Factor Theorem

EXERCISE - 7.1

Q1. Find the remainder (without division) on dividing $f(x)$ by $(x-2)$ where (i) $f(x) = 5x^2 - 7x + 4$ (ii) $f(x) = 2x^3 - 7x^2 + 3$.

Sol. let $x - 2 = 0 \Rightarrow x = 2$

(i) substituting the value of x in $f(x)$

$$f(x) = 5x^2 - 7x + 4$$

$$f(2) = 5(2)^2 - 7(2) + 4 = 20 - 14 + 4 = 10.$$

Hence remainder $= 10$.

(ii) $f(x) = 2x^3 - 7x^2 + 3$

$$f(2) = 2(2)^3 - 7(2)^2 + 3 = 16 - 28 + 3 = -9$$

Hence remainder $= -9$.

Q2. Using remainder theorem, find the remainder on dividing $f(x)$ by $(x+3)$ where (i) $f(x) = 2x^2 - 5x + 1$

(ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

Sol. let $x + 3 = 0 \Rightarrow x = -3$.

Substituting the value of x in $f(x)$.

(i) $f(x) = 2x^2 - 5x + 1$

$$f(-3) = 2(-3)^2 - 5(-3) + 1 = 18 + 15 + 1 = 34.$$

(ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

$$f(-3) = 3(-3)^3 + 7(-3)^2 - 5(-3) + 1$$

$$= -81 + 63 + 15 + 1$$

$$= -2.$$

Q3. Find the remainder (without division) on dividing $f(x)$ by $(2x+1)$ where (i) $f(x) = 4x^2+5x+3$ (ii) $f(x) = 3x^3-7x^2+4x+11$

Sol. let $2x+1=0 \Rightarrow x = \frac{-1}{2}$

Sub. the value of x in $f(x)$.

(i) $f(x) = 4x^2+5x+3$

$f(-\frac{1}{2}) = 4(\frac{-1}{2})^2 + 5(\frac{-1}{2}) + 3 = 4(\frac{1}{4}) - \frac{5}{2} + 3 = 4 - \frac{5}{2} = \frac{3}{2}$

$\therefore$ remainder $= \frac{3}{2}$.

(ii) $f(x) = 3x^3-7x^2+4x+11$

$f(-1/2) = 3(\frac{-1}{2})^3 - 7(\frac{-1}{2})^2 + 4(\frac{-1}{2}) + 11 = \frac{-3}{8} - \frac{7}{4} + 9$

$= \frac{-3-14+72}{8} = \frac{55}{8} = 6\frac{7}{8}$

Q4. (i) Find the remainder (without division) when $2x^3-3x^2+7x-8$ is divided by $(x-1)$.

(ii) Find the remainder (without division) on dividing $3x^2+5x-9$ by $(3x+2)$.

Sol. (i) let $x-1=0 \Rightarrow x=1$

Sub. the value of x in $f(x)$

$f(x) = 2x^3-3x^2+7x-8$

$f(1) = 2(1)^3 - 3(1)^2 + 7(1) - 8 = 2-3+7-8 = -2$.

(ii) let $3x+2=0 \Rightarrow x = \frac{-2}{3}$.

Sub. the value of x in $f(x)$.

$f(x) = 3x^2+5x-9$

$f(-2/3) = 3(\frac{-2}{3})^2 + 5(\frac{-2}{3}) - 9 = \frac{12}{9} - \frac{10}{3} - 9$

$= \frac{12-30-81}{9} = \frac{-99}{9} = -11$

Q5. When $kx^3+9x^2+4x-10$ is divided by $(x+1)$, the remainder is 2. Find the value of k.

Sol. let $x+1=0 \Rightarrow x=-1$.

Sub. the value of x in $f(x)$

$f(x) = kx^3+9x^2+4x-10$

$f(-1) = k(-1)^3+9(-1)^2+4(-1)-10 = -k+9-4-10 = -k-5$

Remainder $=2$, then $-k-5=2 \Rightarrow k=-7$.

Q6. Using remainder theorem, find the value of 'a' if the division of x^3+5x^2-ax+6 by $(x-1)$ leaves the remainder $2a$.

Sol. let $x-1=0 \Rightarrow x=1$

Sub. the value of x in $f(x)$.

