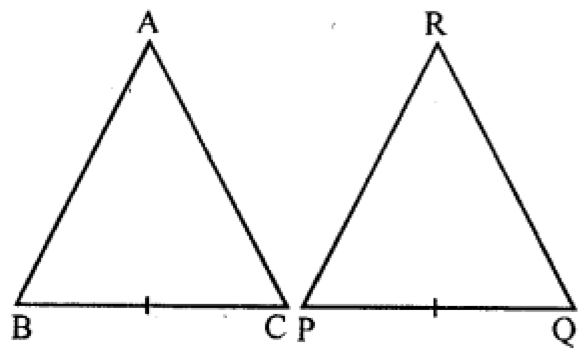
Triangles

Question 1.
It is given that $\triangle ABC \cong \triangle RPQ$. Is it true to say that BC = QR ? Why?
Solution:
$\triangle ABC \cong \triangle RPQ$
$\therefore$ Their corresponding sides and angles are equal



$\therefore$ BC = PQ
$\therefore$ It is not true to say that BC = QR

Question 2.
"If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?
Solution:
No, it is not true statement as the angles should be included angle of there two given sides.

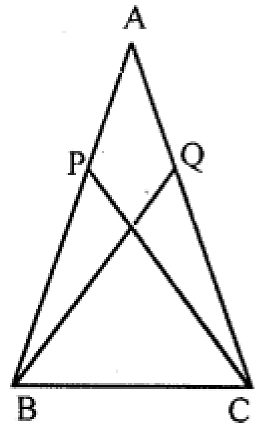
Question 3.
In the given figure, AB=AC and AP=AQ. Prove that
(i) $\triangle APC \cong \triangle AQB$
(ii) CP = BQ
(iii) $\angle APC = \angle AQB$.
Solution:

Given : In the figure, AB = AC, AP = AQ

To prove :

(*i*) $\Delta APC \cong \Delta AQB$ (*ii*) CP = BQ

(*iii*) $\angle APC = \angle AQB$



Proof : In ΔAPC and ΔAQB

AC = AB	(Given)
AP = AQ	(Given)
$\angle A = \angle A$	(Common)
(*i*) $\because \Delta APC \cong \Delta AQB$	(SAS axiom)
(*ii*) BQ = CP	(c.p.c.t.)
(*iii*) $\angle APC = \angle AQB$	(c.p.c.t.)

Question 4.

In the given figure, AB = AC, P and Q are points on BA and CA respectively such that AP = AQ. Prove that

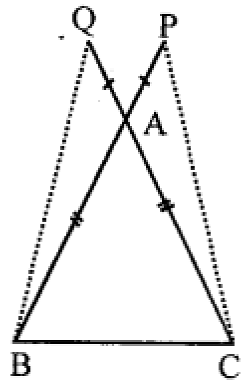
(i) $\Delta APC \cong \Delta AQB$

(ii) CP = BQ

(iii) $\angle ACP = \angle ABQ$.

Solution:

Given : In the given figure, AB = AC
P and Q are point on BA and CA produced respectively such that AP = AQ

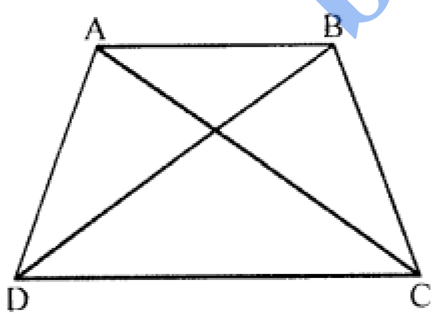


To prove : (*i*) $\Delta APC \cong \Delta AQB$
(*ii*) CP = BQ
(*iii*) $\angle ACP = \angle ABQ$

Proof : In ΔAPC and ΔAQB

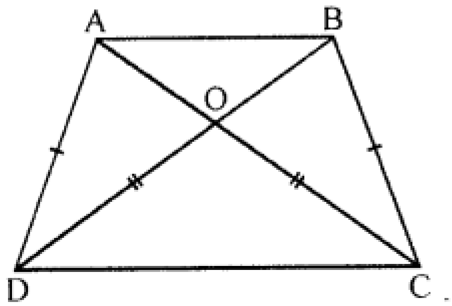
AC = AB (Given)
AP = AQ (Given)
$\angle PAC = \angle QAB$ (Vertically opposite angle)
(*i*) $\therefore \Delta APC \cong \Delta BQP$ (SAS axiom)
$\therefore$ CP = BQ (c.p.c.t.)
$\angle ACP = \angle ABQ$ (c.p.c.t.)

Question 5.
In the given figure, AD = BC and BD = AC. Prove that : ∠ADB = ∠BCA and ∠DAB = ∠CBA.



Solution:

Given : In the figure, AD = BC, BD = AC



To prove :

(*i*) $\angle ADB = \angle BCA$

(*ii*) $\angle DAB = \angle CBA$

Proof : In ΔADB and ΔACB

$AB = AB$	(common)
$AD = BC$	(given)
$DB = AC$	(given)
$\therefore \Delta ADB \cong \Delta ACD$	(SSS axiom)
$\therefore \angle ADB = \angle BCA$	(c.p.c.t.)
$\angle DAB = \angle CBA$	(c.p.c.t.)

Question 6.

In the given figure, ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$. Prove that

(i) $\Delta ABD \cong \Delta BAC$

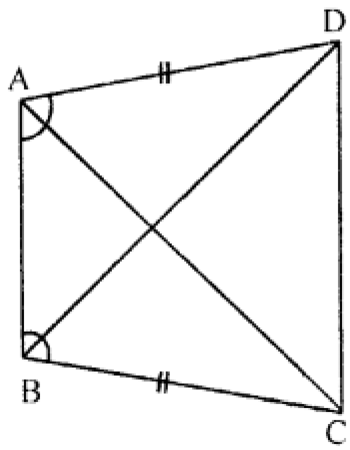
(ii) BD = AC

(iii) $\angle ABD = \angle BAC$.

Solution:

Given : In the figure ABCD is a quadrilateral in which AD = BC

$\angle DAB = \angle CBA$



To prove :

(*i*) $\Delta ABD \cong \Delta BAC$ (*ii*) $BD = AC$

(*iii*) $\angle ABD = \angle BAC$

Proof : In ΔABD and ΔABC

$AB = AB$ (Common)

$\angle DAB = \angle CBA$ (Given)

$AD = BC$ (Given)

(*i*) $\therefore \Delta ABD \cong \Delta ABC$ (SAS axiom)

(*ii*) $\therefore BD = AC$ (c.p.c.t.)

(*iii*) $\angle ABD = \angle BAC$ (c.p.c.t.)

Question 7.

In the given figure, AB = DC and AB || DC. Prove that AD = BC.

Solution:

Given : In the given figure,

AB = DC, AB || DC

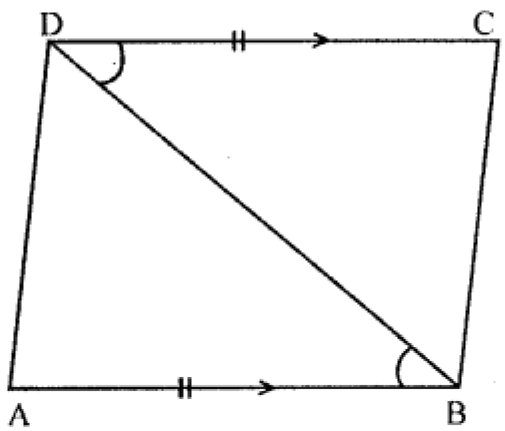
To pove : AD = BC

Proof : $\because$ AB || DC

$\therefore$ $\angle ABD = \angle CDB$ (Alternate angles)

In ΔABD and ΔCDB

AB = DC (Given)



$\angle ABD = \angle CDB$ (Alternate angles)

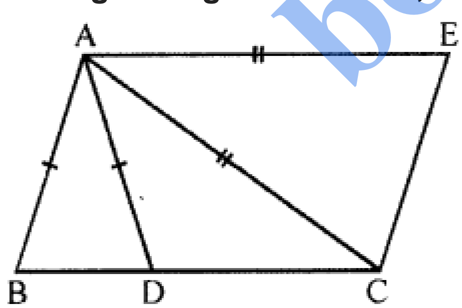
BD = BD (Common)

$\therefore$ $\Delta ABD \cong \Delta CDB$ (SAS axiom)

$\therefore$ AD = BC (c.p.c.t.)

Question 8.

In the given figure. AC = AE, AB = AD and ∠BAD = ∠CAE. Show that BC = DE.



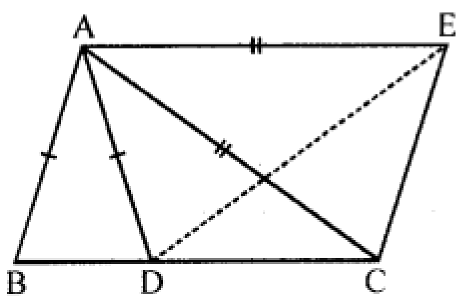
Solution:

Given : In the figure, $AC = AE$, $AB = AD$

$\angle BAD = \angle CAE$

To prove : $BC = DE$

Construction : Join DE.



Proof : In ΔABC and ΔADE

$AB = AD$ (given)

$AC = AE$ (given)

$\angle BAD + \angle DAC = \angle DAC + \angle CAE$

$\Rightarrow \angle BAC = \angle DAE$

$\therefore \Delta ABC \cong \Delta ADE$ (SAS axiom)

$\therefore BC = DE$ (c.p.c.t.)

Question 9.

In the adjoining figure, $AB = CD$, $CE = BF$ and $\angle ACE = \angle DBF$. Prove that
(i) $\Delta ACE \cong \Delta DBF$
(ii) $AE = DF$.

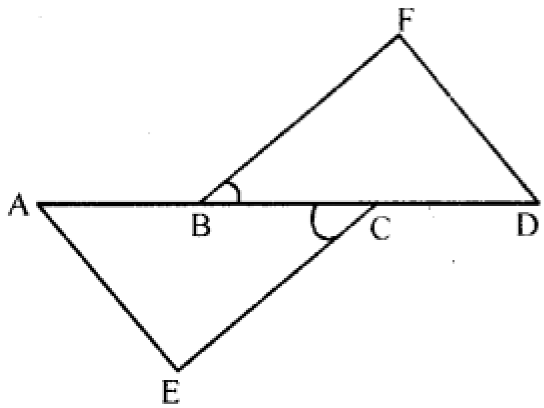
Solution:

Given : In the given figure,

AB = CD

CE = BF

∠ACE = ∠DBF



To prove : (*i*) ΔACE ≅ ΔDBF

(*ii*) AE = DF

Proof : ∵ AB = CD

Adding BC to both sides

AB + BC = BC + CD

⇒ AC = BD

Now in ΔACE and ΔDBF

AC = BD (Proved)

CE = BF (Given)

∠ACE = ∠DBF (Given)

(*i*) ∴ ΔACE ≅ ΔDBF (SAS axiom)

∴ AE = DE (c.p.c.t.)

Question 10.

In the given figure, AB = AC and D is mid-point of BC. Use SSS rule of congruency to show that

(i) ΔABD ≅ ΔACD

(ii) AD is bisector of ∠A

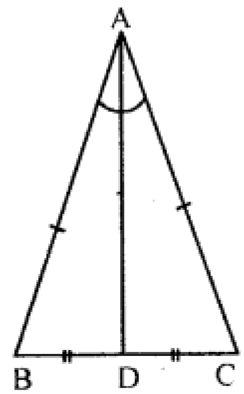
(iii) AD is perpendicular to BC.

Solution:

Given : In the given figure, AB = AC

D is mid point of BC

$\therefore$ BD = DC



To prove :

(i) $\Delta ABD \cong \Delta ACD$

(ii) AD is bisector of $\angle A$

(iii) $AD \perp BC$

Proof : In ΔABD and ΔACD

AB = AC (Given)

BD = DC (Given)

AD = AD (Common)

(i) $\therefore \Delta ABD \cong \Delta ACD$

(ii) $\angle BAD = \angle CAD$ (c.p.c.t.)

$\therefore$ AD is the bisector of $\angle A$

(iii) $\angle ADB = \angle ADC$

But $\angle ADB + \angle ADC = 180^\circ$ (Linear pair)

$\therefore \angle ADB = \angle ADC = 90^\circ$

$\therefore AD \perp BC$

Question 11.

Two line segments AB and CD bisect each other at O. Prove that :

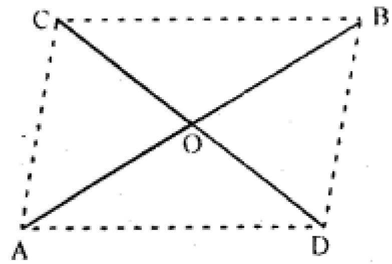
(i) AC = BD

(ii) ∠CAB = ∠ABD

(iii) AD || CB

(iv) AD = CB.

Solution:



In ΔAOC and ΔAOC

$OC = OD$ [AB and CD bisect each other]

and $AO = OB$

Also, $\angle AOC = \angle BOD$ (vertical opposite angles)

$\therefore \Delta AOC \cong \Delta BOD$

(By S.A.S. axiom of congruency)

(*i*) Then $AC = BD$ (c.p.c.t.)

(*ii*) Also $\angle CAO = \angle DBO$ (c.p.c.t.)

i.e. $\angle CAB = \angle ABD$

[$\because \angle CAO = \angle CAB$ and $\angle DBO = \angle ABD$]

(*iii*) We have in *(ii)* part

$\angle CAB = \angle ABD$

But these are Alternate angles

Hence, $AD \parallel CD$

(*iv*) In ΔAOD and ΔBOC

$OC = OD$ and $BO = AO$

(AB and CD bisect each other)

and $\angle BOC = \angle AOD$

(vertical opposite angles)

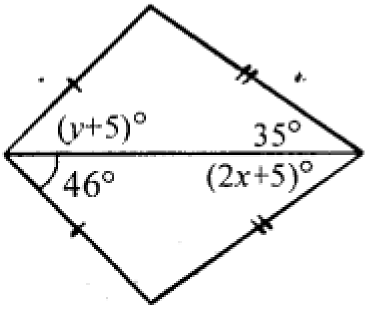
$\therefore \Delta AOD \cong \Delta BOC$

[By S.A.S. axiom of congruency]

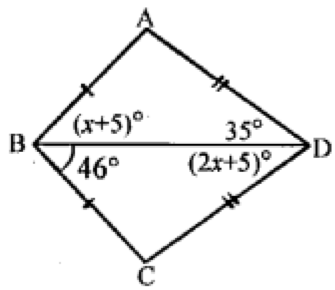
Then, $AD = BC$ (c.p.c.t.)

or $AD = CB$ **(Q.E.D.)**

Question 12.
In each of the following diagrams, find the values of x and y.



Solution:



In ΔABD and ΔBCD,
AB = BC (given)
AD = CD (given)
BD = BD (common)
$\therefore \Delta ABD \cong \Delta BCD$
(By S.S.S axiom of congruency)
$\therefore \angle ABD = \angle CBD$ (c.p.c.t.)
$\Rightarrow y + 5 = 46 \Rightarrow y = 46 - 5 \Rightarrow y = 41$
Also $\angle ADB = \angle BDC$ (c.p.c.t.)
$\Rightarrow 35° = (2x + 5)° \Rightarrow 35 = 2x + 5$
$\Rightarrow 2x + 5 = 35 \Rightarrow 2x = 35 - 5 \Rightarrow 2x = 30$
$\Rightarrow x = \frac{30}{2} \Rightarrow x = 15$

Exercise 10.2

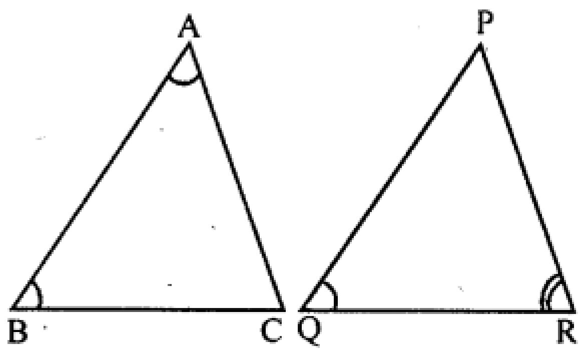
Question 1.
In triangles ABC and PQR, ∠A= ∠Q and ∠B = ∠R. Which side of APQR should be equal to side AB of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:

In ΔABC and ΔPQR

$\angle A = \angle Q$

$\angle B = \angle R$



$AB = QP$

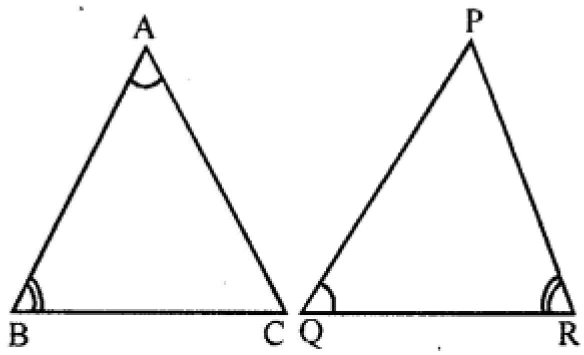
$\because$ Two Δs are congruent of their corresponding two angles and included sides are equal

Question 2.

In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of APQR should be equal to side BC of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:

In ΔABC and ΔPQR



∠A = ∠Q

∠B = ∠R

and their included sides AB and QR will be equal for their congruency

∴ BC = PR (c.p.c.t.)

