6

LINEAR INEQUALITIES

INTRODUCTION

You are all familiar with equations in which two sides of a statement (called left hand side, L.H.S. and right hand side, R.H.S.) are connected by equality sign (=). In fact, you have solved many equations in one and two variables and also solved some statement (or word) problems by translating them in the form of equations. A natural question arises : Is it always possible to translate a statement problem in the form of an equation? Not always! Instead we may get certain statements involving a sign (or symbol) of inequality. These signs are

$<, \ >, \ \leq, \ \geq$

In this chapter, we shall learn how to solve (algebraically or graphically) linear inequalities in one or two variables.

6.1 LINEAR INEQUALITIES IN ONE VARIABLE

Two real numbers or two algebraic expressions connected by the sign $<, >, \leq$ *or* $\geq$ *is called an* ***inequality***.

The statements such as $7 < 11$, $5\frac{1}{2} > -3$, $x < 3$, $x + 5 \leq 7$, $2x - 3 > 8$, $3y + 5 \geq 11$, $\frac{y-3}{2} < 2y + 1$ are all examples of linear inequalities.

The statements $7 < 11$, $5\frac{1}{2} > -3$ are examples of ***numerical inequalities*** and the statements $x < 3$, $x + 5 \leq 7$, $2x - 3 > 8$, $3y + 5 \geq 11$, $\frac{y-3}{2} < 2y + 1$ are examples of ***literal inequalities***.

In general, a linear inequality in one variable can always be written as

$ax + b < 0, \quad ax + b \leq 0, \quad ax + b > 0, \quad ax + b \geq 0$

where a and b are real numbers, $a \neq 0$.

The inequalities $x < 3$, $2x - 3 > 8$ and $\frac{y-3}{2} < 2y + 1$ are called ***strict inequalities*** where as the inequalities $x + 5 \leq 7$, $3y + 5 \geq 11$ are called ***slack inequalities***.

Replacement set. *The set from which values of the variable (involved in the inequality) are chosen is called the* ***replacement set.***

Solution set. *A* ***solution*** *to an inequality is a number (chosen from replacement set) which, when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the* ***solution set*** *of the inequality.*

For example, consider the *inequality* $x < 4$.

	Replacement set	*Solution set*
(*i*)	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}	{1, 2, 3}
(*ii*)	{–1, 0, 1, 2, 5, 8}	{–1, 0, 1, 2}

(*iii*) {– 5, 10} {– 5}

(*iv*) {5, 6, 7, 8, 9, 10} ϕ.

Note that the solution set depends upon the replacement set.

REMARK

*If the replacement set is not given, then we shall take it as **R** (set of real numbers).*

6.1.1 Solving linear inequalities in one variable

The rules for solving inequalities are similar to those for solving equations except for multiplying or dividing by a negative number.

You can do any thing of the following to an inequality :

(*i*) add (or subtract) the same number or expression to both sides.

(*ii*) any number (or expression) can be transposed from one side of an inequality to the other side with the sign of the transposed number (or expression) reversed.

(*iii*) multiply (or divide) both sides by the same positive number.

However, when you multiply or divide by the same **negative** number, the inequality is **reversed**.

For example :

(*i*) If $x < 2$, then $-x > -2$ (Multiplying by – 1)

(*ii*) If $3x - 1 \geq 5$, then $-4\,(3x - 1) \leq -20$ (Multiplying by – 4)

(*iii*) If $-6x \leq 12$, then $x \geq -2$ (Dividing by – 6)

Thus, always reverse the symbol of an inequality when multiplying or dividing by the same negative number.

6.1.2 Procedure to solve a linear inequality in one variable

(*i*) Simplify both sides by removing group symbols and collecting like terms.

(*ii*) Remove fractions (or decimals) by multiplying both sides by an appropriate factor (L.C.M. of fractions or a power of 10 in case of decimals).

(*iii*) Isolate all variable terms on one side and all constants on the other side. Collect like terms wherever possible.

(*iv*) Make the coefficient of the variable 1.

(*v*) Choose the solution set from the replacement set.

ILLUSTRATIVE EXAMPLES

Example 1. *Given $x \in \{-3, -4, -5, -6\}$ and $9 \leq 1 - 2x$, find the possible values of x. Also represent its solution set on the number line.*

Solution. Given $9 \leq 1 - 2x$

$\Rightarrow\ 2x + 9 \leq 1 - 2x + 2x$ (Add $2x$)

$\Rightarrow\ 2x + 9 \leq 1$

$\Rightarrow\ 2x + 9 + (-9) \leq 1 + (-9)$ (Add –9)

$\Rightarrow\ 2x \leq -8$

$\Rightarrow\ x \leq -4$ (Divide by 2)

But $x \in \{-3, -4, -5, -6\}$,

$\therefore$ the solution set is $\{-4, -5, -6\}$.

The solution set is shown by thick dots on the number line.

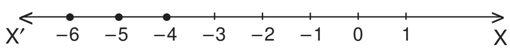


Fig. 6.1.

Example 2. *Solve the inequality* $3 - 2x \geq x - 32$, *given that*

(i) $x \in \mathbf{N}$ *(ii)* $x \in \mathbf{I}$.

Solution. Given $3 - 2x \geq x - 32$

$\Rightarrow \quad (-3) + 3 - 2x \geq x - 32 + (-3)$ (Add -3)

$\Rightarrow \quad -2x \geq x - 35$

$\Rightarrow \quad (-x) - 2x \geq (-x) + x - 35$ (Add $-x$)

$\Rightarrow \quad -3x \geq -35$

$\Rightarrow \quad \left(-\frac{1}{3}\right)(-3x) \leq \left(-\frac{1}{3}\right)(-35)$ (Multiply by $-\frac{1}{3}$ and reverse the symbol)

$\Rightarrow \quad x \leq \frac{35}{3}$.

(*i*) When $x \in \mathbf{N}$, the solution set is $\{1, 2, 3, \ldots, 11\}$.

(*ii*) When $x \in \mathbf{I}$, the solution set is $\{\ldots, -3, -2, -1, 0, 1, 2, \ldots, 11\}$.

Example 3. *Solve* $5x - 3 < 3x + 1$ *when*

(i) x is an integer (ii) x is a real number. *(NCERT)*

Solution. Given $5x - 3 < 3x + 1$

$\Rightarrow \quad 5x < 3x + 1 + 3$ (Transposing 3 to R.H.S.)

$\Rightarrow \quad 5x < 3x + 4$

$\Rightarrow \quad 5x - 3x < 4$ (Transposing $3x$ to L.H.S.)

$\Rightarrow \quad 2x < 4$

$\Rightarrow \quad x < 2$ (Dividing by 2)

(*i*) When x is an integer, the solution set is $\{\ldots, -3, -2, -1, 0, 1\}$.

(*ii*) When x is a real number, the solution of the inequality is given by $x < 2$ *i.e.* the solution set consists of all real number less than 2.

