9

SEQUENCES AND SERIES

INTRODUCTION

Sequences have many important applications in several spheres of human activities. When a collection of objects is arranged in a definite order such that it has an identified first member, second member, third member and so on, we say that the collection is listed in a sequence.

For example :

(*i*) The amount of money in a fixed deposit in a bank over a number of years occur in a sequence.

(*ii*) Depreciated values of certain commodities occur in a sequence.

(*iii*) The population of bacteria at different times forms a sequence.

(*iv*) Consider the number of ancestors *i.e.* parents, grandparents, great grandparents etc. that a person has over 10 generations.

The numbers of person's ancestors for the first, second, third, …, tenth generations are 2, 4, 8, …, 1024. These numbers form a sequence.

Sequences, following specific patterns are called **progressions.** In the previous class, you have already studied about arithmetic progression (A.P.). In this chapter, besides studying more about A.P., we shall also study—arithmetic mean (A.M.), geometric progression (G.P.), geometric mean (G.M.), relationship between A.M. and G.M., arithmetico-geometric series and sum to n terms of special series Σn, Σn^2, Σn^3 etc.

9.1 SEQUENCE

A set of numbers arranged in a definite order according to some definite rule (or rules) is called a ***sequence.*** Each number of the set is called a **term** of the sequence.

A sequence is called ***finite*** *or* ***infinite*** *according as the number of terms in it is finite or infinite.*

The different terms of a sequence are usually denoted by $a_1, a_2, a_3, \ldots$ or by $T_1, T_2, T_3, \ldots$. The subscript (always a natural number) denotes the position of the term in the sequence. The number occurring at the nth place of a sequence *i.e.* a_n is called the *general term* of the sequence.

A finite sequence is described by $a_1, a_2, a_3, \ldots, a_n$ and an infinite sequence is described by $a_1, a_2, a_3, \ldots$ to ∞ .

If all the terms are real, we have a real sequence; if all terms are complex numbers, we have a complex sequence etc.

For example, consider the following sequences :

(*i*) 3, 5, 7, 9, …, 21 (*ii*) 8, 5, 2, −1, −4, … (*iii*) 2, 6, 18, 54, …, 1458

(*iv*) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$ (*v*) 1, 4, 9, 16, … (*vi*) 2, 3, 5, 7, 11, 13, …

(*vii*) 1, 1, 2, 3, 5, 8, 13, …

We observe the following :

(*i*) Here each term is obtained by adding 2 to the previous term.

(*ii*) Here each term is obtained by subtracting 3 from the preceding term.

(*iii*) Here each term is obtained by multiplying the preceding term by 3.

(*iv*) Here each term is obtained by multiplying the preceding term by $-\frac{1}{2}$.

(*v*) Here each term is obtained by squaring the next natural number.

(*vi*) This is the sequence of prime numbers.

(*vii*) Here each term after second term is obtained by adding the previous two terms.

Also note that sequences (*i*) and (*iii*) are finite sequences whereas others are infinite sequences. *Moreover to define a sequence, we need not always have an explicit formula for the nth term.* Till today, nobody has found the formula for nth prime number.

Also note that, in

(*i*) $a_n = a_{n-1} + 2$ (*ii*) $a_n = a_{n-1} - 3$ (*iii*) $a_n = 3a_{n-1}$

(*iv*) $a_n = -\frac{1}{2}a_{n-1}$ (*v*) $a_n = n^2$ (*vii*) $a_n = a_{n-1} + a_{n-2}$ $(n > 2)$

and in (*vi*) we may describe a_n = nth prime number.

If the terms of a sequence can be described by an explicit formula, then the sequence is called a ***progression****.*

Note that the sequences (*i*) to (*v*) given above are all progressions, whereas sequence (*vi*) is not a progression.

The sequence (*vii*) *i.e.* 1, 1, 2, 3, 5, 8, 13, … , is also a progression. It is called **Fibonacci sequence**.

