Chapter 17. Trigonometric Ratios

TRIGONOMETRIC RATIOS

EXERCISE - 16

1

(a) Sol:

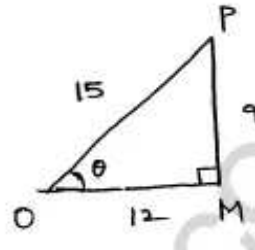By pythagorus theorem

$OP^2 = OM^2 + PM^2$

$15^2 = 12^2 + PM^2$

$225 = 144 + PM^2$

$PM^2 = 225 - 144$

$PM^2 = 81$

$PM = \sqrt{81}$

$PM = 9$

(i) Sol: $\sin\theta = \dfrac{PM}{OP}$

$= \dfrac{9}{15}$

$\sin\theta = \dfrac{3}{5}$

(ii) $\cos\theta = \dfrac{OM}{OP}$

$\cos\theta = \dfrac{12}{15}$

$\cos\theta = \dfrac{4}{5}$

(iii) $\tan\theta = \dfrac{PM}{OM}$

$= \dfrac{9}{12}$

$\tan\theta = \dfrac{3}{4}$

(iv) $\cot\theta = \dfrac{OM}{PM}$

$= \dfrac{12}{9}$

$\cot\theta = \dfrac{4}{3}$

(v) $\sec\theta = \dfrac{OP}{OM}$

$= \dfrac{15}{12}$

$\sec\theta = \dfrac{5}{4}$

(vi) $\csc\theta = \dfrac{OP}{PM}$

$= \dfrac{15}{9}$

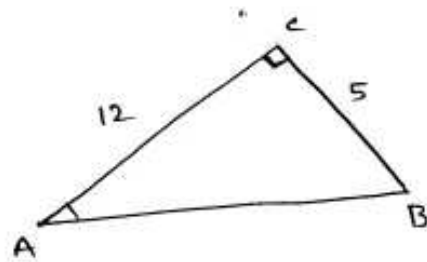$\csc\theta = \dfrac{5}{3}$

(b)

Sol

By Pythagorus theorem

$AB^2 = AC^2 + BC^2$

$= 12^2 + 5^2$

$= 144 + 25$

$AB^2 = 169$

$AB = \sqrt{169}$

$AB = 13$

(i) $\sin A = \dfrac{BC}{AB}$

$\sin A = \dfrac{5}{13}$

(ii) $\cos A = \dfrac{AC}{AB}$

$\cos A = \dfrac{12}{13}$

(iii) $\sin^2 A + \cos^2 A = \left(\dfrac{5}{13}\right)^2 + \left(\dfrac{12}{13}\right)^2$

$= \dfrac{25}{169} + \dfrac{144}{169}$

$= \dfrac{25 + 144}{169}$

$= \dfrac{169}{169}$

$= 1$

$\therefore \sin^2 A + \cos^2 A = 1$

(iv) $\sec^2 A - \tan^2 A$

$\because \sec A = \dfrac{1}{\cos A}$

$= \dfrac{1}{\dfrac{12}{13}}$

$\sec A = \dfrac{13}{12}$

$\because \tan A = \dfrac{\sin A}{\cos A}$

$= \dfrac{\left(\dfrac{5}{13}\right)}{\left(\dfrac{12}{13}\right)}$

$$TanA = \frac{5}{12}$$

$$\therefore Sec^2A - Tan^2A = \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2$$

$$= \frac{169}{144} - \frac{25}{144}$$

$$= \frac{169-25}{144}$$

$$= \frac{144}{144}$$

$$= 1.$$

$$\therefore Sec^2A - Tan^2A = 1.$$

$\frac{2}{Sol}$

(a)

By phytagorus theorem

$\because$ hypotenuse = BC

$$\therefore BC^2 = AB^2 + AC^2$$
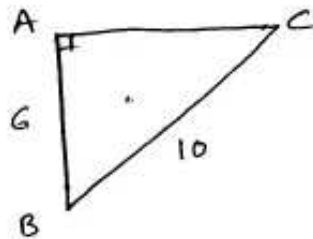
$$10^2 = 6^2 + AC^2$$

$$100 = 36 + AC^2$$

$$AC^2 = 100 - 36$$

$$AC^2 = 64$$

$$AC = \sqrt{64}$$

$$AC = 8.$$

(i) $\quad \sin B = \dfrac{AC}{BC}$

$\qquad\qquad = \dfrac{8}{10}$

$\quad \sin B = \dfrac{4}{5}$

(ii) $\quad \cos C = \dfrac{AC}{BC}$

$\qquad\qquad = \dfrac{8}{10}$

$\quad \cos C = \dfrac{4}{5}$

(iii) $\quad \sin B + \sin C$

$\quad \sin C = \dfrac{AB}{BC}$

$\qquad\qquad = \dfrac{6}{10}$

$\qquad\qquad = \dfrac{3}{5}$

$\therefore \quad \sin B = \dfrac{4}{5}$

$\therefore \quad \sin B + \sin C = \dfrac{4}{5} + \dfrac{3}{5}$

$\qquad\qquad\qquad = \dfrac{4+3}{5}$

$\quad \sin B + \sin C = \dfrac{7}{5}$

(iv) $\sin B \cdot \cos C + \sin C \cdot \cos B$

$\quad \cos B = \dfrac{AB}{BC} = \dfrac{6}{10}$

$\qquad\qquad\qquad = \dfrac{3}{5}$

$\therefore \sin B \cdot \cos C + \sin C \cdot \cos B$

$\Rightarrow \frac{4}{5} \cdot \frac{4}{5} + \frac{3}{5} \cdot \frac{3}{5}$

$\Rightarrow \frac{16}{25} + \frac{9}{25}$

$\Rightarrow \frac{16+9}{25}$

$\Rightarrow \frac{25}{25}$

$\Rightarrow 1$

(b) From Figure.

ΔADC

$\Rightarrow AD^2 + CD^2 = AC^2$

$AD^2 + 5^2 = 13^2$

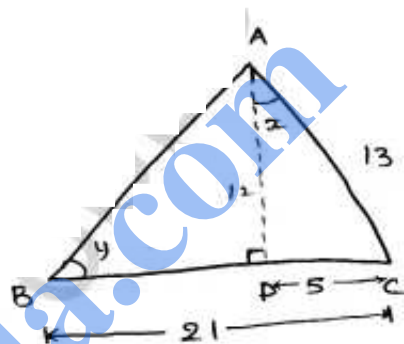$AD^2 + 25 = 169$

$AD^2 = 169 - 25$

$AD^2 = 144$

$AD = \sqrt{144}$

$AD = 12$

From Figure

ΔABD

$BD = BC - CD$

$= 21 - 5$

$BD = 16$

$\therefore AB^2 = BD^2 + AD^2$

$AB^2 = 16^2 + 12^2$

$$AB^2 = 256 + 144$$

$$AB^2 = 400$$

$$AB = \sqrt{400}$$

$$AB = 20.$$

$$\therefore \quad AB = 20 \;,\; AD = 12 \;;$$
$$BD = 16$$
$$AC = 13$$
$$CD = 5$$

