2

RELATIONS AND FUNCTIONS

INTRODUCTION

In daily life, we come across many relations such as Teacher and Student, Mother and Daughter, Book and Cost. In mathematics also, we come across many relations such as

(*i*) number x is square of number y

(*ii*) line l is perpendicular to line m

(*iii*) set A is a proper subset of set B

(*iv*) area of a circle with radius r is πr^2.

In each of these, we notice that a relation involves pairs of objects in a certain order. In this chapter, we will learn how to connect pairs of objects from two sets and then introduce relation between two objects of the pair. Finally, we shall learn about special type of relations called functions. From the beginning of modern mathematics in the 17th century, the concept of function has been at the very centre of mathematical thought. It gives the mathematical rule by which one quantity corresponds to another quantity.

2.1 ORDERED PAIR

*An **ordered pair** is a pair of objects taken in a specific order.*

An ordered pair is written by listing its two members in a specific order, separating them by a comma and enclosing the pair in parentheses. In the ordered pair (a, b), a is called the *first member* (or *component*) and b is called the *second member* (or *component*).

Equality of ordered pairs. *Two ordered pairs* (a, b) *and* (c, d) *are called **equal,** written as* $(a, b) = (c, d)$, *iff* $a = c$ *and* $b = d$.

REMARKS

1. The word ordered implies that the order in which the two elements of the pair occur is meaningful. For example, if we have a sock and a shoe, the order in which they are put on matters. In fact, there are situations in which order is very important and essential.
2. The ordered pairs (a, b) and (b, a) are different unless $a = b$.
3. The two components of an ordered pair may be equal.
4. Note that $\{a, b\} \neq (a, b)$, because $\{a, b\}$ is a set whereas (a, b) is an ordered pair.

2.2 CARTESIAN PRODUCT OF TWO SETS

Let A and B be any two non-empty sets, then the set of all ordered pairs (a, b) *for all* $a \in A$ *and* $b \in B$ *is called the **cartesian product** of A and B. It is written as* $A \times B$ *(read as 'A cross B').*

Symbolically, $A \times B = \{(a, b) : \text{for all } a \in A, b \in B\}$.

For example, let A = {1, 2, 3} and B = {3, 4}, then

A × B = {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)} and

B × A = {(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)}.

From this example, we observe that

(*i*) A × B ≠ B × A. (*ii*) n(A × B) = 6 = n(B × A).

(*iii*) n(A × B) = 6 = 3 × 2 = n(A) × n(B).

REMARK

1. A × B ≠ B × A unless A = B.

2. If A and B are finite sets, then

$$n(A \times B) = n(A) \times n(B) \text{ and } n(A \times B) = n(B \times A).$$

3. A × B = φ when one or both of A, B are empty sets.

4. A × B ≠ φ iff A ≠ φ and B ≠ φ.

5. If A and B are non-empty sets and either A or B is an infinite set, then A × B is also infinite set.

The concept of cartesian product can be extended to more than two sets.

Let A, B and C be any non-empty sets, then the set of all triplets (a, b, c) for all a ∈ A, b ∈ B and c ∈ C is called the ***cartesian product*** *of A, B and C. It is written as A × B × C. Thus,*

$$A \times B \times C = \{(a, b, c) : \text{for all } a \in A, b \in B, c \in C\}.$$

ILLUSTRATIVE EXAMPLES

Example 1. *If the ordered pairs $(x - 1, y + 3)$ and $(2, x + 4)$ are equal, find x and y.*

Solution. $(x - 1, y + 3) = (2, x + 4)$

⇒ $x - 1 = 2$ and $y + 3 = x + 4$

⇒ $x = 3$ and $y = x + 1$

⇒ $x = 3$ and $y = 3 + 1 = 4$.

Hence $x = 3$ and $y = 4$.

Example 2. *If P = {a, b, c} and Q = {d}, form the sets P × Q and Q × P. Are these two cartesian products equal?* (NCERT)

Solution. Given P = {a, b, c} and Q = {d}, by definition of certesian product, we get

P × Q = {(a, d), (b, d), (c, d)}

and Q × P = {(d, a), (d, b), (d, c)}.

By definition of equality of ordered pairs, the pair (a, d) is not equal to the pair (d, a), therefore, P × Q ≠ Q × P.

Example 3. *If A = {1, 2, 3, 4} and $x, y \in$ A, form the set of all ordered pairs (x, y) such that x is a divisor of y.*

Solution. Given A = {1, 2, 3, 4} and $x, y \in$ A.

The set of all ordered pairs (x, y) such that x is a divisor of y

= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}.

Example 4. *Express $\{(x, y) : y + 2x = 5, x, y \in \mathbf{W}\}$ as the set of ordered pairs.*

Solution. Since $y + 2x = 5$ and $x, y \in \mathbf{W}$,

put $x = 0$, $y + 0 = 5 \Rightarrow y = 5$,

$x = 1$, $y + 2 \times 1 = 5 \Rightarrow y = 3$,

$x = 2$, $y + 2 \times 2 = 5 \Rightarrow y = 1$.

For all other values of $x \in \mathbf{W}$, we do not get $y \in \mathbf{W}$.

Hence the required set of ordered pairs is {(0, 5), (1, 3), (2, 1)}.

Example 5. *If* $A = \{1, 5\}$, $B = \{2, 6\}$, $C = \{2, 4\}$, *find* $A \times (B \cup C)$.

Solution. Given $A = \{1, 5\}$, $B = \{2, 6\}$, $C = \{2, 4\}$,

then $B \cup C = \{2, 4, 6\}$.

$\therefore\ A \times (B \cup C) = \{(1, 2), (1, 4), (1, 6), (5, 2), (5, 4), (5, 6)\}$.

Example 6. *If* $A = \{x \mid x \in \mathbf{W}, x < 3\}$, $B = \{x \mid x \in \mathbf{N}, 2 \le x < 4\}$ *and* $C = \{3, 4\}$, *then verify that*

$$(A \cup B) \times C = (A \times C) \cup (B \times C).$$

Solution. Given $A = \{x \mid x \in \mathbf{W}, x < 3\} = \{0, 1, 2\}$

$B = \{x \mid x \in \mathbf{N}, 2 \le x < 4\} = \{2, 3\}$ and $C = \{3, 4\}$

$\Rightarrow\ A \cup B = \{0, 1, 2, 3\}$.

$\therefore\ (A \cup B) \times C = \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$ ...(*i*)

$A \times C = \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4)\}$ and

$B \times C = \{(2, 3), (2, 4), (3, 3), (3, 4)\}$.

$\therefore\ (A \times C) \cup (B \times C) = \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$ ...(*ii*)

From (*i*) and (*ii*), we find that

$$(A \cup B) \times C = (A \times C) \cup (B \times C).$$

Example 7. *If* $A = \{1, 2\}$, $B = \{2, 3\}$ *and* $C = \{0\}$, *form the set* $A \times B \times C$.

Solution. Given $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{0\}$, by def.,

$A \times B \times C = \{(1, 2, 0), (1, 3, 0), (2, 2, 0), (2, 3, 0)\}$.

Example 8. *If* $A = \{-1, 1\}$, *form the set* $A \times A \times A$. *(NCERT)*

Solution. Given $A = \{-1, 1\}$, by def.,

$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),$
$(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$.

Example 9. *If* $\mathbf{R}$ *is the set of all real numbers, what do the cartesian products* $\mathbf{R} \times \mathbf{R}$ *and* $\mathbf{R} \times \mathbf{R} \times \mathbf{R}$ *represent?* *(NCERT)*

Solution. The cartesian product $\mathbf{R} \times \mathbf{R}$ represents the set $\{(x, y) : x, y \in \mathbf{R}\}$ which represents the co-ordinates of all points in two dimensional space.

