Algebraic Expressions and Identities

1. Exercise 10.1

i) Given expression $12x^2yz - 4xy^2$

Terms	Numerical Co-efficient	Literal Co-efficient
$12x^2yz$	12	x^2yz
$-4xy^2$	-4	xy^2

ii) Given expression $8 + mn + nl - lm$

Terms	Numerical Co-efficient	Literal Co-efficient
8	8	-
mn	1	mn
nl	1	nl
$-lm$	-1	lm

iii) Given expression $\frac{x^2}{3} + \frac{y}{6} - xy^2$

Terms	Numerical co-efficient	Literal co-efficient
$\frac{x^2}{3}$	$\frac{1}{3}$	x^2
$\frac{y}{6}$	$\frac{1}{6}$	y
$-xy^2$	-1	xy^2

iv) Given expression $-4p + 2.3q + 1.7r$

Terms	Numerical co-efficient	Literal co-efficient
$-4p$	-4	p
$2.3q$	2.3	q
$1.7r$	1.7	r

2.

i) $5p \times q \times r^2 \rightarrow$ Monomial

ii) $3x^2 \times y \div 2z \rightarrow$ Monomial

iii) $-3+7x^2 \rightarrow$ Binomial

iv) $\frac{5a^2+3b^2+c}{2} \rightarrow$ Trinomial

v) $7x^5 - \frac{3x}{y} \rightarrow$ Binomial

vi) $5p \div 3q - 3p^2q^2 \rightarrow$ Binomial

3.

i) $\frac{2}{5}x^4 - \sqrt{3}x^2 + 5x - 1$

It is polynomial of degree 4

ii) $7x^3 - \frac{3}{x^2} + \sqrt{5}$

due to $-3x^{-2}$ term, It is not called as polynomial

∴ It is NOT Polynomial

iii) $4a^3b^2 - 3ab^4 + 5ab + \frac{2}{3}$

It is a polynomial of degree 5

iv) $2x^2y - \frac{3}{xy} + 5y^3 + \sqrt{3}$

due to negative power in the $-3(xy)^{-1}$

∴ It is NOT a Polynomial

4.

i) Arrange terms for Column method

$$\begin{array}{cccc} ab & -bc & & \\ 0 & +bc & -ca & \\ -ab & 0 & +ca & \\ \hline 0 & +0 & +0 & \\ \hline \end{array}$$

$\therefore\ ab - bc + bc - ca + ca - ab = 0$

ii) Arrange terms in Columns

$$\begin{array}{ccc} 5p^2q^2 & +4pq & +7 \\ -2p^2q^2 & +9pq & +3 \\ \hline 3p^2q^2 & +13pq & +10 \\ \hline \end{array}$$

iii) Arrange terms in Columns

$$\begin{array}{l} l^2 + m^2 + n^2 + 0 + 0 + 0 \\ 0 + 0 + 0 + lm + mn + 0 \\ 0 + 0 + 0 + 0 + mn + nl \\ 0 + 0 + 0 + lm + 0 + nl \\ \hline l^2 + m^2 + n^2 + 2lm + 2mn + 2nl \\ \hline \end{array}$$

iv) Arrange terms in Columns

$$\begin{array}{l} 4x^3 - 7x^2 + 0x + 9 \\ \cancel{3x^3}\,0 + 3x^2 + 5x + 4 \\ 7x^3 + 0 - 11x + 1 \\ 0 + 6x^2 - 13x + 0 \\ \hline 10x^3 + 2x^2 - 29x + 14 \end{array}$$

5.

i) $14a - 5ab + 7b - 5$

$8a + 3ab - 2b + 7$

$(-) \quad (-) \quad (+) \quad -$

$6a - 8ab + 9b - 12$

ii) $12xy - 3yz - 4zx + 5xyz$

$8xy + 4yz + 5zx + 0$

$(-) \quad (-) \quad (-) \quad (-)$

$4xy - 7yz - 9zx + 5xyz$

iii) $4p^2q + 3$

$4p^2q + 3pq + 5pq^2$

iii) $5p^2q - 2pq^2 + 5pq - 11q - 3p + 18$

$4p^2q + 5pq^2 - 3pq + 7q - 8p - 10$

$(-) \quad (-) \quad (+) \quad (-) \quad (+) \quad (+)$

$p^2q - 7pq^2 + 8pq - 18q + 5p + 28$

6. Horizontal method

$3x^2+5xy+7y^2+3 \rightarrow (1)$

$18 - 3p - 11q + 5pq + 2pq^2$

$2x^2 - 4xy - 3y^2 + 7 \rightarrow (2)$

$9x^2 - 8xy + 11y^2 \rightarrow (3)$

$(1) + (2) - (3)$

$(3) - [(1) + (2)]$

$9x^2 - 8xy + 11y^2 - [3x^2+5xy+7y^2+3+2x^2-4xy-3y^2+7]$

$9x^2 - 8xy + 11y^2 - [5x^2 + xy + 4y^2 + 10]$

$4x^2 - 9xy.$

$9x^2 - 8xy + 11y^2 - 5x^2 - xy - 4y^2 - 10$

$4x^2 - 9xy + 7y^2 - 10$

7.

Let $3x^2 + 5xy +$

Let $3a^2 - 5ab - 2b^2 - 3 \rightarrow (1)$

$5a^2 - 7ab - 3b^2 + 3a \rightarrow (2)$

do (2) − (1)

$5a^2 - 7ab - 3b^2 + 3a$

$3a^2 - 5ab - 2b^2 + 0 - 3$

(−) (+) (+) (−) (+)

$2a^2 - 2ab - b^2 + 3a + 3$

8.