(i) $\quad Tan\,x = \dfrac{CD}{AD}$

$\quad\quad Tan\,x = \dfrac{5}{12}$

(ii) $\quad cos\,y = \dfrac{BD}{AB}$

$$= \dfrac{16}{20}$$

$\quad\quad cos\,y = \dfrac{4}{5}$

(iii) $\quad cosec^2y - cot^2y$

$\rightarrow \quad cosec\,y = \dfrac{AB}{AD}$

$$= \dfrac{20}{12}$$

$\quad cosec\,y = \dfrac{5}{3}$

$\Rightarrow \quad cot\,y = \dfrac{BD}{AD}$

$$= \dfrac{16}{12}$$

$$\cot y = \frac{4}{3}$$

$$\therefore \ \csc^2 y - \cot^2 y = \left(\frac{5}{3}\right)^2 - \left(\frac{4}{3}\right)^2$$

$$= \frac{25}{9} - \frac{16}{9}$$

$$= \frac{25-16}{9}$$

$$= \frac{9}{9}$$

$$\therefore \ \csc^2 y - \cot^2 y = 1$$

③

sol:

ⓐ From figure

BD = BC − CD

BD = 21 − 5

BD = 16

From $\triangle ADc$
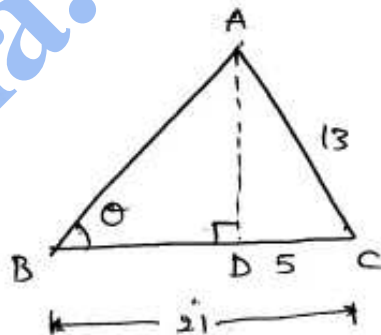
$$AD^2 + DC^2 = AC^2$$

$$AD^2 + 5^2 = 13^2$$

$$AD^2 + 25 = 169$$

$$AD^2 = 169 - 25$$

$$AD^2 = 144$$

$$AD = \sqrt{144}$$

$$AD = 12$$

$$\therefore \ Sec\theta = \frac{AB}{BD}$$

from fig $\triangle ABD$

$$AB^2 = BD^2 + AD^2$$

$$= 16^2 + 12^2$$

$$= 256 + 144$$

$$= 400$$

$$AB^2 = 400$$

$$AB = \sqrt{400}$$

$$AB = 20$$

$$\therefore \ Sec\theta = \frac{20}{16}$$

$$= \frac{5}{4}$$

⑥

Sol:

From figure

$\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$= 3^2 + 4^2$$

$$= 9 + 16$$
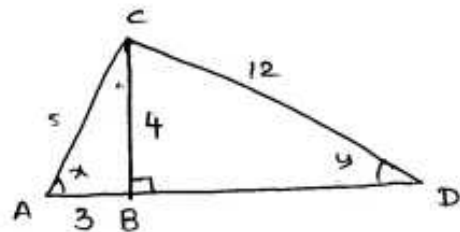
$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$AC = 5$$

From figure

$\triangle BCD$

$$CD^2 = BC^2 + BD^2$$

$$12^2 = 4^2 + BD^2$$

$$144 = 16 + BD^2$$

$$BD^2 = 144 - 16$$

$$= 12\ 8$$

$$BD^2 = 128$$

$$BD = \sqrt{128}$$

$$= \sqrt{16 \times 4 \times 2}$$

$$= 4 \times 2\sqrt{2}$$

$$BD = 8\sqrt{2}$$

$\therefore$

(i) $\quad \sin x = \dfrac{BC}{AC}$

$$\sin x = \dfrac{4}{5}$$

(ii) $\quad \cot x = \dfrac{AB}{BC}$

$$= \dfrac{3}{4}$$

(iii) $\quad \cot^2 x - \text{cosec}^2 x$

$$\because \cot x = \dfrac{3}{4}$$

$$\text{cosec } x = \dfrac{AC}{BC}$$

$$= \dfrac{5}{4}$$

$$\therefore \cot^2 x - \text{cosec}^2 x = \left(\dfrac{3}{4}\right)^2 - \left(\dfrac{5}{4}\right)^2$$

$$= \dfrac{9}{16} - \dfrac{25}{4}$$

$$\therefore \cot^2 x - \csc^2 x = \frac{9-25}{16}$$

$$= \frac{-16}{16}$$

$$= -1$$

(iv) $\sec y = \dfrac{CD}{BD}$

$$= \frac{12}{8\sqrt{2}}$$

$$\sec y = \frac{3}{2\sqrt{2}}$$

(v) $\tan^2 y - \dfrac{1}{\cos^2 y}$

$\because \sec^2 y = \dfrac{1}{\cos^2 y}$

$$= \left(\frac{3}{2\sqrt{2}}\right)^2$$

$$= \frac{9}{4 \times 2}$$

$$\frac{1}{\cos^2 y} = \frac{9}{8}$$

$\therefore \tan^2 y = \left(\dfrac{BC}{BD}\right)^2$

$$= \left(\frac{4}{8\sqrt{2}}\right)^2$$

$$= \frac{16}{64 \times 2}$$

$$\text{Tan}^2 y = \frac{1}{8}$$

$$\therefore \tan^2 y - \frac{1}{\cos^2 y} = \frac{1}{8} - \frac{9}{8}$$

$$= \frac{1-9}{8}$$

$$= \frac{-8}{8}$$

$$= -1$$

④

So.

ⓐ From figure

△ BCD

⇒ $BC^2 = BD^2 + CD^2$
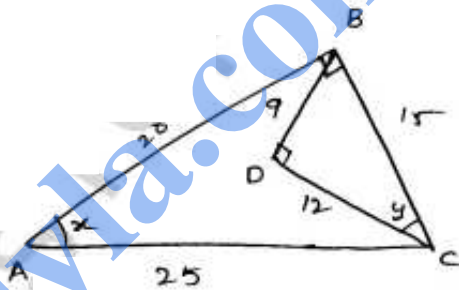
$BC^2 = 9^2 + 12^2$

$= 81 + 144$

$BC^2 = 225$

$BC = \sqrt{225}$

$BC = 15.$

△ABC ⇒ $AB^2 + BC^2 = AC^2$

$AB^2 + 15^2 = 25^2$

$AB^2 = 625 - 225$

$AB^2 = 400$

$AB = \sqrt{400} = 20.$

(i) $2\sin y - \cos y$

∵ $\sin y = \dfrac{BD}{BC}$

$= \dfrac{9}{15}$

∴ $\sin y = \dfrac{3}{5}$

$\cos y = \dfrac{CD}{BC}$

$= \dfrac{12}{15}$

∴ $\cos y = \dfrac{4}{5}$

∴ $2\sin y - \cos y = 2 \cdot \dfrac{3}{5} - \dfrac{4}{5}$

$= \dfrac{6}{5} - \dfrac{4}{5}$

$= \dfrac{6-4}{5}$

$= \dfrac{2}{5}$

(ii) $2\sin x - \cos x$

$\sin x = \dfrac{BC}{AC} = \dfrac{15}{25} = \dfrac{3}{5}$

$\cos x = \dfrac{AB}{AC} = \dfrac{20}{25} = \dfrac{4}{5}$

∴ $2\sin x - \cos x = 2 \cdot \dfrac{3}{5} - \dfrac{4}{5}$

$= \dfrac{6}{5} - \dfrac{4}{5}$

$= \dfrac{6-4}{5} = \dfrac{2}{5}$

(iii)   1- sinx + cosy

∵ sinx = $\frac{3}{5}$

cosy = $\frac{4}{5}$

∴ 1- sinx + cosy = $1 - \frac{3}{5} + \frac{4}{5}$

$= \frac{5-3+4}{5}$

$= \frac{6}{5}$

(iv)   2cosx - 3siny + 4tanx

∵ sinx = $\frac{3}{5}$

cosx = $\frac{4}{5}$

siny = $\frac{3}{5}$

∴ Tanx = $\frac{sinx}{cosx}$

$= \frac{\frac{3}{5}}{\frac{4}{5}}$

Tanx = $\frac{3}{4}$

∴ 2cosx - 3siny + 4tanx

→ $2 \cdot \frac{4}{5} - 3 \cdot \frac{3}{5} + 4 \cdot \frac{3}{4}$

⇒ $\frac{8}{5} - \frac{9}{5} + 3$

→ $\frac{8-9+15}{5}$

⇒ $\frac{14}{5}$

(b)