The cartesian product $\mathbf{R} \times \mathbf{R} \times \mathbf{R}$ represents the set $\{(x, y, z) : x, y, z \in \mathbf{R}\}$ which represents the co-ordinates of all points in three dimensional space.

Example 10. *If* $A \times B = \{(0, 2), (3, -1), (4, 2), (0, -1), (3, 2), (4, -1)\}$, *then find* $B \times A$.

Solution. Clearly, $B \times A$ can be obtained from $A \times B$ by interchanging the components of ordered pairs in $A \times B$.

$\therefore\ B \times A = \{(2, 0), (-1, 3), (2, 4), (-1, 0), (2, 3), (-1, 4)\}$.

Example 11. *If* $A \times B = \{(a, p), (b, q), (c, p), (a, q), (b, p), (c, q)\}$, *find* A *and* B.

Solution. A = set of first components of $A \times B = \{a, b, c\}$,

B = set of second components of $A \times B = \{p, q\}$.

Example 12. *Let* A *and* B *be two sets such that* $n(A) = 5$ *and* $n(B) = 2$. *If* $(a, 1), (b, 5), (c, 5), (d, 1), (e, 5)$ *are in* $A \times B$, *find* A *and* B, *where* a, b, c, d, e *are distinct elements. Also write the remaining elements of* $A \times B$.

Solution. Since a, b, c, d, e are distinct elements and $(a, 1), (b, 5), (c, 5), (d, 1), (e, 5)$ are elements of $A \times B$, therefore,

$a, b, c, d, e \in A$ and $1, 5 \in B$.

But $n(A) = 5$ and $n(B) = 2$,

$\therefore\ A = \{a, b, c, d, e\}$ and $B = \{1, 5\}$.

The remaining elements of $A \times B$ are $(a, 5), (b, 1), (c, 1), (d, 5), (e, 1)$.

Example 13. *The cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.* *(NCERT)*

Solution. Let n (A) = m.

Given $n(A \times A) = 9 \Rightarrow n(A).\ n(A) = 9$

$\Rightarrow \quad m \cdot m = 9 \Rightarrow m^2 = 9 \Rightarrow m = 3$ $\quad (\because m > 0)$

Given $(-1, 0) \in A \times A \Rightarrow -1 \in A$ and $0 \in A$.

Also $(0, 1) \in A \times A \Rightarrow 0 \in A$ and $1 \in A$.

Thus, $-1, 0, 1 \in A$ but $n(A) = 3$.

Therefore, $A = \{-1, 0, 1\}$.

The remaining elements of $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$.

EXERCISE 2.1

Very short answer type questions (1 to 19) :

1. Find a and b if

(*i*) $(a + 1, b - 2) = (3, 1)$ (*ii*) $\left(\frac{a}{3}+1, b-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ (*NCERT*)

(*iii*) $(2a, a + b) = (6, 2)$ (*iv*) $(a + b, 3b - 2) = (7, -5)$.

2. Find x and y if

(*i*) $(4x + 3, y) = (3x + 5, -2)$ (*NCERT Examplar Problems*)

(*ii*) $(x - y, x + y) = (6, 10)$ (*NCERT Examplar Problems*)

(*iii*) $(2x + y, x - y) = (8, 3)$ (*NCERT Examplar Problems*)

(*iv*) $(x - 2, 2y + 1) = (y - 1, x + 2)$

3. If the ordered pairs $(a, -1)$ and $(5, b)$ belong to $\{(x, y) : y = 2x - 3\}$, find the values of a and b.

4. If $P = \{7, 8\}$ and $Q = \{5, 4, 2\}$, find $P \times Q$ and $Q \times P$. (*NCERT*)

5. If $A = \{-1, 0, 1\}$ and $B = \{3, 5\}$, write the following :

(*i*) $A \times B$ (*ii*) $B \times A$ (*iii*) $B \times B$.

6. If $n(A) = 2$ and $B = \{-1, 0, 3\}$, then what is number of elements in $A \times B$?

7. If A is a set such that $n(A) = 3$ and $B = \{3, 4, 5\}$, then what is the number of elements in $A \times B$? (*NCERT*)

8. If $A = \{-3, -1, 0, 4\}$ and $B = \{-1, 0, 1, 2, 3\}$, then write the number of elements in each of the following cartesian products :

(*i*) $A \times B$ (*ii*) $B \times A$ (*iii*) $A \times A$ (*iv*) $B \times B$.

9. If $A = \{1, 2\}$ and $B = \{3, 4\}$, then how many subsets will $A \times B$ have?

10. If $x \in \{2, 3, 5\}$ and $y \in \{2, 4, 6\}$, form the set of all ordered pairs (x, y) such that $x < y$.

11. If $x \in \{-1, 2, 3, 4, 5\}$ and $y \in \{0, 3, 6\}$, form the set of all ordered pairs (x, y) such that $x + y = 5$.

12. If $x \in \{2, 3, 4\}$ and $y \in \{4, 6, 9, 10\}$, form the set of all ordered pairs (x, y) such that x is a factor of y.

13. State whether each of the following statements is true or false. If the statement is false, rewrite the given statement correctly.

(*i*) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m, n), (n, m)\}$.

(*ii*) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in B$ and $y \in A$.

(*iii*) If $A = \{1, 2\}$, $B = \{3, 4\}$, then $A \times (B \cap \phi) = \phi$. (*NCERT*)

14. If $A = \{-1, 0, 1, 2, 3\}$, write the subset S of $A \times A$ such that the second component of the elements of S is 0.

15. If $A \times B = \{(p, q), (p, r), (m, q), (m, r)\}$, find A and B.

16. If $A \times B = \{(-1, 1), (-1, 2), (2, 1), (2, 2), (3, 1), (3, 2)\}$, find A and B.

17. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B, where x, y, z are distinct elements. (*NCERT*)

18. If $A = \{x, y, z\}$ and some elements of $A \times B$ are $(x, 1), (y, 2), (z, 1)$, then write the set B such that $n(A \times B) = 6$.

19. If $A \times B = \{(x, 1), (y, 2), (x, 3), (y, 3), (y, 1), (x, 2)\}$, then find $B \times A$.

20. If $A = \{2, 3, 4\}$ and $B = \{0, 1\}$, find

(*i*) $A \times B$ (*ii*) $B \times A$ (*iii*) $A \times A$ (*iv*) $B \times B$

(*v*) $n(A \times B)$ (*vi*) $n(B \times A)$ (*vii*) $n(A \times A)$ (*viii*) $n(B \times B)$.

Is $A \times B = B \times A$?

21. If $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$, find

(*i*) $A \times B$ (*ii*) $B \times A$

(*iii*) Is $A \times B = B \times A$? (*iv*) Is $n(A \times B) = n(B \times A)$?

(*NCERT Examplar Problems*)

22. If $P = \{x : x < 3, x \in \mathbf{N}\}$ and $Q = \{x : x \le 2, x \in \mathbf{W}\}$, find $(P \cup Q) \times (P \cap Q)$.

(*NCERT Examplar Problems*)

23. Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find

(*i*) $A \times (B \cap C)$ (*ii*) $(A \times B) \cap (A \times C)$

(*iii*) $A \times (B \cup C)$ (*iv*) $(A \times B) \cup (A \times C)$.

24. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$.

Verify that

(*i*) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (*ii*) $A \times C$ is a subset of $B \times D$.