Perimeter of triangle (P) $= 7p^2 - 5p + 11 \rightarrow (1)$

Sides $S_1 = p^2 + 2p - 1 \rightarrow (2)$

$S_2 = 3p^2 - 6p + 3 \rightarrow (3)$

$S_3 = ?$

$P = S_1 + S_2 + S_3$

$S_3 = P - (S_1 + S_2)$

$= 7p^2 - 5p + 11 - [p^2 + 2p - 1 + 3p^2 - 6p + 3]$

$= 7p^2 - 5p + 11 - [4p^2 - 4p + 2]$

$= \cancel{3p^2 - 91}$

$= 7p^2 - 5p + 11 - 4p^2 + 4p - 2$

Hence,

Third side of Triangle $S_3 = 3p^2 - p + 9$

Exercise 10.2

1.

i) $4x^3$ and $-3xy$

$4x^3 \times -3xy$

$(4 \times -3) \times (x^3 \times xy)$

$-12x^4y$

ii) $2xyz$ and 0

$(2xyz) \times 0$

$\underline{\underline{0}}$

iii) $-\frac{2}{3}p^2q$, $\frac{3}{4}pq^2$ and $5pqr$

$\left(-\frac{2}{3}p^2q\right) \times \left(\frac{3}{4}pq^2\right)(5pqr)$

$\left(-\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}_2} \times 5\right) \times (p^2q) \times (pq^2) \times (pqr)$

$-\frac{5}{2}p^4q^4r$

iv) $-7ab$, $-3a^3$ and $-\frac{2}{7}ab^2$

$(-7ab) \times (-3a^3) \times \left(-\frac{2}{7}ab^2\right)$

$\left(-7 \times -3 \times -\frac{2}{7}\right) \times ab \times a^3 \times ab^2$

$\underline{\underline{-6a^5b^3}}$

v) $\frac{-1}{2}x^2, -\frac{3}{5}xy, \frac{2}{3}yz$ and $\frac{5}{7}xyz$

$\left(\frac{-1}{2}x^2\right) \times \left(\frac{-3}{5}xy\right) \times \left(\frac{2}{3}yz\right) \times \left(\frac{5}{7}xyz\right)$

$\left(\frac{-1}{2} \times -\frac{3}{5} \times \frac{2}{3} \times \frac{5}{7}\right) \times x^2 \times xy \times yz \times xyz$

$\frac{1}{7}x^4y^3z^2$

2.
i) $(3x-5y+7z) \times (-3xyz)$

$(3x \times -3xyz) + (-5y \times -3xyz) + (7z \times -3xyz)$

$-9x^2yz + 15xy^2z - 21xyz^2$

ii) $\cancel{2p^2}$ $(2p^2 - 3pq + 5q^2 + 5) \times (-2pq)$

$(2p^2 \times -2pq) + (-3pq \times -2pq) + (5q^2 \times -2pq) +$
$(5 \times -2pq)$

$-4p^3q + 6p^2q^2 - 10pq^3 - 10pq$

iii) $\left(\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3\right) \times (35ab)$

$\left(\frac{2}{3}a^2b \times 35ab\right) + \left(\frac{-4}{5}ab^2 \times 35ab\right) + \left(\frac{2}{7}ab \times 35ab\right) + (3 \times 35ab)$

$\frac{70}{3}a^3b - 28a^2b^3 + 10a^2b^2 + 105ab$

iv) $(4x^2-10xy+7y^2-8x+4y+3)\times(3xy)$

$(4x^2\times 3xy)+(-10xy\times 3xy)+(7y^2\times 3xy)+(-8x\times 3xy)$
$+(4y\times 3xy)+(3\times 3xy)$

$12x^3y-30x^2y^2+21xy^3-24x^2y+12xy^2+9xy$

3.

i) Given length $(l) = p^2q$

breadth $(b) = pq^2$

Rectangle – Area $= l\times b$

$= (p^2q)\times(pq^2)$

Area $= p^3q^3$

ii) Given length $(l) = 5xy$

breadth $(b) = 7xy^2$

Rectangle – Area $= l\times b$

$= (5xy)\times(7xy^2)$

Area $= 35x^2y^3$

4.

i) Given length $(l) = 5ab$

breadth $(b) = 3a^2b$

height $(h) = 7a^4b^2$

Volume of rectangular box $= l\times b\times h$

$= 5ab\times 3a^2b\times 7a^4b^2$

$= (3\times5\times7)\times ab\times a^2b\times a^4b^2$

Volume $= 105a^7b^4$

ii) Given length $(l) = 2pq$

breadth $(b) = 4q^2$

height $(h) = 8rp$

Volume of rectangular box $= l \times b \times h$

$= (2pq) \times (4q^2) \times (8rp)$

$= (2 \times 4 \times 8)\ pq \times q^2 \times rp$

Volume of rectangular box $= \underline{\underline{64p^2q^3r}} = 64p^2q^3r$

5.