$\therefore$ The solution set is $\{x : x \in \mathbf{R}, x < 2\}$ *i.e.* $(-\infty, 2)$.

Example 4. *Solve the following inequalities for real x :*

(i) $3(2 - x) \geq 2(1 - x)$ *(NCERT)* *(ii)* $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$ *(NCERT)*

Solution. (*i*) Given $3(2 - x) \geq 2(1 - x)$

$\Rightarrow \quad 6 - 3x \geq 2 - 2x$

$\Rightarrow \quad 6 - 3x + 2x \geq 2$

$\Rightarrow \quad 6 - x \geq 2$

$\Rightarrow \quad -x \geq 2 - 6$

$\Rightarrow \quad -x \geq -4$

$\Rightarrow \quad x \leq 4$ (Dividing by -1)

$\therefore$ The solution set is $\{x : x \in \mathbf{R}, x \leq 4\}$ *i.e.* $(-\infty, 4]$.

(*ii*) Given $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$.

Multiplying both sides by 60, L.C.M. of fractions, we get

$20(2x - 1) \geq 15(3x - 2) - 12(2 - x)$

$\Rightarrow \quad 40x - 20 \geq 45x - 30 - 24 + 12x$

$\Rightarrow \quad 40x - 20 \geq 57x - 54$

$\Rightarrow \quad 40x - 57x \geq -54 + 20$

$\Rightarrow \quad -17x \geq -34$

$\Rightarrow \quad x \leq 2$ (Dividing by -17)

$\therefore$ The solution set is $\{x : x \in \mathbf{R}, x \leq 2\}$ *i.e.* $(-\infty, 2]$.

Example 5. *Solve* $\frac{2x+1}{3} \geq \frac{3x-2}{5}$, $x \in \mathbf{R}$. *Graph the solution set on the number line.*

Solution. Given $\frac{2x+1}{3} \geq \frac{3x-2}{5}$.

Multiplying both sides by 15, L.C.M. of fractions, we get

$5(2x+1) \geq 3(3x-2) \Rightarrow 10x + 5 \geq 9x - 6$

$\Rightarrow \quad 10x - 9x + 5 \geq -6$

$\Rightarrow \quad x + 5 \geq -6$

$\Rightarrow \quad x \geq -6 - 5$

$\Rightarrow \quad x \geq -11.$

Hence, the solution set is $\{x : x \in \mathbf{R}, x \geq -11\}$ *i.e.* $[-11, \infty)$.

The graph of the solution set is shown by the thick portion of the number line. The solid circle at -11 indicates that the number -11 is included among the solutions.

Fig. 6.2.

Example 6. *Solve* $\frac{x}{4} > \frac{5x-2}{3} - \frac{7x-3}{5}$ *and graph the solution set on the number line.*

(NCERT)

Solution. Given $\frac{x}{4} > \frac{5x-2}{3} - \frac{7x-3}{5}$.

Multiplying both sides by 60, L.C.M. of fractions, we get

$15x > 20(5x-2) - 12(7x-3)$

$\Rightarrow \quad 15x > 100x - 40 - 84x + 36$

$\Rightarrow \quad 15x > 16x - 4$

$\Rightarrow \quad 15x - 16x > -4$

$\Rightarrow \quad -x > -4$

$\Rightarrow \quad x < 4.$

Hence, the solution set is $\{x : x \in \mathbf{R}, x < 4\}$ *i.e.* $(-\infty, 4)$.

The graph of the solution set is shown by thick portion of the number line. The open circle at 4 indicates that the number 4 is not included among the solutions.

Fig. 6.3.

Example 7. *To receive grade 'A' in a mathematics course, one must get an average of atleast 90 marks in five examinations (each of 100 marks). If Ragini's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Ragini must obtain in the fifth examination to get grade 'A' in the course.*

(NCERT)

Solution. Let x be the marks obtained by Ragini in the fifth examination, then

$\frac{87+92+94+95+x}{5} \geq 90$

$\Rightarrow \quad \frac{368+x}{5} \geq 90$

$\Rightarrow \quad 368 + x \geq 450$

$\Rightarrow \quad x \geq 450 - 368$

$\Rightarrow \quad x \geq 82.$

Hence, Ragini must obtain a minimum of 82 marks in the fifth examination to get grade 'A' in the course.

Example 8. *A company manufactures cassettes and its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$ respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?*

(*NCERT Examplar Problems*)

Solution. We know that :

Profit = revenue – cost.

Therefore, to get some profit, revenue > cost

$\Rightarrow \quad 43x > 26000 + 30x \Rightarrow 13x > 26000$

$\Rightarrow \quad x > 2000.$

Hence, the company must sell more than 2000 cassettes.

Example 9. *The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm, find the minimum length of the shortest side.* *(NCERT)*

Solution. Let the length of the shortest side of a triangle be x cm, then the length of longest side = $3x$ cm and the length of the third side = $(3x – 2)$ cm.

$\therefore$ The primeter of the triangle = $(x + 3x + 3x – 2)$ cm = $(7x – 2)$ cm.

According to given, $7x – 2 \geq 61$

$\Rightarrow \quad 7x \geq 61 + 2 \quad \Rightarrow 7x \geq 63$

$\Rightarrow \quad x \geq 9.$

Hence, the minimum length of the shortest side = 9 cm.

EXERCISE 6.1

Very short answer type questions (1 to 13) :

1. Solve the inequality $3x – 5 \leq 13 – x$, where $x \in \{1, 2, 3, \ldots, 10\}$.

2. Solve $11 – 5x > 3 – 2x$, $x \in \mathbf{W}$.

3. If x is a negative integer, find the solution set of $\frac{2}{5} + \frac{1}{5}(2x + 5) \geq 0$.

4. Solve $30x < 200$, when
(*i*) x a natural number (*ii*) x is an integer. (*NCERT*)

5. Solve $–12x > 30$, when
(*i*) x is a natural number (*ii*) x is an integer. (*NCERT*)

6. Solve $2 + 23x < 99 – x$, when
(*i*) x is a natural number (*ii*) x is an integer.

7. Solve $7 – 7x \geq 37$, when
(*i*) x is a natural number (*ii*) x is an integer.

8. Solve $2 < 3x + 7$, when
(*i*) x is an integer (*ii*) x is a real number.

9. Solve $3x + 8 > 2$ when
(*i*) x is an integer (*ii*) x is a real number. (*NCERT*)

10. Solve $7x – 1 < 5x + 3$ when
(*i*) x is an integer (*ii*) x is a real number.

Solve the following (11 to 20) inequalities for real x :

11. (*i*) $4x + 3 \leq 6x + 7$ (*ii*) $3x – 7 > 5x – 1$. (*NCERT*)

12. (*i*) $–(x – 3) + 4 > –2x + 5$ (*ii*) $3(x – 1) \leq 2(x – 3)$. (*NCERT*)

13. (*i*) $2(2x + 3) – 10 \leq 6(x – 2)$ (*ii*) $37 – (3x + 5) \geq 9x – 8(x – 3)$. (*NCERT*)

14. (*i*) $x + \frac{x}{2} + \frac{x}{3} < 11$ (*NCERT*) (*ii*) $\frac{x}{3} > \frac{x}{2} + 1$. (*NCERT*)

15. (*i*) $\frac{4+2x}{3} \geq \frac{x}{2} - 3$ (*ii*) $\frac{5-2x}{3} \leq \frac{x}{6} - 5$. (*NCERT*)

16. (*i*) $\frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$ (*ii*) $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x - 6)$. (*NCERT*)

17. Solve the inequality $3x - 11 < 3$ where $x \in \{1, 2, 3, \ldots, 10\}$. Also represent its solution on a number line.