$f(x) = x^3+5x^2-ax+6$

$f(1) = (1)^3+5(1)^2-a(1)+6 = 12-a$.

remainder $=2a$

$12-a=2a \Rightarrow 3a=12 \Rightarrow a=4$.

Q7. (i) what number must be subtracted from $2x^2-5x$ so that the resulting polynomial leaves the remainder 2 when divided by $2x+1$?

(ii) what number must be added to $2x^3-7x^2+2x$ so that the resulting polynomial leaves the remainder -2 when divided by $2x-3$?

Sol. (i) let 'a' be subtracted from $2x^2-5x$.

Dividing $2x^2-5x$ by $2x+1$.

$$
\begin{array}{r|l}
2x+1 & 2x^2-5x-a \;\; (x-3 \\
 & \underline{\underset{-}{2x^2}\underset{-}{+}\,x} \\
 & \quad -6x-a \\
 & \quad \underline{\underset{+}{-6x}\underset{+}{-}\,3} \\
 & \qquad -a+3
\end{array}
$$

Here remainder is $(3-a)$.

but we are given that remainder is 2.

$3-a=2 \Rightarrow a=1$

$\therefore$ Hence 1 is to be subtracted.

(ii) let 'a' be added to $2x^3-7x^2+2x$ dividing it by $2x-3$, then

$$
\begin{array}{r|l}
2x-3 & 2x^3-7x^2+2x+a \;\; (x^2-2x-2 \\
 & \underline{\underset{-}{2x^3}\underset{+}{-}3x^2} \\
 & \quad -4x^2+2x \\
 & \quad \underline{\underset{+}{-4x^2}\underset{-}{+}6x} \\
 & \qquad -4x+a \\
 & \qquad \underline{\underset{+}{-4x}\underset{-}{+}6} \\
 & \qquad\quad a-6
\end{array}
$$

But remainder is -2, then

$a-6=-2 \Rightarrow a=4$

$\therefore$ Hence 4 is to be added.

Q8. The polynomials kx^3+3x^2-4 and $2x^3-5x+4k$ when divided by $(x+3)$ leave the same remainder. Find the value of k.

Sol. let $x+3=0 \Rightarrow x=-3$.

Sub. the value of x in $f(x)$.

$f(x) = kx^3+3x^2-4$

$f(-3) = k(-3)^3 + 3(-3)^2 - 4 = -27k + 27 - 4$

$= -27k + 23$ ——— (i)

and $f(x) = 2x^3 - 5x + 4k$

$f(-3) = 2(-3)^3 - 5(-3) + 4k = -54 + 15 + 4k$

$= -39 + 4k$ ——— (ii)

Remainder is same in both cases,

from (i) & (ii),

$-27k + 23 = -39 + 4k$

$\Rightarrow -27k - 4k = -39 - 23$

$\Rightarrow -31k = -62 \Rightarrow k = 2$

Q9. By factor theorem, show that $(x+3)$ and $(2x-1)$ are factors of $2x^2 + 5x - 3$.

Sol. let $x + 3 = 0 \Rightarrow x = -3$.

Sub. the value of x in $f(x)$

$f(x) = 2x^2 + 5x - 3$

$f(-3) = 2(-3)^2 + 5(-3) - 3 = 18 - 15 - 3 = 0$

$\therefore$ $x+3$ is a factor of $f(x)$.

Again let $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

Sub. the value of x in $f(x)$

$f(x) = 2x^2 + 5x - 3$

$f(\frac{1}{2}) = 2(\frac{1}{2})^2 + 5(\frac{1}{2}) - 3 = \frac{1}{2} + \frac{5}{2} - 3$

$= 3 - 3 = 0$

$\therefore$ $2x - 1$ is a factor of $f(x)$.

$\therefore$ Hence proved.

Q10. Show that $(x-2)$ is a factor of $3x^2-x-10$. Hence factorise $3x^2-x-10$.

Sol. let $x-2=0 \Rightarrow x=2$.

Sub. the value of x in $f(x)$.

$f(x) = 3x^2 - x - 10$.

$f(2) = 3(2)^2 - 2 - 10 = 12 - 12 = 0$

$\therefore (x-2)$ is a factor of $f(x)$.

