Rectilinear Figures

Exercise 13.1

Question 1.
If two angles of a quadrilateral are 40° and 110° and the other two are in the ratio 3 : 4, find these angles.
Solution:

Sum of four angles of a quadrilateral = 360°
Sum of two given angles = 40° + 110° = 150°
∴ Sum of remaining two angles
= 360° – 150 = 210°
Ratio in these angles = 3 : 4

$$\therefore \text{Third angle} = \frac{210° \times 3}{3+4}$$

$$= \frac{210° \times 3}{7} = 90°$$

$$\text{and fourth angle} = \frac{210° \times 4}{3+4}$$

$$= \frac{210° \times 4}{7} = 120°$$

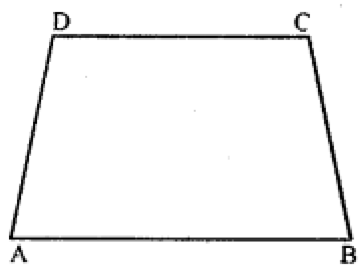
Question 2.
If the angles of a quadrilateral, taken in order, are in the ratio 1 : 2 : 3 : 4, prove that it is a trapezium.
Solution:

In trapezium ABCD

$\angle A : \angle B : \angle C : \angle D = 1 : 2 : 3 : 4$

Sum of angles of the quad. ABCD = 360°

Sum of the ratio's = 1 + 2 + 3 + 4 = 10



$$\therefore \angle A = \frac{360^\circ \times 1}{10} = 36^\circ$$

$$\angle B = \frac{360^\circ \times 2}{10} = 72^\circ$$

$$\angle C = \frac{360^\circ \times 3}{10} = 108^\circ$$

$$\angle D = \frac{360^\circ \times 4}{10} = 144^\circ$$

Now $\angle A + \angle D = 36^\circ + 114^\circ = 180^\circ$

$\because$ $\angle A + \angle D = 180^\circ$ and these are co-interior angles

$\therefore$ AB || DC

Hence ABCD is a trapezium

Question 3.

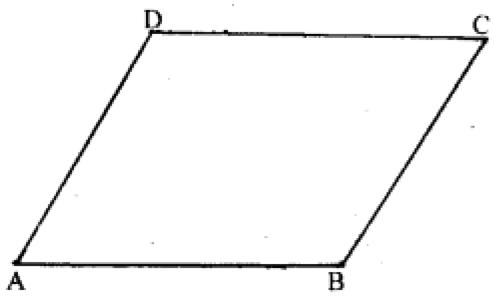
If an angle of a parallelogram is two-thirds of its adjacent angle, find the angles of the parallelogram.

Solution:

Here ABCD is a parallelogram.

Let $\angle A = x°$

then $\angle B = \frac{2}{3}x°$



(given condition an angle of a parallelogram is two third of its adjacent angle.)

$\therefore \angle A + \angle B = 180°$

($\because$ sum of adjacent angle in parallelogram is 180°)

$\Rightarrow x° + \frac{2}{3}x° = 180° \Rightarrow \frac{3x+2x}{3} = 180$

$\Rightarrow \frac{5x}{3} = 180 \Rightarrow 5x = 180 \times 3$

$\Rightarrow x = \frac{180 \times 3}{5} \Rightarrow x = 36 \times 3 \Rightarrow x = 108$

$\therefore \angle A = 108°$

$\angle B = \frac{2}{3} \times 108° = 2 \times 36° = 72°$

$\angle B = \angle D = 72°$

(opposite angle in parallelogram is same)

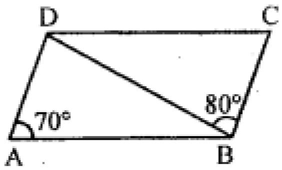
Also, $\angle A = \angle C = 108°$

(opposite angles in parallelogram is same)

Hence, angles of parallelogram are 108°, 72°, 108°, 72°

Question 4.

(a) In figure (1) given below, ABCD is a parallelogram in which ∠DAB = 70°, ∠DBC = 80°. Calculate angles CDB and ADB.
(b) In figure (2) given below, ABCD is a parallelogram. Find the angles of the AAOD.
(c) In figure (3) given below, ABCD is a rhombus. Find the value of x.



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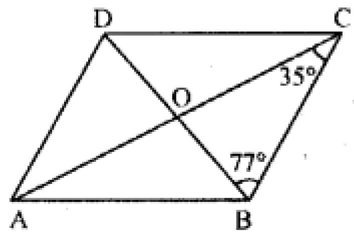

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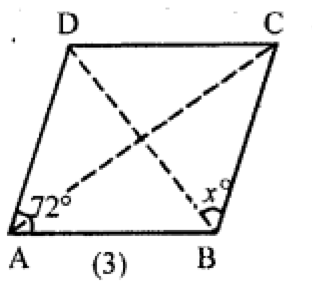


(3)

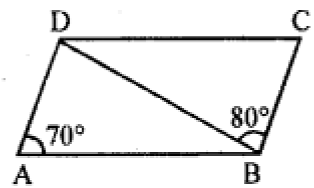
Solution:

(*a*) $\because$ ABCD is || gm

$\therefore$ AB || CD

$\angle ADB = \angle DBC$ (Alternate angles)

$\angle ADB = 80°$ [$\because$ $\angle DBC = 80°$ (given)]



In ΔADB,

$\angle A + \angle ADB + \angle ABD = 180°$

(sum of all angles in a triangle is 180°)

$\Rightarrow$ $70° + 80° + \angle ABD = 180°$

$\Rightarrow$ $150° + \angle ABD = 180°$

$\Rightarrow$ $\angle ABD = 180° - 150°$

$\Rightarrow$ $\angle ABD = 30°$ ....(2)

Now $\angle CDB = \angle ABD$ ....(3)

[$\because$ AB || CD, (Alternate angles)]

From (2) and (3)

$\angle CDB = 30°$ ....(4)

From (1) and (4)

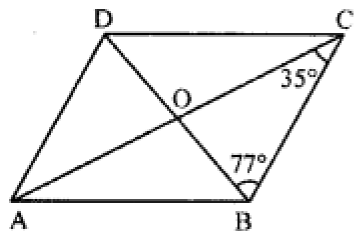
$\angle CDB = 30°$ and $\angle ABD = 80°$

(b) Given $\angle BCO = 35°, \angle CBO = 77°$

In ΔBOC

$\angle BOC + \angle BCO + \angle CBO = 180°$

(Sum of all angles in a triangle is 180°)



$\angle BOC = 180° - 112° = 68°$

Now in ||gm ABCD,

We have,

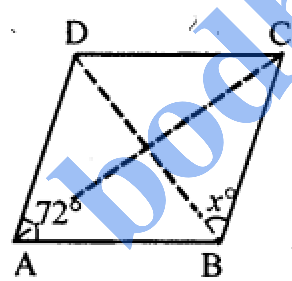
$\angle AOD = \angle BOC$

(vertically opposite angles)

$\therefore \angle AOD = 68°$

(c) ABCD is a rhombus $\angle A + \angle B = 180°$

(In rhombus sum of adjacent angle is 180°)



$\Rightarrow \quad 72° + \angle B = 180° \Rightarrow \quad \angle B = 180° - 72°$

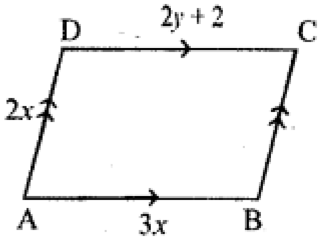
$\Rightarrow \quad \angle B = 108°$

$\therefore \quad x = \frac{1}{2} \angle B = \frac{1}{2} \times 108° = 54°$

Question 5.

(a) In figure (1) given below, ABCD is a parallelogram with perimeter 40. Find the

values of x and y.
(b) In figure (2) given below. ABCD is a parallelogram. Find the values of x and y.
(c) In figure (3) given below. ABCD is a rhombus. Find x and y.
Solution:



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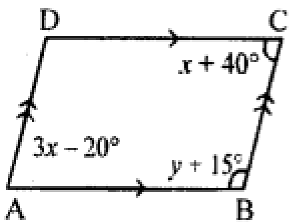

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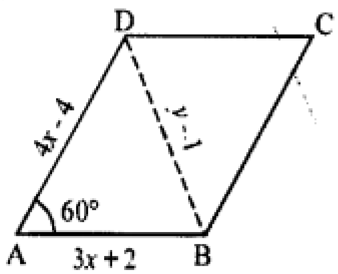


(3)

(*a*) Since ABCD is a parallelogram.

$\therefore$ AB = CD and BC = AD

$\therefore$ $3x = 2y + 2$ (AB = CD)

$3x - 2y = 2$ ....(1)

Also, AB + BC + CD + DA = 40

$\Rightarrow$ $3x + 2x + 2y + 2 + 2x = 40$

$\Rightarrow$ $7x + 2y = 40 - 2$ $\Rightarrow$ $7x + 2y = 38$ ....(2)

Adding (1) and (2),

$$\begin{aligned} 3x - 2y &= 2 \\ 7x + 2y &= 38 \\ \hline 10x \quad &= 40 \end{aligned}$$

$\Rightarrow$ $x = \frac{40}{10} = 4$

Substituting the value of x in (1), we get

$3 \times 4 - 2y = 2$ $\Rightarrow$ $12 - 2y = 2$ $\Rightarrow$ $-2y = 2 - 12$

$\Rightarrow$ $-2y = -10$ $\Rightarrow$ $y = \frac{-10}{-2}$

$\therefore \ y = 5$

Hence, $x = 4, y = 5$ Ans.

(b) In parallelogram ABCD

$\angle A = \angle C$ (opposite angles are same in ||gm)

$\Rightarrow \ 3x - 20° = x + 40° \ \Rightarrow \ 3x - x = 40° + 20°$

$\Rightarrow \ 2x = 60°$

$\Rightarrow \ x = \frac{60°}{2}$

$\Rightarrow \ x = 30°$ .....(1)

Also, $\angle A + \angle B = 180°$

(sum of adjacent angles in ||gm is equal to 180°)

$\Rightarrow \ 3x - 20° + y + 15° = 180°$

$\Rightarrow \ 3x + y - 5° = 180° \ \Rightarrow \ 3x + y = 180° + 5°$

$\Rightarrow \ 3x + y = 185° \ \Rightarrow \ 3 \times 30° + y = 185°$

[Putting the value of x From (1)]

$\Rightarrow \ 90° + y = 185° \ \Rightarrow \ y = 185° - 90°$

$\Rightarrow \ y = 95°$

Hence, $x = 30°, y = 95°$

(*c*) ABCD is a rhombus

$\therefore$ AB = AD

$\Rightarrow$ $3x + 2 = 4x - 4$

$\Rightarrow$ $3x - 4x = -4 - 2$

$\Rightarrow$ $-x = -6$

$\Rightarrow$ $x = 6$ ....(1)

In ΔABD,

$\therefore$ $\angle BAD = 60°$, Also AB = AD

$\therefore$ $\angle ADB = \angle ABD$

$\therefore$ $\angle ADB = \dfrac{180° - \angle BAD}{2}$

$= \dfrac{180° - 60°}{2} = \dfrac{120°}{2} = 60°$

ΔABD is equilateral triangle

($\because$ each angles of this triangle are 60°)

$\therefore$ AB = BD

$\Rightarrow$ $3x + 2 = y - 1$ $\Rightarrow$ $3 \times 6 + 2 = y - 1$

[substituting the value of x from (1)]

$\Rightarrow$ $18 + 2 = y - 1$ $\Rightarrow$ $20 = y - 1$

$\Rightarrow$ $y - 1 = 20$ $\Rightarrow$ $y = 20 + 1$ $\Rightarrow$ $y = 21$

Hence, $x = 6$ and $y = 21$

Question 6.

The diagonals AC and BD of a rectangle > ABCD intersect each other at P. If ∠ABD = 50°, find ∠DPC.

Solution:

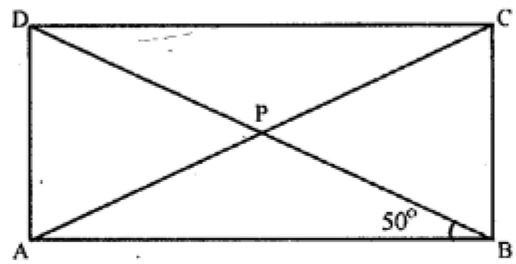
ABCD is a rectangle

Since diagonals of rectangle are same and bisect each other.

$\therefore$ AP = BP

$\therefore$ $\angle PAB = \angle PBA$

(equal sides have equal opposite angles)



$\Rightarrow$ $\angle PAB = 50°$ [$\because$ $\angle PBA = 50°$ (given)]

In ΔAPB,

$\angle APB + \angle ABP + \angle BAP = 180°$

$\Rightarrow$ $\angle APB + 50° + 50° = 180°$

$\Rightarrow$ $\angle APB = 180° - 100°$

$\Rightarrow$ $\angle APB = 80°$ ....(1)

$\therefore$ $\angle DPB = \angle APB$ ....(2)

(vertically opposite angles)

From (1) and (2)

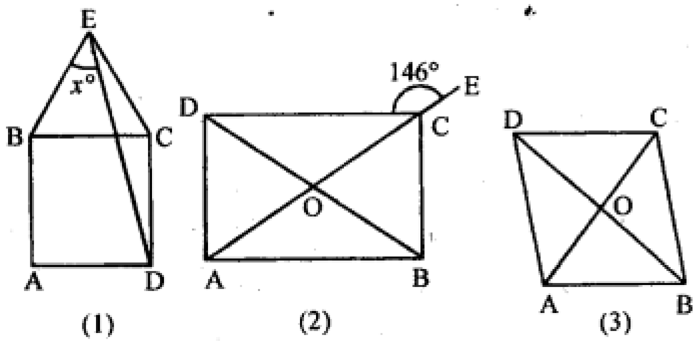
$\angle DPB = 80°$

Question 7.

(a) In figure (1) given below, equilateral triangle EBC surmounts square ABCD. Find angle BED represented by x.

(b) In figure (2) given below, ABCD is a rectangle and diagonals intersect at O. AC is produced to E. If ∠ECD = 146°, find the angles of the △ AOB.

(c) In figure (3) given below, ABCD is rhombus and diagonals intersect at O. If ∠OAB : ∠OBA = 3:2, find the angles of the △ AOD.



