1

SETS

INTRODUCTION

George Cantor, a mathematician, born in Russia and educated in Germany, was the first to realise the importance of sets. The concept of a set is useful in almost every branch of mathematics. In this chapter, you will learn—

- the concept of a set
- representations of a set
- various types of sets
- set relations
- subsets of real numbers—intervals
- Venn diagrams
- operations on sets
- some basic results on cardinality of sets
- practical use of sets in solving problems.

1.1 SETS

In everyday life, we have to deal with *collections* or *aggregates* of objects of one kind or the other. For example, consider the following collections :

(*i*) the collection of even natural numbers less than 15 *i.e.* of the numbers 2, 4, 6, 8, 10, 12, 14.

(*ii*) the collection of vowels in the English alphabet *i.e.* of the letters *a, e, i, o, u*.

(*iii*) all colours of rainbow.

(*iv*) all states of India.

(*v*) all rivers of India.

(*vi*) all prime factors of 330 *i.e.* 2, 3, 5 and 11.

(*vii*) the roots of the equation $x^2 - 2x - 3 = 0$ *i.e.* 3 and -1.

(*viii*) all straight lines (drawn in a particular plane) passing through a given point.

We note that each one of the above collections is a *well-defined* collection of objects. By '*well defined collection of objects*' we mean that given a collection and an object, it should be possible to decide (beyond doubt) whether the object belongs to the given collection or not.

Set. *Any well-defined collection of objects is called a* ***set****. The objects of the set are called its* ***members*** *or* ***elements****.*

Thus, each one of the above collections is a set.

The terms 'objects', 'members' or 'elements' of a set are synonymous and are undefined.

Now, consider the collection of all good books on mathematics.

It is not a well-defined collection, since a mathematics book considered good by one person may not be considered good by another. So, this collection is not a set.

Note that the following collections are not well defined :

(*i*) all intelligent students of class XI of your school.

(*ii*) all big cities of India.

(*iii*) all beautiful girls of India.

(*iv*) five most renowned scientists of the world.

So, none of the above collections is a set.

The sets are usually denoted by capital letters A, B, C etc., and the members of the set are denoted by lower-case letters a, b, c etc.

If x is a member of the set A, we write $x \in$ A (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin$ A (read as 'x does not belong to A'). If x and y both belong to A, we write $x, y \in$ A.

1.1.1 Representations of a set

There are two ways to represent a given set.

1. Roster or tabular form. In this form, we list all the members of the set within braces (curly brackets) and separate these by commas.

For example,

(*i*) the set A of all even natural numbers less than 15 in the *roster form* is written as A = {2, 4, 6, 8, 10, 12, 14}.
Note that 2 ∈ A, 10 ∈ A while 5 ∉ A.

(*ii*) the set S of vowels in the English alphabet in the *tabular form* is written as S = {a, e, i, o, u}.

(*iii*) the set M of months of a year having less than 31 days in the *roster form* is written as M = {February, April, June, September, November}.

(*iv*) the set L of letters in the word 'JODHPUR' in the *tabular form* is written as L = { J, O, D, H, P, U, R }.

REMARKS

1. The order of listing the elements in a set can be changed. Thus, the set {3, 7, 8, 12} may also be written as {7, 3, 8, 12} or {12, 7, 3, 8} etc.
2. If one or more elements of a set are repeated, the set remains the same.
   Thus, the set {a, b, c, b, b, a} is the same as {a, b, c}.
3. While listing the elements of a set, it is sufficient to list its members only once.
   Thus, the set X of letters in the word 'MATHEMATICS' in the tabular form is written as
   X = {M, A, T, H, E, I, C, S}.
4. The roster form enables us to see all the members of a set at a glance. However, if the number of elements in a set is very large, then we represent the set by writing a few elements which clearly indicate the structure of the elements of the set followed (or preceded) by three dots and then writing the last element (if it exists).
   Thus, the set A of odd natural numbers between 50 and 500 in the tabular form can be written as
   A = {51, 53, 55, …, 499}.
   The set P of even integers less than 10 in the roster form can be written as
   P = {…, –4, –2, 0, 2, 4, 6, 8}.

2. Set builder form or rule method. In this form, we write a variable (say x) representing any member of the set which is followed by a colon ' : ' and thereafter we write the property satisfied by each member of the set and then enclose the whole description within braces. If A is a set consisting of elements x having property p, we write A = {x : x has property p}, which is read as 'the set of elements x such that x has the property p'.

The colon ':' stands for the words 'such that'. Sometimes, we use the symbol '|' in place of the colon ':'.

For example,

(*i*) the set A of all even natural numbers less than 15 in the *builder form* is written as A = $\{x : x$ is an even natural number less than 15$\}$.

(*ii*) the set S of vowels in the English alphabet in the *builder form* is written as S = $\{x : x$ is a vowel in the English alphabet$\}$.

(*iii*) the set S = $\{1, 4, 9, 16, 25, \ldots\}$ in the builder form can be written as S = $\{x : x$ is the square of a natural number$\}$.

1.1.2 Kinds of sets

1. Empty set. *A set which does not contain any element is called the* ***empty set*** *or the* ***null set*** *or the* ***void set****.* There is only one such set.

It is denoted by ϕ or $\{\ \}$.

For example,

(*i*) the collection of natural numbers less than 1.

(*ii*) $\{x : 2x + 11 = 3$ and x is a natural number$\}$.

(*iii*) $\{x : x^2 = 9$ and x is an even integer$\}$.

(*iv*) $\{x : x$ is an even prime number greater than 2$\}$.

Each one of these is the empty set.

2. Singleton set. *A set that contains only one element is called a* ***singleton*** (or ***unit***) ***set.***

For example,

(*i*) $\{0\}$.

(*ii*) $\{x : 3x - 1 = 8\}$.

(*iii*) $\{x : x$ is the capital of India$\}$.

Each one of these is a singleton set.

3. Finite set. *A set that contains a limited (definite) number of different elements is called a* ***finite set.***

For example,

(*i*) S = $\{a, e, i, o, u\}$.

(*ii*) A = $\{2, 4, 6, \ldots, 100\}$.

(*iii*) S = $\{x : x$ is the capital of India$\}$.

(*iv*) M = $\{x : x$ is a month of a year$\}$.

(*v*) P = $\{x : x \in \mathbf{N}$ and x is a prime factor of 210$\}$ *i.e.* $\{2, 3, 5, 7\}$.

Each one of these is a finite set.

NOTE

As the empty set has no elements, ϕ is a finite set.

