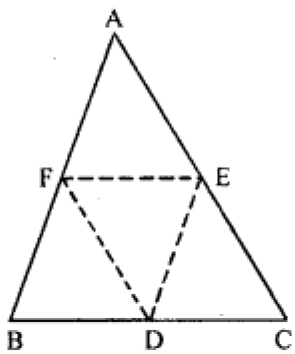
Mid Point Theorem

Question 1.

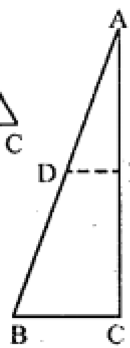
(a) In the figure (1) given below, D, E and F are mid-points of the sides BC, CA and AB respectively of △ ABC. If AB = 6 cm, BC = 4.8 cm and CA= 5.6 cm, find the perimeter of (i) the trapezium FBCE (ii) the triangle DEF.

(b) In the figure (2) given below, D and E are mid-points of the sides AB and AC respectively. If BC = 5.6 cm and∠B = 72°, compute (i) DE (ii)∠ADE.

(c) In the figure (3) given below, D and E are mid-points of AB, BC respectively and DF || BC. Prove that DBEF is a parallelogram. Calculate AC if AF = 2.6 cm.



(1)



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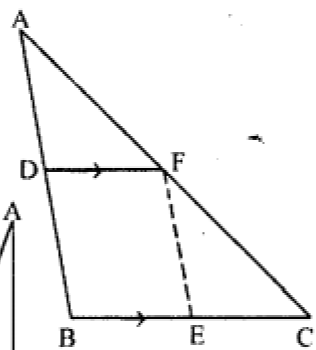
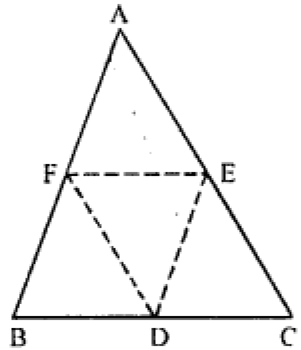


(3)

Solution:

(a) (i) Given : AB = 6 cm, BC = 4.8 cm, and CA = 5.6 cm

Required : The perimeter of trapezium FBCA.



$\because$ F is the mid-point of AB (given)

$\therefore$ $BF = \frac{1}{2} AB = \frac{1}{2} \times 6 \text{ cm}$

$= 3 \text{ cm}$ ....(1)

$\because$ E is the mid-point of AC (given)

$\therefore$ $CE = \frac{1}{2} AC$

$= \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm}$ ...(2)

Now F and E are the mid-point of the AB and CA

$\therefore$ $FE \parallel BC$ and $FE = \frac{1}{2} \times BC$

$\Rightarrow$ $FE = \frac{1}{2} \times 4.8 \text{ cm} = 2.4 \text{ cm}$ ....(3)

$\therefore$ Perimeter of trapezium FBCE

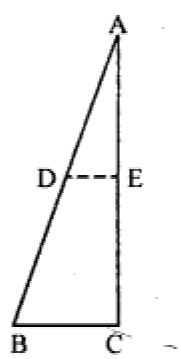
[substituting the value from (1), (2) and (3)]

$= BF + BC + CE + EF$

$= 3 \text{ cm} + 4.8 \text{ cm} + 2.8 \text{ cm} + 2.4 \text{ cm} = 13 \text{ cm}$

Hence, perimeter of trapezium FBCE = 13 cm **Ans.**

(ii) $\because$ D, E and F are the mid-points of the sides BC, CA and AB of Δ ABC respectively.



$\therefore$ $EF \parallel BC$ and $EF = \frac{1}{7} BC$

$\Rightarrow EF = \frac{1}{2} \times 4.8 = 2.4$ cm

Similarly,

$DE = \frac{1}{2} AB = \frac{1}{2} \times 6 \text{ cm} = 3\text{cm}$

and $FD = \frac{1}{2} AC = \frac{1}{2} \times 5.6 \text{ cm} = 2.8\text{cm}$

$\therefore$ Perimeter of Δ DEF

= DE + EF + FD

= 3 cm + 2.4 cm + 2.8 cm = 8.2 cm Ans.

(*b*) **Given :** D and E are mid-point of the sides AB and AC respectively. BC = 5.6 cm and $\angle B = 72°$

Required : (*i*) DE (*ii*) $\angle ADE$

Sol. In ΔABC

$\because$ D and E is mid-point of the sides AB and AC respectively.

$\therefore$ By mid-point theorem

$DE \parallel BC$ and $DE = \frac{1}{2} BC$

(*i*) $DE = \frac{1}{2} BC = \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm}$

(*ii*) $\angle ADE = \angle B$ [(corresponding angles)]

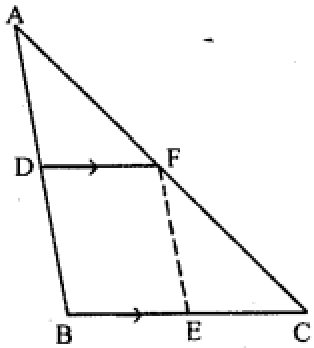
$\therefore \angle ADE = 72°$ [$\because BC \parallel DE$]

[$\angle B = 72°$(given)]

(*c*) **Given :** D and E are mid-points of AB, BC respectively and DF || BC, AF = 2.6 c.m.

To prove : (*i*) BEF is a parallelogram

(*ii*) To calculate the value of AC



Proof : (*i*) In ΔABC

$\because$ D is the mid-point of AB and DF || BC

$\therefore$ F is the mid-point of AC ...(1)

Now, F and E are mid-point of AC and BC respectively.