9.2 SERIES

If the terms of a sequence are connected by plus signs we get a ***series****.*

Thus, if $a_1, a_2, a_3, \ldots$, is a given sequence then the expression $a_1 + a_2 + a_3 + \ldots$ is called the series associated with the given sequence. The series is finite or infinite according as the given sequence is finite or infinite.

From sequences (*i*) to (*v*) given above, we can form following series :

(*i*) $3 + 5 + 7 + \ldots + 21$ (*ii*) $8 + 5 + 2 + (-1) + (-4) + \ldots$

(*iii*) $2 + 6 + 18 + 54 + \ldots + 1458$ (*iv*) $1 + \left(-\frac{1}{2}\right) + \frac{1}{4} + \left(-\frac{1}{8}\right) + \ldots$

(*v*) $1 + 4 + 9 + 16 + \ldots$

If a_n denotes the general term of a sequence, then $a_1 + a_2 + a_3 + \ldots + a_n$ is a series of n terms.

In a series $a_1 + a_2 + a_3 + \ldots + a_k + \ldots$, the sum of first n terms is denoted by S_n.

Thus $S_n = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{k=1}^{n} a_k$.

If S_n denotes the sum of n terms of a sequence, then

$$S_n - S_{n-1} = (a_1 + a_2 + \ldots + a_n) - (a_1 + a_2 + \ldots + a_{n-1}) = a_n.$$

Thus, $a_n = S_n - S_{n-1}$.

REMARK

The word 'series' is referred to the indicated sum not to the sum itself. For example, 1 + 3 + 5 + 7 + 9 is a finite series with five terms. By the words 'sum of a series' will mean the number that results from adding the terms, so the sum of the above series is 25.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the next term of the sequence*

(i) $2, 4, 6, 8$ (ii) $2, 1, \frac{1}{2}, \frac{1}{4}$ (iii) $2, 8, 32, 128$

(iv) $-1, -3, -5, -7$ (v) $1, 8, 27, 64$.

Solution. (*i*) We see that each term is obtained by adding 2 to the previous term. Hence, the next term = 8 + 2 = 10.

(*ii*) Here we see that each term is obtained by multiplying the previous term by $\frac{1}{2}$.

Hence, the next term = $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.

(*iii*) We see that each term is obtained by multiplying the previous term by 4.

Hence, the next term = $128 \times 4 = 512$.

(*iv*) Here each term is obtained by subtracting 2 from the previous term.

Hence, the next term = $-7 - 2 = -9$.

(*v*) We see that terms are cubes of natural numbers — $1^3, 2^3, 3^3, 4^3$. Hence, the next term of the sequence is 5^3 *i.e.* 125.

Example 2. *Write the first four terms of the sequence defined by*

(i) $a_n = 4n^2 + 3$ (ii) $a_n = n$th prime number.

Solution. (*i*) Given $a_n = 4n^2 + 3$. Putting $n = 1, 2, 3, 4$, we get

$a_1 = 4 \times 1^2 + 3 = 7$, $a_2 = 4 \times 2^2 + 3 = 19$,

$a_3 = 4 \times 3^2 + 3 = 39$, $a_4 = 4 \times 4^2 + 3 = 67$.

Hence, the first four terms of the given sequence are 7, 19, 39, 67.

(*ii*) We know that the first four prime numbers are 2, 3, 5, 7.

Hence, the first four terms of the sequence are 2, 3, 5, 7.

Example 3. *Find the 20th term of the sequence defined by* $a_n = \frac{n(n-2)}{n+3}$. (NCERT)

Solution. Given $a_n = \frac{n(n-2)}{n+3}$, putting $n = 20$, we get

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20 \times 18}{23} = \frac{360}{23}.$$

Example 4. *Find the 13th and 14th terms of the sequence defined by*

$$a_n = \begin{cases} n^2, \text{ when } n \text{ is even} \\ n^2 + 1, \text{ when } n \text{ is odd} \end{cases}.$$

Solution. As 13 is odd, $a_{13} = n^2 + 1 = 13^2 + 1 = 169 + 1 = 170$;
and as 14 is even, $a_{14} = n^2 = 14^2 = 196$.