4. Infinite set. *A set that contains an unlimited number of different elements is called an* ***infinite set.*** In other words, a set which is not finite is called an infinite set.

For example,

(*i*) the set of even natural numbers *i.e.* $\{2, 4, 6, \ldots\}$.

(*ii*) $\{x : x \in \mathbf{N}$ and x is prime$\}$ *i.e.* $\{2, 3, 5, 7, 11, 13, \ldots\}$.

(*iii*) the set of all points on a line segment.

(*iv*) the set of all straight lines (drawn in a particular plane) passing through a given point.

Each one of these is an infinite set.

NOTE

All infinite sets cannot be written in the roster form. For example, the set of real numbers cannot be written in this form because the elements of this set do not follow any pattern.

1.1.3 Cardinal number (or order) of a finite set

The number of different elements in a finite set A is called the ***cardinal number (or order)*** *of A, and it is denoted by* n(A) or O(A).

For example,

(*i*) let A = {a, e, i, o, u}, then n(A) = 5.

(*ii*) let A be the set of letters in the word SCHOOL
i.e. A = {S, C, H, O, L}, then n(A) = 5.

(*iii*) let A = {x : x is a prime factor of 60} *i.e.* A = {2, 3, 5}, then n(A) = 3.

(*iv*) let D = {x : x is a digit in our number system}
i.e. D = {0, 1, 2, ..., 9}, then n(D) = 10.

NOTE

The cardinal number of the empty set is zero and the cardinal number of a singleton set is one. *The cardinal number of an infinite set is never defined.*

1.1.4 Some standard sets of numbers

(*i*) **Natural numbers.** The set of natural (or counting) numbers is denoted by **N**. Thus **N** = {1, 2, 3, ...}.

(*ii*) **Whole numbers.** The set of whole numbers is denoted by **W**. Thus
W = {0, 1, 2, 3, ...}.

(*iii*) **Integers.** The set of all integers is denoted by **I** or **Z**. Thus
I = {..., –2, –1, 0, 1, 2, ...}.

(*iv*) **Rational numbers.** Any number which can be expressed in the form $\frac{p}{q}$ where $p, q \in$ **I** and $q \neq 0$ is called a rational number. Thus $\frac{2}{3}, \frac{-7}{5}, 6, \frac{6}{8}, -7$ etc. are rational numbers. The set of rational numbers is denoted by **Q**.

(*v*) **Real numbers**. All rational as well as irrational numbers are real numbers. Thus –3, 0, 5, $\frac{5}{3}, -\frac{7}{2}, \sqrt{2}, -2+\sqrt{3}, \sqrt[3]{2}$ etc. are all real numbers. The set of real numbers is denoted by **R**.

(*vi*) **Irrational numbers.** The set of irrational numbers is denoted by **T**. Thus, **T** = {x : $x \in$ **R** and $x \notin$ **Q**} *i.e.* **T** is the set of all real numbers that are not rational. So, $\sqrt{2}, \sqrt{3}, -\sqrt[3]{5}$, π are members of **T**.

(*vii*) **Positive rational numbers.** The set of positive rational numbers is denoted by $\mathbf{Q}^+$.

(*viii*) **Positive real numbers.** The set of positive real numbers is denoted by $\mathbf{R}^+$.

ILLUSTRATIVE EXAMPLES

Example 1. *State whether the statement 'collection of competent school teachers in Delhi is a set' is true or false. Justify your answer.*

Solution. False, because the collection of competent school teachers in Delhi is not well-defined. A particular teacher considered competent by one person might be considered incompetent by another.

Example 2. *Write the following sets in the roster form :*

(*i*) $A = \{x \mid x \in \mathbf{N} \text{ and } 4 < x \leq 10\}$.

(*ii*) H = {x | x *is a letter in the word 'ARITHMETIC'*}.

(*iii*) $B = \{x \mid x \in \mathbf{N} \text{ and } 5 < x^2 < 50\}$.

(*iv*) $A = \{x : x \in \mathbf{I} \text{ and } x^2 < 20\}$.

(*v*) $S = \{x : x \text{ is a solution of the equation } x^2 - x - 6 = 0\}$.

(*vi*) $B = \left\{x : x = \dfrac{2n-1}{n+2}, n \in \mathbf{W} \text{ and } n < 4\right\}$.

(*vii*) $A = \{x : x \text{ is a two digit number such that the sum of its digits is } 9\}$.

(*viii*) $P = \{x \mid x \text{ is a positive integer less than 10 and } 2^x - 1 \text{ is an odd integer}\}$.

(*NCERT Examplar Problems*)

Solution. (*i*) A = {5, 6, 7, 8, 9, 10}.

(*ii*) H = {A, R, I, T, H, M, E, C}.

(*iii*) We know that the squares of natural numbers 3, 4, 5, 6, 7 lie between 5 and 50, therefore, the set A in roster form is A = {3, 4, 5, 6, 7}.

(*iv*) We know that the squares of integers 0, ± 1, ± 2, ± 3, ± 4 are less than 20, therefore, the set A in roster form is A = {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4}.

(*v*) The given equation is $x^2 - x - 6 = 0$

$\Rightarrow \quad (x - 3)(x + 2) = 0$

$\Rightarrow \quad x - 3 = 0$ or $x + 2 = 0 \quad \Rightarrow x = 3, -2$.

$\therefore$ The set S in the roster form is S = {3, – 2}.

(*vi*) As $n \in \mathbf{W}$ and $n < 4$, $n = 0, 1, 2, 3$.

Also $x = \dfrac{2n-1}{n+2}$, putting $n = 0, 1, 2, 3$, we get

$$x = -\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, 1.$$

Therefore, the set A in roster form is $A = \left\{-\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{3}{4}, 1\right\}$.

(*vii*) As x is a two digit number and the sum of whose digits is 9, such numbers are 18, 27, 36, 45, 54, 63, 72, 81, 90.

Therefore, the set A in the roster form is

A = {18, 27, 36, 45, 54, 63, 72, 81, 90}.

(*viii*) $2^x - 1$ is always an odd positive integer for all positive integral values of x. In particular, $2^x - 1$ is an odd integer for $x = 1, 2, 3, \ldots, 9$.

$\therefore$ P = {1, 2, 3, 4, 5, 6, 7, 8, 9}.