$\therefore$ EF || AB ....(2)

Now, DF || BC

$\Rightarrow$ DF || BE ....(3)

$\because$ EF || AB [From (2)]

$\Rightarrow$ EF || DB ....(4)

From (3) and (4), DBEF is a parallelogram

(*ii*) $\because$ F is mid-point of AC

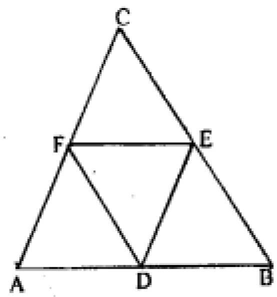
$\therefore$ $AC = 2 \times AF = 2 \times 2.6 \text{ cm} = 5.2 \text{ cm}.$

Question 2.

Prove that the four triangles formed by joining in pairs the mid-points of the sides c of a triangle are congruent to each other.

Solution:

Given: In Δ ABC, D, E and r,
F are mid-points of AB, BC and CA respectively. Join DE, EF and FD.



To prove :

$\Delta ADF \cong \Delta DBE \cong \Delta ECF \cong \Delta DEF$.

Proof : In ΔABC, D and E are mid-point of AB and BC respectively

$\therefore$ DE || AC or FC

Similarly, DF || EC

$\therefore$ DECF is a parallelogram.

$\therefore$ Diagonal FE divides the parallelogram DECF in two congruent triangle DEF and CEF.

$\therefore$ $\Delta DEF \cong \Delta FCF$ ....(1)

Similarly we can prove that,

$\Delta DBE \cong \Delta DEF$ ...(2)

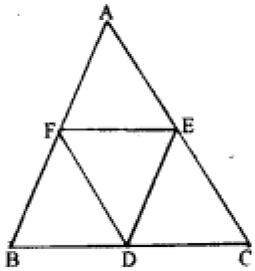
and $\Delta DEF \cong \Delta ADF$ ....(3)

From (1), (2) and (3),

$\Delta ADF \cong \Delta DBE \cong \Delta ECF \cong \Delta DEF$

(Q.E.D.)

Question 3.
If D, E and F are mid-points of sides AB, BC and CA respectively of an isosceles triangle ABC, prove that ΔDEF is also F, isosceles.
Solution:
Given : ABC is an isosceles triangle in which AB = AC



D, E and F are mid point of the sides BC, CA and AB respectively D, E, F are joined

To prove : ΔDEF is an isosceles triangle.

Proof : D and E are the mid points of BC and AC

$\therefore$ $DE \parallel AB$ and $DE = \frac{1}{2} AB$ ....(1)

Again, D and F are the mid-points of BC and AB respectively.

$\therefore$ $DF \parallel AC$ and $DF = \frac{1}{2} AC$ ....(2)

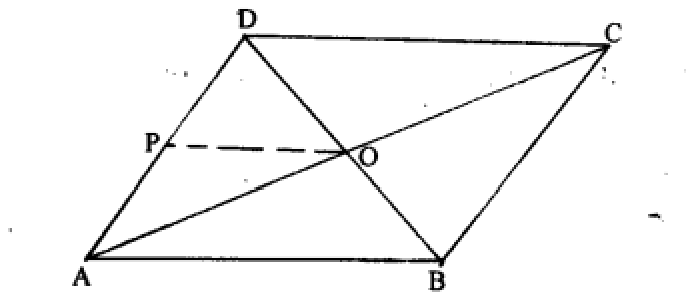
$\because$ $AB = BC$ (given)

$\therefore$ $DE = DF$

$\therefore$ ΔDEF is an isosceles trianlge

(Q.E.D.)

Question 4.
The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the mid-point of AD, prove that
(i) PQ || AB
(ii) PO=$\frac{1}{2}$CD.
Solution:
(i) Given : ABCD is a parallelogram in which diagonals AC and BD intersect each other. At point O, P is the mid-point of AD. Join OP.



To Prove : (*i*) PQ || AB (*ii*) $PQ = \frac{1}{2}$ CD.

Proof : We know that in parallelogram diagonals bisect each other.

∴ BO = OD

i.e. O is the mid-point of BD

Now, in Δ ABD,

P and O is the mid-point of AD and BD respectively

∴ PO || AB and $PO = \frac{1}{2}$ AB ...(1)

i.e. PO || AB [Proved (*i*) part]

(*ii*) Now ∵ ABCD is a parallelogram

∴ AB = CD ...(2)

From (1) and (2),

$PO = \frac{1}{2}$ CD **(Q.E.D.)**

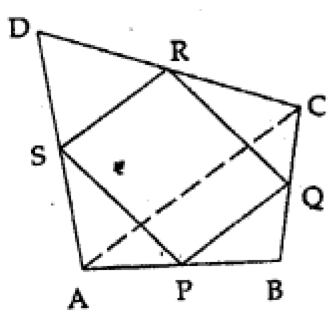
Question 5.

In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA respectively. AC is its diagonal. Show that

(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.



Solution:

Given : In quadrilateral ABCD
P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively AC is the diagonal.

To prove : (*i*) SR || AC and SR = $\frac{1}{2}$ AC

(*ii*) PQ = SR

(*iii*) PQRS is a parallelogram

Proof : (*i*) In ΔADC

S and R are the mid-points of AD and DC

∴ SR || AC and SR = $\frac{1}{2}$ AC...(*i*)

(Mid-points theorm)

(*ii*) Similarly in ΔABC,

P and Q are mid-points of AB and BC

PQ || AC and PQ = $\frac{1}{2}$ AC ...(*ii*)

From (*i*) and (*ii*),

PQ = SR and PQ || SR

(*iii*) ∵ PQ = SR and PQ || SR

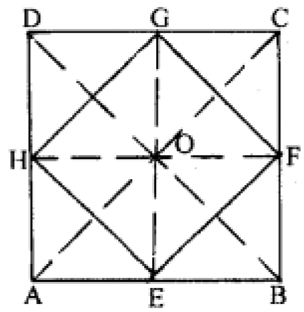
∴ PQRS is a parallelogram

Question 6.

