Trigonometric Identities

EXERCISE – 19.1

Prove the following trigonometrical identities (1 to 19):

Q1. (i) $\sin\theta\cot\theta + \sin\theta\operatorname{cosec}\theta = 1 + \cos\theta$

(ii) $\dfrac{1}{1+\tan^2\theta} + \dfrac{1}{1+\cot^2\theta} = 1$

(iii) $\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

Sol. (i) LHS $= \sin\theta\cot\theta + \sin\theta\operatorname{cosec}\theta$

$= \sin\theta \times \dfrac{\cos\theta}{\sin\theta} + \sin\theta \times \dfrac{1}{\sin\theta}$

$= \cos\theta + 1 = 1 + \cos\theta =$ RHS

(ii) LHS $= \dfrac{1}{1+\tan^2\theta} + \dfrac{1}{1+\cot^2\theta} = \dfrac{1}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} + \dfrac{1}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$

$= \dfrac{1}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} + \dfrac{1}{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$

$= \dfrac{\cos^2\theta}{\cos^2\theta+\sin^2\theta} + \dfrac{\sin^2\theta}{\sin^2\theta+\cos^2\theta}$

$= \dfrac{\cos^2\theta+\sin^2\theta}{\sin^2\theta+\cos^2\theta}$

$= 1 =$ RHS.

(iii) LHS $= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$

$= (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2(\sin^2\theta)(\cos^2\theta)$

$= (\sin^2\theta + \cos^2\theta)^2 \quad (\because \sin^2\theta+\cos^2\theta = 1)$

$= 1^2 = 1$ $\therefore$ Hence proved.

Q2. (i) $\cot^2 A - \dfrac{1}{\sin^2 A} + 1 = 0$

(ii) $\sec A(1-\sin A)(\sec A + \tan A) = 1$

Sol. (i) LHS $= \cot^2 A - \dfrac{1}{\sin^2 A} + 1$

$$= \frac{\cos^2 A}{\sin^2 A} - \frac{1}{\sin^2 A} + 1 = \frac{\cos^2 A - 1}{\sin^2 A} + 1 = \frac{-(1-\cos^2 A)}{\sin^2 A} + 1$$

$$= -\frac{\sin^2 A}{\sin^2 A} + 1 = -1+1 = 0 = \text{RHS}.$$

(ii) LHS $= \sec A(1-\sin A)(\sec A + \tan A)$

$$= \frac{1}{\cos A}(1-\sin A)\left[\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right]$$

$$= \frac{1-\sin A}{\cos A} \times \frac{1+\sin A}{\cos A} = \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1 = \text{RHS}.$$

Q3. (i) $\dfrac{1}{1+\cos A} + \dfrac{1}{1-\cos A} = 2\operatorname{cosec}^2 A$

(ii) $\dfrac{1}{\sec A + \tan A} + \dfrac{1}{\sec A - \tan A} = 2\sec A$

Sol. (i) LHS $= \dfrac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)} = \dfrac{2}{1-\cos^2 A} = \dfrac{2}{\sin^2 A}$

$$= 2\operatorname{cosec}^2 A = \text{RHS}.$$

(ii) LHS $= \dfrac{1}{\sec A + \tan A} + \dfrac{1}{\sec A - \tan A}$

$$= \frac{(\sec A - \tan A) + (\sec A + \tan A)}{(\sec A + \tan A)(\sec A - \tan A)} = \frac{2\sec A}{\sec^2 A - \tan^2 A}$$

$$= \frac{2\sec A}{1} \quad \{\because \sec^2 A - \tan^2 A = 1\}$$

$= 2\sec A = \text{RHS}.$

Q4. (i) $\dfrac{\sin A}{1+\cos A} = \dfrac{1-\cos A}{\sin A}$

(ii) $\dfrac{1-\tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Sol. (i) LHS $= \dfrac{\sin A}{1+\cos A} = \dfrac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)} = \dfrac{\sin A(1-\cos A)}{1-\cos^2 A}$

$= \dfrac{\sin A(1-\cos A)}{\sin^2 A} \quad (\because 1-\cos^2 A = \sin^2 A)$

$= \dfrac{1-\cos A}{\sin A} = RHS.$

(ii) LHS $= \dfrac{1-\tan^2 A}{\cot^2 A - 1} = \dfrac{1-\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A}{\sin^2 A} - 1} = \dfrac{\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A - \sin^2 A}{\sin^2 A}}$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \dfrac{\sin^2 A}{\cos^2 A - \sin^2 A} = \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A = RHS.$

Q5. (i) $\dfrac{\sec A - 1}{\sec A + 1} = \dfrac{1-\cos A}{1+\cos A}$

(ii) $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$

Sol. (i) LHS $= \dfrac{\sec A - 1}{\sec A + 1} = \dfrac{\dfrac{1-\cos A}{\cos A}}{\dfrac{1+\cos A}{\cos A}} = \dfrac{1-\cos A}{\cos A} \times \dfrac{\cos A}{1+\cos A}$