i) $x^2(3-2x+x^2)$

$3x^2 - 2x^3 + x^4$

For $x=1$

$3 \times 1^2 - 2 \times 1^3 + 1^4$

$3 \times 1 - 2 \times 1 + 1$

$3-2+1$

$4-2$

$\underline{\underline{2}}$ for $x=1$

For $x=-1$

$3x^2 - 2x^3 + x^4$

$3 \times (-1)^2 + (-2)(-1)^3 + (-1)^4$

$3 \times 1 + (-2 \times -1) + 1$

$3 + 2 + 1$

$\underline{\underline{6}}$ for $x=-1$

For $x = \frac{2}{3}$

$$3x^2 - 2x^3 + x^4$$

$$3\times\left(\frac{2}{3}\right)^2 - 2\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4$$

$$3\times\frac{4}{9} - 2\times\frac{8}{27} + \frac{16}{81}$$

$$\frac{12}{9} - \frac{16}{27} + \frac{16}{81}$$

$$\frac{108 - 48 + 16}{81}$$

$$\frac{76}{81} \text{ for } x = \frac{2}{3}$$

For $x = -\frac{1}{2}$

$$3x^2 - 2x^3 + x^4$$

$$3\times\left(-\frac{1}{2}\right)^2 - 2\times\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^4$$

$$3\times\frac{1}{4} - 2\times\left(-\frac{1}{8}\right) + \frac{1}{16}$$

$$\frac{3}{4} + \frac{2}{8} + \frac{1}{16}$$

$$\frac{12 + 4 + 1}{16}$$

$$\frac{17}{16} \text{ for } x = -\frac{1}{2}$$

ii) $5xy(3x+4y-7) - 3y(xy - x^2 + 9) - 8$

$(5xy\times3x) + (5xy\times4y) - 5xy\times7 + (-3y\times xy) + (-3y\times -x^2)$
$+ (-3y\times9) - 8$

$15x^2y + 20xy^2 - 35xy - 3xy^2 + 3x^2y - 27y - 8$

$\underline{\underline{18x^2y + 17xy^2 - 62xy - 8}}$

For $x=2$, $y=-1$

$18(2^2)(-1) + (17\times2\times(-1)^2) - (62\times2\times-1) - 8$

$-72 + 34 + 124 - 8$

$\underline{\underline{78}}$

6.

i) First expression $= 4p(2-p^2) = 8p - 4p^3$

Second expression $= 8p^3 - 3p$

Required sum $= (8p - 4p^3) + (8p^3 - 3p)$

$= 4p^3 + 5p$

ii) First expression $= 7xy(8x+2y-3) = 56x^2y + 14xy^2 - 21xy.$

Second expression $= 3y(4x^2y - 5xy + 8xy^2)$

$= 12x^2y^2 - 15xy^2 + 24xy^3$

Required sum $= (56x^2y + 14xy^2 - 21xy) + (12x^2y^2 - 15xy^2 + 24xy^3)$

ii) First expression $= 7xy(8x+2y-3) = 56x^2y + 14xy^2 - 21xy$

Second expression $= 4xy^2(3y - 7x + 8) = 12xy^3 - 28x^2y^2 + 32xy^2$

Required sum $= (56x^2y + 14xy^2 - 21xy) + (12xy^3 - 28x^2y^2 + 32xy^2)$

$= 12xy^3 - 28x^2y^2 + 56x^2y + 46xy^2 - 21xy$

7.

i) First expression $= 6x(x-y+z) - 3y(x+y-z)$

$= 6x^2 - 6xy + 6xz - 3xy - 3y^2 + 3yz$

$= 6x^2 + 6xz - 6xy + 3yz - 3y^2 \to (1)$

Second expression $= 2z(-x+y+z)$

$= -2xz - 2yz + 2z^2 \to (2)$

$(2) - (1) = (-2xz - 2yz + 2z^2) - (6x^2 + 6xz - 6xy + 3yz - 3y^2)$

$= -2xz - 2yz + 2z^2 - 6x^2 - 6xz + 6xy - 3yz + 3y^2$

$= -6x^2 + 3y^2 + 2z^2 + 6xy - 5yz - 8xz$

ii) ~~7xy(~~ First expression $= 7xy(x^2 - 2xy + 3y^2) - 8x(x^2y - 4xy + 7xy^2)$

$= 7x^3y - 14x^2y^2 + 21xy^3 - 8x^3y + 32x^2y - 56x^2y^2$

$= -7x^3y + 21xy^3 - 8x^3y - 70x^2y^2 + 32x^2y$

First expression $= -x^3y - 8x^3y - 70x^2y^2 + 32x^2y + 21xy^3 \to (1)$

Second expression $= 3y(4x^2y - 5xy + 8xy^2)$

$= 12x^2y^2 - 15xy^2 + 24xy^3 \rightarrow (2)$

$(2) - (1) = 12x^2y^2 - 15xy^2 + 24xy^3 - (-x^3y + 21xy^3 - 70x^2y^2 + 32x^2y)$

$= 12x^2y^2 - 15xy^2 + 24xy^3 + x^3y - 21xy^3 + 70x^2y^2 - 32x^2y$

$= 82x^2y^2 + x^3y + 3xy^3 - 15xy^2 - 32x^2y$

$= x^3y + 3xy^2 + 82x^2y^2 - 15xy^2 - 32x^2y$

Exercise 10.3

1.

i) $(5x-2)(3x+4)$

$5x(3x+4)-2(3x+4)$

$15x^2+20x-6x-8$

$15x^2+14x-8$

ii) $(ax+b)(cx+d)$

$ax(cx+d)+b(cx+d)$

$acx^2+adx+bcx+bd$

$acx^2+(ad+bc)x+bd$

iii) $(4p-7)(2-3p)$

$4p(2-3p)-7(2-3p)$

$8p-12p^2-14+21p$

$-12p^2+29p-14$

iv) $(2x^2+3)(3x-5)$

$2x^2(3x-5)+3(3x-5)$

$6x^3-10x^2+9x-15$

$6x^3-10x^2+9x-15$

v) $(1.5a - 2.5b)(1.5a + 2.5b)$

$1.5a(1.5a + 2.5b) - 2.5b(1.5a + 2.5b)$

$(1.5 \times 1.5)a^2 + (1.5 \times 2.5)ab - (2.5 \times 1.5)ab - (2.5 \times 2.5)b^2$

$2.25a^2 + 3.75ab - 3.75ab - 6.25b^2$

$2.25a^2 + 0 + 6.25b^2$

$\underline{\underline{2.25a^2 - 6.25b^2}}$

vi) $\left(\frac{3}{7}p^2 + 4q^2\right)\left(7\left(p^2 - \frac{3}{4}q^2\right)\right)$

$\frac{3}{7}p^2 \times \left(7p^2 - \frac{21}{4}q^2\right) + 4q^2\left(7p^2 - \frac{21}{4}q^2\right)$

$3p^4 - \frac{9}{4}q^2p^2 + 28p^2q^2 - 21q^4$

$3p^4 + \frac{103}{4}p^2q^2 - 21q^4$

2.