Show that the quadrilateral formed by joining the mid-points of the adjacent sides of a square, is also a square,

Solution:

Given : A square ABCD in which E, F, G and H are mid-points of AB, BC, CD and DA respectively join EF, FG, GH and HE.



To Prove : EFGH is a square

Construction : Join AC and BD.

Proof : In ΔACD, G and H are mid-points of CD and AC respectively.

$\therefore$ GH || AC and GH = $\frac{1}{2}$ AC ...(1)

Now, in ΔABC, E and F are mid-points of AB and BC respectively.

$\therefore$ EF || AC and EF = $\frac{1}{2}$ AC ...(2)

From (1) and (2),

EF || GH and EF = GH = $\frac{1}{2}$ AC ....(3)

Similarly, we can prove that

EF || GH and EH = GF = $\frac{1}{2}$ BD

But AC = BD (∵ Diagonals of square are equal)

Dividing both sides by 2,

$\frac{1}{2}$ AC = $\frac{1}{2}$ BD ...(4)

From (3) and (4),

EF = GH = EH = GF ...(5)

$\therefore$ EFGH is a parallelogram

Now, in ΔGOH and ΔGOF

OH = OF

(Diagonals of parallelogram bisect each other)

OG = OG (Common)

GH = GF [From (5)]

$\therefore \quad \Delta GOH \cong \Delta GOF$

[By S.S.S. axiom of congruency]

$\therefore \quad \angle GOH = \angle GOF$ (c.p.c.t.)

Now $\angle GOH + \angle GOF = 180^\circ$ (Linear pair)

or $\angle GOH + \angle GOH = 180^\circ$

or $2\angle GOH$

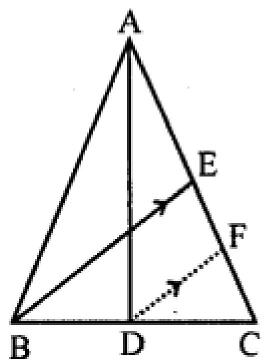
$\therefore \quad \angle GOH = \frac{180^\circ}{2} = 90^\circ$

$\therefore$ Diagonals of parallelogram ABCD bisect and perpendicular to each other.

$\therefore$ EFGH is a square **(Q.E.D.)**

Question 7.

In the adjoining figure, AD and BE are medians of ∆ABC. If DF U BE, prove that CF = $\frac{1}{4}$ AC.



Solution:

Given : In the given figure,
AD and BE are the medians of ΔABC
$DF \parallel BE$ is drawn

To prove : $CF = \frac{1}{4} AC$

Proof :
In ΔBCE
$\therefore$ D is the mid-point of BC and $DF \parallel BE$
$\therefore$ F is the mid-points of EC

$\Rightarrow CF = \frac{1}{2} EC$ ...(*i*)

$\because$ E is the mid-point of AC

$\therefore EC = \frac{1}{2} AC$ ...(*ii*)

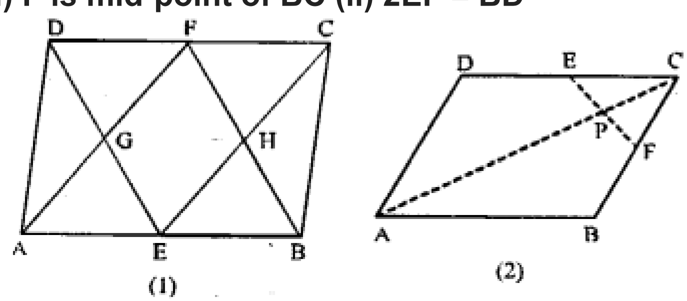
From (*i*) and (*ii*),

$CF = \frac{1}{2} EC = \frac{1}{2}\left(\frac{1}{2} AC\right)$

$= \frac{1}{4} AC$

Question 8.

(a) In the figure (1) given below, ABCD is a parallelogram. E and F are mid-points of the sides AB and CO respectively. The straight lines AF and BF meet the straight lines ED and EC in points G and H respectively. Prove that
(i) $\Delta HEB = \Delta HCF$
(ii) GEHF is a parallelogram.
(b) In the diagram (2) given below, ABCD is a parallelogram. E is mid-point of CD and P is a point on AC such that PC = $\frac{1}{4}$ AC. EP produced meets BC at F. Prove that
(i) F is mid-point of BC (ii) 2EF = BD



Solution:

Given : ABCD is a parallelogram. E and F are mid-point of the side AB and CD respectively.

To prove :

(*i*) $\Delta HEB \cong \Delta HCF$

(*ii*) GEHF is a parallelogram.

Proof : (*i*) ABCD is a parallelogram.

FC || BE

$\therefore \quad \angle CEB = \angle FCE$ (Alternate angles)

$\Rightarrow \quad \angle HEB = \angle FCH$ ....(1)

Also $\angle EBF = \angle CFB$ (Alternate angles)

$\Rightarrow \quad \angle EBH = \angle CFM$ ....(2)

E and F are mid-points of AB and CD respectively.