18. Solve $5 - 4x > 2 - 3x$, $x \in \mathbf{W}$. Also represent its solution on the number line.

19. List the solution set of $\frac{11-2x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}$, $x \in \mathbf{N}$.

20. If $x \in \mathbf{W}$, find the solution set of $\frac{3}{5}x - \frac{2x-1}{3} > 1$.

21. Solve the following inequalities and show the graph of the solution in each case on the number line :

(*i*) $7x + 3 < 5x + 9$ (*NCERT*) (*ii*) $3(1 - x) < 2(x + 4)$ (*NCERT*)

(*iii*) $x + 5 \leq 2x + 3$ (*iv*) $\frac{4x-10}{3} \leq \frac{5x-7}{2}$

(*v*) $\frac{3x}{5} - \frac{2x-1}{3} > 1$ (*vi*) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. (*NCERT*)

22. Solve $3(2 - x) \geq 2(1 - x)$, given that $x \in \mathbf{R}^+$ where $\mathbf{R}^+ = \{x : x \in \mathbf{R}$ and $x > 0\}$. Also represent the solution on the number line.

23. The marks obtained by a student in two tests were 62 and 48. Find the number of minimum marks he should get in the third test to have an average of atleast 60 marks. (*NCERT*)

24. The marks obtained by Sukriti in first two unit tests are 75 and 70. Find the number of minimum marks she should get in the third unit test to have an average of atleast 60 marks. (*NCERT*)

25. The cost and revenue functions of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$ respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit? (*NCERT Examplar Problems*)

6.2 SOLUTION OF A SYSTEM OF LINEAR INEQUALITIES IN ONE VARIABLE

To solve a system of linear inequalities in one variable, proceed as follows :

(*i*) Solve each linear inequalities separately.

(*ii*) Find the values of the variable which satisfy all the given linear inequalities.

(*iii*) The common values of the variable form the required solution of the given system of linear inequalities.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve the following system of linear inequalities :*

$3x - 1 \geq 5,\ 2x - 3 > 7.$

Solution. We solve each inequality separately.

$3x - 1 \geq 5 \Rightarrow 3x \geq 6 \Rightarrow x \geq 2$ …(*i*)

and $2x - 3 > 7 \Rightarrow 2x > 10 \Rightarrow x > 5$ …(*ii*)

From (*i*) and (*ii*), we find that the values of x which satisfy both the given inequalities are given by $x > 5$.

Hence, the solution set is $\{x : x \in \mathbf{R}, x > 5\}$ *i.e.* $(5, \infty)$.

REMARK

If we draw the graphs of (*i*) and (*ii*) on the number line, we see that the values of x common to both the inequalities are given by $x > 5$.

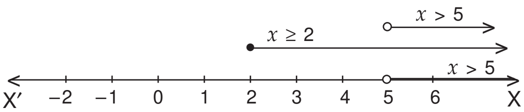


Fig. 6.4.

Example 2. *Solve the following system of linear inequalities :*

$4x - 5 < 11, \quad -3x - 4 \geq 8.$

Solution. We solve each inequality separately.

$$4x - 5 < 11 \Rightarrow 4x < 16 \Rightarrow x < 4 \quad \ldots(i)$$

and $-3x - 4 \geq 8 \Rightarrow -3x \geq 12 \Rightarrow x \leq -4$ ...(*ii*)

From (*i*) and (*ii*), we find that the values of x which satisfy both the given inequalities are given by $x \leq -4$.

Hence, the solution set is $\{x : x \in \mathbf{R}, x \leq -4\}$ *i.e.* $(-\infty, -4]$.

REMARK

If we draw the graphs of (*i*) and (*ii*) on the number line, we see that the values of x common to both the inequalities are given by $x \leq -4$.

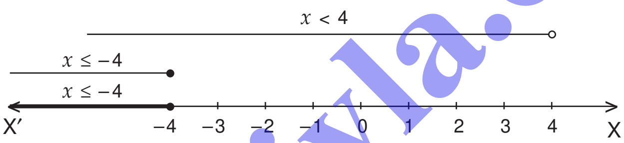


Fig. 6.5.

Example 3. *Solve the following system of linear inequalities :*

$2(x + 1) \leq x + 5$, $3(x + 2) > 2 - x$ and represent the solution graphically on the number line. *(NCERT)*

Solution. We solve each inequality separately.

$$2(x + 1) \leq x + 5 \Rightarrow 2x + 2 \leq x + 5 \Rightarrow 2x - x \leq 5 - 2$$

$$\Rightarrow \quad x \leq 3 \quad \ldots(i)$$

and $3(x + 2) > 2 - x \Rightarrow 3x + 6 > 2 - x \Rightarrow 3x + x > 2 - 6$

$$\Rightarrow \quad 4x > -4 \Rightarrow x > -1 \Rightarrow -1 < x \quad \ldots(ii)$$

From (*i*) and (*ii*), we find that the values of x which satisfy both the given inequalities are given by $-1 < x \leq 3$.

Hence, the solution set is $\{x : x \in \mathbf{R}, -1 < x \leq 3\}$ *i.e.* $(-1, 3]$.

If we draw the graphs of (*i*) and (*ii*) on the number line, we see that the values of x common to both the inequalities are given by $-1 < x \leq 3$.

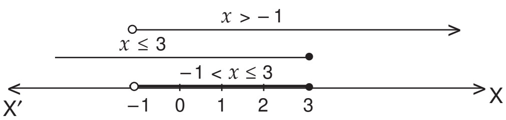


Fig. 6.6.

Example 4. *Solve* $2 \le 3x - 4 \le 5$. *(NCERT)*

Solution. Given $2 \le 3x - 4 \le 5$

$\Rightarrow \quad 2 + 4 \le 3x - 4 + 4 \le 5 + 4$ (Add 4)

$\Rightarrow \quad 6 \le 3x \le 9$

$\Rightarrow \quad 2 \le x \le 3$ (Dividing 3)

Hence, the solution set is [2, 3].

REMARK

In fact, the given inequality $2 \le 3x - 4 \le 5$ consists of a system of two inequalities : $2 \le 3x - 4$, $3x - 4 \le 5$. We have solved these two inequalities simultaneously.