(i) $(x - 2y + 3)(x + 2y)$

$$\begin{array}{r} x - 2y + 3 \\ x + 2y \\ \hline 2xy - 4y^2 + 6y \\ x^2 - 2xy + 0 + 0 + 3x \\ \hline x^2 + 0 - 4y^2 + 6y + 3x \end{array}$$

$\underline{\underline{x^2 + 3x + 6y - 4y^2}}$

ii) $(3-5x+2x^2)(4x-5)$

$$\begin{array}{r} 3-5x+2x^2 \\ 4x-5 \\ \hline \end{array}$$

$$\begin{array}{rrrr} & 2x^2 & -5x & +3 \\ & & 4x & -5 \\ \hline 8x^3 & -20x^2 & +12x & \\ & -10x^2 & -25x & -15 \\ \hline 8x^3 & -30x^2 & -13x & -15 \\ \hline \end{array}$$

iii)

ঙ. i) $(3x^2-2x-1)(2x^2+x-5)$

$$\begin{array}{rrrrr} & & 3x^2 & -2x & -1 \\ & & 2x^2 & +x & -5 \\ \hline 6x^4 & -4x^3 & -2x^2 & & \\ & +3x^3 & -2x^2 & -x & \\ & & -15x^2 & +10x & +5 \\ \hline 6x^4 & -x^3 & -19x^2 & +9x & +5 \\ \hline \end{array}$$

ii) $(2-3y-5y^2)(2y-1+3y^2)$

$$
\begin{array}{r}
-5y^2-3y+2 \\
3y^2+2y-1 \\
\hline
-15y^4-9y^3+6y^2 \qquad\qquad \\
-10y^3-6y^2+4y \qquad \\
+5y^2+3y-2 \\
\hline
-15y^4-19y^3+5y^2+7y-2
\end{array}
$$

4.

i) $(x^2+3)(x-3)+9$

$x^2(x-3)+3(x-3)+9$

$x^3-3x^2+3x-9+9$

x^3-3x^2+3x

ii) $(x+3)(x-3)(x+4)(x-4)$

$[x(x-3)+3(x-3)][x(x-4)+4(x-4)]$

$[x^2-3x+3x-9][x^2-4x+4x-16]$

$[x^2-9][x^2-16]$

$x^2(x^2-16)-9(x^2-16)$

$x^4-16x^2-9x^2+144$

x^4-25x^4+144

iii) $(x+5)(x+6)(x+7)$

$[(x+5)(x+6)](x+7)$

$[x(x+6)+5(x+6)](x+7)$

$(x^2+6x+5x+30)(x+7)$

$(x^2+6x+5x+30)x+(x^2+6x+5x+30)7$

$(x^2+11x+30)x+(x^2+11x+30)7$

$x^3+11x^2+30x+7x^2+77x+210$

$\underline{\underline{x^3+18x^2+107x+210}}$

iv) $(p+q-2r)(2p-q+r)-4qr$

$p(2p-q+r)+q(2p-q+r)-2r(2p-q+r)-4qr$

$2p^2-pq+pr+2pq-q^2+qr-4pr+2qr-2r^2-4qr$

$2p^2-q^2-2r^2+pq-qr-3pr$

v) $(p+q)(r+s)+(p-q)(r-s)-2(pr+qs)$

$p(r+s)+q(r+s)+p(r-s)-q(r-s)-2pr-2qs$

$pr+ps+qr+qs+pr-ps-qr+qs-2pr-2qs$

$2pr-2pr+ps-ps+qr-qr+2qs-2qs$

$0+0+0+0=\underline{0}$

vi) $(x+y+z)(x-y+z)+(x+y-z)(-x+y+z)-4zx$

$x(x-y+z)+y(x-y+z)+z(x-y+z)+x(-x+y+z)$

$+y(-x+y+z)-z(-x+y+z)-4zx$

$x^2-xy+xz+xy-y^2+yz+xz-yz+z^2-x^2+xy+xz$

$-xy+y^2+yz+xz-yz-z^2-4zx$

$x^2-x^2+2xy-2xy+4xz-4xz-y^2+y^2+2yz-2yz$

$+z^2-z^2$

$0+0+0+0+0+0=\underline{0}$

5\. Sides of rectangle $S_1=5x^2+25xy+4y^2$

$S_2=2x^2-2xy+3y^2$

Area of rectangle $= S_1\times S_2$

$A=(5x^2+25xy+4y^2)(2x^2-2xy+3y^2)$

$A=\underline{10x^4+40x^3y-27x^2y^2+67xy^3+12y^4}$

$$
\begin{array}{r}
5x^2+25xy+4y^2 \\
2x^2-2xy+3y^2 \\
\hline
10x^4+50x^3y+8x^2y^2 \qquad\qquad\qquad \\
-10x^3y-50x^2y^2-8xy^3 \qquad\quad \\
+15x^2y^2+75xy^3+12y^4 \\
\hline
10x^4+40x^3y-27x^2y^2+67xy^3+12y^4 \\
\hline
\end{array}
$$