Solution:

(a) Since EBC is an equilateral triangle

EB = BC = EC

$\therefore$ EB = BC = EC ....(1)

Also, ABCD is a square

AB = BC = CD = AD ....(2)

From (1) and (2),

EB = EC = AB = BC = CD = AD ....(3)

In ΔECD,

$\angle ECD = \angle BCD + \angle ECB$

(BEC is an equilateral triangle)

$\Rightarrow \angle ECD = 90° + 60° = 150°$ ....(4)

Also, EC = CD [From (3)]

$\therefore \angle DEC = \angle CDE$ ....(5)

$\therefore \angle ECD + \angle DEC + \angle CDE = 180°$

(sum of all angles in a triangle is 180°)

$\Rightarrow 150° + \angle DEC + \angle DEC = 180°$

(using (4) and (5))

$\Rightarrow 2\angle DEC = 180° - 150° \Rightarrow 2\angle DEC = 30°$

$\Rightarrow \angle DEC = \frac{30°}{2} \Rightarrow \angle DEC = 15°$ ....(6)

Now $\angle BEC = 60°$ (BEC is an equilateral triangle)

$\Rightarrow \angle BED + \angle DEC = 60° \Rightarrow x° + 15° = 60°$

[From (6)]

$\Rightarrow x = 60° - 15° \Rightarrow x = 45°$

Hence, value of $x = 45°$

(*b*) Since ABCD is a rectangle

$\angle ECD = 146°$ (given)

$\therefore$ ACE is a st. line

$\therefore$ $146° + \angle ACD = 180°$

(linear pair)

$\Rightarrow$ $\angle ACD = 180° - 146°$

$\Rightarrow$ $\angle ACD = 34°$ ....(1)

$\therefore$ $\angle CAB = \angle ACD$ (Alternate angles) ...(2)

[$\because$ AB || CD]

From (1) and (2)

$\Rightarrow$ $\angle CAB = 34°$ $\Rightarrow$ $\angle OAB = 34°$ ....(3)

In $\angle AOB$

AO = OB

(In rectangle diagonals are same & bisect each other)

$\Rightarrow$ $\angle OAB = \angle OBA$ ...(4)

(equal sides have equal angles opposite to them)

From (3) and (4),

$\angle OBA = 34°$ ....(5)

$\therefore \angle AOB + \angle OBA + \angle OAB = 180°$

(Sum of all angles in a triangle is 180°)

$\Rightarrow \angle AOB + 34° + 34° = 180°$ [using (3) and (5)]

$\Rightarrow \angle AOB + 68° = 180°$

$\Rightarrow \angle AOB = 180° - 68° \Rightarrow \angle AOB = 112°$

Hence, $\angle AOB = 112°$, $\angle OAB = 34°$

and $\angle OBA = 34°$

(*c*) Here ABCD is a rhombus and diagonals intersect at O.

and $\angle OAB : \angle OBA = 3 : 2$

Let $\angle OAB = 2x°$

then $\angle OBA = 2x°$

We know that diagonals of rhombus intersect at right angles.

$\therefore \angle OAB = 90°$ in ΔAOB

$\therefore \angle OAB + \angle OBA = 180°$

$\Rightarrow 90° + 3x° + 2x° = 180° \Rightarrow 90° + 5x° = 180°$

$\Rightarrow 5x° = 180° - 90° \Rightarrow x° = \frac{90°}{5}$

$\Rightarrow x° = 18°$

$\therefore \angle OAB = 3x° = 3 \times 18° = 54°$

$\angle OBA = 2x° = 2 \times 18° = 36°$

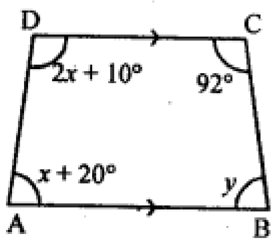
and $\angle AOB = 90°$

Question 8.

(a) In figure (1) given below, ABCD is a trapezium. Find the values of x and y.

(b) In figure (2) given below, ABCD is an isosceles trapezium. Find the values of x and.y.

(c) In figure (3) given below, ABCD is a kite and diagonals intersect at O. If ∠DAB = 112° and ∠DCB = 64°, find ∠ODC and ∠OBA.



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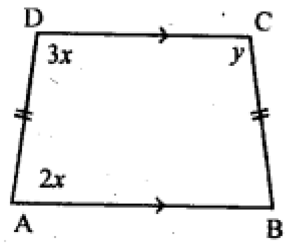

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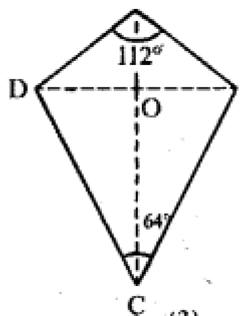


(3)

Solution:

(a) **Given :** ABCD is a trapezium

$\angle A = x + 20°$, $\angle B = y$, $\angle C = 92°$, $\angle D = 2x + 10°$

Required : Value of x and y.

Since ABCD is a trapezium.

Sol. $\angle B + \angle C = 180°$

($\because$ AB || DC)

$\Rightarrow \quad y + 92° = 180°$

$\Rightarrow \quad y = 180° - 92° \quad \Rightarrow \quad y = 88°$

Also, $\angle A + \angle D = 180°$

$\Rightarrow \quad x + 20° + 2x + 10° = 180°$

$\Rightarrow \quad 3x + 30° = 180°$

$\Rightarrow \quad 3x = 180° - 30° \quad \Rightarrow \quad 3x = 150°$

$$\Rightarrow \quad x = \frac{150°}{3} \quad \Rightarrow \quad x = 50°$$

Hence, value of $x = 50°$ and $y = 88°$

(b) Given : ABCD is an isosceles trapezium BC = AD

$\angle A = 2x$, $\angle C = y$, $\angle D = 3x$

Required : Value of x and y.

Sol. Since ABCD is a trapezium and AB || DC

$\therefore \quad \angle A + \angle D = 180°$

$\Rightarrow \quad 2x + 3x = 180°$

$\Rightarrow \quad 5x = 180°$

$$\Rightarrow \quad x = \frac{180°}{5} = 36°$$

$\therefore \quad x = 36°$ ....(1)

Also, AB = BC and AB || DC

$\therefore \quad \angle A + \angle C = 180° \quad \Rightarrow \quad 2x + y = 180°$

$\Rightarrow \quad 2 \times 36° + y = 180°$

[substituting the value of x from (1)]

$\Rightarrow \quad 72° + y = 180° \quad \Rightarrow \quad y = 180° - 72°$

$\Rightarrow \quad y = 108°$

Hence, value of $x = 72°$ and $y = 108°$

(c) Given : ABCD is a kite and diagonals intersect at O.

$\angle DAB = 112°$ and

$\angle DCB = 64°$

Required : $\angle ODC$ and $\angle OBA$

Sol. : $\therefore$ AC diagonal of kite ABCD

$\therefore \quad \angle DOC = \frac{64}{2}° = 32°$

$\therefore \angle DOC = 90°$

(diagonal of kites bisect at right angles)

In $\angle OCD$,

$\therefore \angle ODC = 180° - (\angle DCO + \angle DOC)$

$= 180° - (32° + 90°) = 180° - 122° = 58°$

In ΔDAB,

$\angle OAB = \frac{112°}{2} = 56°$

$\angle OAB = 90°$

(diagonals of kites bisect at right angles)

In ΔOAB

$\angle OBA = 180° - (\angle OAB + \angle AOB)$

$= 180° - (56° + 90°) = 180° - 146° = 34°$

Hence, $\angle ODC = 58°$ and $\angle OBA = 34°$

Question 9.

(i) Prove that each angle of a rectangle is 90°.

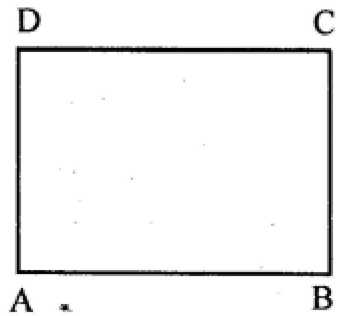
(ii) If the angle of a quadrilateral are equal, prove that it is a rectangle.

(iii) If the diagonals of a rhombus are equal, prove that it is a square.

(iv) Prove that every diagonal of a rhombus bisects the angles at the vertices.

Solution:

(*i*) A rectangle ABCD



To prove : Each angle of rectangle = 90°

Proof : $\because$ Opposite angles of a rectangle are equal

$\therefore$ $\angle A = \angle C$ and $\angle B = \angle D$

But $\angle A + \angle B + \angle C + \angle D = 360°$

(Sum of angles of a quadrilateral)

$\Rightarrow$ $\angle A + \angle B + \angle A + \angle B = 360°$

$\Rightarrow$ $2(\angle A + \angle B) = 360°$

$\Rightarrow$ $\angle A + \angle B = \frac{360°}{2} = 180°$

But $\angle A + \angle B$ (Angles of a rectangle)

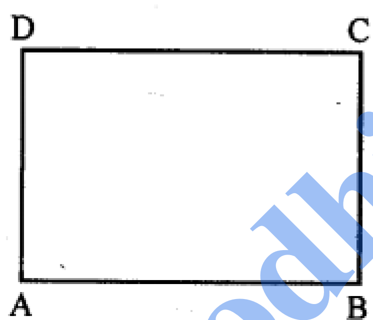
$\therefore$ $\angle A = \angle B = 90°$

Hence $\angle A = \angle B = \angle C = \angle D = 90°$

(*ii*) **Given :** In quadrilateral ABCD,

$\angle A = \angle B = \angle C = \angle D$

To prove : ABCD is a rectangle



Proof : $\angle A = \angle B = \angle C = \angle D$

$\Rightarrow$ $\angle A = \angle C$ and $\angle B = \angle D$

But these are opposite angles of the quadrilateral

$\therefore$ ABCD is a parallelogram

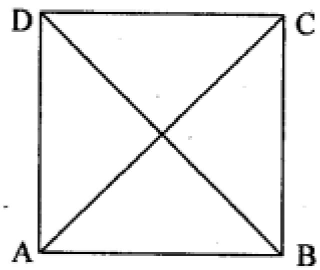
$\because$ $\angle A = \angle B = \angle C = \angle D = 90°$

Hence ABCD is a rectangle

Hence proved.

... proved.

(*iii*) **Given :** Δ ABCD is a rhombus in which AC = BD



To Prove : ABCD is a square.

Proof : In ΔABC and ΔDCB,

$AB = DC$ (ABCD is a rhombus)

$BC = BC$ (common)

and $AC = BD$ (given)

$\therefore \Delta ABC \cong \Delta DCB$

(By S.S.S. axiom of congruency)

$\therefore \angle ABC = \angle DBC$ (c.p.c.t.)

But these are angle made by transversal BC on the same side of parallel Lines AB and CD

$\therefore \angle ABC + \angle DBC = 180°$

$\therefore \angle ABC = 90°$

$\therefore$ ABCD is a square **(Q.E.D.)**

(*iv*) AC and BD bisects $\angle A$, $\angle C$ and $\angle B$, $\angle D$ respectively.

Proof :

Statements	**Reasons**
(1) In ΔAOD and ΔCOD	
$AD = CD$	(each side or rhombus is same)
$OD = OD$	(common)
$AO = OC$	(diagonal of rhombus bisect each other)
(2) $\Delta AOD \cong \Delta COD$	[S.S.S.]
(3) $\angle AOD = \angle COD$	[c.p.c.t.]
(4) $\angle AOD + \angle COD = 180°$	AOC is a st. line

$\Rightarrow \angle AOD + \angle COD = 180°$ By (3)

$\Rightarrow 2\angle AOD = 180° \Rightarrow \angle AOD = \frac{180°}{2}$

$\Rightarrow \quad \angle AOD = 90°$

(5) $\angle COD = 90°$ By (3) and (4)

$\therefore$ OD $\perp$ AC $\Rightarrow$ BD $\perp$ AC

(6) $\angle ADO = \angle CDO$ (c.p.c.t.)

$\Rightarrow$ OD bisect $\angle D$ $\Rightarrow$ BD bisect $\angle D$

Similarly we can prove that BD bisect $\angle B$.

and AC bisect the $\angle A$ and $\angle C$.

Question 10.

ABCD is a parallelogram. If the diagonal AC bisects ∠A, then prove that:

(i) AC bisects ∠C

(ii) ABCD is a rhombus

(iii) AC ⊥ BD.

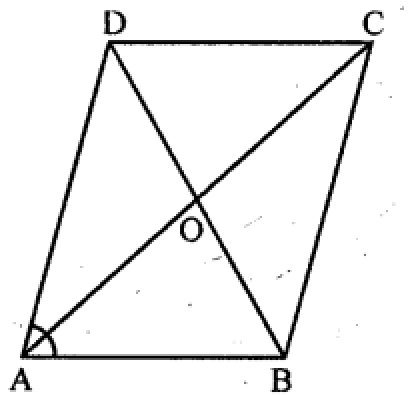
Solution:

Given : In parallelogram ABCD, diagonal AC bisects ∠A

To prove : (*i*) AC bisects ∠C

(*ii*) ABCD is a rhombus

(*iii*) AC ⊥ BD



Proof : (*i*) ∵ AB ∥ CD (opposite sides of a ∥gm)

∴ ∠DCA = ∠CAB (Alternate angles)

Similarly ∠DAC = ∠DCB

But ∠CAB = ∠DAC (∵ AC bisects ∠A)

∴ ∠DCA = ∠ACB

∴ AC bisects ∠C

(*iii*) ∵ AC bisects ∠A and ∠C

and ∠A = ∠C

∴ ABCD is a rhombus

(*iii*) ∵ AC and BD are the diagonals of a rhombus

∴ AC and BD bisect each other at right angles

Hence AC ⊥ BD

Hence proved.

Question 11.

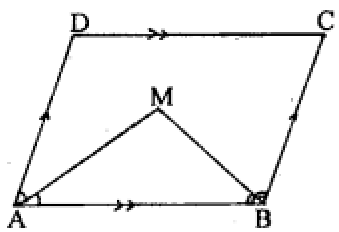
(i) Prove that bisectors of any two adjacent angles of a parallelogram are at right angles.
(ii) Prove that bisectors of any two opposite angles of a parallelogram are parallel.
(iii) If the diagonals of a quadrilateral are equal and bisect each other at right

angles, then prove that it is a square.

Solution:

(*i*) Given AM bisect angle A and BM bisects angle B of ∥ gm ABCD

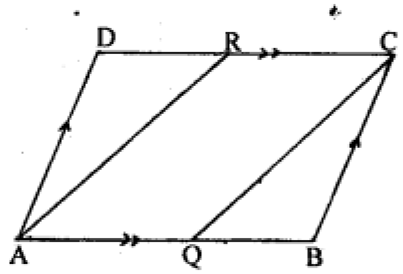
To Prove : $\angle AMB = 90^\circ$



Proof :

Statements	**Reasons**
(1) $\angle A + \angle B = 180^\circ$	AD ∥ BC and AB is the transversal.
(2) $\frac{1}{2}(\angle A + \angle B) = \frac{180^\circ}{2}$	Multiplying both sides by $\frac{1}{2}$
$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$	
$\Rightarrow \angle MAB + \angle MBA = 90^\circ$	(*i*) AM bisects $\angle A$ $\therefore \frac{1}{2}\angle A = \angle MAB$ (*ii*) BM bisects $\angle B$ $\therefore \frac{1}{2}\angle B = \angle MBA$
(3) In ΔAMB,	
$\angle AMB + \angle MAB + \angle MBA = 180^\circ$	Sum of angles of a triangle is equal to 180°
$\Rightarrow \angle AMB + (\angle MAB + \angle MAB) = 180^\circ$	
(4) $\angle AMB + 90^\circ = 180^\circ$	From (2) and (3)
$\Rightarrow \angle AMB = 180^\circ - 90^\circ$	
$\Rightarrow \angle AMB = 90^\circ$	**(Q.E.D.)**

(*ii*) Given : a || gm ABCD in which bisector AR of $\angle A$ meets DC in R and bisector CQ of $\angle C$ meets AB in Q.



