Ratios and Proportions

EXERCISE-7.1

Solution-01:

Weight of Alloy = $27\frac{1}{2}$ kg + $2\frac{3}{4}$ kg

= $\frac{55}{2}$ kg + $\frac{11}{4}$ kg = $\frac{121}{4}$ kg.

weight of tin = $2\frac{3}{4}$ kg

= $\frac{11}{4}$ kg.

∴ Ratio of tin to the alloy = $\frac{\frac{11}{4}\text{ kg}}{\frac{121}{4}\text{ kg}}$

= $\frac{11}{121}$

= 1 : 11.

Solution-02:

(i) Required ratio = $\frac{2}{3} \times \frac{4}{9}$

= $\frac{8}{27}$ = 8 : 27

(ii) Required ratio = $\frac{4}{5} \times \frac{35}{7} \times \frac{9}{11}$

= $\frac{36}{77}$ = 36 : 77

(iii) Required ratio = $\frac{(a-b)}{(a+b)} \times \frac{(a+b)^2}{(a^2+b^2)} \times \frac{(a^2-b^2)}{(a^2-b^2)}$

= 1 : 1

Solution-03:

(i) The duplicate ratio of $2:3$ is $2^2:3^2$ i.e $4:9$

(ii) The duplicate ratio of $\sqrt{5}:7$ is $(\sqrt{5})^2:7^2$ i.e $5:49$

(iii) The duplicate ratio of $5a:6b$ is $(5a)^2:(6b)^2$

i.e $(25a^2):(36b^2)$

Solution-04:

(i) The triplicate ratio of $3:4$ is $3^3:4^3$

i.e $27:64$.

(ii) The triplicate ratio of $\frac{1}{2}:\frac{1}{3}$ is $\left(\frac{1}{2}\right)^3:\left(\frac{1}{3}\right)^3$

i.e $\frac{1}{8}:\frac{1}{27}$

(iii) The triplicate ratio of $1^3:2^3$ is $(1)^3:(8)^3$

i.e $1:512$.

Solution-05:-

(i) The sub duplicate ratio of $9:16$ is $\sqrt{9}:\sqrt{16}$

i.e $3:4$

(ii) The sub duplicate ratio of $\frac{1}{4}:\frac{1}{9}$ is $\sqrt{\frac{1}{4}}:\sqrt{\frac{1}{9}}$

i.e $\frac{1}{2}:\frac{1}{3}$

(iii) The sub duplicate ratio of $9a^2:49b^2$ is $\sqrt{(9a^2)}:\sqrt{49b^2}$

i.e $\sqrt{(3a)^2}:\sqrt{(7b)^2}$

$\therefore$ Sub duplicate ratio is $3a:7b$.

Solution-06:

(i) The subtriplicate ratio of $1:216$ i.e $\sqrt[3]{1} : \sqrt[3]{216}$

$$\sqrt[3]{(1)^3} : \sqrt[3]{(6)^3}$$

i.e $1:6$

(ii) The subtriplicate ratio of $\frac{1}{8} : \frac{1}{125}$

i.e $\sqrt[3]{\left(\frac{1}{2}\right)^3} : \sqrt[3]{\left(\frac{1}{5}\right)^3}$

i.e $\frac{1}{2} : \frac{1}{5}$

(iii) The subtriplicate ratio of $27a^3 : 64b^3$

i.e $\sqrt[3]{(3a)^3} : \sqrt[3]{(4b)^3}$

i.e $3a : 4b$.

Solution-07:

(i) The reciprocal ratio of $4:7$ is $7:4$

(ii) The reciprocal ratio of $3^2 : 4^2$ is $4^2 : 3^2$

(iii) The reciprocal ratio of $\frac{1}{9} : 2$ is $2 : \frac{1}{9}$.