Example 3. *Write the following sets in the builder form :*

(*i*) *the counting numbers which are multiples of 6 and less than 50.*

(*ii*) *the fractions whose numerator is 1 and whose denominator is a counting number less than 10.*

(*iii*) *the set of all positive integers whose cube is odd.*

(*iv*) $A = \left\{\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \ldots, \dfrac{9}{10}\right\}$.

(*v*) $B = \left\{1, \dfrac{1}{4}, \dfrac{1}{9}, \dfrac{1}{16}, \dfrac{1}{25}, \ldots\right\}$.

Solution. (*i*) $\{x : x \text{ is a multiple of 6 and } 0 < x < 50\}$.

(*ii*) $\left\{\dfrac{1}{x} : x \text{ is a counting number and } x < 10\right\}$.

(*iii*) Here, we are to consider only positive integers. As the cube of an even positive integer is an even positive integer and the cube of an odd positive integer is an odd positive integer, therefore, the members of the required set are all positive odd integers. Hence, in builder form the required set can be written as $\{x : x \text{ is an odd positive integer}\}$

i.e. $\{x : x = 2k + 1 \text{ and } k \in \mathbf{W}\}$.

(*iv*) Here, we observe that each member in the given set has numerator one less than the denominator. Also the numerator begins with 1 and ends with 9.

Hence, the set A in the builder form can be written as

$$A = \left\{x : x = \frac{n}{n+1}, n \in \mathbf{N} \text{ and } 1 \le x \le 9\right\}.$$

(*v*) $$B = \left\{x : x = \frac{1}{n^2}, n \in \mathbf{N}\right\}.$$

Example 4. *Match each of the set on the left described in roster form with the same set on the right described in set builder form :*

(*i*) {3, 4, 5, 6, 7}	(*a*) {*x* : *x* is a solution of $x^2 + x - 2 = 0$}
(*ii*) {1, 3, 5, 7, 9}	(*b*) {*x* : *x* is a letter in the word TEACHER}
(*iii*) {A, C, H, R, T, E}	(*c*) {*x* : *x* is an odd natural number less than 10}
(*iv*) {1, –2}	(*d*) {$x : x \in \mathbf{N}$ and $2 < x \le 7$}.

Solution. In (*d*), $x \in \mathbf{N}$ and $2 < x \le 7$, so the values of *x* are 3, 4, 5, 6, 7 and hence (*i*) matches (*d*).

In (*c*), *x* is an odd natural number and less than 10, so the values of *x* are 1, 3, 5, 7, 9 and hence (*ii*) matches (*c*).

In (*b*), there are 7 letters in the word TEACHER and the letter E is repeated, so (*iii*) matches (*b*).

In (*a*), $x^2 + x - 2 = 0 \Rightarrow (x - 1)(x + 2) = 0 \Rightarrow x = 1, -2$, so (*iv*) matches (*a*).

Example 5. *State which of the following statements are true and which are false. Justify your answer.*

(*i*) $31 \notin$ {*x* : *x* has exactly two positive factors}.

(*ii*) $77 \in$ {*x* : *x* has exactly four positive factors}.

(*iii*) $28 \in$ {*x* : the sum of all positive factors of *x* is 2*x*}. (*NCERT Examplar Problems*)

Solution. (*i*) False; since 31 has exactly two positive factors, 1 and 31, 31 belongs to the set.

(*ii*) True; since 77 has exactly four positive factors, 1, 7, 11 and 77, 77 belongs to the set.

(*iii*) True; since the sum of positive factors of $28 = 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28$.

Example 6. *State which of the following sets are finite or infinite. In case of finite sets, mention the cardinal number :*

(*i*) $A = \{x : x \in \mathbf{N} \text{ and } x^2 = 9\}$

(*ii*) $B = \{x : x \in \mathbf{W} \text{ and } 2x - 1 = 0\}$

(*iii*) $C = \{x : x \in \mathbf{N} \text{ and } x^2 - 3x + 2 = 0\}$

(*iv*) $D = \{x : x \in \mathbf{N}$ and *x* is prime}

(*v*) $E = \{x : x \in \mathbf{N}$ and *x* is odd}

(*vi*) F = {*x* : *x* is a month of a year having less than 31 days}

(*vii*) $G = \{x : x \in \mathbf{I} \text{ and } x > -3\}$.

Solution. (*i*) Given $x^2 = 9 \Rightarrow x = 3, -3$ but $x \in \mathbf{N}$,

$\therefore$ A = {3}, which is a finite set. $n(A) = 1$.

(*ii*) Given $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$ but $x \in \mathbf{W}$,

$\therefore$ B = ϕ, which is a finite set. $n(\phi) = 0$.

(*iii*) Given $x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$ but $x \in \mathbf{N}$,

$\therefore$ C = {1, 2}, which is a finite set. $n(C) = 2$.

(*iv*) D = {$x : x \in \mathbf{N}$ and *x* is prime} = {2, 3, 5, 7, 11, 13, ...}.

Since prime numbers are infinite in number, D is an infinite set.

(*v*) E = {$x : x \in \mathbf{N}$ and *x* is odd} = {1, 3, 5, 7, 9, 11, ...}.

Since odd numbers are infinite in number, E is an infinite set.

(*vi*) F = $\{x : x$ is month of a year having less than 31 days$\}$
= {February, April, June, September, November}, which is a finite set.
$n(F) = 5$.

(*vii*) $G = \{x : x \in \mathbf{I}$ and $x > -3\} = \{-2, -1, 0, 1, 2, 3, \ldots\}$, which is an infinite set.

EXERCISE 1.1

Very short answer type questions (1 to 8) :

† 1. State which of the given collection of objects is a set :

(*i*) A collection of popular cinema actors of India.
(*ii*) The collection of even natural numbers less than 51.
(*iii*) The collection of counting numbers less than 1.
(*iv*) Collection of interesting books written by Shakespeare.
(*v*) The collection of novels written by Munshi Prem Chand. (*NCERT*)
(*vi*) The collection of 10 most talented students of your school.
(*vii*) Collection of all rivers flowing in India.
(*viii*) Collection of 5 rivers flowing in India.
(*ix*) Collection of all rational numbers which lie between -1 and 1.
(*x*) A team of eleven best cricketers of the world. (*NCERT*)
(*xi*) A collection of most dangerous animals of the world. (*NCERT*)

2. If A = {3, 5, 7, 9, 11}, then write which of the following statements are true. If a statement is not true, mention why.

(*i*) $3 \in A$ (*ii*) $5, 9 \in A$ (*iii*) $8 \notin A$
(*iv*) $9 \notin A$ (*v*) $\{3\} \in A$ (*vi*) $\{5, 7\} \in A$.