Example 5. *Solve* $6 \le -3(2x - 4) < 12$. *(NCERT)*

Solution. Given $6 \le -3(2x - 4) < 12$

$\Rightarrow \quad 6 \le -6x + 12 < 12$

$\Rightarrow \quad 6 - 12 \le -6x + 12 - 12 < 12 - 12$ (Adding −12)

$\Rightarrow \quad -6 \le -6x < 0$

$\Rightarrow \quad 1 \ge x > 0$ (Divide by −6)

which can be written as $0 < x \le 1$.

Hence, the solution set is (0, 1].

Example 6. *Solve the following inequalities :*

(i) $-12 < 4 - \frac{3x}{-5} \le 2$ *(NCERT)* *(ii)* $-3 \le 4 - \frac{7}{2}x \le 18$ *(NCERT)*

Solution. (*i*) Given $-12 < 4 - \frac{3x}{-5} \le 2$

$\Rightarrow \quad -12 - 4 < \frac{3x}{5} \le 2 - 4$

$\Rightarrow \quad -16 < \frac{3x}{5} \le -2$

$\Rightarrow \quad -16 \times \frac{5}{3} < \frac{5}{3} \times \frac{3x}{5} \le -2 \times \frac{5}{3}$ $\left(\text{Multiplying by } \frac{5}{3}\right)$

$\Rightarrow \quad -\frac{80}{3} < x \le -\frac{10}{3}$.

Hence, the solution is $\left(-\frac{80}{3}, -\frac{10}{3}\right]$.

(*ii*) Given $-3 \le 4 - \frac{7}{2}x \le 18$

$\Rightarrow \quad -3 - 4 \le -\frac{7}{2}x \le 18 - 4$

$\Rightarrow \quad -7 \le -\frac{7}{2}x \le 14$

$\Rightarrow \quad 2 \ge x \ge -4$ $\left(\text{Dividing by } -\frac{7}{2}\right)$

$\Rightarrow \quad -4 \le x \le 2$.

Hence, the solution set is [– 4, 2].

Example 7. *Find the values of x, which satisfy the inequality* $-\frac{1}{5} \le \frac{3x}{10} + 1 < \frac{2}{5}$, $x \in \mathbf{R}$. *Graph the solution set on the number line.*

Solution. Given $-\frac{1}{5} \le \frac{3x}{10} + 1 < \frac{2}{5}$.

Multiplying throughout by 10, L.C.M. of fractions, we get

$-2 \le 3x + 10 < 4$

$\Rightarrow$ $-2 - 10 \le 3x + 10 + (-10) < 4 + (-10)$ (Adding -10)

$\Rightarrow$ $-12 \le 3x < -6$

$\Rightarrow$ $-4 \le x < -2$ (Dividing by 3)

Hence, the solution set is $\{x : x \in \mathbf{R}, -4 \le x < -2\}$ *i.e.* $[-4, -2)$.

The graph of the solution set is shown by the thick portion of the number line. The open circle at -2 indicates that the number -2 is not included among the solutions.

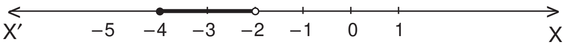


Fig. 6.7.

Example 8. *Solve the following inequality, and graph the solution set on the number line :*

$$2y - 3 < y + 2 \le 3y + 5.$$

Solution. Given $2y - 3 < y + 2 \le 3y + 5$, $y \in \mathbf{R}$

$\Rightarrow$ $2y - 3 < y + 2$ and $y + 2 \le 3y + 5$

$\Rightarrow$ $2y < y + 5$ and $y \le 3y + 3$

$\Rightarrow$ $y < 5$ and $-2y \le 3$

$\Rightarrow$ $y < 5$ and $y \ge -\frac{3}{2}$

$\Rightarrow$ $y < 5$ and $-\frac{3}{2} \le y$ $\Rightarrow -\frac{3}{2} \le y < 5$.

$\therefore$ The solution set is $= \left\{y : y \in \mathbf{R}, -\frac{3}{2} \le y < 5\right\}$ *i.e.* $\left[-\frac{3}{2}, 5\right)$.

The graph of the solution set is shown by the thick portion of the number line.

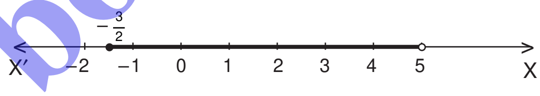


Fig. 6.8.

Example 9. *Solve the following system of linear inequalities :*

$$2(2x + 3) - 10 < 6(x - 2), \quad \frac{2x - 3}{4} + 6 \ge 4 + \frac{4x}{3}.$$

Solution. We solve each inequality separately.

$2(2x + 3) - 10 < 6(x - 2) \Rightarrow 4x + 6 - 10 < 6x - 12$

$\Rightarrow$ $4x - 6x < -12 - 6 + 10 \Rightarrow -2x < -8$

$\Rightarrow$ $x > 4$ ...(*i*)

and $\frac{2x - 3}{4} + 6 \ge 4 + \frac{4x}{3}$ $\Rightarrow$ $\frac{2x - 3}{4} - \frac{4x}{3} \ge -2$

$\Rightarrow$ $6x - 9 - 16x \ge -24$ (Multiplying both sides by 12)

$\Rightarrow$ $-10x \ge -24 + 9$

$\Rightarrow$ $-10x \ge -13 \Rightarrow x \le \frac{13}{10}$

$\Rightarrow$ $x \le 1.3$ ...(*ii*)

From (*i*) and (*ii*), we find that there is no value of x which satisfy both the inequalities. Thus, there are no real numbers which satisfy the given system of inequalities.

Hence, the solution set = ϕ.

REMARK

If we draw the graphs of (*i*) and (*ii*) on the number line, we see that there are no values of x which are common to both the given inequalities.

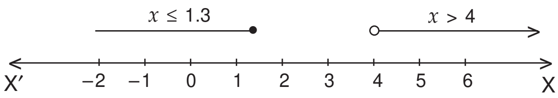


Fig. 6.9.

Example 10. *An electrician can be paid under two schemes as given below :*

Scheme I : ₹ 500 and ₹ 70 per hour.

Scheme II : ₹ 120 per hour.

If the job takes x hours, for what value of x does the scheme I give the electrician better wages?

Solution. As the job takes x hours,

under scheme I, wage of the electrician = ₹ $(500 + 70x)$

under scheme II, wage of the electrician = ₹ $120x$.

The scheme I will give the electrician better wages if

$500 + 70x > 120x$

$\Rightarrow \quad 500 > 120x - 70x$

$\Rightarrow \quad 500 > 50x$

$\Rightarrow \quad 10 > x \Rightarrow x < 10.$

Hence, the value of x should be less than 10.

Example 11. *Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.* *(NCERT)*

Solution. Let x be the smaller of the two consecutive even positive integers, then the other consecutive even integer is $x + 2$.