Dividing $(3x^2-x-10)$ by $(x-2)$, we get

$$\begin{array}{r|l} x-2) & 3x^2 - x - 10 \,(3x+5 \\ & \underline{-\; 3x^2 \mp 6x} \\ & \quad 5x - 10 \\ & \underline{-\; 5x \mp 10} \\ & \quad (0) \end{array}$$

$\therefore 3x^2 - x - 10 = (x-2)(3x+5)$

Q11. Show that $(x-1)$ is a factor of x^3-5x^2-x+5. Hence factorise x^3-5x^2-x+5.

Sol. let $x-1=0 \Rightarrow x=1$

Sub. the value of x in $f(x)$.

$f(x) = x^3 - 5x^2 - x + 5$

$f(1) = (1)^3 - 5(1)^2 - (1) + 5 = 1 - 5 - 1 + 5 = 0$

$\therefore (x-1)$ is a factor of $x^3 - 5x^2 - x + 5$.

Now dividing $f(x)$ by $(x-1)$, we get

$$x-1\overline{)\,x^3-5x^2-x+5\,}\,(x^2-4x-5$$
$$\underline{\overset{-}{}x^3\overset{+}{-}x^2}$$
$$-4x^2-x$$
$$\underline{\overset{+}{-}4x^2\overset{-}{+}4x}$$
$$-5x+5$$
$$\underline{\overset{+}{-}5x\overset{-}{+}5}$$
$$0$$

$$\therefore x^3-5x^2-x+5=(x-1)(x^2-4x-5)=(x-1)(x^2-5x+x-5)$$
$$=(x-1)[x(x-5)+1(x-5)]$$
$$=(x-1)(x+1)(x-5)$$

Q12. Show that $(x-3)$ is a factor of $x^3-7x^2+15x-9$. Hence factorise $x^3-7x^2+15x-9$.

Sol. let $x-3=0 \Rightarrow x=3$.

Sub. the value of x in $f(x)$.

$$f(x)=x^3-7x^2+15x-9$$
$$f(3)=(3)^3-7(3)^2+15(3)-9=27-63+45-9=0$$

$\therefore (x-3)$ is a factor of $f(x)$.

Now dividing by $(x-3)$, we get

$$x-3\overline{)\,x^3-7x^2+15x-9\,}\,(x^2-4x+3$$
$$\underline{\overset{-}{}x^3\overset{+}{-}3x^2}$$
$$-4x^2+15x$$
$$\underline{\overset{+}{-}4x^2\overset{-}{+}12x}$$
$$3x-9$$
$$\underline{\overset{-}{}3x\overset{+}{-}9}$$
$$0$$

$$\therefore x^3-7x^2+15x-9=(x-3)(x^2-4x+3)$$

$= (x-3)[x^2 - x - 3x + 3]$

$= (x-3)[x(x-1) - 3(x-1)]$

$= (x-3)(x-3)(x-1)$

$= (x-3)^2(x-1)$

Q13. Show that $(2x+1)$ is a factor of $4x^3+12x^2+11x+3$. Hence factorise $4x^3+12x^2+11x+3$.

Sol. let $2x+1=0 \Rightarrow x = -1/2$

Sub. the value of x in $f(x)$.

$f(x) = 4x^3 + 12x^2 + 11x + 3.$

$f(-\frac{1}{2}) = 4\left(\frac{-1}{2}\right)^3 + 12\left(\frac{-1}{2}\right)^2 + 11\left(\frac{-1}{2}\right) + 3$

$= \frac{-1}{2} + 3 - \frac{11}{2} + 3 = -6 + 6 = 0$

$\therefore (2x+1)$ is a factor of $4x^3+12x^2+11x+3$.

Now dividing $f(x)$ by $(2x+1)$, we get

$$
\begin{array}{r|l}
2x+1 & 4x^3+12x^2+11x+3 \;(2x^2+5x+3 \\
 & -\;4x^3+2x^2 \\ \hline
 & 10x^2+11x \\
 & -\;10x^2+5x \\ \hline
 & 6x+3 \\
 & -\;6x+3 \\ \hline
 & 0
\end{array}
$$

$\therefore 4x^3+12x^2+11x+3 = (2x+1)(2x^2+5x+3)$

$= (2x+1)(2x^2+2x+3x+3)$

$= (2x+1)[2x(x+1)+3(x+1)]$

$= (x+1)(2x+1)(2x+3).$

Q14. Show that $(2x+7)$ is a factor of $2x^3+5x^2-11x-14$. Hence factorise the given expression completely, using the factor theorem.