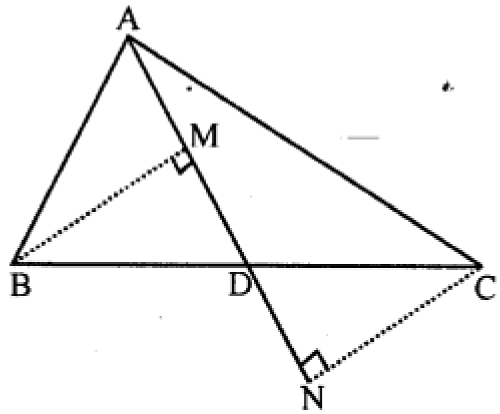
Question 3.

"If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent". Is the statement true? Why?

Solution:

The given statement can be true only if the corresponding (included) sides are equal otherwise not.

Question 4.
In the given figure, AD is median of △ABC, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN.
Solution:



Given : In ΔABC, AD is median BM and CN are perpendicular to AD form B and C respectively

To prove : BM = CN

Proof : In ΔBMD and ΔCND

BD = CD (AD is median)

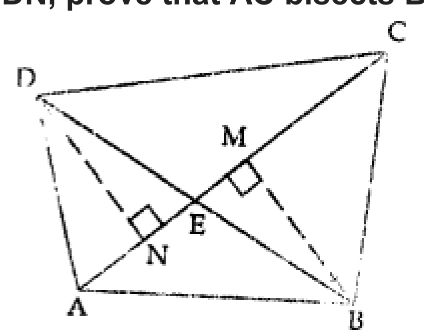
$\angle M = \angle N$ (each 90°)

$\angle BDM = \angle CDN$ (Vertically opposite angles)

$\therefore$ $\Delta BMD \cong \Delta CND$ (AAS axiom)

$\therefore$ BM = CN (c.p.c.t.)

Question 5.
In the given figure, BM and DN are perpendiculars to the line segment AC. If BM = DN, prove that AC bisects BD.



Solution:

Given : In the figure, BM and DN are perpendicular to AC

BM = DN

To prove : AC bisects BD *i.e.*, BE = ED

Construction : Join BD which intersects AC at E

Proof : In ΔBEM and ΔDEN

BM = DN (Given)

∠M = ∠N (each 90°)

∠DEN = ∠BEM

(Vertically opposite angles)

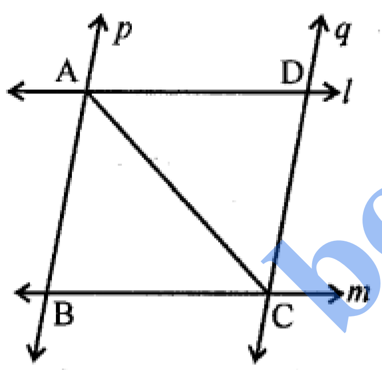
∴ ΔBEM ≅ ΔDEN (AAS axiom)

∴ BE = ED

⇒ AC bisects BD

Question 6.

In the given figure, l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC ≅ ΔCDA.



Solution:

In the given figure, two lines l and m are parallel to each other and lines p and q are also a pair of parallel lines intersecting each othat at A, B, C and D. AC is joined.

To prove : $\Delta ABC \cong \Delta CDA$

Proof : In ΔABC and ΔCDA

$AC = AC$ (Common)

$\angle ACB = \angle CAD$ (Alternate angles)

$\angle BAC = \angle ACD$ (Alternate angles)

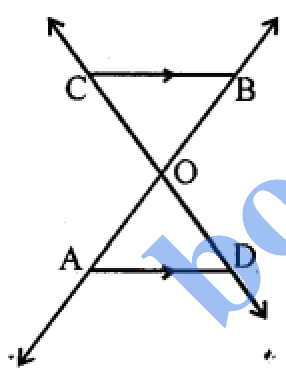
$\therefore$ $\Delta ABC \cong \Delta DCA$ (ASA axiom)

Question 7.

In the given figure, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid-point of both the line segments AB and CD.

Solution:

Given : In the given figure, lines AB and CD intersect each other at O such that BC || AD and BC = DA



To prove : O is the mid point of AB and CD

Proof : ΔAOD and ΔBOC

$AD = BC$ (Given)

$\angle OAD = \angle OBC$ (Alternate angles)

$\angle ODA = \angle OCB$ (Alternate angles)

$\therefore$ $\Delta AOD \cong \Delta BOC$ (SAS axiom)

$\therefore$ $OA = OB$ and $OD = OC$

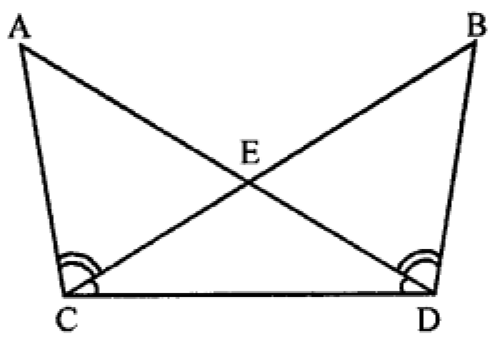
$\therefore$ O is the mid-point of AB and CD

Question 8.

In the given figure, ∠BCD = ∠ADC and ∠BCA = ∠ADB. Show that
(i) ΔACD ≅ ΔBDC
(ii) BC = AD
(iii) ∠A = ∠B.

Solution:

Given : In the given figure,
∠BCD = ∠ADC
∠BCA = ∠ADB



To prove :
(*i*) ΔACD ≅ ΔBDC (*ii*) BC = AD
(*iii*) ∠A = ∠B

Proof : ∵ ∠BCA = ∠ADB
and ∠BCD = ∠ADC
Adding we get,
∠BCA + ∠BCD = ∠ADB + ∠ADC
⇒ ∠ACD = ∠BDC
Now in ΔACD and ΔBDC
CD = CD (Common)
∠ACD = ∠BDC (Proved)
∠ADC = ∠BCD (Given)
(*i*) ∴ ΔACD ≅ ΔBDC (ASA axiom)
∴ AD = BC (c.p.c.t.)
∠A = ∠B (c.p.c.t.)

Question 9.

In the given figure, ∠ABC = ∠ACB, D and E are points on the sides AC and AB respectively such that BE = CD. Prove that
(i) ΔEBC ≅ ΔDCB
(ii) ΔOEB ≅ ΔODC

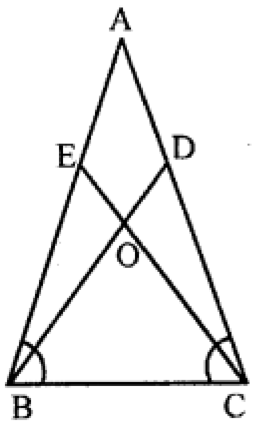
(iii) OB = OC.

Solution:

Given : In the given figure,

$\angle ABC = \angle ACB$

D and E are the points on AC and AB such



To prove : (*i*) $\Delta EBC \cong \Delta DCB$

(*ii*) $\Delta OEB \cong \Delta ODC$

(*iii*) OB = OC

Proof : In ΔABC,

$\because \angle ABC = \angle ACB$

$\therefore$ AC = AB (Sides opposite to equal angles)

In ΔEBC and ΔDCB,

EB = DC (Given)

BC = BC (Common)

$\angle CBD = \angle DCB$ ($\because \angle ABC = \angle ACB$)

(*i*) $\because \Delta EBC \cong \Delta DCB$ (SAS axiom)

$\angle ECB = \angle DBC$ (c.p.c.t.)

Now in ΔOEB and ΔODC

BE = CD (Given)

$\angle EBO = \angle DCO$

$\{\because \angle ABC - \angle DBC = \angle ACB - \angle OCB\}$

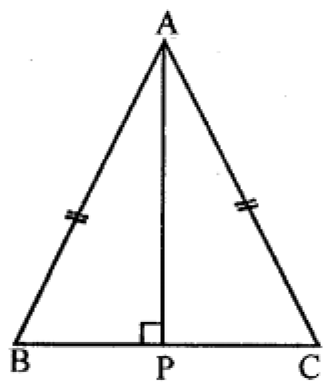
$\angle EOB = \angle DOC$

(*ii*) $\therefore \Delta OEB \cong \Delta ODC$ (AAS axiom)

(*iii*) $\therefore$ OB = OC (c.p.c.t.)

Question 10.
ABC is an isosceles triangle with AB=AC. Draw AP ⊥ BC to show that ∠B = ∠C.
Solution:

Given : ΔABC is an isosceles triangle with
$AB = AC$
$AP \perp BC$
To prove : $\angle B = \angle C$
Proof : In right ΔAPB and ΔAPC
Side $AP = AP$ (Common)



Hyp. $AB = AC$ (Given)
$\therefore$ $\Delta APB \cong \Delta APC$ (RHS axiom)
$\therefore$ $\angle B = \angle C$ (c.p.c.t.)

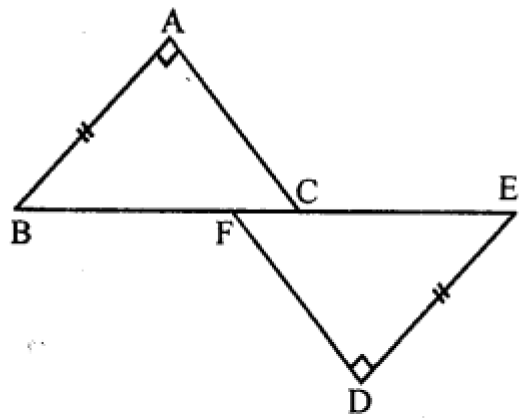
Question 11.
In the given figure, BA ⊥ AC, DE⊥ DF such that BA = DE and BF = EC.

Solution:

Given : In the given figure,

BA ⊥ AC, DE ⊥ DF

BA = DE, BF = EC



To prove : ΔABC ≅ ΔDEF

Proof : ∵ BF = CE

Adding FC both sides

BF + FC = FC + CE

⇒ BC = EF

Now in right ΔABC and ΔDEF

Side AB = DE (Given)

Hyp. BC = EF (Proved)

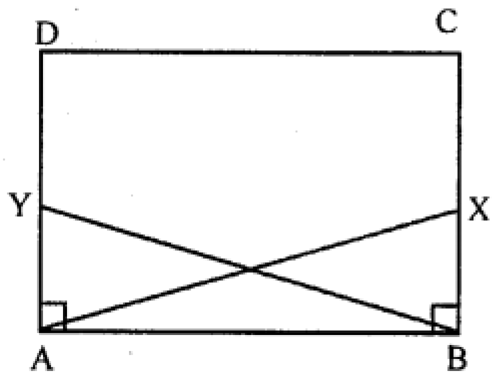
∴ ΔABC ≅ ΔDEF

Question 12.

ABCD is a rectanige. X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠BAY = ∠ABX.

Solution:

Given : In rectangle ABCD, X and Y are points on the sides AD and BC respectively such that AY = BX



To prove : BY = AX and ∠BAY = ∠ABX

Proof : In ΔABX and ΔABY

AB = AB	(Common)
∠A = ∠B	(Each 90°)
BX = AY	(Given)
∴ ΔABX ≅ ΔABY	(SAS axiom)
AX = BY	(c.p.c.t.)
or BY AX	
and ∠AXB = ∠BYA	(c.p.c.t.)

Question 13.

(a) In the figure (1) given below, QX, RX are bisectors of angles PQR and PRQ respectively of A PQR. If XS⊥ QR and XT ⊥ PQ, prove that

(i) ΔXTQ ≅ ΔXSQ

(ii) PX bisects the angle P.

(b) In the figure (2) given below, AB || DC and ∠C = ∠D. Prove that

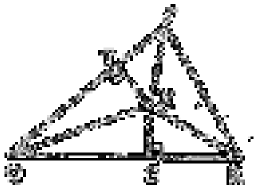
(i) AD = BC

(ii) AC = BD.

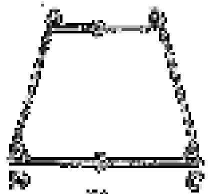
(c) In the figure (3) given below, BA || DF and CA II EG and BD = EC . Prove that, .

(i) BG = DF

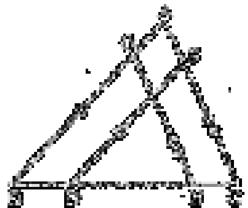
(ii) EG = CF.



(i)



(ii)



(iii)

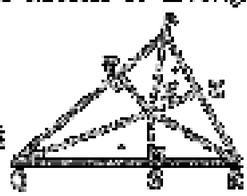
Solution:

(i) Given. In ΔPQR, QX is the bisector of $\angle PQR$ and RX is the bisector of $\angle PRQ$, $XS \perp QR$ and $XT \perp PQ$.

To prove.

(i) $\Delta XTQ \cong \Delta XSQ$

(ii) PX bisects the angle P.



(i)

Construction. Draw $XU \perp PR$.

Proof. In ΔXTQ and ΔXSQ

$\angle QTX = \angle QSX$ (each $90°$)

$\because$ [$XT \perp QP$ (given) and $XS \perp QR$] (given)

$\angle TQX = \angle SQX$ (QX is bisector of $\angle PQR$)

$QX = QX$ (Common)

$\therefore \Delta XTQ \cong \Delta XSQ$ (By A.A.S. axiom of congruency)

$\therefore XT = XS$ (c.p.c.t.) ... (i)

Now in ΔXSR and ΔXUR

$\angle XSR = \angle XUR$ (each $90°$)

[$\because \angle XSR = 90°$ (given) $\angle XUR = 90°$ by construction]

$\angle XRS = \angle XRU$

[RX is the bisector of $\angle PRQ$]

$RX = RX$ (common)

$\therefore \Delta XSR \cong \Delta XUR$ (By A.A.S. axiom of congruency)

$\therefore XS = XU$ (c.p.c.t.) ... (ii)

From (i) and (ii), we get

$XT = XU$ ... (iii)

In ΔXTP and ΔXUP,

$\angle XTP = \angle XUP = 90°$

$$\begin{bmatrix} \angle XTP = 90^\circ \text{ (given)} \\ \angle XTP = 90^\circ \text{ (construction)} \end{bmatrix}$$

(hyp) XP = (hyp) XP (common

XT = XZ [From (3)

$\therefore$ $\Delta XTP \cong \Delta XZP$

[By R.H.S. axiom of congruency

$\therefore$ $\angle XPT = \angle XPZ$ (c.p.c.t.

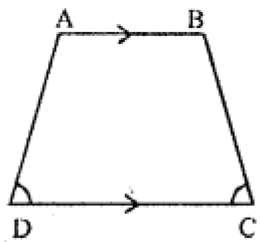
$\therefore$ PX bisects the angle P. **(Q.E.D.**

(*b*) In following figure

Given. AB || DC and $\angle C = \angle D$

To prove. (*i*) AD = BC

(*ii*) AC = BD



(2)

Construction. Draw AE $\perp$ CD, BF $\perp$ CD an join A to C and B to D.

Proof. (*i*) In ΔAED and ΔBCF

$\angle AED = \angle BFC$ (each 90°

[By construction AE $\perp$ CD and BF $\perp$ CD]

$\angle D = \angle C$ (giver

AE = BF

[Distance between parallel lines are same]

$\therefore$ $\Delta AED \cong \Delta BCF$

(By A.A.S. axiom of congruency

AD = BC (c.p.c.t.) .... (1

(*ii*) In ΔACD and ΔBCD

$\angle D = \angle C$ (Given

DC = DC (Commor

AD = BC [From (1)

$\therefore$ $\Delta ACD \cong \Delta BCD$

(By S.A.S. axiom of congruency

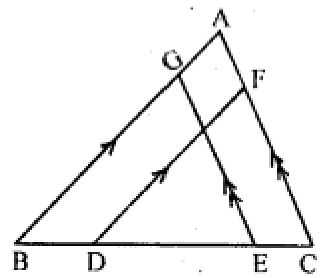
$\therefore$ AC = BD (c.p.c.t.) **(Q.E.D.**

(*c*) In following figure

Given. BA || DF and CA || EG and BD = EC

To prove. (*i*) BG = DF

(*ii*) EG = CF



(3)

Proof. (*i*) In ΔBEG and ΔDCF

$\angle B = \angle D$

($\because$ BA || DF, corresponding angles equal)

$\angle E = \angle C$

($\because$ CA || EG corresponding angles equal)

and BE = BC – EC = BC – BD = DC

i.e. BE = DC

$\therefore$ $\Delta BEG \cong \Delta DCF$

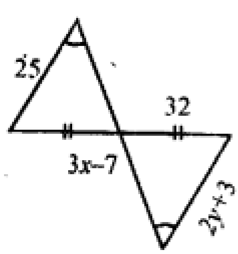
(By A.S.A. axiom of congruency)

$\therefore$ BG = DF (c.p.c.t.)

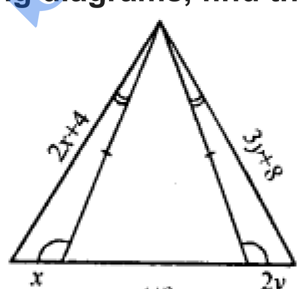
(*iii*) EG = CF (c.p.c.t.) **(Q.E.D.)**

Question 14.

In each of the following diagrams, find the values of x and y.



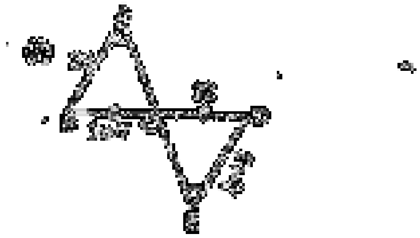
(*i*)



(*ii*)

Solution:

(i)



In ΔABC and ΔCDE

$BC = CD$ (Given)

$\angle BAC = \angle CDE$ (Given)

$\angle ACB = \angle DCE$ (Vertically opposite angles)

$\therefore \Delta ABC \cong \Delta CDE$

(By A.A.S. criterion of congruence)

$\therefore$ $BC = CE$ (c.p.c.t.)