3. Use the roster method to represent the following sets :

(*i*) The counting numbers which are multiples of 6 and less than 50.
(*ii*) The fractions whose numerator is 1, and whose denominator is a counting number less than 10.
(*iii*) $\{x : x \in \mathbf{N}$ and x is a prime factor of 84$\}$.
(*iv*) The set of odd integers lying between -4 and 8.
(*v*) The set of all natural numbers x for which $x + 6$ is less than 10.
(*vi*) The set of all integers x for which $x + 6$ is greater than 10.
(*vii*) The set of all integers x for which $x + 6$ is less than 10.
(*viii*) The set of all integers x for which $\frac{60}{x}$ is a natural number.
(*ix*) $\{t : t^3 = t, t \in \mathbf{R}\}$ (*NCERT Examplar Problems*)
(*x*) $\left\{x : \frac{x-2}{x+3} = 3, x \in \mathbf{R}\right\}$ (*NCERT Examplar Problems*)
(*xi*) $\{x : x^4 - 5x^2 + 6 = 0, x \in \mathbf{R}\}$ (*NCERT Examplar Problems*)
(*xii*) $\left\{x : x \in \mathbf{I}, -\frac{1}{2} < x < \frac{9}{2}\right\}$. (*NCERT*)
(*xiii*) $\{x : x \in \mathbf{N}$ and $4x - 3 \le 15\}$.
(*xiv*) $\{x : x \in \mathbf{N}, x^2 < 40\}$
(*xv*) $\{x : x \in \mathbf{Z}$ and $x^2 < 16\}$.
(*xvi*) The set of all digits in our *decimal system.*
(*xvii*) The set of all letters in the word TRIGONOMETRY.
(*xviii*) The set of all vowels in the English alphabet which precede *q*.
(*xix*) $\{x : x$ is a consonant in the English alphabet which precedes $k\}$. (*NCERT*)

† If a question contains more than one parts, then for one mark question only one part may be asked in the exam.

4. Write the following sets in the builder form :

(*i*) {1, 3, 5, 7, 9, 11, 13}
(*ii*) {2, 4, 6, 8, …}
(*iii*) {3, 6, 9, 12, 15}
(*iv*) {2, 4, 8, 16, 32, 64}
(*v*) {5, 25, 125, 625}
(*vi*) {1, 4, 9, 16, …, 100}
(*vii*) {1, 4, 9, 16, 25, …}
(*viii*) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right\}$.

5. Which of the following are examples of the null set?

(*i*) Set of even prime numbers.
(*ii*) Set of odd natural numbers divisible by 2.
(*iii*) Set of all Indian kids 5 metres tall.
(*iv*) $\{x : x \in \mathbf{N}, x < 5 \text{ and } x > 8\}$.
(*v*) {*x* : *x* is a point common to any two parallel straight lines}.
(*vi*) {*x* : *x* is a student of your school presently studying in both classes XI and XII}.

6. Which of the following sets are finite or infinite?

(*i*) The set of days of a week.
(*ii*) The set of numbers which are multiples of 7.
(*iii*) The set of animals living on Earth.
(*iv*) The set of consonants in the English alphabet.
(*v*) The set of circles drawn in a plane.
(*vi*) The set of prime numbers which are less than one crore.

7. Find the cardinal number of the following sets :

(*i*) { }
(*ii*) {0}
(*iii*) A = {1, 2, 2, 1, 3}
(*iv*) The set of all Indians having 8 legs.
(*v*) The set of all letters in the word PRINCIPAL
(*vi*) The set of all vowels in the word PRINCIPAL.

8. (*i*) Write the cardinal number of the set A, where A = {*x* : *x* is a two digit number, sum of whose digits is 8}.

(*ii*) Write the cardinal number of the set of all integers *x* for which $\frac{30}{x}$ is a natural number.

(*iii*) What is the cardinal number of the set X, where X = {*x* : *x* is a letter in the word 'CHANDIGARH'}?

(*iv*) If S = {*x* : *x* is a positive multiple of 3 less than 100} and
P = {*x* : *x* is a prime number less than 20}, then write $n(\text{S}) + n(\text{P})$.

(*NCERT Examplar Problems*)

9. Match each of the sets on the left described in roster form with the same set on the right described in set builder form :

(*i*) {2, 3}	(*a*) $\{x : x \in \mathbf{N}$ and is a divisor of 6}
(*ii*) {5, –5}	(*b*) $\{x : x \in \mathbf{N}$ and is a prime divisor of 6}
(*iii*) {1, 3, 5}	(*c*) {*x* : *x* is a letter in the word LITTLE}
(*iv*) {1, 2, 3, 6}	(*d*) {*x* : *x* is an odd natural number less than 6}
(*v*) {T, E, L, I}	(*e*) {*x* : *x* is a root of the equation $x^2 - 25 = 0$}.

10. State which of the following statements are true and which are false. Justify your answer.

(*i*) $37 \notin \{x : x$ has exactly two positive factors} (*NCERT Examplar Problems*)
(*ii*) $35 \in \{x : x$ has exactly four positive factors}
(*iii*) $128 \in \{y :$ the sum of all positive factors of y is $2y\}$.
(*iv*) $7747 \in \{t : t$ is a multiple of 37} (*NCERT Examplar Problems*)

11. Classify the following sets into finite set and infinite set. In case of finite sets, mention the cardinal number.

(*i*) A = $\{x : x \in \mathbf{I}, x < 5\}$.

(*ii*) A = $\{x : x \in \mathbf{W}$, x is divisible by 4 and 9$\}$.

(*iii*) P = $\{x : x$ is an even prime number $> 2\}$.

(*iv*) F = $\{x : x \in \mathbf{N}$ and x is a factor of 84$\}$.

(*v*) B = $\{x : x$ is a two digit number, sum of whose digits is 12$\}$.

(*vi*) C = $\{x : x \in \mathbf{W}, 3x - 7 \le 8\}$.

(*vii*) $\{x : x = 5n, n \in \mathbf{N}$ and $x < 20\}$.

(*viii*) $\{x : x = 5n, n \in \mathbf{I}$ and $x < 20\}$.

(*ix*) $\left\{x : x = \frac{n}{n+1}, n \in \mathbf{W} \text{ and } n \le 10\right\}$.

(*x*) $\left\{x : x = \frac{2n}{n+3}, n \in \mathbf{N} \text{ and } 5 < n < 20\right\}$.