Example 5. *Find the first five terms of the sequence given by $a_1 = 2$, $a_2 = 3 + a_1$ and $a_n = 2a_{n-1} + 5$ for $n > 2$. Also write the corresponding series.* (NCERT)

Solution. Here $a_1 = 2$, $a_2 = 3 + a_1 = 3 + 2 = 5$.

Given $a_n = 2a_{n-1} + 5$ for $n > 2$, putting $n = 3, 4, 5$, we get

$a_3 = 2a_2 + 5 = 2 \times 5 + 5 = 15$

$a_4 = 2a_3 + 5 = 2 \times 15 + 5 = 35$

$a_5 = 2a_4 + 5 = 2 \times 35 + 5 = 75$.

Hence, the first five terms of the given sequence are 2, 5, 15, 35, 75.

The corresponding series is $2 + 5 + 15 + 35 + 75 + \ldots$.

Example 6. *The Fibonacci sequence is defined by* $a_1 = a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ *for* $n > 2$, *find* $\frac{a_{n+1}}{a_n}$ *for* $n = 1, 2, 3, 4, 5$. *(NCERT)*

Solution. Given $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for $n > 2$ ...(*i*)

Putting $n = 3, 4, 5$ and 6 in (*i*), we get

$$a_3 = a_2 + a_1 = 1 + 1 = 2,$$
$$a_4 = a_3 + a_2 = 2 + 1 = 3,$$
$$a_5 = a_4 + a_3 = 3 + 2 = 5 \text{ and}$$
$$a_6 = a_5 + a_4 = 5 + 3 = 8.$$

Putting $n = 1, 2, 3, 4$ and 5 in $\frac{a_{n+1}}{a_n}$, we get

$$\frac{a_2}{a_1}, \frac{a_3}{a_2}, \frac{a_4}{a_3}, \frac{a_5}{a_4}, \frac{a_6}{a_5}$$

i.e. $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$ *i.e.* $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$.

Example 7. (*i*) *Find the first 3 terms of the series* $\Sigma(-1)^{n+1}\, 3^{-n}$.

(*ii*) *The sum of* n *terms of a series is* $n^2 + 3n$ *for all values of* n. *Find the first 3 terms of the series. Also find its 10th term.*

Solution. (*i*) nth term of the given series $a_n = (-1)^{n+1}\, 3^{-n}$.

$\therefore$ First term $= a_1 = (-1)^{1+1}\, 3^{-1} = \frac{1}{3}$.

Second term $= a_2 = (-1)^{2+1}\, 3^{-2} = -\frac{1}{9}$.

Third term $= a_3 = (-1)^{3+1}\, 3^{-3} = \frac{1}{27}$.

Hence, first 3 terms of the given series are $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}$.

(*ii*) Given $S_n = n^2 + 3n \Rightarrow S_{n-1} = (n-1)^2 + 3(n-1) = n^2 + n - 2$

$\therefore \quad a_n = S_n - S_{n-1} = n^2 + 3n - (n^2 + n - 2) = 2n + 2$.

Putting $n = 1, 2, 3$ and 10, we get

$$a_1 = 2 \times 1 + 2 = 4,\ a_2 = 2 \times 2 + 2 = 6,\ a_3 = 2 \times 3 + 2 = 8$$

and $a_{10} = 2 \times 10 + 2 = 22$.

Hence, the first three terms are 4, 6, 8 and the 10th term is 22.