$\Rightarrow$ $3x - 7 = [illegible]$ $\Rightarrow$ $3x = [illegible] + 7$ $\Rightarrow$ $3x = [illegible]$

$\Rightarrow$ $x = \frac{[illegible]}{3}$ $\Rightarrow$ $x = [illegible]$

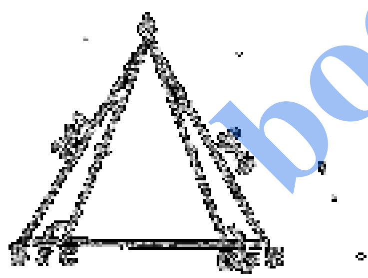
Also, $AB = DE$ (c.p.c.t.)

$\Rightarrow$ $[illegible]$ $\Rightarrow$ $2y + 5 = [illegible]$

$\Rightarrow$ $2y = [illegible] - 5$ $\Rightarrow$ $2y = [illegible]$ $\Rightarrow$ $y = [illegible]$

Hence, $x = [illegible]$ and $y = [illegible]$

(ii)



In ΔABC and ΔABD

$AC = AD$ (Given)

$\angle BAC = \angle DAB$ (Given)

$\angle ACB = \angle ADB$ (Given)

$\therefore$ $\Delta ABC \cong \Delta ABD$

(By A.A.S. criterion of congruence)

$\therefore$ $[illegible]$ (c.p.c.t.)

[illegible]

Also, [illegible] (given)

[illegible]

Multiplying equation (ii) by [illegible] and subtracting equation (i), we get

[illegible]

Subtracting, [illegible]

[illegible]

Substituting the value of y in equation (ii) we get

[illegible]

Hence, [illegible]

Exercise 10.3

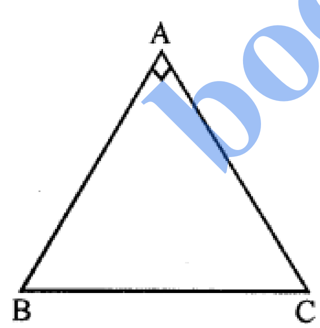
Question 1.

ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

In right ΔABC, $\angle A = 90°$

$\therefore \angle B + \angle C = 180° - \angle A$

$= 180° - 90° = 90°$



$\because AB = AC$

$\therefore \angle C = \angle B$ (Angles opposite to equal sides)

$\therefore \angle B + \angle B = 90° \Rightarrow 2\angle B = 90°$

$\therefore \angle B = \frac{90°}{2} = 45°$

$\therefore \angle B = \angle C = 45°$

Solution:

Question 2.
Show that the angles of an equilateral triangle are 60° each.
Solution:

ΔABC is an equilateral triangle

$\therefore$ $AB = BC = CA$

$\therefore$ $\angle A = \angle B = \angle C$ (Opposite to equal sides)

But $\angle A + \angle B + \angle C = 180°$ (Sum of angls of a triangle)

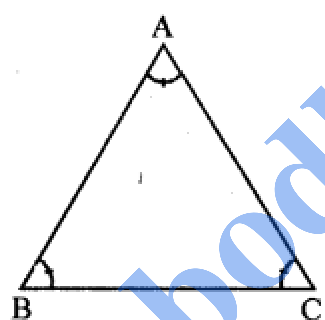
$\therefore$ $\angle A + \angle A + \angle A = 180°$

$$\Rightarrow 3\angle A = 180° \Rightarrow \angle A = \frac{180°}{3} = 60°$$

$\therefore$ $\angle A = \angle B = \angle C = 60°$

Question 3.
Show that every equiangular triangle is equilateral.
Solution:

ΔABC is an equaiangular



$\therefore$ $\angle A = \angle B = \angle C$

In ΔABC

$\because$ $\angle B = \angle C$

$\therefore$ $AC = AB$ (Sides opposite to equal angles) ...(*i*)

Similarly, $\angle C = \angle A$

$\therefore$ $BC = AB$ ...(*ii*)

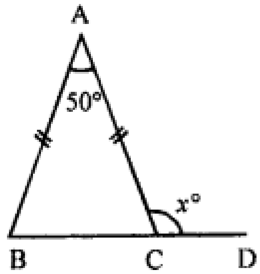
From (*i*) and (*ii*)

$AB = BC = AC$

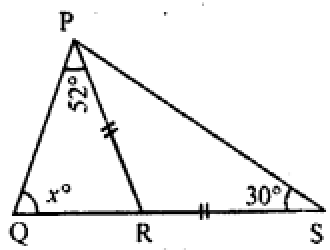
$\therefore$ ΔABC is an equilateral triangle

Question 4.

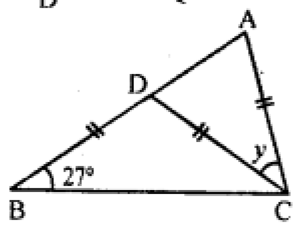
In the following diagrams, find the value of x:



(*i*)



(*ii*)



(*iii*)

Solution:

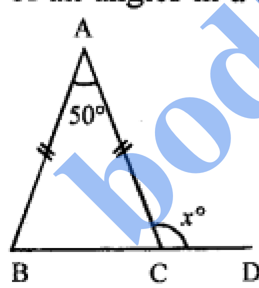
(*iii*)

(*i*) In following diagram given that AB = AC

i.e. $\angle B = \angle ACB$ (angles opposite to equal sides in a triangles are equal)

Now, $\angle A + \angle B + \angle ACB = 180°$

(sum of all angles in a triangle is 180°)



(*i*)

$\Rightarrow \quad 50° + \angle B + \angle B = 180°$

$[\because \angle A = 50° \text{ (given) } \angle B = \angle ACB]$

$\Rightarrow \quad 50° + 2\angle B = 180° \Rightarrow 2\angle B = 180° - 50°$

$\Rightarrow \quad 2\angle B = 130° \quad \Rightarrow \quad \angle B = \dfrac{130}{2} = 65°$

$\therefore \quad \angle ACB = 65°$

Also, $\angle ACB + x° = 180°$ (Linear pair)

$65° + x° = 180° \quad \Rightarrow \quad x° = 180° - 65°$

$\Rightarrow \quad x° = 115°$

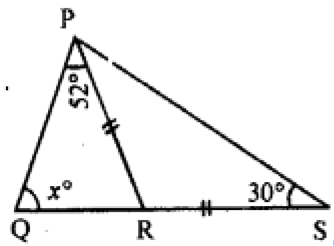
Hence, value of $x = 115$

(ii) In ΔPRS,

Given that $PR = RS$

$\therefore \quad \angle PSR = \angle RPS$

(angles opposite in a triangle, equal sides are equal)



(*ii*)

$\Rightarrow \quad 30° = \angle RPS \quad \Rightarrow \quad \angle RPS = 30°$ .... (1)

$\angle QPS = \angle QPR + \angle RPS$

$\Rightarrow \quad \angle QPS = 52° + 30°$

(Given, $\angle QPR = 52°$ and from (1), $\angle RPS = 30°$)

$\Rightarrow \quad \angle QPS = 82°$ .... (2)

Now, in ΔPQS

$\angle QPS + \angle QSP + PQS = 180°$

(sum of all angles in a triangles is 180°)

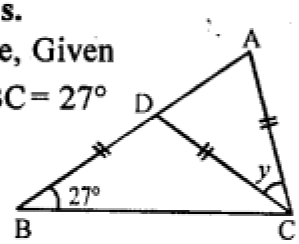
$\Rightarrow \quad 82° + 30° + x° = 180°$

[From (2) $\angle QPS = 82°$ and $\angle QSP = 30°$ (given)]

$\Rightarrow \quad 112° + x° = 180° \quad \Rightarrow \quad x° = 180° - 112°$

Hence, value of $x =$ **68 Ans.**

(*iii*) In the following figure, Given that, $BD = CD = AC$ and $\angle DBC = 27°$



Now, in Δ BCD

BD = CD (given)

$\angle DBC = \angle BCD$ ..... (1)

(In a triangle sides opposite equal angles are equal)

Also, $\angle DBC = 27°$ (given) .... (2)

From (1) and (2), we get

$\angle BCD = 27°$ ..... (3)

Now, ext. $\angle CDA = \angle DBC + \angle BCD$

[exterior angle is equal to sum of two interior opposite angles]

$\Rightarrow$ ext. $\angle CDA = 27° + 27°$ [From (2) and (3)]

$\Rightarrow \quad \angle CDA = 54°$ ..... (4)

In ΔACD,

AC = CD (given)

$\angle CAD = \angle CDA$ (In a triangle, angles opposite to equal sides are equal)

$\angle CAD = 54°$ [From (4)] .... (5)

Also, in ΔACD

$\angle CAD + \angle CDA + \angle ACD = 180°$

(sum of all angles in a triangle is 180°)

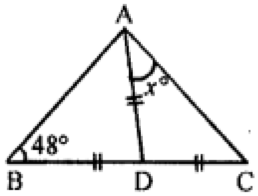
$\Rightarrow \quad 54° + 54° + y = 180°$ [From (4) and (5)]

$\Rightarrow \quad 108° + y = 180° \quad \Rightarrow \quad y = 180° - 108°$

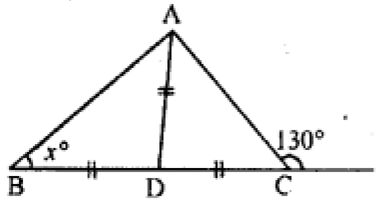
$\Rightarrow \quad y = 72°$

Hence, value of $y = 72°$

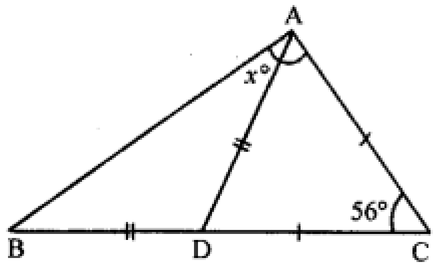
Question 5.
In the following diagrams, find the value of x:



(i)



(ii)

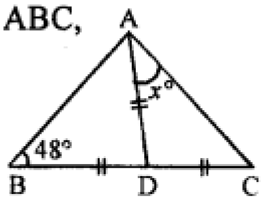


(iii)

Solution:

(i) In the following figure,

Given. In ΔABC,



(i)

$AD = BD = CD$.

$\angle B = 48°$, $\angle DAC = x°$

Now, in ΔABD

$BD = AD$ (given)

$\angle BAD = \angle B$ .... (1)

(angles opposite equal sides in a triangle are equal).

$\angle B = 48°$ .... (2)

Now, in ΔABD

From (1) and (2) $\angle BAD = 48°$ ......
(3)

Exterior $\angle ADC = \angle B + \angle BAD$

(In a triangle exterior angle is equal to sum of two interior opposite angles)

$\angle ADC = 48° + 48° \Rightarrow \angle ADC = 96°$ ... (4)

Now, in Δ ADC

AD = DC (given)

$\therefore \angle C = \angle DAC$ .... (5)

(In a triangle, angles opposite equal sides are equal)

$\angle DAC = x°$ (given).... (6)

From (5) and (6)

$\angle C = x°$ .... (7)

Now, in ΔADC

$\angle C + \angle ADC + \angle DAC = 180°$

(sum of all the angles in a triangle is 180°)

$\Rightarrow x° + 96° + x° = 180°$ [From 4, 6 and 7]

$\Rightarrow 2x° = 180° - 96° \Rightarrow 2x° = \frac{84}{2} \Rightarrow x° =$
42°

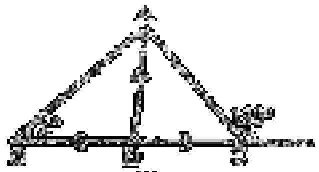
Hence, value of x = 42

(*ii*) Given in ΔABC,

Exterior $\angle ACE = 130°$ and AD = BD = DC

To calculate the value of x.

Now, $\angle ACD + ACE = 180°$ .... (1)



(ii)

(∵ DCE is a straight line)

$\angle ACE = 150°$ ... (2)

From (1) and (2),

$\angle ACD + 150° = 180° \Rightarrow \angle ACD = 180° - 150°$

$\Rightarrow \angle ACD = 30°$ ... (3)

Now, in ΔABC,

$AB = AC$ (given)

$\therefore \angle ABC = \angle ACB$ ... (4)

(In a triangle, angles opposite equal sides are equal)

From (3) and (4),

$\angle ABC = 30°$ ... (5)

Now, in ΔABC

$\angle ABC + \angle ACB + \angle BAC = 180°$

(sum of all angles in a triangle is 180°)

$\angle BAC + 30° + 30° = 180°$ [From (3), (5)]

$\angle BAC + 60° = 180° \Rightarrow \angle BAC = 120°$ ... (6)

Also, $\angle ADC = \angle BAD + \angle ABD$ ... (7)

(In a triangle, exterior angle is equal to sum of two interior opposite angles)

$BD = AD$ (given)

$\therefore \angle BAD = \angle ABD$ ... (8)

(In a triangle, angles opposite equal sides are equal)

From (7) and (8),

$\angle ADC = \angle ABD + \angle ABD$

$\angle ADC = 2\angle ABD$ ... (9)

From (6) and (9),

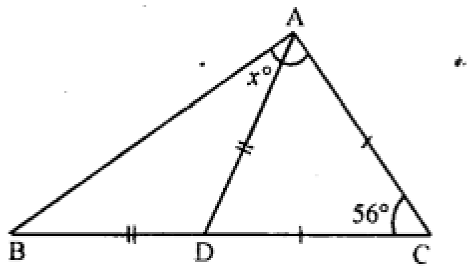
$90° = 2\angle ABD \Rightarrow 90° = 2x°$

[$\angle ABD = x°$ (given)]

$\Rightarrow 2x° = 90° \Rightarrow x° = \frac{90°}{2} = 45°$

Hence, value of x = 45 Ans.

(iii) In following figure,



(iii)

Given that AC = CD , AD = BD

and $\angle BAC = x°$, $\angle ACD = 56°$

o evaluate the value of x.

low, in Δ ACD

C = CD (given)

$\angle ADC = \angle DAC$ .... (1)

In a triangle, angles opposite to equal sides are qual)

lso, $\angle ADC + \angle DAC + 56° = 180°$

(sum of all angles in a triangle is 180°)

$\Rightarrow \angle DAC + \angle DAC + 56° = 180°$

[From equation (1) $\angle ADC = \angle DAC$]

$\Rightarrow 2\angle DAC + 56° = 180°$

$\Rightarrow 2\angle DAC = 180° - 56° \Rightarrow 2\angle DAC = 124°$

$\Rightarrow \angle DAC = \frac{124°}{2} \Rightarrow \angle DAC = 62°$ .... (2)

.e. $\angle ADC = 62°$ ...... (3)

[From (1) $\angle DAC = \angle ADC$]

low, in ΔABD

D = BD (given)

$\angle ABD = \angle BAD$ ...... (4)

In a triangle, angles opposite equal sides are qual)

But ext. $\angle ADC = \angle ABD + \angle BAD$

$\Rightarrow 62° = \angle BAD + \angle BAD$ [From (3) and (4)]

$\Rightarrow 62° = 2\angle BAD \Rightarrow \angle BAD = 62°$

$\Rightarrow \angle BAD = \frac{62°}{2} \Rightarrow \angle BAD = 31°$

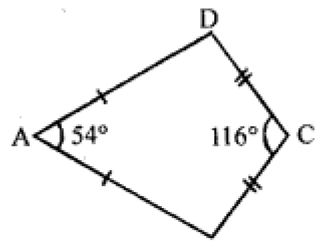
... (5)

Now, from figure, $x° = \angle BAD + \angle DAC$

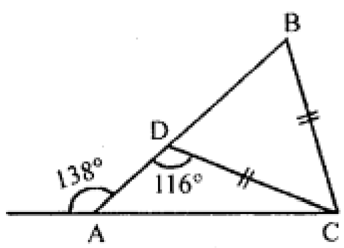
$x° = 31° + 62° \Rightarrow x° = 93°$ [From (4) and (5)]

Hence, value of $x = 93$

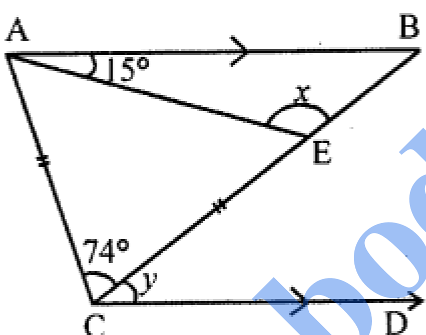
Question 6.
(a) In the figure (1) given below, AB = AD, BC = DC. Find ∠ ABC.
(b)In the figure (2) given below, BC = CD. Find ∠ACB.
(c) In the figure (3) given below, AB || CD and CA = CE. If ∠ACE = 74° and ∠BAE =15°, find the values of x and y.
Solution:



(1)



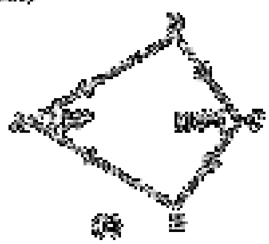
(2)



(3)

[illegible] In figure, Given that $AB = AD$, $BC = DC$,
$\angle DAB = 64^\circ$ and $\angle BCD = 110^\circ$

To find $\angle ABC$



Construction: Join BD

Here, In $\triangle BCD$

$BC = DC$ (given)

$\angle CBD = \angle CDB$ ... (i)

(In a triangle, equal sides have equal angles opposite to them)

Also, $\angle BCD + \angle CBD + \angle CDB = 180^\circ$

(sum of all angles in a triangle is 180°)

$\Rightarrow \quad 110^\circ + \angle CBD + \angle CDB = 180^\circ$

$[\angle BCD = 110^\circ$ (given) and from (i), $\angle CBD = \angle CDB]$

$\Rightarrow \quad 110^\circ + 2\angle CBD = 180^\circ$

$\Rightarrow \quad 2\angle CBD = 180^\circ - 110^\circ = 70^\circ \quad \Rightarrow \quad 2\angle CBD = 70^\circ$

$\Rightarrow \quad \angle CBD = \frac{70^\circ}{2} = 35^\circ$ ... (ii)

Now, In $\triangle ABD$, $AB = AD$

(given)

$\angle ABD = \angle ADB$ ... (iii)

(In a triangle equal sides have equal angles opposite to them)

Also, $\angle BAD + \angle ABD + \angle ADB = 180^\circ$

(sum of all angles in a triangle is 180°)

$\Rightarrow \quad 64^\circ + \angle ABD + \angle ADB = 180^\circ$

$[\angle BAD = 64^\circ$ (given) and from (iii) $\angle ABD = \angle ADB]$

$\Rightarrow \quad 64^\circ + 2\angle ABD = 180^\circ$

$\Rightarrow \quad 2\angle ABD = 180^\circ - 64^\circ \quad \Rightarrow \quad 2\angle ABD = 116^\circ$

$\Rightarrow \quad \angle ABD = \frac{116^\circ}{2} \quad \Rightarrow \quad \angle ABD = 58^\circ$

... (iv)

From figure it is clear that,

$\angle ABC = \angle CBD + \angle ABD$

$\Rightarrow \quad \angle ABC = 35^\circ + 58^\circ$ [using (ii) and (iv)]

$\Rightarrow \quad \angle ABC = $ [illegible]

(ii) In following figure

Given, $BC = CD$, $\angle ADE = $ [illegible],

$\angle EBD = $ [illegible]

To find, $\angle ACB$.

