some values of x and y (*e.g.* $x = 2$, $y = 3$) while it may be false for some values (*e.g.* $x = -2$, $y = 1$). Hence second sentence is not a statement. Third sentence may by true for some values of x and y (*e.g.* $x = 3$, $y = 5$) while it may be false for some values (*e.g.* $x = -3$, $y = 5$). Hence third sentence is not a statement.

Sentences (*iv*), (*v*), (*vi*) are not statements because they are command, question and exclamation respectively.

Sentences (*vii*) and (*viii*) are *not* statements. Sentences containing words like today, yesterday, tomorrow are not considered statements.

Sentence (*ix*) is not a statement, as it is not clear who is 'he'. Similarly, (*x*) is not a statement as it is not clear what is 'here'. Hence sentences containing pronouns where person referred to are not clearly specified and sentences containing unspecified places ('here', 'there') etc. are *not* mathematically acceptable statements.

Can you tell which of the following are propositions (statements) and which are not ?

1. Ob la di ob la da.
2. Left, Right, Left !
3. Good morning to all.
4. Raise your hands !
5. Are you going to Delhi ?
6. Have you ever gone to America?
7. May God bless you!
8. How tall is Bobby ?
9. Bobby is 3 metres tall.
10. Sania plays tennis.
11. The sun is a star.
12. All roses are white.
13. All integers are natural numbers.

14. All squares are rectangles.
15. $5 + 3 = 10$.
16. $5 + 3 = 8$.
17. 9 is greater than 14.
18. There are 40 days in a month.

(**Answer.** Last 10 are propositions, first 8 are not).

By convention, propositions are represented by lower case letters p, q, r etc. (with or without subscripts). 'True' or 'False' (abbreviated T and F respectively) is called the *truth value* or *logical value* of any statement according as the statement is true or false.

Thus the statements

'The sum of 3 and 5 is 8',

and '$3 + 5 \geq 7$'

are true and have truth value "True" or "T",

while the statements

'$3 + 5 = 10$'

and 'Every set is finite'

are false and have truth value "False" or "F".

Now consider the sentence "$x + 3 = 5$". The truth value of this sentence can not be found till we are told the value of x. Such sentences are called *"open sentences". They are not statements.* Consider the sentences "$x + 0 = x$" and "$(x - 1)(x + 1) = x^2 - 1$". Regardless of the value of x, they always have truth value "True". Hence, *algebraic identities are statements* (*propositions*).

EXERCISE 14.1

Very short answer type questions (1 to 3) :

1. Identify which of the following are statements (propositions) :

(*i*) Krishna is black. (*ii*) How black is Krishna?
(*iii*) Listen to me ! (*iv*) A triangle has four sides.
(*v*) The moon revolves around the earth. (*vi*) Prime factors of 6 are 2 and 3.
(*vii*) Do you know the prime factors of 6? (*viii*) Do your homework.
(*ix*) $x^2 + 5x + 6 = 0$. (*x*) $x^2 + 5x + 6 = (x + 2)(x + 3)$.
(*xi*) $2 + 2 = 4$. (*xii*) $2 + 2 = 5$.
(*xiii*) $2 + 5 < 11$. (*xiv*) The earth is a star.

2. Which of the following sentences are statements? Give reasons for your answer.

(*i*) Mathematics is fun. (*ii*) The sun is a star.
(*iii*) There is no rain without clouds. (*iv*) How far is Mumbai from here?
(*v*) Mathematics is difficult. (*NCERT*)
(*vi*) The square of an even integer is an even integer.
(*vii*) Answer this question. (*NCERT*) (*viii*) Today is a windy day. (*NCERT*)
(*ix*) The sides of a rhombus have equal lengths.
(*x*) There are 35 days in a month. (*NCERT*) (*xi*) All real numbers are complex numbers. (*NCERT*)
(*xii*) All complex numbers are real numbers. (*xiii*) There are twelve days in a week.
(*xiv*) The square of an odd integer is an even integer.
(*xv*) Tomorrow is holiday. (*xvi*) $x - 12 = 5$.
(*xvii*) She is a mathematics graduate.
(*xviii*) Every square is a rectangle. (*NCERT Examplar Problems*)
(*xix*) $\sin^2 x + \cos^2 x = 0$. (*NCERT Examplar Problems*)

3. State the truth values of the following :

(*i*) There are only finite number of rational numbers.

(*ii*) The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in \mathbf{R}$, has always two real roots.

(*iii*) Sky is red.

(*iv*) New Delhi is the capital of India.

(*v*) Every rectangle is a square.