As both are larger than 5 and their sum is less than 23, we get

$x > 5$ and $x + (x + 2) < 23$

$\Rightarrow \quad x > 5$ and $2x + 2 < 23$

$\Rightarrow \quad x > 5$ and $2x < 21$

$\Rightarrow \quad x > 5$ and $x < \dfrac{21}{2}$

$\Rightarrow \quad 5 < x$ and $x < 10.5$

$\Rightarrow \quad 5 < x < 10.5.$

Since x is even positive integer and it lies between 5 and 10.5, the posible values of x are 6, 8 and 10. Then the corresponding values of the other even integer *i.e.* $x + 2$ are 8, 10 and 12.

Hence, the required pairs are 6, 8; 8, 10; 10, 12.

Example 12. *Find all pairs of consecutive odd positive integers, both of which are larger than 10, such that their sum is less than 40.* *(NCERT)*

Solution. Let x be the smaller of the two consecutive odd positive integers, then the other consecutive odd integer is $x + 2$.

As both are larger than 10 and their sum is less than 40, we get

$x > 10$ and $x + (x + 2) < 40$

$\Rightarrow \quad x > 10$ and $2x + 2 < 40$

$\Rightarrow \quad x > 10$ and $2x < 38$

$\Rightarrow \quad 10 < x$ and $x < 19$

$\Rightarrow \quad 10 < x < 19.$

Since x is odd positive integer and it lies between 10 and 19, the possible values of x are 11, 13, 15 and 17. Then the corresponding values of the other odd integer *i.e.* $x + 2$ are 13, 15, 17 and 19.

Hence, the required pairs are 11, 13; 13, 15; 15, 17; 17, 19.

Example 13. *An acid solution is to be kept in between 30° and 35° Celsius. What is the range of temperature in degree Fahrenheit if conversion formula is given by* $C = \frac{5}{9}(F - 32)$ *where C and F represent temperature in degree Celsius and degree Fahrenheit respectively.* *(NCERT)*

Solution. Given $30 < C < 35$ ...(*i*)

Conversion formula is $C = \frac{5}{9}(F - 32)$, putting this value of C in (*i*), we get

$$30 < \frac{5}{9}(F - 32) < 35$$

$\Rightarrow \quad 6 < \frac{1}{9}(F - 32) < 7$ (Divide by 5)

$\Rightarrow \quad 54 < F - 32 < 63$ (Multiply by 9)

$\Rightarrow \quad 86 < F < 95.$ (Add 32)

Hence, the required range of temperature is between 86° and 95° Fahrenheit.

Example 14. *IQ of a person is given by the formula* $IQ = \frac{MA}{CA} \times 100$, *where MA is mental age and CA is chronical age. If* $80 \le IQ \le 140$ *for a group of 12 years old children, find the range of their mental age.* *(NCERT)*

Solution. Given $IQ = \frac{MA}{CA} \times 100$ and $CA = 12$ years,

$\therefore \quad IQ = \frac{MA}{12} \times 100 = \frac{25}{3} MA.$

Also, $80 \le IQ \le 140$ (Given)

$\Rightarrow \quad 80 \le \frac{25}{3} MA \le 140$

$\Rightarrow \quad \frac{3}{25} \times 80 \le MA \le \frac{3}{25} \times 140$ $\left(\text{Multiplying by } \frac{3}{25}\right)$

$\Rightarrow \quad \frac{48}{5} \le MA \le \frac{84}{5}$

$\Rightarrow \quad 9.6 \le MA \le 16.8.$

Hence, the mental age lies between 9.6 years and 16.8 years *i.e.* the mental age is atleast 9.6 years but not more than 16.8 years.

Example 15. *The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.*

(*NCERT Examplar Problems*)

Solution. The first two pH readings are 8.48 and 8.35. Let the third pH reading be x, then the average of the three readings $= \frac{8.48 + 8.35 + x}{3} = \frac{16.83 + x}{3}$.

The pH reading is considered normal if it lies between 8.2 and 8.5.

$\therefore \quad 8.2 < \frac{16.83 + x}{3} < 8.5$

$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$

$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83$

$\Rightarrow \quad 7.77 < x < 8.67$

Hence, the third pH reading must lie between 7.77 and 8.67.

Example 16. *In drilling world's deepest hole it was found that the temperature T in degree celsius, x km below the earth's surface was given by T = 30 + 25(x – 3), 3 ≤ x ≤ 15. At what depth will temperature be between 155°C and 205°C?* *(NCERT Examplar Problems)*

Solution. Given T = 30 + 25(x – 3), 3 ≤ x ≤ 15, where T is the temperature in degree celsius.

The temperatue will be between 155°C and 205°C if

$155 < T < 205 \Rightarrow 155 < 30 + 25(x - 3) < 205$

$\Rightarrow \quad 125 < 25(x - 3) < 175 \Rightarrow 5 < x - 3 < 7$

$\Rightarrow \quad 8 < x < 10$

Hence, the depth will be between 8 km and 10 km.

Example 17. *A man wants to cut three lengths from a single piece of cardboard of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if the third piece is to be atleast 5 cm longer than the second?* *(NCERT)*

Solution. Let the length of the shortest board be x cm, then the lengths of the second and third pieces are (x + 3) cm and 2x cm respectively.

According to given,

$x + (x + 3) + 2x \leq 91$ and $2x \geq (x + 3) + 5$

$\Rightarrow \quad 4x + 3 \leq 91$ and $2x \geq x + 8$

$\Rightarrow \quad 4x \leq 88$ and $x \geq 8$

$\Rightarrow \quad x \leq 22$ and $x \geq 8$

$\Rightarrow \quad 8 \leq x \leq 22.$

Hence, the shortest piece must be atleast 8 cm long but not more than 22 cm long.

Example 18. *A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of 8% solution, how many litres of 2% solution will have to be added?* *(NCERT)*

Solution. Let x litres of 2% boric acid solution be added to 640 litres of 8% boric acid solution.

Total mixture = (x + 640) litres.

According to given,

2% of x + 8% of 640 > 4% of (x + 640)

and 2% of x + 8% of 640 < 6% of (x + 640)

$\Rightarrow \quad \frac{2}{100}x + \frac{8}{100} \times 640 > \frac{4}{100}(x + 640)$ and $\frac{2}{100}x + \frac{8}{100} \times 640 < \frac{6}{100}(x + 640)$

$\Rightarrow \quad 2x + 8 \times 640 > 4x + 4 \times 640$ and $2x + 8 \times 640 < 6x + 6 \times 640$

$\Rightarrow \quad 4 \times 640 > 2x$ and $2 \times 640 < 4x$

$\Rightarrow \quad 1280 > x$ and $320 < x$

$\Rightarrow \quad x < 1280$ and $320 < x$

$\Rightarrow \quad 320 < x < 1280.$

Hence, the number of litres of 2% boric acid solution must be more than 320 but less than 1280.

EXERCISE 6.2

Solve the following (1 to 6) system of inequalities :

1. (*i*) $3x - 1 \geq 5,\ x + 2 > -1$ (*ii*) $2x - 7 < 11,\ \ 3x + 4 < -5$.