Example 8. *If for a sequence,* $S_n = 2(3^n - 1)$, *find its first four terms.*

Solution. Given $S_n = 2(3^n - 1) \Rightarrow S_{n-1} = 2(3^{n-1} - 1)$.

$$\therefore \quad a_n = S_n - S_{n-1} = 2(3^n - 1) - 2(3^{n-1} - 1)$$
$$= 2(3^n - 3^{n-1}) = 2 \times 3^{n-1}(3 - 1) = 4 \times 3^{n-1}.$$

Putting $n = 1, 2, 3, 4$, we get

$$a_1 = 4 \times 3^0 = 4,\ a_2 = 4 \times 3^1 = 12,\ a_3 = 4 \times 3^2 = 36 \text{ and } a_4 = 4 \times 3^3 = 108.$$

Hence, the first four terms of the sequence are 4, 12, 36, 108.

Example 9. (*i*) *Write* $\sum_{k=1}^{n}(k^2 + 1)$ *in expanded form.*

(*ii*) *Write the series* $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{n}{n+1}$ *in sigma notation.*

Solution. (*i*) Putting $k = 1, 2, 3, 4, \ldots, n$ in $(k^2 + 1)$, we get $2, 5, 10, 17, \ldots, n^2 + 1$

Hence, $\sum_{k=1}^{n} (k^2 + 1) = 2 + 5 + 10 + 17 + \ldots + (n^2 + 1)$.

(*ii*) We see that kth term of series $= \dfrac{k}{k+1}$.

Hence, the given series can be written as $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \ldots + \dfrac{n}{n+1} = \sum_{k=1}^{n} \dfrac{k}{k+1}$.

EXERCISE 9.1

Very short answer type questions (1 to 7) :

1. Give an example of a sequence which is not a progression.
2. Which term of the sequence given by $a_n = n^2 + 2n + 1$, $n \in \mathbf{N}$, is 361 ?
3. Write the first three terms of the sequence whose nth term is given by $a_n = (-1)^{n-1} 5^{-n}$.
4. If a sequence is given by $a_1 = 2$, $a_2 = 3 + a_1$ and $a_n = 2a_{n-1} - 1$ for $n > 2$. Then write the corresponding series upto 4 terms.
5. Write the next term of each of the following sequences :

   (*i*) $3, 1, -1, -3, \ldots$ (*ii*) $5, -\dfrac{5}{2}, \dfrac{5}{4}, -\dfrac{5}{8}, \ldots$
6. Write the next term of each of the following sequences :

   (*i*) $0, 2, 6, 12, 20, \ldots$ (*ii*) $6, 9, 16, 27, 42, \ldots$ (*iii*) $1, 5, 14, 30, 55, \ldots$

   Hint. (*iii*) $5 = 1 + 2^2$, $14 = 5 + 3^2$, $30 = 14 + 4^2$, …
7. Write the eleventh term of the following sequence :

   1, 1, 2, 3, 5, 8, 13, 21, 34, …
8. Write first 5 terms of the following sequences whose nth terms are given by :

   (*i*) $a_n = 2n + 5$ (*NCERT*) (*ii*) $a_n = \dfrac{2n-3}{6}$ (*NCERT*)

   (*iii*) $a_n = n(n-1)$ (*iv*) $a_n = n(n+2)$ (*NCERT*)

   (*v*) $a_n = 2^n$ (*NCERT*) (*vi*) $a_n = \dfrac{n}{n+1}$ (*NCERT*)

   (*vii*) $a_n = (-1)^{n-1} 5^{n+1}$ (*NCERT*) (*viii*) $a_n = \dfrac{n(n^2+5)}{4}$ (*NCERT*)

   (*ix*) $a_n = \dfrac{n^2+1}{2n-3}$.
9. Find the indicated term(s) in each of the following sequences whose nth terms are :

   (*i*) $a_n = 4n - 3$; a_{17}, a_{24} (*NCERT*) (*ii*) $a_n = \dfrac{n^2}{2^n}$; a_5, a_7 (*NCERT*)