(*vi*) Zero is a complex number.

(*vii*) $2 + 2 = 4$.

(*viii*) $2 + 2 = 5$.

(*ix*) $x - 0 = x$.

(*x*) The equation $x^2 - 1 = 0$ has two roots, $+1$ and -1.

(*xi*) Pigs fly.

(*xii*) $x^3 + 1 = (x + 1)(x^2 - x + 1)$.

4. Fill in the blanks :

(*i*) Logic is the study of ....

(*ii*) Axiomatic (mathematical) approach to the study of logic was first used by ....

(*iii*) Imperative, exclamatory and interrogative sentences are not ....

(*iv*) The law of the excluded middle says that ....

(*v*) An open sentence is a statement. True or False? ....

(*vi*) Algebraic identities are statements. True or False? ....

5. Give five examples of sentences which are not statements. Give reason for the answers.

14.2 NEGATION OF A STATEMENT

The denial of a statement is called the *negation* of the statement.

Given the statement p : "Raja is happy", consider the following statements :

(*i*) Raja is not happy.

(*ii*) Raja is unhappy.

(*iii*) Raja is sad.

(*iv*) It is false that Raja is happy.

(*v*) It is not true that Raja is happy.

(*vi*) It is not the case that Raja is happy.

Each of these statements is opposite in meaning to the given statement.

This is called negation of proposition p, and is denoted by $\sim p$, p' or $\bar{p}$. As the symbol $\sim$ applies to a single statement, it is sometimes called a *modifier* rather than *connective.*

Thus, negation of "Sky is purple" is "Sky is not purple"; negation of "$3 + 3 = 5$" is "$3 + 3 \neq 5$" or "It is false that $3 + 3 = 5$"; negation of "$x \in$ A" is "$x \notin$ A", and so on.

REMARK

The negative of the statements that contain the words like "for all", "there exists", "some" or "for every" can be tricky. For example :

(*i*) The negation of "All mathematicians are men" is not

"All mathematicians are not men".

Infact, the negation of the given statement is

"Not all mathematicians are men"

or "There exists a mathematician who is not a man"

or "It is false that all mathematicians are men".

(*ii*) The negation of "There exists a dog which does not bite" is

"There does not exist a dog which does not bite"

or "All dogs bite".

Now consider the truth/false value of following statements :

Statement	*Value*
Paris is in France	True
2 + 2 = 4	True
Sky is purple	False
Giraffes are short	False

Now consider the negations of these statements (propositions) and their truth/false value :

(Negated) statement	*Value*
Paris is not in France	False
2 + 2 ≠ 4	False
Sky is not purple	True
Giraffes are not short	True

Notice what happened. Negation turns a true statement into a false statement, and a false statement into a true statement. In other words, if p is true, then $\sim p$ is false; if p is false, then $\sim p$ is true. This can be written in the form of a table as

p	$\sim p$
T	F
F	T

Such tables are called **truth tables**. Note that logical value 'T' stands for 'True' and logical value 'F' stands for 'False'.

Can you write the negation of following sentences alongwith their logical value (T or F) ?

(*i*) 1 + 1 = 2

(*ii*) 1 + 1 = 0

(*iii*) $2 \in \mathbf{N}$

(*iv*) New Delhi is the capital of India.

(*v*) Pigs fly.

(*vi*) All natural numbers are integers.

ILLUSTRATIVE EXAMPLES

Example 1. *Write the negation of the following statements. Also check whether the resulting statements are true or false.*

(i) $\sqrt{7}$ *is rational.* *(NCERT)*

(ii) Sum of 3 and 4 is 9. (*NCERT*)

(iii) Every natural number is greater than 0. (*NCERT*)

(iv) Australia is a continent. (*NCERT*)

Solution. (*i*) Negation of the given statement is :

It is false that $\sqrt{7}$ is rational.

It may also be written as

$\sqrt{7}$ is not rational.

or as

$\sqrt{7}$ is irrational

We know that this is true. (Notice that original statement was false).

(*ii*) Negation of the given statement is :

It is false that sum of 3 and 4 is 9.

It may also be written as :

Sum of 3 and 4 is not 9.

We know that this is true. (Notice that original statement was false.)

(*iii*) Negation of given statement is :

It is false that every natural number is greater than 0.

It can also be written as :

There exists atleast one natural number which is not greater than 0.

We know that this is false. (Notice that original statement was true).

(*iv*) Negation of given statement is :

It is false that Australia is a continent.

It can also be written as :

Australia is not a continent.

We know that this is false. (Notice that original statement was true).