2. (*i*) $4 - 5x > -11,\ 4x + 11 \leq -13$ (*ii*) $5x + 1 > -24,\ 5x - 1 < 24$. (*NCERT*)

3. (*i*) $7x - 8 < 4x + 7,\ -\frac{x}{2} > 4$ (*ii*) $x + 2 \leq 5,\ 3x - 4 > -2 + x$.

4. (*i*) $3x - 7 < 5 + x,\ 11 - 5x \leq 1$ (*NCERT*) (*ii*) $4x + 5 > 3x,\ -(x + 3) + 4 \leq -2x + 5$.

5. (*i*) $-2 - \frac{x}{4} \leq \frac{1 + x}{3},\ 3 - x < 4(x - 3)$ (*ii*) $5x - 7 < 3(x + 3),\ 1 - \frac{3x}{2} \geq x - 4$.

6. (*i*) $-4x + 1 \geq 0,\ 3 - 4x < 0$ (*ii*) $4x + 3 \geq 2x + 17,\ 3x - 5 < -2$.

7. Solve the following system of inequalities :

(*i*) $3x - 7 > 2(x - 6),\ 6 - x > 11 - 2x$ (*ii*) $5(2x - 7) - 3(2x + 3) \leq 0,\ 2x + 19 \leq 6x + 47$.

Also graph their solutions on the number line. (*NCERT*)

Solve the following (8 to 12) inequalities :

8. (*i*) $-8 \leq 5x - 3 < 7$ (*NCERT*) (*ii*) $-2 \leq 6x - 1 < 2$.

9. (*i*) $-2 < 1 - 3x < 7$ (*ii*) $-12 < 3x - 5 \leq 4$.

10. (*i*) $7 \leq \frac{3x + 11}{2} \leq 11$ (*NCERT*) (*ii*) $-5 \leq \frac{5 - 3x}{2} \leq 8$. (*NCERT*)

11. (*i*) $-15 < \frac{3(x - 2)}{5} \leq 0$ (*NCERT*) (*ii*) $-5 \leq \frac{2 - 3x}{4} \leq 9$. (*NCERT Examplar Problems*)

12. (*i*) $-12 \leq \frac{4 - 3x}{-5} < 2$ (*ii*) $3(2 - x) \leq 2x + 1 < 15$.

13. Solve the inequality and represent the solution on the number line :

$$-\frac{2}{3} < -\frac{x}{3} + 1 \leq \frac{2}{3},\ x \in \mathbf{R}.$$

14. Find the range of values of x, which satisfy $-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in \mathbf{R}$. Graph these values of x on the real number line.

15. Find the range of values of x which satisfies $-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3},\ x \in \mathbf{R}$. Graph the values of x on the number line.

16. Find all pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11. (*NCERT*)

Hint. Let x be the smaller of the two consecutive odd positive integers, then the other odd integer is $x + 2$.

As both are smaller than 10 and their sum is more than 11, we get $x + 2 < 10$ and $x + (x + 2) > 11$.

17. A solution is to be kept between 68° and 77° Fahrenheit. What is the range in temperature in degree Celsius if the conversion formula is given by $F = \frac{9}{5}C + 32$ where C and F represent temperature in degree Celsius and degree Fahrenheit respectively. (*NCERT*)

18. A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

(*NCERT Examplar Problems*)

19. A manufacturer has 600 litres of 12% solution of acid. How many litres of 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18% acid? (*NCERT*)

Hint. Let x litres of 30% acid solution be added to 600 litres of 12% acid solution, then

15% of $(x + 600) <$ 30% of x + 12% of 600 < 18% of $(x + 600)$.

20. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content? (*NCERT*)

Hint. Let x litres of water be added to 1125 litres of 45% acid solution, then 25% of $(x + 1125) <$ 45% of 1125 < 30% of $(x + 1125)$.

6.3 GRAPHICAL SOLUTION OF LINEAR INEQUALITIES

A statement of any one of the following types:

(*i*) $x < a$ (*ii*) $x \le a$ (*iii*) $x > a$ (*iv*) $x \ge a$ or

(*i*) $y < b$ (*ii*) $y \le b$ (*iii*) $y > b$ (*iv*) $y \ge b$

where a, b are real numbers, is called a ***linear inequality*** in one variable x or y.

A statement of any one of the following types :

(*i*) $ax + by < c$ (*ii*) $ax + by \le c$ (*iii*) $ax + by > c$ (*iv*) $ax + by \ge c$

where a, b, c are real numbers and a, b are non-zero, is called a ***linear inequality*** in two variables x and y.

We know that a straight line l divides the cartesian plane into two parts. Each part is called a *half plane*. A vertical line divides the plane into left half and right half planes and a non-vertical line divides the plane into lower half and upper half planes (see fig. 6.10 (*i*) and (*ii*)).

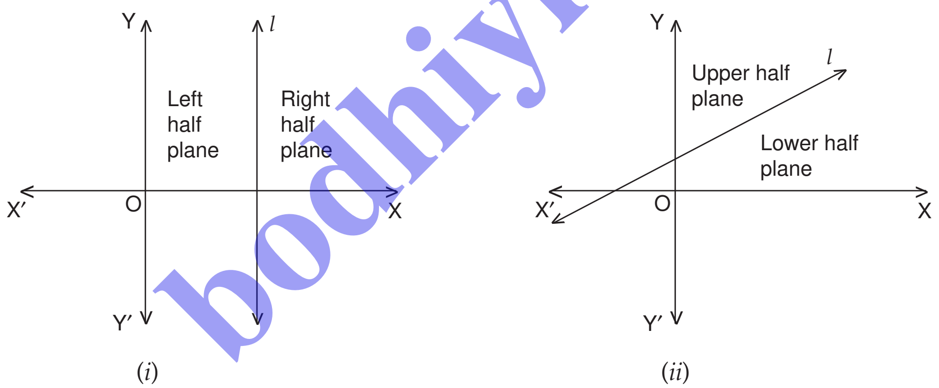


Fig. 6.10.

Graphical solution of a linear inequality

An ordered pair (α, β) of real numbers may or may not satisfy a given inequality (in one or two variables).

The set of all ordered pairs of real numbers which satisfy a given inequality is called the ***solution set*** *of the given inequality.*

Since there is one-one correspondence between the ordered pairs of real numbers and the points of a co-ordinate plane, therefore, we can represent the solution set of a given inequality (in one or two variables) by the points of a co-ordinate plane.

The region of the plane containing all the points whose co-ordinates satisfy a given inequality is called the ***solution region*** *(or* ***graph****) of the inequality.*

To find the graphical solution of an inequality in one variable

(*i*) Draw the straight line $x = a$ (or $y = b$) as the case may be.

(*ii*) The straight line $x = a$ divides the co-ordinate plane into two halves.

(*iii*) One half is the graph of $x < a$ and the other half is the graph of $x > a$. Shade the solution region of the given inequality.

(*iv*) If an inequality is of the form $x \leq a$ or $x \geq a$, then the points on the line $x = a$ are also included in the solution region and draw a dark line in the solution region.