25. If $A = \{x : x \in \mathbf{W}, x < 2\}$, $B = \{x : x \in \mathbf{N}, 1 < x < 5\}$ and $C = \{3, 5\}$, find

(*i*) $A \times (B \cap C)$ (*ii*) $A \times (B \cup C)$. (*NCERT Examplar Problems*)

26. If $A = \{x : x \in \mathbf{N}$ and $x \le 3\}$, $B = \{x : x \in \mathbf{I}, -1 \le x \le 1\}$ and $C = \{1, 2\}$, verify that

(*i*) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (*ii*) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(*iii*) $(A - B) \times C = A \times C - B \times C$.

27. If $A = \{1, 2\}$ and $B = \{3, 4\}$, write $A \times B$. How many subsets will $A \times B$ have? List them. (*NCERT*)

Hint. $n(A \times B) = n(A) \times n(B) = 2 \times 2 = 4$, number of subsets of $A \times B = 2^4$.

28. If $P = \{1, 2\}$, form the set $P \times P \times P$.

29. If $A = \{a, b, c\}$ and some elements of $A \times B$ are $(a, p), (b, q), (c, p)$, write the set B and find the remaining ordered pairs of $A \times B$ such that $n(A \times B) = 6$.

30. If A, B are two sets such that $n(A \times B) = 6$ and some elements of $A \times B$ are $(-1, 2)$, $(2, 3)$, $(4, 3)$, then find $A \times B$ and $B \times A$.

31. The cartesian product $A \times A$ has nine elements among which are found $(-2, 0)$ and $(1, 0)$, then find the set A and the cartesian product $A \times A$.

32. Given $B = \{2, 3, 5\}$ and some elements of $A \times B$ are $(a, 2), (b, 3), (c, 5)$. Find the set A and the remaining ordered pairs of $A \times B$ such that $A \times B$ is least.

2.3 RELATIONS

In everyday life, we frequently speak of relations between two or more objects. To learn the concept properly, consider the following examples :

(*i*) Let A = {1, 2, 3, 5} and B = {2, 4}, then

$A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4), (5, 2), (5, 4)\}$.

We can obtain a subset of A × B by introducing a relation 'is less than' between the elements of the sets A and B.

If we write R for the relation '*is less than*', then we get

$$1 \text{ R } 2,\ 1 \text{ R } 4,\ 2 \text{ R } 4,\ 3 \text{ R } 4.$$

Omitting the letter R between the above pairs of numbers and writing these pairs of numbers as ordered pairs, the above information can be written as a set of ordered pairs R where

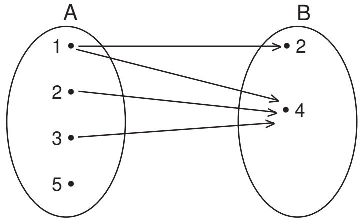


Fig. 2.1.

$$R = \{(1, 2), (1, 4), (2, 4), (3, 4)\}$$
$$= \{(x, y) : x \in A, y \in B, x < y\}.$$

Thus the relation '*is less than*' from the set A to the set B gives rise to a subset R of A × B such that $(x, y) \in R$ iff $x \text{ R } y$ *i.e.* iff $x < y$.

(*ii*) Let A = {2, 3, 5, 9} and B = {4, 6, 9, 15, 25}. There is a relation '*is a divisor of* ' between the elements of the sets A and B. If we write R for the relation '*is a divisor of* ', then we get

2 R 4, 2 R 6, 3 R 6, 3 R 9, 3 R 15, 5 R 15, 5 R 25, 9 R 9.

This can be written as a set of ordered pairs R where

$$R = \{(2, 4), (2, 6), (3, 6), (3, 9), (3, 15), (5, 15), (5, 25), (9, 9)\}$$
$$= \{(x, y) : x \in A, y \in B, x \text{ is a divisor of } y\}.$$

Thus, the relation '*is a divisor of* ' from the set A to the set B gives rise to a subset R of A × B such that $(x, y) \in R$ iff $x \text{ R } y$ *i.e.* iff x is a divisor of y.

(*iii*) Let **N** be the set of natural numbers. There is a relation '*has as its square*' from the set **N** to **N**. If we write R for the relation '*has as its square*', then we get

1 R 1, 2 R 4, 3 R 9, 4 R 16, 5 R 25, ……

This can be written as a set of ordered pairs R where

$$R = \{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), \ldots\ldots\}$$
$$= \{(x, y) : x, y \in \mathbf{N}, y = x^2\}.$$

Thus, R is a subset of **N** × **N** such that $(x, y) \in R$ iff $x \text{ R } y$ *i.e.* iff $y = x^2$.

The above examples lead to :

Definition. *If A, B are any two (non-empty) sets, then any subset of A × B is called a* ***relation*** *from A to B.*

Let R be a relation from A to B. If R = ϕ, then R is called the *empty relation* and if R = A × B, then R is called the *universal relation*.

If R is a relation from A to B and if $(a, b) \in R$, then we write $a \text{ R } b$ and say that a is related to b and if $(a, b) \notin R$, then we write $a \not\text{R}\, b$ and say that a is not related to b.

In particular, if A is any (non-empty) set, then any subset of A × A is called a ***relation on A****.*

2.3.1 Representation of a relation

1. Roster form. In this form, a relation is represented by the set of all ordered pairs which belong to the given relation.

For example, let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, …, 20}, and let R be the relation 'has as its square' from A to B, then

$$R = \{(1, 1), (2, 4), (3, 9), (4, 16)\}.$$

2. Set-builder form. In this form, the relation is represented as $\{(x, y) : x \in A, y \in B, x \ldots y\}$, the blank is to be replaced by the rule which associates x and y.

For example, let A = {1, 3, 4, 5, 7}, B = {2, 4, 6, 8} and

R = {(1, 2), (3, 4), (5, 6), (7, 8)} then R in the builder form can be written as

$$R = \{(x, y) : x \in A, y \in B, x \text{ is one less than } y\}.$$

3. By arrow diagram. In this form, the relation is represented by drawing arrows from first components to the second components of all ordered pairs which belong to the given relation.

For example, let A = {1, 2, 3, 5}, B = {2, 3, 4} and R be the relation 'is greater than' from A to B, then

R = {(3, 2), (5, 2), (5, 3), (5, 4)}.

This relation R from A to B can be represented by the arrow diagram, shown in fig. 2.2.

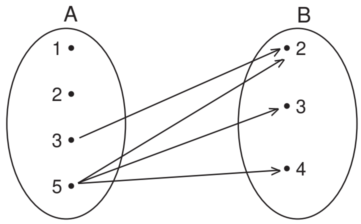


Fig. 2.2.

2.3.2 Domain and range of a relation

Let A, B be any two (non-empty) sets and R be a relation from A to B, then the ***domain*** *of the relation R, is the set of all first components of the ordered pairs which belong to R, and the* ***range*** *of the relation R is the set of all second components of the ordered pairs which belong to R. Thus,*

domain of R $= \{x : x \in A, (x, y) \in R$ *for some* $y \in B\}$ *and*

range of R $= \{y : y \in B, (x, y) \in R$ *for some* $x \in A\}$.

If R is a relation from A to B, then B is called ***codomain*** *of R.*

For example,

let A = {1, 3, 4, 5, 7}, B = {2, 4, 6, 8} and R be the relation *'is one less than'* from A to B, then

R = {(1, 2), (3, 4), (5, 6), (7, 8)}. Here,

domain of R = {1, 3, 5, 7} and range of R = {2, 4, 6, 8}.

In this example, note that range of R = B = codomain of R.