To Prove : AR || CQ

Proof :

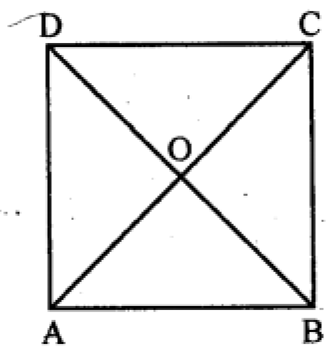
Statements	Reasons
(1) In ∥ gm ABCD	
$\angle A = \angle C$	opposite angles of ∥ gm are equal.
$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$	multiplying both sides by $\frac{1}{2}$.
$\Rightarrow \angle DAR = \angle BCQ$	(*i*) AR is bisector of $\frac{1}{2}\angle A = \angle DAR$ (*ii*) CQ is bisector of $\frac{1}{2}\angle C = \angle BCQ$
(2) In ΔADR and ΔCBQ	
$\angle DAR = \angle BCQ$	Proved in (1)
$AD = BC$	opposite sides of ∥ gm ABCD are equal.
$\angle D = \angle B$	opposite sides of ∥ gm ABCD are equal.
$\therefore \Delta ADR \cong \Delta CBQ$	[By A.S.A. axiom of congruency]
$\therefore \angle DRA = \angle BCQ$	[c.p.c.t.]
(3) $\angle DRA = \angle RAQ$	Alternate angles [DC ∥ AB, ∵ ABCD is a ∥ gm]
(4) $\angle RAQ = \angle BCQ$	From (2) and (3)

But these are corresponding angles

∴ AR ∥ CQ **(Q.E.D.)**

(*iii*) **Given :** In quadrilateral ABCD, diagonals AC and BD are equal and bisect each other at right angles

To prove : ABCD is a square



Proof : In ΔAOB and ΔCOD

$AO = OC$ (given)

$BO = OD$ (given)

$\angle AOB = \angle COD$ (vertically opposite angles)

$\therefore$ $\Delta AOB \cong \Delta COD$ (SAS axiom)

$\therefore$ $AB = CD$

and $\angle OAB = \angle OCD$

But these are alternate angles

$\therefore$ $AB \parallel CD$

$\therefore$ ABCD is a parallelogram

$\because$ In a parallelogram, the diagonal bisect each other and are equal

$\therefore$ ABCD is a square

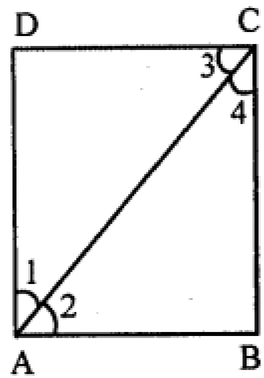
Question 12.

(i) If ABCD is a rectangle in which the diagonal BD bisect ∠B, then show that ABCD is a square.

(ii) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:

(*i*) ABCD is a rectangle and its diagonals AC bisects ∠A and ∠C



To prove : ABCD is a square

Proof : ∵ Opposite sides of a rectangle are equal and each angle is 90°

∵ AC bisects ∠A and ∠C

∴ ∠1 = ∠2 and ∠3 = ∠4

But ∠A = ∠C = 90°

∴ ∠2 = 45° and ∠4 = 45°

∴ AB = BC (Opposite sides of equal angles)

But AB = CD and BC = AD

∴ AB = BC = CD = DA

∴ ABCD is a square

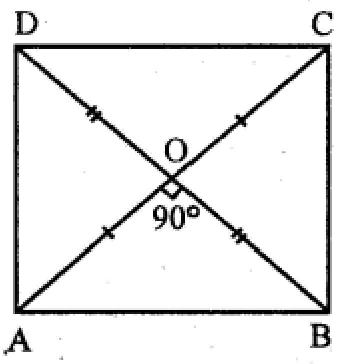
(*ii*) In quadrilateral ABCD diagonals AC and BD are equal and bisect each other at right angle

To prove : ABCD is a square

Proof : In ΔAOB and ΔBOC

AO = CO

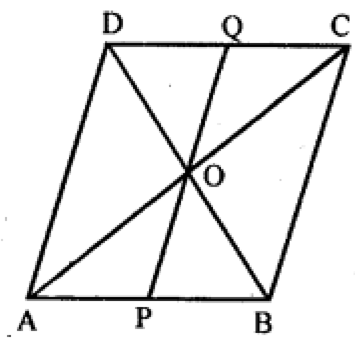
(Diagonals bisect each other at right angle)



OB = OB (Common)
∠AOB = ∠COB (Each 90°)
∴ ΔAOB ≅ ΔBOC (SAS axiom)
∴ AB = BC ...(*i*)
Similarly in ΔBOC and ΔCOD
OB = OD
(Diagonals bisect each other at right angles)
OC = OC (Common)
∠BOC = ∠COD (Each 90°)
∴ ΔBOC ≅ ΔCOD
∴ BC = CD ...(*ii*)
From (*i*) and (*ii*),
AB = BC = CD = DA
∴ ABCD is a square

Question 13.
P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.
Solution:

ABCD is a parallelogram P and Q are the points on AB and DC. Diagonals AC and BD intersect each other at O.



To prove : OP = OQ

Proof : ∵ Diagonals of ||gm ABCD bisect each other at O

∴ AO = OC and BO = OD

Now in ΔAOP and ΔCOQ

AO = OC (Proved)

∠OAP = ∠OCQ (Alternate angles)

∠AOP = ∠COQ (Vertically opposite angles)

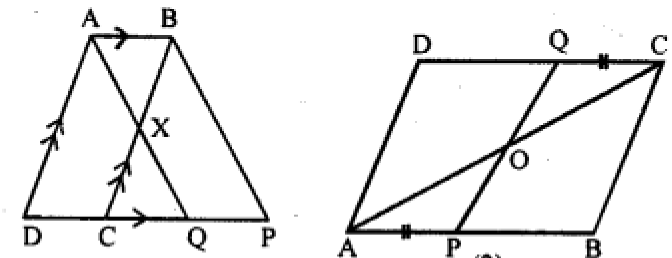
∴ ΔAOP ≅ ΔCOQ (SAS axiom)

∴ OP = OQ

Hence O bisects PQ

Question 14.

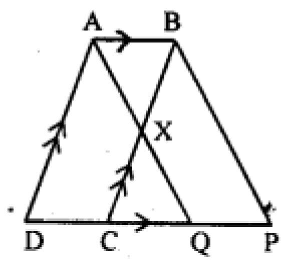
(a) In figure (1) given below, ABCD is a parallelogram and X is mid-point of BC. The line AX produced meets DC produced at Q. The parallelogram ABPQ is completed. Prove that:
(i) the triangles ABX and QCX are congruent;
(ii)DC = CQ = QP
(b) In figure (2) given below, points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that AP = CQ. Show that AC and PQ bisect each other.



(1) (2)

Solution:

(a) **Given :** ABCD is a parallelogram and X is mid-point of BC. The line AX produced meets DC produced at Q and ABPQ is a || gm.



To Prove : (*i*) $\Delta ABX \cong \Delta QCX$

(*ii*) DC = CQ = QP

Proof :

Statements	**Reasons**
(1) In ΔABX and ΔQCX	
BX = XC	X is the mid-point of BC

$\angle AXB = \angle CXQ$ vertically opposite angles

$\angle XCQ = \angle XBA$ Alternate angle

($\because AB \parallel CQ$)

$\therefore \Delta ABX \cong \Delta QCX$ [A.S.A.]

(2) $\therefore CQ = AB$ [c.p.c.t.]

(3) $AB = DC$ ABCD is a $\parallel$ gm

(4) $AB = QP$ ABPQ is a $\parallel$ gm

(5) $DC = CQ = QP$ From (2), (3) and (4)

(Q.E.D.)

(b) In $\parallel$gm ABCD,

P and Q are points on AB and CD respectively PQ and AC intersect each other at O and AP = CQ

To prove : AC and PQ bisect each other

i.e., AO = OC, PO = OQ

Proof : In ΔAOP and ΔCOQ

$AP = CQ$ (Given)

$\angle AOP = \angle COQ$ (Vertically opposite angles)

$\angle OAP = \angle OCQ$ (Alternate angles)

$\therefore \Delta AOP \cong \Delta COQ$ (AAS axiom)

$\therefore OP = OQ$ (c.p.c.t.)

and $OA = OC$ (c.p.c.t.)

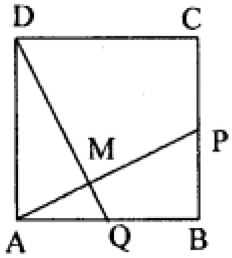
Hence AC and PQ bisect each other.

Question 15.

ABCD is a square. A is joined to a point P on BC and D is joined to a point Q on AB. If AP=DQ, prove that AP and DQ are perpendicular to each other.

Solution:

Given : ABCD is a square. P is any point on BC and Q is any point on AB and these points are taken such that AP = DQ.



To Prove : AP ⊥ DQ.

Proof :

Statements	**Reasons**
(1) In ΔABP and ΔADQ	
AP = DQ	given
AD = AB	ABCD is a square
$\angle DAQ = \angle ABP$	ABCD is a square and each 90°
$\therefore \Delta ABP \cong \Delta ADQ$	[R.H.S. axiom of congruency]
$\therefore \angle BAP = \angle ADQ$	
(2) But $\angle BAD = 90°$	each angle of square is 90°
(3) $\angle BAD = \angle BAP + \angle PAD$	
$90° = \angle BAP + \angle PAD$	From (2)
$\Rightarrow \angle BAP + \angle PAD = 90°$	
$\Rightarrow \angle PAD + \angle ADQ = 90°$	From (1)
(4) In ΔADM,	
$\angle MAD + \angle ADM + \angle AMD = 180°$	Sum of all angles in a triangle is 180°
$\Rightarrow 90° + \angle AMD = 180°$	From (3)
$\Rightarrow \angle AMD = 180° - 90°$	
$\Rightarrow \angle AMD = 90°$	
$\therefore$ DM ⊥ AP	
$\Rightarrow$ DQ ⊥ AP	

Hence, AP ⊥ DQ **(Q.E.D.)**

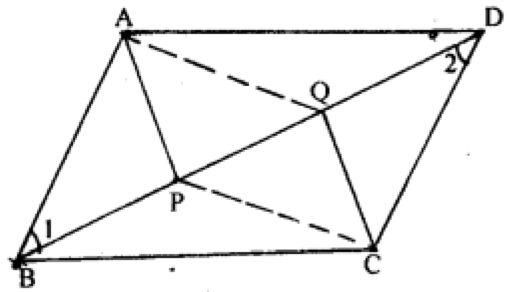
Question 16.

If P and Q are points of trisection of the diagonal BD of a parallelogram ABCD, prove that CQ || AP.

Solution:

Given : ABCD is a || gm in which BP = PQ = QD

To Prove : CQ || AP



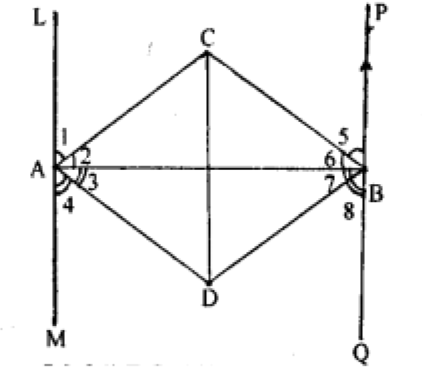
Proof :

Statements	**Reasons**
(1) In || gm ABCD	
AB = CD	opposite sides of || gm are equal.
(2) In || gm ABCD	
AB = CD	From (1)
and BD is the transversal	
$\therefore \angle 1 = \angle 2$.	Alternate angles.
(3) In ΔABP and ΔDCQ,	
AB = CD	opposite sides of || gm are equal.
$\angle 1 = \angle 2$	From (2)
BP = QD	given
$\therefore \Delta ABP \cong \Delta DCQ$	[S.A.S. axiom of congruency]
$\therefore$ AP = QC	[c.p.c.t.]
Also $\angle APB = \angle DQC$	[c.p.c.t.]
$\Rightarrow -\angle APB = -\angle DQC$	multiplying both sides by (–1)
$\Rightarrow 180° - \angle APB = 180° - \angle DQC$	Adding 180° both sides
$\angle APQ = \angle CQP$	
But these are alternate angles.	

$\therefore$ AP || QC $\Rightarrow$ CQ || AP **(Q.E.D.)**

Question 17.

A transversal cuts two parallel lines at A and B. The two interior angles at A are bisected and so are the two interior angles at B ; the four bisectors form a quadrilateral ABCD. Prove that
(i) ABCD is a rectangle.
(ii) CD is parallel to the original parallel lines.



Solution:

Given : LM || PQ AB transversal line cut ∠M at A and PQ at B.

AC, AD, BC and BD is the bisector of ∠LAB, ∠BAM, ∠PAB and ∠ABQ respectively.

AC and BC intersect at C and AD and BD intersect at D. A quadrilateral ABCD is formed.

To Prove : (*i*) ABCD is a rectangle

(*ii*) CD || LM and PQ

Proof :

Statements	Reasons
(1) $\angle LAB + \angle BAM = 180°$	LAM is a st. line

$\Rightarrow \frac{1}{2}(\angle LAB + \angle BAM) = 90^\circ$	Multiplying both sides by $\frac{1}{2}$.
$\Rightarrow \frac{1}{2}\angle LAB + \frac{1}{2}\angle BAM = 90^\circ$	
$\Rightarrow \angle 2 + \angle 3 = 90^\circ$	AC & AD is bisector of $\angle LAB$ & $\angle BAM$ respectively. $\therefore \frac{1}{2}\angle LAB = \angle 2$ and $\frac{1}{2}\angle LAB = \angle 3$
$\Rightarrow \angle CAD = 90^\circ$	
$\Rightarrow \angle A = 90^\circ$	
(2) Similarly, $\angle PBA + \angle QBA = 180^\circ$	PBQ is a st. line
$\Rightarrow \frac{1}{2}\angle PBA + \frac{1}{2}\angle QBA$	Multiplying both sides by $\frac{1}{2}$
$\Rightarrow \angle 6 + \angle 7 = 90^\circ$	$\because$ BC and BD is bisector of $\angle PBA$ and $\angle QBA$ respectively. $\frac{1}{2}\angle PBA = \angle 6$, $\frac{1}{2}\angle QBA = \angle 7$

$\Rightarrow \quad \angle CBD = 90^\circ$

$\Rightarrow \quad \angle B = 90^\circ$

(3) $\therefore \angle LAB + \angle ABP = 180^\circ$	Sum of co-interior angles is 180° [LM ‖ PQ given]
$\frac{1}{2}\angle LAB + \frac{1}{2}\angle ABP = 90^\circ$	Multiplying both sides by $\frac{1}{2}$
$\angle 2 + \angle 6 = 90^\circ$	$\therefore$ AC and BC is bisector of $\angle LAB$ and $\angle PBA$ respectively. $\therefore \frac{1}{2}\angle LAB = \angle 2$ and $\frac{1}{2}\angle APB = \angle 6$

(4) In ΔACB

$\angle 2 + \angle 6 + \angle C = 180^\circ$ — Sum of all angles in a triangle is 180°

$\Rightarrow (\angle 2 + \angle 6) + \angle C = 180^\circ$

$\Rightarrow 90^\circ + \angle C = 180^\circ$ — using (6)

$\Rightarrow \angle C = 90^\circ$

(5) $\therefore \angle MAB + \angle ABQ = 180^\circ$ — Sum of co-interior angles is 180° [(LM || PQ) given]

$\Rightarrow \frac{1}{2} \angle MAB + \frac{1}{2} \angle ABQ = \frac{180^\circ}{2}$ — Multiplying both sides by $\frac{1}{2}$.

$\Rightarrow \angle 3 + \angle 7 = 90^\circ.$ — $\because$ AD and BD bisect the $\angle MAB$ and $\angle ABQ$

$\therefore \frac{1}{2} \angle MAB = \angle 3$ and $\frac{1}{2} \angle ABQ = \angle 7$

(6) In ΔADB,

$\because \angle 3 + \angle 7 + \angle D = 180^\circ$ — Sum of all angles in a triangle is 180°

$\Rightarrow (\angle 3 + \angle 7) + \angle D = 180^\circ$

$\Rightarrow 90^\circ + \angle D = 180^\circ$ — From (5)

$\Rightarrow \angle D = 180^\circ - 90^\circ$

$\Rightarrow \angle D = 90^\circ$

(7) $\angle LAB + \angle BAM$ — From (1) and (3)

$= \angle BAM = \angle ABP$

$\Rightarrow \frac{1}{2} \angle BAM = \frac{1}{2} \angle ABP$ — Multiplying both sides by $\frac{1}{2}$

$\Rightarrow \angle 3 = \angle 6$ — $\because$ AD and BC is bisector of $\angle BAM$ & $\angle ABP$ respectively.