Example 2. *Write the negations of the following statements :*

(i) Everyone in France speaks French.

(ii) Both the diagonals of a rectangle have the same length. *(NCERT)*

(iii) There does not exist a quadrilateral which has all sides equal. *(NCERT)*

Solution. (*i*) It is false that everyone in France speaks french.

Alternatively, it can be written as there exists atleast one person in France who does not speak French.

(*ii*) It is false that both the diagonals of a rectangle have the same length.

Alternatively, it can be written as there is atleast one rectangle whose both diagonals do not have the same length.

(*iii*) It is not the case that there does not exist a quadrilateral which has all sides equal.

Alternatively, it can be written as there exists a quadrilateral which has all sides equal.

EXERCISE 14.2

Very short answer type questions (1 to 4) :

1. Write the negation of the following statements :

(*i*) The number 17 is prime. (*NCERT Examplar Problems*)

(*ii*) $\sqrt{5}$ is a rational number. (*NCERT Examplar Problems*)

(*iii*) $\sqrt{2}$ is not a complex number. (*NCERT*)

(*iv*) 2 + 7 = 6. (*NCERT Examplar Problems*)

(*v*) The number 2 is greater than 7. (*NCERT*)

(*vi*) Cow has four legs. (*NCERT Examplar Problems*)

(*vii*) A leap year has 366 days. (*NCERT Examplar Problems*)

(*viii*) Every natural number is an integer. (*NCERT*)

(*ix*) Every real number is an irrational number. (*NCERT Examplar Problems*)

(*x*) All triangles are not equilateral. (*NCERT*)

(*xi*) All similar triangles are congruent. (*NCERT Examplar Problems*)

(*xii*) Area of every circle is the same as the perimeter of the circle. (*NCERT Examplar Problems*)

2. Are the following pairs of statements negations of each other?

(*i*) The number x is not a rational number.

The number x is not an irrational number. (*NCERT*)

(*ii*) The number x is not a rational number.

The number x is an irrational number.

3. Write the negation of "All men are mortal"

(*i*) Using "There exists atleast..." (*ii*) Without using "There exists..."

4. Write down the negation of the statement "All the sides of an equilateral triangle are of the same length".

5. Find out whether the negations of the statements given in question 1 are true or false. Also write the truth value of the original statements.

6. Which of the following statements are negation of the statement "All politicians are honest"?

(*i*) It is not true that all politicians are honest.

(*ii*) All politicians are not honest.

(*iii*) Not all politicians are honest.

(*iv*) There exists a politician which is not honest.

Explain the importance of honesty in politics. ***(Value Based)***

7. Which of the following are the negation of the statement "π is not a rational number" ?

(*i*) π is a rational number. (*ii*) π is an irrational number.

(*iii*) π is not an irrational number.

14.3 COMPOUND STATEMENTS

Simple statement. A statement which cannot be broken into two or more statements is called a ***simple statement.*** For example :

(*i*) Roses are red (*ii*) New Delhi is the capital of India

(*iii*) Every set is a finite set

are all simple statements.

Compound statement. A statement that can be formed by combining two or more simple statements is called a ***compound statement***.

Each statement of a compound statement is called a **component statement**.

Consider the following statement :

p : Sam is very smart or he is very lucky.

This statement is actually made up of two (component) statements connected by "*or*" :

p : Sam is very smart.

r : Sam is very lucky.

Now consider the following statement :

p : Delhi is in India and Islamabad is in Pakistan.

This statement is made up of two simple statements connected by "*and*".

p : Delhi is in India.

r : Islamabad is in Pakistan.

Similarly, the sentence "Navin likes tennis and cricket" is made of two simple statements "Navin likes tennis" and "Navin likes cricket" connected by "and".

ILLUSTRATIVE EXAMPLES

Example 1. *Find the component statements of the following compound statements :*

(i) Zero is a positive number or a negative number. *(NCERT)*

(ii) There is something wrong with the bulb or the wiring.

(iii) Sun is bigger than earth and earth is bigger than moon.

(iv) All rational numbers are real and all real numbers are complex. *(NCERT)*

Solution. (*i*) The component statements are :

p : Zero is a positive number.

q : Zero is a negative number.

The connecting word is "or".

(*ii*) The component statements are :

p : There is something wrong with the bulb.

q : There is something wrong with the wiring.

The connecting word is "or".

(*iii*) The component statements are :

p : Sun is bigger than earth.

q : Earth is bigger than moon.

The connecting word is "and".

(*iv*) The component statements are :

p : All rational numbers are real.

q : All real numbers are complex.

The connecting word is "and".