Solution-08:

Given ratios are $\frac{2}{3}$, $\frac{17}{21}$, $\frac{11}{14}$ and $\frac{5}{7}$

We convert them into equivalent like fractions

L.C.M of 3, 21, 14 and 7 is

$$\begin{array}{r|l} 7 & 3, 21, 14, 7 \\ 3 & 3, 3, 2, 1 \\ & 1, 1, 2, 1 \end{array}$$

L.C.M $= 21 \times 2 = 42$.

$\therefore \quad A : D = \frac{A}{B} \times \frac{B}{D}$

$= \frac{2}{3} \times \frac{24}{35}$

$= \frac{2 \times 8}{35}$

$\therefore$ Req. ratio $= \frac{16}{35}$

(ii) Given that,

$\frac{x}{y} = \frac{2}{3}$ & $\frac{y}{z} = \frac{4}{7}$

L.C.M of two values of y is 12

i.e 3 and 4 is 12

Thus, $\frac{x}{y} = \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

$\frac{y}{z} = \frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$

$\therefore x : y : z$ is $8 : 12 : 21$ is the simplest form

Solution-ID:

(ii) Given that,

$3A = 4B = 6C$

$\Rightarrow 3A = 4B$

$\Rightarrow \frac{A}{B} = \frac{4}{3}$

$4B = 6C$

$\Rightarrow \frac{B}{C} = \frac{6}{4} = \frac{3}{2}$

$\therefore$ $A : D = \frac{B}{D} \times \frac{A}{B} \times \frac{B}{D}$

$= \frac{2}{3} \times \frac{28}{30} \times \frac{24}{35}$ (8)

$= \frac{16}{35}$.

Required ratio A:D is 16:35

9 (ii)

Given that $x:y = 2:3$, $y:z = 4:7$

L.C.M of two value of y is 12

i.e 3 and 4 is 12

Thus, $x : y = 2:3 = 8:12$

$y:z = 4:7 = 12:21$

$\therefore$ $x:y:z = 8:12:21$, which is in simplest form

Solution - 10:

(i) $A : B = \frac{1}{4} : \frac{1}{5} = \frac{\frac{1}{4}}{\frac{1}{5}} = \frac{5}{4}$

$B : C = \frac{1}{1} : \frac{1}{6} = \frac{6}{7}$

L.C.M of two values of B is 4 and 6 $\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$

i.e $\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$ L.C.M = 24

Thus, $A : B = \frac{1}{4} \times \frac{1}{7} : \frac{1}{5} \times \frac{1}{7} = \frac{1}{28} : \frac{1}{35}$

∴ Required ratio A : B : C is 4 : 3 : 2

(i) Given that,

$$A : B = \frac{1}{4} : \frac{1}{5} = \frac{5}{4}$$

$$B : C = \frac{1}{7} : \frac{1}{6} = \frac{6}{7}$$

L.C.M of two values of B is 4 and 6 is 24

i.e $4 \times 6 = 24$

$$\therefore A : B = \frac{5 \times 6}{4 \times 6} = \frac{30}{24}$$

$$B : C = \frac{6 \times 4}{7 \times 4} = \frac{24}{28}$$

∴ Required ratio is A : B : C = 30 : 24 : 28

in simplest form 15 : 12 : 14

Solution- II :-

$$\Rightarrow \frac{3x + 5y}{3x - 5y} = \frac{7}{3}$$

$$\Rightarrow 3(3x + 5y) = 7(3x - 5y)$$

$$\Rightarrow 9x + 15y = 21x - 35y$$

$$= 21x - 9x = 15y + 35y$$

$$\Rightarrow 12x = 50y$$

$$\Rightarrow \frac{x}{y} = \frac{50}{12} = \frac{25}{6}.$$

(ii) $a : b = 3 : 11$

$a = 3$

$b = 11$

$$\therefore \frac{15a + 3b}{9a + 5b} = \frac{15(3) - 3(11)}{9(3) + 5(11)}$$

$$= \frac{45 - 33}{27 + 55}$$

$$= \frac{12}{82}$$

$$= \frac{6}{41}$$

Solution-12:

Given that

$$\frac{(4x^2 + xy)}{(3xy - y^2)} = \frac{12}{5}$$

$$\Rightarrow 5(4x^2 + xy) = 12(3xy - y^2)$$

$$\Rightarrow 20x^2 + 5xy = 36xy - 12y^2$$

$$\Rightarrow 20x^2 + 12y^2 - 31xy = 0$$

$$\Rightarrow 20x^2 + 12y^2 - 15xy - 16xy = 0$$

$$\Rightarrow 20x^2 - 16xy + 15xy + 12y^2 = 0$$

$$\Rightarrow 4x(5x - 4y) - 3y(5x - 4y) = 0$$

$$\Rightarrow (4x - 3y)(5x - 4y) = 0$$

$$\therefore 4x = 3y \ \& \ 5x = 4y$$

EXERCISE - 7.2

Solution-01:

(i) Given that

$10 : 35 = x : 42$

$\Rightarrow \frac{10}{35} = \frac{x}{42}$

$\Rightarrow 42 \times 10 = x \times 35$ [$\because$ By cross multiplication]

$\Rightarrow x \times 35 = 420$

$\Rightarrow x = \frac{420}{35}$

$\Rightarrow x = 12.$

(ii) Given that

$3 : x = 24 : 2$

$\Rightarrow \frac{3}{x} = \frac{24}{2}$

$\Rightarrow$ By doing cross multiplication

$24 \times x = 3 \times 2$

$\Rightarrow 24x = 6$

$\Rightarrow x = \frac{6}{24}$

$\Rightarrow x = \frac{1}{4}.$

(iii) Given that

$25 : 15 = x : 3$

$\Rightarrow \frac{25}{15} = \frac{x}{3}$

$\Rightarrow 15x = 75 \quad \Rightarrow x = 5.$

(iv) Given that

$x : 50 = 3 : 2$

$\Rightarrow x : 50 = \frac{3}{2}$

$\Rightarrow x \times 2 = 50 \times 3$

$\Rightarrow 2x = 150$

$\Rightarrow x = \frac{150}{2}$

$\Rightarrow x = 75.$

Solution-02:

(i) Given that,

3, 12, 15 and Let 4th proportion be 'x'

$\frac{3}{12} = \frac{15}{x}$

$\Rightarrow 3x = 12 \times 15$

$\Rightarrow x = \frac{12 \times 15}{3}$

$\Rightarrow x = 4 \times 15$

$\Rightarrow x = 60.$

(ii) Given that

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ Let 4th proportion be 'x' then

$\Rightarrow \frac{1}{3} : \frac{1}{4} = \frac{1}{5} : x$

$\Rightarrow \frac{4}{3} = \frac{1}{5x}$

$\Rightarrow 20x = 3 \quad \Rightarrow x = \frac{3}{20}.$

(iii) Given that.

1.5, 2.5, 4.5 Let 4th proportion be 'x' then.

$1.5 : 2.5 = 4.5 : x$

$\Rightarrow \frac{1.5}{2.5} = \frac{4.5}{x}$

$\Rightarrow x \times 1.5 = 4.5 \times 2.5$

$\Rightarrow x = \frac{4.5 \times 2.5}{1.5}$

$\Rightarrow x = 3 \times 2.5$

$\Rightarrow x = 7.5$.

(iv) Given that.

9.6 kg, 7.2 kg, 28.8 kg Let 4th proportion be 'x' then

$\Rightarrow$ 9.6 kg : 7.2 kg = 28.8 kg : x kg

$\Rightarrow \frac{9.6\ kg}{7.2\ kg} = \frac{28.8\ kg}{x\ kg}$

$\Rightarrow 9.6 \times x = 7.2 \times 28.8$

$\Rightarrow 9.x = \frac{7.2 \times 28.8}{9.6}$

$\Rightarrow x = 7.2 \times 3$

$\Rightarrow x = 22.6$.