Sol:

(ii) By pythagorus theorem

$$AC^2 = BC^2 + AB^2$$

$$5^2 = 3^2 + AB^2$$

$$25 = 9 + AB^2$$
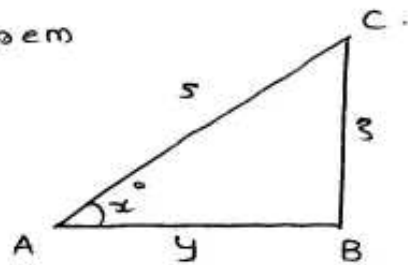
$$AB^2 = 25 - 9$$

$$AB^2 = 16$$
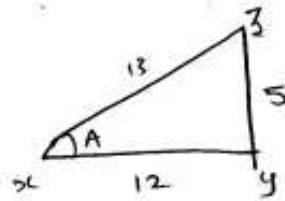
$$AB = \sqrt{16}$$

$$AB = 4$$

$$\therefore AB = y = 4.$$

(i)

$$\sin x° = \frac{BC}{AC}$$

$$\sin x° = \frac{3}{5}$$

5

Sol:

Given $\tan A = \frac{yz}{xy} = \frac{5}{12}$

By pythagorus theorem

$$xy^2 + yz^2 = xz^2$$

$$12^2 + 5^2 = xz^2$$

$$\therefore 144 + 25 = xz^2$$

$$xz^2 = 169$$

$$xz = 13.$$

(i) $\cos A = \dfrac{xy}{x3}$

$= \dfrac{12}{13}$

(ii) $\operatorname{cosec} A - \cot A$

$\operatorname{cosec} A = \dfrac{x3}{y2}$

$= \dfrac{13}{5}$

$\cot A = \dfrac{xy}{y3}$

$= \dfrac{12}{5}$

$\therefore \operatorname{cosec} A - \cot A = \dfrac{13}{5} - \dfrac{12}{5}$

$= \dfrac{13-12}{5}$

$= \dfrac{1}{5}$

$\dfrac{6}{\text{sol:}}$

(a) Given $AB = 7$

$BC - AC = 1$

By pythagorus theorem
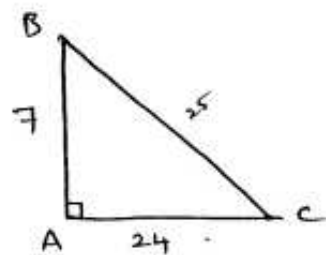
$BC^2 = AB^2 + AC^2$

$\therefore BC = 1 + AC$

$\therefore (1+AC)^2 = 7^2 + AC^2$

$1 + AC^2 + 2AC = 49 + AC^2$

$2AC = 49 - 1$

$$2AC = 48$$

$$AC = 24$$

(i) $\quad sinC = \dfrac{AB}{BC}$

$\because$ from given $\quad BC - AC = 1$

$$BC - 24 = 1$$

$$BC = 1 + 24$$

$$BC = 25$$

$$sinC = \dfrac{7}{25}$$
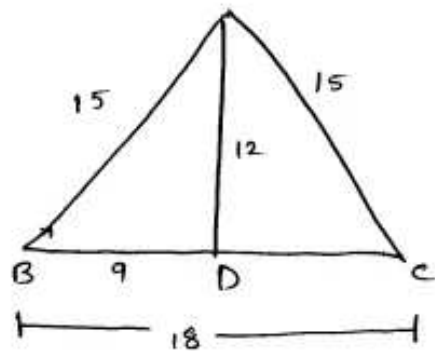
(ii) $\quad TanB = \dfrac{AC}{AB}$

$$= \dfrac{24}{7}$$

7 sol:

(i) $\quad Cos\angle ABC = \dfrac{BD}{BA}$

$$= \dfrac{\cancel{9}^{3}}{\cancel{15}_{5}}$$

$$= \dfrac{3}{5}$$

(li) $\sin \angle ACB = \dfrac{AD}{AC}$

$\qquad\qquad = \dfrac{12}{15}$

$\qquad\qquad = \dfrac{4}{5}$

⑥
ⓑ
sol

Given $PQ = 40$

$\qquad PR + QR = 50$

By pythagorous theorem

$PR^2 = PQ^2 + QR^2$

$(50 - QR)^2 = PQ^2 + QR^2$

$2500 + QR^2 - 100QR = 40^2 + QR^2$

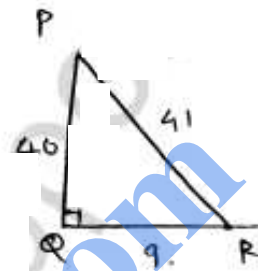$2500 - 1600 = 100 QR$

$100 QR = 900$

$QR = \dfrac{900}{100}$

$QR = 9.$

Given $\qquad PR + QR = 50$

$\qquad\qquad PR + 9 = 50$

$\qquad\qquad PR = 50 - 9$

$\qquad\qquad PR = 41$

(i) $\sin P = \dfrac{QR}{PR}$

$= \dfrac{9}{41}$

(ii) $\cos P = \dfrac{PQ}{PR}$

$= \dfrac{40}{41}$

(iii) $\tan R = \dfrac{\sin R}{\cos R}$

$\because \sin R = \dfrac{PQ}{PR}$

$= \dfrac{40}{41}$

$\cos R = \dfrac{QR}{PR}$

$= \dfrac{9}{41}$

$\therefore \tan R = \dfrac{\frac{40}{41}}{\frac{9}{41}}$

$= \dfrac{40}{9}$

$\therefore \tan R = \dfrac{40}{9}$.
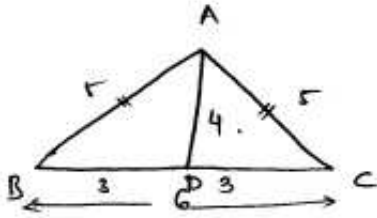
Given AB = AC = 5 cm