$\therefore \quad BE = \frac{1}{2} AB$ ...(3)

and $\quad CF = \frac{1}{2} CD$ ...(4)

But ABCD is a parallelogram

$\therefore \quad AB = CD$

$\frac{1}{2}$ AB = $\frac{1}{2}$ CD (Dividing both sides by $\frac{1}{2}$)

BE = CF [From (3) and (4)] ....(5)

Now, in ΔHEB and ΔHCF

∠HEB = ∠FCH [From (1)]

∠EBH = ∠CFH [From (2)]

BE = CF [From (5)]

∴ ΔHEB ≅ ΔHCF

(By A.S.A axiom of congruency)

[(*i*) part is proved]

(***ii***) Since E and F are mid-points of AB and CD

∴ AE = CF [∵ AB = CD]

Now AE || CF (given)

∴ AE = CF and AE || CF

∴ AECF is a || gm.

Now, G and H points on the AF and CE respectively.

∴ GF || EH ....(6)

Similarly we can prove that GFHE is a || gm.

Now point G and H on the line DE and BF respectively.

∴ GE || HF ....(7)

From (6) and (7)

GEHF is a parallelogram. **(Q.E.D.)**

(*b*) **Given :** ABCD is a parallelogram in which E is the mid-point of DC and P is a point on AC such that PC = $\frac{1}{4}$ AC. EP produced meets BC at F.

To Prove : (*i*) F is the mid-point of BC.

(*ii*) 2 EF = BD

Construction : Join BD to intersect AC at O.

Proof : Diagonals of || gm bisect each other.

∴ AO = CO

But $CP = \frac{1}{4} AC$ (given)

$\therefore \quad CP = \frac{1}{4}(2\,CO) \Rightarrow \quad CP = \frac{1}{2} CO$

i.e. P is mid-point of CO.

$\therefore$ In ΔCOD, E and P are mid-points of DC & CO.

$\therefore \quad EP \parallel DO$

i.e. $EF \parallel DO$

Further, in ΔCBD, E is mid-point of DC and $EF \parallel BD$

$\therefore$ F is the mid-point of BC and $EF = \frac{1}{2} BD$

i.e. $2\,EF = BD$. **(Q.E.D.)**

Question 9.

ABC is an isosceles triangle with AB = AC. D, E and F are mid-points of the sides BC, AB and AC respectively. Prove that the line segment AD is perpendicular to EF and is bisected by it.

Solution:

Given : ABC is an isosceles triangle with AB = AC. D, E and F are mid-points of the sides BC, AB and AC respectively.

To prove : $AD \perp EF$ and AD bisect the EF.

Proof : In ΔABD and ΔACD

$\angle ABD = \angle ACD$ (ABC is an isosceles triangle)

$BD = BC$ (given D is mid-point of BC)

$AB = AC$ (given)

$\therefore \Delta ABD \cong \Delta ACD$

(By S.A.S. axiom of congruency)

$\therefore \angle ADB = \angle AOC$ (c.p.c.t.)

Also, $\angle ADB + \angle AOC = 180^\circ$ (Linear pair)

$\Rightarrow \angle ADB + \angle ADB = 180^\circ$ (By above)

$\Rightarrow 2\angle ADB = \frac{180^\circ}{2} \Rightarrow \angle ADB = 90^\circ$

$\therefore AD \perp BC$ ...(1)

Now D and E are mid-points of BC and AB (given)

$\therefore DE \parallel AF$ ...(2)

Again D and F are mid-point of BC and AC

$\therefore EF \parallel AD$ ....(3)

From (2) and (3)

AEDF is a || gm.

$\because$ Diagonals of a parallelogram bisect each other

$\therefore$ AD and EF bisect each other

From (1) and (3)

$AD \perp EF$ ($EF \parallel BC$) **(Q.E.D.)**

Question 10.

(a) In the quadrilateral (1) given below, AB || DC, E and F are mid-points of AD and BD respectively. Prove that:

(i) G is mid-point of BC (ii) $EG = \frac{1}{2}(AB + DC)$.

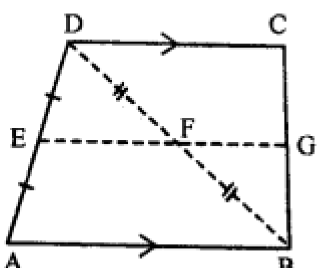
(b) In the quadrilateral (2) given below, AB || DC || EG. If E is mid-point of AD prove that :

(i) G is mid-point of BC (ii) 2EG = AB + CD.

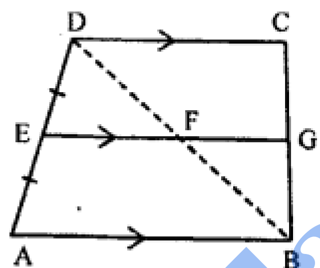
(c) In the quadrilateral (3) given below, AB || DC. E and F are mid-point of non-parallel sides AD and BC respectively. Calculate :

(i) EF if AB = 6 cm and DC = 4 cm

(ii) AB if DC = 8 cm and EF = 9 cm.



(1)



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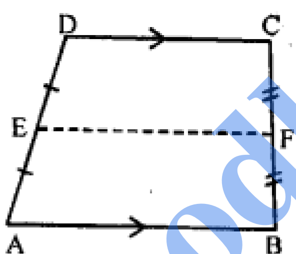
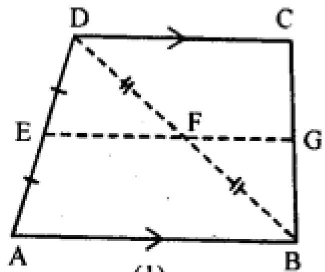


(3)

Solution:

(*a*) **Given :** AB ∥ DC, E and F are mid-points of AD and BD respectively



(I)

To Prove :

(*i*) G is mid-point of BC

(*ii*) $EG = \frac{1}{2}(AB + DC)$.