ILLUSTRATIVE EXAMPLES

Example 1. *If A and B are finite sets such that n(A) = m and n(B) = k, find the number of relations from A to B.*

Solution. Given $n(A) = m$ and $n(B) = k$

$\therefore$ $n(A \times B) = n(A) \times n(B) = mk.$

$\therefore$ The number of subsets of $A \times B = 2^{mk}$

($\because$ if $n(A) = m$, then the number of subsets of $A = 2^m$)

Since every subset of $A \times B$ is a relation from A to B, therefore, the number of relations from A to B $= 2^{mk}$.

Example 2. *If a relation R = {(0, 0), (2, 4), (–1, –2), (3, 6), (1, 2)}, then*

(i) write domain of R. *(ii) write range of R.*

(iii) write R in the builder form. *(iv) represent R by an arrow diagram.*

Solution. Given R = {(0, 0), (2, 4), (–1, –2), (3, 6), (1, 2)}.

(*i*) Domain of R = {0, 2, –1, 3, 1}.

(*ii*) Range of R = {0, 4, –2, 6, 2}.

(*iii*) R in the builder form can be written as
$R = \{(x, y) : x \in \mathbf{I}, -1 \le x \le 3, y = 2x\}.$

(*iv*) The relation R can be represented by the arrow diagram, shown in fig. 2.3.

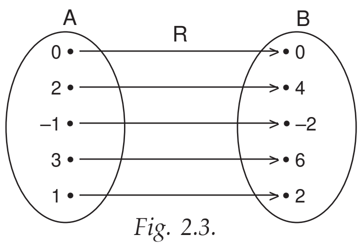


Fig. 2.3.

Example 3. *If A = {–1, 2, 5, 8}, B = {0, 1, 3, 6, 7} and R be the relation 'is one less than' from A to B, then*

(i) find R as a set of ordered pairs. *(ii) find domain and range of R.*

Solution. (*i*) Given A = {–1, 2, 5, 8}, B = {0, 1, 3, 6, 7} and R is the relation *'is one less than'* from A to B, therefore,

R = {(–1, 0), (2, 3), (5, 6)}.

(*ii*) Domain of R = {–1, 2, 5} and range of R = {0, 3, 6}.

Example 4. *If A = {1, 2, 3}, B = {1, 2, 3, 4} and R = {(x, y): (x, y) $\in$ A $\times$ B, y = x + 1}, then*

(i) find A $\times$ B. *(ii) write R in roster form.*

(iii) write domain and range of R. *(iv) represent R by an arrow diagram.*

Solution. (*i*) {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)}.

(*ii*) R = {(1, 2), (2, 3), (3, 4)}.

(*iii*) Domain of R = {1, 2, 3} and range of R = {2, 3, 4}.

(*iv*) The relation R can be represented by the arrow diagram shown in fig. 2.4.

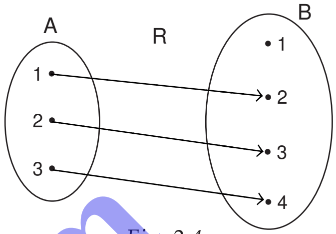


Fig. 2.4.

Example 5. *If A = {1, 2, 3, 4, …, 14} and a relation R is defined from A to A by*

R = {(x, y) : 3x – y = 0, x, y $\in$ A}.

(i) Write R in roster form. *(ii) Write its domain, codomain and range.*

(iii) Depict this relationship by an arrow diagram. *(NCERT)*

Solution. Given A = {1, 2, 3, 4, …, 14}, relation R from A to A is defined by $3x - y = 0$ *i.e.* $y = 3x$, $x, y \in$ A.

(*i*) R = {(1, 3), (2, 6), (3, 9), (4, 12)}.

(*ii*) Domain = {1, 2, 3, 4},

codomain = {1, 2, 3, …, 14} = A

and range = {3, 6, 9, 12}.

(*iii*) The relation R can be represented by the arrow diagram shown in fig. 2.5.

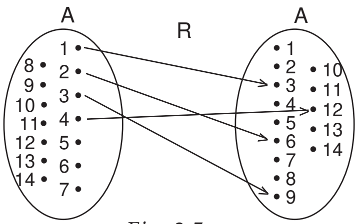


Fig. 2.5.

Example 6. *The adjoining diagram shows a relation between the sets P and Q. Write this relation*

(i) in roster form

(ii) in set builder form.

What is its domain and range? *(NCERT)*

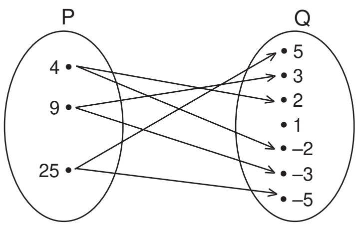


Fig. 2.6.

Solution. (*i*) In roster form,

R = {(4, 2), (4, –2), (9, 3), (9, –3), (25, 5), (25, –5)}

(*ii*) In set builder form, R = $\{(x, y) : x = y^2, x \in \text{P}, y \in \text{Q}\}$

Domain of this relation = {4, 9, 25} and its range = {2, –2, 3, –3, 5, –5}.

Example 7. *Find the domain and the range of the relation R defined by*

R = {(x + 1, x + 3) : x $\in$ {0, 1, 2, 3, 4, 5}}.

Solution. Given $x \in \{0, 1, 2, 3, 4, 5\}$,

put $x = 0$, $x + 1 = 0 + 1 = 1$ and $x + 3 = 0 + 3 = 3$,

$x = 1$, $x + 1 = 1 + 1 = 2$ and $x + 3 = 1 + 3 = 4$,

$x = 2,\ \ x + 1 = 2 + 1 = 3$ and $x + 3 = 2 + 3 = 5$,

$x = 3,\ \ x + 1 = 3 + 1 = 4$ and $x + 3 = 3 + 3 = 6$,

$x = 4,\ \ x + 1 = 4 + 1 = 5$ and $x + 3 = 4 + 3 = 7$,

$x = 5,\ \ x + 1 = 5 + 1 = 6$ and $x + 3 = 5 + 3 = 8$,

Hence R = {(1, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 8)}.

$\therefore$ Domain of R = {1, 2, 3, 4, 5, 6} and range of R = {3, 4, 5, 6, 7, 8}.

Example 8. *If $R = \{(x, y) : x, y \in$ **W**$, 2x + y = 8\}$,* then

(i) find the domain and the range of R.

(ii) write R as a set of ordered pairs.

Solution. (*i*) Given $2x + y = 8$ and $x, y \in$ **W**,

put $x = 0,\ 2 \times 0 + y = 8 \quad \Rightarrow \quad y = 8$,

$x = 1,\ 2 \times 1 + y = 8 \quad \Rightarrow \quad y = 6$,

$x = 2,\ 2 \times 2 + y = 8 \quad \Rightarrow \quad y = 4$,

$x = 3,\ 2 \times 3 + y = 8 \quad \Rightarrow \quad y = 2$,

$x = 4,\ 2 \times 4 + y = 8 \quad \Rightarrow \quad y = 0$.

For all other values of $x \in$ **W**, we do not get $y \in$ **W**.

$\therefore$ Domain of R = {0, 1, 2, 3, 4} and range of R = {8, 6, 4, 2, 0}.

(*ii*) R as a set of ordered pairs can be written as

R = {(0, 8), (1, 6), (2, 4), (3, 2), (4, 0)}.

Example 9. *Find the domain and the range of the relation R given by*

$$R = \{(x, y) : y = x + \frac{6}{x}, \text{ where } x, y \in \mathbf{N} \text{ and } x < 6\} \qquad (\textit{NCERT Examplar Problems})$$

Solution. Given $y = x + \frac{6}{x}$, $x, y \in$ **N** and $x < 6$.