BC = 6

(i) $\sin c = \dfrac{AD}{AC}$

$= \dfrac{4}{5}$

(ii) $\tan B = \dfrac{AD}{BD}$

$= \dfrac{4}{3}$

(iii) $\tan c - \cot B$

∵ $\tan c = \dfrac{AD}{DC} = \dfrac{4}{3}$

$\cot B = \dfrac{BD}{AD} = \dfrac{3}{4}$

∴⟹ $\tan c - \cot B = \dfrac{4}{3} - \dfrac{3}{4}$

$= \dfrac{16-9}{12}$

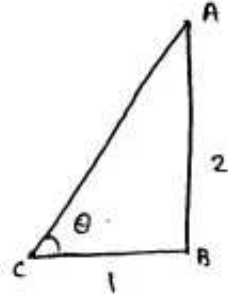$= \dfrac{7}{12}$

(b) Given AB = 2
            BC = 1

∴ $\sin\theta = \dfrac{AB}{AC}$

$\tan\theta = \dfrac{AB}{BC}$

∴ By pythagorous theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 2^2 + 1^2$

$AC^2 = 4 + 1$

$AC^2 = 5$

$AC = \sqrt{5}$

∴ $\sin^2\theta = \left(\dfrac{AB}{AC}\right)^2 = \left(\dfrac{2}{\sqrt{5}}\right)^2 = \dfrac{4}{5}$

$\tan^2\theta = \left(\dfrac{AB}{BC}\right)^2 = \left(\dfrac{2}{1}\right)^2 = 4$

∴ $\sin^2\theta + \tan^2\theta = \dfrac{4}{5} + 4$

$= \dfrac{4+20}{5}$

$= \dfrac{24}{5}$

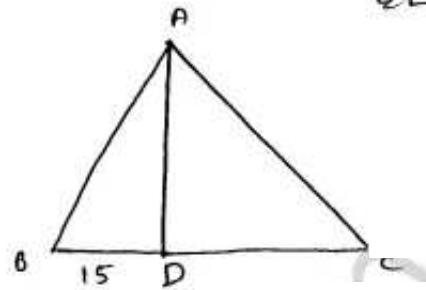$\sin^2\theta + \tan^2\theta = 4\dfrac{4}{5}$

(c)

Given BD = 15

$\sin B = \dfrac{4}{5}$

$\tan C = 1$

∵ $\sin B = \dfrac{4}{5} \times \dfrac{5}{5}$

$\sin B = \dfrac{AD}{AB} = \dfrac{20}{25}$

(i) AD = 20

∵ $\tan C \Rightarrow \dfrac{\sin C}{\cos C} = 1$

$\Rightarrow \dfrac{AD}{DC} = 1$

∴ AD = DC

∴ DC = 20

In ΔACD ; By Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$= 20^2 + 20^2$

$= 400 + 400$

$AC^2 = 800$

$AC = \sqrt{800}$

$= \sqrt{400 \times 2}$

$AC = 20\sqrt{2}$

(ii) $\tan^2 B - \dfrac{1}{\cos^2 B} = -1$.

$\therefore$ LHS $\Rightarrow$ $\tan^2 B - \dfrac{1}{\cos^2 B}$

$\tan^2 B = \left(\dfrac{AD}{BD}\right)^2 = \left(\dfrac{20}{15}\right)^2 = \left(\dfrac{4}{3}\right)^2$

$= \dfrac{16}{9}$

$\cos^2 B = \left(\dfrac{BD}{AB}\right)^2 = \left(\dfrac{15}{25}\right)^2 = \left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$

$\therefore \tan^2 B - \dfrac{1}{\cos^2 B} \Rightarrow \dfrac{16}{9} - \dfrac{1}{\left(\dfrac{9}{25}\right)}$

$\Rightarrow \dfrac{16}{9} - \dfrac{25}{9}$

$\Rightarrow \dfrac{16-25}{9}$

$\Rightarrow -1$.

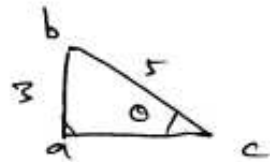$\therefore$ LHS = RHS

Hence proved.

$\dfrac{9}{81}$ ①

Given
$\sin\theta = \dfrac{3}{5}$

$\therefore$ From pythagorus theorem

$bc^2 = ab^2 + ac^2$

$5^2 = 3^2 + ac^2$

$25 = 9 + ac^2$

$$ac^2 = 25 - 9$$

$$ac^2 = 16$$

$$ac = \sqrt{16}$$

$$ac = 4$$

$$\therefore \cos\theta = \frac{ac}{bc}$$

$$= \frac{4}{5}$$

(ii)

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{3}{8}}{\frac{4}{8}}$$

$$= \frac{3}{4}$$

⑩ Sol: Given that $\tan\theta = \frac{5}{12}$

By pythagorus theorem
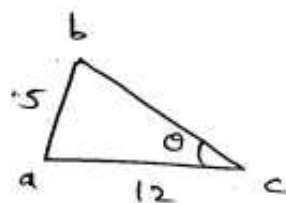

$$bc^2 = ab^2 + ac^2$$

$$= 5^2 + 12^2$$

$$= 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$\therefore bc = 13.$$

$\sin\theta = \dfrac{ab}{bc}$

$= \dfrac{5}{13}$

$\cos\theta = \dfrac{ac}{bc}$

$= \dfrac{12}{13}$

⑪

Sol:

Given $\sin\theta = \dfrac{6}{10}$

by pythagorus theorem

$bc^2 = ab^2 + ac^2$

$10^2 = 6^2 + ac^2$

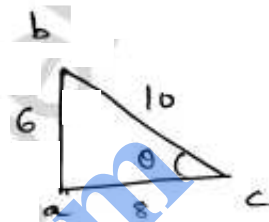$100 = 36 + ac^2$

$ac^2 = 100 - 36$

$ac^2 = 64$

$ac = \sqrt{64}$

$ac = 8$

$\therefore \cos\theta = \dfrac{ac}{bc} = \dfrac{8}{10}$

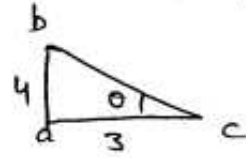$\tan\theta = \dfrac{ab}{ac} = \dfrac{6}{8}$

$\therefore \cos\theta + \tan\theta = \dfrac{8}{10} + \dfrac{6}{8} = \dfrac{64 + 60}{80}$

$= \dfrac{124}{80} \Rightarrow \dfrac{31}{20} = 1\dfrac{11}{20}$

(12)

Sol.

Given $\tan\theta = \dfrac{4}{3}$

by pythagorous theorem

$$ab^2 + ac^2 = bc^2$$

$$4^2 + 3^2 = bc^2$$

$$16 + 9 = bc^2$$

$$bc^2 = 25$$

$$bc = \sqrt{25}$$

$$bc = 5.$$

$$\therefore \sin\theta = \dfrac{ab}{bc} = \dfrac{4}{5}$$

$$\cos\theta = \dfrac{ac}{bc} = \dfrac{3}{5}$$

$$\therefore \sin\theta + \cos\theta = \dfrac{4}{5} + \dfrac{3}{5}$$

$$= \dfrac{4+3}{5}$$
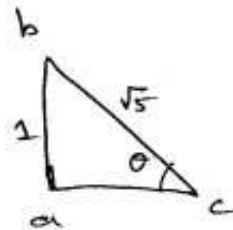
$$= \dfrac{7}{5}$$

(13)

Sol.