$\therefore \frac{1}{2} \angle BAM = \angle 3$

and $\frac{1}{2} \angle ABP = \angle 6$

Similarly $\angle 2 = \angle 7$

(8) In ΔABC and ΔABD

$\angle 2 = \angle 7$	From (7)
AB = AB	common
$\angle 6 = \angle 3$	From (7)
$\therefore \Delta ABC \cong \Delta ABD$	[By A.S.A. axiom of congruency]
$\therefore$ AC = DB	[c.p.c.t.]
Also CB = AD	[c.p.c.t.]
(9) $\angle A = \angle B = \angle C = \angle D = 90°$	From (1), (2), (4) and (6)
AC = DB	Proved in (8)
CB = AD	Proved in (8)
$\therefore$ ABCD is a rectangle.	
(10) $\because$ ABCD is a rectangle	From (9)
OA = OD	Diagonals of rectangle bisect each other.
(11) In ΔAOD	
OA = OD	From (10)
$\therefore \angle 9 = \angle 3$	Angles opposite to equal sides are equal.
(12) $\angle 3 = \angle 4$	AD bisects $\angle MAB$
(13) $\angle 9 = \angle 4$	From (11) and (12)

But these are alternate angles.

$\therefore$ OD || LM

$\Rightarrow$ CD || LM

Similarly we can prove that

$\angle 10 = \angle 8$

But these are alternate angles.

$\therefore$ OD || PQ

$\Rightarrow$ CD || PQ.

(14) CD || LM	Proved in (13)
CD || PQ	Proved in (19)

(Q.E.D.)

Question 18.

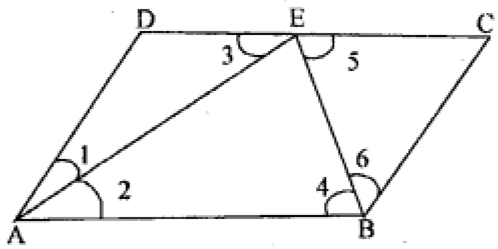
In a parallelogram ABCD, the bisector of ∠A meets DC in E and AB = 2 AD. Prove that

(i) BE bisects ∠B

(ii) ∠AEB = a right angle.

Solution:

Given : ABCD is a || gm in which bisectors of angle A and B meets in E and AB = 2 AD.



To Prove : (*i*) BE bisects ∠B

(*ii*) ∠AEB = a right angle *i.e.* ∠AEB = 90°

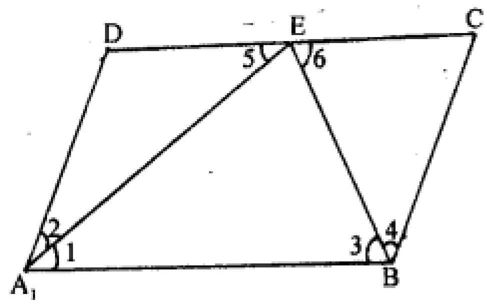
Proof :

Statements	Reasons
(1) In ‖ gm ABCD	
∠1 = ∠2	AD bisector of ∠A .
(2) AB ‖ DC	
and AE is the transversal	
∴ ∠2 = ∠3	(alternate angles)
(3) ∠1 = ∠2	From (1) and (2)
(4) In Δ ADE	
∠1 = ∠3	Prove in (3),
∴ DE = AD	Sides opposite equal angles are equal
⇒ AD = DE	
(5) AB = 2 AD	given
$\Rightarrow \frac{AB}{2} = AD$	
$\Rightarrow \frac{AB}{2} = DE$	using (4)
$\Rightarrow \frac{DC}{2} = DE$	AB = DC (∵ opposite sides of ‖ gm are equal)
∴ E is the mid-point of D	
∴ DE = EC	

(6) AD = BC	opposite sides of ∥ gm are equal.
(7) DE = BC	From (4) and (6)
(8) EC = BC	From (5) and (7)
(9) In ΔBCE	
EC = BC	Proved in (8)
$\therefore \angle 6 = \angle 5$	Angles opposite equal sides are equal
(10) AB ∥ DC and BE is the transversal	
$\therefore \angle 4 = \angle 5$	Alternate angles.
(11) $\angle 4 = \angle 6$	From (9) and (10)
$\therefore$ BE is bisector of $\angle B$	
(12) $\angle A + \angle B = 180°$	Sum of co-interior angles is equal to 180° (AD ∥ BC)
$\frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{180°}{2}$	Multiplying both sides by $\frac{1}{2}$
$\angle 2 + \angle 4 = 90°$	AE is bisector of $\angle A$ and BE is bisector of $\angle B$.
(13) In ΔAPB,	
$\angle AEB + \angle 2 + \angle 4 = 180°$	
$\Rightarrow \angle AEB + 90° = 180°$	From (12)
$\Rightarrow \angle AEB = 180° - 90°$	
$\Rightarrow \angle AEB = 90°$	

(Q.E.D.)

Question 19.
ABCD is a parallelogram, bisectors of angles A and B meet at E which lie on DC. Prove that AB
Solution:



Given : ABCD is a parallelogram in which bisector of $\angle A$ and $\angle B$ meets DC in E

To Prove : AB = 2 AD

Proof :

Statements	**Reasons**
(1) In parallelogram ABCD AB ∥ DC $\angle 1 = \angle 5$	Alternate angles (∵ AE is transversal)
(2) $\angle 1 = \angle 2$	AE is bisector of $\angle A$ (given)
(3) $\angle 2 = \angle 5$	From (1) and (2)
In ΔAED, DE = AD	equal angles have equal sides oppo- -site to them.
(4) $\angle 3 = \angle 6$	Alternate angles
(5) $\angle 3 = \angle 4$	[∵ BE is bisector of $\angle B$ (given)]

(6) $\angle 4 = \angle 6$	From (4) and (5)
In ΔBCE	
$BC = EC$	equal angles have equal sides opposite to them.
(7) $AD = BC$	opposite sides of ‖ gm are equal.
(8) $AD = DE = EC$	From (3), (6) and (7)
(9) $AB = DC$	opposite sides of ‖ gm are equal.
$AB = DE + EC$	
$AB = AD + AD$	From (8)
$AB = 2\,AD$	

(Q.E.D.)

Question 20.

ABCD is a square and the diagonals intersect at O. If P is a point on AB such that AO =AP, prove that 3 ∠POB = ∠AOP.

Solution:

Given : ABCD is a square and the diagonals intersect at O. P is a point on AB such that AO = AP.

To Prove : 3 $\angle POB = \angle AOP$

Proof :

Statements	**Reasons**
(1) In square ABCD AC is a diagonal $\therefore \angle CAB = 45°$ $\Rightarrow \angle OAP = 45°$	In square diagonals make 45° with side.
(2) In ΔAOP.	
$\angle OAP = 45°$	From (1)
AO = AP	equal side have a equal angles opposite to them.
$\therefore \angle AOP + \angle APO + \angle OAP = 180°$	Sum of all angles in a triangle is 180°
$\angle AOP + \angle AOP + 45° = 180°$	
$2\angle AOP = 180° - 45°$	
$2\angle AOP = 135°$	

$\angle AOP = \frac{135^\circ}{2}$

(3) $\angle AOB = 90^\circ$ In square ABCD diagonals bisect at right angles.

$\Rightarrow \angle AOP + \angle POB = 90^\circ$

$\Rightarrow \frac{135^\circ}{2} + \angle POB = 90^\circ$ From (2)

$\Rightarrow \angle POB = 90^\circ - \frac{135^\circ}{2}$

$\Rightarrow \angle POB = \frac{180^\circ - 135^\circ}{2}$

$\Rightarrow \angle POB = \frac{45^\circ}{2}$

$3\angle POB = \frac{135^\circ}{2}$ Multiplying both sides by 3,

(4) $\angle AOP = 3\ \angle POB$ From (2) and (3)

(Q.E.D.)

Question 21.

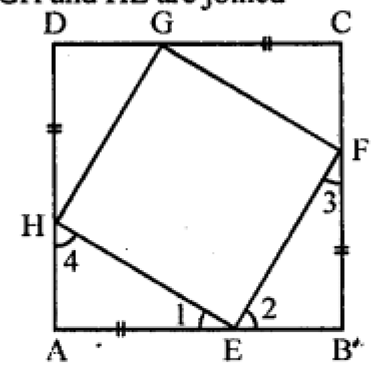
ABCD is a square. E, F, G and H are points on the sides AB, BC, CD and DA respectively such that AE = BF = CG = DH. Prove that EFGH is a square.

Solution:

Given : ABCD is a square in which E, F, G and H are points on AB, BC, CD and DA

Such that AE = BF = CG = DH

EF, FG, GH and HE are joined



To prove : EFGH is a square

Prove : $\because$ AE = BF = CG = DH

$\therefore$ EB = FC = GD = HA

Now in ΔAEH and ΔBFE

AE = BF (given)

AH = EB (proved)

$\angle A = \angle B$ (each 90°)

$\therefore$ $\Delta AEH \cong \Delta BFE$ (S.A.S. axiom)

$\therefore$ EH = EF (c.p.c.t.)

and $\angle 4 = \angle 2$ (c.p.c.t.)

But $\angle 1 + \angle 4 = 90°$

$\therefore$ $\angle 1 + \angle 2 = 90°$ ($\because \angle 4 = \angle 2$)

$\therefore$ $\angle HEF = 90°$

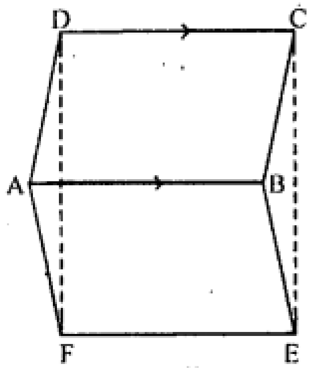
Hence EFGH is a square.

Hence proved.

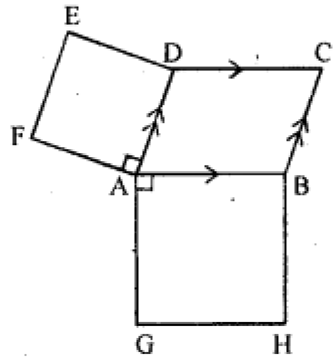
Question 22.

(a) In the Figure (1) given below, ABCD and ABEF are parallelograms. Prove that
(i) CDFE is a parallelogram
(ii) FD = EC
(iii) Δ AFD = ΔBEC.
(b) In the figure (2) given below, ABCD is a parallelogram, ADEF and AGHB are two squares. Prove that FG = AC

Solution:



(1)



(2)

(*a*) Given : ABCD and ABEF are || gms

To Prove :(*i*) CDEF is || gm

(*ii*) FD = EC

(*iii*) $\Delta AFD \cong \Delta BEC$

Proof :

Statements	**Reasons**
(1) DC || AB and DC = AB	ABCD is a || gm
(2) FE || AB and FE = AB	ABEF is a || gm
(3) DC || FE and DC = FE	From (1) and (2)
∴ CDFE is a || gm	If a pair of opposite sides of a quadrilateral are parallel and equal
It is a || gm.	
(4) CDFE is a || gm FD = EC	opposite sides of || gm CDFE are equal.
(5) In ΔAFD and ΔBEC	opposite sides || gm
AD = BC	ABCD are equal.
AF = BE	opposite sides of ||gm ABEF are equal.

FD = EC	From (4)
$\therefore \Delta AFD \cong \Delta BEC$	[By S.S.S. axiom of congruency]
	(Q.E.D.)

(*b*) **Given :** ABCD is a || gm, ADEF and AGHB are two squares.

To Prove : FG = AC

Proof :

Statements	**Reasons**
(1) $\angle FAG + 90° + 90° + \angle BAD = 36°$	At a point total angle is 360°
$\Rightarrow \angle FAG = 36° - 90° - 90° - \angle BAD$	
$\Rightarrow \angle FAG = 180° - \angle BAD$	ABCD is a || gm
(2) $\angle B + \angle BAD = 180°$	Sum of adjacent angle in ||gm is equal to 180°
$\Rightarrow \angle B = 180° - \angle BAD$	
(3) $\angle FAG = \angle B$	From (1) and (3)
(4) In ΔAFG and ΔABC AF = BC	FA DE and ABCD both are square on the same base DA.
Similarly AG = AB	
$\angle FAG = \angle B$	From (3)
$\therefore \Delta AFG \cong \Delta ABC$	[By S.A.S. axiom of congruency]
$\therefore$ FG = AC	[c.p.c.t.]
	(Q.E.D.)

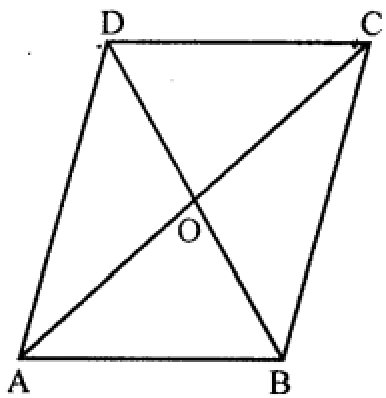
Question 23.

ABCD is a rhombus in which ∠A = 60°. Find the ratio AC : BD.

Solution:

Let each side of the rhombus ABCD = a

$\because \angle A = 60°$



$\therefore$ ΔABD is an equilateral triangle

$\therefore \quad BD = AB = a$

$\because$ The diagonals of a rhombus bisect each other at right angles,

$\therefore$ In right Δ AOB,

$AO^2 + OB^2 = AB^2$

$$\Rightarrow AO^2 = AB^2 - OB^2 = a^2 - \left(\frac{1}{2}a\right)^2$$

$$= a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$$

$$\therefore \quad AO = \sqrt{\frac{3}{4}a^2} = \frac{\sqrt{3}}{2}a$$

But $\quad AC = 2\,AO = 2 \times \frac{\sqrt{3}}{2}a = \sqrt{3}\,a$

Now $\quad AC : BD = \sqrt{3}\,a : a = \sqrt{3} : 1.$

Exercise 13.2

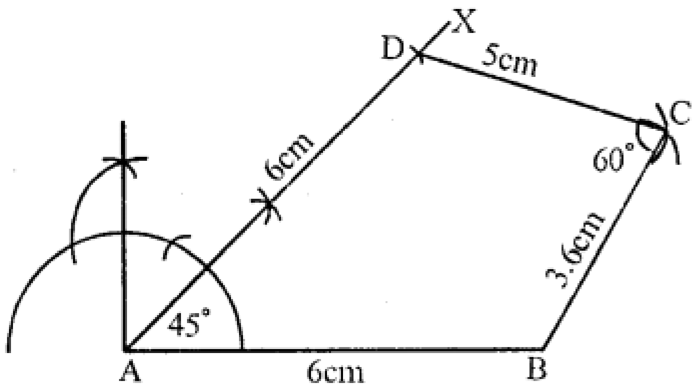
Question 1.

Using ruler and compasses only, construct the quadrilateral ABCD in which ∠ BAD = 45°, AD = AB = 6cm, BC = 3.6cm, CD = 5cm. Measure ∠ BCD.

Solution:

Steps of construction :

(i) draw a line segment AB = 6cm



(ii) At A, draw a ray AX making an angle of 45° and cut off AD = 6cm

(iii) With centre B and radius 3.6cm, and with centre D and radius 5cm, draw two arcs intersecting each other at C.

(iv) Join BC and DC,

ABCD is the required quadrilateral.

On measuring ∠BCD, it is 60°.

Question 2.

