

Matrices

EXERCISE-9.1

Q1 classify the following matrices:

(i) $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$

(ii) $[2 \ 3 \ -7]$

(iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$

(vi) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Sol.

(i) It is square matrix of order 2

(ii) It is row matrix of order 1×3

(iii) It is column matrix of order 3×1

(iv) It is matrix of order 3×2

(v) It is matrix of order 2×3

(vi) It is zero matrix of order 2×3 .

Q2.

(i) If a matrix has 4 elements, what are the possible orders it can have?

(ii) If a matrix has 8 elements, what are the possible orders it can have?

Sol.

(i) It can have 1×4 , 4×1 or 2×2 order

(ii) It can have 1×8 , 8×1 , 2×4 or 4×2 order.

Q3.

Construct a 2×2 matrix whose elements a_{ij} are given by.

(i) $a_{ij} = 2i - j$ (ii) $a_{ij} = i \cdot j$

Sol.

(i) It can be $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

(ii) It can be $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Q4. Find the values of x and y if $\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Sol. Comparing corresponding elements,

$$2x + y = 5 \longrightarrow (i)$$

$$3x - 2y = 4 \longrightarrow (ii)$$

Multiply (i) by 2 and (ii) by 1

$$4x + 2y = 10$$

$$3x - 2y = 4$$

Adding we get, $7x = 14 \Rightarrow x = 2$

Substituting the value of x in (i)

$$2 \times 2 + y = 5 \Rightarrow y = 1$$

Hence $x = 2, y = 1$

Q5. Find the values of x, y and z if $\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$

Sol. Comparing the corresponding elements of equal determinants,

$$x+2 = -5 \Rightarrow x = -7$$

$$5z = -20 \Rightarrow z = -4$$

$$y+y = 6 \Rightarrow y+y-6=0 \Rightarrow y^2+3y-2y-6=0$$

$$\Rightarrow y(y+3)-2(y+3)=0 \Rightarrow (y+3)(y-2)=0$$

Either $y+3=0$, then $y=-3$ or $y-2=0$, then $y=2$

Hence $x = -7, y = -3, z = -4$

Q6. Find the values of x, y, a and b if $\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$

Sol. Comparing corresponding elements,

$$x-2 = 3 \Rightarrow x = 5$$

$$y = 1$$

$$a+2b = 5 \longrightarrow (i)$$

$$3a-b = 1 \longrightarrow (ii)$$

Multiplying (i) by 1 and (ii) by 2

$$a+2b = 5$$

$$6a-2b = 2$$

Adding, we get $7a = 7 \Rightarrow a = 1$

Substitute the value of 'a' in (i)

$$1+2b = 5 \Rightarrow b = 2$$

Hence $x = 5, y = 1, a = 1, b = 2$.

Q7. Find the values of x, y, a and b if $\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$

Sol. Comparing the corresponding elements:

$$3x+4y = 2 \rightarrow (i)$$

$$x-2y = 4 \rightarrow (ii)$$

Multiply (i) by 1 and (ii) by 2

$$3x+4y = 2$$

$$2x-4y = 8$$

Adding, we get $5x = 10 \Rightarrow x = 2$

Substituting the value of x in (i)

$$3 \times 2 + 4y = 2 \Rightarrow 4y = -4 \Rightarrow y = -1$$

$$a+b = 5 \rightarrow (iii)$$

$$2a-b = -5 \rightarrow (iv)$$

Adding we get $3a = 0 \Rightarrow a = 0$

Substitute the value of 'a' in (iii)

$$0+b = 5 \Rightarrow b = 5$$

Hence $x = 2, y = -1, a = 0, b = 5$

EXERCISE - 9.2

Q1. Given that $M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$, find $M+2N$.

Sol. $M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$, $N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

$$M+2N = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$$

Q2. If $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, find $2A-3B$.

Sol. $2A-3B = 2 \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} + 3 \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$

Q3. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, find $3A+4B$.

Sol. $3A+4B = 3 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + 4 \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 20 \\ 18 & 13 \end{bmatrix}$

Q4. Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$ (i) find the matrix $2A+B$.