Given $\csc\theta = \sqrt{5}$

$$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{bc}{ab} = \dfrac{\sqrt{5}}{1}.$$

$$\therefore bc = \sqrt{5} \; ; \; ab = 1$$

By pythagorus theorem ;

$$bc^2 = ab^2 + ac^2$$

$$(\sqrt{5})^2 = 1^2 + ac^2$$

$$5 = 1 + ac^2$$

$$ac^2 = 5 - 1$$

$$ac^2 = 4$$

$$ac = \sqrt{4}$$

$$ac = 2.$$

$$\therefore \cot\theta - \cos\theta = \frac{ac}{ab} - \frac{ac}{bc}$$

$$= \frac{2}{1} - \frac{2}{\sqrt{5}}$$

$$= \frac{2\sqrt{5} - 2}{\sqrt{5}}$$

$$= \frac{2(\sqrt{5} - 1)}{\sqrt{5}}$$

Solution - 14 :-

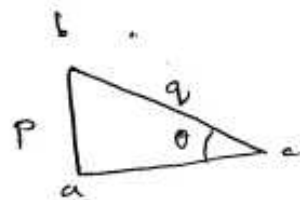Given $\sin\theta = \dfrac{p}{q}$

by pythagorus theorem

$$q^2 = p^2 + ac^2$$

$$ac^2 = q^2 - p^2$$

$$ac = \sqrt{q^2 - p^2}$$

$$\cos\theta = \frac{ac}{bc} = \frac{\sqrt{-p^2 + q^2}}{q}$$

$$\therefore \cos\theta + \sin\theta = \frac{\sqrt{q^2 - p^2}}{q} + \frac{p}{q} \quad = \frac{p + \sqrt{q^2 - p^2}}{q}.$$

Solution-15

Given $\tan\theta = \dfrac{8}{15}$

by pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = 8^2 + 15^2$$

$$bc^2 = 64 + 225$$

$$bc^2 = 289$$

$$bc = \sqrt{289}$$

$$bc = 17$$

$\therefore ab = 8 ; ac = 15 ; bc = 17$

$$\sec\theta = \frac{bc}{ac} = \frac{17}{15}$$
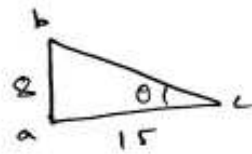
$$\csc\theta = \frac{bc}{ab} = \frac{17}{8}$$

$\therefore \sec\theta + \csc\theta = \dfrac{17}{15} + \dfrac{17}{8}$

$$= \frac{17 \times 8 + 17 \times 15}{120}$$

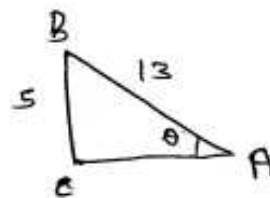$$= \frac{391}{120}$$

$$= 3\frac{31}{120}$$

Solution-16 :

Given $13 \sin A = 5$

$$\sin A = \frac{5}{13}$$

by pythagorus theorem

$$BA^2 = CB^2 + AC^2$$

$$\rightarrow 13^2 = 5^2 + AC^2$$

$$169 = 25 + AC^2$$

$$AC^2 = 169 - 25$$

$$AC^2 = 144$$

$$AC = \sqrt{144}$$

$$AC = 12$$

$$\therefore \quad CB = 5 \; ; \; BA = 13 \; ; \; AC = 12$$

$$\sin A = \frac{5}{13}$$

$$\cos A = \frac{12}{13}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{\left(\frac{5}{13}\right)}{\left(\frac{12}{13}\right)}$$

$$\Rightarrow \frac{5}{12}$$

$$\therefore \quad \frac{5\sin A - 2\cos A}{\tan A} = \frac{5\left(\frac{5}{13}\right) - 2\left(\frac{12}{13}\right)}{\left(\frac{5}{12}\right)}$$

$$= \frac{\frac{25}{13} - \frac{24}{13}}{\frac{5}{12}}$$

$$= \frac{\frac{25-24}{13}}{\frac{5}{12}}$$

$$= \frac{1}{13} \times \frac{12}{5} \quad \Rightarrow \frac{12}{65}$$

Solution -17

Given $\operatorname{cosec} A = \sqrt{2}$

$\operatorname{cosec} A = \dfrac{1}{\sin A} = \sqrt{2}$

$\therefore \quad \sin A = \dfrac{1}{\sqrt{2}}$

By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$(\sqrt{2})^2 = 1^2 + ac^2$

$2 = 1 + ac^2$

$ac^2 = 2-1$

$ac^2 = 1$

$ac = \sqrt{1}$

$ac = 1$

$\therefore \quad ac = 1 \ ; \ bc = \sqrt{2} \ ; \ ab = 1$

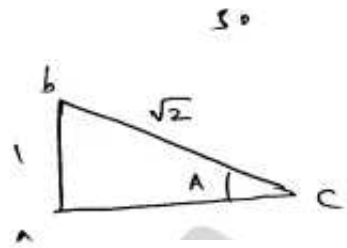$\therefore \quad \sin A = \dfrac{1}{\sqrt{2}}$

$\cos A = \dfrac{1}{\sqrt{2}}$

$\operatorname{Tan} A = \dfrac{\sin A}{\cos A} = \dfrac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} = 1$

$\cot A = \dfrac{1}{\operatorname{Tan} A}$

$= \dfrac{1}{1}$

$\cot A = 1$

$\therefore \dfrac{2\sin^2 A + 3\cot^2 A}{\tan^2 A - \cos^2 A}$

$\Rightarrow \dfrac{2\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 3\,(1)^2}{(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow \dfrac{2\cdot\frac{1}{2} + 3\cdot(1)}{1 - \frac{1}{2}}$

$\Rightarrow \dfrac{1+3}{\left(\frac{2-1}{2}\right)}$

$\Rightarrow \dfrac{4\times 2}{1}$

$\Rightarrow 8.$

Solution 18 :-

Given ABCD is a rhombus
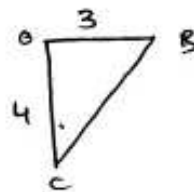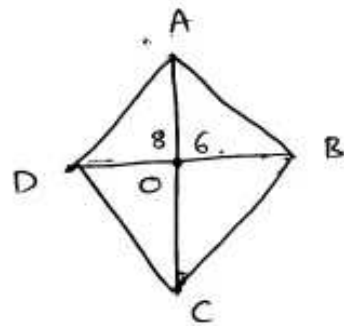
$AC = 8$; $BD = 6$

from figure

$\triangle OBC$;

By pythagoras theorem

$BC^2 = OC^2 + OB^2$

$= 3^2 + 4^2$

$= 9 + 16 = 25$

$BC = \sqrt{25} = 5$

$$\therefore \quad Sin \angle OCB = \frac{OB}{BC}$$

$$= \frac{3}{5}.$$

Solution-19 :-

Given $tan\theta = \frac{5}{12}$

By pythagoras theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = 5^2 + 12^2$$

$$bc^2 = 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$bc = 13$$

$\therefore \quad bc = 13 \; ; \; ab = 5 \; ; \; ac = 12$

$$\therefore \quad Sin\theta = \frac{ab}{bc} = \frac{5}{13}$$

$$cos\theta = \frac{ac}{bc} = \frac{12}{13}$$

$$\therefore \quad \frac{cos\theta + sin\theta}{cos\theta - sin\theta} = \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$

$$= \frac{\frac{12+5}{13}}{\frac{12-5}{13}} = \frac{17}{7} = 2\frac{3}{7} //$$

Solution- 20 :