1.2 SET RELATIONS

1.2.1 Equivalent sets

Two (finite) sets A and B are called ***equivalent*** *if they have the same number of elements.*

Thus two finite sets A and B are equivalent, written as A ↔ B (read as A is equivalent to B), if $n(A) = n(B)$.

For example,

(*i*) Let A = $\{a, b, c, d, e\}$ and B = $\{2, 3, 5, 7, 9\}$, then $n(A) = 5 = n(B)$. So A ↔ B.

(*ii*) Let A = $\{x : x$ is a colour of rainbow$\}$ and B = $\{x \mid x \in W, x < 7\}$, then $n(A) = 7 = n(B)$. So A ↔ B.

(*iii*) Let P = $\{x : x$ is a letter in the word 'FLOWER'$\}$
and Q = $\{x : x$ is a letter in the word 'FOLLOWER'$\}$,
then $n(P) = 6 = n(Q)$ because each set = {F, L, O, W, E, R}.
So P ↔ Q.

1.2.2 Equal sets

Two sets A and B are said to be ***equal*** *if they have exactly the same elements.* We write it as A = B.

Thus A = B if every member of A is a member of B and every member of B is a member of A.

If A and B are not equal, we write it as A ≠ B.

For example,

(*i*) A = {1, 2} and B = {2, 1, 1, 2, 1}, then A = B.

(*ii*) Let P = $\{x; x$ is a vowel in the word 'EQUALITY'$\}$ and
Q = $\{x : x$ is a vowel in the word 'QUATITATIVE'$\}$, then
P = Q because each set = {E, U, A, I}.

(*iii*) Let A = $\{-3, -2, -1, 0, 1, 2, 3\}$ and B = $\{x : x \in \mathbf{I}$ and $x^2 < 10\}$, then A = B.

(*iv*) Let P = $\{x : x \in \mathbf{N}$ and $x^2 - 2 = 0\}$ and Q = $\{x : x$ is a triangle having 5 sides$\}$, then P = Q because each set = ϕ.

REMARK

If A, B are finite sets and A = B, then $n(A) = n(B)$ so A ↔ B *i.e.* two finite equal sets are always equivalent but the converse may not be true. For example, let A = {2, 3, 5} and B = {2, 3, 4} then $n(A) = 3 = n(B)$, so A ↔ B but A ≠ B.

Thus, two (finite) equal sets are always equivalent but two equivalent sets may not be equal.

1.2.3 Subset

Let A, B be any two sets, then A is called a ***subset*** *of B if every member of A is also a member of B.* We write it as $A \subset B$ (read as 'A is a subset of B' or 'A is contained in B').

Thus $A \subset B$ *if* $x \in A$ *implies* $x \in B$.

If $A \subset B$ *i.e.* A is contained in B, we may also say that B contains A or B is a **superset** of A. We write it as $B \supset A$ (read as 'B contains A' or 'B is a superset of A').

If there exists atleast one element in A which is not a member of B, then A is not a subset of B and we write it as $A \not\subset B$.

For example,

(*i*) let A = {2, 3, 5} and B = {1, 2, 3, 5, 6}. Since every member of A is also a member of B, $A \subset B$. Note that $1 \in B$ but $1 \notin A$, so $B \not\subset A$.

(*ii*) let A = {*a*, *e*, *i*, *o*, *u*} and B = {*a*, *b*, *c*, *d*, *e*}. Note that $i \in A$ but $i \notin B$, so $A \not\subset B$. Also $b \in B$ but $b \notin A$, so $B \not\subset A$.

(*iii*) let P be the set of letters in the word 'SCHOOL' and Q be the set of letters in the word 'SCHOLAR'. In roster form, P = {S, C, H, O, L} and Q = {S, C, H, O, L, A, R}. Clearly $P \subset Q$ while $Q \not\subset P$.

(*iv*) let A = {x : x is a divisor of 56} and B = {x : x is a prime divisor of 56}, then

A = {1, 2, 4, 7, 8, 14, 28, 56} and B = {2, 7}. It is easy to see that $B \subset A$ while $A \not\subset B$.

(*v*) let A = {1, 3, 5, 3, 1} and B = {x : x is an odd natural number less than 6}.

It is easy to see that $A \subset B$ and $B \subset A$. In fact, A = B.

Proper subset. *Let A be any set and B be a non-empty set, then A is called a* ***proper subset*** *of B if every member of A is also a member of B and there exists atleast one element in B which is not a member of A.*

If A is a proper subset of B, we write it as $A \subset B$, $A \neq B$.

In the above example, in (*i*) A is a proper subset of B, in (*iii*) P is a proper subset of Q and in (*iv*) B is a proper subset of A.

REMARK

If two sets A and B are equal *i.e.* A = B, then $A \subset B$ and $B \subset A$. Conversely, if $A \subset B$ and $B \subset A$, then A = B. Thus A = B if and only if for every $a \in A \Rightarrow a \in B$ and for every $b \in B \Rightarrow b \in A$.

NOTE

Let A be any set, then

(1) $A \subset A$ *i.e.* every set is a subset of itself, but not a proper subset. A subset which is not a proper subset is called an **improper subset.**

(2) Every set has only one *improper subset.*

(3) Since the empty set has no elements, $\phi \subset A$ *i.e.* the empty set is a subset of every set.

(4) Empty set is a proper subset of every set except itself.

Subsets of a set

(*i*) Let A = {*a*}, then the subsets of A are ϕ, A.

Note that n (A) = 1, number of subsets of A = 2 = 2^1.

(*ii*) Let A = {1, 2}, then the subsets of A are ϕ, {1}, {2}, A. Note that n (A) = 2, number of subsets of A = 4 = 2^2.

(*iii*) Let A = {1, 2, 3}, then the subsets of A are

ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, A.

Note that n (A) = 3, number of subsets of A = 8 = 2^3.

REMARK

If A is a set with n (A) = m, then the number of subsets of A = 2^m and the number of proper subsets of A = $2^m - 1$.

1.2.4 Power set

The set formed by all the subsets of a given set A is called the ***power set*** *of A, it is denoted by P(A).*

For example, let A = {1, 2}, then P(A) = {ϕ, {1}, {2}, A}.

REMARK

If A is a set with n (A) = m, then n (P(A)) = 2^m.

1.2.5 Universal set

A set that contains all the elements under consideration in a given problem is called ***universal set.*** It is denoted by ξ or U.

It is a kind of 'parent set'. Every set under discussion is a subset of universal set.

Note that the choice of universal set is not unique. Universal set may vary from one problem to another. Therefore, we shall always specify universal set in a given problem.