Exercise 10.4

1.

i)

$$-39pq^2r^5 \div -24p^3q^3r$$

$$= \frac{-39pq^2r^5}{-24p^3q^3r}$$

$$= \frac{13}{8}\frac{r^4}{p^2q}$$

ii)

$$\frac{-3}{4}a^2b^3 \div \frac{6}{7}a^3b^2 = \frac{\frac{-3}{4}a^2b^3}{\frac{6}{7}a^3b^2}$$

$$= \frac{-3}{4} \times \frac{7}{6} \times \frac{b}{a}$$

$$= \frac{-7}{8}\frac{b}{a}$$

2. i)

$$\begin{array}{r|l} & 3x^3 - \frac{8}{3}x^2 - 4 \\ \hline 3x & 9x^4 - 8x^3 - 12x + 3 \\ & \underline{(-)\,9x^4} \\ & 0 - 8x^3 \\ & \underline{(+)\,-8x^3} \\ & 0 - 12x \\ & \underline{(+)\,-12x} \\ & 0 + 3 \end{array}$$

Quotient $= 3x^3 - \frac{8}{3}x^2 - 4$; Remainder $= 3$

ii)

$$\begin{array}{r|l} & -7q^2+16pq \\ -2p^2q & 14p^2q^3-32p^3q^2+15pq^2-22p+18q \\ & \underline{(-)14p^2q^3} \\ & 0-32p^3q^2+15pq^2-22p+18q \\ & \underline{\overset{(+)}{-}32p^3q^2} \\ & 0+15pq^2-22p+18q \end{array}$$

Quotient $= -7q^2+16pq$, Remainder $= +15pq^2-22p+18q$

3.

i) $6x^2+$

$$\begin{array}{r|l} & 3x+5 \\ 2x+1 & 6x^2+13x+5 \\ & \underline{\overset{(-)}{}6x^2\overset{(-)}{+}3x} \\ & +10x+5 \\ & \underline{\overset{(-)}{}10x\overset{(-)}{+}5} \\ & 0 \end{array}$$

Quotient $= 3x+5$, Remainder $= 0$

ii)

$$\begin{array}{r|l} & y^2-y-1 \\ 1+y & y^3+1 \\ & \underline{y^3+0+y^2} \\ & -y^2+1 \\ & \underline{-y^2+0-y} \\ & 1-y \\ & \underline{\overset{(+)}{-}1\overset{(+)}{-}y} \\ & 2 \end{array}$$

Quotient $= y^2-y-1$; Remainder $= 2$

iii)

$$
\begin{array}{r|l}
 & -2x+3 \\
\hline
x+1 & -2x^2+x+5 \\
 & -2x^2-2x \\
 & (+)\quad (+) \\
\hline
 & 3x+5 \\
 & +3x+3 \\
 & -\quad - \\
\hline
 & 2 \\
\hline
\end{array}
$$

Quotient $= -2x+3$, Remainder $= 2$

iv)

$$
\begin{array}{r|l}
 & x^2-4x+4 \\
\hline
x-2 & x^3-6x^2+12x-8 \\
 & x^3-2x^2 \\
 & (-)\quad (+) \\
\hline
 & -4x^2+12x-8 \\
 & -4x^2+8x \\
 & (+)\quad (-) \\
\hline
 & 4x-8 \\
 & 4x-8 \\
 & (-)\quad (+) \\
\hline
 & 0 \\
\hline
\end{array}
$$

Quotient $= x^2-4x+4$, Remainder $= 0$

4. i)

$$
\begin{array}{r|l}
 & 2x^2+5x+3 \\
\hline
3x-7 & 6x^3+x^2-26x-25 \\
 & 6x^3-14x^2 \\
 & (-)\quad (+) \\
\hline
 & 15x^2-26x-25 \\
 & 15x^2-35x \\
 & (-)\quad (+) \\
\hline
 & 9x-25 \\
 & 9x-21 \\
 & (-)\quad (+) \\
\hline
 & -4 \\
\hline
\end{array}
$$

Quotient $= 2x^2+5x+3$, Remainder $= -4$

ii)

$$
\begin{array}{r}
m^2 - 5m - 5 \\
m-1 \overline{\big) \, m^3 - 6m^2 + 7} \\
\underset{(-)}{m^3} \; \underset{(+)}{- m^2} \\
\hline
-5m^2 + 7 \\
\underset{(+)}{-5m^2} \underset{(-)}{+0} \underset{(-)}{+5m} \\
\hline
7 - 5m \\
\underset{(-)}{5} \underset{(+)}{-5m} \\
\hline
2 \\
\hline
\end{array}
$$

Quotient $= m^2 - 5m - 5$, Remainder $= 2$

5. i)

$$
\begin{array}{r}
a+1 \\
a^2+a+1 \overline{\big) \, a^3 + 2a^2 + 2a + 1} \\
\underset{(-)}{a^3} \underset{(-)}{+ a^2} \underset{(-)}{+ a} \\
\hline
a^2 + a + 1 \\
\underset{(-)}{a^2} \underset{(-)}{+ a} \underset{(-)}{+ 1} \\
\hline
0 \\
\hline
\end{array}
$$