Sol. let $2x+7=0 \Rightarrow x = \frac{-7}{2}$

$$f\left(\frac{-7}{2}\right) = 2\left(\frac{-7}{2}\right)^3 + 5\left(\frac{-7}{2}\right)^2 - 11\left(\frac{-7}{2}\right) - 14$$

$$= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14$$

$$= \frac{-343+245+154-56}{4}$$

$$= 0$$

Hence, $(2x+7)$ is the factor of the given expression.

on dividing $2x^3+5x^2-11x-14=0$ by $2x+7$, we get

$$2x+7\,\big)\,2x^3+5x^2-11x-14\,\big(\,x^2-x-2$$

$$\underline{\begin{array}{l} 2x^3+7x^2 \\ -\quad\;\; - \end{array}}$$

$$-2x^2-11x$$

$$\underline{\begin{array}{l} -2x^2-7x \\ +\quad\;\;\; + \end{array}}$$

$$-4x-14$$

$$\underline{\begin{array}{l} -4x-14 \\ +\quad\;\; + \end{array}}$$

$$0$$

$$\therefore 2x^3+5x^2-11x-14 = (2x+7)(x^2-x-2)$$

$$= (2x+7)(x^2-2x+x-2)$$

$$= (2x+7)[x(x-2)+1(x-2)]$$

$$= (x+1)(x-2)(2x+7)$$

Q15. Use factor theorem to factorise the following polynomials completely: (i) x^3+2x^2-5x-6 (ii) $x^3-13x-12$.

Sol. (i) $f(x) = x^3+2x^2-5x-6$

let $x=-1$, then

$f(-1) = (-1)^3+2(-1)^2-5(-1)-6$

$= -1+2+5-6 = 0$

$\therefore (x+1)$ is a factor of $f(x)$.

Now dividing $f(x)$ by $x+1$, we get

$$
\begin{array}{r|l}
x+1 & x^3+2x^2-5x-6 \;(x^2+x-6 \\
 & \underline{-x^3 \mp x^2} \\
 & \quad x^2-5x \\
 & \quad \underline{-x^2 \mp x} \\
 & \qquad -6x-6 \\
 & \qquad \underline{-6x-6} \\
 & \qquad\quad \underline{\;+\quad +\;} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore f(x) = (x+1)(x^2+x-6) = (x+1)(x^2+3x-2x-6)$

$= (x+1)[x(x+3)-2(x+3)]$

$= (x+1)(x-2)(x+3)$

(ii) $x^3-13x-12$

$f(x) = x^3-13x-12$ —— (i)

putting $x=-1$ in (i), we get

$f(-1) = (-1)^3-13(-1)-12 = -1+13-12 = 0$

$\therefore (x+1)$ is a factor of $f(x)$.

on dividing $x^3-13x-12$ by $(x+1)$, we get

$$
\begin{array}{l}
x+1 \overline{)\, x^3 - 13x - 12 \,}\, (x^2 - x - 12 \\
\underline{\;\; x^3 + x^2 \;\;} \\
\;\;\;\; -x^2 - 13x \\
\;\;\;\; \underline{-x^2 - x} \\
\;\;\;\;\;\;\;\; -12x - 12 \\
\;\;\;\;\;\;\;\; \underline{-12x - 12} \\
\;\;\;\;\;\;\;\;\;\;\;\; 0
\end{array}
$$

$$\therefore x^3 - 13x - 12 = (x+1)(x^2 - x - 12)$$
$$= (x+1)(x^2 - 4x + 3x - 12)$$
$$= (x+1)[x(x-4) + 3(x-4)]$$
$$= (x+1)(x+3)(x-4)$$

Q16. If $(2x+1)$ is a factor of $6x^3 + 5x^2 + ax - 2$, find the value of a.