When $x = 1$, $y = 1 + \frac{6}{1} = 7$ and $7 \in$ **N**, so $(1, 7) \in$ R;

when $x = 2$, $y = 2 + \frac{6}{2} = 5$ and $5 \in$ **N**, so $(2, 5) \in$ R;

when $x = 3$, $y = 3 + \frac{6}{3} = 5$ and $5 \in$ **N**, so $(3, 5) \in$ R;

when $x = 4$, $y = 4 + \frac{6}{4} \notin$ **N**, and

when $x = 5$, $y = 5 + \frac{6}{5} \notin$ **N**

$\therefore$ R = {(1, 7), (2, 5), (3, 5)}.

Domain of R = {1, 2, 3} and range of R = {7, 5}.

Example 10. *Find the linear relation between the components of the ordered pairs of the relation R where R = {(2, 1), (4, 7), (1, −2), …}.*

Solution. Given R = {(2, 1), (4, 7), (1, −2), …}.

Let $y = ax + b$ be the linear relation between the components of R.

Since $(2, 1) \in$ R, $\quad \therefore \ y = ax + b \ \Rightarrow \ 1 = 2a + b$ …(*i*)

Also $(4, 7) \in$ R, $\quad \therefore \ y = ax + b \ \Rightarrow \ 7 = 4a + b$ …(*ii*)

Subtracting (*i*) from (*ii*), we get $2a = 6 \ \Rightarrow \ a = 3$.

Substituting $a = 3$ in (*i*), we get $1 = 6 + b \ \Rightarrow \ b = -5$.

Substituting these values of a and b in $y = ax + b$, we get $y = 3x - 5$, which is the required linear relation between the components of the given relation.

Example 11. *If A = {1, 2, 3, 5}, B = {4, 6, 9} and a relation R from A to B is defined by R = {(x, y) : the difference between x and y is odd, $x \in A$, $y \in B$}. Then*

(i) write R in the roster form (NCERT)

(ii) represent R by an arrow diagram.

Solution. (*i*) Given A = {1, 2, 3, 5}, B = {4, 6, 9} and R is relation from A to B given by

R = {(x, y) : $x - y$ is odd, $x \in$ A, $y \in$ B}, therefore

R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

(*ii*) The relation R can be represented by the arrow diagram shown in fig 2.7.

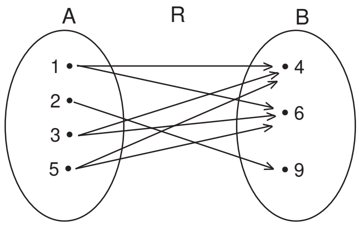


Fig. 2.7.

Example 12. *Let A = {3, 5} and B = {7, 11}. Let R = {(a, b) : $a \in A$, $b \in B$, $a - b$ is odd}. Show that R is an empty relation from A to B.*

Solution. Here A × B = {(3, 7), (3, 11), (5, 7), (5, 11)}. Since none of the numbers 3 – 7, 3 – 11, 5 – 7, 5 – 11 is an odd number, therefore, none of pairs (3, 7), (3, 11), (5, 7) and (5, 11) belongs to R.

Hence R is an empty relation.

Example 13. *If A = {2, 4, 6, 9}, B = {4, 6, 18, 27, 54} and a relation R from A to B is defined by R = {(a, b) : $a \in A$, $b \in B$, a is a factor of b and $a < b$}, then find R in Roster form. Also find its domain and range.* (*NCERT Examplar Problems*)

Solution. Given A = {2, 4, 6, 9}, B = {4, 6, 18, 27, 54} and

R = {(a, b) : $a \in$ A, $b \in$ B, a is a factor of b and $a < b$}.

Since 2 is a factor of 4 and 2 < 4, so (2, 4) ∈ R.

Similarly, (2, 6), (2, 18), (2, 54) ∈ R.

Also (6, 18), (6, 54), (9, 18), (9, 27), (9, 54) ∈ R.

∴ R = {(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54), (9, 18), (9, 27), (9, 54)}.

Domain of R = {2, 6, 9}, range of R = {4, 6, 18, 27, 54}.

Example 14. *Let R be a relation from **Q** to **Q** defined by*

$$R = \{(a, b) : a, b \in \mathbf{Q} \text{ and } a - b \in \mathbf{Z}\}. \text{ Show that}$$

*(i) $(a, a) \in R$ for all $a \in$ **Q***

(ii) $(a, b) \in R$ implies that $(b, a) \in R$

(iii) $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$. (NCERT)

Solution. (*i*) For all $a \in \mathbf{Q}$, $a - a = 0$ and $0 \in \mathbf{Z}$, it implies that $(a, a) \in$ R.

(*ii*) Given $(a, b) \in$ R $\Rightarrow a - b \in \mathbf{Z} \Rightarrow -(a - b) \in \mathbf{Z}$

$\Rightarrow b - a \in \mathbf{Z} \Rightarrow (b, a) \in$ R.

(*iii*) Given $(a, b) \in$ R and $(b, c) \in$ R $\Rightarrow a - b \in \mathbf{Z}$ and $b - c \in \mathbf{Z}$

$\Rightarrow ((a - b) + (b - c)) \in \mathbf{Z} \Rightarrow a - c \in \mathbf{Z} \Rightarrow (a, c) \in$ R.

Example 15. *Let R be a relation from **N** to **N** defined by*

$R = \{(a, b) : a, b \in \mathbf{N}$ and $a = b^2\}$. Are the following true?

*(i) $(a, a) \in R$ for all $a \in$ **N***

(ii) $(a, b) \in R$ implies $(b, a) \in R$

(iii) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$. (NCERT)

Solution. (*i*) No; let $a = 2$, as $2 \neq 2^2$, so $(2, 2) \notin$ R.

(*ii*) No; let $a = 4$, $b = 2$. As $4 = 2^2$, so $(4, 2) \in$ R but $2 \neq 4^2$. So $(2, 4) \notin$ R.

(*iii*) No; let $a = 16$, $b = 4$ and $c = 2$. As $16 = 4^2$ and $4 = 2^2$, so $(16, 4) \in$ R and $(4, 2) \in$ R but $16 \neq 2^2$, so $(16, 2) \notin$ R.

EXERCISE 2.2

Very short answer type questions (1 to 10) :

1. If A and B are two sets such that n (A) = 2 and n (B) = 3, find the number of relations from

(*i*) A to B (*ii*) B to A (*iii*) A to A.

2. Let A = {1, 2} and B = {3, 4}. Find

(*i*) A × B (*ii*) number of relations from A to B. (*NCERT*)

3. Let A = {1, 2} and B = $\{x, y, z\}$, find the number of relations from

(*i*) A to A (*ii*) A to B (*iii*) B to A (*iv*) B to B.

4. If a relation R = {(– 2, 1), (0, 2), (3, 1), (0, – 1), (4, 2), (5, 1)}, then write its domain and range.

5. If A = {2, 3, 5}, B = {2, 4, 6} and R is the relation from A to B defined by R = $\{(x, y) : x \in A, y \in B \text{ and } x < y\}$, then write R in the roster form.

6. If A = {1, 3, 5, 7, 8} and B = {2, 3, 4, 6, 8, 10} and R be the relation **'is one less than'** from A to B, then write R in the roster form.

7. If A = {2, 3, 4}, B = {4, 6, 9, 10} and R = $\{(x, y) : (x, y) \in A \times B$ such that x is a factor of $y\}$, then write R in roster form.