(*v*) If an inequality is of the form $x < a$ or $x > a$, then the points on the line $x = a$ are not be included in the solution region and draw a broken line in the solution region.

For example :

1. Let us consider the inequality $x \geq 1$ in one variable.

Draw the straight line $x = 1$, which is a vertical line. It divides the plane into two halves. The points which lie either to the right of the line $x = 1$ or on the line $x = 1$ satisfy the given inequality $x \geq 1$. The graph (or solution region) of the inequality $x \geq 1$ is shown shaded in fig. 6.11.

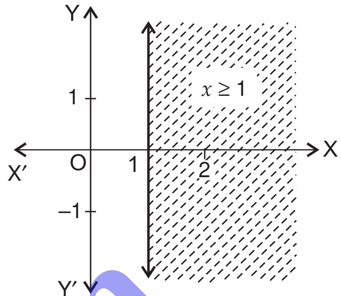


Fig. 6.11.

2. Let us consider the inequality $2y - 3 < 0$ in one variable.

Draw the straight line $2y - 3 = 0$ *i.e.* $y = \frac{3}{2}$, which is a horizontal line. It divides the plane into two halves. The points which lie below the line $y = \frac{3}{2}$ satisfy the given inequality $2y - 3 < 0$. Hence the graph (or solution region) is shown shaded in fig. 6.12.

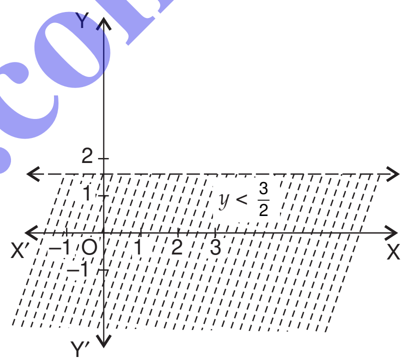


Fig. 6.12.

To find the graphical solution of an inequality in two variables

(*i*) Draw the straight line $ax + by = c$.

(*ii*) The straight line $ax + by = c$ divides the co-ordinate plane into two halves.

(*iii*) One half is the graph of $ax + by < c$ and the other half is the graph of $ax + by > c$.

(*iv*) In order to identify the half plane represented by an inequality, take any point (α, β) not lying on the line $ax + by = c$ and check whether it satisfies the inequality or not. If it satisfies, then the inequality represents the half plane containing the point and shade this region; otherwise, the inequality represents that half plane which does not contain the point within it. For convenience, we take the point (0, 0). However, if (0, 0) lies on the line then take any other point of the plane not lying on the line.

(*v*) If an inequality is of the form $ax + by \leq c$ or $ax + by \geq c$, then the points on the line $ax + by = c$ are also included in the solution region and draw a dark line in the solution region.

(*vi*) If an inequality is of the form $ax + by < c$ or $ax + by > c$, then the points on the line $ax + by = c$ are not to be included in the solution region and draw a broken line in the solution region.

For example :

1. Let us consider the inequality $3x + 4y \geq 12$ in two variables.

First, draw the straight line $3x + 4y = 12$ ...(*i*)

To draw the line (*i*), on putting $y = 0$, we get $x = 4$ and on putting $x = 0$, we get $y = 3$. Therefore, the straight line (*i*) passes through the points A (4, 0) and B (0, 3).

The straight line divides the co-ordinate plane into two halves. Further, we note that the point O (0, 0) lies below the line AB and it *does not satisfy* the given inequality $3x + 4y \geq 12$

$$(\because\ 3 \times 0 + 4 \times 0 = 0 < 12).$$

Hence, the graph of given inequality $3x + 4y \geq 12$ is that part of co-ordinate plane which lies above the line AB (including the points on the line AB). The required graph (or solution set) of the given inequality is shown shaded in fig. 6.13.

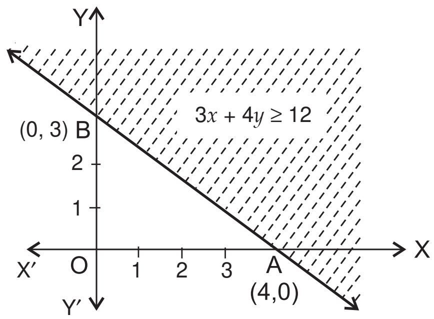


Fig. 6.13.

2. Consider the inequality $3x - 4y < 12$ in two variables.

First draw the straight line $3x - 4y = 12$ ...(*i*)

To draw the line (*i*), on putting $y = 0$, we get $x = 4$ and on putting $x = 0$, we get $y = -3$. Therefore, the line passes through the points A (4, 0) and B (0, –3).

The straight line divides the co-ordinate plane into two halves. Further, we note that the point (0, 0) lies above the line AB and it satisfies the given inequality $3x - 4y < 12$

$$(\because\ 3 \times 0 - 4 \times 0 = 0 < 12).$$

Hence, the graph of the given inequality $3x - 4y < 12$ is that part of the co-ordinate plane which lies above the line AB (excluding the points on the line AB). The required graph (or the solution set) of the given inequality is shown shaded in fig. 6.14. The dotted (or broken) line AB indicates that the points on the line AB are not included in the solution set.

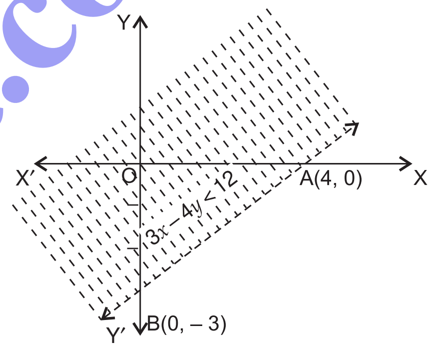


Fig. 6.14.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve the inequality $2x - 3y > 6$ graphically.* *(NCERT)*

Solution. The given inequality is $2x - 3y > 6$ ...(*i*)

Let us draw the graph of $2x - 3y = 6$ *i.e.* $y = \dfrac{2x - 6}{3}$.

Table of values

x	0	3
y	–2	0

Draw the straight line $2x - 3y = 6$, which passes through the points A (0, –2) and B(3, 0). The line divides the plane into two parts.

Putting $x = 0$, $y = 0$ in (*i*), we get

$2.0 - 3.0 > 6$ *i.e.* $0 > 6$, which is not true. Therefore, the graph of (*i*) consists of that part of the plane divided by the straight line $2x - 3y = 6$ which does not contain the origin. The required graph (or solution region) of the given inequality is shown shaded in fig. 6.15. The dotted (or broken) line AB indicates that the points on the line are not included in the solution region.

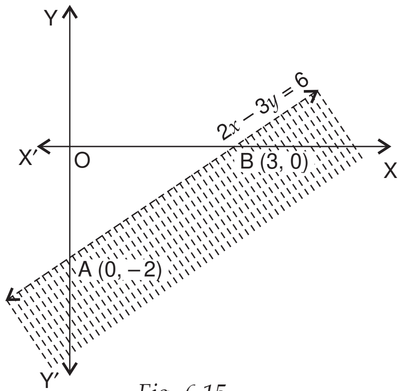


Fig. 6.15.