(ii) find the matrix C such that $C+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol. (i) $2A+B = 2 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$

(ii) $C+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$

$$C = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Q5. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$, find $A+2B-3C$.

sol.

$$\begin{aligned} A+2B-3C &= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2 \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix} \end{aligned}$$

Q6. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$, find the matrix X if

(i) $3A+X=B$ (ii) $X-3B=2A$

sol.

(i) $3A+X=B \Rightarrow X=B-3A$

$$X = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -4 & -5 \end{bmatrix}$$

(ii) $X-3B=2A \Rightarrow X=2A+3B$

$$X = 2 \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$$

Q7. Solve the matrix equation $\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - 3X = \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$

sol.

$$\begin{aligned} \begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} &= 3X \Rightarrow \begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = 3X \\ \Rightarrow X &= \frac{1}{3} \begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} \end{aligned}$$

Q8. If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M .

sol.

Given that

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix}$$

$$2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$$

$$M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$$

Q9. If $A = \begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix}$, find matrix X such that

$$3A + 5B - 2X = 0$$

sol. $3A + 5B - 2X = 0 \Rightarrow 2X = 3A + 5B$

$$\Rightarrow 2X = 3 \begin{bmatrix} 9 & 1 \\ 5 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 5 \\ 7 & -11 \end{bmatrix} = \begin{bmatrix} 27 & 3 \\ 15 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 25 \\ 35 & -55 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 32 & 28 \\ 50 & -46 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 16 & 14 \\ 25 & -23 \end{bmatrix}$$

Q10. find x and y if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

sol. $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \rightarrow (i)$

$$x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \rightarrow (ii)$$

Adding (i) and (ii) we get $2x = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow 2x = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Subtracting (ii) from (i)

$$2y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 2y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \Rightarrow y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Q11 (i) If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find the values of x and y .

(ii) If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$, find the values of x , y and z

sol. (i) $\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

Comparing the corresponding elements,

$$8+y=0 \Rightarrow y=-8$$

$$2x+1=5 \Rightarrow x=2$$

$$\text{Hence } x=2, y=-8$$

(ii) Given that $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow z=7, 8+y=0 \Rightarrow y=-8, 2x+1=5 \Rightarrow x=2$$

$$\text{Hence } x=2, y=-8, z=7$$

Q12. If $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

Sol. $\Rightarrow \begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 2-2-0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a+1 & b+2 \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements

$$a+1=5 \Rightarrow a=4$$

$$b+2=0 \Rightarrow b=-2$$

$$-c=3 \Rightarrow c=-3$$

Q13. If $A = \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 \\ 1 & b \end{bmatrix}$, $C = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$ and $5A+2B=C$,

find the values of a , b and c .

Sol.

$$A = \begin{pmatrix} 2 & a \\ -3 & 5 \end{pmatrix}, B = \begin{pmatrix} -2 & 3 \\ 7 & b \end{pmatrix}, C = \begin{pmatrix} c & 9 \\ -1 & -11 \end{pmatrix}$$

$$\text{Now } 5A + 2B = C$$

$$\Rightarrow 5 \begin{pmatrix} 2 & a \\ -3 & 5 \end{pmatrix} + 2 \begin{pmatrix} -2 & 3 \\ 7 & b \end{pmatrix} = \begin{pmatrix} c & 9 \\ -1 & -11 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 10 & 5a \\ -15 & 25 \end{pmatrix} + \begin{pmatrix} -4 & 6 \\ 14 & 2b \end{pmatrix} = \begin{pmatrix} c & 9 \\ -1 & -11 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 6 & 5a+6 \\ -1 & 25+2b \end{pmatrix} = \begin{pmatrix} c & 9 \\ -1 & -11 \end{pmatrix}$$

Comparing the corresponding elements of equal determinants,

$$c = 6$$

$$5a + 6 = 9 \Rightarrow 5a = 3 \Rightarrow a = \frac{3}{5}$$

$$25 + 2b = -11 \Rightarrow 2b = -36 \Rightarrow b = -18$$

\therefore Hence $a = \frac{3}{5}$, $b = -18$, $c = 6$.