Now, $\angle ADB + \angle BDC = 180^\circ$ (Linear pair)

$\Rightarrow$ [illegible] $+ \angle BDC = 180^\circ$

[$\angle ADB = $ [illegible] given]

$\Rightarrow \quad \angle BDC = 180^\circ - $ [illegible]

$\Rightarrow \quad \angle BDC = $ [illegible] ...(1)

In ΔBCD, $BC = CD$ (given)

$\therefore \quad \angle BDC = \angle B$ ...(2)

(In a triangle equal sides have equal angles opposite to them)

From (1) and (2),

$\angle B = $ [illegible] ...(3)

Now, $\angle EAB = \angle B + \angle ACB$

(exterior angle is equal to sum of two interior opposite angles)

$\Rightarrow \quad 120^\circ = $ [illegible] $+ \angle ACB$

[$\angle EAB = 120^\circ$ (given) from (3), $\angle B = $ [illegible]]

$\Rightarrow$ [illegible] $+ \angle ACB = 120^\circ$

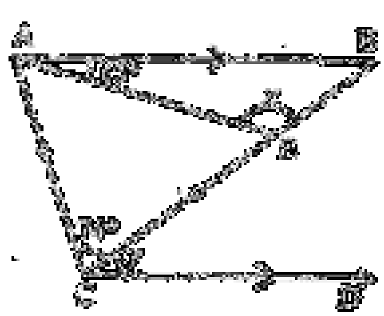
$\Rightarrow \quad \angle ACB = 120^\circ - $ [illegible]

$\Rightarrow \quad \angle ACB = $ [illegible]

(iii) In the figure, $AB \parallel CD$

$CA = CE$

$\angle ACE = $ [illegible], $\angle BAE = $ [illegible]



In ΔAEC

$AC = CE$

$\therefore \angle CAE = \angle CEA$

But $ACE = 74°$

$\therefore \angle CAE + \angle CEA = 180° - 74° = 106°$

$$\therefore \angle CAE = \angle CEA = \frac{106°}{2} = 53°$$

Ext. $\angle AEB = \angle CAE + \angle ACE$

$\Rightarrow x = 53° + 74° = 127°$

$\because AB \parallel CD$

$\therefore \angle CAB + \angle ACD = 180°$

(Sum of cointerior angles)

$\Rightarrow 15° + 53° + 74° + y° = 180°$

$\Rightarrow 142° + y = 180°$

$\Rightarrow y = 180° - 142° = 38°$

Question 7.

In $\triangle ABC$, $AB = AC$, $\angle A = (5x + 20)°$ and each of the base angle is $\frac{2}{5}$ th of $\angle A$. Find the measure of $\angle A$.

Solution:

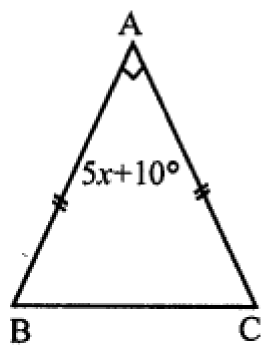
Given : In ΔABC, $AB = AC$

$\angle A = (5x + 20)°$

$\angle B = \angle C = \frac{2}{5}(\angle A)$

$= \frac{2}{5}(5x + 20)°$

$= 2(x + 4)° = 2x + 8$



$\therefore$ But sum of angles of a triangle = 180°

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 5x + 20 + 2x + 8 + 2x + 8 = 180°$

$9x + 36 = 180°$

$9x = 180 - 36 = 144$

$x = \frac{144}{9} = 16$

$\therefore \angle A = 5x + 20 = 5 \times 16 + 20$

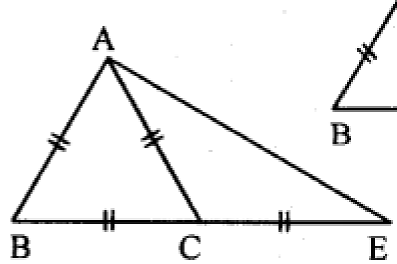
$= 80° + 20° = 100°$

Question 8.

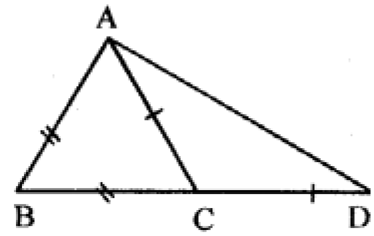
(a) In the figure (1) given below, ABC is an equilateral triangle. Base BC is produced to E, such that BC'= CE. Calculate ∠ACE and ∠AEC.

(b) In the figure (2) given below, prove that ∠ BAD : ∠ ADB = 3 : 1.

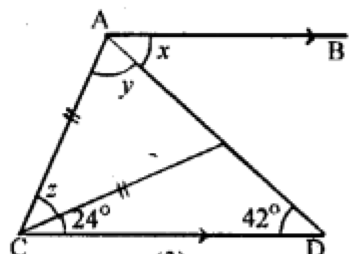
(c) In the figure (3) given below, AB || CD. Find the values of x, y and ∠.



(1)



(2)



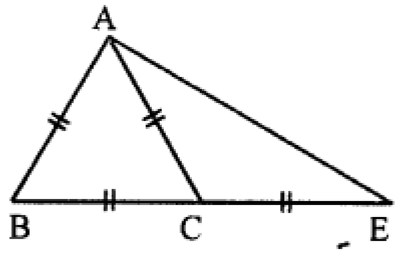
(3)

Solution:

(*a*) In following figure,

Given. ABC is an equilateral triangle BC = CE

To find. $\angle ACE$ and $\angle AEC$.



(1)

As given that ABC is an equilateral triangle,

i.e. $\angle BAC = \angle B = \angle ACB = 60°$ .... (1)

(each angle of an equilateral triangle is 60°)

Now, $\angle ACE = \angle BAC + CB$

(exterior angle is equal to sum of two interior opposite angles)

$\Rightarrow \quad \angle ACE = 60° + 60°$ [By (1)]

$\Rightarrow \quad \angle ACE = 120°$

Now, in ΔACE

Given, AC = CE ($\because$ AC = BC = CE)

$\angle CAE = \angle AEC$ .... (2)

(In a triangle equal sides have equal angles opposite to them)

Also, $\angle CAE + \angle AEC + 120° = 80°$
(sum of all angles in a triangle is 180°)

$\Rightarrow \quad \angle AEC + \angle AEC + 120° = 180 \quad$ [By (2)]

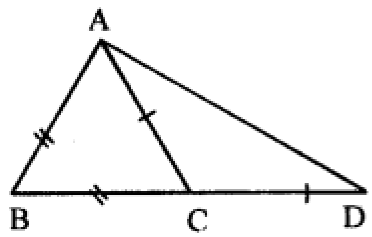
$\Rightarrow \quad 2\angle AEC = 180° - 120° \Rightarrow 2\angle AEC = 60°$

$\Rightarrow \quad 2\angle AEC = \frac{60°}{2} = 30°$

Hence, $\angle ACE = 120°$ and $\angle AEC = 30°$

(b) In following figure

Given. ΔABD, AC meets BD in C. AB = BC, AC = CD.



(2)

To prove. $\angle BAD : \angle ADB = 3 : 1$

Proof. In ΔABC,

AB = BC (Given)

$\therefore \angle ACB = \angle BAC$ .... (1)

(In a triangle, equal angles opposite to them)

In ΔACD,

AC = CD (Given)

$\therefore \angle ADC = \angle CAD$

(In a triangle, equal sides have equal angles opposite to them)

$\Rightarrow \quad \angle CAD = \angle ADC$ .... (2)

From, Adding (1) and (2), we get

$\angle BAC + \angle CAD = \angle ACB + \angle ADC$

$\angle BAD = \angle ACB + \angle ADC$ .... (3)

Now, in ΔACD

Exterior $\angle ACB = \angle CAD + \angle ADC$ ....(4)

(In an triangle, exterior angle is equal to sum of two interior opposite angles)

$\therefore \quad \angle ACB = \angle ADC + \angle ADC$

[From (2) and (4)]

$\Rightarrow \quad \angle ACB = 2\angle ADC \qquad \text{.... (5)}$

Now, $\angle BAD = 2\angle ADC + \angle ADC$

[From (3) and (4)]

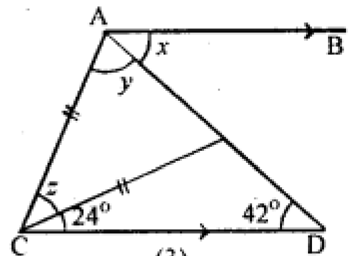
$\Rightarrow \quad \angle BAD = 3\angle ADC \quad \Rightarrow \quad \dfrac{\angle BAD}{\angle ADC} = \dfrac{3}{1}$

$\Rightarrow \quad \angle BAD : \angle ADC = 3 : 1$ **(Q.E.D.)**

(c) In following figure,

Given. $AB \parallel CD$, $\angle ECD = 24°$, $\angle CDE = 42°$.

To find. The value of x, y and z.



(3)

Now, in ΔCDE,

ext $\angle CEA = 24° + 42°$ [In a triangle exterior angle is equal to sum of two interior opposite angles]

$\angle CEA = 66°$ .... (1)

Now, in ΔACE

$AC = CE$ (Given)

$\therefore \quad \angle CAE = \angle CEA$

(In a triangle equal side have equal angles opposite to them)

$y = 66°$ (By equation (1) ....(2)

Also, $y + z + \angle CEA = 180°$

(sum of all angles in a triangle is 180°)

$\Rightarrow \quad 66° + z + 66° = 180°$

[From equation (1) and (2)]

$\Rightarrow \quad z + 132° = 180° \Rightarrow \quad z = 180° - 132°$

$\Rightarrow \quad z = 48°$ .... (3)

Given that, $AB \parallel CD$

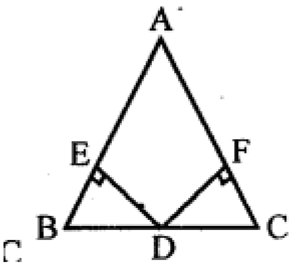
$\therefore \quad \angle x = \angle ADC$ (alternate angles)

$x = 42°$ (4)

Hence, from (2), (3) and (4) equation gives $x = 42°$, $y = 66°$ and $z = 48°$

Question 9.

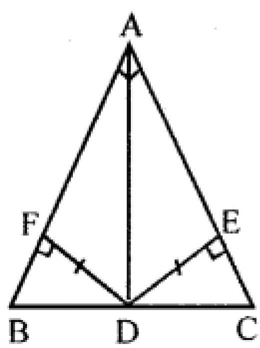
In the given figure, D is mid-point of BC, DE and DF are perpendiculars to AB and AC respectively such that DE = DF. Prove that ABC is an isosceles triangle.



Solution:

Given : In ΔABC,
D is the mid-point of BC

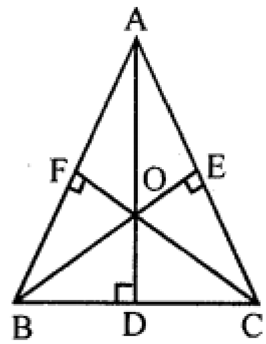
$DE \perp AB$, $DF \perp AC$
$DE = DE$



To prove : ΔABC is an isosceles triangle
Proof : In right ΔBED and ΔCDF
Hypotenuse $BD = DC$ (D is mid-point)
Side $DF = DE$ (Given)
$\therefore$ $\Delta BED \cong \Delta CDF$ (RHS axiom)
$\therefore$ $\angle B = \angle C$
$\Rightarrow$ $AB = AC$ (Sides opposite to equal angles)
$\therefore$ ΔABC is an isosceles triangle

Question 10.
In the given figure, AD, BE and CF arc altitudes of $\triangle ABC$. If AD = BE = CF, prove that ABC is an equilateral triangle.
Solution:

Given : In the figure given,
AD, BE and CF are altitudes of ΔABC and
$AD = BE = CF$



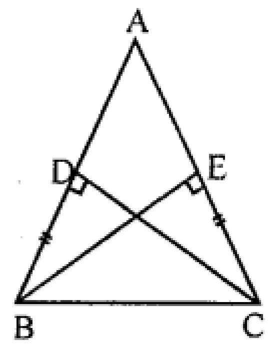
To prove : ΔABC is an equilateral triangle
Proof : In the right ΔBEC and ΔBFC

Hypotenuse $BC = BC$ (Common)
Side $BE = CF$ (Given)
$\therefore$ $\Delta BEC \cong \Delta BFC$ (RHS axiom)
$\therefore$ $\angle C = \angle B$ (c.p.c.t.)
$AB = AC$ (Sides opposite to equal angles) ...(*i*)

Similarly we can prove that $\Delta CFA \cong \Delta ADC$
$\therefore$ $\angle A = \angle C$
$\therefore$ $AB = BC$ ...(*ii*)
From (*i*) and (*ii*),
$AB = BC = AC$
$\therefore$ ΔABC is an equilateral triangle

Question 11.
In a triangle ABC, AB = AC, D and E are points on the sides AB and AC respectively such that BD = CE. Show that:
(i) $\triangle DBC \cong \triangle ECB$
(ii) $\angle DCB = \angle EBC$
(iii) OB = OC,where O is the point of intersection of BE and CD.
Solution:

Given : In ΔABC, $AB = AC$
D and E are points on the sides AB and AC respectively such that $BD = CE$

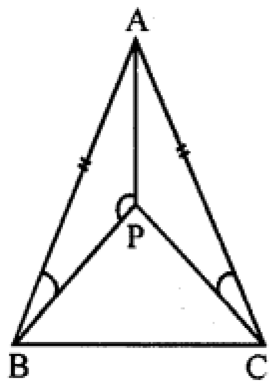


To prove : (*i*) $\Delta DBC \cong \Delta ECB$
(*ii*) $\angle DCB = \angle EBC$
$OB = OC$ where O is the point of intersection of BE and CD.
CD and BE are joined
Proof : In ΔABC, $AB = AC$
and $BD = CE$
$\therefore$ $\angle C = \angle B$ (Opposite to equal sides)
In ΔDBC and ΔECB
$BC = BC$ (Common)
$BD = CE$ (Given)
$\angle B = \angle C$ (Proved)
(*i*) $\therefore$ $\Delta DBC \cong \Delta ECB$ (SAS axiom)
(*ii*) $\therefore$ $\angle DCB = \angle EBC$ (c.p.c.t.)
(*iii*) In ΔOBD and ΔOCE
$\angle D = \angle E$ (each $= 90^{\circ}$)
$DB = EC$ (given)
$\angle DOB = \angle EOC$ (vertically oppositive angles)
$\therefore$ $\Delta OBD \cong \Delta OCE$ (A.A.S. Axiom)
$\therefore$ $OB = OC$ (c.p.c.t.)