8. If A = {2, 3, 4, 5, 6} and R is a relation from A to A defined by R = $\{(x, y) : y = x + 1, x, y \in A\}$, then list the elements of R.

9. If A = {1, 2, 3, …, 17} and R is a relation on A defined by R = $\{(x, y) : 3x - y = 0, x, y \in A\}$, then write R in the roster form.

10. Write the following relations in the roster form:

(*i*) R = $\{(x, x^3) : x$ is a prime number less than 10} (*NCERT*)

(*ii*) R = $\{(x - 2, x^2) : x$ is a prime number less than 10}.

11. Let A = {1, 2, 3, 4} and B = $\{x, y, z\}$. Let R be a relation from A to B defined by R = $\{(1, x), (1, z), (3, x), (4, y)\}$.

(*i*) Find the domain and range of R. (*ii*) Represent R by an arrow diagram.

12. Let a relation R = {(1, – 1), (2, 0), (3, 1), (4, 2), (5, 3)}, then

(*i*) write the domain and the range of R. (*ii*) write R in the builder form.

13. Let A = {2, 4, 6}, B = {4, 6, 18} and R be the relation *'is a factor of'* from A to B. Find R as a set of ordered pairs and represent it by an arrow diagram.

14. Given R = $\{(x, y) ;\ y = x - 3, x, y \in \mathbf{Z}\}$. State which of the ordered pairs belong to the given relation :

(*i*) (5, 2) (*ii*) (1, 2) (*iii*) (0, – 3) (*iv*) (7, – 4) (*v*) (– 4, 1).

15. Given A = {1, 2, 3, 4, 5} and R = $\{(x, y) : x \in A, y \in A\}$. Find the set of ordered pairs which satisfy the conditions given below :

(*i*) $x + y = 5$ (*ii*) $x + y < 5$ (*iii*) $x + y > 8$. (*NCERT Examplar Problems*)

16. (*i*) If R = $\{(x, y) : x, y \in \mathbf{W}, x^2 + y^2 = 25\}$, then find the domain and the range of R. Also write R in Roster form. (*NCERT Examplar Problems*)

(*ii*) If R = $\{(x, y) : x, y \in \mathbf{Z}, x^2 + y^2 = 64\}$, then write R in Roster form. (*NCERT Examplar Problems*)

17. Define a relation R on the set **N** of natural numbers by

$R = \{(x, y) : y = x + 5, x < 4, x, y \in \mathbf{N}\}$. Depict this relationship using

(*i*) roster form (*ii*) an arrow diagram

Write down the domain and range of R. (*NCERT*)

18. Determine the domain and the range of the relation R defined by

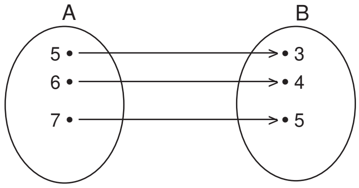
$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$. (*NCERT*)

19. The adjoining diagram shows a relationship between the sets P and Q. Write this relation in

(*i*) roster form

(*ii*) set builder form.

What is its domain and range ? (*NCERT*)



20. Let $A = \{1, 2, 3, 4, 5, 6\}$ and R be the relation defined on A by

$R = \{(a, b) : a \in A, b \in A, a$ divides $b\}$, then

(*i*) list the elements of R (*ii*) find the domain of R

(*iii*) find the range of R. (*NCERT*)

21. Determine the domain and the range of the following relations :

(*i*) $\{(x, y) : x \in \mathbf{N}, y \in \mathbf{N}$ and $x + y = 10\}$

(*ii*) $\{(x, y) : x \in \mathbf{N}, x < 5, y = 3\}$.

22. Let R be the relation on **N** defined by

$R = \{(a, b) : a \in \mathbf{N}, b \in \mathbf{N}$ and $a + 3b = 12\}$, then

(*i*) list the elements of R

(*ii*) find the domain of R

(*iii*) find the range of R.

23. If R is the relation on **N** defined by

$R = \{(x, y) : y = x + \frac{12}{x}, x, y \in \mathbf{N}\}$, then find

(*i*) R in Roster form (*ii*) domain of R (*iii*) range of R.

24. If $A = \{2, 3, 6, 10\}$ and $B = \{1, 6, 10\}$, find the elements of the subset of $A \times B$ corresponding to the relation R 'is less than'. Also represent this relation by an arrow diagram.

25. Write down the domain and the range of the relation $(x, y) : x = 3y$ and x and y are natural numbers less than 10.

26. Let $A = \{-2, -1, 0, 1, 2\}$, list the ordered pairs satisfying each of the following relations on A :

(*i*) *'is greater than'.* (*ii*) *'is the square of '.* (*iii*) *'is the negative of '.*

27. If $A = \{1, 3, 5, 6\}$ and $B = \{3, 4, 5\}$, write the relation R as a set of ordered pairs if

(*i*) $R = \{(x, y) : (x, y) \in A \times B : x + y$ is even$\}$

(*ii*) $R = \{(x, y) : (x, y) \in A \times B : xy$ is odd$\}$.

28. Let R be the relation on **Z** defined by

$R = \{(x, y) : x, y \in \mathbf{Z}, x - y$ is an odd integer$\}$.

Find the domain and range of R. (*NCERT*)

29. Let $R = \{(x, y) : x, y \in \mathbf{Z}, y = 2x - 4\}$. If $(a, -2)$ and $(4, b^2)$ belong to R, find the values of a and b.

30. Find the linear relation between the components of the ordered pairs of the relation R where

(*i*) $R = \{(-1, -1), (0, 2), (1, 5), \ldots\}$. (*ii*) $R = \{(0, 2), (-1, 5), (2, -4), \ldots\}$.

2.4 FUNCTIONS

A function is a special case of a relation. To be specific, let X, Y be two non-empty sets and R (or f) be a relation from X to Y, then R may not relate an element of X to an element of Y or it may relate an element of X to more than one element of Y. But a function relates each element of X to a unique element of Y. We have :

Definition. *If X, Y are two non-empty sets then a subset f of X × Y is called a* ***function*** (*or* ***mapping*** *or* ***map***) *from X to Y iff for each $x \in X$, there exists a unique $y \in Y$ such that $(x, y) \in f$. It is written as $f : X \to Y$.*

Thus, a subset f of X × Y is called a function from X to Y iff

(i) for each $x \in X$, there exists $y \in Y$ such that $(x, y) \in f$ and

(ii) no two different ordered pairs have the same first component.

In other words, a function from X to Y is a rule (or correspondence) which associates to each element x of X, a unique element y of Y.

Image of an element. *The unique element $y \in Y$ is called the* ***image*** *of the element x of X under the function $f : X \to Y$. It is denoted by $f(x)$ i.e. $y = f(x)$. The element y is also called the* ***value*** *of the function f at x.*

2.4.1 Domain and range of a function

Let f be a function from X to Y, then the set X is called the ***domain*** *of the function f and the set Y is called the* ***codomain.***

The set consisting of all the images of the elements of X under the function f is called the ***range*** *of f. It is denoted by f (X).* Thus,

$$\text{range of } f = \{f(x) : \text{for all } x \in X\}.$$

Note that range of f is a subset of Y (codomain) which may or may not be equal to Y.

For example :

(1) Let X = {1, 2, 3, 4, 5}, Y = {0, 1, 2, 3, 5, 7, 9, 11, 13} and

(*i*) f = {(1, 1), (2, 0), (3, 7), (4, 9), (5, 13)}, then f is a function from X to Y because each element of X has a unique image in Y.

Range of f = {1, 0, 7, 9, 13}.