Sol. let $2x+1 = 0 \Rightarrow x = -\frac{1}{2}$

Sub. the value of x in $f(x)$

$$\therefore f(x) = 6x^3 + 5x^2 + ax - 2$$

$$f\left(-\tfrac{1}{2}\right) = 6\left(-\tfrac{1}{2}\right)^3 + 5\left(-\tfrac{1}{2}\right)^2 + a\left(-\tfrac{1}{2}\right) - 2$$
$$= -\frac{3}{4} + \frac{5}{4} - \frac{a}{2} - 2$$
$$= \frac{-3 + 5 - 2a - 8}{4} = \frac{-6 - 2a}{4}$$

$\therefore 2x+1$ is a factor of $f(x)$

Remainder $= 0$

$$\therefore \frac{-6-2a}{4} = 0 \Rightarrow -6 - 2a = 0 \Rightarrow 2a = -6$$
$$\Rightarrow a = -3.$$

Q17. If $(3x-2)$ is a factor of $3x^3 - kx^2 + 21x - 10$, find the value of k.

Sol. let $3x - 2 = 0 \Rightarrow x = \frac{2}{3}$.

Sub. the value of x in $f(x)$,

$f(x) = 3x^3 - kx^2 + 21x - 10$

$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21\left(\frac{2}{3}\right) - 10$

$= \frac{8}{9} - \frac{4k}{9} + 14 - 10$

$= \frac{8}{9} - \frac{4k}{9} + 4 = \frac{8 - 4k + 36}{9} = \frac{44 - 4k}{9}$

$\because$ Remainder is '0'

$\therefore \frac{44 - 4k}{9} = 0 \Rightarrow 44 - 4k = 0 \Rightarrow k = 11.$

Q18. what number must be added to $4x^3 - 8x^2 + 3x$ so that the resulting polynomial has a factor $2x+1$?

Sol. let 'a' be added to $4x^3 - 8x^2 + 3x$, then

$f(x) = 4x^3 - 8x^2 + 3x + a$.

$\therefore (2x+1)$ is a factor of $f(x)$, $f(x) = 0$

Now $2x+1 = 0 \Rightarrow x = -\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + a$

$= -\frac{1}{2} - 2 - \frac{3}{2} + a = -4 + a$

$\therefore$ $2x+1$ is a factor.

$f\left(-\frac{1}{2}\right) = 0 \Rightarrow -4 + a = 0$

$\Rightarrow a = 4$

$\therefore$ Hence 4 is to be added.

Q19. If $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$, then find the value of p. factorize the above Expression completely.

Sol. let $x - 2 = 0 \Rightarrow x = 2$

Now $f(x) = 2x^3 - x^2 + px - 2$

$$f(2) = 2(2)^3 - (2)^2 + p(2) - 2$$
$$= 16 - 4 + 2p - 2$$
$$= 2p + 10$$

$\because f(2) = 0$, then $2p + 10 = 0 \Rightarrow p = -5$

Now the polynomial will be $2x^3 - x^2 - 5x - 2$

$$2x^3 - x^2 - 5x - 2 = (x-2)(2x^2 + 3x + 1)$$
$$= (x-2)(2x^2 + 2x + x + 1)$$
$$= (x-2)[2x(x+1) + 1(x+1)]$$
$$= (x+1)(x-2)(2x+1).$$

$x - 2)\ 2x^3 - x^2 - 5x - 2\ (2x^2 + 3x + 1$

$\quad 2x^3 - 4x^2$

$\quad - \quad +$

$\quad 3x^2 - 5x$

$\quad 3x^2 - 6x$

$\quad - \quad +$

$\quad x - 2$

$\quad x - 2$

$\quad - \quad +$

$\quad 0$

Q20. Find the value of the constants a and b, if $(x-2)$ and $(x+3)$ are both factors of the Expression $x^3 + ax^2 + bx - 12$

Sol. let $x - 2 = 0 \Rightarrow x = 2$.

Sub. the value of x in $f(x)$

$$f(x) = x^3 + ax^2 + bx - 12.$$

$f(2) = (2)^3 + a(2)^2 + b(2) - 12$

$= 8 + 4a + 2b - 12$

$= 4a + 2b - 4$

$\therefore$ $x - 2$ is a factor

$\therefore 4a + 2b - 4 = 0 \Rightarrow 4a + 2b = 4$

$\Rightarrow 2a + b = 2$ ——— (i)

Again let $x + 3 = 0$, then $x = -3$

Sub the value of x in $f(x)$

$f(x) = x^3 + ax^2 + bx - 12$

$f(-3) = (-3)^3 + a(-3)^2 + b(-3) - 12$

$= -27 + 9a - 3b - 12$

$= +9a - 3b - 39$

$x + 3$ is a factor of $f(x)$

$\therefore -39 + 9a - 3b = 0 \Rightarrow 9a - 3b = 39$

$\Rightarrow 3a - b = 13$ ——— (ii)

Adding (i) & (ii), $5a = 15 \Rightarrow a = 3$.