$= \dfrac{1-\cos A}{1+\cos A} = RHS.$

(ii) LHS $= \sec^2 A + \operatorname{cosec}^2 A = \dfrac{1}{\cos^2 A} + \dfrac{1}{\sin^2 A} = \dfrac{\sin^2 A + \cos^2 A}{(\sin^2 A)(\cos^2 A)}$

$= \dfrac{1}{\sin^2 A \cos^2 A} = \sec^2 A \operatorname{cosec}^2 A = RHS.$

Q6. (i) $\frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A} = 2\sec A$

(ii) $\frac{\tan A}{\sec A-1} + \frac{\tan A}{\sec A+1} = 2\,\mathrm{cosec}\,A$

Sol. (i) $LHS = \frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A} = \frac{(1+\sin A)(1+\sin A)+\cos^2 A}{\cos A(1+\sin A)}$

$= \frac{1+\sin A+\sin A+\sin^2 A+\cos^2 A}{\cos A(1+\sin A)} = \frac{1+2\sin A+1}{\cos A(1+\sin A)}$

$= \frac{2(1+\sin A)}{\cos A(1+\sin A)} = \frac{2}{\cos A} = 2\sec A$

(ii) $LHS = \frac{\tan A}{\sec A-1} + \frac{\tan A}{\sec A+1} = \tan A\left[\frac{1}{\sec A-1} + \frac{1}{\sec A+1}\right]$

$= \tan A\left[\frac{\sec A+1+\sec A-1}{(\sec A-1)(\sec A+1)}\right] = \frac{\tan A \times 2\sec A}{\sec^2 A-1}$

$= \frac{2\sec A \cdot \tan A}{\tan^2 A} = \frac{2\sec A}{\tan A} = \frac{2}{\sin A}$

$= 2\,\mathrm{cosec}\,A = RHS.$

Q7. (i) $\frac{\mathrm{cosec}\,A}{\mathrm{cosec}\,A-1} + \frac{\mathrm{cosec}\,A}{\mathrm{cosec}\,A+1} = 2\sec^2 A$

(ii) $\cot A - \tan A = \frac{2\cos^2 A-1}{\sin A\cos A}$

(iii) $\frac{\cot A-1}{2-\sec^2 A} = \frac{\cot A}{1+\tan A}$

Sol. (i) $LHS = \frac{\mathrm{cosec}\,A}{\mathrm{cosec}\,A-1} + \frac{\mathrm{cosec}\,A}{\mathrm{cosec}\,A+1} = \mathrm{cosec}\,A\left[\frac{1}{\mathrm{cosec}\,A-1} + \frac{1}{\mathrm{cosec}\,A+1}\right]$

$= \mathrm{cosec}\,A\left[\frac{\mathrm{cosec}\,A+1+\mathrm{cosec}\,A-1}{(\mathrm{cosec}\,A-1)(\mathrm{cosec}\,A+1)}\right]$

$= \frac{\mathrm{cosec}\,A\,(2\,\mathrm{cosec}\,A)}{\mathrm{cosec}^2 A-1}$

$$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} = \frac{2\sin^2 A}{\sin^2 A \times \cos^2 A} = \frac{2}{\cos^2 A}$$

$$= 2\sec^2 A = RHS.$$

(ii) $LHS = \cot A - \tan A = \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} = \frac{\cos^2 A - \sin^2 A}{\sin A \cos A}$

$$= \frac{\cos^2 A - (1 - \cos^2 A)}{\sin A \cos A} = \frac{2\cos^2 A - 1}{\sin A \cos A} = RHS.$$

(iii) $LHS = \frac{\cot A - 1}{2 - \sec^2 A} = \frac{\frac{\cos A}{\sin A} - 1}{2 - \frac{1}{\cos^2 A}} = \frac{\frac{\cos A - \sin A}{\sin A}}{\frac{2\cos^2 A - 1}{\cos^2 A}}$

$$= \frac{\cos A - \sin A}{\sin A} \times \frac{\cos^2 A}{2\cos^2 A - 1} = \frac{\cos^2 A(\cos A - \sin A)}{\sin A(2\cos^2 A - 1)}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A[2\cos^2 A - (\sin^2 A + \cos^2 A)]} = \frac{\cos^2 A(\cos A - \sin A)}{\sin A[2\cos^2 A - \sin^2 A - \cos^2 A]}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A(\cos^2 A - \sin^2 A)} = \frac{\cos^2 A(\cos A - \sin A)}{\sin A(\cos A + \sin A)(\cos A - \sin A)}$$

$$= \frac{\cos^2 A}{\sin A(\cos A + \sin A)}$$

$RHS = \frac{\cot A}{1 + \tan A} = \frac{\frac{\cos A}{\sin A}}{1 + \frac{\sin A}{\cos A}} = \frac{\frac{\cos A}{\sin A}}{\frac{\cos A + \sin A}{\cos A}}$

$$= \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A + \sin A}$$

$$= \frac{\cos^2 A}{\sin A(\cos A + \sin A)}$$

$\therefore LHS = RHS = \frac{\cos^2 A}{\sin A(\cos A + \sin A)}$

Q8. (i) $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

(ii) $\dfrac{\cos\theta}{1-\tan\theta} - \dfrac{\sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$