Note that some elements of Y are not associated with any element of X.

(*ii*) f = {(1, 3), (2, 3), (3, 5), (4, 7), (5, 5)}, then f is a function from X to Y because each element of X has a unique image in Y.

Range of f = {3, 5, 7}.

Note that the second components may repeat.

(*iii*) f = {(1, 5), (2, 7), (4, 9), (5, 0)}, then f is not a function from X to Y because the element 3 of X has no image in Y.

(*iv*) f = {(1, 1), (1, 2), (2, 3), (3, 5), (4, 7), (5, 11)}, then f is not a function because the different pairs (1, 1) and (1, 2) have same first component *i.e.* the element 1 of X has two different images.

(2) Let X = {a, b, c, d} and Y = {p, q, r, s, t}, then

(*i*) the rule depicted by the adjoining arrow diagram represents a function from X to Y because each element of X has a unique image in Y.

Range of the function = {p, q, r, t}.

Note that the element s of Y is not associated with any element of X.

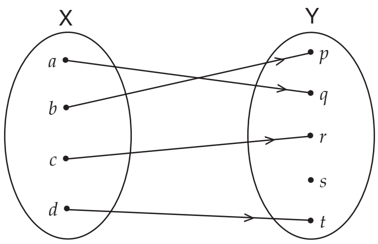


Fig. 2.8.

(*ii*) the rule depicted by the adjoining arrow diagram represents a function from X to Y because each element of X has a unique image in Y.

Range of the function = $\{p, r, s\}$.

Note that the elements a and d of X have the same image s in Y.

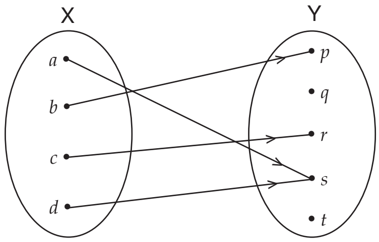


Fig. 2.9.

(*iii*) the rule depicted by the adjoining arrow diagram does not represent a function from X to Y because the element a of X has two different images p and r in Y.

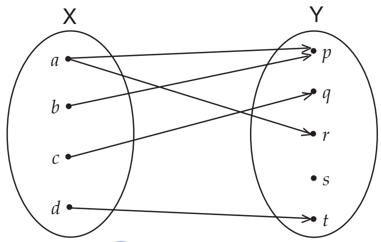


Fig. 2.10.

(*iv*) the rule depicted by the adjoining arrow diagram does not represent a function from X to Y because the element c of X has no image in Y.

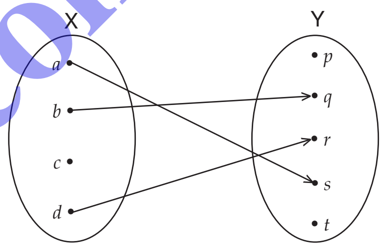


Fig. 2.11.

2.4.2 Main features of a function

Let f be a function from X to Y, then

(*i*) to every $x \in$ X, there exists a unique element $y \in$ Y such that $y = f(x)$.

(*ii*) no element of X can have more than one images in Y.

(*iii*) there may be elements of Y which are not associated with any element of X.

(*iv*) distinct elements of X may have same image in Y.

(*v*) function f is determined when $f(x)$ is known for all $x \in$ X.

ILLUSTRATIVE EXAMPLES

Example 1. *Which of the following relations are functions? Give reasons.*

(i) $R = \{(2, 1), (3, 1), (4, 2)\}$ *(ii)* $R = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$

(iii) $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$. *(NCERT)*

Solution. (*i*) Domain of R = {2, 3, 4}. We note that each element of the domain of R has a unique image, therefore, the relation R is a function.

(*ii*) Domain of R = {2, 3, 4}. We note that the element 2 of the domain of R has two different images 2 and 4, therefore, the relation R is not a function.

(*iii*) Domain of R = {1, 2, 3, 4, 5, 6}. We note that each element of the domain of R has a unique image, therefore, the relation R is a function.

Example 2. *If A = {1, 2, 3} and f, g, h and s are relations corresponding to the subsets of A × A indicated against them, which of f, g, h and s are functions ? In case of a function, find its domain and range.*

(*i*) $f = \{(2, 1), (3, 3)\}$ (*ii*) $g = \{(1, 2), (1, 3), (2, 3), (3, 1)\}$

(*iii*) $h = \{(1, 3), (2, 1), (3, 2)\}$ (*iv*) $s = \{(1, 2), (2, 2), (3, 1)\}$.

Solution. (*i*) f is not a function because the element 1 of A does not appear as the first component of ordered pairs of f, so 1 has no image in A.

(*ii*) g is not a function because the different pairs (1, 2) and (1, 3) of g have same first component *i.e.* the element 1 of A has two different images in A.

(*iii*) h is a function because each element of A has a unique image in A.

Domain of $h = \{1, 2, 3\} = A$ and range of $h = \{3, 1, 2\} = A$.

(*iv*) s is a function because each element of A has a unique image in A.

Domain of $s = \{1, 2, 3\} = A$ and range of $s = \{2, 1\}$.

Example 3. *Consider the following diagrams carefully and state whether they represent functions. Give reasons for your answer. In case of a function, write its domain and range.*

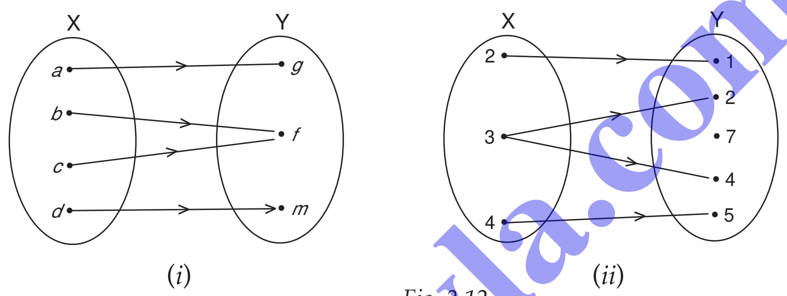


Fig. 2.12.

Solution. (*i*) The given diagram represents a function because each element of the set $\{a, b, c, d\}$ has a unique image in the set $\{g, f, m\}$.

Its domain = $\{a, b, c, d\}$ and range = $\{g, f, m\}$.

(*ii*) The given diagram does not represent a function because the element 3 of the set {2, 3, 4} has two different images 2 and 4 in the set {1, 2, 4, 5, 7}.

Example 4. *Let* **N** *be the set of natural numbers and the relation R be defined on* **N** *by*

$R = \{(x, y) : y = 2x, x, y \in \mathbf{N}\}$.

What is the domain, codomain and range of R ? Is this relation a function ? (NCERT)

Solution. Given $R = \{(x, y) : y = 2x, x, y \in \mathbf{N}\}$.

∴ Domain of R = **N**, codomain of R = **N**

and range of R is the set of even natural numbers.

Since every natural number x has a unique image $2x$, therefore, the relation R is a function.

Example 5. *A relation 'f' is defined by* $f : x \to x^2 - 2$, *where* $x \in \{-1, -2, 0, 2\}$.

(*i*) *List the elements of f.* (*ii*) *Is f a function?*

Solution. Relation f is defined by $f : x \to x^2 - 2$ *i.e.* $f(x) = x^2 - 2$, where $x \in [-1, -2, 0, 2]$

(*i*) $f(-1) = (-1)^2 - 2 = 1 - 2 = -1,$

$f(-2) = (-2)^2 - 2 = 4 - 2 = 2,$

$f(0) = 0^2 - 2 = 0 - 2 = -2,$

$f(2) = 2^2 - 2 = 4 - 2 = 2.$

∴ $f = \{(-1, -1), (-2, 2), (0, -2), (2, 2)\}$

(*ii*) We note that each element of the domain of f has a unique image, therefore, the relation f is a function.