Solution - 04

(i) Given that

5, 10 then let Third proportion be 'x' then

$$\Rightarrow \frac{5}{10} = \frac{10}{x}$$

$$\Rightarrow 5x = 100$$

$$\Rightarrow x = \frac{100}{5}$$

$$\Rightarrow x = 20.$$

(ii) 0.24, 0.6

Let the third proportion be 'x' then

$$\Rightarrow \frac{0.24}{0.6} = \frac{0.6}{x}$$

$$\Rightarrow x \times 0.24 = 0.6 \times 0.6$$

$$\Rightarrow x = \frac{0.36}{0.24}$$

$$\Rightarrow x = \frac{3}{2}$$

(iii) ₹3, ₹12

Let the third proportion be 'x' then

$$\Rightarrow \frac{₹3}{₹12} = \frac{₹12}{x}$$

$$\Rightarrow ₹3x = 12 \times 12$$

$$\Rightarrow x = \frac{144}{3}$$

$$\Rightarrow x = 48.$$

(iv) Given that

$5\frac{1}{4}$ and 7

$\Rightarrow 5\frac{1}{4} = 5.25$ & 7

Let the third proportion be 'x' then

$\Rightarrow \frac{5.25}{7} = \frac{7}{x}$

$\Rightarrow \frac{5.25}{7} = \frac{7}{x}$

$\Rightarrow 5.25x = 49$

$\Rightarrow x = \frac{49}{5.25}$

$\Rightarrow x = \frac{4900}{525}$

$\Rightarrow x = \frac{196}{21}$.

Solution-04

(i) Given numbers are 5 & 80

∴ mean proportion of 5 & 80 $= \sqrt{5 \times 80}$

$= \sqrt{400}$

$= \sqrt{4 \times 100}$

$= \sqrt{4} \times \sqrt{100}$

$= 2 \times 10$

$= 20$.

∴ mean proportion of 5 and 80 is 20.

(ii) Given numbers are $\frac{1}{12}$ and $\frac{1}{75}$.

mean proportion of $\frac{1}{12}$ and $\frac{1}{75}$ is $\sqrt{\frac{1}{12} \times \frac{1}{75}}$

$= \sqrt{\frac{1}{900}}$

$= \sqrt{\left(\frac{1}{30}\right)^2}$

$= \frac{1}{30}$.

(iii) Given that

mean proportion of 8.1 and 2.5 is $\sqrt{8.1 \times 2.5}$

$= \sqrt{\frac{81 \times 25}{100}}$

$= \sqrt{\frac{(9)^2 \times (5)^2}{(10)^2}}$

$= \frac{45}{10}$

$= 4.5$.

(iv) Given that

proportions are $(a-b)$ & $(a^3 - a^2b)$.

mean proportion is $\sqrt{(a-b)(a-b)a^2}$

$= \sqrt{(a-b)^2 a^2}$

$= a(a-b)$

mean proportion is $a(a-b)$.

Solution-05.

Given that a, 12, 16 and b are in continued proportion

$$\Rightarrow \frac{a}{12} = \frac{12}{16} = \frac{16}{b}$$

$$\Rightarrow \frac{a}{12} = \frac{12}{16}$$

$$\Rightarrow a = \frac{144}{16}$$

$$\Rightarrow a = 9$$

$$\Rightarrow b = \frac{16 \times 16}{12}$$

$$\Rightarrow b = \frac{256}{12}$$

$$\Rightarrow b = \frac{64}{3}$$

Solution-06

Given numbers are 5, 11, 19 and 37.