   (*iii*) $a_n = (-1)^{n-1} n^3$; a_9 (*NCERT*) (*iv*) $a_n = (n-1)(2-n)(3+n)$; a_1, a_2, a_{20}. (*NCERT*)
10. Find the 18th and 25th terms of the sequence defined by

$$T_n = \begin{cases} n(n+2), & \text{if } n \text{ is even natural number} \\ \dfrac{4n}{n^2+1}, & \text{if } n \text{ is odd natural number.} \end{cases}$$

11. Find the first five terms of each of the following sequences and obtain the corresponding series :

    (*i*) $a_1 = 1$, $a_n = a_{n-1} + 2$, $n \geq 2$

    (*ii*) $a_1 = 3$, $a_n = 3a_{n-1} + 2$, for all $n > 1$ (*NCERT*)

    (*iii*) $a_1 = -1$, $a_n = \dfrac{a_{n-1}}{n}$ for $n \geq 2$ (*NCERT*)

    (*iv*) $a_1 = a_2 = 2$, $a_n = a_{n-1} - 1$ for $n > 2$.
12. If the sum of n terms of a sequence is given by $S_n = 2n^2 + 3n$ for all $n \in \mathbf{N}$, find the first 4 terms. Also find its 20th term.

13. First term of a sequence is 1 and the $(n + 1)$th term is obtained by adding $(n + 1)$ to the nth term for all natural numbers n. Find the sixth term of the sequence.

Hint. $a_{n+1} = a_n + (n + 1)$ for all natural numbers n.

9.3 ARITHMETIC PROGRESSION (A.P.)

A sequence (finite or infinite) is called an ***arithmetic progression*** *(abbreviated A.P.) iff the difference of any term from its preceding term is constant.*

This constant is usually denoted by d and is called **common difference.**

Thus $a_1, a_2, \ldots, a_n$ or $a_1, a_2, a_3, \ldots$ is an A.P. iff $a_{k+1} - a_k = d$, a constant (independent of k) for $k = 1, 2, \ldots, n - 1$ or $k = 1, 2, 3, \ldots$ as the case may be.

It follows that, in an A.P., $a_{n+1} = a_n + d$ *i.e.* any term (except the first) is obtained by adding the fixed number d to its preceding term.

If the sequence $a_1, a_2, a_3, \ldots, a_n$ is an A.P., then the series

$a_1 + a_2 + a_3 + \ldots + a_n$ is called an **arithmetic series**.

General term of an A.P.

Let a be the first term and d be the common difference of an A.P., then the A.P. is

$a, a + d, a + 2d, \ldots$ and its nth term $= a + (n - 1)\, d$.

Hence, general term $\boldsymbol{a_n = a + (n - 1)\, d}$.

Last term of an A.P.

If the last term of an A.P. consisting of n terms is denoted by l, then

$$\boldsymbol{l = a + (n - 1)\, d}.$$

The nth term from the end of a finite A.P.

(*i*) If $a, a + d, a + 2d, \ldots$ is a finite A.P. consisting of m terms, then the nth term from the end

$= (m - n + 1)$th term from beginning

$= a + (\overline{m - n + 1} - 1)\, d = a + (m - n)\, d$.

(*ii*) If $a, a + d, a + 2d, \ldots\ldots$ is a finite A.P. with last term l, then the nth term from end

$= l + (n - 1)(-d) = l - (n - 1)\, d$.

For, when we look at the terms of the given A.P. from the last and move towards beginning we find that the sequence is an A.P. with common difference $-d$ and first term as l, therefore, nth term from the end of the given A.P. $= l + (n - 1)(-d) = l - (n - 1)\, d$.

Sum of n terms of an A.P.