Proof :

In ΔABD, DF = BF (∵ F is mid-point of BD)

Also E is the mid-point of AD (given)

$\therefore$ EF || AB and EF = $\frac{1}{2}$ AB ....(1)

$\Rightarrow$ EG || CD [AB || CD (given)]

Now F is mid-point of BD and FG || DC

$\therefore$ G is mid-point of BC.

$\Rightarrow$ FG = $\frac{1}{2}$ DC ....(2)

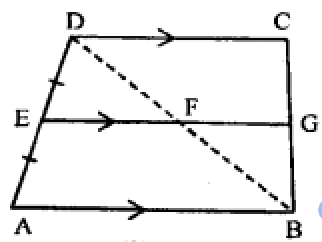
Adding (1) and (2), we get

EF + FG = $\frac{1}{2}$ AB + $\frac{1}{2}$ DC $\Rightarrow$ EG = $\frac{1}{2}$ (AB + DC)

Hence, the result.

(*b*) **Given :** Quadrilateral ABCD in which AB||DC||EG. E is mid-point of AD.

To prove : (*i*) G is mid-point of BC.



(2)

(*ii*) 2 EG = AB + CD

Proof : $\because$ AB || DC (given)

and EG || AB (given)

$\Rightarrow$ EG || DC

In ΔDAB,

E is mid-point of AD and EG || AB (given)

$\therefore$ F is the mid-point of BD and EF = $\frac{1}{2}$ AB .... (1)

In ΔBCD,

F is mid-point of BD and FG || DC

$\Rightarrow$ FG = $\frac{1}{2}$ CD ....(2)

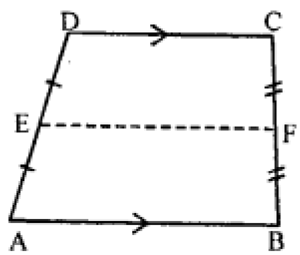
Adding (1) and (2)

$EF + FG = \frac{1}{2}AB \ \frac{1}{2}CD$

$EG = \frac{1}{2}(AB + CD)$ **(Q.E.D.)**

(*c*) Given : A quadrilateral in which AB || DC, E and F are mid-points of non parallel sides AD and BC respectively.

Required : (*i*) EF if AB = 6 cm and DC = 4 cm
(*ii*) AB if DC = 8 cm and EF = 9 cm



(3)

Now, The length of line segment joining the mid-points of two non-parallel sides is half the sum of the lengths of the parallel sides.

∵ E and F are mid-points of AD and BC respectively.

$\therefore \ EF = \frac{1}{2}(AB + CD)$ ...(1)

(*i*) AB = 6 cm and DC = 4 cm

Putting these in (1), we get

$EF = \frac{1}{2}(6 + 4) = \frac{1}{2} \times 10 = 5$ cm **Ans.**

(*ii*) DC = 8 cm and EF = 9 cm

Putting these in (1), we get

$EF = \frac{1}{2}(AB + DC) \Rightarrow 9 = \frac{1}{2}(AB + 8)$

$\Rightarrow 18 = AB + 8 \Rightarrow 18 - 8 = AB$

$\because$ AB = 10 cm **Ans.**

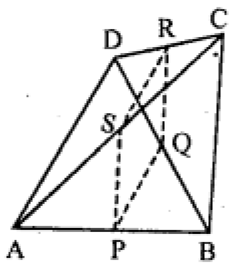
Question 11.

(a) In the quadrilateral (1) given below, AD = BC, P, Q, R and S are mid-points of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.

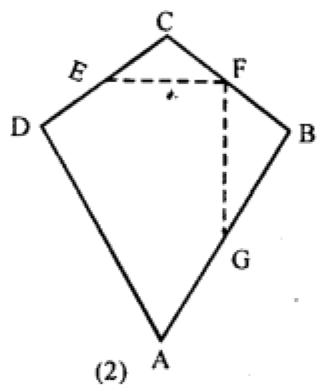
(b) In the figure (2) given below, ABCD is a kite in which BC = CD, AB = AD, E, F, G are mid-points of CD, BC and AB respectively. Prove that:

(i) ∠EFG = 90

(ii) The line drawn through G and parallel to FE bisects DA.



(1)

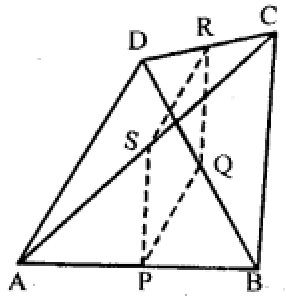


(2)

Solution:

AD = BC. P, Q, R and S are mid-points of AB, BD, CD and AC respectively.

To Prove : PQRS is a rhombus.



Proof : In ΔABD, P and Q are mid-points of AB and BD respectively

(given)

$\therefore$ PQ || AD and PQ

$= \frac{1}{2}$ AB ...(1)

Again in ΔBCD, R and Q are mid-points of DC and BD respectively (given)

$\therefore$ RQ || BC and RQ $= \frac{1}{2}$ BC ....(2)

Also P and S are mid-points of AB and AC respectively (given)

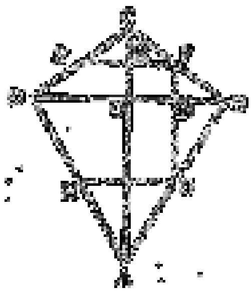
PS || BC and PS $= \frac{1}{2}$ BC ....(3)

$\because$ AD = BC. (given)

[illegible]

[illegible]

Given : [illegible] is a kite in which [illegible], [illegible]. E, F, G are mid-points of CD, BC and AB respectively.



To Prove : (*i*) $\angle EFG = 90^\circ$

(***ii***) The line drawn through G and parallel to FE bisects DA.

Construction : Join AC and BD and Draw GH through G parallel to FE.