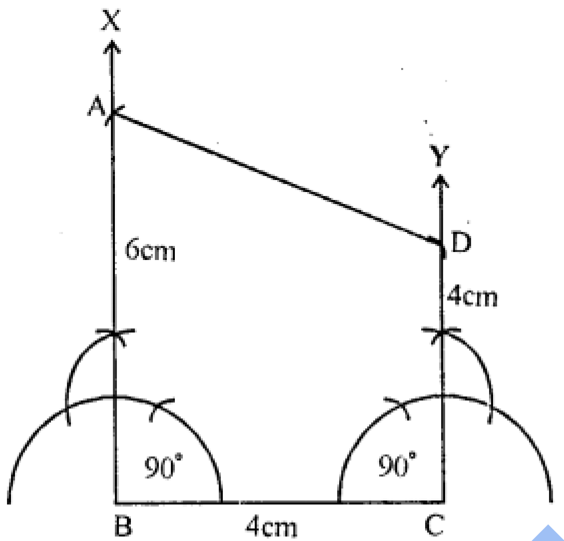
Draw a quadrilateral ABCD with AB = 6cm, BC = 4cm, CD = 4 cm and ∠ ABC = ∠ BCD = 90°

Solution:

Steps of construction :

(i) Draw a line segment BC = 4cm.

(ii) At B and C draw rays BX and CY making an angle of 90° each



(iii) From BX, cut off BA = 6cm and from CY, cut off CD = 4cm

(iv) Join AD,

ABCD is the required quadrilateral

Question 3.

Using ruler and compasses only, construct the quadrilateral ABCD given that AB = 5 cm, BC = 2.5 cm, CD = 6 cm, ∠BAD = 90° and the diagonal AC = 5.5 cm.

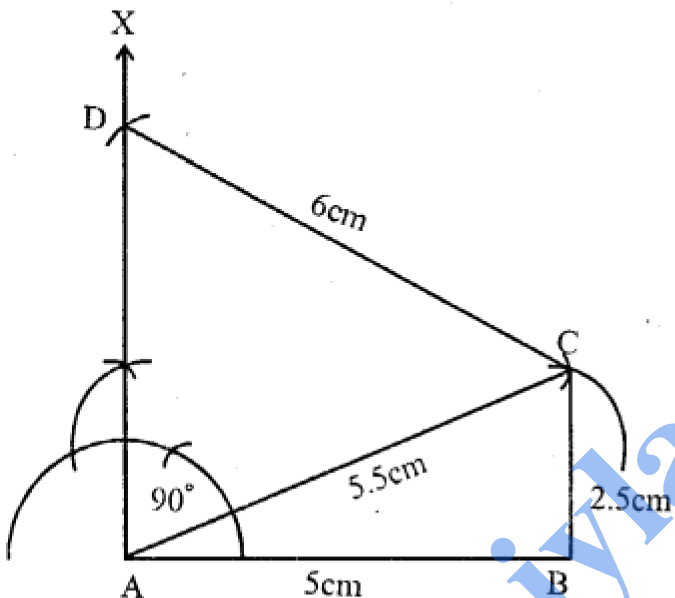
Solution:

Steps of construction :

(i) Draw a line segment AB = 5cm.

(ii) With centre A and radius 5.5 cm and with centre B and radius 2.5 cm draw arcs which intersect each other at C.

(iii) Join AC and BC.



(iv) at A, draw a ray AX making an angle of 90°.

(v) With centre C and radius 6cm, draw an arc intersecting AX at D

(v) Join CD

ABCD is the required quadrilateral.

Question 4.

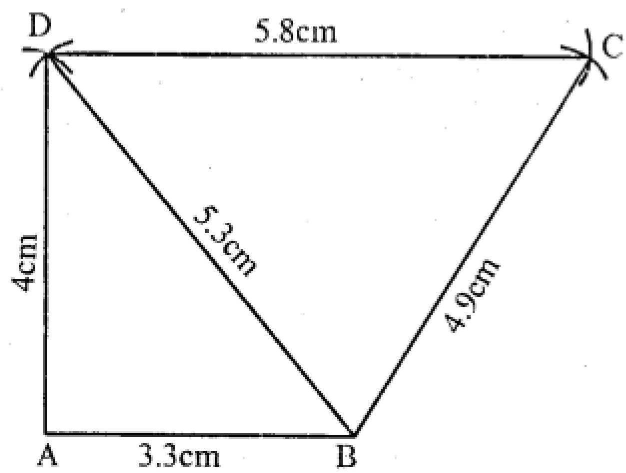
Construct a quadrilateral ABCD in which AB = 3.3 cm, BC = 4.9 cm, CD = 5.8 cm, DA = 4 cm and BD = 5.3 cm.

Solution:

Steps of construction :

(i) Draw a line segment AB = 3.3 cm

(ii) With centre A and radius 4 cm, and with centre B and radius 5.3 cm, draw ares intersecting each other at D.



(iii) Join AD and BD.

(iv) With centre B and radius 4.9 cm and with centre D and radius 5.8cm, draw arcs intersecting each other at C.

(v) Join BC and DC.

ABCD is the required quadrilateral.

Question 5.

Construct a trapezium ABCD in which AD || BC, AB = CD = 3 cm, BC = 5.2cm and AD = 4 cm

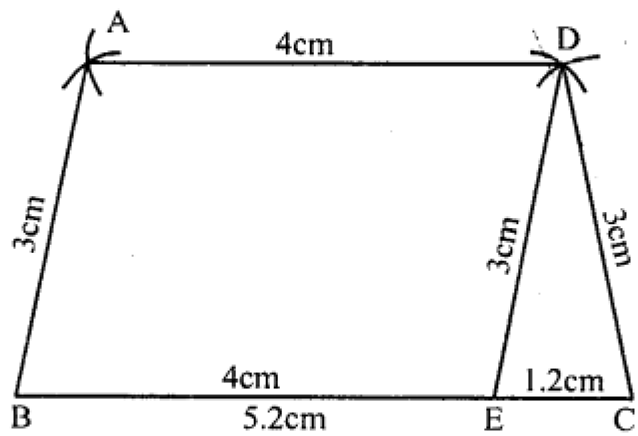
Solution:

Steps of construction :

(i) Draw a line segment BC = 5.2cm

(ii) From BC, cut off BE = AD = 4cm

(iii) With centre E and C, and radius 3 cm, draw arcs intersecting each other at D.



(iv) Join ED and CD.

(v) With centre D and radius 4cm and with centre B and radius 3 cm, draw arcs intersecting each other at A.

(vi) Join BA and DA.

ABCD is the required trapezium.

Question 6.

Construct a trapezium ABCD in which AD || BC, ∠B= 60°, AB = 5 cm. BC = 6.2 cm and CD = 4.8 cm.

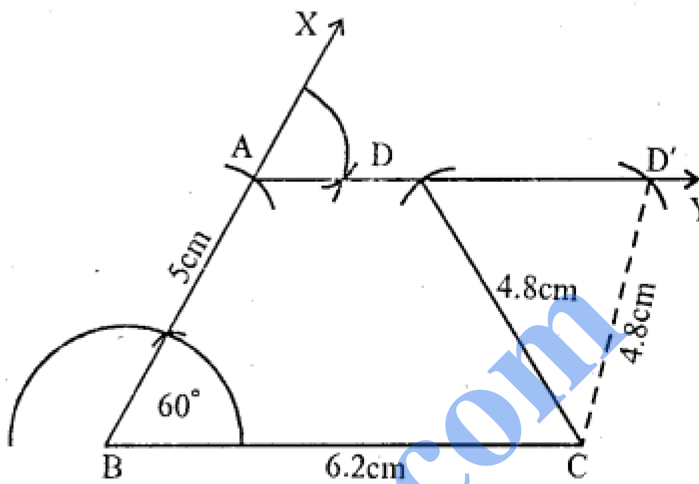
Solution:

Steps of construction.

(i) Draw a line segment BC = 6.2 cm.

(ii) At B, draw a ray BX making an angle of 60° and cut off AB = 5cm.

(iii) From A, draw a line AY parallel to BC.



(iv) With centre C and radius 4.8cm, draw an arc which intersects AY at D and D′.

(v) Join CD and CD′

Then ABCD and ABCD′ are the required two trapezium.

Question 7.

Using ruler and compasses only, construct a parallelogram ABCD with AB = 5.1 cm, BC = 7 cm and ∠ABC = 75°.

Solution:

Steps of construction.

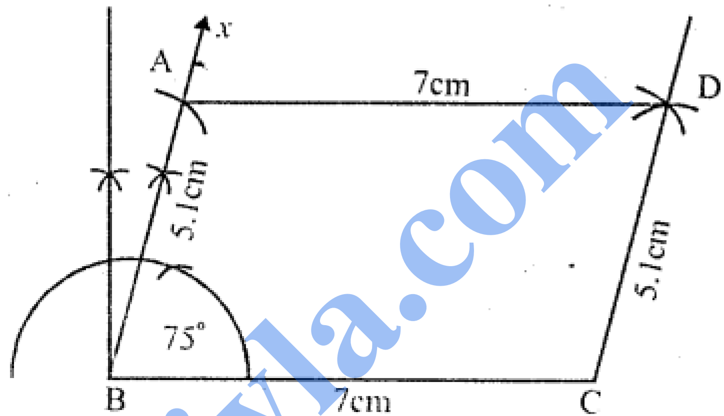
(i) Draw a line segment BC = 7 cm.

(ii) A to B, draw a ray Bx making an angle of 75° and cut off AB = 5.1 cm.

(iii) With centre A and radius 7 cm with centre C and radius 5.1 cm, draw arcs intersecting each other at D.

(iv) Join AD and CD.

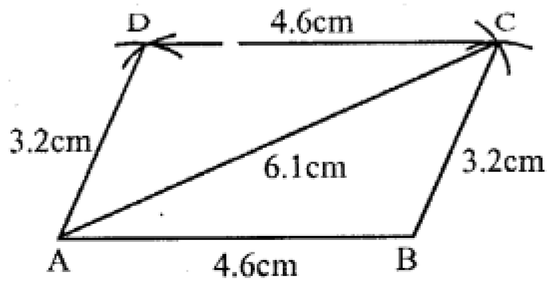
ABCD is the required parallelogram.



Question 8.

Using ruler and compasses only, construct a parallelogram ABCD in which AB = 4.6 cm, BC = 3.2 cm and AC = 6.1 cm.

Solution:



(i) Draw a line segment AB = 4.6 cm

(ii) With centre A and raduis 6.1 cm and with centre B and raduis 3.2 cm, draw arcs intersecting each other at C.

(iii) Join AC and BC.

(iv) Again with centre A and raduis 3.2 cm and with centre C and raduis 4.6 cm, draw arcs intersecting each other at D.

(v) Join AD and CD.

Then ABCD is the required parallelogram.

Question 9.

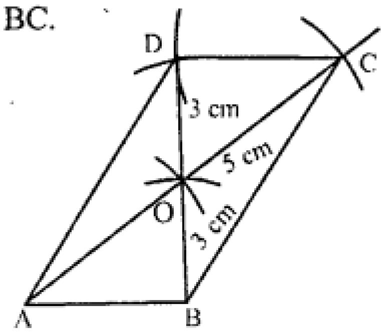
Using ruler and compasses, construct a parallelogram ABCD give that AB = 4 cm, AC = 10 cm, BD = 6 cm. Measure BC.

Solution:

Given : AB = 4 cm, AC = 10 cm, BD = 6 cm

Required : (*i*) To construct a parallelogram ABCD.

(*ii*) Length of BC.



Steps of Construction :

1. Construct triangle OAB such that

$OA = \frac{1}{2} \times AC = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$

$OB = \frac{1}{2} \times BD = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$

(Since diagonals of || gm bisect each other) and AB = 4 cm.

2. Produce AO to C such that OA = OC = 5 cm
3. Produce BO to D such that OB = OD = 3 cm
4. Join AD, BC, and CD.
5. ABCD is the required parallelogram.
6. Measure BC which is equal to 7.2 cm.

Question 10.

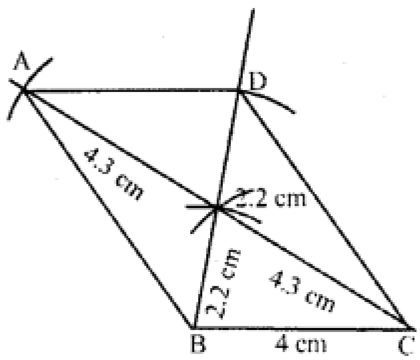
Using ruler and compasses only, construct a parallelogram ABCD such that BC = 4 cm, diagonal AC = 8.6 cm and diagonal BD = 4.4 cm. Measure the side AB.

Solution:

Given : BC = 4 cm, diagonal AC = 8.6 cm and diagonal BP = 4.4 cm

Required : (*i*) To construct a parallelogram

(*ii*) Measurement the side AB.



Steps of Construction :

1. Construct triangle OBC such that

$OB = \frac{1}{2} \times BD = \frac{1}{2} \times 4.4 \text{ cm} = 2.2 \text{ cm}$

$OC = \frac{1}{2} \times AC = \frac{1}{2} \times 8.6 \text{ cm} = 4.3 \text{ cm}$

(Since diagonals of || gm bisect each other) and BC = 4 cm

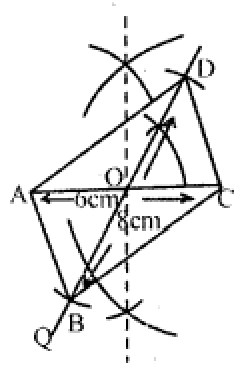
2. Produce BO to D such that BO = OD = 2.2 cm
3. Produce CO to A such that CO = OA = 4.3 cm
4. Join AB, AD and CD
5. ABCD is the required parallelogram
6. Measure the side AB, AB = 5.6 cm

Question 11.

Use ruler and compasses to construct a parallelogram with diagonals 6 cm and 8 cm in length having given the acute angle between them is 60°. Measure one of the longer sides.

Solution:

Given : Diagonal AC = 6 cm. Diagonal BD = 8 cm
Angle between the diagonals = 60°
Required : (*i*) To construct a parallelogram.
(*ii*) To measure one of longer side.



Steps of Construction :
1. Draw AC = 6 cm.
2. Find the mid-point O of AC.
(∴ Diagonals of || gm bisect each other)
3. Draw line POQ such that ∠POC = 60° and

$OB = OD = \frac{1}{2} BD = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}.$

∴ From OP cut OD = 4 cm and from OQ cut OB = 4 cm.
4. Join AB, BC, CD and DA.
5. ABCD is the required parallelogram.
6. Measure the length of side AD = 6.1 cm.

Question 12.
Using ruler and compasses only, draw a parallelogram whose diagonals are 4 cm and 6 cm long and contain an angle of 75°. Measure and write down the length of one of the shorter sides of the parallelogram.
Solution:

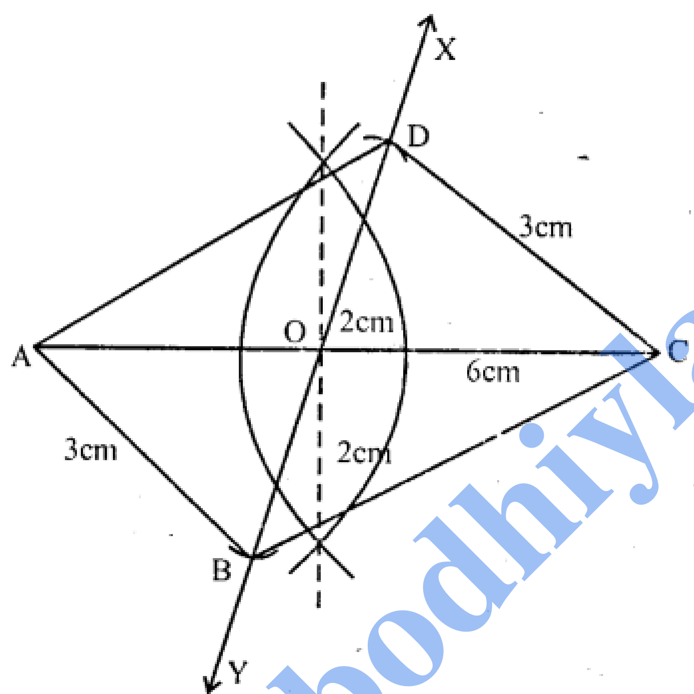
Steps of construction :

(i) Draw a line segment AC = 6cm.