Let a be the first term, d the common difference and l the last term of an A.P. If S_n denotes the sum of its first n terms, then

$$S_n = \frac{n}{2}(2a + (n - 1)\, d) \;\text{ or }\; S_n = \frac{n}{2}(a + l).$$

REMARKS

1. If the sum of n terms of an A.P. is denoted by S_n, then its common difference $= S_2 - 2S_1$.
2. Three numbers a, b and c are in A.P. iff $b - a = c - b$ *i.e.* iff $2b = a + c$.
3. Some problems involve 3, 4 or 5 numbers in A.P.

If the sum of the numbers is given, then in an A.P.,

(i) three numbers are taken as $a - d, a, a + d$

(ii) four numbers are taken as $a - 3d, a - d, a + d, a + 3d$.

(iii) five numbers are taken as $a - 2d, a - d, a, a + d, a + 2d$.

This considerably simplifies the calculations of a and d.

4. The sum of the terms equidistant from the beginning and the end of an A.P. is always the same and equals to the sum of the first and the last terms.

9.3.1 ***If the terms of an arithmetic progression (A.P.) are increased, decreased, multiplied or divided by the same non-zero constant, they remain in arithmetic progression.***

Proof. Consider the A.P. $a, a + d, a + 2d, a + 3d, \ldots$ (1)

(*i*) If each term of (1) is increased by a constant k, we obtain the sequence $a + k, a + d + k, a + 2d + k, a + 3d + k, \ldots$ *i.e.* $a + k, a + k + d, a + k + 2d, a + k + 3d, \ldots$ which is clearly an A.P. whose first term is $a + k$ and common difference is d.

(*ii*) If each term of (1) is decreased by a constant k, we obtain the sequence $a - k, a + d - k, a + 2d - k, \ldots$ *i.e.* $a - k, a - k + d, a - k + 2d, a - k + 3d, \ldots$ which is clearly an A.P. whose first term is $a - k$ and common difference is d.

(*iii*) If each term of (1) is multiplied by a constant k, we obtain the sequence $ak, (a + d)k, (a + 2d)k, \ldots$ *i.e.* $ak, ak + dk, ak + 2dk, ak + 3dk \ldots$ which is clearly an A.P. with first term ak and common difference dk.

(*iv*) When each term of (1) is divided by $k (\neq 0)$, we obtain the sequence

$$\frac{a}{k}, \frac{a}{k} + \frac{d}{k}, \frac{a}{k} + 2\left(\frac{d}{k}\right), \frac{a}{k} + 3\left(\frac{d}{k}\right), \ldots$$

which is clearly an A.P. with first term $\frac{a}{k}$ and common difference $\frac{d}{k}$.

ILLUSTRATIVE EXAMPLES

Example 1. *The fourth term of A.P. is equal to 3 times its first term and seventh term exceeds twice the third term by 1. Find the first term and the common difference.*

Solution. Let a be the first term and d be the common difference.

Now $a_4 = 3a_1 \Rightarrow a + 3d = 3a \Rightarrow 3d = 2a$ …(*i*)

$a_7 = 2a_3 + 1 \Rightarrow a + 6d = 2(a + 2d) + 1 \Rightarrow 2d = a + 1$ …(*ii*)

Solving (*i*) and (*ii*) simultaneously, we get $a = 3, d = 2$.

Hence the first term of the given sequence is 3 and common difference is 2.

Example 2. *In a sequence, the nth term is $a_n = 2n^2 + 3$. Show that it is not an A.P.*

Solution. Given $a_n = 2n^2 + 3 \Rightarrow a_{n+1} = 2(n + 1)^2 + 3$.

$$\begin{aligned} \therefore \quad a_{n+1} - a_n &= (2(n + 1)^2 + 3) - (2n^2 + 3) \\ &= (2(n^2 + 2n + 1) + 3) - (2n^2 + 3) \\ &= 4n + 2, \text{ which depends upon } n \text{ and is not constant.} \end{aligned}$$

Hence the given sequence is not an A.P.