Proof : (*i*) Diagonals of a kite intersect at right angle

$\therefore \angle MON = 90^\circ$ ....(1)

In ΔBCD

E and F are mid-points of CD and BC respectively

$\therefore$ EF || DB and $EF = \frac{1}{2} DB$ ....(2)

$\therefore$ EF || DB $\Rightarrow$ MF || ON

$\therefore \angle MON + \angle MFN = 180^\circ$

$\Rightarrow 90^\circ + \angle MFN = 180^\circ$

$\Rightarrow \angle MFN = 180^\circ - 90^\circ \Rightarrow \angle MFN = 90^\circ$

$\Rightarrow \angle EFG = 90^\circ$ (Proved)

(***ii***) In ΔABD

G is mid-point of AB and HG || DB

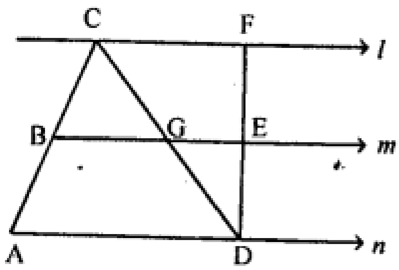
[From (2), EF || DB and EF || HG (given)]

$\Rightarrow$ HG || DB

$\therefore$ H is mid-point of DA

Hence, the line drawn through G and parallel to FE bisects DA. **(Q.E.D.)**

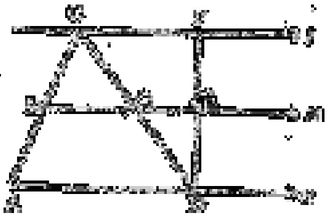
Question 12.
In the adjoining figure, the lines l, m and n are parallel to each other, and G is mid-point of CD. Calculate:
(i) BG if AD = 6 cm
(ii) CF if GE = 2.3 cm
(iii) AB if BC = 2.4 cm
(iv) ED if FD = 4.4 cm.



Solution:

Given : The straight line *l*, *m* and *n* are parallel to each other. G is the mid-point of CD.

To Calculate : (*i*) BG if AD = 6 cm



[illegible]

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 6):

Question 1.

In a △ABC, AB = 3 cm, BC = 4 cm and CA = 5 cm. IfD and E are mid-points of AB and BC respectively, then the length of DE is

(a) 1.5 cm

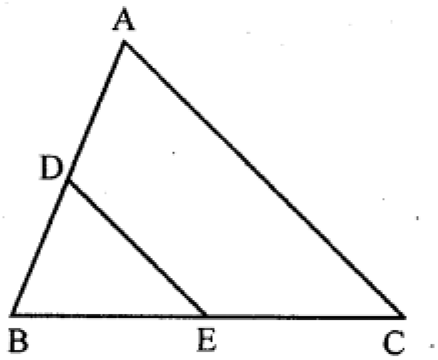
(b) 2 cm

(c) 2.5 cm

(d) 3.5 cm

Solution:

In △ABC, D and E are the mid-points of sides AB and BC



$$\therefore DE = \frac{1}{2} AC$$

But AC = 5 cm

$$\therefore DE = \frac{1}{2} \times 5 = \frac{5}{2} cm = 2.5 cm \qquad (c)$$

Question 2.

In the given figure, ABCD is a rectangle in which AB = 6 cm and AD = 8 cm. If P and Q are mid-points of the sides BC and CD respectively, then the length of PQ is

(a) 7 cm
(b) 5 cm
(c) 4 cm
(d) 3 cm

Solution:

In the given figure,

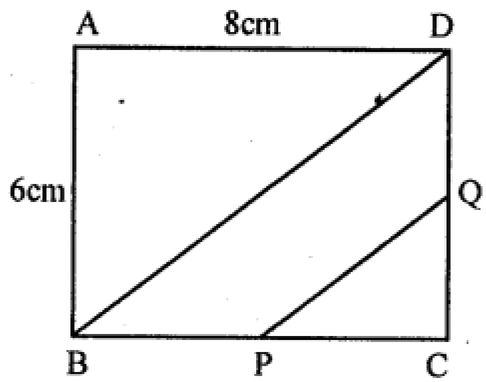
ABCD is a rectangle AB = 6 cm, AD = 8 cm

P and Q are mid-points of BC and CD

$\therefore$ $PQ = \frac{1}{2} BD$

But $BD = \sqrt{BC^2 + CD^2}$

(Pythagoras Theorem)



$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$ cm

$= \sqrt{100} = 10$ cm

$\therefore$ $PQ = \frac{1}{2} BD = \frac{1}{2} \times 10 = 5$ cm (b)

Question 3.

D and E are mid-points of the sides AB and AC of △ABC and O is any point on the side BC. O is joined to A. If P and Q are mid-points of OB and OC respectively, then DEQP is

(a) a square

(b) a rectangle

(c) a rhombus

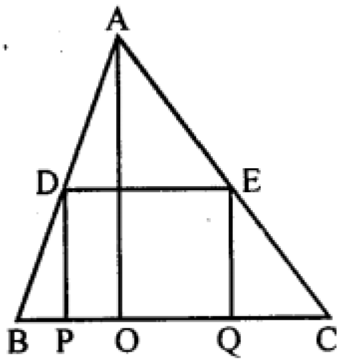
(d) a parallelogram

Solution:

D and E are mid-points of sides AB and AC respectively of AABC O is any point on BC and AO is joined P and Q are mid-points of OB and OC, EQ and DP are joined

D and E are mid-points of sides AB and AC respectively of ΔABC

O is any point on BC and AO is joined P and Q are mid-points of OB and OC, EQ and DP are joined



$\because$ D and E are the mid-points of AB and AC

$\therefore$ $DE = \frac{1}{2} BC$ and $DE \parallel BC$ ...(*i*)