EXERCISE - 9.3

Q1. If $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$, find AB and BA . Is $AB = BA$?

Sol. $AB = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2-15 & -2+10 \\ 1-9 & -1+6 \end{pmatrix} = \begin{pmatrix} -13 & 8 \\ -8 & 5 \end{pmatrix}$

and $BA = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2-1 & 5-3 \\ -6+2 & -15+6 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -4 & -9 \end{pmatrix}$

hence $AB \neq BA$.

Q2. If $P = \begin{pmatrix} 4 & 6 \\ 2 & -8 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$, find $2PQ$?

Sol. $2PQ = 2 \begin{pmatrix} 4 & 6 \\ 2 & -8 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix} = 2 \begin{pmatrix} 8-6 & -12+6 \\ 4-8 & -6-8 \end{pmatrix} = 2 \begin{pmatrix} 2 & -6 \\ -4 & -14 \end{pmatrix}$
 $= \begin{pmatrix} 4 & -12 \\ -8 & -28 \end{pmatrix}$

Q3. If $A = \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}$, compute $(-A)^2$.

Sol. $A = \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}$, $-A = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$

$(-A)^2 = (-A)(A) = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 4+1 & -2-3 \\ -2-3 & 1+9 \end{pmatrix} = \begin{pmatrix} 5 & -5 \\ -5 & 10 \end{pmatrix}$

Q4. Given $A = \begin{pmatrix} 1 & 1 \\ 8 & 3 \end{pmatrix}$, evaluate $A^2 - 4A$.

Sol. $A^2 - 4A = \begin{pmatrix} 1 & 1 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 8 & 3 \end{pmatrix} - 4 \begin{pmatrix} 1 & 1 \\ 8 & 3 \end{pmatrix}$
 $= \begin{pmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{pmatrix} - \begin{pmatrix} 4 & 4 \\ 32 & 12 \end{pmatrix}$
 $= \begin{pmatrix} 9 & 4 \\ 32 & 17 \end{pmatrix} - \begin{pmatrix} 4 & 4 \\ 32 & 12 \end{pmatrix}$
 $= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

Q8. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 1 \\ 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & -3 \\ 0 & 1 \end{pmatrix}$, find each of the following and state if they are equal.

(i) $CA+B$ (ii) $A+CB$.

Sol.

$$(i) CA = \begin{pmatrix} -2 & -3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -11 & -16 \\ 3 & 4 \end{pmatrix}$$

$$CA+B = \begin{pmatrix} -11 & -16 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 6 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -5 & -15 \\ 4 & 5 \end{pmatrix}$$

$$(ii) A+CB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -2 & -3 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 6 & 1 \\ 1 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -15 & -5 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -14 & -3 \\ 4 & 5 \end{pmatrix}$$

We can say that $CA+B \neq A+CB$.

Q9.

If $A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix}$, find $2B-A^2$.

Sol.

$$2B = 2 \begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ -4 & 2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$$

$$2B-A^2 = \begin{pmatrix} 6 & 4 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ -4 & 5 \end{pmatrix}$$

Q10.

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 5 & 1 \\ 7 & 4 \end{pmatrix}$, compute

(i) $A(B+C)$ (ii) $(B+C)A$.

Sol.

$$A(B+C) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 11 & 6 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 7 & 2 \\ 11 & 6 \end{pmatrix} = \begin{pmatrix} 29 & 14 \\ 65 & 30 \end{pmatrix}$$

$$(B+C) = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 5 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 11 & 6 \end{pmatrix}$$

$$(B+C)A = \begin{pmatrix} 7 & 2 \\ 11 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 13 & 22 \\ 29 & 46 \end{pmatrix}$$

Q11. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$, find the matrix $C(B-A)$.

sol. $B-A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

$$C(B-A) = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}$$

Q12. Let $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$. find $A^2 + AB + B^2$.