Quotient $= a+1$, Remainder $= 0$

ii)

$$
\begin{array}{r}
4x - 3 \\
3x^2-2x+5 \overline{\big) \, 12x^3 - 17x^2 + 26x - 18} \\
\underset{(-)}{12x^3} \underset{(+)}{- 8x^2} \underset{(-)}{+ 20x} \\
\hline
-9x^2 + 6x - 18 \\
\underset{(+)}{-9x^2} \underset{(-)}{+ 6x} \underset{(+)}{- 15} \\
\hline
-3 \\
\hline
\end{array}
$$

Quotient $= 4x - 3$, Remainder $= -3$

6. A Given Area of rectangle $= 8x^2 - 45y^2 + 18xy$

One side $S_1 = 4x + 15y$

Other side $S_2 = ?$

$A = S_1 \times S_2$

$S_2 = A \div S_1$

$$
\begin{array}{r|l}
 & 2x - 3y \\
4x+15y & 8x^2 - 45y^2 + 18xy \\
 & 8x^2 + 0 \quad + 30xy \\
 & (-) \quad (-) \quad (-) \\
\hline
 & -45y^2 - 12xy \\
 & -45y^2 - 12xy \\
 & (+) \quad (+) \\
\hline
 & 0
\end{array}
$$

Quotient $= 2x - 3y$, Remainder $= 0$

$\therefore$ length of other side of rectangle

$S_2 = 2x - 3y$

Exercise 10.5

1\. i) $(3x+5)(3x+5)$

$$(3x+5)^2 = (3x)^2 + 2(3x).5 + 5^2 \quad (\because (a+b)^2 = a^2+2ab+b^2)$$

$$= 9x^2 + 30x + 25$$

ii) $(9y-5)(9y-5)$

$$(9y-5)^2 = (9y)^2 - 2(9y)5 + (+5)^2 \quad (\because (a-b)^2 = a^2-2ab+b^2)$$

$$= 81y^2 - 90y + 25$$

iii) $(4x+11y)(4x-11y)$

$$(4x)^2 - (11y)^2 \quad (\because (a+b)(a-b) = a^2-b^2)$$

$$16x^2 - 121y^2$$

iv) $\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right)$

$$\left(\frac{3}{2}m\right)^2 - \left(\frac{2}{3}n\right)^2 \quad (\because (a+b)(a-b) = a^2-b^2)$$

$$\frac{9}{4}m^2 - \frac{4}{9}n^2$$

v) $\left(\frac{2}{a} + \frac{5}{b}\right)\left(\frac{2}{a} + \frac{5}{b}\right)$

$$\left(\frac{2}{a} + \frac{5}{b}\right)^2 = \left(\frac{2}{a}\right)^2 + 2\cdot\frac{2}{a}\cdot\frac{5}{b} + \left(\frac{5}{b}\right)^2 \quad (\because (a+b)^2 = a^2+2ab+b^2)$$

$$= \frac{4}{a^2} + \frac{20}{ab} + \frac{25}{b^2}$$

vi) $\left(\frac{p^2}{2}+\frac{2}{q^2}\right)\left(\frac{p^2}{2}-\frac{2}{q^2}\right)$

$\left(\frac{p^2}{2}\right)^2-\left(\frac{2}{q^2}\right)^2$ $\left(\because (a+b)(a-b) = a^2-b^2\right)$

$\frac{p^4}{4}-\frac{4}{q^4}$

2.

i. $81^2 = (80+1)^2$

$= 80^2+2\cdot80\cdot1+1^2$ $\left(\because (a+b)^2 = a^2+2ab+b^2\right)$

$= 6400+160+1$

$81^2 = 6561$

ii. $97^2 = (100-3)^2$

$= (100)^2 - 2\cdot100\cdot3+3^2$ $\left(\because (a-b)^2 = a^2-2ab+b^2\right)$

$= 10000 - 600 + 9$

$= 9409$

iii) $105^2 = (100+5)^2$

$= 100^2+2\cdot100\cdot5+5^2$ $\left(\because (a+b)^2 = a^2+2ab+b^2\right)$

$= 10000+1000+25$

$= 11025$

iv) $997^2 = (1000-3)^2$

$= (1000)^2 - 2\cdot1000\cdot3+3^2$ $\left(\because (a-b)^2 = a^2-2ab+b^2\right)$

$= 1000000 - 6000 + 9$

$997^2 = 994009$

v) $6.1^2 = (6+0.1)^2$

$= 6^2 + 2 \cdot 6 (0.1) + 0.1^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$

$= 36 + 1.2 + 0.01$

$6.1^2 = 37.21$

vi) $496 \times 504 = (500-4)(500+4)$

$= (500)^2 - 4^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$

$= 250000 - 16$

$496 \times 504 = 249984$

vii) $20.5 \times 19.5 = (20+0.5)(20-0.5)$

$= 20^2 - 0.5^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$

$= 400 - 0.25$

$20.5 \times 19.5 = 399.75$

viii) $9.6^2 = (10-0.4)^2$

$= 10^2 - 2 \cdot 10 (0.4) + (0.4)^2 \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$

$= 100 - 8 + 0.16$

$9.6^2 = 92.16$

3.

i. $(pq+5r)^2 = (pq)^2 + 2\cdot pq\cdot 5r + (5r)^2 \quad (\because (a+b)^2 = a^2+b^2+2ab)$

$= p^2q^2 + 10pqr + 25r^2$

ii) $\left(\frac{5}{2}a - \frac{3}{5}\cdot b\right)^2 = \left(\frac{5}{2}\cdot a\right)^2 - 2\cdot \frac{5}{2}\cdot a\cdot \frac{3}{5}\cdot b + \left(\frac{3}{5}b\right)^2$