Given $5\cos A - 12\sin A = 0$

$5\cos A = 12\sin A$

$\dfrac{5}{12} = \dfrac{\sin A}{\cos A}$

$\dfrac{5}{12} = \tan A$

By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$bc^2 = 5^2 + 12^2$

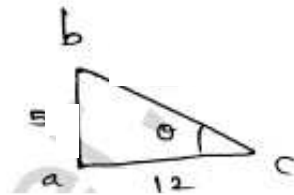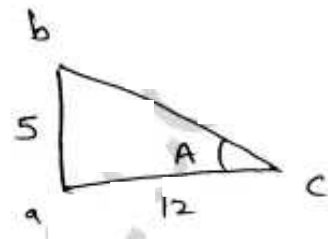$bc^2 = 25 + 144$

$bc^2 = 169$

$bc = \sqrt{169}$

$bc = 13$

$\therefore ab = 5$; $bc = 13$; $ac = 12$

$\sin A = \dfrac{ab}{bc} = \dfrac{5}{13}$

$\cos A = \dfrac{ac}{bc} = \dfrac{12}{13}$

$\therefore \dfrac{\sin A + \cos A}{2\cos A - \sin A} = \dfrac{\dfrac{5}{13} + \dfrac{12}{13}}{2 \cdot \dfrac{12}{13} - \dfrac{5}{13}}$

$= \dfrac{(5+12)/13}{(24-5)/13}$

$= \dfrac{17}{19}$
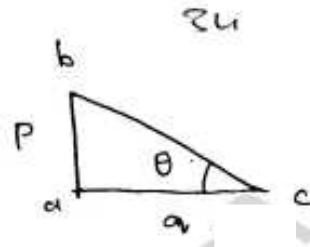
Solution - 21

Given $\tan\theta = \dfrac{p}{q}$

by pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = p^2 + q^2$$

$$bc = \sqrt{p^2+q^2}$$

$\therefore \quad \sin\theta = \dfrac{ab}{bc} = \dfrac{p}{\sqrt{p^2+q^2}}$

$\cos\theta = \dfrac{ac}{bc} = \dfrac{q}{\sqrt{p^2+q^2}}$

$\therefore \quad \dfrac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \dfrac{p \cdot \dfrac{p}{\sqrt{p^2+q^2}} - q \cdot \dfrac{q}{\sqrt{p^2+q^2}}}{p \cdot \dfrac{p}{\sqrt{p^2+q^2}} + q \cdot \dfrac{q}{\sqrt{p^2+q^2}}}$

$$= \dfrac{\dfrac{p^2 - q^2}{\sqrt{p^2+q^2}}}{\dfrac{p^2+q^2}{\sqrt{p^2+q^2}}}$$

$$= \dfrac{p^2 - q^2}{p^2 + q^2}$$

Solution - 22 :

Given $3\cot\theta = 4$

$\cot\theta = \dfrac{4}{3}$

By pythagoras theorem

$bc^2 = ab^2 + ac^2$

$bc^2 = 3^2 + 4^2$

$bc^2 = 9 + 16$

$bc^2 = 25$

$bc = \sqrt{25}$

$bc = 5$

$\therefore\ \sin\theta = \dfrac{ab}{bc} = \dfrac{3}{5}$

$\cos\theta = \dfrac{ac}{bc} = \dfrac{4}{5}$

$\therefore\ \dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \dfrac{5\cdot\dfrac{3}{5} - 3\cdot\dfrac{4}{5}}{5\cdot\dfrac{3}{5} + 3\cdot\dfrac{4}{5}}$

$= \dfrac{(15 - 12)/5}{(15 + 12)/5}$

$= \dfrac{3}{27}$

$= \dfrac{1}{9}$

Solution - 23 (i)

Given $5\cos\theta - 12\sin\theta = 0$

$$5\cos\theta = 12\sin\theta$$

$$\frac{5}{12} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{5}{12} = \tan\theta$$
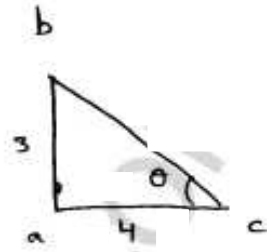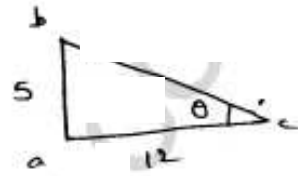
By pythagorus theorem

$$bc^2 = ab^2 + ac^2$$

$$bc^2 = 5^2 + 12^2$$

$$bc^2 = 25 + 144$$

$$bc^2 = 169$$

$$bc = \sqrt{169}$$

$$bc = 13$$

$$\therefore \sin\theta = \frac{ab}{bc} = \frac{5}{13}$$

$$\cos\theta = \frac{ac}{bc} = \frac{12}{13}$$

$$\therefore \frac{\sin\theta + \cos\theta}{2\cos\theta - \sin\theta} = \frac{\frac{5}{13} + \frac{12}{13}}{2 \cdot \frac{12}{13} - \frac{5}{13}}$$
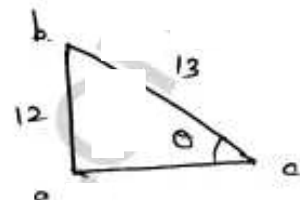
$$= \frac{\frac{5+12}{13}}{\frac{24-5}{13}}$$

$$= \frac{17}{16} \,/\!/$$

Solution-23 (ii) :-

Given $\csc \theta = \dfrac{13}{12}$

$\Rightarrow \csc \theta = \dfrac{1}{\sin \theta} = \dfrac{13}{12}$

$\therefore \sin \theta = \dfrac{12}{13}$

By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$13^2 = 12^2 + ac^2$

$169 = 144 + ac^2$

$169 - 144 = ac^2$

$25 = ac^2$

$ac = \sqrt{25}$

$ac = 5$

$\therefore \sin \theta = \dfrac{12}{13}$

$\cos \theta = \dfrac{ac}{bc} = \dfrac{5}{13}$

$\therefore \dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \dfrac{2 \cdot \dfrac{12}{13} - 3 \cdot \dfrac{5}{13}}{4 \cdot \dfrac{12}{13} - 9 \cdot \dfrac{5}{13}}$

$= \dfrac{\dfrac{24 - 15}{13}}{\dfrac{48 - 45}{13}}$

$= \dfrac{9}{3} = 3$

Solution - 24 :-

Given $5\sin\theta = 3$

$\sin\theta = \dfrac{3}{5}$

By pythagorus theorem

$bc^2 = ab^2 + ac^2$

$5^2 = 3^2 + ac^2$

$25 = 9 + ac^2$

$ac^2 = 25 - 9$

$ac^2 = 16$

$ac = \sqrt{16}$

$ac = 4.$

$\cos\theta = \dfrac{ac}{bc} = \dfrac{4}{5}$

$\therefore \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{5}{4}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3/\cancel{5}}{4/\cancel{5}} = \dfrac{3}{4}$

$\therefore \dfrac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \dfrac{\dfrac{5}{4} - \dfrac{3}{4}}{\dfrac{5}{4} + \dfrac{3}{4}}$