Question 12.
ABC is an isosceles triangle in which AB = AC. P is any point in the interior of

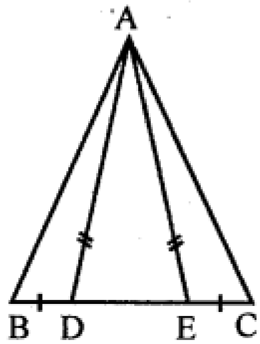
ΔABC such that ∠ABP = ∠ACP. Prove that
(a) BP = CP
(b) AP bisects ∠BAC.
Solution:

Given : In an isosceles ΔABC, AB = AC
P is any point inside the ΔABC such that
∠ABP = ∠ACP
To prove : (a) BP = CP



(b) AP bisects ∠BAC
Proof : In ΔAPB and ΔAPC
AP = AP (Common)
AB = AC (Given)
∠ABP = ∠ACP (Given)
∴ ΔAPB ≅ ΔAPC (SSA axiom)
(*i*) ∴ BP = CP (c.p.c.t.)
and ∠BAP = ∠CAP (c.p.c.t.)
∴ AP bisects ∠BAC

Question 13.
In the adjoining figure, D and E are points on the side BC of ΔABC such that BD = EC and AD = AE. Show that ΔABD ≅ ΔACE.
Solution:



Given : In the given figure, D and E are the points on the sides BC of ΔABC,

$BD = EC$ and $AD = AE$

To prove : $\Delta ABD \cong \Delta ACE$

Proof : $\because$ In ΔADE

$\angle ADE = \angle AED$

$\therefore$ $\angle AED = \angle ADE$

But $\angle ADE + \angle ADB = 180°$ (Linear pair)

and $\angle AED + \angle AEC = 180°$ (Linear pair)

$\therefore$ $\angle ADB = \angle AEC$ ($\because \angle ADE = \angle AED$)

Now in ΔABD and ΔACE

$AD = AE$ (Given)

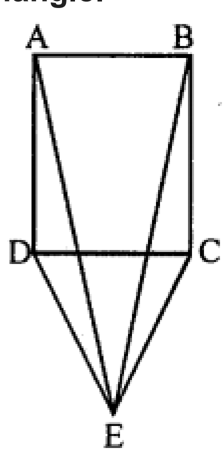
$BD = CE$ (Given)

$\angle ADB = \angle AEC$ (Proved)

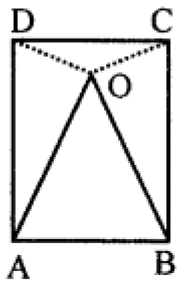
$\therefore$ $\Delta ABD \cong \Delta ACE$ (SAS axiom)

Question 14.

(a) In the figure (i) given below, CDE is an equilateral triangle formed on a side CD of a square ABCD. Show that $\triangle ADE \cong \triangle BCE$ and hence, AEB is an isosceles triangle.



(b) In the figure (ii) given below, O is a point in the interior of a square ABCD such that OAB is an equilateral trianlge. Show that OCD is an isosceles triangle.



Solution:

(a) **Given :** In the figure, CDE is an equilateral triangle on the side CD of square ABCD

AE and BE are joined

To prove : (*i*) $\Delta ADE \cong \Delta BCE$

(*ii*) ΔAEB is an isosceles triangle.

Proof : $\because$ Each angle of a square is 90° and each angle of an equilateral triangle is 60°

$\therefore$ $\angle ADE = \angle ADC + \angle CDE$

$= 90° + 60° = 150°$

Similarly, $\angle BCE = 90° + 60° = 150°$

Now in ΔADE and ΔBCE

$AD = BC$ (Sides of a square)

$DE = CE$ (Sides of an equilateral triangle)

$\angle ADE = \angle BCE$ (Each 150°)

(*i*) $\therefore$ $\Delta ADE \cong \Delta BCE$ (SAS axiom)

(*ii*) $\therefore$ $AE = BE$

Now in ΔAEB,

$AE = BE$ (Proved)

$\therefore$ ΔAEB is an isosceles triangle

(b) **Given :** In the figure, O is a point in interior of the square ABCD such that OAB is an equilateral triangle.

To prove : ΔOCD is an isosceles triangle

Proof : $\because$ ΔOAB is an equilateral triangle

$\therefore$ $OA = OB = AB$

$\angle OAD = \angle DAB - \angle OAB$

$= 90° - 60° = 30°$

Similarly, $\angle OBC = 30°$

Now in ΔOAD and ΔOBC

$OA = OB$ (Sides of equilateral triangle)

$AD = BC$ (Sides of a square)

$\angle OAD = \angle OBC$ (Each $= 30°$)

$\therefore \Delta OAD \cong \Delta OBC$ (SAS axiom)

$\therefore OD = OC$ (c.p.c.t.)

Now in ΔOCD,

$OD = OC$

$\therefore \Delta OCD$ is an isosceles triangle

Question 15.

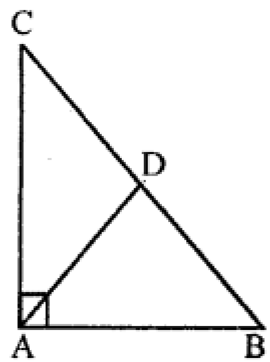
In the given figure, ABC is a right triangle with AB = AC. Bisector of ∠A meets BC at D. Prove that BC = 2AD.

Solution:

In the given figure, ΔABC is a right angled triangle, right angle at A

AB = AC

Bisector of $\angle A$ meets BC at D



To prove : BC = 2AD

Proof : In right ΔABC, $\angle A = 90°$ and AB = AC

$\therefore\ \angle B = \angle C = \dfrac{90°}{2} = 45°\ \ (\because \angle B + \angle C = 90°)$

$\because$ AD is bisector of $\angle A$

$\therefore\ \angle DAB = \angle DAC = \dfrac{90°}{2} = 45°$

Now in ΔADB

$\angle DAB = \angle B$ (Each 45°)

$\therefore$ AD = DB ...(*i*)

Similarly we can prove that in ΔADC,

$\angle DAC = \angle C = 45°$

$\therefore$ AD = DC ...(*ii*)

Adding (*i*) and (*ii*),

Adding (*i*) and (*ii*),

AD + AD = DB + DC = BD + DC

$\Rightarrow$ 2AD = BC

Hence BC = 2AD

Exercise 10.4

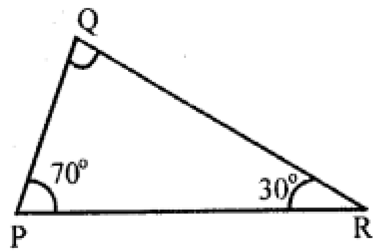
Question 1.
In △PQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is longest? Give reason for your answer.
Solution:

In ΔPQR, $\angle P = 70°$, $\angle R = 30°$

But $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow 70° + 30° + \angle Q = 180°$

$\Rightarrow 100° + \angle Q = 180°$



$\therefore \angle Q = 180° - 100° = 80°$

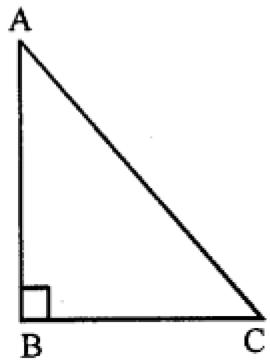
$\therefore \angle Q = 80°$ the greatest angle

$\therefore$ Its opposite side PR is the longest side
(Side opposite to greatest angle is longest)

Question 2.
Show that in a right angled triangle, the hypotenuse is the longest side.

Solution:

Given : In right angled ΔABC, $\angle B = 90°$



To prove : AC is the longest side

Proof : In ΔABC,

$\because$ $\angle B = 90°$

$\therefore$ $\angle A$ and $\angle C$ are acute angles

i.e., less than 90°

$\therefore$ $\angle B$ is the greatest angle

or $\angle B > \angle C$ and $\angle B > \angle A$

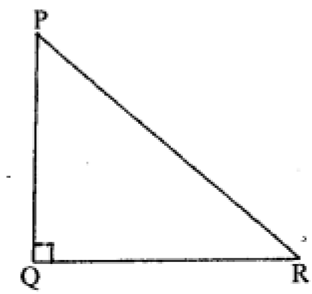
$\therefore$ $AC > AB$ and $AC > BC$

Hence AC is the longest side

Question 3.

PQR is a right angle triangle at Q and PQ : QR = 3:2. Which is the least angle.

Solution:



Here, PQR is a right angle triangle at Q. Also given that

PQ : QR = 3 : 2

Let PQ = $3x$, then, QR = $2x$

It is clear that QR is the least side.

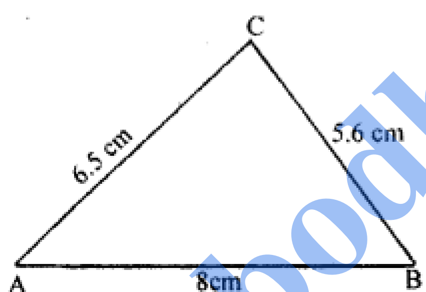
Then, we know that the least angle has least side opposite to it.

Hence, $\angle P$ is the least angle.

Question 4.

In $\triangle$ ABC, AB = 8 cm, BC = 5.6 cm and CA = 6.5 cm. Which is (i) the greatest angle ?
(ii) the smallest angle ?

Solution:



Given that AB = 8 cm, BC = 5.6 cm, CA = 6.5 cm.

Here AB is the greatest side

Then $\angle C$ is the greatest angle

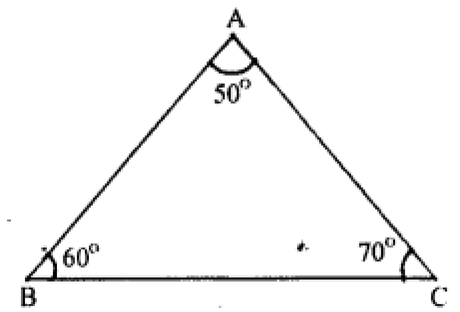
($\therefore$ the greater side has greater angle opposite to it)

Also, BC is the least side

then $\angle A$ is the least angle

($\because$ the least side has least angle opposite to it)

Question 5.
In $\triangle ABC$, $\angle A = 50°$, $\angle B = 60°$, Arrange the sides of the triangle in ascending order.
Solution:



Given in a ΔABC,

$\angle A = 50°$, $\angle B = 60°$

$\angle C = 180 - (\angle A + \angle B)$

[sum of all angles in a triangle is 180°]

$\Rightarrow \quad \angle C = 180° - (50° + 60°)$

$\Rightarrow \quad \angle C = 180° - 110° \quad \Rightarrow \quad \angle C = 70°$

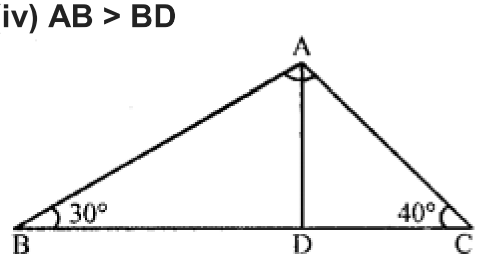
Now, $\angle A < \angle B < \angle C$

$BC < CA < AB$

($\because$ greater angles has greater side opposite to it)

Hence, sides of ΔABC in ascending order as BC, CA, AB.

Question 6.
In figure given alongside, $\angle B = 30°$, $\angle C = 40°$ and the bisector of $\angle A$ meets BC at D. Show
(i) BD > AD
(ii) DC > AD
(iii) AC > DC
(iv) AB > BD



Solution:

Given : In ΔABC, $\angle B = 30°$, $\angle C = 40°$
and bisector of $\angle A$ meets BC at D

To prove :

(*i*) $BD > AD$ (*ii*) $DC > AD$

(*iii*) $AC > DC$ (*iv*) $AB > BD$

Proof : In ΔABC,

$\angle B = 30°$ and $\angle C = 40°$

$\therefore \angle BAC = 180° - (30° + 40°) = 180° - 70° = 110°$

$\because$ AD is bisector of $\angle A$

$\therefore \angle BAD = \angle CAD = \dfrac{110°}{2} = 55°$

(*i*) Now in ΔABD,

$\because \angle BAD > \angle B$

$\therefore BD > AD$

(*ii*) In ΔACD,

$\angle CAD > \angle C$

$DC > AD$

(*iii*) $\angle ADC = 180° - (40° + 55°) = 180° - 95° = 85°$

In ΔADC,

$\because \angle ADC > \angle CAD$

$\therefore AC > DC$

(*iv*) Similarly,

$\angle ADB = 180° - \angle ADC = 180° - 85° = 95°$

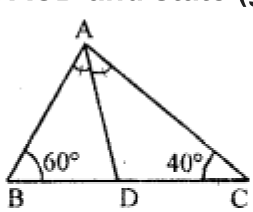
$\therefore$ In ΔADB

$AB > BD$ Hence proved.

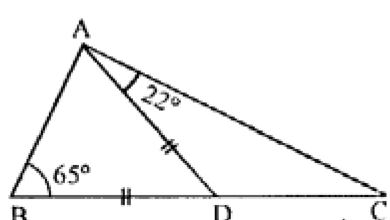
Question 7.

(a) In the figure (1) given below, AD bisects ∠A. Arrange AB, BD and DC in the descending order of their lengths.

(b) In the figure (2) given below, ∠ ABD = 65°, ∠DAC = 22° and AD = BD. Calculate ∠ ACD and state (giving reasons) which is greater : BD or DC ?



(1)

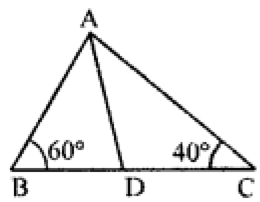


(2)

Solution:

(*a*) **Given.** In ΔABC, AD bisects $\angle A$, $\angle B = 60°$ and $\angle C = 40°$

To arrange. AB , BD and DC in the descending order.



(1)

In Δ ABC

$\angle BAC + \angle B + \angle C = 180°$

[sum of all angles in a triangle is 180°]

$\Rightarrow \quad \angle BAC + 60° + 40° = 180°$

[From given , $\angle B = 60°$, $\angle C = 40°$

$\Rightarrow \quad \angle BAC = 180° - 100° \quad \Rightarrow \quad \angle BAC = 80°$

$\therefore$ AD bisects $\angle A$

$\therefore \quad \angle BAD = \angle DAC = \frac{1}{2} \times \angle BAC$

$\Rightarrow \quad \angle BAD = \angle DAC = \frac{1}{2} \times 80°$

$\angle BAD = \angle DAC = 40°$ ..... (1)

In ΔABD, ext. $\angle ADC = \angle B + \angle BAD$

[In a triangle exterior angle is equal to sum of opposite interior angles]

$\therefore \quad \angle ADC = 60° + 40°$

$\Rightarrow \quad \angle ADC = 100°$ ..... (2)

Similarly, In ΔACD, ΔADB
$= 40° + 40° = 80°$ ..... (3)

Now, $\angle ADB = 80°$ [From (3)]

$\angle BAD = 40°$ [From (2)]

$\angle DAC = 40°$ [From (1)]

Now, $\angle ADB > \angle DAC = \angle BAD$

[$\because$ $80° > 40° = 40°$]

Hence, AB, DC, BD in the descending order of their lengths

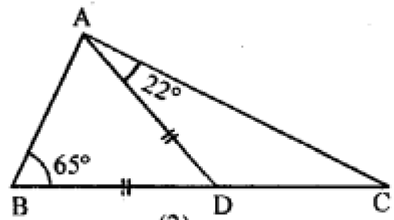
(**Note :** It can also written as AB, BD, DC in the descending order ∵ DC = BD)

(*b*) Given. In ΔABC, $\angle ABD = 65°$

$\angle DAC = 22°$, and AD = BD.

To calculate the $\angle ACD$ and say which is greater, BD or DC.

Now, in ΔABD



(2)

$\therefore$ AD = BD (given)

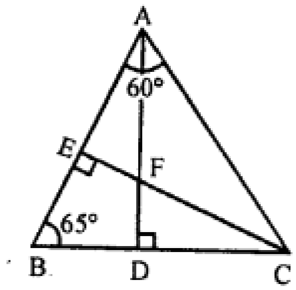
$\therefore \angle ABD = \angle BAD$ ..... (1)
(In a triangle, equal sides have equal angles opposite to them)
Also, $\angle ABD = 65°$ ..... (2)
From (1) and (2), we get
$\angle BAD = 65°$ ..... (3)
In ΔABC,
$\angle A + \angle B + \angle C = 180°$
[sum of all angles in a triangle is 180°]
$(BAD + \angle DAC) + \angle B + \angle C = 180°$
[$\therefore \angle A = \angle BAD + \angle DAC$]
$\Rightarrow \angle BAD + \angle DAC + \angle B + \angle ACD = 180°$
[$\therefore \angle C = \angle ACD$]
$\Rightarrow 65° + 22° + 65° + \angle ACD = 180°$
(Substituting the value of $\angle BAD$, $\angle DAC$ & $\angle B$)
$\Rightarrow 152° + \angle ACD = 180°$
$\Rightarrow \angle ACD = 180° - 152° \Rightarrow \angle ACD =$ 28° Now, $\angle BAD = 65°$ [From (3)]
and $\angle CAD = 22°$ (Given)
$\therefore \angle BAD > \angle CAD$
$\therefore BD > DC$
[Greater angle has greater opposite side]
Hence, BD is greater than DC.

Question 8.
(a) In the figure (1) given below, prove that (i) CF> AF (ii) DC>DF.
(b) In the figure (2) given below, AB = AC.
Prove that AB > CD.
(c) In the figure (3) given below, AC = CD. Prove that BC < CD.

(a) Given. In ΔABC, $AD \perp BC$, $CF \perp AB$.

AD and $\angle E$ intersect at F.