$(\because (a-b)^2 = a^2 - 2ab + b^2)$

$= \frac{25}{4}a^2 - 3ab + \frac{9}{25}b^2$

iii) $(\sqrt{2}\cdot a + \sqrt{3}\cdot b)^2 = (\sqrt{2}\cdot a)^2 + 2\sqrt{2}\cdot a\cdot \sqrt{3}\cdot b + (\sqrt{3}\cdot b)^2$

$(\because (a+b)^2 = a^2 + 2ab + b^2)$

$= 2a^2 + 2\sqrt{6}ab + 3b^2$

iv) $\left(\frac{2x}{3y} - \frac{3y}{2x}\right)^2 = \left(\frac{2x}{3y}\right)^2 - 2\cdot \frac{2x}{3y}\cdot \frac{3y}{2x} + \left(\frac{3y}{2x}\right)^2$

$= \frac{4x^2}{9y^2} - 2 + \frac{9y^2}{4x^2} \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$

4.

i. $(x+7)(x+3) = x^2 + (7+3)x + 7\times3 \quad (\because (x+a)(x+b) = x^2 + (a+b)x + ab)$

$= x^2 + 10x + 21$

ii. $(3x+4)(3x-5) = (3x)^2 + (4+(-5))(3x) + 4x-5$

$(\because (x+a)(x+b) = x^2 + (a+b)x + ab)$

$= 9x^2 - 3x - 20$

iii. $(p^2+2q)(p^2-3q) = (p^2)^2 + (2q+(-3q))p^2 + 2q\times-3q$

$(\because (x+a)(x+b) = x^2 + (a+b)x + ab)$

$= p^4 - p^2q^2 - 6q^2$

$= p^4 - p^2q - 6q^2$

iv. $(abc+3)(abc-5) = (abc)^2 + (3+(-5)).abc + 3x-5$

$(\because (x+a)(x+b) = x^2 + (a+b).x + ab)$

$= (abc)^2 - 2abc - 15$

5.

i. $203\times204 = (200+3)(200+4)$

$= (200)^2 + (3+4)200 + 3\times4 \quad (\because (x+a)(x+b) = x^2 + (a+b)x + ab)$

$= 40000 + 1400 + 12$

$= 41412$

ii. $8.2\times8.7 = (8+0.2)(8+0.7)$

$= 8^2 + (0.2+0.7)8 + 0.2\times0.7 \quad (\because (x+a)(x+b) = x^2 + (a+b).x + ab)$

$= 64 + 7.2 + 0.14$

$= 71.34$

iii. $107 \times 93 = (100+7)(100-7)$

$= (100)^2 + (7+(-7)) \cdot 100 + 7 \times -7$

$(\because (x+a)(x+b) = x^2 + (a+b) \cdot x + ab)$

$= 10000 + 0 \cdot 100 - 49$

$= 9951$

6.

i. $53^2 - 47^2 = (53+47)(53-47)$ $(\because a^2 - b^2 = (a+b)(a-b))$

$= (100)(6)$

$= 600$

ii. $(2.05)^2 - (0.95)^2 = (2.05+0.95)(2.05-0.95)$

$= 3 \times 0.1$

$= 0.3$

iii. $(14.3)^2 - (5.7)^2 = (14.3+4.7)(14.3-4.7)$

$= (19)(9.6)$

$= 182.4$

7.

i. $(2x+5y)^2 + (2x-5y)^2$

$(2x)^2 + (5y)^2 + 2 \cdot 2x \cdot 5y + (2x)^2 + (5y)^2 - 2 \cdot 2x \cdot 5y$

$2(2x)^2 + 2(5y)^2$ $(\because (a+b)^2 = a^2 + b^2 + 2ab$

$(a-b)^2 = a^2 + b^2 - 2ab$

$2 \cdot [4x^2 + 25y^2]$

$\underline{\underline{8x^2 + 50y^2}}$

ii) $\left(\frac{7}{2}a-\frac{5}{2}b\right)^2-\left(\frac{5}{2}a-\frac{7}{2}b\right)^2$ III

$\left(\frac{7}{2}a\right)^2+\left(\frac{5}{2}b\right)^2-2\cdot\frac{7}{2}a\cdot\frac{5}{2}\cdot b-\left[\left(\frac{5}{2}a\right)^2+\left(\frac{7}{2}b\right)^2-2\cdot\frac{5}{2}\cdot a\cdot\frac{7}{2}b\right]$

$\frac{49}{4}a^2+\frac{25}{4}b^2-\cancel{2\cdot\frac{7}{2}\cdot a\cdot\frac{5}{2}\cdot b}-\frac{25}{4}a^2-\frac{49}{4}b^2+\cancel{2\cdot\frac{5}{2}\cdot a\cdot\frac{7}{2}\cdot b}$

$\left(\frac{49}{4}-\frac{25}{4}\right)a^2+\left(\frac{25}{4}-\frac{49}{4}\right)b^2$

$\frac{24}{4}\cdot a^2-\frac{24}{4}\cdot b^2$

$6(a^2-b^2)$

iii) $(p^2-q^2r)^2+2p^2q^2r$

$(p^2)^2-2\cdot p^2\cdot q^2\cdot r+(q^2r)^2+2p^2q^2\cdot r$

$p^4-\cancel{2p^2q^2\cdot r}+q^4\cdot r^2+\cancel{2p^2q^2\cdot r}$ $\left(\because (a+b)^2=a^2+2ab+b^2\right.$
$\left.(a-b)^2=a^2-2ab+b^2\right)$