ANSWERS

EXERCISE 6.1

1. {1, 2, 3, 4} **2.** {0, 1, 2} **3.** {– 3, – 2, – 1}

4. (*i*) {1, 2, 3, 4, 5, 6} (*ii*) {… , – 3, – 2, – 1, 0, 1, 2, 3, 4, 5, 6}

5. (*i*) φ (*ii*) {… , – 5, – 4, – 3} **6.** (*i*) {1, 2, 3, 4} (*ii*) {… , – 1, 0, 1, 2, 3, 4}

7. (*i*) φ (*ii*) {… , – 7, – 6, – 5}

8. (*i*) {– 1, 0, 1, 2, …} (*ii*) $\left(-\frac{5}{3}, \infty\right)$ **9.** (*i*) {– 1, 0, 1, 2, …} (*ii*) (– 2, ∞)

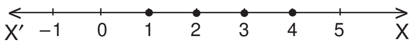
10. (*i*) {…, – 2, – 1, 0, 1} (*ii*) (– ∞, 2) **11.** (*i*) [– 2, ∞) (*ii*) (– ∞, – 3)

12. (*i*) (– 2, ∞) (*ii*) (– ∞, – 3] **13.** (*i*) [4, ∞) (*ii*) (– ∞, 2]

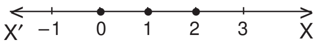
14. (*i*) (– ∞, 6) (*ii*) (– ∞, – 6) **15.** (*i*) [– 26, ∞) (*ii*) [8, ∞)

16. (*i*) (– ∞, 6.3] (*ii*) (– ∞, 120]

17. {1, 2, 3, 4}

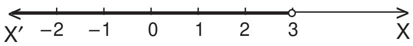


18. {0, 1, 2}

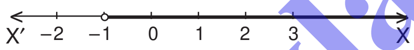


19. {1, 2, 3, …, 13} **20.** φ

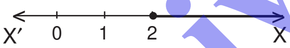
21. (*i*) (– ∞, 3)



(*ii*) (– 1, ∞)



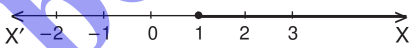
(*iii*) [2, ∞)



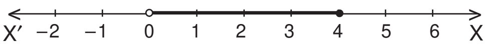
(*iv*) $\left[\frac{1}{7}, \infty\right)$

(*v*) (– ∞, – 10)

(*vi*) [1, ∞)



22. (0, 4]



23. 70 **24.** 35 **25.** More than 50.

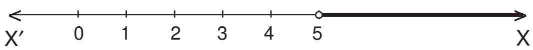
EXERCISE 6.2

1. (*i*) [2, ∞) (*ii*) (– ∞, – 3) **2.** (*i*) (– ∞, – 6] (*ii*) (– 5, 5)

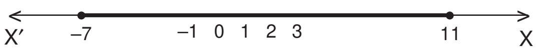
3. (*i*) (– ∞, – 8) (*ii*) (1, 3] **4.** (*i*) [2, 6) (*ii*) (– 5, 4]

5. (*i*) (3, ∞) (*ii*) (– ∞, 2] **6.** (*i*) φ (*ii*) φ

7. (*i*) (5, ∞)



(*ii*) [– 7, 11]

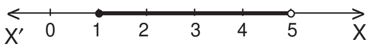


8. (*i*) [– 1, 2) (*ii*) $\left[-\frac{1}{6}, \frac{1}{2}\right)$ **9.** (*i*) (– 2, 1) (*ii*) $\left(-\frac{7}{3}, \frac{1}{3}\right]$

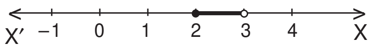
10. (*i*) $\left[1, \frac{11}{3}\right]$ (*ii*) $\left[-\frac{11}{3}, 5\right]$ **11.** (*i*) $(-23, 2]$ (*ii*) $\left[-\frac{34}{3}, \frac{22}{3}\right]$

12. (*i*) $\left[-\frac{56}{3}, \frac{14}{3}\right)$ (*ii*) $[1, 7)$

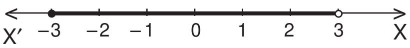
13. $[1, 5)$



14. $[2, 3)$



15. $[-3, 3)$



16. 5, 7; 7, 9 **17.** Between 20°C and 25°C

18. Between 104° F and 113° F

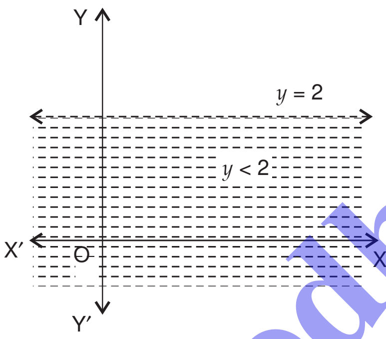
19. More than 120 litres but less than 300 litres

20. More than 562.5 litres but less than 900 litres

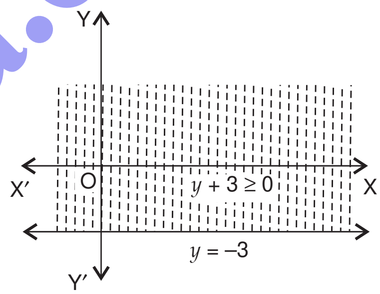
EXERCISE 6.3

The solution set consists of all points in the shaded part of the co-ordinate plane. The dotted line indicates that the points on the line are not included in the solution set.

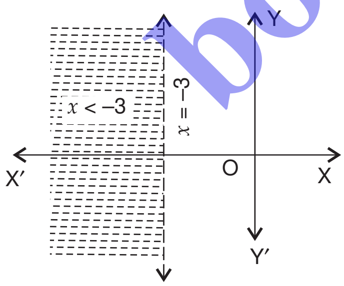
1.



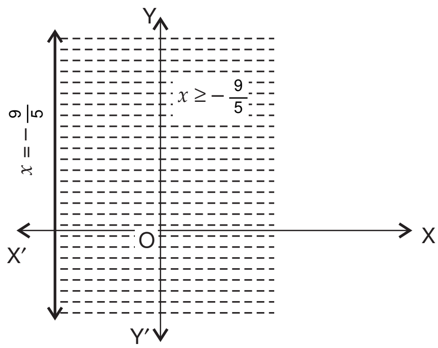
2.



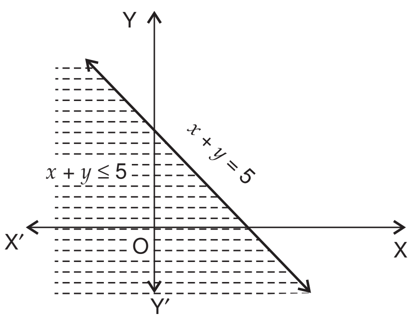
3.



4.



5.



6.