$= \dfrac{\dfrac{5-3}{4}}{\dfrac{5+3}{4}}$

$= \dfrac{2}{8}$

$= \dfrac{1}{4}.$
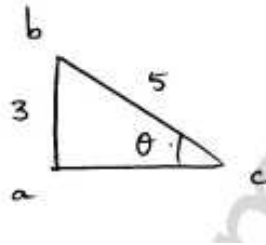
Solution - 25 :-

Given $\quad \sin\theta = \cos\theta$

Then $\quad 2\tan^2\theta + \sin^2\theta - 1$

$\Rightarrow \quad 2\left(\dfrac{\sin\theta}{\cos\theta}\right)^2 + \sin^2\theta - 1$

$\Rightarrow \quad 2\left(\dfrac{\sin\theta}{\sin\theta}\right)^2 + \sin^2\theta - 1$

$\Rightarrow \quad 2(1)^2 + \sin^2\theta - 1$

$\Rightarrow \quad 2 + \sin^2\theta - 1$

$\Rightarrow \quad \sin^2\theta + 1$

from given $\quad \sin\theta = \cos\theta$

so $\Rightarrow \theta = 45$

$\therefore \quad \sin 45 = \cos 45$

$\dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\therefore \quad 2\tan^2\theta + \sin^2\theta - 1 \quad \Rightarrow \quad \sin^2\theta + 1$

$\Rightarrow \quad \left(\dfrac{1}{\sqrt{2}}\right)^2 + 1$

$\Rightarrow \quad \dfrac{1}{2} + 1$

$\Rightarrow \quad \dfrac{3}{2}$

$\therefore \quad 2\tan^2\theta + \sin^2\theta - 1 = \dfrac{3}{2}$

Solution - 26 (i)

LHS $\Rightarrow$ $\cos\theta \cdot \tan\theta$

$\rightarrow$ $\cos\theta \cdot \dfrac{\sin\theta}{\cos\theta}$

$\rightarrow$ $\sin\theta$

RHS $=$ $\sin\theta$

$\therefore$ LHS $=$ RHS.

$\left(\because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \right)$

Solution - 26 (ii)

LHS $\Rightarrow$ $\sin\theta \cdot \cot\theta$

$\rightarrow$ $\sin\theta \cdot \dfrac{\cos\theta}{\sin\theta}$

$\rightarrow$ $\cos\theta$

$\therefore$ LHS $=$ RHS.

$\because \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Solution - 26 (iii)

LHS $\Rightarrow$ $\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$

$\rightarrow$ $\dfrac{\sin^2\theta + \cos\theta \cdot \cos\theta}{\cos\theta}$

$\rightarrow$ $\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$\Rightarrow$ $\dfrac{1}{\cos\theta}$

$\therefore$ LHS $=$ RHS.

$\left(\because \sin^2\theta + \cos^2\theta = 1 \right)$

Solution- 27 :-

Given ∠C = 90°

$\tan A = \dfrac{3}{4}$
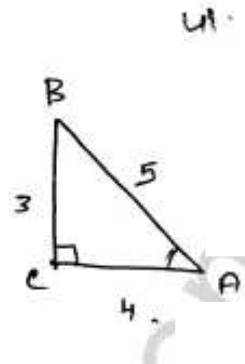
By pythagorus theorem

$AB^2 = BC^2 + AC^2$

$AB^2 = 3^2 + 4^2$

$AB^2 = 9 + 16$

$AB^2 = 25$

$AB = \sqrt{25}$

$AB = 5$

∴ $\sin A = \dfrac{BC}{AB} = \dfrac{3}{5}$

$\cos A = \dfrac{AC}{AB} = \dfrac{4}{5}$

$\cos B = \dfrac{BC}{AB} = \dfrac{3}{5}$

$\sin B = \dfrac{AC}{AB} = \dfrac{4}{5}$

∴ $\sin A \cos B + \cos A \sin B \Rightarrow \dfrac{3}{5} \cdot \dfrac{3}{5} + \dfrac{4}{5} \cdot \dfrac{4}{5}$

$\Rightarrow \dfrac{9}{25} + \dfrac{16}{25}$

$\Rightarrow \dfrac{9+16}{25}$

$\Rightarrow \dfrac{25}{25}$

$= 1$

∴ LHS = RHS.

Solution - 28 :- ⓐ

Given.

In △ABC, △BRS

AB = 18 cm

BC = 7.5 cm.

RS = 5 cm

∠BSR = x°

∠SAB = y°
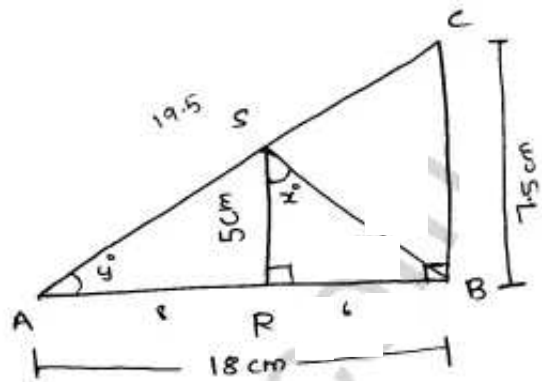
From Hint ; AR = 12 cm

RB = 6 cm.

AC = 19.5

(i) $\tan x° = \dfrac{RB}{SR} = \dfrac{6}{5}$

(ii) $\sin y = \dfrac{BC}{AC}$

$= \dfrac{7.5}{19.5} \times \dfrac{10}{10}$

$= \dfrac{75}{195} = \dfrac{5}{13}$

$= \dfrac{5}{13}$

∴ $\sin y = \dfrac{5}{13}$

Solution - 28 (b) :-
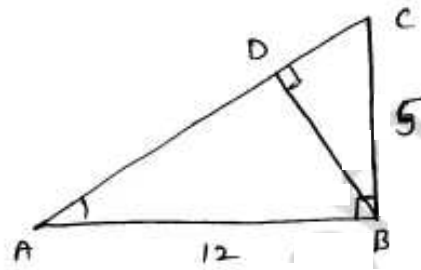
(i) From fig

By pythagorus theorem

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169}$$

$$AC = 13.$$

$$\therefore \cos \angle CBD = \frac{AB}{AC}$$

$$= \frac{12}{13}$$

(ii) $\cot \angle ABD = \dfrac{BC}{AC}$

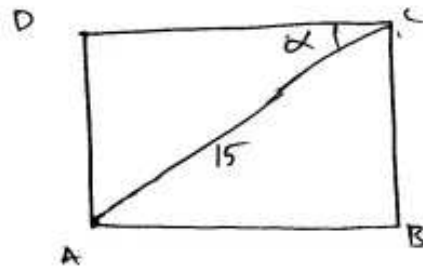$$\dfrac{5}{13}.$$

Solution - 24 :-

Given ABCD is rectangle

$$AC = 15$$

$$\angle ACD = \alpha \quad ; \quad \cot \alpha = \frac{3}{2}$$

$$\cot \alpha = \frac{CD}{AD} = \frac{3}{2}$$

$$\therefore CD = \frac{3}{2} AD$$

From $\triangle ACD$

$$AC^2 = AD^2 + CD^2$$
$$15^2 = AD^2 + \left(\frac{3}{2} AD\right)^2$$
$$225 = AD^2 + \frac{9 AD^2}{4}$$
$$225 = \frac{4 AD^2 + 9 AD^2}{4}$$
$$225 \times 4 = 13 AD^2$$
$$AD^2 = \frac{225 \times 4}{13}$$
$$AD = \sqrt{\frac{225 \times 4}{13}}$$
$$AD = \frac{15 \times 2}{\sqrt{13}}$$
$$AD = \frac{30}{\sqrt{13}}$$
$$\therefore CD = \frac{3}{2} AD$$
$$= \frac{3}{2} \cdot \frac{30}{\sqrt{3}}$$
$$= \frac{45}{\sqrt{3}}$$