Let the number to abe added 'x' then

$5+x, 11+x, 19+x$ and $37+x$ are in proportion.

$$\Rightarrow \frac{5+x}{11+x} = \frac{19+x}{37+x}$$

$$\Rightarrow (5+x)(37+x) = (19+x)(11+x)$$

$$\Rightarrow 185 + 5x + 37x + x^2 = 209 + 11x + x^2 + 19x$$

$$\Rightarrow 42x - 30x = 209 - 185$$

$$\Rightarrow 12x = 24$$

$$\Rightarrow x = 2.$$

Solution-07:

Given that numbers are 23, 30, 57 and 78

Let the number 'x' be subtracted to be in proportion

$$\Rightarrow \frac{23-x}{30-x} = \frac{57-x}{78-x}$$

$$\Rightarrow (23-x)(78-x) = (57-x)(30-x)$$

$$\Rightarrow 1794 - 23x - 78x + x^2 = 1710 - 57x - 30x + x^2$$

$$\Rightarrow 1794 - 1710 = -87x + 101x$$

$$\Rightarrow 14x = 84$$

$$\Rightarrow x = \frac{84}{14} = 6$$

Solution-08:

Given that,

$$\frac{2x-1}{5x-6} = \frac{6x+2}{15x-9}$$

$$\Rightarrow (2x-1)(15x-9) = (6x+2)(5x-6)$$

$$\Rightarrow 30x^2 - 15x + 9 - 18x = 30x^2 - 36x + 10x - 12$$

$$\Rightarrow -33x + 9 = -26x - 12$$

$$\Rightarrow 33x - 26x = 12 + 9$$

$$\Rightarrow 7x = 21$$

$$\Rightarrow x = \frac{21}{7}$$

$$\Rightarrow x = 3.$$

$$\frac{2}{3} = \frac{2\times14}{3\times14} = \frac{28}{42}$$

$$\frac{17}{21} = \frac{17\times2}{21\times2} = \frac{34}{42}$$

$$\frac{11}{14} = \frac{11\times3}{14\times3} = \frac{33}{42}$$

$$\frac{5}{7} = \frac{5\times6}{7\times6} = \frac{30}{42}$$

∴ The given ratio are $\frac{28}{42}, \frac{34}{42}, \frac{33}{42}, \frac{30}{42}$

Hence, the given ratios in the ascending order of magnitude

$$\frac{28}{42}, \frac{30}{42}, \frac{33}{42}, \frac{34}{42}$$

i.e 2 : 3, 5 : 7, 11 : 14, 17 : 21.

Solution-09 :

Given that,

A : B = 2 : 3

B : C = 4 : 5

C : D = 6 : 7

then B : D = $\frac{B\times C}{C\times D}$

$= \frac{4\times6}{5\times7}$

$= \frac{4}{5}\times\frac{6}{7} = \frac{24}{35}$

Solution-09:

Mean proportion of $x+2$ and $x+9$ is

$$\sqrt{(x+2)(x+9)} = x+5$$

Rooting on b/s we get

$$\Rightarrow \left[\sqrt{(x+2)(x+9)}\right]^2 = [x+5]^2$$

$$\Rightarrow (x+2)(x+9) = x^2+25+2(5)(x)$$

$$\Rightarrow x^2+2x+9x+18 = x^2+10x+25$$

$$\Rightarrow 11x-10x = 25-18$$

$$\Rightarrow x = 7.$$

Solution-10.

Given that

$$\frac{16+y}{26+y} = \frac{26+y}{40+y} = \frac{40+y}{x+y}$$

$$\Rightarrow \frac{26+y}{40+y} = \frac{40+y}{x+y}$$

$$\Rightarrow 26x = 1600 \qquad (26+y)(x+y) = (40+y)^2$$

$$\Rightarrow x = \frac{1600}{26} \qquad 26x+xy+26y = 1600+80y$$

$$\Rightarrow 26x+xy-54y-1600 = 0$$

$$\Rightarrow \frac{16+y}{26+y} = \frac{40+y}{x+y}$$

$$\Rightarrow (16+y)(x+y) = (40+y)(26+y)$$

$$\Rightarrow 16x+xy+16y+y^2 = 1040+xy+26y+y^2$$

$$\Rightarrow 16x = 10y+1040.$$