sol. $A^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ -2 & -3 \end{pmatrix}$$

$$A^2 + AB + B^2 = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ 5 & 4 \end{pmatrix}$$

Q13. Let $A = \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 3 \\ 1 & -3 \end{pmatrix}$. find $A^2 - A + BC$.

sol. $A^2 - A + BC = \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix} - \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix} + \begin{pmatrix} 2 & -6 \\ -3 & 6 \end{pmatrix}$

$$= \begin{pmatrix} 4-4+2 & -2+2-6 \\ 6-6-3 & -3+3+6 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -6 \\ -3 & 6 \end{pmatrix}$$

Q14. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find A^2 and A^3 . Also state which of these is equal to $-A$.

Sol. $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

from above it is clear that $A^3 = A$.

Q15. If $\alpha = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, show that $6\alpha - \alpha^2 = 9I$, where I is the unit matrix.

Sol. $\alpha^2 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$

$$\begin{aligned} \text{LHS} &= 6\alpha - \alpha^2 = 6 \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \\ &= 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I = \text{RHS.} \end{aligned}$$

Hence proved.

Q16. Show that $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is a solution of the matrix equation $\alpha^2 - 2\alpha - 3I = 0$ where I is the unit matrix of order 2.

Sol. $\alpha^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

$$\begin{aligned} \text{NOW } \alpha^2 - 2\alpha - 3I &= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$\therefore \alpha^2 - 2\alpha - 3I = 0$ Hence proved.

Q17. Find the matrix X of order 2×2 which satisfies the equation:

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Sol.

$$\Rightarrow \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

Q18. If $A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$, find the value of x so that $A^2 = 0$.

Sol.

$$A^2 = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix} \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix} = \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix}$$

$$\because A^2 = 0$$

$$\Rightarrow \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

on comparing $1+x = 0 \Rightarrow x = -1$

Q19. If $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$, find the value of x .

Sol.

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-3 \\ 0+0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

Comparing the corresponding element

$$x = -1$$

Q20. Find x and y if $\begin{pmatrix} -3 & 2 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ y \end{pmatrix}$

Sol. $\begin{pmatrix} -3 & 2 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ y \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} -3x+4 \\ 0-10 \end{pmatrix} = \begin{pmatrix} -5 \\ y \end{pmatrix}$$

Comparing the corresponding elements,

$$-3x+4 = -5 \Rightarrow -3x = -9 \Rightarrow x = 3$$

$$-10 = y \Rightarrow y = -10$$

Q21. Find the values of x and y if $\begin{pmatrix} x+y & y \\ 2x & x-y \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Sol. $\begin{pmatrix} x+y & y \\ 2x & x-y \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 2x+2y & -y \\ 4x & -x+y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Comparing the corresponding elements,

$$2x+y = 3 \quad \text{--- (i)}$$

$$3x+y = 2 \quad \text{--- (ii)}$$

on subtracting, we get

$$-x = 1 \Rightarrow x = -1$$

Substituting the value of x in (i)

$$2(-1)+y = 3 \Rightarrow y = 5$$

Q22. Find x and y if $\begin{pmatrix} 2 & x \\ y & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 7 \end{pmatrix}$

Sol. $\begin{pmatrix} 2+x & 4+x \\ y+3 & 2y+3 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 7 \end{pmatrix}$

Comparing the corresponding elements

$$2+x = 3 \Rightarrow x = 1$$

$$y+3 = 5 \Rightarrow y = 2$$

Hence $x = 1, y = 2$

Q23. If $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 9 & 0 \end{pmatrix}$, find the values of x and y .

Sol.

$$\begin{pmatrix} x+0 & 0+2y \\ 3x+0 & 0+3y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 9 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x & 2y \\ 3x & 3y \end{pmatrix} = \begin{pmatrix} x & 0 \\ 9 & 0 \end{pmatrix}$$

Comparing the corresponding elements

$$2y = 0 \Rightarrow y = 0$$

$$3x = 9 \Rightarrow x = 3$$

Q24. If $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ write down the values of a, b, c & d .