Example 2. *Find the component statements of the following and check whether they are true or not :*

(i) All prime numbers are either even or odd. *(NCERT)*

(ii) A square is a quadrilateral and its four sides are equal. *(NCERT)*

(iii) Chandigarh is the capital of Haryana and U.P. (*NCERT Examplar Problems*)

(iv) 100 is divisible by 3, 5 and 11. *(NCERT)*

Solution. (*i*) The component statements are :

p : All prime numbers are even.

q : All prime numbers are odd.

Both these statements are false. Here the connecting word is "or".

(*ii*) The component statements are :

p : A square is a quadrilateral.

q : A square has all its four sides equal.

Both these statements are true. Here the connecting word is "and".

(*iii*) The component statements are :

p : Chandigarh is the capital of Haryana.

q : Chandigarh is the capital of U.P.

The first statement is true while the second statement is false. Here the connecting word is "and".

(*iv*) The component statements are :

p : 100 is divisible by 3.

q : 100 is divisible by 5.

q : 100 is divisible by 11.

The second statement is true while first and third are false. Here the connecting word is "and".

EXERCISE 14.3

Very short answer type questions :

1. Find the component statements of the following compound statements. Clearly mention the connecting word.

(*i*) Jack and Jill went up the hill. (*NCERT Examplar Problems*)

(*ii*) Chennai is in India and is the capital of Tamil Nadu. (*NCERT Examplar Problems*)

(*iii*) $\sqrt{7}$ is a rational number or an irrational number. (*NCERT Examplar Problems*)

(*iv*) Number 7 is odd and prime. (*NCERT Examplar Problems*)

(*v*) A rectangle is a quadrilateral or a 5-sided polygon. (*NCERT Examplar Problems*)

2. Find the component statements of the following compound statements and check whether they are true or false.
   (*i*) All integers are positive or negative.
   (*ii*) $\sqrt{2}$ is a rational number or an irrational number.
   (*iii*) A student who has passed Mathematics or Computer Science can go for MCA.
   (*iv*) 36 is a multiple of 2, 6 and 8.

14.4 LOGICAL CONNECTIVES AND QUANTIFIERS

In Arithmetic, you are familiar with operations on numbers, like + (addition), × (multiplication), ÷ (division) etc. In sets, you have used operations like complement ($\overline{A}$, A^c etc.), union ($A \cup B$), intersection ($A \cap B$) etc. In mathematical reasoning, simple statements can be connected by using connectives like 'And', 'Or', 'Implies' etc.

14.4.1 The connective 'And' — Conjunction ($p \wedge q$)

If any two simple statements are combined by the word "and" to form a compound statement, then the resulting statement is called the **conjunction** of the original statements. Symbollically, the conjunction of the two statements p and q is denoted by $p \wedge q$. The elements p and q of $p \wedge q$ are called its **conjucts**.

If p is 'Krishna is rich' and q is 'Sudama is poor', then $p \wedge q$ is 'Krishna is rich and Sudama is poor'. Note that the conjuncts can be switched around to mean the same thing. Thus 'Sudama is poor and Krishna is rich' means the same as previous sentence. In other words, $q \wedge p$ means the same thing as $p \wedge q$. In ordinary conversation, we use conjunctions a lot, but often in disguised form. Some typical examples of disguised forms are :

Type	*Disguised form*
p, but q	It is raining, but the sun is shining.
p, although q	It is raining, altough the sun is shining.
p, besides q	Sam is smart, besides he is intelligent.
p, however q	Bob is hard working, however he is stupid.
p, whereas q	Krishna is rich, whereas Sudama is poor.

In addition to these, sentences such as 'Mango and Banana are fruits' can be rephrased as 'Mango is a fruit and Banana is a fruit'.

Please note that the word 'And' in a statement does *not* always mean that it can be broken into two substatements. Consider the following sentence :

Oil and water do not mix well.

Here the word 'And' is not a connective, and given statement cannot be broken into two substatements — it is a simple statement.

Logical value of conjunction

$p \wedge q$ is true only if both the components p and q are true, otherwise it is false. Consider the following four statements :

(*i*) The sun rises in the east and sets in the west.
(*ii*) The sun rises in the north and sets in the south.
(*iii*) $5 > 3$ and $7 < 5$.
(*iv*) $2 + 2 = 5$ and London is in England.

Here only the statement (*i*) is true as both of its components are true. Other three statements are false as atleast one of the substatements is false.

Truth table of a conjunction is

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Thus, a compound statement with connective(s) 'AND' is true only if all of its component statements are true; otherwise, it is false.