Sol. (i) $LHS = \tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \sin^2\theta \times \dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \cdot \sin^2\theta = RHS.$

(ii) $LHS = \dfrac{\cos\theta}{1-\tan\theta} - \dfrac{\sin^2\theta}{\cos\theta-\sin\theta} = \dfrac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} - \dfrac{\sin^2\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} - \dfrac{\sin^2\theta}{\cos\theta-\sin\theta} = \dfrac{\cos^2\theta}{\cos\theta-\sin\theta} - \dfrac{\sin^2\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta} = \dfrac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta-\sin\theta} = \cos\theta+\sin\theta = RHS.$

Q9. (i) $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

(ii) $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{\sin A}{1+\cos A}$

Sol. (i) $LHS = 1 - \dfrac{\cos^2\theta}{1+\sin\theta} = 1 - \dfrac{(1-\sin^2\theta)}{1+\sin\theta} = 1 - \dfrac{(1+\sin\theta)(1-\sin\theta)}{1+\sin\theta}$

$= 1 - 1 + \sin\theta = \sin\theta = RHS.$

(ii) $LHS = \sqrt{\dfrac{1-\cos A}{1+\cos A}} = \sqrt{\dfrac{(1-\cos A)(1+\cos A)}{(1+\cos A)(1+\cos A)}} = \sqrt{\dfrac{1-\cos^2 A}{(1+\cos A)^2}}$

$= \sqrt{\dfrac{\sin^2 A}{(1+\cos A)^2}} = \dfrac{\sin A}{1+\cos A} = RHS.$

Q10. (i) $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

(ii) $\dfrac{\sin\theta\tan\theta}{1-\cos\theta} = 1+\sec\theta$.

Sol. (i) $LHS = \sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\dfrac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}} = \sqrt{\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\dfrac{(1+\cos\theta)^2}{\sin^2\theta}} = \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \operatorname{cosec}\theta + \cot\theta = RHS.$

(ii) $LHS = \dfrac{\sin\theta\tan\theta}{1-\cos\theta} = \dfrac{\sin\theta \times \frac{\sin\theta}{\cos\theta}}{1-\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta(1-\cos\theta)}$

$= \dfrac{1-\cos^2\theta}{\cos\theta(1-\cos\theta)} = \dfrac{(1+\cos\theta)(1-\cos\theta)}{\cos\theta(1-\cos\theta)} = \dfrac{1+\cos\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta} = 1+\sec\theta = RHS.$

Q11. (i) $\sin^4\theta - \cos^4\theta = 1-2\cos^2\theta$

(ii) $\sec^4\theta - \tan^4\theta = 1+2\tan^2\theta$.

Sol. (i) $LHS = \sin^4\theta - \cos^4\theta = (\sin^2\theta)^2 - (\cos^2\theta)^2$

$= (\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta) = (1)(1-\cos^2\theta-\cos^2\theta)$

$= 1-2\cos^2\theta = RHS.$

(ii) $LHS = \sec^4\theta - \tan^4\theta = (\sec^2\theta)^2 - (\tan^2\theta)^2$

$= (\sec^2\theta+\tan^2\theta)(\sec^2\theta-\tan^2\theta) = (\sec^2\theta+\tan^2\theta)(1)$

$= 1+\tan^2\theta+\tan^2\theta$

$= 1+2\tan^2\theta = RHS.$

Q12. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Sol. $LHS = (\sin^2\theta)^3 + (\cos^2\theta)^3 \qquad \{\because a^3+b^3 = (a+b)^3 - 3ab(a+b)\}$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= (1)^3 - 3\sin^2\theta\cos^2\theta(1) \qquad \{\because \sin^2\theta + \cos^2\theta = 1\}$

$= 1 - 3\sin^2\theta\cos^2\theta.$

Q13. (i) $\dfrac{1+\tan A}{\sin A} + \dfrac{1+\cot A}{\cos A} = 2(\sec A + \operatorname{cosec} A)$

(ii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = \tan^2 A + \cot^2 A + 7$

Sol. (i) $LHS = \dfrac{1+\tan A}{\sin A} + \dfrac{1+\cot A}{\cos A}$

$= \dfrac{1+\frac{\sin A}{\cos A}}{\sin A} + \dfrac{1+\frac{\cos A}{\sin A}}{\cos A}$

$= \dfrac{\cos A + \sin A}{\cos A\sin A} + \dfrac{\sin A + \cos A}{\sin A\cos A}$

$= 2\left[\dfrac{\cos A + \sin A}{\cos A\sin A}\right] = 2\left[\dfrac{\cos A}{\cos A\sin A} + \dfrac{\sin A}{\cos A\sin A}\right]$

$= 2\left[\dfrac{1}{\sin A} + \dfrac{1}{\cos A}\right] = 2[\operatorname{cosec} A + \sec A] = RHS.$