Sub. the value of 'a' in (i)

$2(3) + b = 2 \Rightarrow b = -4$.

$\therefore$ Hence $a = 3$, $b = -4$.

Q21. If $(x+2)$ and $(x-3)$ are factors of $x^3 + ax + b$, find the values of a and b. factorize the given Expression.

Sol.

let $x + 2 = 0 \Rightarrow x = -2$.

Sub. the value of x in $f(x)$.

$f(x) = x^3 + ax + b$.

$f(-2) = (-2)^3 + a(-2) + b$

$= -8 - 2a + b$

$\therefore$ $x+2$ is a factor

$\therefore$ Remainder is zero.

$-8 - 2a + b = 0 \Rightarrow 2a - b = -8$ —— (i)

Again let $x - 3 = 0 \Rightarrow x = 3$

Sub. the value of x in $f(x)$,

$f(x) = x^3 + ax + b$

$f(3) = (3)^3 + a(3) + b = 3a + b + 27$

$(x-3)$ is a factor, remainder $= 0$

$3a + b + 27 = 0 \Rightarrow 3a + b = -27$ —— (ii)

Adding (i) & (ii), $5a = -35 \Rightarrow a = -7$

Sub. the value of 'a' in (i), $2(-7) - b = -8$

$\Rightarrow -14 - b = -8 \Rightarrow b = -6$

$\therefore$ Hence $a = -7$, $b = -6$

$(x+2)$ and $(x-3)$ are the factors of $x^3 + ax + b$

$\Rightarrow x^3 - 7x - 6$.

Now dividing $x^3 - 7x - 6$ by $(x+2)(x-3)$ or $x^2 - x - 6$,

we get

$$
\begin{array}{r|l}
x^2 - x - 6 \,) & x^3 - 7x - 6 \,(\, x + 1 \\
 & \underline{-\; x^3 - x^2 - 6x} \\
 & \quad\;\; x^2 - x - 6 \\
 & \underline{-\; x^2 - x - 6} \\
 & \quad\quad 0
\end{array}
$$

$\therefore x^3 - 7x - 6 = (x+1)(x+2)(x-3)$

Q22. $(x-2)$ is a factor of the expression x^3+ax^2+bx+6. When this expression is divided by $(x-3)$, it leaves the remainder 3. Find the values of a and b.

Sol. As it is given that $(x-2)$ is a factor of the expression

$$x^3+ax^2+bx+6 \quad \text{—— (i)}$$

$$f(x) = x^3+ax^2+bx+6$$

$$f(2) = (2)^3+a(2)^2+b(2)+6 = 4a+2b+14$$

$\because$ $(x-2)$ is a factor, remainder $=0$

$$4a+2b+14=0 \Rightarrow 2a+b+7=0$$

$$\Rightarrow 2a+b=-7 \quad \text{—— (ii)}$$

When Exp (i) is divided by $(x-3)$, it leaves the remainder 3.

$$x-3)\ x^3+ax^2+bx+6\ (x^2+(3+a)x+(9+3a+b)$$

$$\underline{x^3-3x^2}$$
$$\quad -\ +$$

$$(3+a)x^2+bx$$

$$\underline{(3+a)x^2-3(3+a)x}$$
$$\quad -\ +$$

$$(9+3a+b)x+6$$

$$\underline{(9+3a+b)x-3(9+3a+b)}$$
$$\quad -\ +$$

$$33+9a+3b.$$

Remainder $= 33+9a+3b = 3$ (given)

$$\Rightarrow 9a+3b=-30 \Rightarrow 3a+b=-10 \quad \text{—— (iii)}.$$

Subtracting (iii) from (ii),

$$2a+b=-7$$
$$\underline{3a+b=-10}$$
$$\quad -\ \ -\ \ \ +$$
$$-a=3 \Rightarrow a=-3.$$

put $a = -3$ in (iii), we get

$b = -1$

$\therefore$ Hence $a = -3, \ b = -1$.