Sol.

$$\Rightarrow \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} a+0 & 0+b \\ c+0 & 0+d \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Comparing the corresponding elements,

$$a = 3, b = 4, c = 2, d = 5$$

Q25. Find the value of x given that $A^2 = B$ where $A = \begin{pmatrix} 2 & 12 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & x \\ 0 & 1 \end{pmatrix}$

Sol.

Given that $A^2 = B$

$$\Rightarrow \begin{pmatrix} 2 & 12 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 12 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & x \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 4 & x \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4 & 36 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & x \\ 0 & 1 \end{pmatrix}$$

on comparing, $x = 36$.

Q26. If $A = \begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 36 \\ 0 & 1 \end{pmatrix}$, find the value of x , given that $A^2 = B$.

sol. $A^2 = \begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4+0 & 2x+x \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 4 & 3x \\ 0 & 1 \end{pmatrix}$

$\therefore A^2 = B$

$\begin{pmatrix} 4 & 3x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 36 \\ 0 & 1 \end{pmatrix}$

Comparing the corresponding elements

$3x = 36 \Rightarrow x = 12$.

Q27. Evaluate x, y if $\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 2x \\ 1 \end{pmatrix} + 2 \begin{pmatrix} -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 4y \end{pmatrix}$

sol. $\Rightarrow \begin{pmatrix} 6x-2 \\ -2x+4 \end{pmatrix} + \begin{pmatrix} -8 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \\ 4y \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 6x-10 \\ -2x+14 \end{pmatrix} = \begin{pmatrix} 8 \\ 4y \end{pmatrix}$

Comparing the corresponding elements,

$6x-10 = 8 \Rightarrow 6x = 18 \Rightarrow x = 3$

$-2x+14 = 4y \Rightarrow -6+14 = 4y \Rightarrow 4y = 8 \Rightarrow y = 2$.

\therefore Hence $x = 3, y = 2$

Q28. If $\begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} b & 11 \\ 4 & c \end{pmatrix}$, find a, b and c .

sol. $\Rightarrow \begin{pmatrix} 4a-3 & 3a+2 \\ 4+0 & 3+0 \end{pmatrix} = \begin{pmatrix} b & 11 \\ 4 & c \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 4a-3 & 3a+2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} b & 11 \\ 4 & c \end{pmatrix}$

Comparing the corresponding elements

$3a+2 = 11 \Rightarrow a = 3$

$b = 4a-3 = 4(3)-3 = 9$

$c = 3$

Q29. If $A = \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{pmatrix}$, find the value of x if $AB = BA$.

Sol.

$$AB = \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2+0 & x-2 \\ 0+0 & 0+\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 & x-2 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & x \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2+0 & 8-x \\ 0+0 & 0+\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 & 8-x \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$\therefore AB = BA$$

$$\begin{pmatrix} 2 & x-2 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 & 8-x \\ 0 & \frac{1}{2} \end{pmatrix}$$

Comparing the corresponding elements

$$x-2 = 8-x \Rightarrow 2x = 10 \Rightarrow x = 5$$

Q30. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, find x, y so that $A^2 = xA + yI$.

Sol.

$$A^2 = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix}$$

$$\therefore A^2 = xA + yI$$

$$\Rightarrow \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix} = x \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} + y \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix} = \begin{pmatrix} 2x & 3x \\ x & 2x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 0 & y \end{pmatrix} = \begin{pmatrix} 2x+y & 3x \\ x & 2x+y \end{pmatrix}$$

comparing the corresponding elements

$$3x = 12 \Rightarrow x = 4$$

$$2x+y = 7 \Rightarrow 2(4)+y = 7 \Rightarrow y = -1$$

Q31.

Given that $A = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ and that $AB = A+B$, find the values of a, b and c .

Sol.