ILLUSTRATIVE EXAMPLE

Example. *Write the component statements of the following compound statements and check whether the compound statement is true or false.*

(i) A point occupies a position and its location can be determined. *(NCERT)*

(ii) $2 + 2 = 5$, whereas $3 + 3 = 6$.

(iii) 42 is divisible by 5, 6 and 7.

(iv) 42 is a multiple of 2, 3 and 7.

Solution. (*i*) The component statements of given statement are :

p : A point occupies a positon.

q : A point's location can be determined.

Here the connecting word is "and".

As both the component statements are true, the given compound statement is true.

(*ii*) The given conjuction is in disguised form — the word "whereas" can be replaced by "and". Hence the component statements are :

p : $2 + 2 = 5$

q : $3 + 3 = 6$.

As the first statement is false, the given compound statement is false.

(*iii*) The given compound statement is made up of three component statements connected by 'and' :

p : 42 is divisible by 5

q : 42 is divisible by 6

r : 42 is divisible by 7.

Second and third component statements are true but the first statement is false. Hence the given compound statement is false.

(*iv*) The given compound statement is made up of three component statements connected by 'and'

p : 42 is a multiple of 2

q : 42 is a multiple of 3

r : 42 is a multiple of 7.

All these three statements are true. Hence the given compound statement is true.

14.4.2 The connective "or" — Disjunction ($p \vee q$)

If any two simple statements are combined by the word "or" to form a compound statement, then the resulting statement is called the **disjunction** of the original statements. Symbolically, the disjunction of the two statements p and q is denoted by $p \vee q$. The elements p and q of $p \vee q$ are called its **conjucts.**

If p is "The sun shines" and q is '6 is prime', then $p \vee q$ is 'The sun shines or 6 is prime'.

REMARK

The use of the word "or" in English language is ambiguous.

10. Rewrite each of the following statements in the form "p if and only if q."

(*i*) If you watch television then your mind is free and if your mind is free, then you watch television.

(*ii*) For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.

(*iii*) If a quadrilateral is equiangular, then it is a rectangle, and vice versa.

11. Given statements in (*i*) and (*ii*), identify the statements given below as contrapositive or converse of each other.

(*i*) (*a*) If you live in Delhi, then you have winter clothes.

(*b*) If you do not have winter clothes, then you do not live in Delhi.

(*c*) If you have winter clothes, then you live in Delhi.

(*ii*) (*a*) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

(*b*) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

(*c*) If the diagonals of a quadrilateral bisects each other, then it is a parallelogram.

12. Write the following statement in five different ways, conveying the same meaning :

If a triangle is equiangular, then it is an obtuse angled triangle.

13. Determine whether the arguments used to check the validity of the following statement is correct.

The statement "If x^2 is irrational then x is irrational" is true because the number $x^2 = \pi^2$ is irrational, therefore $x = \pi$ is irrational.

ANSWERS

EXERCISE 14.1

1. Only (*i*), (*iv*), (*v*), (*vi*), (*x*), (*xi*), (*xii*), (*xiii*), (*xiv*), (*xviii*) and (*xix*) are statements

2. Only (*ii*), (*iii*), (*vi*), (*ix*), (*x*), (*xi*), (*xii*), (*xiii*) and (*xiv*) are statements

3. (*i*) F (*ii*) F (*iii*) F (*iv*) T (*v*) F (*vi*) T (*vii*) T (*viii*) F (*ix*) T (*x*) T (*xi*) F (*xii*) T

4. (*i*) reasoning (*ii*) George Boole (*iii*) statements (propostions)

(*iv*) a statement cannot be both true and false at the same time

(*v*) False (*vi*) True

EXERCISE 14.2

1. (*i*) The number 17 is not prime (*ii*) $\sqrt{5}$ is not a rational number

(*iii*) $\sqrt{2}$ is a complex number (*iv*) $2 + 7 \neq 6$

(*v*) The number 2 is not greater than 7 (*vi*) Cow has not four legs

(*vii*) A leap year has not 366 days.

(*viii*) It is false that every natural number is an integer.

Alternatively **:** There exists a natural number which is not an integer.

(*ix*) There exists a real number which is not an irrational number.

(*x*) It is false that all triangles are not equilateral.

(*xi*) There exist similar triangles which are not congruent.

(*xii*) There exists a circle whose area is not same as the perimeter of the circle.

2. (*i*) Yes (*ii*) No

3. (*i*) There exists atleast one man who is not mortal

(*ii*) Not all men or mortal

Alternatively **:** It is false that all men are mortal

4. There exists an equilateral triangle all of whose sides are not of the same length