ANSWERS

EXERCISE 2.1

1. (*i*) $a = 2, b = 3$ (*ii*) $a = 2, b = 1$ (*iii*) $a = 3, b = -1$ (*iv*) $a = 8, b = -1$

2. (*i*) $x = 2, y = -2$ (*ii*) $x = 8, y = 2$

   (*iii*) $x = \frac{11}{3}, y = \frac{2}{3}$ (*iv*) $x = 3, y = 2$

3. $a = 1, b = 7$

4. $P \times Q = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$ and

   $Q \times P = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

5. (*i*) $\{(-1, 3), (-1, 5), (0, 3), (0, 5), (1, 3), (1, 5)\}$

   (*ii*) $\{(3, -1), (3, 0), (3, 1), (5, -1), (5, 0), (5, 1)\}$

   (*iii*) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$

6. 6 **7.** 9 **8.** (*i*) 20 (*ii*) 20 (*iii*) 16 (*iv*) 25

9. 16 **10.** $\{(2, 4), (2, 6), (3, 4), (3, 6), (5, 6)\}$

11. $\{(-1, 6), (2, 3), (5, 0)\}$ **12.** $\{(2, 4), (2, 6), (2, 10), (3, 6), (3, 9), (4, 4)\}$

13. (*i*) False; $P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$

    (*ii*) False; if A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$

    (*iii*) True.

14. $S = \{(-1, 0), (0, 0), (1, 0), (2, 0), (3, 0)\}$ **15.** $A = \{p, m\}$ and $B = \{q, r\}$

16. $A = \{-1, 2, 3\}$ and $B = \{1, 2\}$ **17.** $A = \{x, y, z\}$ and $B = \{1, 2\}$

18. $B = \{1, 2\}$ **19.** $B \times A = \{(1, x), (2, y), (3, x), (3, y), (1, y), (2, x)\}$

20. (*i*) $\{(2, 0), (2, 1), (3, 0), (3, 1), (4, 0), (4, 1)\}$

    (*ii*) $\{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

    (*iii*) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$

    (*iv*) $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$

    (*v*) 6 (*vi*) 6 (*vii*) 9 (*viii*) 4 ; No

21. (*i*) $\{(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)\}$

    (*ii*) $\{(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2), (7, 3), (7, 4), (9, 1), (9, 2), (9, 3), (9, 4)\}$

    (*iii*) No (*iv*) Yes

22. $[(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

23. (*i*) $\{(1, 4), (2, 4), (3, 4)\}$ (*ii*) $\{(1, 4), (2, 4), (3, 4)\}$

    (*iii*) $\{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

    (*iv*) $\{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

25. (*i*) $\{(0, 3), (1, 3)\}$

    (*ii*) $\{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

27. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$; 16;

    ϕ, $\{(1, 3)\}$, $\{(1, 4)\}$, $\{(2, 3)\}$, $\{(2, 4)\}$, $\{(1, 3), (1, 4)\}$, $\{(1, 3), (2, 3)\}$, $\{(1, 3), (2, 4)\}$, $\{(1, 4), (2, 3)\}$, $\{(1, 4), (2, 4)\}$, $\{(2, 3), (2, 4)\}$, $\{(1, 3), (1, 4), (2, 3)\}$, $\{(1, 3), (1, 4), (2, 4)\}$, $\{(1, 3), (2, 3), (2, 4)\}$, $\{(1, 4), (2, 3), (2, 4)\}$, $A \times B$.

28. $\{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$

29. $B = \{p, q\}$; (a, q), (b, p), (c, q)

30. $A \times B = \{(-1, 2), (-1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$,

    $B \times A = \{(2, -1), (3, -1), (2, 2), (3, 2), (2, 4), (3, 4)\}$

31. A = {–2, 0, 1}, A × A = {(–2, –2), (– 2, 0), (– 2, 1), (0, –2), (0, 0), (0, 1), (1, –2), (1, 0), (1, 1)}

32. A = {a, b, c} ; (a, 3), (a, 5), (b, 2), (b, 5), (c, 2), (c, 3)

EXERCISE 2.2

1. (*i*) 64 (*ii*) 64 (*iii*) 16 **2.** {(1, 3), (1, 4), (2, 3), (2, 4)}; 16

3. (*i*) 16 (*ii*) 64 (*iii*) 64 (*iv*) 512 **4.** Domain = {–2, 0, 3, 4, 5} and range = {1, 2, –1}

5. R = {(2, 4), (2, 6), (3, 4), (3, 6), (5, 6)} **6.** R = {(1, 2), (3, 4), (5, 6), (7, 8)}

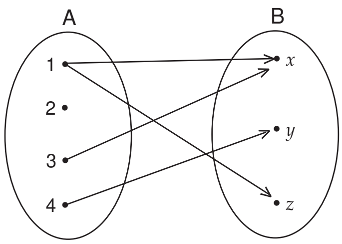
7. R = {(2, 4), (2, 6), (2, 10), (3, 6), (3, 9), (4, 4)} **8.** {(2, 3), (3, 4), (4, 5), (5, 6)}

9. R = {(1, 3), (2, 6), (3, 9), (4, 12), (5, 15)}

10. (*i*) R = {(2, 8), (3, 27), (5, 125), (7, 343)} (*ii*) R = {(0, 4), (1, 9), (3, 25), (5, 49)}

11. (*i*) Domain = {1, 3, 4} and range = {x, y, z}

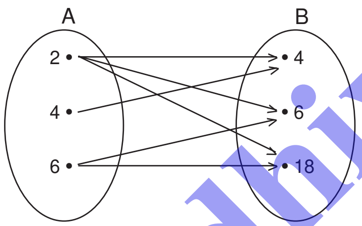
(*ii*)



12. (*i*) Domain of R = {1, 2, 3, 4, 5}, range of R = {–1, 0, 1, 2, 3}

(*ii*) R = {(x, y) : $x \in$ **N**, $1 \le x \le 5$, $y = x - 2$}

13. R = {(2, 4), (2, 6), (2, 18), (4, 4), (6, 6), (6, 18)}



14. (*i*) and (*iii*)

15. (*i*) {(1, 4), (2, 3), (3, 2), (4, 1)} (*ii*) {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}

(*iii*) {(4, 5), (5, 4), (5, 5)}

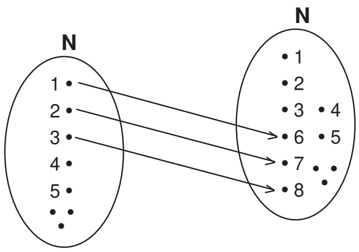
16. (*i*) Domain of R = {0, 3, 4, 5}, range of R = {5, 4, 3, 0}

and R = {(0, 5), (3, 4), (4, 3), (5, 0)}

(*ii*) R = {(0, 8), (0, –8), (8, 0), (–8, 0)}

17. (*i*) {(1, 6), (2, 7), (3, 8)}

(*ii*)



Domain = {1, 2, 3} and range = {6, 7, 8}

18. Domain = {0, 1, 2, 3, 4, 5} and range = {5, 6, 7, 8, 9, 10}

19. (*i*) {(5, 3), (6, 4), (7, 5)}

(*ii*) {(x, y) : $y = x - 2$, $x \in$ **N**, $5 \le x \le 7$}

Domain = {5, 6, 7} and range = {3, 4, 5}