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

$$AB = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 3a+0 & 3b+0 \\ 0+0 & 0+4c \end{pmatrix} = \begin{pmatrix} 3a & 3b \\ 0 & 4c \end{pmatrix}$$

$$A+B = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 3+a & b \\ 0 & 4+c \end{pmatrix}$$

$$\therefore AB = A+B$$

$$\begin{pmatrix} 3a & 3b \\ 0 & 4c \end{pmatrix} = \begin{pmatrix} 3+a & b \\ 0 & 4+c \end{pmatrix}$$

Comparing the corresponding elements

$$3a = 3+a \Rightarrow 2a = 3 \Rightarrow a = \frac{3}{2}$$

$$3b = b \Rightarrow b = 0$$

$$4c = 4+c \Rightarrow c = \frac{4}{3}$$

Q32 If $P = \begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix}$, $Q = \begin{pmatrix} 3 & x \\ y & 2 \end{pmatrix}$, find x, y such that $PQ = O$

sol.

$$PQ = \begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} 3 & x \\ y & 2 \end{pmatrix} = \begin{pmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{pmatrix}$$

$$\therefore PQ = O$$

$$\begin{pmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Comparing the corresponding elements

$$6+6y = 0 \Rightarrow y = -1$$

$$2x+12 = 0 \Rightarrow x = -6$$

Q33 Let $M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = [1 \ 2]$ where M is a matrix.

(i) State the order of the Matrix M .

(ii) Find the matrix M .

sol.

(i) M is of the order 1×2

$$\text{let } M = [x \ y]$$

$$[x \ y] \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = [1 \ 2]$$

$$\Rightarrow [x+0 \quad x+2y] = [1 \quad 2]$$

Comparing the corresponding elements

$$x = 1$$

$$x+2y = 2 \Rightarrow 1+2y = 2 \Rightarrow y = \frac{1}{2}$$

$$\therefore M = \left[1 \quad \frac{1}{2} \right]$$

Q34. Given $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$, write down (i) The order of the matrix X.
(ii) The matrix X.

Sol.

(i) order of matrix X is 2×1

$$\text{let } X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{then } \begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x-2y \\ x+4y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

Comparing the corresponding elements

$$8x-2y = 12 \quad \text{--- (i)}$$

$$x+4y = 10 \quad \text{--- (ii)}$$

Multiply (i) by 2 and (ii) by 1.

$$16x-4y = 24$$

$$x+4y = 10$$

on Adding Δ $17x = 34 \Rightarrow x = 2$

Substituting the value of x in (i)

$$8 \times 2 - 2y = 12 \Rightarrow 16 - 2y = 12 \Rightarrow y = 2.$$

$$\therefore X = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Q35. Solve the matrix equation $\begin{pmatrix} 4 \\ 1 \end{pmatrix} x = \begin{pmatrix} -4 & 8 \\ -1 & 2 \end{pmatrix}$

Sol. let matrix $x = \begin{pmatrix} x & y \end{pmatrix}$

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} x & y \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ -1 & 2 \end{pmatrix}$$

Comparing the corresponding elements

$$4x = -4 \Rightarrow x = -1$$

$$4y = 8 \Rightarrow y = 2$$

$$\therefore x = \begin{pmatrix} -1 & 2 \end{pmatrix}$$

Q36. If $A = \begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find matrix C such that $AC = B$.

Sol. let matrix $C = \begin{pmatrix} x \\ y \end{pmatrix}$

$$AC = \begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - y \\ -4x + 5y \end{pmatrix}$$

But $AC = B$

$$\begin{pmatrix} 2x - y \\ -4x + 5y \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Comparing the corresponding elements

$$2x - y = -3 \quad \text{--- (i)}$$

$$-4x + 5y = 2 \quad \text{--- (ii)}$$

Multiply (i) by 5 and (ii) by 1

$$10x - 5y = -15$$

$$-4x + 5y = 2$$

on Adding, we get $6x = -13 \Rightarrow x = -13/6$

substituting the value of x in (i), $2(-13/6) - y = -3$

$$\Rightarrow y = -4/3.$$

$$\therefore \text{Matrix } C = \begin{pmatrix} -13/6 \\ -4/3 \end{pmatrix}$$