(ii) $LHS = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin^2 A\operatorname{cosec}^2 A + \cos^2 A + \sec^2 A + 2\cos^2 A\sec^2 A$

$= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2(1) + 2(1)$

$= 1 + 2 + 2 + (1+\cot^2 A) + (1+\tan^2 A)$

$= 7 + \tan^2 A + \cot^2 A$

$= \tan^2 A + \cot^2 A + 7. = RHS.$

$$= \frac{[\cos\theta + (1-\sin\theta)]^2}{\cos^2\theta - (1-\sin\theta)^2} = \frac{(\cos\theta + 1 - \sin\theta)^2}{\cos^2\theta - (1+\sin^2\theta - 2\sin\theta)}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{\cos^2\theta - 1 - \sin^2\theta + 2\sin\theta}$$

$$= \frac{1 + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{1 - \sin^2\theta - 1 - \sin^2\theta + 2\sin\theta}$$

$$= \frac{2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{2\sin\theta - 2\sin^2\theta}$$

$$= \frac{2(1+\cos\theta) - 2\sin\theta(1+\cos\theta)}{2\sin\theta(1-\sin\theta)}$$

$$= \frac{(1+\cos\theta)\,2(1-\sin\theta)}{2\sin\theta(1-\sin\theta)}$$

$$= \frac{1+\cos\theta}{\sin\theta} = RHS.$$

(ii) $LHS = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\sin\theta} = \frac{\sin\theta}{\frac{\cos\theta+1}{\sin\theta}} = \frac{\sin^2\theta}{1+\cos\theta}$

$$= \frac{1-\cos^2\theta}{1+\cos\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{1+\cos\theta} = \cancel{RHS}\ 1-\cos\theta = RHS$$

$$RHS = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}} = 2 + \frac{\sin^2\theta}{\cos\theta - 1}$$

$$= \frac{2\cos\theta - 2 + \sin^2\theta}{\cos\theta - 1} = \frac{2\cos\theta - 2 + (1-\cos^2\theta)}{\cos\theta - 1}$$

$$= \frac{2(\cos\theta - 1) + (1+\cos\theta)(1-\cos\theta)}{\cos\theta - 1}$$

$$= \frac{(\cos\theta - 1)(2 - \cos\theta - 1)}{\cos\theta - 1} = 1 - \cos\theta$$

$\therefore$ LHS = RHS.

Q16. (i) $(\sin\theta + \cos\theta)(\sec\theta + \operatorname{cosec}\theta) = 2 + \sec\theta \operatorname{cosec}\theta$

(ii) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Sol.

(i) LHS $= (\sin\theta + \cos\theta)(\sec\theta + \operatorname{cosec}\theta)$

$$= (\sin\theta + \cos\theta)\left[\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right] = \frac{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 + \sec\theta\operatorname{cosec}\theta = \text{RHS}.$$

(ii) LHS $= (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)(\tan\theta + \cot\theta)$$

$$= \frac{1 - \sin^2\theta}{\sin\theta} \cdot \frac{1 - \cos^2\theta}{\cos\theta}(\tan\theta + \cot\theta)$$

$$= \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta}(\tan\theta + \cot\theta)$$

$$= \sin\theta \cdot \cos\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta \cdot \cos\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}\right)$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

$$= \text{RHS}.$$

$$= \frac{-\sin^2 A - \sin A}{\cos A(1+\sin A)} = \frac{-\sin A(1+\sin A)}{\cos A(1+\sin A)} = -\tan A.$$

$$\text{RHS} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A} = \frac{1}{\cos A} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\cos A}{1-\sin A} = \frac{1 - \sin A - \cos^2 A}{\cos A(1-\sin A)}$$

$$= \frac{-\sin A + \sin^2 A}{\cos A(1-\sin A)} = \frac{-\sin A(1-\sin A)}{\cos A(1-\sin A)}$$

$$= \frac{-\sin A}{\cos A} = -\tan A$$

$\therefore$ LHS = RHS.

(ii) $$\text{LHS} = \frac{\cot A + \tan B}{\cot B + \tan A} = \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}}$$

$$= \frac{\frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \sin B}} = \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}}$$

$$= \frac{\sin B \cos A}{\sin A \cos B} = \tan B \cot A = \text{RHS}.$$

(iii) $$\text{LHS} = \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B}{\cos^2 A \cos^2 B} - \frac{\cos^2 A \sin^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$= \sec^2 A - \sec^2 B$$

$$= \text{RHS}.$$

Q19. (i) $\frac{\sec\theta - 1}{\sec\theta + 1} = (\cot\theta - \operatorname{cosec}\theta)^2$

(ii) $\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}$

Sol. (ii) LHS $= \frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta} = \frac{\frac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}}{\frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta}}$

$= \frac{\sin\theta(1+\cos\theta)}{\sin\theta(1-\cos\theta)} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}} = \frac{\frac{\sec\theta+1}{\sec\theta}}{\frac{\sec\theta-1}{\sec\theta}}$

$= \frac{\sec\theta+1}{\sec\theta-1}$ = RHS.