$\angle BAC = 60°$, $\angle ABC = 65°$



(1)

To prove. (*i*) CF > AF (*ii*) DC > DF

Proof. (*i*) In ΔAEC,

$\angle B + \angle BEC + \angle BCE = 180°$ ..... (1)

(sum of angles of a triangle = 180°)

$\angle B = 65°$ (Given) ..... (2)

$\angle BEC = 90°$ [(CE $\perp$ AB) Given] ..... (3)

Putting these value in equation (1), we get

$65° + 90° + \angle BCE = 180°$

$\Rightarrow \quad 155° + \angle BCE = 180° \quad \Rightarrow \quad \angle BCE = 25°$

$\Rightarrow \quad \angle DCF = 25° \quad [BCE = \angle DCF]$ ..... (4)

Now in ΔCDF,

$\angle DCF + \angle FDC + \angle CFD = 180°$

Solution:

[sum of all angles in a triangle is $180°$]

$\Rightarrow \quad 25 + 90° + \angle CFD = 180°$

[From (4) $\angle DCF = 25°$ & $AD \perp BC$, $\angle FDC = 90°$]

$\Rightarrow \quad 115° + \angle CFD = 180°$

$\Rightarrow \quad \angle CFD = 180° - 115° \angle CFD = 65°$ ..... (5)

Also, $\angle AFC + \angle CFD = 180°$

[AFD is a straight line]

$\Rightarrow \quad \angle AFC + 65° = 180°$

$\Rightarrow \quad \angle AFC = 180 - 65° (\angle CFD = 65°)$

$\Rightarrow \quad \angle AFC = 115°$ .... (6)

Now, in ACE,

$\angle ACE + \angle CEA + \angle BAC = 180°$

[sum of all angles in a triangle is $180°$]

$\Rightarrow \quad \angle ACE + 90° + 60° = 180°$

[$\because \angle CEA = 90°, \angle BAC = 60°$]

$\Rightarrow \quad \angle ACE + 150° = 180°$

$\Rightarrow \quad \angle ACE = 180° - 150° \Rightarrow \angle ACE = 30°$... (7)

Now, in ΔAFC,

$\angle AFC + \angle ACF + \angle FAC = 180°$

[sum of all angles in a triangle is $180°$]

$\Rightarrow \quad 115° + 30° + \angle FAC = 180°$ (By (6) and (7))

$\Rightarrow \quad 145° + \angle FAC = 180°$

$\Rightarrow \angle FAC = 180° - 145°$

$\Rightarrow \quad \angle FAC = 35°$ .... (8)

Now, in ΔAFC,

$\angle FAC = 35°$ [From equation (8)]

$\angle ACF = 30°$ [From equation (7)]

$\therefore \angle FAC > \angle ACF$ $(35° > 30°)$

$\therefore CF > AF$

[Greater angle has greater side opposite to it]

Now, in ΔCDF,

$\angle DCF = 25°$ [From equation (4)]

$\angle CFD = 65°$ [From equation (5)]

$\therefore \angle CFD > DCF$ $(\because 65° > 25°)$

$\therefore DC > DF$.

[greater angle has greater side opposite to it]

(Q.E.D.)

(*b*) Given. In ΔABD, AC meets BD in C.

$\angle B = 70°$, $\angle D = 40°$ $AB = AC$.

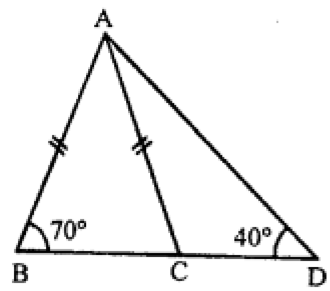
To prove. $AB > CD$.

Proof. In ΔABC,

$AB = AC$ (given)

$\therefore \angle ACB = \angle B$ ...... (1)

(In a triangle, equal sides have equal angles opposite to them)



(2)

Also, $\angle B = 70°$ [Given] .... (2)

From (i) and (ii), we get

$\angle ACB + \angle ACD = 180°$ [BCD is a st. line]

$\Rightarrow$ $70° + \angle ACD = 180°$ [From equation (3)]

$\Rightarrow$ $\angle ACD = 180° - 70°$

$\Rightarrow$ $\angle ACD = 110°$ ..... (4)

Now, in ΔACD,

$\angle CAD + \angle ACD + \angle D = 180°$

[sum of all angles in a triangle is 180°]

$\Rightarrow \quad \angle CAD + 110° + 40° = 180°$ [From (4)

$\angle ACD = 110°$ and $\angle D = 40°$ (given)]

$\Rightarrow \quad \angle CAD + 150° = 180°$

$\Rightarrow \quad \angle CAD = 180° - 150°$

$\Rightarrow \quad \angle CAD = 30°$ ..... (5)

Now, in ΔACD

$\angle ACD = 110°$ [From equation (4)]

$\angle CAD = 30°$ [From equation (5)]

$\angle D = 40°$ (given)

$\therefore \quad \angle D > \angle CAD$ ($40° > 30°$)

$\therefore \quad AC > CD$

[Greater angle has greater side opposite to it]

$\Rightarrow \quad AB > CD$ [$\because AB = AC$ given)]

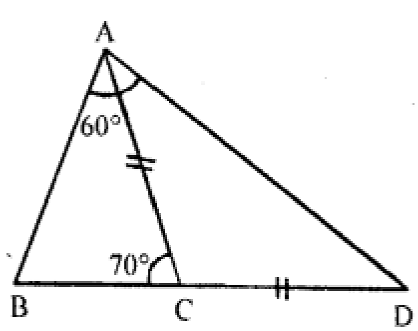
(Q.E.D.)

(*c*) **Given.** In ΔACD, $AC = CD$, $\angle BAD = 60°$, $\angle ACB = 70°$

To prove. $BC < CD$.

Proof. In ΔACD,

$\therefore \quad AC = CD,$ (Given)



(3)

$\therefore \angle CAD = \angle CDA$ ..... (1)

[In a triangle If two sides are equal, then angles opposite to them are also equal]

Also, $\angle ACB = 70°$ ..... (2)

Now, $\angle ACB = \angle CAD + \angle CDA$

[exterior angle is equal to sum of two interior opposite angles]

$\Rightarrow \quad 70° = \angle CAD + \angle CAD$

[From (1) and (2)]

$\Rightarrow \quad 70° = 2\angle CAD$

$\Rightarrow 2\angle CAD = 70°$

$$\Rightarrow \quad \angle CAD = \frac{70°}{2} = 35°$$

$\because \quad \angle BAD = 60°$ (given)

$\therefore \angle BAC = \angle BAD - \angle CAD$

$= 60° - 35° = 25°$

$\therefore \quad \angle BAC < \angle CAD$ $[\because 25° < 35°]$

$\therefore \quad BC < CD$

[Greater angles has greater side opposite to it].

(Q.E.D.)

Question 9.

(a) In the figure (i) given below, ∠B < ∠A and ∠C < ∠D. Show that AD < BC. (b) In the figure (ii) given below, D is any point on the side BC of ∆ABC. If AB > AC, show that AB > AD.

Solution:

(a) In the given figure,

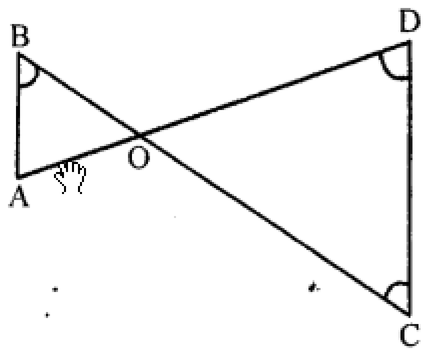
$\angle B < \angle A$ and $\angle C < \angle D$

To prove : $AD < BC$

Proof : In ΔABO

$\angle B < \angle A$ (Given)

$\therefore$ $AO < BO$ ...(*i*)



Similarly in ΔOCD

$\angle C < \angle D$ (Given)

$\therefore$ $OD < OC$

$\therefore$ Adding (*i*) and (*ii*)

$AO + OD < BO + OC$

$\Rightarrow$ $AD < BC$

Hence $AD < BC$

(b) In the given figure,

D is any point on BC of ΔABC

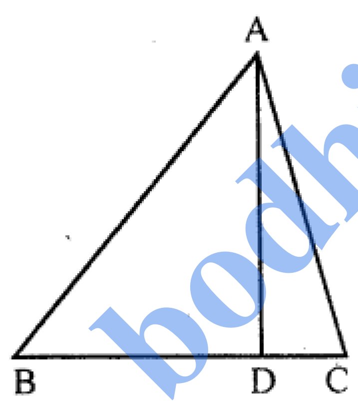
$AB > AC$

To prove : $AB > AD$

Proof : $\because$ In ΔABC

$AB > AC$

$\angle C > \angle B$



In ΔABD

Ext. $\angle ADC > \angle B$

$\therefore$ $\angle ADC > \angle C$

($\because \angle C > \angle B$)

$\therefore$ $AC > AD$ ...(*i*)

But $AB > AC$

(Given) ...(*ii*)

$\therefore$ From (*i*) and (*ii*),

$AB > AD$

Question 10.

(i) Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer,

(ii) Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

(iii) Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Solution:

(i) Length of sides of a triangle are 4 cm, 3 cm and 7 cm

We know that sum of any two sides of a triangle is greatar than its third side But 4 + 3 = 7 cm

Which is not possible

Hence to construction of a triangle with sides 4 cm, 3 cm and 7 cm is not possible.

(ii) Length of sides of a triangle are 9 cm, 7 cm and 17 cm

We know that sum of any two sides of a triangle is greater than its third side Now 9 + 7 = 16 < 17 ∴ It is not possible to construct a triangle with these sides.

(iii) Length of sides of a triangle are 8 cm, 7 cm and 4 cm We know that sum of any two sides of a triangle is greater than its third side Now 7 + 4 = 11 > 8

Yes, It is possible to construct a triangle with these sides.

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 18):

Question 1.

Which of the following is not a criterion for congruency of triangles?

(a) SAS

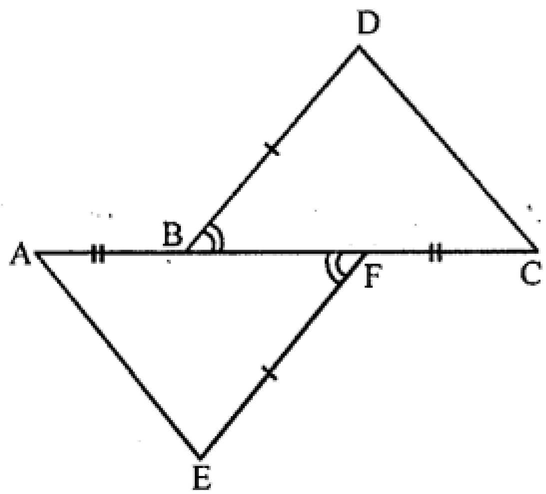
(b) ASA

(c) SSA

(d) SSS

Solution:
Criteria of congruency of two triangles 'SSA' is not the criterion. **(c)**

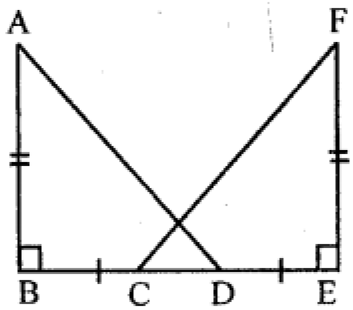
Question 2.
In the adjoining figure, AB = FC, EF=BD and ∠AFE = ∠CBD. Then the rule by which ΔAFE = ΔCBD is
(a) SAS
(b) ASA
(c) SSS
(d) AAS



Solution:
In the figure given,
ΔAFE ≅ ΔCBD by SAS axiom
AB + BF = BF + FC (∵ AB = FC)
⇒ AF = BC
EF = BD
∠AFE = ∠CBD (b)

Question 3.
In the adjoining figure, AB ⊥ BE and FE ⊥ BE. If AB = FE and BC = DE, then
(a) ΔABD ≅ ΔEFC
(b) ΔABD ≅ ΔFEC
(c) ΔABD ≅ ΔECF
(d) ΔABD ≅ ΔCEF
Solution:

In the figure given,



$AB \perp BE$ and $FE \perp BE$

$AB = FE$, $BC + CD = CD + DE$

$(\because BC = DE)$

$\Rightarrow AB = FE$ and $BD = CE$, $\angle B = \angle E$

(Each $90°$)

$\therefore \Delta ABD \cong \Delta FEC$ (b)

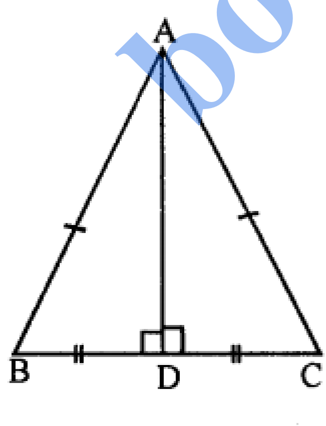
Question 4.

In the adjoining figure, AB=AC and AD is median of ΔABC, then AADC is equal to
(a) 60°
(b) 120°
(c) 90°
(d) 75°

Solution:

In the given figure, $AB = AC$

AD is median of ΔABC

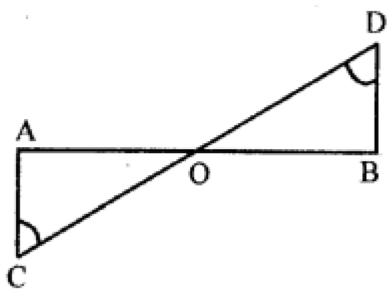


$\therefore$ D is mid-point $\Rightarrow BD = DC$

$\therefore AD \perp BC$

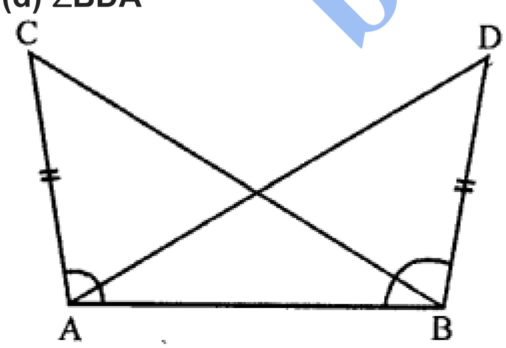
$\therefore \angle ADC = 90°$ (c)

Question 5.
In the adjoining figure, O is mid point of AB. If ∠ACO = ∠BDO, then ∠OAC is equal to
(a) ∠OCA
(b) ∠ODB
(c) ∠OBD
(d) ∠BOD
Solution:
In the given figure, O is mid-point of AB,
∠ACO = ∠BDO
∠AOC = ∠BOD
(Vertically opposite angles)



∴ ΔOAC ≅ ΔOBD (AAS)
∴ ∠OAC = ∠OBD (c)

Question 6.
In the adjoining figure, AC = BD. If ∠CAB = ∠DBA, then ∠ACB is equal to
(a) ∠BAD
(b) ∠ABC
(c) ∠ABD
(d) ∠BDA



Solution:

In the figure, $AC = BD$

$\angle CAB = \angle DBA$

$AB = AB$ (Common)

$\therefore$ $\Delta ABC \cong \Delta ABD$ (SAS axiom)

$\therefore$ $\angle ACB = \angle BDA$ (c.p.c.t.) (d)

Question 7.

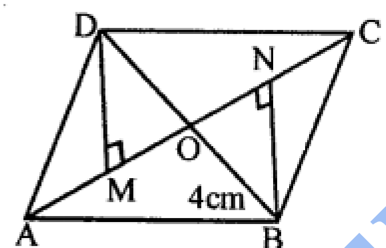
In the adjoining figure, ABCD is a quadrilateral in which BN and DM are drawn perpendiculars to AC such that BN = DM. If OB = 4 cm, then BD is

(a) 6 cm
(b) 8 cm
(c) 10 cm
(d) 12 cm

Solution:

In the given figure,

ABCD is a quadrilateral



$BN \perp AC$, $DM \perp AC$

$BN = DM$, $OB = 4$ cm

In ΔONB and ΔOMD

$BN = DM$

$\angle N = \angle M$ (Each 90°)

$\angle BON = DOM$ (Vertically opposite angles)

$\therefore$ $\Delta ONB \cong \Delta OMD$

$\therefore$ $OB = OD$

But $OB = 4$ cm

$\therefore$ $BD = BO + OD = 4 + 4 = 8$ cm (b)

Question 8.

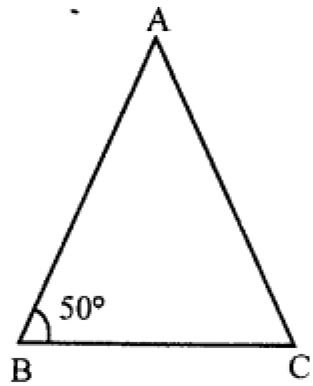
In $\triangle ABC$, AB = AC and $\angle B = 50°$. Then $\angle C$ is equal to

(a) 40°
(b) 50°

(c) 80°
(d) 130°

Solution:

In ΔABC, $AB = AC$



$\therefore \angle C = \angle B$ (Angles opposite to equal sides)

$\angle B = 50°$

$\therefore \angle C = 50°$ (b)

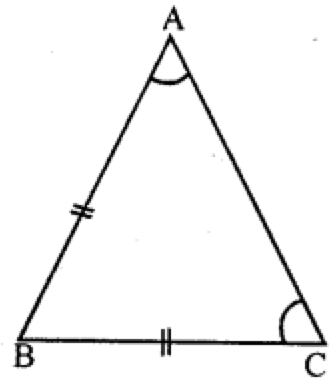
Question 9.

In ΔABC, $BC = AB$ and $\angle B = 80°$. Then $\angle A$ is equal to

(a) 80°
(b) 40°
(c) 50°
(d) 100°

Solution:

In $\Delta ABC = BC = AB$



$\therefore \angle A = \angle C$ (Angles opposite to equal sides)

$\angle B = 80°$

$\therefore \angle A + \angle C = 180° - 80° = 100°$

But $\angle A = \angle C = 100°$

and $2\angle A$

$\angle A = \frac{100°}{2} = 50°$ (c)

Question 10.

In $\triangle PQR$, $\angle R = \angle P$, QR = 4 cm and PR = 5 cm. Then the length of PQ is

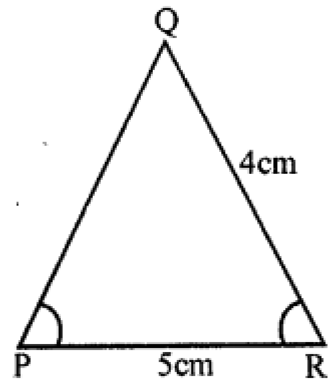
(a) 4 cm
(b) 5 cm
(c) 2 cm
(d) 2.5 cm

Solution:

In ΔPQR

$\angle R = \angle P$, $QR = 4$ cm

$PR = 5$ cm



$\because \angle P = \angle R$

$PQ = QR$

$\therefore$ (Sides opposite to equal angles)

$\therefore PQ = 4$ cm (a)

Question 11.