Example 3. *In a sequence, the sum of first n terms is $S_n = nP + \frac{1}{2}n(n - 1)Q$ where P, Q are constants. Show that the sequence is an A.P. Find its first term, common difference and 100th term.* *(NCERT)*

Solution. Given $S_n = nP + \frac{1}{2}n(n - 1)Q$,

$\therefore \quad S_{n-1} = (n - 1)P + \frac{1}{2}(n - 1)(n - 2)Q$.

$\therefore \quad a_n = S_n - S_{n-1} = (n - (n - 1))P + \frac{1}{2}[n(n - 1) - (n - 1)(n - 2)]Q$

$= P + \frac{1}{2}(n - 1)(n - (n - 2))Q = P + \frac{1}{2}(n - 1) \times 2Q = P + (n - 1)Q$

$\Rightarrow \quad a_{n+1} = P + nQ$.

$\therefore \quad a_{n+1} - a_n = (P + nQ) - (P + (n - 1)Q) = Q$, which is constant.

Hence the given sequence is an A.P. with common difference Q.

$a_1 = P$ and $a_{100} = P + 99Q$.

Example 4. *Which term of the sequence 25,* $24\frac{1}{4}$, $23\frac{1}{2}$, $22\frac{3}{4}$, *... is the first negative term ?*

Solution. The given sequence is an A.P. with common difference $d = -\frac{3}{4}$ and first term $a = 25$.

Let nth term of the given A.P. be the *first negative term,* then

$$a_n < 0 \Rightarrow 25 + (n-1)\left(-\frac{3}{4}\right) < 0$$

$$\Rightarrow \quad \frac{103}{4} - \frac{3n}{4} < 0 \Rightarrow 103 - 3n < 0 \Rightarrow 103 < 3n$$

$$\Rightarrow \quad 3n > 103 \Rightarrow n > \frac{103}{3} \text{ i.e. } n > 34\frac{1}{3}.$$

Since 35 is the least natural number satisfying $n > 34\frac{1}{3} \Rightarrow n = 35$.

Hence, 35th term of the given sequence is the first negative term.

Example 5. *Which term of the sequence* $12 + 8i$, $10 + 7i$, $8 + 6i$, *... is*

(*i*) *real* (*ii*) *purely imaginary ?*

Solution. The given sequence is an A.P. with common difference $d = -2 - i$ and first term $a = 12 + 8i$.

$\therefore \quad a_n$ (general term) $= a + (n-1)\,d = (12 + 8i) + (n-1)(-2 - i)$

$= (12 - 2n + 2) + (8 - n + 1)\,i = (14 - 2n) + (9 - n)\,i.$

(*i*) Let nth term of the given sequence be real

$\Rightarrow \quad (14 - 2n) + (9 - n)\,i$ is real $\Rightarrow 9 - n = 0 \Rightarrow n = 9$.

Hence, 9th term of the given sequence is real.

(*ii*) Let nth term of the given sequence be purely imaginary

$\Rightarrow \quad (14 - 2n) + (9 - n)\,i$ is purely imaginary

$\Rightarrow \quad 14 - 2n = 0 \Rightarrow n = 7$.

Hence, 7th term of the given sequence is purely imaginary.

Example 6. *How many terms are identical in the two Arithmetic progressions 2, 4, 6, 8, ... upto 100 terms and 3, 6, 9, 12, ... upto 80 terms.*

Solution. 100th term of the A.P. 2, 4, 6, 8, ...

$= 2 + (100 - 1)\,2 = 200$

and 80th term of the A.P. 3, 6, 9, 12, ...

$= 3 + (80 - 1)\,3 = 240.$

Let n terms be identical in the two given Arithmetic progressions.

The sequence of identical terms is 6, 12, 18, ... which is an A.P. with first term 6 and common difference 6.

Its nth term $= 6 + (n - 1)\,6 = 6n$.

Since the last term *i.e.* nth term of the sequence of identical terms can't be greater than 200,

$$6n \leq 200 \Rightarrow n \leq \frac{100}{3} \Rightarrow n = 33.$$

Hence, 33 terms are identical.