Eliminate θ between the equations (20 to 22)

Q20. $x = a\sec\theta$, $y = b\tan\theta$

Sol. $\frac{x}{a} = \sec\theta$, $\frac{y}{b} = \tan\theta$

on squaring, $\frac{x^2}{a^2} = \sec^2\theta$, $\frac{y^2}{b^2} = \tan^2\theta$

on subtracting, $\sec^2\theta - \tan^2\theta = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Q21. If $x = h + a\cos\theta$, $y = k + b\sin\theta$. then prove that

$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$

Sol. Given that $x = h + a\cos\theta$, $y = k + b\sin\theta$

$\Rightarrow \frac{x-h}{a} = \cos\theta$, $\frac{y-k}{b} = \sin\theta$.

As we know, $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$

Q22. If $\sec\theta + \tan\theta = m$ and $\sec\theta - \tan\theta = n$ then prove that $nm = 1$

Sol. $\sec\theta + \tan\theta = m, \quad \sec\theta - \tan\theta = n$ ——(i) (given)

As we know, $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$$

Now from (i), $(n)(m) = 1 \Rightarrow nm = 1$

Q23. If $2\sin A - 1 = 0$, show that $\sin 3A = 3\sin A - 4\sin^3 A$.

Sol. Given $2\sin A - 1 = 0$ ——(i)

To show $\sin 3A = 3\sin A - 4\sin^3 A$

from (i), $\sin A = \frac{1}{2} \Rightarrow A = 30^\circ$

Now LHS $= \sin 3A = \sin(3\times 30^\circ) = \sin 90^\circ = 1$

RHS $= 3\sin A - 4\sin^3 A = 3\sin 30^\circ - 4\sin^3 30^\circ = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - 4\left(\frac{1}{8}\right) = \frac{3}{2} - \frac{1}{2} = 1$

$\therefore$ Hence LHS = RHS.

~~Q24.~~ When $0^\circ < \theta < 90^\circ$, solve the following equations (24 to 26):

Q24. (i) $2\sin^2\theta = \frac{1}{2}$

(ii) $2\cos 3\theta = 1$

Sol. (i) Given that $2\sin^2\theta = \frac{1}{2} \Rightarrow \sin^2\theta = \frac{1}{4} \Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$

(ii) Given that $2\cos 3\theta = 1 \Rightarrow \cos 3\theta = \frac{1}{2} \Rightarrow \cos 3\theta = \cos 60^\circ$

$\Rightarrow 3\theta = 60^\circ \Rightarrow \theta = 20^\circ$.

Q25. (i) $4\cos^2\theta - 3 = 0$

(ii) $\sin^2\theta - \frac{1}{2}\sin\theta = 0$

Sol. (i) Given that $4\cos^2\theta - 3 = 0 \Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \cos\theta = \cos 30^\circ \Rightarrow \theta = 30^\circ$

(ii) Given that $\sin^2\theta - \frac{1}{2}\sin\theta = 0$

$\Rightarrow \sin^2\theta = \frac{1}{2}\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \sin\theta = \sin 30^\circ$

$\Rightarrow \theta = 30^\circ$.

Q26. (i) $2\cos^2\theta + \sin\theta - 2 = 0$

(ii) $3\cos\theta = 2\sin^2\theta$

Sol. (i) $2\cos^2\theta + \sin\theta - 2 = 0$

$\Rightarrow 2(1 - \sin^2\theta) + \sin\theta - 2 = 0$

$\Rightarrow 2 - 2\sin^2\theta + \sin\theta - 2 = 0$

$\Rightarrow -2\sin^2\theta + \sin\theta = 0$

$\Rightarrow \sin\theta = 2\sin^2\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin 30^\circ$

$\Rightarrow \theta = 30^\circ$

(ii) $3\cos\theta = 2\sin^2\theta$

$\Rightarrow 3\cos\theta = 2(1-\cos^2\theta)$

$\Rightarrow 3\cos\theta = 2 - 2\cos^2\theta$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow 2\cos\theta(\cos\theta+2) - 1(\cos\theta+2) = 0$

$\Rightarrow (2\cos\theta-1)(\cos\theta+2) = 0$

either $\cos\theta + 2 = 0$ then $\cos\theta = -2$. but it is not possible as it is negative.

or $2\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos 60^\circ$

$\Rightarrow \theta = 60^\circ$.