For example,

(*i*) For A = {b, c, g, m, u}, universal set may be {x : x is a letter in English alphabet}.

(*ii*) For A = {x : $x \in$ **N**, $3 \le x < 12$}, universal set may be {1, 2, 3, ..., 20} or **N**.

(*iii*) For A = {Earth, Mars}, universal set may be {x : x is a planet of our solar system}.

1.2.6 Subsets of real numbers

We know some standard sets of numbers. These sets are subsets of the set of real numbers. It is easy to see that :

$$\mathbf{N} \subset \mathbf{W} \subset \mathbf{I} \subset \mathbf{Q} \subset \mathbf{R}, \mathbf{T} \subset \mathbf{R}, \mathbf{N} \not\subset \mathbf{T}.$$

Also $\mathbf{Q}^+ \subset \mathbf{Q} \subset \mathbf{R}$ and $\mathbf{R}^+ \subset \mathbf{R}$.

Intervals as subsets of R

Intervals are some special types of subsets of the set of real numbers.

Let a and b be two (distinct) real numbers and $a < b$.

The set of all real numbers lying between a and b is said to form an ***open interval***. It is denoted by (a, b). Precisely,

$$(a, b) = \{x : x \in \mathbf{R}, a < x < b\}.$$

The number a is called the **left end point** of the interval and the number b is called the **right end point**. It may be noted that the open interval (a, b) does not contain the left and right end points a and b.

Geometrically, let the points A and B on the real axis represent the real numbers a and b respectively, then the open interval (a, b) is the set of all points to the right of A and to the left of B. It is represented on the real axis as follows :

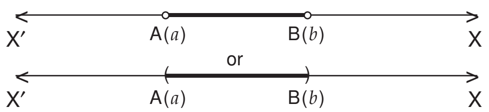

Fig. 1.1.

The set of all real numbers lying between a and b and including the numbers a and b is said to form a ***closed interval***. It is denoted by [a, b]. Precisely,

$$[a, b] = \{x : x \in \mathbf{R}, a \le x \le b\}.$$

The closed interval $[a, b]$ is represented on the real axis as follows :

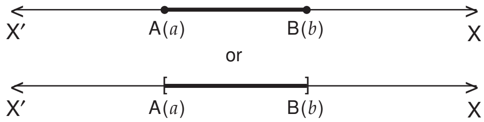

Fig. 1.2.

The set of all real numbers lying between a and b, and including the number b is said to form an ***open-closed interval****.* This interval is open on the left but closed on the right, it is denoted by $(a, b]$. Precisely,

$$(a, b] = \{x : x \in \mathbf{R},\ a < x \le b\}.$$

It is represented on the real axis as follows :

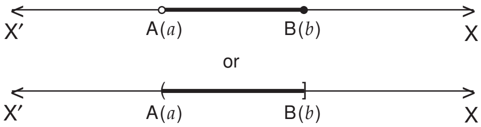

Fig. 1.3.

The set of all real numbers lying between a and b, and including the number a is said to form a ***closed-open interval****.* This interval is closed on the left but open on the right, it is denoted by $[a, b)$. Precisely,

$$[a, b) = \{x : x \in \mathbf{R},\ a \le x < b\}.$$

It is represented on the real axis as follows :

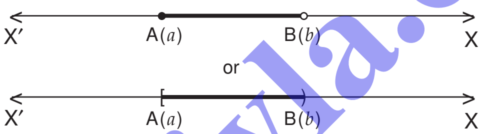

Fig. 1.4.

The number $(b - a)$ is called the **length** of any of the intervals (a, b), $[a, b]$, $(a, b]$ or (a, b).

The intervals introduced above are all **finite intervals**. We shall also need **infinite intervals**.

The set of all real numbers x such that x > a form an ***infinite interval****.* It is denoted by (a, ∞). Precisely,

$$(a, \infty) = \{x : x \in \mathbf{R},\ x > a\}.$$

It is represented on the real axis as follows :

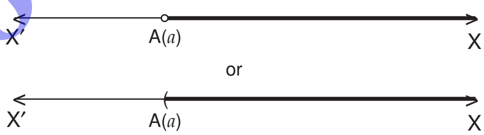

Fig. 1.5.

The set of all real numbers x such that x ≥ a form an ***infinite interval****.* It is denoted by $[a, \infty)$. Precisely,

$$[a, \infty) = \{x : x \in \mathbf{R},\ x \ge a\}.$$

It is represented on the real axis as follows :

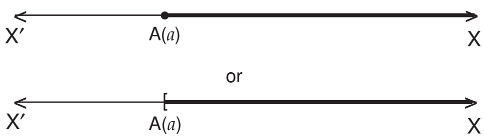

Fig. 1.6.

The set of all real numbers x such that x < a form an ***infinite interval****.* It is denoted by $(-\infty, a)$. Precisely,

$$(-\infty, a) = \{x : x \in \mathbf{R},\ x < a\}.$$

It is represented on the real axis as follows :

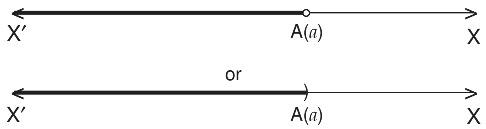

Fig. 1.7.

The set of all real numbers x such that x ≤ a form an ***infinite interval***. It is denoted by $(-\infty, a]$. Precisely,

$$(-\infty, a] = \{x : x \in \mathbf{R}, x \le a\}.$$

It is represented on the real axis as follows :

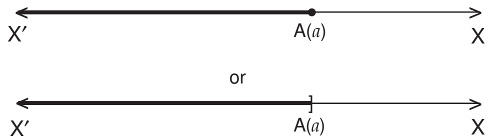

Fig. 1.8.

The set of all real numbers is an ***infinite interval***. It is denoted by $(-\infty, \infty)$. Precisely,

$$(-\infty, \infty) = \{x : x \in \mathbf{R}\}.$$

The real axis itself represents this interval.

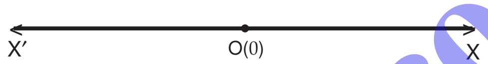

Fig. 1.9.

REMARK

It may be noted that '∞' (read as infinity) *is not a number* and cannot be treated as such. It is a symbol representing largeness without any bound *i.e.* greater than any positive real number however large.

Similarly, '−∞' represents smallness without any bound *i.e.* smaller than any negative real number however small.