$\therefore$ P and Q are mid-points of BO and OC

$\therefore$ $PQ = PO + OQ$

$= \frac{1}{2} BO + \frac{1}{2} OC = \frac{1}{2}(BO + OC)$

$= \frac{1}{2} BC$ ...(*ii*)

$= DE$

From EQPD is a $\parallel$gm (d)

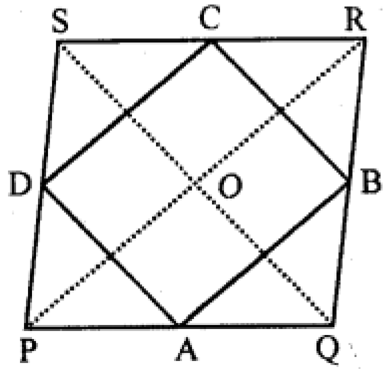
Question 4.

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectanlge if

(a) PQRS is a parallelogram
(b) PQRS is a rectangle
(c) the diagonals of PQRS are perpendicular to each other
(d) the diagonals of PQRS are equal.

Solution:

A, B, C and D are the mid-points of the sides PQ, QR, RS and SP respectively



The quadrilateral so formed by joining the mid-points A, B, C, D is ABCD

ABCD will be rectangle, if the diagonals of PQRS bisect each other

i.e., PR and QS bisect each other (c)

Question 5.

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a rhombus if

(a) ABCD is a parallelogram

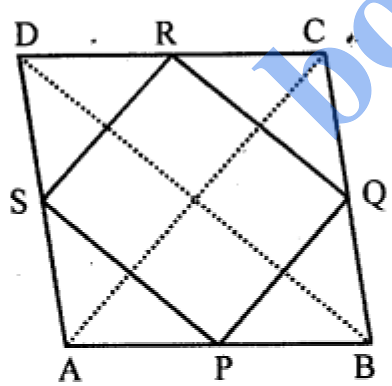
(b) ABCD is a rhombus

(c) the diagonals of ABCD are equal

(d) the diagonals of ABCD are perpendicular to each other.

Solution:

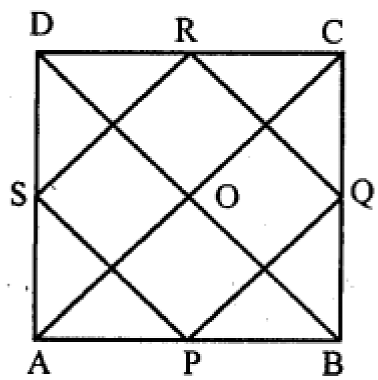
P, Q, R and S are the mid-points of the quadrilateral ABCD and a quadrilateral is formed by joining the mid-points in order



PQRS will be a rhombus if the diagonals of ABCD are equal

i.e., AC = BD (c)

Question 6.
The figure formed by joining the mid points of the sides of a quadrilateral ABCD, taken in order, is a square only if
(a) ABCD is a rhombus r
(b) diagonals of ABCD are equal
(c) diagonals of ABCD are perpendicular to each other
(d) diagonals of ABCD are equal and perpendicular to each other.
Solution:
P, Q, R and S are the mid-points of the quadrilateral ABCD and a quadrilateral is formed by joining them in order. The quadrilateral so formed will be a square if the diagonals of ABCD are equal and perpendicular to each other.



i.e., AC and BD are equal and bisect it perpendicular. (d)

Chapter Test

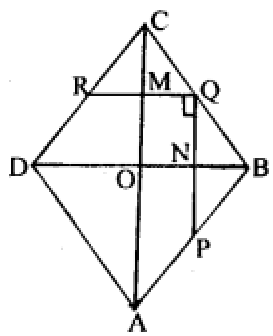
Question 1.
ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD respectively. Prove that PQ ⊥ QR.
Solution:

Given : ABCD is a rhombus with P, Q and R as mid-points of AB, BC and CD respectively.

To Prove : $PQ \perp QR$

Construction : Join AC & BD.

Proof : Diagonals of rhombus intersect at right angle.



$\therefore \angle MON = 90^\circ$ ....(1)

In ΔBCD

Q and R are mid-points of BC and CD respectively.

$\therefore RQ \parallel DB$ and $RQ = \frac{1}{2} DB$ ....(2)

$\therefore RQ \parallel DB \Rightarrow MQ \parallel ON$

$\therefore \angle MQN + \angle MON = 180^\circ$

$\Rightarrow \angle MQN + 90^\circ = 180^\circ \Rightarrow \angle MQN = 180^\circ - 90^\circ$

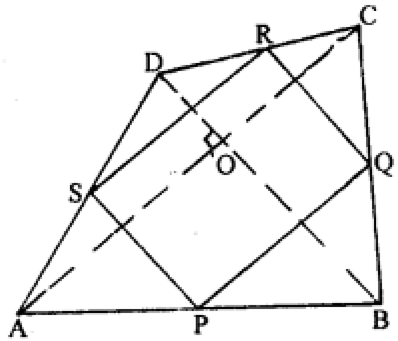
$\Rightarrow \angle MQN = 90^\circ \Rightarrow NQ \perp MQ$

or $PQ \perp QR$ **(Q.E.D.)**

Question 2.

The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral formed by joining the mid-points of its adjacent sides is a rectangle.

Solution:



Ans. Given : ABCD is a quadrilateral in which diagonals AC and BD are perpendicular to each other. P, Q, R and S are mid-points of AB, BC, CD and DA respectively.

To prove : PQRS is a rectangle.