EXERCISE – 19.2

Without using trigonometric tables, evaluate the following (1 to 4):

Q1. (i) $\dfrac{\cos 18^\circ}{\sin 72^\circ}$ (ii) $\dfrac{\tan 41^\circ}{\cot 49^\circ}$ (iii) $\dfrac{\operatorname{cosec} 17^\circ 30'}{\sec 73^\circ 30'}$

Sol. (i) $\dfrac{\cos 18^\circ}{\sin 72^\circ} = \dfrac{\cos 18^\circ}{\sin(90^\circ - 18^\circ)} = \dfrac{\cos 18^\circ}{\sin 18^\circ} = 1$

(ii) $\dfrac{\tan 41^\circ}{\cot 49^\circ} = \dfrac{\tan 41^\circ}{\cot(90^\circ - 41^\circ)} = \dfrac{\tan 41^\circ}{\tan 41^\circ} = 1$

(iii) $\dfrac{\operatorname{cosec} 17^\circ 30'}{\sec 73^\circ} = \dfrac{\operatorname{cosec} 17^\circ 30'}{\sec(90^\circ - 17^\circ 30')} = \dfrac{\operatorname{cosec} 17^\circ 30'}{\operatorname{cosec} 17^\circ 30'} = 1$

Q2. (i) $\dfrac{\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right)$ (iv) $\dfrac{\cos 75^\circ}{\sin 15^\circ} + \dfrac{\sin 12^\circ}{\cos 78^\circ} - \dfrac{\cos 18^\circ}{\sin 72^\circ}$

(ii) $\left(\dfrac{\sin 49^\circ}{\cos 41^\circ}\right) + \left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right)$ (v) $\dfrac{\sin 25^\circ}{\sec 65^\circ} + \dfrac{\cos 25^\circ}{\operatorname{cosec} 65^\circ}$

(iii) $\dfrac{\sin 72^\circ}{\cos 18^\circ} - \dfrac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

Sol. (i) $\dfrac{\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right) = \dfrac{\cot 40^\circ}{\cot 40^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\cos 35^\circ}\right) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$.

(ii) $\dfrac{\sin 49^\circ}{\cos 41^\circ} + \dfrac{\cos 35^\circ}{\sin 55^\circ} = \dfrac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ} + \dfrac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ}$

$= \dfrac{\cos 41^\circ}{\cos 41^\circ} + \dfrac{\sin 55^\circ}{\sin 55^\circ} = 1 + 1 = 2.$

(iii) $\dfrac{\sin 72^\circ}{\cos 18^\circ} - \dfrac{\sec 32^\circ}{\operatorname{cosec} 58^\circ} = \dfrac{\sin 72^\circ}{\cos(90^\circ - 72^\circ)} - \dfrac{\sec 32^\circ}{\operatorname{cosec}(90^\circ - 32^\circ)}$

$= \dfrac{\sin 72^\circ}{\sin 72^\circ} - \dfrac{\sec 32^\circ}{\sec 32^\circ} = 1 - 1 = 0$

(iv) $\dfrac{\cos 75^\circ}{\sin 15^\circ} + \dfrac{\sin 12^\circ}{\cos 78^\circ} - \dfrac{\cos 18^\circ}{\sin 72^\circ}$

$= \dfrac{\cos 75^\circ}{\sin(90^\circ - 75^\circ)} + \dfrac{\sin 12^\circ}{\cos(90^\circ - 12^\circ)} - \dfrac{\cos 18^\circ}{\sin(90^\circ - 18^\circ)}$

$= \dfrac{\cos 75^\circ}{\cos 75^\circ} + \dfrac{\sin 12^\circ}{\sin 12^\circ} - \dfrac{\cos 18^\circ}{\cos 18^\circ}$

$= 1 + 1 - 1 = 1$

(v) $\dfrac{\sin 25^\circ}{\sec 65^\circ} + \dfrac{\cos 25^\circ}{\operatorname{cosec} 65^\circ} = \sin 25^\circ \cdot \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin(90^\circ - 65^\circ) \cdot \cos 65^\circ + \cos(90^\circ - 65^\circ) \cdot \sin 65^\circ$

$= \cos 65^\circ \cdot \cos 65^\circ + \sin 65^\circ \cdot \sin 65^\circ$

$= \cos^2 65^\circ + \sin^2 65^\circ \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Q3. (i) $\sin 62^\circ - \cos 28^\circ$ (ii) $\operatorname{cosec} 35^\circ - \sec 55^\circ$

Sol. (i) $\sin(90^\circ - 28^\circ) - \cos 28^\circ = \cos 28^\circ - \cos 28^\circ = 0$

(ii) $\operatorname{cosec} 35^\circ - \sec(90^\circ - 35^\circ) = \operatorname{cosec} 35^\circ - \operatorname{cosec} 35^\circ = 0$

Q4. $7 \cdot \dfrac{\sin 27^\circ}{\cos 63^\circ} + \dfrac{3 \cos 21^\circ}{\sin 69^\circ} - 6 \cdot \dfrac{\tan 36^\circ}{\cot 54^\circ}$

Sol. $= \dfrac{7 \sin 27^\circ}{\cos(90^\circ - 27^\circ)} + \dfrac{3 \cos 21^\circ}{\sin(90^\circ - 21^\circ)} - 6 \cdot \dfrac{\tan 36^\circ}{\cot(90^\circ - 36^\circ)}$

$= 7 \dfrac{\sin 27^\circ}{\sin 27^\circ} + 3 \cdot \dfrac{\cos 21^\circ}{\cos 21^\circ} - 6 \cdot \dfrac{\tan 36^\circ}{\tan 36^\circ}$