(ii) Bisect AC at O.

(iii) At O, draw a ray XY making an angle of 75° at O.

(iv) From OX and OY, cut off OD = OB = $\frac{4}{2}$ = 2 cm



(v) Join AB, BC, CD and DA

Then ABCD is the required parallelogram

On measuring one of the shorter sides,

AB = CD = 3cm.

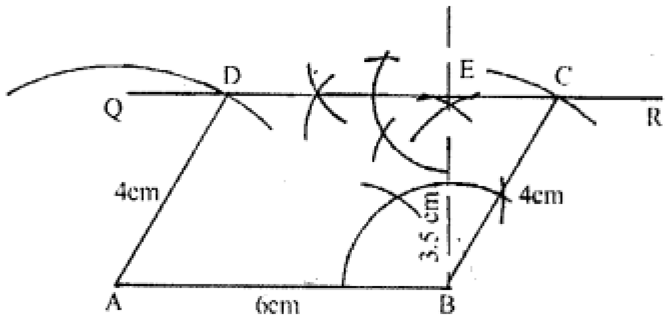
Question 13.

Using ruler and compasses only, construct a parallelogram ABCD with AB = 6 cm, altitude = 3.5 cm and side BC = 4 cm. Measure the acute angles of the parallelogram.

Solution:

Given : AB = 6 cm Altitude = 3.5 cm and BC = 4 cm.

Required : (*i*) To construct a parallelogram ABCD.
(*ii*) To measure the acute angle of parallelogram.

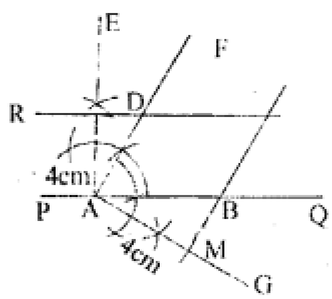


Steps of Construction :

1. Draw AB = 6 cm.
2. At B, draw BP ⊥ AB.
3. From BP, cut BE = 3.5 cm = height of || gm.
4. Through E draw QR parallel to AB.
5. With B as centre and radius BC = 4 cm draw an arc which cuts QR at C.
6. Since opposite sides of || gm are equal
∴ AD = BC = 4 cm.
∴ With A as centre and radius = 4 cm draw an arc which cut QR at D.
7. ∴ ABCD is the required parallelogram.
8. To measure the acute angle of parallelogram which is equal to 61°.

Question 14.
The perpendicular distances between the pairs of opposite sides of a parallelogram ABCD are 3 cm and 4 cm and one of its angles measures 60°. Using ruler and compasses only, construct ABCD.
Solution:

Given : $\angle BAD = 60°$
height be 3 cm and 4 cm from AB and BC respectively (say)
Required : To construct a parallelogram ABCD.



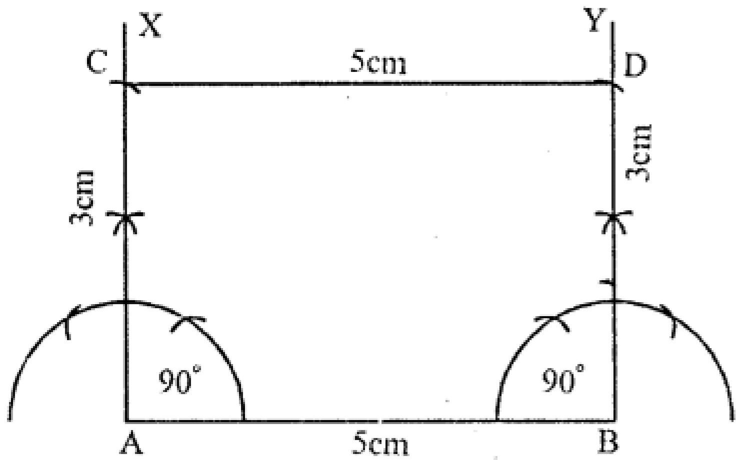
Steps of Construction :
1. Draw a st. line PQ, take a point A on it.
2. At A, construct $\angle QAF = 60°$.
3. At A, draw AE ⊥ PQ from AE cut off AN = 3cm
4. Through N draw a st. line parallel to PQ to meet AF at D.
5. At A, draw AG ⊥ AD, from AG cut off AM = 4 cm.
6. Through M, draw a st. line parallel to AD to meet AQ in B and ND in C. Then ABCD is the required parallelogram.

Question 15.
Using ruler and compasses, construct a rectangle ABCD with AB = 5cm and AD = 3 cm.

Solution:

Steps of construction :

1. Draw a st. line AB = 5cm
2. At A and B construct $\angle XAB$ and $\angle YBA = 90°$.
3. From A and B cut off AC and BD = 3 cm each
4. Join CD
5. ABCD is the required rectangle



Question 16.

Using ruler and compasses only, construct a rectangle each of whose diagonals measures 6cm and the diagonals intersect at an angle of 45°.

Solution:

Steps of construction.

(i) Draw a line segment AC = 6cm

(ii) Bisect it at O

(iii) At O, draw a ray XY making an angle of 45° at O.

(iv) From XY, cut off

$$OB = OD = \frac{6}{2} = 3 \text{ cm each}$$

(v) Join AB, BC, CD and DA

Then ABCD is the required rectangle.

Question 17.

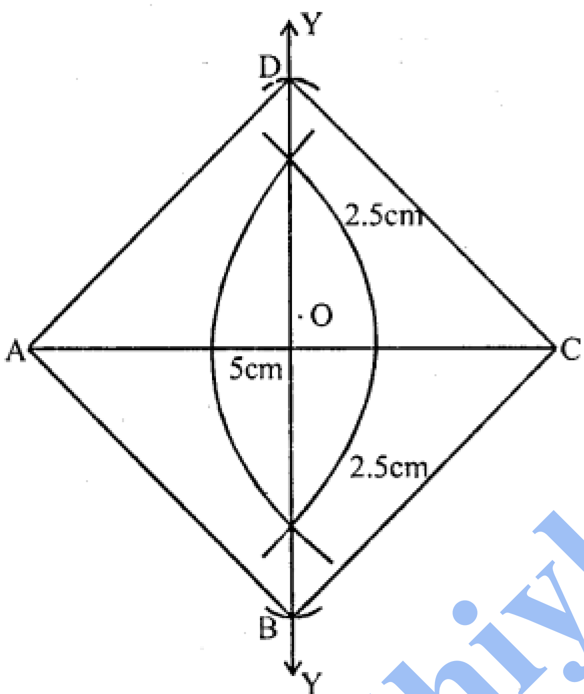
Using ruler and compasses only, construct a square having a diagonal of length 5cm. Measure its sides correct to the nearest millimeter.

Solution:

Steps of construction :

(i) Draw a line segment AC = 5cm

(ii) Draw its perpendicular bisector XY bisecting it at O



(iii) From XY, cut off

$$OB = OD = \frac{5}{2} = 2.5 \text{ cm}$$

(iv) Join AB, BC, CD and DA.

ABCD is the required square

On measuring its sides,

each side = 3.6 cm (approximately)

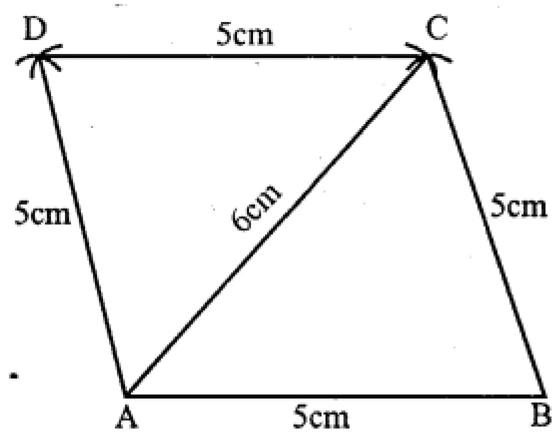
Question 18.

Using ruler and compasses only construct A rhombus ABCD given that AB 5cm, AC = 6cm measure ∠BAD.

Solution:

Steps of construction.

(i) Draw a line segment AB = 5cm



(ii) With centre A and radius 6cm, with centre B and radius 5cm, draw arcs intersecting each other at C.

(iii) Join AC and BC

(iv) With centre A and C and radius 5cm, draw arcs intersecting eachother at D

(v) Join AD and CD.

Then ABCD is a rhombus

On measuring, ∠BAD = 106°

Question 19.

Using ruler and compasses only, construct rhombus ABCD with sides of length 4cm and diagonal AC of length 5 cm. Measure ∠ABC.

Solution:

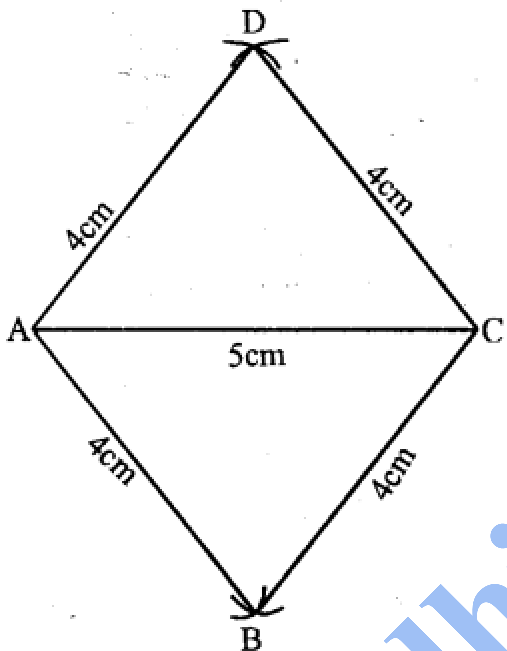
Steps of construction :

(i) Draw a line segment AC = 5cm

(ii) With centre A and C and radius 4cm, draw arcs intersecting each other above and below AC at D and B.

(iii) Join AB, BC, CD and DA

ABCD is the required rhombus.



Question 20.
Construct a rhombus PQRS whose diagonals PR and QS are 8cip and 6cm respectively.
Solution:

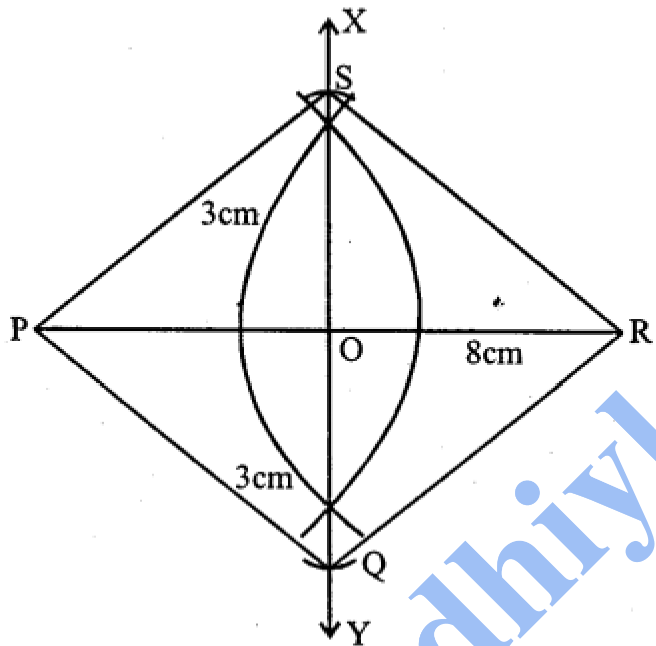
Steps of construction :

(i) Draw a line segment PR = 8cm

(ii) Draw its perpendicular bisector XY intersecting it at O.

(iii) From XY, cut off OQ = OS

$$= \frac{6}{2} = 3\text{cm each.}$$



(iv) Join PQ, QR, RS and SP

Then PQRS is the required rhombus.

Question 21.

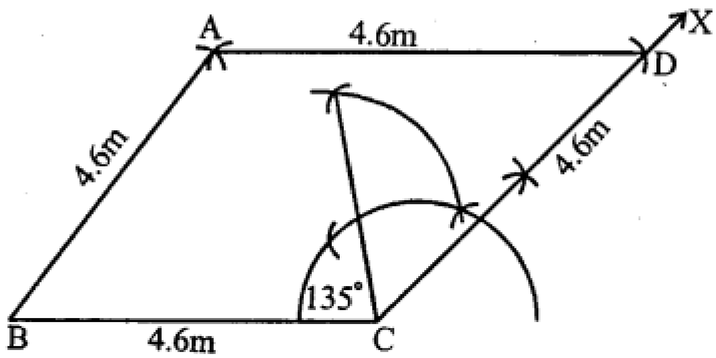
Construct a rhombus ABCD of side 4.6 cm and ∠BCD = 135°, by using ruler and compasses only.

Solution:

Steps of construction :

(i) Draw a line segment BC = 4.6 cm.

(ii) At C, draw a ray CX making an angle of 135° and cut off CD = 4.6 cm.



(iii) With centres B and D, and radius 4.6 cm draw arcs intersecting each other at A.

(iv) Join BA, DA

Then ABCD is the required rhombus.

Question 22.

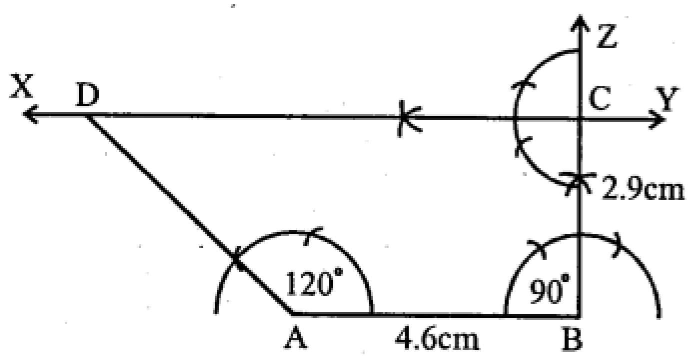
Construct a trapezium in which AB || CD, AB = 4.6 cm, ∠ ABC = 90°, ∠ DAB = 120° and the distance between parallel sides is 2.9 cm.

Solution:

Steps of construction :

(i) Draw a line segment AB = 4.6 cm

(ii) At B, draw a ray BZ making an angle of 90° and cut off BC = 2.9 cm (distance between AB and CD)



(iii) At C, draw a parallel line XY to AB.

(iv) At A, draw a ray making an angle of 120° meeting XY at D.

Then ABCD is the required trapezium.

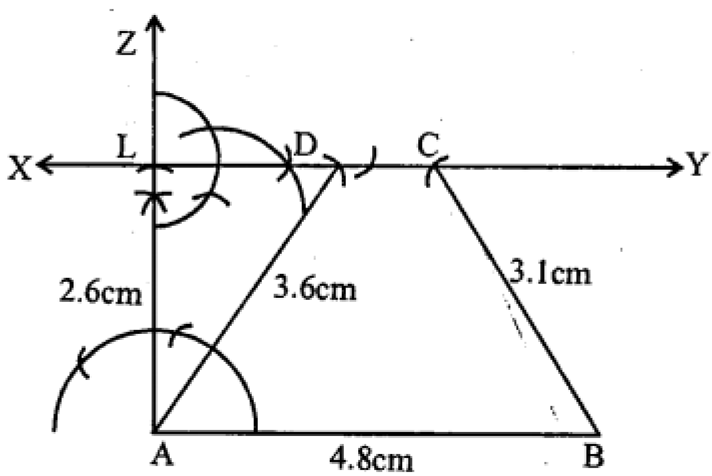
Question 23.