ILLUSTRATIVE EXAMPLES

Example 1. *Find all pairs of equal sets (if any) :*

$A = \{0\}$, $B = \{x : x < 5 \text{ and } x > 15\}$,

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,

$E = \{x : x$ *is a positive integral root of the equation* $x^2 - 2x - 15 = 0\}$.

Solution. The given sets are :

A = {0}, B = ϕ

C = {5}, D = {5, −5}

E = {5} $(\because x^2 - 2x - 15 = 0 \Rightarrow (x - 5)(x + 3) = 0 \Rightarrow x = 5, -3)$

Here, we find that the only pair of equal sets is C and E.

Example 2. *Consider the following sets :*

ϕ, A = {1, 3}, B = {1, 5, 9} and C = {1, 2, 3, 5, 7, 9}.

Insert the correct symbol ⊂ *or* ⊄ *between the following pairs of sets :*

(*i*) *ϕ … B* (*ii*) *A … B* (*iii*) *A … C* (*iv*) *B … C.* *(NCERT)*

Solution. (*i*) Since ϕ is subset of every set, ϕ ⊂ B.

(*ii*) Since 3 ∈ A and 3 ∉ B, therefore, A ⊄ B.

(*iii*) Since every member of A is a member of C, A ⊂ C.

(*iv*) Since every member of B is member of C, B ⊂ C.

Example 3. *State whether each of the following statements is true or false for the sets A, B and C where*

$A = \{x : x$ *is a letter of the word 'BOWL'*$\}$

$B = \{x : x$ *is a letter of the word 'ELBOW'*$\}$

$C = \{x : x$ *is a letter of the word 'BELLOW'*$\}$

(*i*) $A \subset B$ (*ii*) $B \supset C$ (*iii*) $B = C$ (*iv*) $B \leftrightarrow C$

(*v*) *A is a proper subset of B* (*vi*) *B is a proper subset of C.*

Solution. The given sets in the roster form are

A = {B, O, W, L},

B = {E, L, B, O, W} and

C = {B, E, L, O, W}

(*i*) true (*ii*) true (*iii*) true (*iv*) true (*v*) true (*vi*) false.

Example 4. *Let* $\xi = \{1, 2, 3, \ldots, 50\}$, $A = \{x : x$ *is divisible by 2 and 3*$\}$,

$B = \{x : x = n^2, n \in \mathbf{N}\}$ *and* $C = \{x : x$ *is a factor of 42*$\}$, *then*

(*i*) *write the sets A, B and C in roster form.*

(*ii*) *state* $n(A)$, $n(B)$ *and* $n(C)$.

(*iii*) *state whether* $A \leftrightarrow B$.

(*iv*) *state whether* $A \leftrightarrow C$.

Solution. (*i*) Here $\xi = \{1, 2, 3, \ldots, 50\}$.

It is understood that A, B and C are subsets of ξ, so the members of these sets are to be taken only from ξ.

The sets A, B and C in roster form are

A = {6, 12, 18, 24, 30, 36, 42, 48},

B = {1, 4, 9, 16, 25, 36, 49} and

C = {1, 2, 3, 6, 7, 14, 21, 42}.

(*ii*) Note that $n(A) = 8$, $n(B) = 7$ and $n(C) = 8$.

(*iii*) No; because $n(A) \neq n(B)$.

(*iv*) Yes; because $n(A) = n(C)$. Note that $A \neq C$.

Example 5. *Let A, B and C be three sets.*

(*i*) *If* $A \in B$ *and* $B \subset C$, *is it true that* $A \subset C$? *If not, give an example.* (NCERT)

(*ii*) *If* $A \subset B$ *and* $B \in C$, *is it true that* $A \in C$? *If not, give an example.* (NCERT)

Solution. (*i*) No. Let A = {1}, B = {{1}, 2} and C = {{1}, 2, 3}.

Here $A \in B$ and $B \subset C$ but $A \not\subset C$ because $1 \in A$ and $1 \notin C$.

(*ii*) No. Let A = {1}, B = {1, 2} and C = {{1, 2}, 3}.

Here $A \subset B$ and $B \in C$ but $A \notin C$.

Example 6. *Let* $A = \{x : x$ *is a letter in the word 'GEORGE CANTOR'*$\}$ *and* $B = \{x : x$ *is a vowel in the word 'GEORGE CANTOR'*$\}$, *then*

(*i*) *write the sets A, B in the tabular form.* (*ii*) *state* $n(A)$ *and* $n(B)$.

(*iii*) *write the number of proper subsets of A.* (*iv*) *write the power set of B.*

Solution. (*i*) A = {G, E, O, R, C, A, N, T} and B = {E, O, A}.

(*ii*) $n(A) = 8$ and $n(B) = 3$.

(*iii*) The number of proper subsets of A $= 2^8 - 1 = 256 - 1 = 255$.

(*iv*) P(B) = {ϕ, {E}, {O}, {A}, {E, O}, {E, A}, {O, A}, B}.

Example 7. *Let* $A = \{1, 2, 3\}$, $B = \{2, 4\}$ *and* $C = \{1, 2, 3, 4\}$. *Find all sets X such that*

(*i*) $X \subset A$ *and* $X \subset B$

(*ii*) $X \subset C$ *but* $X \not\subset A$.

14. There are 200 individuals with a skin disorder; 120 had been exposed to the chemical C_1, 50 to the chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(*i*) chemical C_1 but not chemical C_2

(*ii*) chemical C_2 but not chemical C_1

(*iii*) chemical C_1 or chemical C_2. (*NCERT*)

15. In an examination, 56 per cent of the candidates failed in English and 48 per cent failed in Science. If 18 per cent failed in both English and Science, find the percentage of those who passed in both the subjects.

16. From amongst the 6000 literate individuals of a city, 50% read newspaper A, 45% read newspaper B and 25% read neither A nor B. How many individuals read both the newspapers A as well as B ?

17. In a beauty contest, half the number of judges voted for Miss A, $\frac{2}{3}$ of them voted for Miss B, 10 voted for both and 6 did not vote for either Miss A or Miss B. Find how many judges, in all, were present there.

18. In a group of 50 students, the number of students studying French, English and Sanskrit were found to be as follows :

French = 17, English = 13, Sanskrit = 15;

French and English = 9, English and Sanskrit = 4, French and Sanskrit = 5;

English, French and Sanskrit = 3.