$= 7 + 3 - 6$

$= 4.$

Q5. Express each of the following interms of trigonometric ratios of angles between 0° to 45° :

(i) $\tan 81^\circ + \cos 72^\circ$ (ii) $\cot 49^\circ + \operatorname{cosec} 87^\circ$

Sol. (i) $\tan 81^\circ + \cos 72^\circ = \tan(90^\circ - 9^\circ) + \cos(90^\circ - 18^\circ)$

$= \cot 9^\circ + \sin 18^\circ$

(ii) $\cot 49^\circ + \operatorname{cosec} 87^\circ = \cot(90^\circ - 41^\circ) + \operatorname{cosec}(90^\circ - 3^\circ)$

$= \tan 41^\circ + \sec 3^\circ.$

Without using trigonometric tables, prove that (6 to 10) :

Q6. (i) $\sin^2 28^\circ - \cos^2 62^\circ = 0$ (ii) $\cos^2 25^\circ + \cos^2 65^\circ = 1$

(iii) $\operatorname{cosec}^2 67^\circ - \tan^2 23^\circ = 1$ (iv) $\sec^2 22^\circ - \cot^2 68^\circ = 1$

Sol. (i) $\sin^2 28^\circ - \cos^2 62^\circ = \sin^2 28^\circ - \cos^2(90^\circ - 28^\circ)$

$= \sin^2 28^\circ - \sin^2 28^\circ = 0 = \text{RHS}.$

(ii) $\text{LHS} = \cos^2 25^\circ + \cos^2 65^\circ = \cos^2 25^\circ + \sin^2 25^\circ = 1 = \text{RHS}.$

(iii) $\text{LHS} = \operatorname{cosec}^2 67^\circ - \tan^2 23^\circ = \operatorname{cosec}^2 67^\circ - \tan^2(90^\circ - 67^\circ)$

$= \operatorname{cosec}^2 67^\circ - \cot^2 67^\circ = 1 = \text{RHS}.$

(iv) $\text{LHS} = \sec^2 22^\circ - \cot^2 68^\circ = \sec^2 22^\circ - \cot^2(90^\circ - 22^\circ)$

$= \sec^2 22^\circ - \tan^2 22^\circ = 1 = \text{RHS}.$

Q7. (i) $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1$

(ii) $\sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ = 2$

Sol (i) $LHS = \sin 63^\circ \cos(90^\circ - 63^\circ) + \cos 63^\circ \sin(90^\circ - 63^\circ)$

$= \sin 63^\circ \sin 63^\circ + \cos 63^\circ \cos 63^\circ$

$= \sin^2 63^\circ + \cos^2 63^\circ = 1 = RHS.$

(ii) $LHS = \sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ$

$= \frac{1}{\cos 31^\circ} \cdot \sin 59^\circ + \cos 31^\circ \frac{1}{\sin 59^\circ} = \frac{\sin 59^\circ}{\cos 31^\circ} + \frac{\cos 31^\circ}{\sin 59^\circ}$

$= \frac{\sin 59^\circ}{\sin 59^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} = 1 + 1 = 2 = RHS.$

Q8. (i) $\sec 70^\circ \sin 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ = 0$

(ii) $\sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ = 0$

Sol. (i) $LHS = \sec 70^\circ \sin 20^\circ - \cos 20^\circ \operatorname{cosec} 70^\circ$

$= \frac{\sin 20^\circ}{\cos 70^\circ} - \frac{\cos 20^\circ}{\sin 70^\circ} = \frac{\sin 20^\circ}{\sin 20^\circ} - \frac{\cos 20^\circ}{\cos 20^\circ} = 1 - 1$

$= 0 = RHS.$

(ii) $LHS = \sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ$

$= \sin^2 20^\circ + \sin^2(90^\circ - 20^\circ) - \tan^2 45^\circ$

$= \sin^2 20^\circ + \cos^2 20^\circ - \tan^2 45^\circ = 1 - 1 = 0 = RHS.$

Q9. (i) $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 = 0$

(ii) $2 \frac{\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ} = 1$

Sol. (i) $LHS = \frac{\cot 54^\circ}{\tan(90^\circ - 54^\circ)} + \frac{\tan 20^\circ}{\cot(90^\circ - 20^\circ)} - 2$

$= \frac{\cot 54^\circ}{\cot 54^\circ} + \frac{\tan 20^\circ}{\tan 20^\circ} - 2 = 1 + 1 - 2 = 0 = RHS.$

(ii) LHS $= 2\dfrac{\tan 53^\circ}{\cot 37^\circ} - \dfrac{\cot 80^\circ}{\tan 10^\circ} = 2\dfrac{\tan 53^\circ}{\cot(90^\circ-53^\circ)} - \dfrac{\cot 80^\circ}{\tan(90^\circ-10^\circ)}$

$= 2\dfrac{\tan 53^\circ}{\tan 53^\circ} - \dfrac{\cot 80^\circ}{\cot 80^\circ} = 2(1) - 1 = 1 =$ RHS.