$\underline{\underline{p^4+q^4r^2}}$

8. LHS

i. $(4x+7y)^2-(4x-7y)^2$

$(4x)^2+(7y)^2+2\cdot4x\cdot7y-\left[(4x)^2+(7y)^2-2\cdot4x\cdot7y\right]$

$(\because (a-b)^2=a^2+b^2-2ab)$

$\cancel{(4x)^2}+\cancel{(7y)^2}+2\cdot4x\cdot7y-\cancel{(4x)^2}-\cancel{(7y)^2}+2\cdot4x\cdot7y$

$4\cdot4x\cdot7y$

$\underline{\underline{112xy}}=R.H.S$

ii. $\left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq$ $\boxed{IV}$

$$\left(\frac{3}{7}p\right)^2 + \left(\frac{7}{6}q\right)^2 - 2\cdot\frac{3}{7}p\cdot\frac{7}{6}\cdot q + pq$$

$$\left(\because (a-b)^2 = a^2 + b^2 - 2ab\right)$$

$$\frac{9}{49}p^2 + \frac{49}{36}q^2 - \cancel{pq} + \cancel{pq}$$

$$\frac{9}{49}\cdot p^2 + \frac{49}{36}q^2 = R.H.S$$

$$\therefore L.H.S = R.H.S$$

iii. $L.H.S = (p-q)(p+q) + (q-r)(q+r) + (r-p)(r+p)$

$$= \cancel{p(p+q) - q(p+q) + q(q+r)}$$

$$= p^2 - q^2 + q^2 - r^2 + r^2 - p^2$$

$$\left(\because (a+b)(a-b) = a^2 - b^2\right)$$

$$= 0 = R.H.S$$

$$\therefore LHS = RHS$$

9. given $\left(x+\frac{1}{x}\right) = 2$

Squaring on both sides

i) $\left(x+\frac{1}{x}\right)^2 = 2^2$

$x^2+2\cdot x\cdot\frac{1}{x}+\left(\frac{1}{x}\right)^2 = 4 \left(\because (a+b)^2 = a^2+b^2+2ab\right)$

$x^2+2+\frac{1}{x^2} = 4$

$x^2+\frac{1}{x^2} = 4-2$

$x^2+\frac{1}{x^2} = 2$

ii) Again Squaring on both sides

$\left(x^2+\frac{1}{x^2}\right)^2 = 2^2$

$(x^2)^2+2\cdot x^2\cdot\frac{1}{x^2}+\left(\frac{1}{x^2}\right)^2 = 4$

$x^4+2+\frac{1}{x^4} = 4$

$x^4+\frac{1}{x^4} = 4-2$

$x^4+\frac{1}{x^4} = 2$

10. $x - \frac{1}{x} = 7$ VI

i) Squaring on both sides

$$\left(x-\frac{1}{x}\right)^2 = 7^2$$

$$x^2 - 2\cdot x\cdot\frac{1}{x} + \left(\frac{1}{x}\right)^2 = 49 \quad (\because (a-b)^2 = a^2-2ab+b^2)$$

$$x^2 - 2 + \frac{1}{x^2} = 49$$

$$x^2 + \frac{1}{x^2} = 49+2$$

$$x^2 + \frac{1}{x^2} = 51$$

ii) $x^2 + \frac{1}{x^2} = 51$

Squaring on both sides

$$\left(x^2+\frac{1}{x^2}\right)^2 = 51^2$$

$$(x^2)^2 + 2\cdot x^2\cdot\frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 2601$$

$$x^4 + 2 + \frac{1}{x^4} = 2601$$

$$x^4 + \frac{1}{x^4} = 2601-2$$

$$x^4 + \frac{1}{x^4} = 2599$$

11. $x^2+\frac{1}{x^2}=23$

i. $x^2+\frac{1}{x^2}=23$

Adding '2' on both sides

$$x^2+\frac{1}{x^2}+2=23+2$$

$x+\frac{1}{x})$

$$(x)^2+\left(\frac{1}{x}\right)^2+2\cdot x\cdot\frac{1}{x}=25 \quad \left(\because a^2+b^2+2ab=(a+b)^2\right)$$

$$\left(x+\frac{1}{x}\right)^2=25$$

$$x+\frac{1}{x}=5$$

ii. $x^2+\frac{1}{x^2}=23$

Substract '2' on both sides

$$x^2+\frac{1}{x^2}-2=23-2$$

$$(x)^2+\left(\frac{1}{x}\right)^2-2\cdot x\cdot\frac{1}{x}=21$$

$$\left(x-\frac{1}{x}\right)^2=21 \quad \left(\because a^2+b^2-2ab=(a-b)^2\right)$$

$$x-\frac{1}{x}=\sqrt{21}$$

$$x-\frac{1}{x}=3\sqrt{3}$$

12. given $a+b=9$, $ab=10$ VI

Squaring on both sides

$(a+b)^2 = 9^2$

$a^2+b^2+2ab = 81$

$a^2+b^2+2\times10 = 81$ ($\because$ given $ab=10$)

$a^2+b^2+20=81$

$\underline{\underline{a^2+b^2=61}}$

13. given $a-b=6$, $a^2+b^2=42$

$a-b=6$

Squaring on both sides

$(a-b)^2 = 6^2$

$a^2+b^2-2ab = 36$

$42-2ab=36$ ($\because$ $a^2+b^2=42$)

$42-36=2ab$

$2ab=6$

$\boxed{ab=3}$

14. given $a^2+b^2=41$, $ab=4$

i) Consider $(a+b)^2 = a^2+b^2+2ab$

$(a+b)^2 = 41+2\times4 = 41+8$

$(a+b)^2 = 49$

$a+b = 7$

ii) Consider $(a-b)^2 = a^2+b^2-2ab$

$= 41-2\times4 = 41-8 = 33$.

$(a-b)^2 = 33$

$a-b = \sqrt{33}$