Construct a trapezium ABCD when one of parallel sides AB = 4.8 cm, height = 2.6cm, BC = 3.1 cm and AD = 3.6 cm.

Solution:

Steps of construction :

(i) Draw a line segment AB = 4.8cm



(ii) At A draw a ray AZ making an angle of 90° and cut off AL = 2.6cm.

(iii) At L, draw a line XY parallel to AB.

(iv) With centre A and radius 3.6cm and with centre B and radius 3.1 cm, draw arcs intersecting XY at D and C respectively.

(iv) Join AD, BC

Then ABCD is the required trapezium.

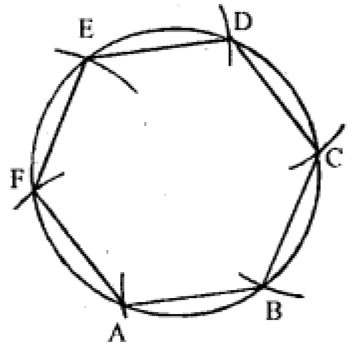
Question 24.

Construct a regular hexagon of side 2.5 cm.

Solution:

Given : Each side of regular Hexagon = 2.5 cm

Required : To construct a regular Hexagon.



Steps of Construction :

1. With O as centre and radius = 2.5 cm, draw a circle.
2. Take any point A on the circumference of circle.
3. With A as centre and radius equal to 2.5 cm, draw an arc which cuts the circumference in B.
4. With B as centre and radius = 2.5 cm, draw an arc which circumference of circle at C.
5. With C as centre and radius = 2.5 cm draw an arc which cuts circumference of circle at D.
6. With D as centre and radius = 2.5 cm draw an arc which cuts circumference of circle at E.
7. With E as centre and radius = 2.5 cm draw an arc which cuts circumference of circle at F.
8. Join AB, BC, CD, DE, EF and FA.
9. ABCDEF is the required Hexagon.

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 12):

Question 1.

Three angles of a quadrilateral are 75°, 90° and 75°. The fourth angle is

(a) 90°

(b) 95°

(c) 105°

(d) 120°

Solution:

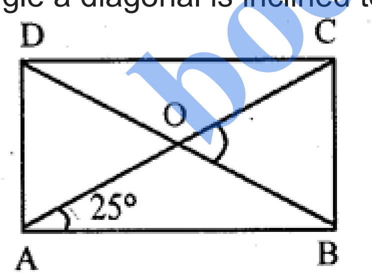
Sum of 4 angles of a quadrilateral = 360° Sum of three angles = 75° + 90° + 75° = 240°

Fourth angle = 360° – 240° = 120° **(d)**

Question 2.
A quadrilateral ABCD is a trapezium if
(a) AB = DC
(b) AD = BC
(c) ∠A + ∠C = 180°
(d) ∠B + ∠C = 180°
Solution:
A quadrilateral ABCD is a trapezium if ∠B + ∠C= 180°
(Sum of co-interior angles) **(d)**

Question 3.
If PQRS is a parallelogram, then ∠Q – ∠S is equal to
(a) 90°
(b) 120°
(c) 0°
(d) 180°
Solution:
PQRS is a parallelogram ∠Q – ∠S = 0
(∵ Opposite angles of a parallelogram, are equal) **(c)**

Question 4.
A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is
(a) 55°
(b) 50°
(c) 40°
(d) 25°
Solution:
In a rectangle a diagonal is inclined to one side of the rectangle is 25°



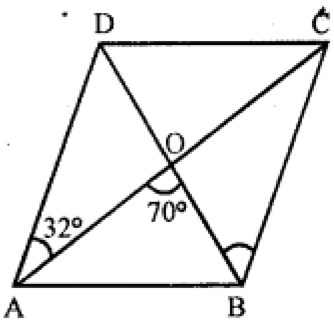
i.e. ∠OAB = 25°
But OA = OB
∴ ∠OBA = 25°
But Ext. ∠COB = ∠OAB + ∠OBA
= 25° + 25° = 50° (c)

Question 5.
ABCD is a rhombus such that ∠ACB = 40°. Then ∠ADB is

(a) 40°
(b) 45°
(c) 50°
(d) 60°
Solution:

Question 6.
The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If ∠D AC = 32° and ∠AOB = 70°, then ∠DBC is equal to
(a) 24°
(b) 86°
(c) 38°
(d) 32°
Solution:

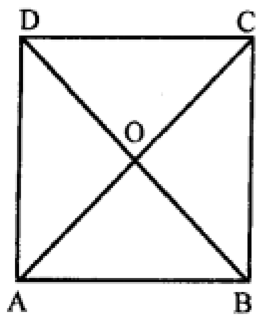
Diagonals AC and BD of parallelogram ABCD intersect each other at O



$\angle DAC = 32°$, $\angle AOB = 70°$
$\angle ADO = 70° - 32°$ ($\because$ Ext. $\angle AOB = 70°$)
$= 38°$
But $\angle DBC = \angle ADO$ or $\angle ADB$
(Alternate angles)
$\therefore$ $\angle DBC = 38°$ (c)

Question 7.
If the diagonals of a square ABCD intersect each other at O, then ∆OAB is
(a) an equilateral triangle
(b) a right angled but not an isosceles triangle
(c) an isosceles but not right angled triangle
(d) an isosceles right angled triangle

Solution:
Diagonals of square ABCD intersect each other at O
(∵ Diagonals of a square bisect each other at right angles)
(∵ $\angle AOB = 90°$ and AO = BO)



ΔOAB is an isosceles. (d)

Question 8.
If the diagonals of a quadrilateral PQRS bisect each other, then the quadrilateral PQRS must be a
(a) parallelogram
(b) rhombus
(c) rectangle
(d) square
Solution:
Diagonals of a quadrilateral PQRS bisect each other, then quadrilateral must be a parallelogram.
(∵ A rhombus, rectangle and square are also parallelogram) **(a)**

Question 9.
If the diagonals of a quadrilateral PQRS bisect each other at right angles, then the quadrilateral PQRS must be a
(a) parallelogram
(b) rectangle
(c) rhombus
(d) square
Solution:
Diagonals of quadrilateral PQRS bisect each other at right angles, then quadrilateral PQRS [must be a rhombus.
(∵ Square is also a rhombus with each angle equal to 90°) **(c)**

Question 10.
Which of the following statement is true for a parallelogram?
(a) Its diagonals are equal.
(b) Its diagonals are perpendicular to each other.

(c) The diagonals divide the parallelogram into four congruent triangles.
(d) The diagonals bisect each other.
Solution:
For a parallelogram an the statement 'The diagoanls bisect each other' is true. **(d)**

Question 11.
Which of the following is not true for a parallelogram?
(a) opposite sides are equal
(b) opposite angles are equal
(c) opposite angles are bisected by the diagonals
(d) diagonals bisect each other
Solution:
The statement that in a parallelogram, .the opposite angles are bisected by the diagonals, is not true in each case. **(c)**

Question 12.
A quadrilateral in which the diagonals are equal and bisect each other at right angles is a
(a) rectangle which is not a square
(b) rhombus which is not a square
(c) kite which is not a square
(d) square
Solution:
In a quadrilateral, if diagonals are equal and bisect each other at right angles, is a square. **(d)**

Chapter Test

Question P.Q.
The interior angles of a polygon add upto 4320°. How many sides does the polygon have ?
Solution:
Sum of interior angles of a polygon
$= (2n-4) \times 90°$

$\Rightarrow \quad 4320° = (2n-4) \times 90°$

$\Rightarrow \quad \frac{4320°}{90°} = (2n-4) \quad \Rightarrow \quad \frac{432}{9} = 2n-4$

$\Rightarrow \quad 48 = 2n-4 \quad \Rightarrow \quad 48+4 = 2n \quad \Rightarrow \quad 52 = 2n$

$\Rightarrow \quad 2n = 52 \quad \Rightarrow \quad n = \frac{52}{2} = 26$

Hence, the polygon have 26 sides.

Question P.Q.

If the ratio of an interior angle to the exterior angle of a regular polygon is 5:1, find the number of sides.

Solution:

The ratio of an interior angle to the exterior angle of a regular polygon = 5 : 1

$$\Rightarrow \frac{(2n-4)\times 90^\circ}{n} : \frac{360}{n} = 5:1$$

$$\Rightarrow (2n-4)\times 90^\circ : 360 = 5:1$$

$$\Rightarrow \frac{(2n-4)\times 90^\circ}{360} = \frac{5}{1} \Rightarrow \frac{2n-4}{4} = \frac{5}{1}$$

$$\Rightarrow 2n-4 = 5\times 4 \Rightarrow 2n-4 = 20$$

$$\Rightarrow 2n = 20+4 \Rightarrow 2n = 24 \Rightarrow n = \frac{24}{2}$$

$$\Rightarrow n = 12$$

Hence, number of sides of regular polygon = 12.

Question P.Q.

In a pentagon ABCDE, BC || ED and ∠B: ∠A : ∠E =3:4:5. Find ∠A.

Solution:

$\because$ BC || ED

$\therefore \angle C + \angle D = 180°$ (Co-interior angles)

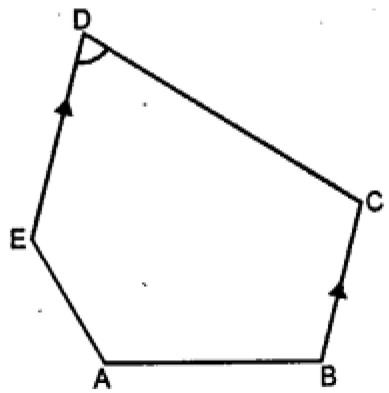
But $\angle A + \angle B + \angle C + \angle D + \angle E = 540°$

$\therefore \angle A + \angle B + \angle E \neq 180° = 540°$

$\Rightarrow \angle A + \angle B + \angle E = 540° - 180° = 360°$

But $\angle B : \angle A = \angle E = 3:4:5$

Let $\angle B = 3x, \angle A = 4x$ and $\angle E = 5x$



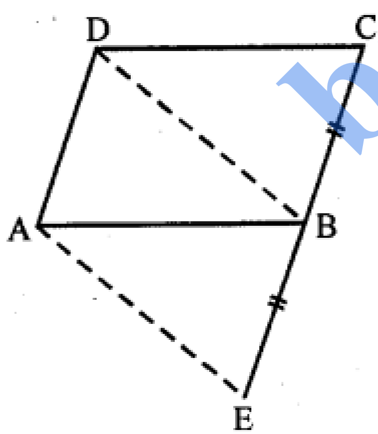
$\therefore 3x + 4x + 5x = 360° \Rightarrow 12x = 360°$

$\Rightarrow x = \frac{360°}{12} = 30°$

$\therefore A = 4x = 4 \times 30° = 120°$ **Ans.**

Question 1.

In the given figure, ABCD is a parallelogram. CB is produced to E such that BE=BC. Prove that AEBD is a parallelogram.



Solution:

In the figure, ABCD is a ||gm side CB is produced to E such that BE = BC

BD and AE are joined

To prove : AEBD is a parallelogram

Proof : In ΔAEB and ΔBDC

EB = BC (Given)

∠ABE = ∠DCB (Corresponding angles)

AB = DC (Opposite sides of ||gm)

∴ ΔAEB ≅ ΔBDC (SAS axiom)

∴ AE = DB (c.p.c.t.)

But AD = CB = BE (Given)

∵ The opposite sides are equal and ∠AEB = ∠DBC (c.p.c.t.)

But these are corresponding angle

∴ AEBD is a parallelogram

Question 2.

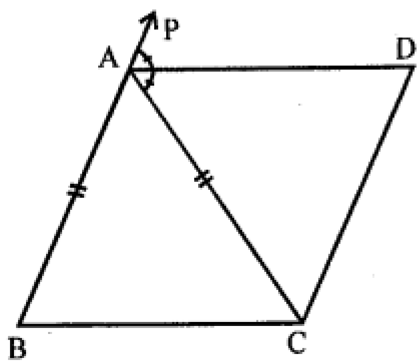
In the given figure, ABC is an isosceles triangle in which AB=AC. AD bisects exterior angle PAC and CD || BA. Show that

(i) ∠DAC=∠BCA

(ii) ABCD is a parallelogram.

Solution:

Given : In isosceles ΔABC, $AB = AC$.
AD is the bisector of ext. $\angle PAC$ and
$CD \parallel BA$



To prove : (*i*) $\angle DAC = \angle BCA$

(*ii*) ABCD is a ||gm

Proof : In ΔABC

$\because$ $AB = AC$ (Given)

$\therefore$ $\angle C = \angle B$

(Angles opposite to equal sides)

$\because$ Ext. $\angle PAC = \angle B + \angle C$

$= \angle C + \angle C = 2\angle C = 2\angle BCA$

$\therefore$ $2\angle DAC = 2\angle BCA$

$\angle DAC = \angle BCA$

But these are alternate angles

$\therefore$ $AD \parallel BC$

But $AB \parallel AC$ (Given)

$\therefore$ ABCD is a ||gm

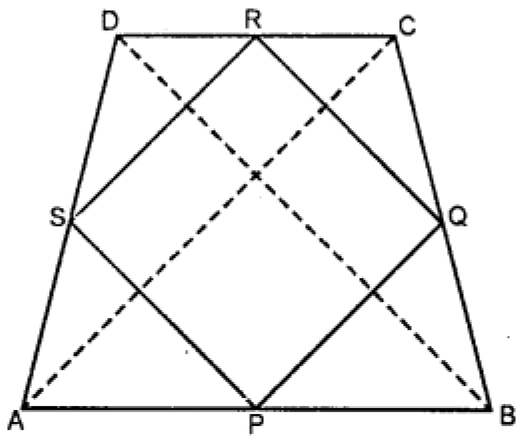
Question 3.

Prove that the quadrilateral obtained by joining the mid-points of an isosceles trapezium is a rhombus.

Solution:

Given. ABCD is an isosceles trapezium in which AB | | DC and AD = BC

P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively PQ, QR, RS and SP are joined.



To Prove. PQRS is a rhombus.

Constructions. Join AC and BD.

Proof. $\because$ ABCD is an isosceles trapezium

$\therefore$ Its diagnoals are equal

$\therefore$ AC = BD

Now in ΔABC,

P and Q are the mid-points of AB and BC

$\therefore$ PQ | | AC and $PQ = \frac{1}{2}AC$ ...(*i*)

Similarly in ΔADC,

S and R mid-points of CD and AD

$\therefore$ SR | | AC and $SR = \frac{1}{2}AC$ ...(*ii*)

from (*i*) and (*ii*)

PQ | | SR and PQ = SR

$\therefore$ PQRS is a parallelogram

Now in ΔAPS and ΔBPQ,

AP = BP (P is mid-point of AB)

AS = BQ (Half of equal sides)

$\angle A = \angle B$

($\because$ ABCD is isosceles trapezium)

$\because \quad \Delta APS \cong BPQ$

$\therefore \quad PS = PQ$

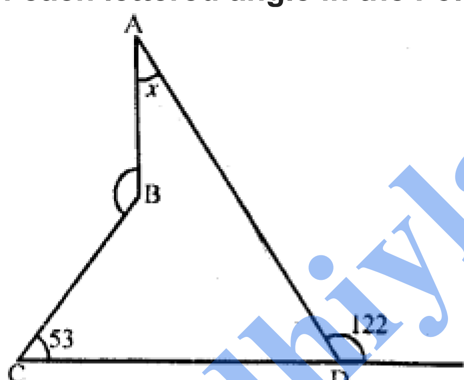
But there are the adjacent sides of a parallelogram

$\therefore$ Sides of PQRS are equal

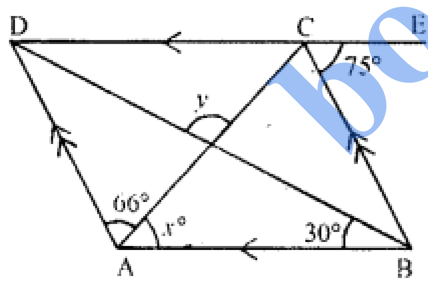
Hence PQRS is a rhombus.