Q10. (i) $\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ = 0$

(ii) $\dfrac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$

Sol. (i) LHS $= \dfrac{\cos 70^\circ}{\sin(90^\circ-70^\circ)} + \dfrac{\cos 59^\circ}{\sin(90^\circ-59^\circ)} - 8\sin^2 30^\circ$

$= \dfrac{\cos 70^\circ}{\cos 70^\circ} + \dfrac{\cos 59^\circ}{\cos 59^\circ} - 8\sin^2 30^\circ$

$= 1 + 1 - 8\left(\dfrac{1}{4}\right) = 2 - 2 = 0 =$ RHS.

(ii) LHS $= \dfrac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = \dfrac{\cos 80^\circ}{\sin(90^\circ-80^\circ)} + \cos 59^\circ \operatorname{cosec} 31^\circ$

$= \dfrac{\cos 80^\circ}{\cos 80^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} = 1 + \dfrac{\cos 59^\circ}{\cos 59^\circ} = 1 + 1 = 2 =$ RHS.

Q11. prove the following:

(i) $\dfrac{\cos\theta}{\sin(90^\circ-\theta)} + \dfrac{\sin\theta}{\cos(90^\circ-\theta)} = 2$

(ii) $\cos\theta \sin(90^\circ-\theta) + \sin\theta \cos(90^\circ-\theta) = 1$

(iii) $\dfrac{\tan\theta}{\tan(90^\circ-\theta)} + \dfrac{\sin(90^\circ-\theta)}{\cos\theta} = \sec^2\theta$

(iv) $\dfrac{\tan(90^\circ-A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

Sol. (i) $LHS = \dfrac{\cos\theta}{\sin(90^\circ-\theta)} + \dfrac{\sin\theta}{\cos(90^\circ-\theta)} = \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\sin\theta}$

$= 1 + 1 = 2 = RHS.$

(ii) $LHS = \cancel{\tan\theta}\ \cos\theta \sin(90^\circ-\theta) + \sin\theta \cos(90^\circ-\theta)$

$= \cos\theta \cdot \cos\theta + \sin\theta \cdot \sin\theta = \cos^2\theta + \sin^2\theta = 1 = RHS.$

(iii) $LHS = \dfrac{\tan\theta}{\tan(90^\circ-\theta)} + \dfrac{\sin(90^\circ-\theta)}{\cos\theta} = \dfrac{\tan\theta}{\cot\theta} + \dfrac{\cos\theta}{\cos\theta}$

$= \tan\theta \cdot \tan\theta + 1 = \tan^2\theta + 1 = \sec^2\theta = RHS.$

(iv) $LHS = \dfrac{\tan(90^\circ-A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A$

$= \dfrac{\cot A \cdot \cot A - \cos^2 A \cdot \operatorname{cosec}^2 A}{\operatorname{cosec}^2 A} = \dfrac{\cot^2 A - \dfrac{\cos^2 A}{\sin^2 A}}{\operatorname{cosec}^2 A}$

$= \dfrac{\cot^2 A - \cot^2 A}{\operatorname{cosec}^2 A} = 0 = RHS.$

Q12. Prove the following:

(i) $\dfrac{\cos(90^\circ-A)\sin(90^\circ-A)}{\tan(90^\circ-A)} = 1 - \cos^2 A.$

(ii) $\dfrac{\sin(90^\circ-A)}{\operatorname{cosec}(90^\circ-A)} + \dfrac{\cos(90^\circ-A)}{\sec(90^\circ-A)} = 1$

Sol. (i) $LHS = \dfrac{\cos(90^\circ-A)\sin(90^\circ-A)}{\tan(90^\circ-A)} = \dfrac{\sin A \cdot \cos A}{\cot A}$

$= \dfrac{\sin A \cos A \cdot \sin A}{\cos A}$

$= \sin^2 A$

$= 1 - \cos^2 A = RHS.$

Q14. Find the value of $\theta\,(0^\circ < \theta < 90^\circ)$ if :

(i) $\cos 63^\circ \sec(90^\circ - \theta) = 1$

(ii) $\tan 35^\circ \cot(90^\circ - \theta) = 1$

Sol.

(i) $\cos 63^\circ \sec(90^\circ - \theta) = 1 \Rightarrow \cos 63^\circ = \dfrac{1}{\sec(90^\circ - \theta)}$

$\Rightarrow \cos 63^\circ = \cos(90^\circ - \theta)$

$\therefore 63^\circ = 90^\circ - \theta \Rightarrow \theta = 90^\circ - 63^\circ = 27^\circ$

(ii) $\tan 35^\circ . \cot(90^\circ - \theta) = 1$

$\Rightarrow \tan 35^\circ = \dfrac{1}{\cot(90^\circ - \theta)}$

$\Rightarrow \tan 35^\circ = \tan(90^\circ - \theta)$

$\therefore 35^\circ = 90^\circ - \theta$

$\Rightarrow \theta = 90^\circ - 35^\circ = 55^\circ$

$\therefore \theta = 55^\circ$.