In $\triangle ABC$ and APQR, AB = AC, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are

(a) isosceles but not congruent

(b) isosceles and congruent

(c) congruent but isosceles

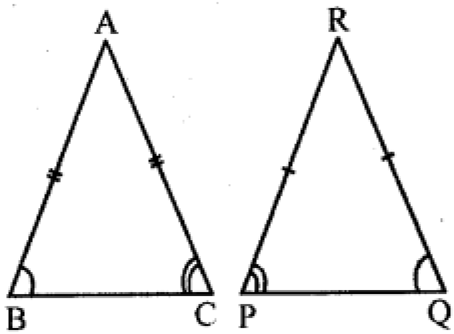
(d) neither congruent nor isosceles

Solution:

In ΔABC and ΔPQR

$AB = AC$, $\angle C = \angle P$

$\angle B = \angle Q$



$\because$ In ΔABC, $AB = AC$

$\angle C = \angle B$ (Opposite to equal sides)

But $\angle C = \angle P$ and $\angle B = \angle Q$

$\therefore$ $\angle P = \angle Q$

$\therefore$ $RQ = PR$

$\therefore$ ΔRPQ is an isosceles triangle but not congruent (a)

Question 12.

Two sides of a triangle are of lenghts 5 cm and 1.5 cm. The length of the third side of the triangle can not be

(a) 3.6 cm
(b) 4.1 cm
(c) 3.8 cm
(d) 3.4 cm

Solution:

In a triangle, two sides are 5 and 1.5 cm.

$\because$ Sum of any two sides of a triangle is greater than its third side

$\therefore$ Third side $< (5 + 1.5)$ cm

$\Rightarrow$ Third side < 6.5 cm

or third side $+ 1.5 > 5$ cm

or third side $> 5 - 1.5 = 3.5$ cm

$\therefore$ Third side cannot be equal to 3.4 cm (d)

Question 13.

If a, b, c are the lengths of the sides of a trianlge, then

(a) a – b > c
(b) c > a + b
(c) c = a + b
(d) c < A + B
Solution:
a, b, c are the lengths of the sides of a trianlge than a + b> c or c < a + b
(Sum of any two sides is greater than its third side) **(d)**

Question 14.
It is not possible to construct a triangle when the lengths of its sides are
(a) 6 cm, 7 cm, 8 cm
(b) 4 cm, 6 cm, 6 cm
(c) 5.3 cm, 2.2 cm, 3.1 cm
(d) 9.3 cm, 5.2 cm, 7.4 cm
Solution:
We know that sum of any two sides of a triangle is greater than its third side 2.2 + 3.1 = 5.3 ⇒ 5.3 = 5.3 is not possible (c)

Question 15.
In ∆PQR, if ∠R> ∠Q, then
(a) QR > PR
(b) PQ > PR
(c) PQ < PR
(d) QR < PR
Solution:
In ∆PQR, ∠R> ∠Q
∴ PQ > PR **(b)**

Question 16.
If triangle PQR is right angled at Q, then
(a) PR = PQ
(b) PR < PQ
(c) PR < QR
(d) PR > PQ

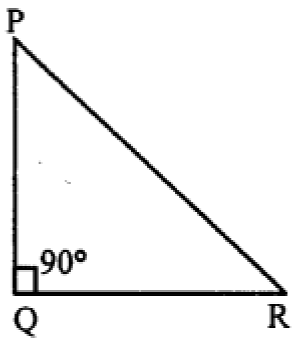
Solution:

In right angled ΔPQR,

$\angle Q = 90°$

Side opposite to greater angle is greater

$\therefore$ PR > PQ (d)



Question 17.

If triangle ABC is obtuse angled and $\angle C$ is obtuse, then

(a) AB > BC

(b) AB = BC

(c) AB < BC

(d) AC > AB

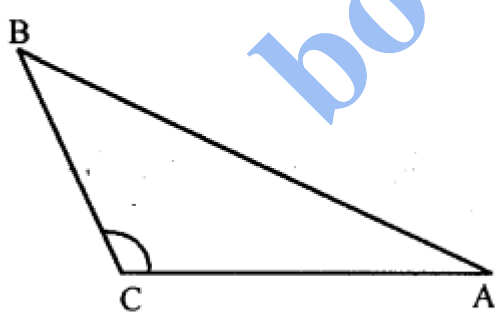
Solution:

In ΔABC, $\angle C$ is obtuse angle

AB > BC

(Side opposite to greater angle is greater)

(a)



Question P.Q.

A triangle can be constructed when the lengths of its three sides are

(a) 7 cm, 3 cm, 4 cm

(b) 3.6 cm, 11.5 cm, 6.9 cm

(c) 5.2 cm, 7.6 cm, 4.7 cm

(d) 33 mm, 8.5 cm, 49 mm

Solution:
We know that in a triangle, if sum of any two sides is greater than its third side, it is possible to construct it 5.2 cm, 7.6 cm, 4.7 cm is only possible. **(c)**

Question P.Q.
A unique triangle cannot be constructed if its
(a) three angles are given
(b) two angles and one side is given
(c) three sides are given
(d) two sides and the included angle is given
Solution:
A unique triangle cannot be constructed if its three angle are given, **(a)**

Question 18.
If the lengths of two sides of an isosceles are 4 cm and 10 cm, then the length of the third side is
(a) 4 cm
(b) 10 cm
(c) 7 cm
(d) 14 cm
Solution:
Lengths of two sides of an isosceles triangle are 4 cm and 10 cm, then length of the third side is 10 cm
(Sum of any two sides of a triangle is greater than its third side and 4 cm is not possible as 4 + 4 > 10 cm.

Chapter Test

Question 1.
In triangles ABC and DEF, ∠A = ∠D, ∠B = ∠E and AB = EF. Will the two triangles be congruent? Give reasons for your answer.
Solution:
In ΔABC and ΔDEF
$\angle A = \angle D$
$\angle B = \angle E$
$AB = EF$
In ΔABC, two angles and included side is given but in ΔDEF, corresponding angles are equal but side is not included of there angle.
$\therefore$ Triangles cannot be congruent.

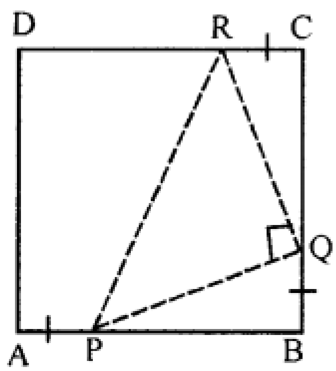
Question 2.

In the given figure, ABCD is a square. P, Q and R are points on the sides AB, BC and CD respectively such that AP= BQ = CR and ∠PQR = 90°. Prove that

(a) ΔPBQ ≅ ΔQCR

(b) PQ = QR

(c) ∠PRQ = 45°



Solution:

Given : In the given figure, ABCD is a square P, Q and R are the points on the sides AB, BC and CD respectively such that

$AP = BQ = CR$, $\angle PQR = 90°$

To prove : (a) $\Delta PBQ \cong \Delta QCR$

(b) $PQ = QR$

(c) $\angle PRQ = 45°$

Proof : $\because AB = BC = CD$ (Sides of square)

and $AP = BQ = CR$ (Given)

Subtracting, we get

$AB - AP = BC - BQ = CD - CR$

$\Rightarrow PB = QC = RD$

Now in ΔPBQ and $\angle QCR$

$PB = QC$ (Proved)

$BQ = CR$ (Given)

$\angle B = \angle C$ (Each 90°)

$\therefore \Delta PBQ \cong \Delta QCR$ (SAS axiom)

$\therefore PQ = QR$ (c.p.c.t.)

But $\angle PQR = 90°$ (Given)

$\angle RPQ = \angle PRQ$

(Angles opposite to equal angles)

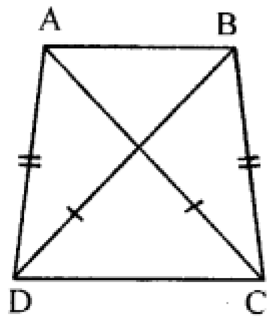
But $\angle RPQ + \angle PRQ = 90°$

$$\angle RPQ = \angle PRQ = \frac{90°}{2} = 45°$$

Question 3.

In the given figure, AD = BC and BD = AC. Prove that ∠ADB = ∠BCA.

Solution:



Given : In the figure,

AD = BC, BD = AC

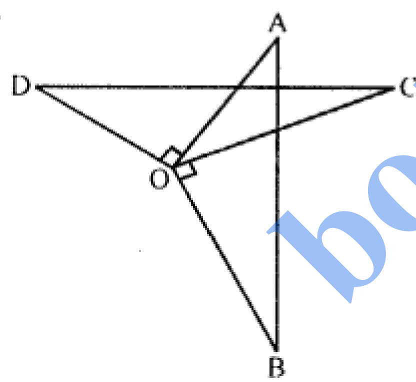
To prove : ∠ADB = ∠BCA

Proof : In ΔADB and ΔACB

AB = AB	(Common)
AD = BC	(Given)
BD = AC	(Given)
∴ ΔADB ≅ ΔACB	(SSS axiom)
∴ ∠ADB = ∠BCA	(c.p.c.t.)

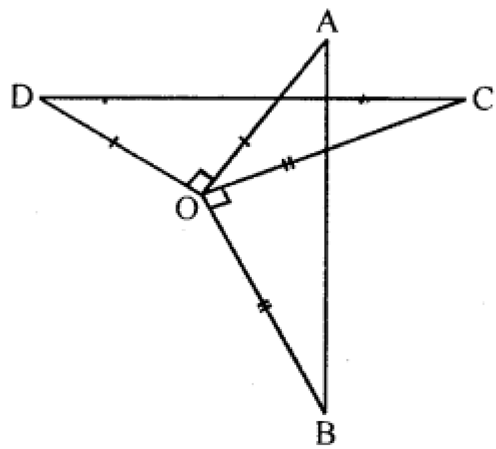
Question 4.

In the given figure, OA ⊥ OD, OC X OB, OD = OA and OB = OC. Prove that AB = CD.



Solution:

Given : In the figure, OA ⊥ OD, OC ⊥ OB.
OD = OA, OB = OC



To prove : AB = CD

Proof : $\angle AOD = \angle COB$ (each 90°)

Adding $\angle AOC$ (both sides)

$\angle AOD + \angle AOC = \angle AOC + \angle COB$

$\Rightarrow \angle COD = \angle AOB$

Now, in ΔAOB and ΔDOC

OA = OD (given)

OB = OC (given)

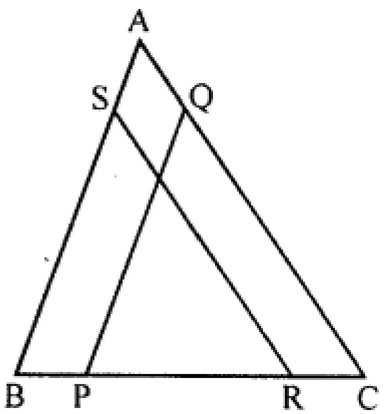
$\angle AOB = \angle COD$ (proved)

$\therefore \Delta AOB \cong \Delta DOC$ (SAS axiom)

$\therefore$ AB = CD (c.p.c.t.)

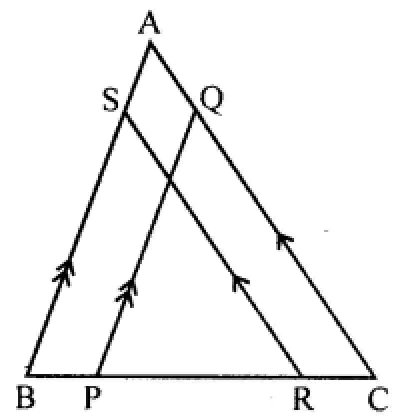
Question 5.

In the given figure, PQ || BA and RS CA. If BP = RC, prove that:
(i) $\Delta BSR \cong \Delta PQC$
(ii) BS = PQ
(iii) RS = CQ.



Solution:

Given : In the given figure,
PQ || BA, RS || CA
BP = RC

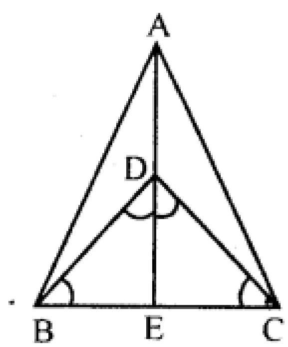


To prove :
(*i*) $\Delta BSR \cong \Delta PQC$ (*ii*) BS = PQ
(*iii*) RS = CQ

Proof : BP = RC
$\because$ BC – RC = BC – BP
$\therefore$ BR = PC
Now, in ΔBSR and ΔPQC
$\angle B = \angle P$ (corresponding angles)
$\angle R = \angle C$ (corresponding angles)
BR = PC (proved)
$\therefore \Delta BSR \cong \Delta PQC$ (ASA axiom)
$\therefore$ BS = PQ (c.p.c.t.)
RS = CQ (c.p.c.t.)

Question 6.
In the given figure, AB = AC, D is a point in the interior of $\triangle ABC$ such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of $\triangle ABC$.
Solution:

Given : In the figure given, AB = AC
D is a point in the interior of ΔABC
Such that $\angle DBC = \angle DCB$
To prove : AD bisects $\angle BAC$
Construction : Join AD and produced it to BC in E
Proof : In ΔABC,
AB = AC
$\therefore$ $\angle B = \angle C$ (Angles opposite to equal sides)
and $\angle DBC = \angle DCB$ (Given)
Subtracting, we get
$\angle B - \angle DBC = \angle C - \angle DCB$



$\Rightarrow$ $\angle ABD = \angle ACD$
Now in ΔABD and ΔACD
AD = AD (Common)
$\angle ABD = \angle ACD$ (Proved)
AB = AC (Given)
$\therefore$ $\Delta ABD \cong \Delta ACD$ (SAS axiom)

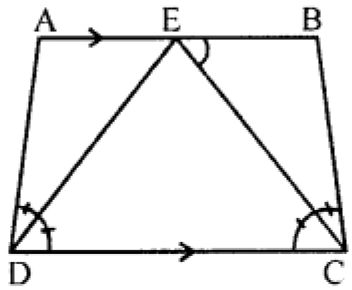
$\therefore$ $\angle BAD = \angle CAD$ (c.p.c.t.)
$\therefore$ AD is bisector of $\angle BAC$

Question 7.
In the adjoining figure, AB || DC. CE and DE bisects ∠BCD and ∠ADC respectively. Prove that AB = AD + BC.

Solution:

Given : In the given figure, AB || DC

CE and DE bisects ∠BCD and ∠ADC respectively



To prove : AB = AD + BC

Proof : ∵ AD || DC and ED is the transversal

∴ ∠AED = ∠EDC (Alternate angles)

= ∠ADC (∵ ED is bisector of ∠ADC)

∴ AD = AE ...(*i*)

(Sides opposite to equal angles)

Similarly,

∠BEC = ∠ECD = ∠ECB

∴ BC = EB ...(*ii*)

Adding (*i*) and (*ii*),

AD + BC = AE + EB = AB

∴ AB = AD + BC

Question 8.

In △ABC, D is a point on BC such that AD is the bisector of ∠BAC. CE is drawn parallel to DA to meet BD produced at E. Prove that △CAE is isosceles

Solution:

Given : In ΔABC,

D is a point on BC such that AD is the bisector of $\angle BAC$

$CE \parallel DA$ to meet BD produced at E

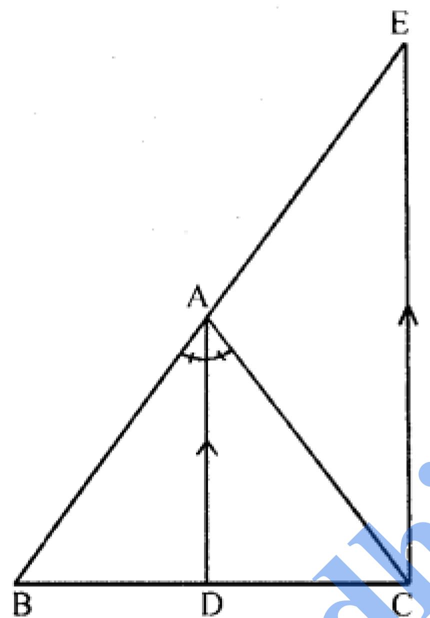
To prove : ΔCAE is an isosceles

Proof : $\because AD \parallel EC$ and AC is its transversal

$\therefore \angle DAC = \angle ACE$ (Alternate angles)

and $\angle BAD = \angle CEA$

(Corresponding angles)



But $\angle BAD = \angle DAC$

($\because$ AD is bisector of $\angle BAC$)

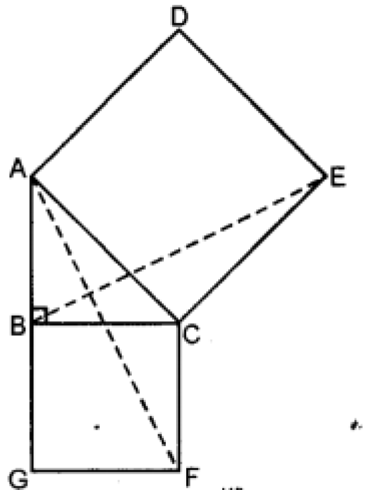
$\therefore \angle ACE = \angle CAE$

$AE = AC$ (Sides opposite to equal angles)

$\therefore \Delta ACE$ is an isosceles triangle.

Question 9.