Proof : P and Q are mid-points of AB and BC (given)

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC ....(1)

Again S and R are mid-points of AD and DC (given)

$\therefore$ SR || AC and SR = $\frac{1}{2}$ AC ....(2)

From (1) and (2)

PQ || SR and PQ = SR

$\therefore$ PQRS is a parallelogram

Further AC and BD intersect at right angles

$\therefore$ SP || BD and BD $\perp$ AC.

$\therefore$ SP $\perp$ AC

i.e. SP $\perp$ SR

i.e. $\angle RSP = 90^\circ$

$\therefore$ $\angle RSP = \angle SRQ = \angle RQS = \angle SPQ = 90^\circ$

$\therefore$ PQRS is a rectangle **(Q.E.D.)**

Question 3.

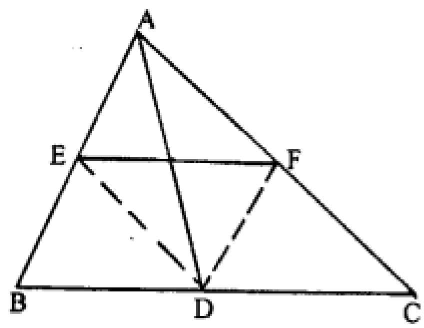
If D, E, F are mid-points of the sides BC, CA and AB respectively of a △ ABC, Prove that AD and FE bisect each other.

Solution:

Given : D, E, F are mid-points of the sides BC, CA and AB respectively of a ΔABC

To Prove : AD and FE bisect each other.

Const : Join ED and FD



Proof : D and E are mid-points of BC and AB respectively (given).

$\therefore$ DE || AC $\Rightarrow$ DE || AF ....(1)

Again D and F are mid-points of BC and AC respectively (given)

$\therefore$ DF || AB $\Rightarrow$ DF || AE ....(2)

From (1) and (2)

ADEF is a ||gm

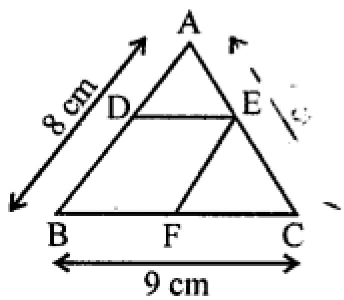
$\because$ Diagonals of a ||gm bisect each other

$\therefore$ AD and EF bisect each other.

Hence, the result. **(Q.E.D.)**

Question 4.
In △ABC, D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If AB = 8 cm and BC = 9 cm, find the perimeter of the parallelogram BDEF.
Solution:
In △ABC, D and E are the mid-points of

sides AB and AC respectively. DE is joined and from E, EF || AB is drawn AB = 8 cm and BC = 9 cm.



To prove.

(*i*) BDEI is a parallelogram.

(*ii*) Find the perimeter of BDEF

Proof : In ΔABC,

∵ B and E are the mid-points of AB and AC respectively

$\therefore DE \parallel BC$ and $DE = \frac{1}{2}BC$

$\because EF \parallel AB$

∴ DEFB is a parallelogram.

$\therefore DE = BF$

$\because DE = \frac{1}{2}BC = \frac{1}{2} \times 9 = 4{\cdot}5$ cm

$EF = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4$ cm.

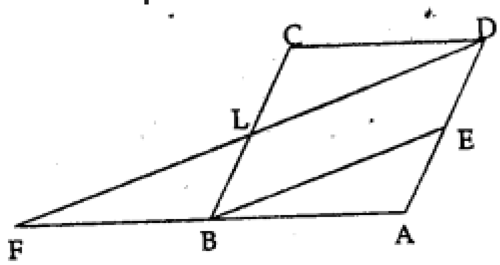
∴ Perimeter of BDEF = 2 (DE + EF)

= 2 (4·5 + 4)

= 8·5 × 2 = 17 cm. **Ans.**

Question 5.

In the given figure, ABCD is a parallelogram and E is mid-point of AD. DL EB meets AB produced at F. Prove that B is mid-point of AF and EB = LF.



Solution:

Given In the figure

ABCD is a parallelogram

E is mid-point of AD

DL || EB meets AB produced at F

To prove : EB = LF

B is mid-point of AF

Proof : ∵ BC || AD and BE || LD

∴ BEDL is a parallelogram

∴ BE = LD and BL = AE

∵ E is mid-point of AD

∴ L is mid-point of BC

In ΔFAD,

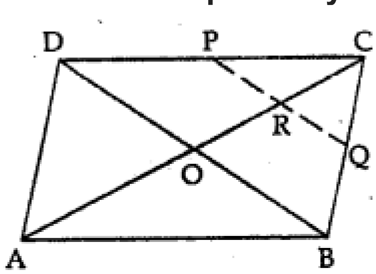
E is mid-point of AD and BE || LD at FLD

∴ B is mid point of AF

∵ $EB = \frac{1}{2} FD = LF$

Question 6.

In the given figure, ABCD is a parallelogram. If P and Q are mid-points of sides CD and BC respectively. Show that CR = $\frac{1}{2}$ AC.



Solution:

Given : In the figure, ABCD is a parallelogram P and Q are the mid-points of sides CD and BC respectively.

To prove : $CR = \frac{1}{4} AC$

Construction : Join AC and BD.

Proof : In ||gm ABCD, diagonals AC and BD bisect each other at O

$AO = OC$ or $OC = \frac{1}{2} AC$ ...(*i*)

In ΔBCD,

P and Q are mid points of CD and BC

$\therefore$ PQ || BD

$\because$ In ΔBCO,

Q is mid-point of BC and PQ || OB

$\therefore$ R is mid-point of CO

$\therefore CR = \frac{1}{2} OC = \frac{1}{2}\left(\frac{1}{2} BC\right)$

$\therefore CR = \frac{1}{4} BC$