Q23 If $ax^3 + 3x^2 + bx - 3$ has a factor $(2x+3)$ and leaves remainder -3 when divided by $(x+2)$. find the values of a and b. with these values of a and b, factorize the given expression.

Sol. let $2x+3 = 0$ then $2x = -3 \Rightarrow x = \frac{-3}{2}$

Sub. the value of x in $f(x)$,

$f(x) = ax^3 + 3x^2 + bx - 3$

$f\left(\frac{-3}{2}\right) = a\left(\frac{-3}{2}\right)^3 + 3\left(\frac{-3}{2}\right)^2 + b\left(\frac{-3}{2}\right) - 3$

$= \frac{-27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3$

$\therefore$ $2x+3$ is a factor of $f(x)$, remainder $= 0$

$\frac{-27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3 = 0$

$\Rightarrow -27a + 54 - 12b - 24 = 0$

$\Rightarrow -27a - 12b = -30$

$\Rightarrow 9a + 4b = 10$ —— (i)

Again let $x+2 = 0 \Rightarrow x = -2$.

Sub. the value of x in $f(x)$.

$f(x) = ax^3 + 3x^2 + bx - 3$

$f(-2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$

$= -8a + 12 - 2b - 3$

$= -8a - 2b + 9$

$\therefore$ remainder $= -3$

$-8a - 2b + 9 = -3$

$\Rightarrow -8a - 2b = -12$

$\Rightarrow \quad 4a + b = 6 \quad$ —— (ii)

Multiply (ii) by 4 $\Rightarrow \quad 16a + 4b = 24$

(i) $\Rightarrow \quad \underline{-9a \mp 4b = -10}$

$7a = 14$

$\Rightarrow a = 2.$

Sub. the value of 'a' in (i)

$9(2) + 4b = 10 \Rightarrow 4b = -8 \Rightarrow b = -2.$

$\therefore$ Hence $a = 2,\ b = -2$

$f(x) = ax^3 + 3x^2 + bx - 3 = 2x^3 + 3x^2 - 2x - 3$

$\therefore$ $2x+3$ is a factor

$\therefore$ Dividing $f(x)$ by $x+2$

$$
\begin{array}{r l}
2x+3 \,\big)\, 2x^3 + 3x^2 - 2x - 3 \,\big(\, x^2 - 1 \\
\underline{-\,2x^3 + 3x^2} \\
-2x - 3 \\
\underline{-2x - 3} \\
+\quad + \\
0
\end{array}
$$

$\therefore 2x^3 + 3x^2 - 2x - 3 = (2x+3)(x^2-1)$

$= (2x+3)(x+1)(x-1)$

Q24. Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$. If $x-2$ is a factor of $f(x)$ but leaves the remainder -15 when it divides $g(x)$, find the values of a and b with these values of a and b, factorise the Expression $f(x) + g(x) + 4x^2 + 7x$.

Sol. Given $f(x) = ax^2+bx+2$, $g(x) = bx^2+ax+1$

As $x-2$ is a factor of $f(x)$, $f(2)=0$

$\Rightarrow a(2)^2+b(2)+2=0$

$\Rightarrow 4a+2b+2=0$

$\Rightarrow 2a+b+1=0$ ——— (1)

when $g(x)$ divide by $(x-2)$, leaves remainder -15

$g(2) = -15$

$\Rightarrow b(2)^2+a(2)+1 = -15$

$\Rightarrow 4b+2a+16=0$

$\Rightarrow 2b+a+8=0$ ——— (2)

from (1) & (2),

(1) $\times 2 \Rightarrow 4a+2b+2=0$

(2) $\times 1 \Rightarrow a+2b+8=0$

$\quad - \quad - \quad -$

$3a-6=0 \Rightarrow a=2.$

put the 'a' value in (1), $2(2)+b+1=0$

$\Rightarrow b=-5.$

Now, $f(x) = ax^2+bx+2 = 2x^2-5x+2$

$g(x) = +bx^2+ax+1 = -5x^2+2x+1$

So, $f(x)+g(x)+4x^2+7x$

$= 2x^2-5x+2-5x^2+2x+1+4x^2+7x$

$= x^2+4x+3$

$= x^2+3x+x+3$

$= x(x+3)+1(x+3)$

$= (x+1)(x+3).$