Hence proved.

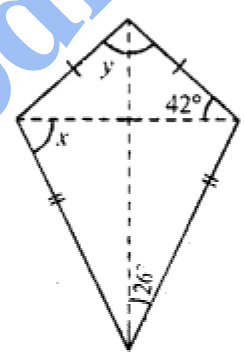
Question 4.
Find the size of each lettered angle in the Following Figures.



(i)



(ii)

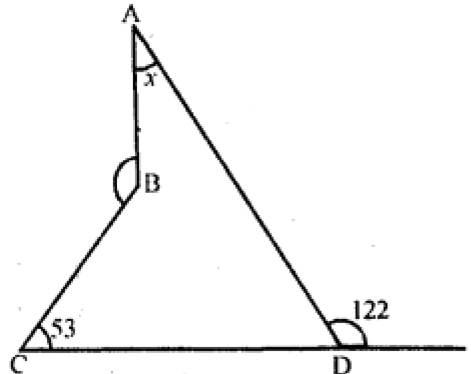


(iii)

Solution:

(*i*) $\because$ CDE is a st. line

$\therefore \quad \angle ADE + \angle ADC = 180°$



(*i*)

$122° + \angle ADC = 180°$

$\angle ADC = 180° - 122°°$

$\angle ADC = 58°$ ....(1)

$\angle ABC = 360° - 140° = 220°$

(At any point the angle is 360°) ...(2)

Now, in quadrilateral ABCD,

$\angle ADC + \angle BCD + \angle BAD + \angle ABC = 360°$

$\Rightarrow \quad 58° + 53° + x + 220° = 36o°$

[using (1) and (2)]

$\Rightarrow \quad 331° + x = 360° \quad \Rightarrow \quad x = 360° - 331°$

$\Rightarrow \quad x = 29°$ **Ans.**

(*ii*) $\because$ DE || AB (given)

$\therefore \quad \angle ECB = \angle CBA$ (Alternate angles)

$\Rightarrow \quad 75° = \angle CBA$

$\therefore \quad \angle CBA = 75°$

$\because$ AD || BC (given)

$\therefore \quad (x + 66°) + (75°) = 180°$

(co-interior angles are supplementary)

$\Rightarrow \quad x + 66° + 75° = 180° \quad \Rightarrow \quad x + 141° = 180°$

$\Rightarrow \quad x = 180° - 141°$

$\therefore \quad x = 39°$ ...(1)

Now, in ΔAMB,

Now, in Δ AMB,

$x + 30° + \angle AMB = 180°$

(sum of all angles in a triangle is 180°)

$\Rightarrow \quad 39° + 30° + \angle AMB = 180°$ [From (1)]

$\Rightarrow \quad 69° + \angle AMB = 180°$

$\Rightarrow \quad \angle AMB = 180° - 69°$

$\Rightarrow \quad \angle AMB = 111°$ ....(2)

$\because \quad \angle AMB = y$ (vertically opposite angles)

$\Rightarrow \quad 111° = y$ [From (2)]

$\therefore \quad y = 111°$

Hence, $x = 39°$ and $y = 111°$

(*iii*) In Δ ABD

AB = AD (given)

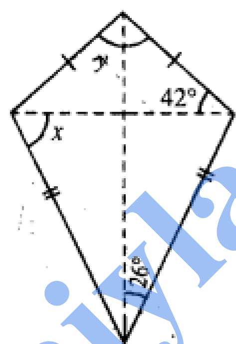
$\angle ABD = \angle ADB$

($\because$ equal sides have equal angles opposite to them)

$\Rightarrow \quad \angle ABD = 42°$

[$\because \angle ADB = 42°$ (given)]

$\because \quad \angle ABD + \angle ADB + \angle BAD = 180°$



(*iii*)

(Sum of all angles in a triangle is 180°)

$\Rightarrow \quad 42° + 42° + y = 180° \quad \Rightarrow \quad 84° + y = 180°$

$\Rightarrow \quad y = 180° - 84° \quad \Rightarrow \quad y = 96°$

$\angle BCD = 2 \times 26° = 52°$

In $\angle BCD$

$\because \quad BC = CD$ (given)

$\therefore \quad \angle CBD = \angle CDB = x$

[equal side have equal angles opposite to them]

$\therefore \quad \angle CBD + \angle CDB + \angle BCD = 180°$

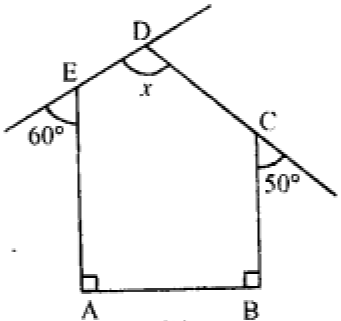
$\Rightarrow \quad x + x + 52° = 180° \quad \Rightarrow \quad 2x = 180° - 52°$

$\Rightarrow \quad 2x = 128° \quad \Rightarrow \quad x = \dfrac{128°}{2} \quad \Rightarrow \quad x = 64°$

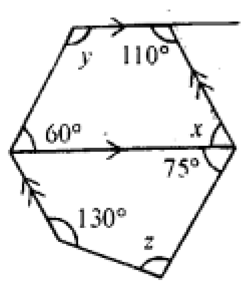
Hence, $x = 64°$ and $y = 90°$

Question 5.

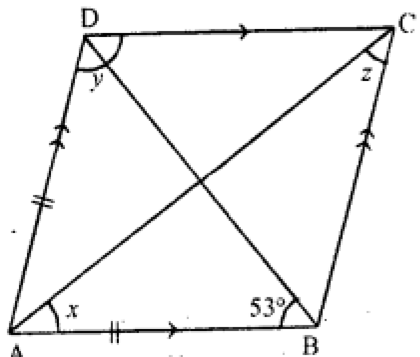
Find the size of each lettered angle in the following figures :



(ii)



(iii)



(i)

Solution:

(i) Here AB || CD and BC || AD (given)

$\therefore$ ABCD is a || gm

$\therefore$ $y = 2 \times \angle ABD$

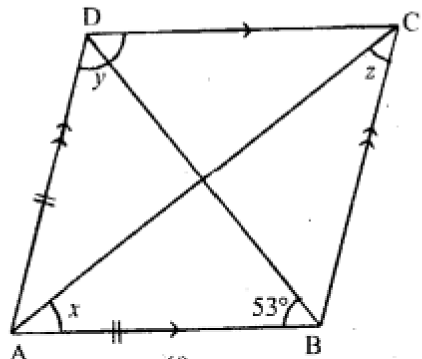
$\Rightarrow$ $y = 2 \times 53° = 106°$ ....(1)

Also, $y + \angle DAB = 180°$

$\Rightarrow$ $106° + \angle DAB = 180°$

$\Rightarrow$ $\angle DAB = 180° - 106°$ $\Rightarrow$ $\angle DAB = 74°$

$\therefore$ $x = \frac{1}{2} \angle DAB$ ($\because$ AC bisect $\angle DAB$)



(i)

$\Rightarrow \quad x = \frac{1}{2} \times 74° = 37°$

and $\angle DAC = x = 37$ ....(2)

$\therefore \quad \angle DAC = z$ (Alternate angles) ....(3)

From (2) and (3),

$z = 37°$

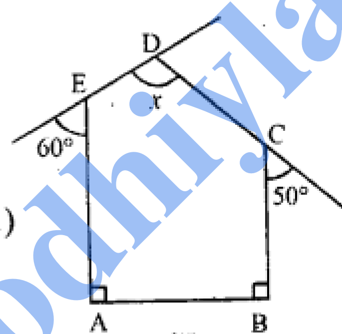
Hence, $x = 37°, y = 106°, z = 37°$

(ii) $\because$ ED is a st. line

$\therefore \quad 60° + \angle AED = 180°$
(linear pair)

$\Rightarrow \quad \angle AED = 180° - 60°$

$\Rightarrow \quad \angle AED = 120°$ ....(1)



(ii)

$\because$ CD is a st. line

$\therefore \quad 50° + \angle BCD = 180°$
(linear pair)

$\Rightarrow \quad \angle BCD = 180° - 50°$

$\Rightarrow \quad \angle BCD = 130°$ ....(2)

In pentagon ABCDE

$\angle A + \angle B + \angle AED + \angle BCD + x = 540°$

(Sum of interior angles in pentagon is 540°)

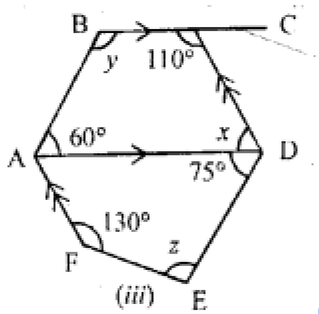
$\Rightarrow \quad 90° + 90° + 120° + 130° + x = 540°$

$\Rightarrow \quad 430° + x = 540° \quad \Rightarrow \quad x = 540° - 430°$

$\Rightarrow \quad x = 110°$

Hence, value of $x = 110°$

(iii) In given figure, AD||BC (given)



(*iii*)

$\therefore \quad 60° + y = 180°$ and $x + 110° = 180°$

$\Rightarrow \quad y = 180° - 60°$ and $x = 180° - 110°$

$\Rightarrow \quad y = 120°$ and $x = 70°$

$\because$ CD||AF (given)

$\therefore \quad \angle FAD = x$ (Alternate angles)

$\Rightarrow \quad \angle FAD = 70°$ ....(1)

In quadrilateral ADEF,

$\angle FAD + 75° + z + 130° = 360°$

$\Rightarrow \quad 70° + 75° + z + 130° = 360°$ [using (1)]

$\Rightarrow \quad 275° + z = 360° \quad \Rightarrow \quad z = 85°$

Hence, $x = 70°$, $y = 120°$ and $z = 85°$

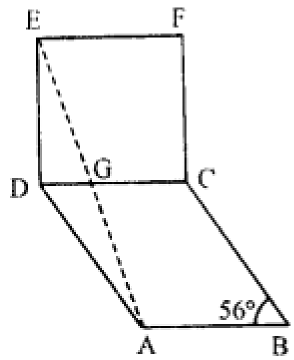
Question 6.

In the adjoining figure, ABCD is a rhombus and DCFE is a square. If ∠ABC = 56°, find

(i) ∠DAG

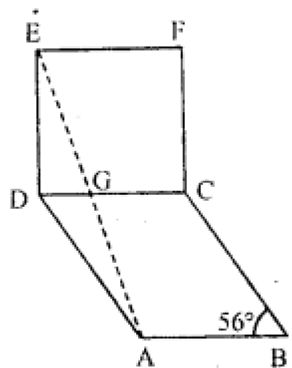
(ii) ∠FEG

(iii) ∠GAC
(iv) ∠AGC.



Solution:

Here ABCD and DCFE is a rhombus and square respectively.



$\therefore \quad AB = BC = DC = AD$ ....(1)

Also $DC = EF = FC = EF$ ....(2)

From (1) and (2),

$AB = BC = DC = AD = EF = FC = EF$ ....(3)

$\angle ABC = 56°$ (given)

$\angle ADC = 56°$

(opposite angle in rhombus are equal)

$\therefore \angle EDA = \angle EDC + \angle ADC = 90° + 56° = 146°$

In ΔADE,

$DE = AD$ [From (3)]

$\angle DEA = \angle DAE$

(equal sides have equal opposite angles)

$$\angle DEA = \angle DAG = \frac{180° - \angle EDA}{2}$$

$$= \frac{180° - 146°}{2} = \frac{34°}{2} = 17°$$

$\Rightarrow \quad \angle DAG = 17°$

Also, $\angle DEG = 17°$

$\therefore \quad \angle FEG = \angle E - \angle DEG$

$= 90° - 17° = 73°$

In rhombus ABCD,

$\angle DAB = 180° - 56° = 124°$

$\angle DAC = \frac{124°}{2}$ ($\because$ AC diagonals bisect the $\angle A$)

$\angle DAC = 62°$

$\therefore \angle GAC = \angle DAC - \angle DAG$

$= 62° - 17° = 45°$

In ΔEDG,

$\angle D + \angle DEG + \angle DGE = 180°$

(Sum of all angles in a triangle is 180°)

$\Rightarrow \quad 90° + 17° + \angle DGE = 180°$

$\Rightarrow \quad \angle DGE = 180° - 107° = 73°$ ....(4)

Hence, $\angle AGC = \angle DGE$ ....(5)

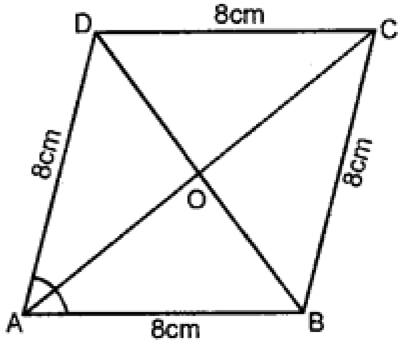
(vertically opposite angles)

From (4) and (5)

$\angle AGC = 73°$

Question 7.
If one angle of a rhombus is 60° and the length of a side is 8 cm, find the lengths of its diagonals.
Solution:
Each side of rhombus ABCD is 8 cm.

$\therefore$ AB = BC = CD = DA = 8 cm.



Let $\angle A = 60°$

$\therefore$ ΔABD is an equilateral triangle

$\therefore$ AB = BD = AD = 8 cm.

$\because$ Diagonals of a rhombus bisect each other eight angles.

$\therefore$ AO = OC, BO = OD = 4 cm.

and $\angle AOB = 90°$

Now in right ΔAOB,

$$AB^2 = AO^2 + OB^2$$ (Pythagoras Theorem)

$\Rightarrow$ $(8)^2 = AO^2 + (4)^2$

$\Rightarrow$ $64 = AO^2 + 16$

$\Rightarrow$ $AO^2 = 64 - 16 = 48 = 16 + 3$

$\therefore$ $AO = \sqrt{16 \times 3} = 4\sqrt{3}$ cm.

But AC = 2 AO

$\therefore$ $AC = 2 \times 4\sqrt{3} = 8\sqrt{3}$ cm

Question 8.
Using ruler and compasses only, construct a parallelogram ABCD with AB = 5 cm, AD = 2.5 cm and ∠BAD = 45°. If the bisector of ∠BAD meets DC at E, prove that ∠AEB is a right angle.

Solution:

Given : AB = 5 cm, AD = 2.5 cm and ∠BAD = 45°.

Required : (*i*) To construct a parallelogram ABCD.

(*ii*) If the bisector of ∠BAD meets DC at E then prove that ∠AEB = 90°.

Steps of Construction :

1. Draw AB = 5.0 cm.
2. Draw ∠BAX = 45° on side AB.
3. Take A as centre and radius 2.5 cm cut the line AX at D.
4. Take B as centre and radius 2.5 cm draw an arc.
5. Take D as centre and radius equal to 5.0 cm cut the arc of step (4) at C.
6. Join BC and CD.
7. ABCD is the required parallelogram.
8. Draw the bisector of ∠BAD, which meets DC at E.
9. Join BE.
10. Measure the ∠AEB which is equal to 90°.