$\therefore$ Area of $\triangle ACD$ + Area of $\triangle ABC$ = Area of rectangle ABCD.

$$\therefore CD = \frac{45}{\sqrt{13}} = AB$$
$$AD = \frac{30}{\sqrt{13}} = BC$$

$\therefore$ Area of $\square ABCD = CD \times AD. = \frac{45}{\sqrt{13}} \times \frac{30}{\sqrt{13}}$

$$\text{Area} = \frac{45 \times 30}{\sqrt{13} \cdot \sqrt{13}}$$

$$= \frac{1350}{13}$$

$$= 103\frac{11}{13} \ cm^2.$$

$$\therefore \ \text{Perimeter} = 2(AB + BC)$$

$$= 2\left(\frac{45}{\sqrt{13}} + \frac{30}{\sqrt{13}}\right)$$

$$= 2\left(\frac{45 + 30}{\sqrt{13}}\right)$$

$$= 2 \times \frac{75}{\sqrt{13}}$$

$$= \frac{150}{\sqrt{13}}$$

Solution - 30 :-

ⓐ

From $\triangle BCD$

$\Rightarrow$ By pythagorus theorem

$$BD^2 = BC^2 + CD^2$$

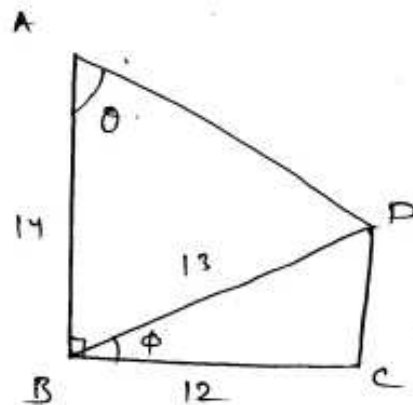$$13^2 = 12^2 + CD^2$$

$$169 = 144 + CD^2$$

$$CD^2 = 169 - 144$$

$$CD^2 = 25$$

$$CD = \sqrt{25} = \underline{5}$$

(i) $\sin\phi = \dfrac{CD}{BD}$

$\qquad = \dfrac{5}{13}$

(ii) $\tan\theta = \dfrac{DE}{AE}$

$\qquad = \dfrac{12}{9} = \dfrac{4}{3}$

Solution - 30 (b) :-

$\tan\theta = \dfrac{4}{3}$

$\therefore \sin\theta = \dfrac{DE}{AD}$

$AD = \dfrac{12}{\sin\theta}$
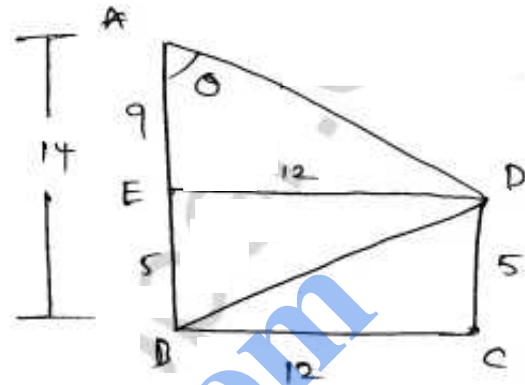
(or) $\cos\theta = \dfrac{AE}{AD}$

$AD = \dfrac{9}{\cos\theta}$

Solution - 31 :-(1)

LHS $\Rightarrow (\sin A + \cos A)^2 + (\sin A - \cos A)^2$

$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin A\cos A$

$\left(\because (a+b)^2 = a^2 + b^2 + 2ab \atop (a-b)^2 = a^2 + b^2 - 2ab \right)$

$\Rightarrow \quad 2\sin^2 A + 2\cos^2 A$

$\Rightarrow \quad 2(\sin^2 A + \cos^2 A)$

$\Rightarrow \quad 2(1) = 2$

$\therefore$ LHS = RHS.

Solution - 31 (ii) :-

LHS $\Rightarrow$ $\cot^2 A - \dfrac{1}{\sin^2 A} + 1$

$\Rightarrow$ $\dfrac{\cos^2 A}{\sin^2 A} - \dfrac{1}{\sin^2 A} + 1$

$\Rightarrow$ $\dfrac{\cos^2 A - 1 + \sin^2 A}{\sin^2 A}$

$\Rightarrow$ $\dfrac{(\cos^2 A + \sin^2 A) - 1}{\sin^2 A}$

$\Rightarrow$ $\dfrac{1 - 1}{\sin^2 A}$

$\Rightarrow$ 0 = RHS.

Solution - 31 (iii) :-

LHS $\Rightarrow$ $\dfrac{1}{1 + \tan^2 A} + \dfrac{1}{1 + \cot^2 A}$

$\Rightarrow$ $\dfrac{1}{1 + \dfrac{\sin^2 A}{\cos^2 A}} + \dfrac{1}{1 + \dfrac{\cos^2 A}{\sin^2 A}}$

$\Rightarrow$ $\dfrac{1}{\dfrac{\cos^2 + \sin^2 A}{\cos^2 A}} + \dfrac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin^2 A}}$

$\Rightarrow$ $\dfrac{\cos^2 A}{(1)} + \dfrac{\sin^2 A}{(1)}$

$\Rightarrow$ $\cos^2 A + \sin^2 A$

$\Rightarrow$ 1 = RHS //.

Solution – 32 :

Given $\sqrt{\dfrac{1-\sin^2\theta}{1-\cos^2\theta}}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \sqrt{\dfrac{\cos^2\theta}{\sin^2\theta}}$

$\Rightarrow \sqrt{\cot^2\theta}$

$\Rightarrow \cot\theta$

Solution – 33 :–

Given $\sin\theta + \operatorname{cosec}\theta = 2$

Squaring on both sides.

$(\sin\theta + \operatorname{cosec}\theta)^2 = 2^2$

$\sin^2\theta + \operatorname{cosec}^2\theta + 2 \cdot \sin\theta \cdot \operatorname{cosec}\theta = 4$

$\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta \dfrac{1}{\sin\theta} = 4$

$\sin^2\theta + \operatorname{cosec}^2\theta = 4 - 2$

$\underline{\sin^2\theta + \operatorname{cosec}^2\theta = 2}$

Solution – 34 :–

Given $x = a\cos\theta + b\sin\theta$

$y = a\sin\theta - b\cos\theta$

squaring on b.s.

$x^2 = (a\cos\theta + b\sin\theta)^2$

$$x^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta \quad ---(1)$$

$$\because \quad y = a\sin\theta - b\cos\theta$$

Squaring on b.s.

$$y^2 = (a\sin\theta - b\cos\theta)^2$$

$$y^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta \quad ---(2)$$

① + ②

$$\rightarrow x^2 + y^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta +$$
$$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$

$$= a^2(1) + b^2(1)$$

$$= a^2 + b^2$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.