NOTE

If finally we get $n \leq m$, then $n = m$ if m is an integer and $n =$ an integer just less than m if m is not an integer.

ANSWERS

EXERCISE 9.1

1. 2, 3, 5, 7, 11, 13, 17, … **2.** 18th **3.** $\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}$

4. 2 + 5 + 9 + 17 **5.** (*i*) –5 (*ii*) $\frac{5}{16}$

6. (*i*) 30 (*ii*) 61 (*iii*) 91 **7.** 89

8. (*i*) 7, 9, 11, 13, 15 (*ii*) $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$ (*iii*) 0, 2, 6, 12, 20 (*iv*) 3, 8, 15, 24, 35

(*v*) 2, 4, 8, 16, 32 (*vi*) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ (*vii*) 25, –125, 625, –3125, 15625

(*viii*) $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21, \frac{75}{2}$ (*ix*) $-2, 5, \frac{10}{3}, \frac{17}{5}, \frac{26}{7}$

9. (*i*) 65, 93 (*ii*) $\frac{25}{32}, \frac{49}{128}$ (*iii*) 729 (*iv*) 0, 0, –7866

10. 360, $\frac{50}{313}$

11. (*i*) 1, 3, 5, 7, 9; 1 + 3 + 5 + 7 + 9 + …

(*ii*) 3, 11, 35, 107, 323; 3 + 11 + 35 + 107 + 323 + …

(*iii*) $-1, -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{24}, -\frac{1}{120}$; $(-1) + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right) + \left(-\frac{1}{24}\right) + \left(-\frac{1}{120}\right) + \ldots$

(*iv*) 2, 2, 1, 0, –1; 2 + 2 + 1 + 0 + (–1) + … .

12. 5, 9, 13, 17; 81 **13.** 21

EXERCISE 9.2

1. (*i*) $3\frac{3}{4}$ (*ii*) $\sqrt{300}$ **2.** 67 **3.** 28th **4.** (*i*) 10th (*ii*) 8th

5. 55 **6.** 2 **7.** 10th **8.** 33 **9.** 27

10. 1 **12.** 15 **13.** n^2 **14.** ₹ 65 **15.** 6

16. 0 **17.** 30 **18.** 2475 **19.** 21st **21.** (*i*) $6r - 1$ (*ii*) $2q$

23. $-\frac{1}{4}$ **25.** (*i*) $m + n - p$ (*ii*) 0. **26.** $\frac{b(r-q) - c(r-p)}{p-q}$

28. 158 **29.** 27 **30.** ₹ 245 **31.** ₹ 1600 **32.** 399

33. $\frac{n}{2}(5n + 7)$ **34.** 8; 4 **35.** 6 or 12 **36.** $x = 29$ **37.** 0

38. 0 **39.** 1002001 **40.** 98450 **41.** 8729 **42.** 1210

43. 867 **44.** (*i*) –14, –11, –8 (*ii*) 2; 26 **45.** (*i*) $\frac{7}{16}$ (*ii*) $-\frac{23}{25}$

56. (*i*) 5, 7, 9 (*ii*) –4, –1, 2 (*iii*) 7, 8, 9 (*iv*) 6, 10, 14 (*v*) 2, 6, 10, 14

57. (*i*) ₹ 1400 **58.** (*i*) ₹ 8080 (*ii*) ₹ 83520 **59.** ₹ 7900000

60. ₹ 5000; Saving is important for individuals as it provides security against unexpected expenses. It helps in the progress of the country and the development of economy.

61. (*i*) 550 (*ii*) 4375

(*iii*) 775; Keeping 'save environment' and 'conservation of exhaustible resources' in mind, the manually operated machine should be promoted so that energy could be saved. The manufacturer uses his/her wisdom to create more employment for villagers by using hand operated machines.