Find the number of students who study :

(*i*) French only

(*ii*) French and Sanskrit but not English

(*iii*) English only

(*iv*) French and English but not Sanskrit

(*v*) Sanskrit only

(*vi*) English and Sanskrit but not French

(*vii*) atleast one of the three languages

(*viii*) none of the three languages. (*NCERT Examplar Problems*)

ANSWERS

EXERCISE 1.1

1. (*ii*), (*iii*), (*v*), (*vii*) and (*ix*) are sets

2. (*i*), (*ii*), (*iii*) are true

(*iv*) is not true because $9 \in A$

(*v*) is not true because {3} is a set and not an element

(*vi*) is not true because {5, 7} is a set

3. (*i*) {6, 12, 18, 24, 30, 36, 42, 48} (*ii*) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}\right\}$

(*iii*) {2, 3, 7} (*iv*) {–3, –1, 1, 3, 5, 7} (*v*) {1, 2, 3}

(*vi*) {5, 6, 7, …} (*vii*) {…, –1, 0, 1, 2, 3} (*viii*) {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

(*ix*) {0, 1, –1} (*x*) $\left\{-\frac{11}{2}\right\}$ (*xi*) $\{\sqrt{2}, -\sqrt{2}, \sqrt{3}, -\sqrt{3}\}$

(*xii*) {0, 1, 2, 3, 4} (*xiii*) {1, 2, 3, 4} (*xiv*) {1, 2, 3, 4, 5, 6}

(*xv*) {–3, –2, –1, 0, 1, 2, 3} (*xvi*) {0, 1, 2, …, 9}

(*xvii*) {T, R, I, G, O, N, M, E, Y} (*xviii*) {*a*, *e*, *i*, *o*}

(*xix*) {*b*, *c*, *d*, *f*, *g*, *h*, *j* }.

4. (*i*) $\{x : x$ is an odd natural number less than $14\}$
(*ii*) $\{x : x$ is an even natural number$\}$ (*iii*) $\{x : x = 3n, n \in \mathbf{N}$ and $n \le 5\}$
(*iv*) $\{x : x = 2^n, n \in \mathbf{N}$ and $n \le 6\}$ (*v*) $\{x : x = 5^n : n \in \mathbf{N}$ and $n \le 4\}$
(*vi*) $\{x : x = n^2, n \in \mathbf{N}$ and $n \le 10\}$ (*vii*) $\{x : x = n^2, n \in \mathbf{N}\}$
(*viii*) $\left\{x : x = \dfrac{n}{n+1}, n \in \mathbf{N}\right\}$
5. (*ii*), (*iii*), (*iv*), (*v*), (*vi*)
6. (*i*) finite (*ii*) infinite (*iii*) finite (*iv*) finite (*v*) infinite (*vi*) finite
7. (*i*) 0 (*ii*) 1 (*iii*) 3 (*iv*) 0 (*v*) 7 (*vi*) 2
8. (*i*) 8 (*ii*) 8 (*iii*) 8 (*iv*) 41
9. (*i*) matches (*b*), (*ii*) matches (*e*), (*iii*) matches (*d*), (*iv*) matches (*a*), (*v*) matches (*c*).
10. (*i*) False (*ii*) True (*iii*) False (*iv*) False
11. (*i*) infinite (*ii*) infinite (*iii*) finite; 0 (*iv*) finite; 12 (*v*) finite; 7
(*vi*) finite; 6 (*vii*) finite; 3 (*viii*) infinite (*ix*) finite, 11 (*x*) finite; 14

EXERCISE 1.2

1. (*i*) A = B (*ii*) A ≠ B (*iii*) A ≠ B (*iv*) A ≠ B (*v*) A ≠ B (*vi*) A = B
(*vii*) A = B (*viii*) A ≠ B
2. (*i*) False (*ii*) False (*iii*) True (*iv*) False (*v*) True (*vi*) False
(*vii*) False (*viii*) True (*ix*) False (*x*) True (*xi*) False
3. (*i*) $\in$ (*ii*) $\notin$ (*iii*) $\subset$ (*iv*) = (*v*) = (*vi*) $\supset$ (*vii*) $\notin$
4. (*i*) False (*ii*) False (*iii*) True (*iv*) False (*v*) True (*vi*) True
5. (*i*) False (*ii*) False (*iii*) False (*iv*) False (*v*) False (*vi*) True
6. (*i*), (*v*), (*vi*), (*viii*), (*ix*) and (*xi*)
7. A = {2, 3}
8. (*i*) No; let A = {1} and B = {{1}, 2}
(*ii*) No; let A = {1, 2} and B = {2, 3}, here $1 \in A$, $A \not\subset B$ but $1 \notin B$
(*iii*) No; let A = {1, 2}, B = {2, 3} and C = {1, 2, 4}
9. (*i*) ϕ, $\{a\}$ (*ii*) ϕ, $\{a\}$, $\{b\}$, $\{a, b\}$ (*iii*) ϕ
10. $\{\phi, \{-1\}, \{0\}, \{2\}, \{-1, 0\}, \{-1, 2\}, \{0, 2\}, \{-1, 0, 2\}\}$
11. 31 **12.** 7 **13.** 4
14. (*i*) $(-3, 5]$ (*ii*) $(-5, -1)$ (*iii*) $[2, 7]$ (*iv*) $[0, 3)$
(*v*) $(-\infty, 5]$ (*vi*) $(-\infty, -3)$ (*vii*) $[-2, \infty)$
15. (*i*) $\{x : x \in \mathbf{R}, -2 < x \le 0\}$ (*ii*) $\{x : x \in \mathbf{R}, 2 < x < 7\}$
(*iii*) $\{x : x \in \mathbf{R}, -5 \le x \le -2\}$ (*iv*) $\{x : x \in \mathbf{R}, -9 \le x < 4\}$
(*v*) $\{x : x \in \mathbf{R}, x > 3\}$ (*vi*) $\{x : x \in \mathbf{R}, x \le -1\}$
(*vii*) $\{x : x \in \mathbf{R}, x < 4\}$
16. A = {1, 2, 3, 4, 5, 6, 7} and B = {1, 2, 3, 4, 5, 6}
17. (*i*) A = {0, 6, 12, 18, 24, 30, 36, 42, 48}, B = {0, 7, 14, 21, 28, 35, 42, 49}
and C = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36}
(*ii*) 9; 8; 11
18. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
19. (*i*) X = ϕ, {2}
(*ii*) X = {1}, {3}, {1, 2}, {1, 4}, {2, 3}, {3, 4}, {1, 3}, {1, 2, 4}, {2, 3, 4}, {1, 2, 3}, {1, 3, 4}, {1, 2, 3, 4}