In the figure (ii) given below, ABC is a right angled triangle at B, ADEC and BCFG are squares. Prove that AF = BE.



(ii)

Solution:

Given. In right ΔABC, $\angle B = 90°$

ADEC and BCFG are squares on the sides AC and BC of ΔABC respectively AF and BE are joined.

To prove. AE = BE

Proof. $\angle ACE = \angle BCF$ (each $90°$)

Adding $\angle ACB$ both sides

$\angle ACB + \angle ACE = \angle ACB + \angle BCF$

$\Rightarrow \quad \angle BCE = \angle ACF$

Now in ΔBCE and ΔACF,

$CF = AC$ (sides of a square)

$BC = CF$ (sides of a square)

$\angle BCE = \angle ACF$ (proved)

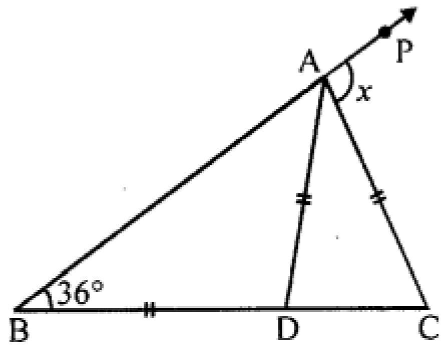
$\therefore \Delta BCE \cong \Delta ACF$ (SAS postulate)

$\therefore BE = AF$ (c.p.c.t.)

Hence proved.

Question 10.

In the given figure, BD = AD = AC. If ∠ABD = 36°, find the value of x .



Solution:

Given : In the figure, BD = AD = AC

$\angle ABD = 36°$

To find : Measure of x.

Proof : In ΔABD,

$AD = BD$ (given)

$\therefore \angle ABD = \angle BAD = 36°$ $(\because \angle ABD = 36°)$

$\therefore$ Ext. $\angle ADC = \angle ABD + \angle BAD$ (sum of interior opposite angles)

$= 36° + 36° = 72°$

But in ΔADC

$AD = AC$

$\therefore \angle ADC = \angle ACD = 72°$

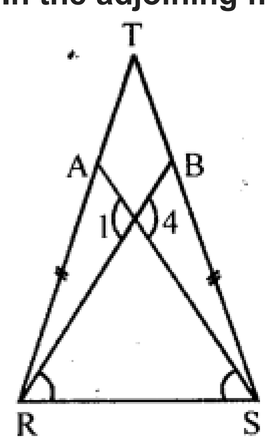
and Ext. $\angle PBC = \angle ABC + \angle ACD$

$= 36° + 72° = 108°$

$\therefore x = 108°$

Question 11.

In the adjoining figure, TR = TS, ∠1 = 2∠2 and ∠4 = 2∠3. Prove that RB = SA.



Solution:

Given: In the figure , RST is a triangle

TR = TS,

$\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$

To prove : RB = SA

Proof : $\angle 1 = \angle 4$ (Vertically opposite angles)

But $2\angle 2 = \angle 1$ and $2\angle 3 = 4$

$\therefore$ $2\angle 2 = 2\angle 3$

$\therefore$ $\angle 2 = \angle 3$

$\therefore$ But $\angle TRS = \angle TSR$ ($\because$ TR = TS given)

$\therefore$ $\angle TRS - \angle BRS = \angle TSR - \angle ASR$

$\Rightarrow$ $\angle ARB = \angle BSA$

Now in ΔRBT and ΔSAT

$\angle T = \angle T$ (Common)

TR = TS (Given)

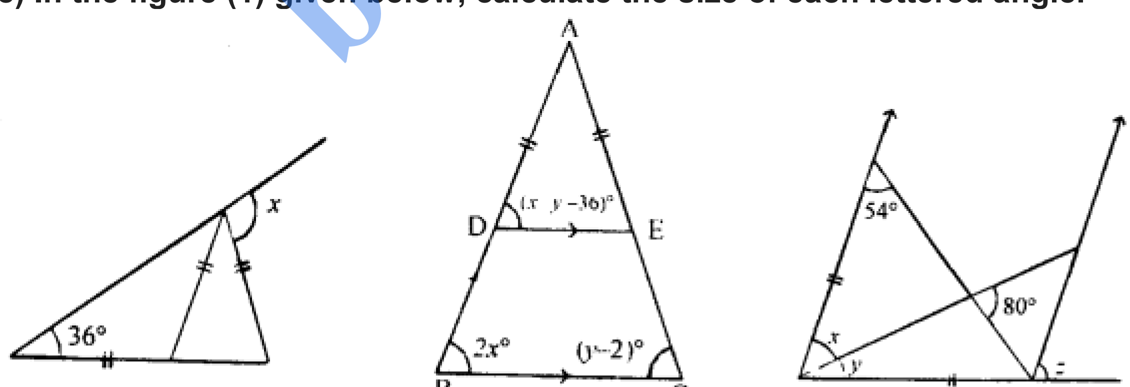
and $\angle TRB = \angle TSA$ (Proved)

$\therefore$ $\Delta RBT \cong \Delta SAT$ (SAS axiom)

$\therefore$ RB = SA (c.p.c.t.)

Question 12.

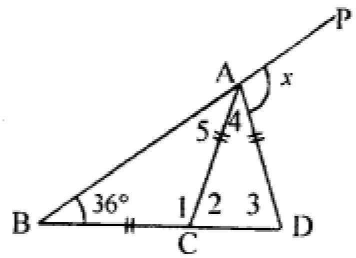
(a) In the figure (1) given below, find the value of x.
(b) In the figure (2) given below, AB = AC and DE || BC. Calculate
(i)x
(ii) y
(iii) ∠BAC
(c) In the figure (1) given below, calculate the size of each lettered angle.



(1) (2) (3)

Solution:

(*a*) We have to calculate the value of x.



Now, in ΔABC

$\angle 5 = 36°$ ..... (1)

Also, $36° + \angle 1 + \angle 5 = 180°$ [$\because AC = BC$]

[sum of all angles in a triangle is 180°]

$\Rightarrow$ $36° + \angle 1 + 36° = 180°$ [from (1)]

$\Rightarrow$ $72° + \angle 1 = 180°$ $\Rightarrow$ $\angle 1 = 180° - 72°$

$\Rightarrow$ $\angle 1 = 108°$ ..... (2)

Also, $\angle 1 + \angle 2 = 180°$ (Linear pair)

$\Rightarrow$ $108° + \angle 2 = 180°$ [From (2)]

$\Rightarrow$ $\angle 2 = 180° - 108°$ $\Rightarrow$ $\angle 2 = 72°$ ..... (3)

Also, $\angle 2 = \angle 3$ (AC = AD)

$\therefore$ $\angle 3 = 72°$ [From (3)]..... (4)

Now, in ΔACD

$\angle 2 + \angle 3 + \angle 4 = 180°$

[sum of all angles in a triangle is 180°]

$\Rightarrow$ $72° + 72° + \angle 4 = 180°$ [From (3) and (4)]

$\Rightarrow$ $144° + \angle 4 = 180°$ $\Rightarrow$ $\angle 4 = 180° - 144°$

$\Rightarrow$ $\angle 4 = 36°$ ..... (5)

$\therefore$ ABP is a St. line

$\therefore$ $\angle 5 + \angle 4 + x = 180°$

$36° + 36° + x = 180°$ [From (1) and (5)]

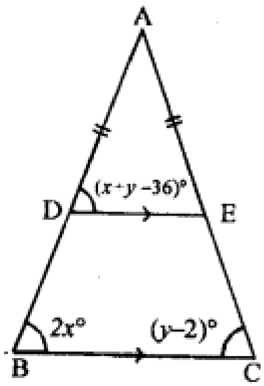
$72° + x = 180$ $\Rightarrow$ $x = 108°$

Hence, value of $x = 108°$ **Ans.**

(*b*) **Given.** AB = AC, and DE $\parallel$ BC

$\angle ADE = (x + y - 36)°$

$\angle ABC = 2x°$ and $\angle ACB = (y - 2)°$



To Calculate. (*i*) x (*ii*) y (*iii*) $\angle BAC$

Now, in ΔABC

$\therefore$ AB = AC

$2x = y - 2$

[In a triangle equal sides here equal angle opposite to them]

$2x - y = -2$ ..... (1)

$\therefore$ DE || BC,

$x + y - 36 = 2x$ [corresponding angles]

$\Rightarrow x + y - 2x = 36 \Rightarrow -x + y = 36$ ..... (2)

From equation (1) and (2),

$$2x - y = -2$$
$$-x + y = 36$$

Adding, $x = 34$

Substituting the value of x in equation (1), we get

$2 \times 34 - y = -2 \Rightarrow 68 - y = -2$

$\Rightarrow 68 + 2 = y \Rightarrow 70 = y \Rightarrow y = 70$

Hence, value of $x = 34°$

and value of $y = 70°$

(*iii*) In ΔABC

$\angle BAC + 2x° + (y-2)° = 180°$

[sum of all angles in a triangle is 180°]

$\Rightarrow \angle BAC + 2 \times 34° + (70 - 2)° = 180°$

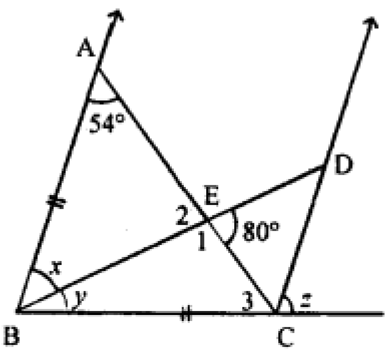
(Substituting the value of x and y)

$\Rightarrow \angle BAC + 68° + 68° = 180°$

$\Rightarrow \angle BAC = 180° - 136° \Rightarrow \angle BAC = 44°$

Hence, value of $\angle BAC = 44°$ **Ans.**

(*c*) Given. $\angle BAE = 54°$, $\angle DEC = 80°$ and AB = BC.



To calculate. The value of x, y and z.

Now $\angle 2 = 80°$ ..... (1)

(vertically opposite angles $\therefore$ AC and BD cut at point E)

In ΔABE,

$54° + x + \angle 2 = 180°$

(sum of all angles in triangle is 180°)

$\Rightarrow \quad 54° + x + 80° = 180° \qquad (\because \angle 2 = 80°)$

$\Rightarrow \quad 134° + x = 180° \quad \Rightarrow \quad x = 180° - 134°$

$\Rightarrow \quad x = 46°$

Now, $\angle 1 + 80° = 180°$ (Linear pair)

$\angle 1 = 180° - 80° \quad \Rightarrow \quad \angle 1 = 100°$ ..... (2)

Also, AB = BC (given)

$\angle 3 = 54°$

(In a triangle equal sides have equal angles)

Now, in ΔABC

$54° + (x + y) + \angle 3 = 180°$

(substituting the value of x and $\angle 3$)

$\Rightarrow \quad 154° + y = 180° \quad \Rightarrow \quad y = 180° - 154°$

$\Rightarrow \quad y = 26°$ ..... (3)

$\therefore$ AB || CD, $\quad \therefore x + y = z$

[corresponding angles]

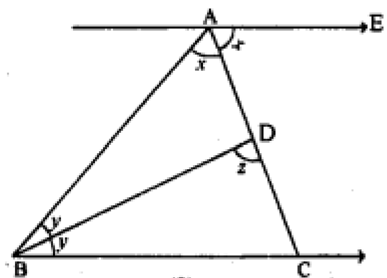
$\Rightarrow \quad 46° + 26° = z$ [From (2) and (3)]

$\Rightarrow \quad z = 46° + 26° \quad \Rightarrow \quad z = 72°$

Hence, value of $x = 46°$, $y = 26°$ and $z = 72°$

Question 13.

(a) In the figure (1) given below, AD = BD = DC and ∠ACD = 35°. Show that
(i) AC > DC (ii) AB > AD.
(b) In the figure (2) given below, prove that
(i) x + y = 90° (ii) z = 90° (iii) AB = BC



(2)

Solution:

(a) Given : In the figure given,

AD = BD = DC

∠ACD = 35°

To prove : (*i*) AC > DC, (*ii*) AB > AD

Proof : In Δ ADC, AD = DC

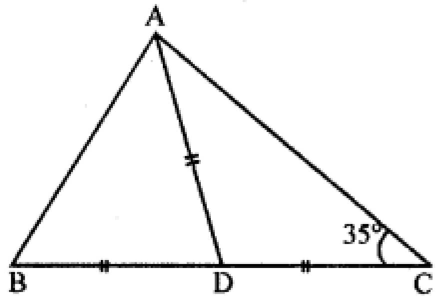
$\therefore$ ∠DAC = ∠DCA = 35°

$\Rightarrow$ ∠ADC = 180° – (∠DAC + ∠DCA)

$\therefore$ ∠ADC = 180° – (35° + 35°)

= 180° – 70° = 110°

and Ext. ∠ADB = ∠DAC + ∠DCA = 35° + 35° = 70°



$\because$ AD = BD

$\angle BAD = \angle ABD$

But $\angle BAD + \angle ABD = 180° - \angle ADB$

$\Rightarrow \angle ABD + \angle ABD = 180° - 70° = 110°$

$$\Rightarrow 2\angle ABD = 110° \Rightarrow \angle ABD = \frac{110°}{2} = 55°$$

(*i*) Now $\because \angle ADC > \angle DAC$

$\therefore$ AC > DC

and $\angle ADB > \angle ABD$

$\therefore$ AB > AD

(*b*) Given. $\angle EAC = \angle BAC = x$

$\angle ABD = \angle DBC = y$

$\angle BDC = z$

To prove. (*i*) $x + y = 90°$ (*ii*) $z = 90°$

(*iii*) AB = BC

Proof. (*i*) $\therefore$ AE || BC

$\therefore \angle ACB = x$ [Alternate angles] ..... (1)

In ΔABC

$x + (y + y) + \angle ACB = 180°$

[sum of all angles in a triangle is 180°]

$\Rightarrow x + 2y + x = 180°$ [From (1)]

$\Rightarrow 2x + 2y = 180°$

$\Rightarrow 2(x + y) = 180°$ **(proved)** ..... (2)

$\Rightarrow x + y = 90°$

(*ii*) Now, in ΔBCD,

$y + z + \angle BCD = 180°$

[sum of all angles in a triangle is 180°]

$\Rightarrow \quad y + z + x = 180°$

$\Rightarrow \quad 90° + z = 180°$ [From (2), $x + y = 90°$]

$\Rightarrow \quad z = 90°$ **(proved)** ..... (3)

(*iii*) In ΔABC

$\angle BAC = \angle BAC = x$ (each same value)

$\therefore$ AB = CB

(In a triangle equal angles has equal sides)

(proved)

Question 14.

In the given figure, ABC and DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

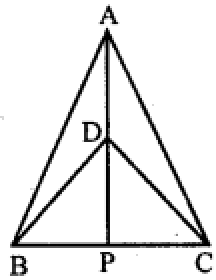
(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.

Solution:

Given : In the figure, two isosceles triangles ABC and DBC are on the same base BC. With vertices A and D on the same side of BC.

AD is joined and produced to meet BC at P.



To prove :

(*i*) $\Delta ABD \cong \Delta ACD$

(*ii*) $\Delta ABP \cong \Delta ACP$

(*iii*) AP bisects $\angle A$ as well as $\angle D$

(*iv*) AP is the perpendicular bisector of BC

Proof : $\because$ ΔABC and ΔDBC are isosceles

AB = AC and DB = DC

(*i*) Now in ΔABD and ΔACP

AB = AC (Proved)

DB = DC (Proved)

AD = AD (Common)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (SSS axiom)

$\therefore$ $\angle BAD = \angle CAD$ (c.p.c.t.)

$\therefore$ ADP bisects $\angle A$

and $\angle ADB = \angle ADC$ (c.p.c.t.)

But $\angle ADB + \angle BDP = \angle CAD + \angle CDP = 180°$

$\therefore$ $\angle BDP = \angle CDP$

$\therefore$ ADP bisects $\angle D$ also

Now in ΔAPB and ΔACD

AB = AC (Given)

AP = AP (Common)

and $\angle BAD = \angle CAD$ (Proved)

$\therefore$ $\angle APB \cong \Delta ACP$ (SAS axiom)

$\therefore$ BP = CP (c.p.c.t.)

and $\angle APB = \angle APC$

But $\angle APB + \angle APC = 180°$ (Linear pair)

$\therefore$ $\angle APB = \angle APC = 90°$

and BP = PC

$\therefore$ AP is perpendicular bisector of BC

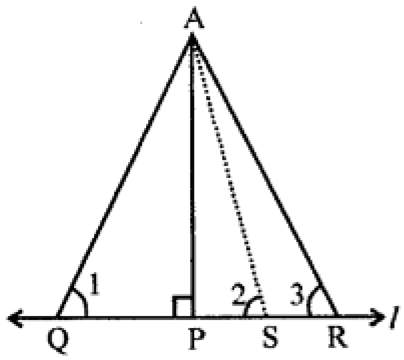
Question 15.

In the given figure, AP ⊥ l and PR > PQ. Show that AR > AQ.

Solution:

Given : In the given figure,

$AP \perp l$ and $PR > PQ$



To prove : $AR > AQ$

Construction : Take a point S on l,

Such that $PS = PQ$

Join A and S

Proof : In ΔAQP and ΔASP

$AP = AP$ (Common)

$QP = SP$ (Given)

$\angle APQ = \angle APS$ (Each 90°)

$\therefore \Delta APQ \cong \Delta APS$ (SAS axiom)

$\therefore \angle 1 = \angle 2$

$AQ = AS$ (Sides opposite to equal angles)

In ΔASR

Ext. $ASP > \angle ARS$

$\Rightarrow \angle 2 > \angle 3$

$\Rightarrow \angle 1 > \angle 3$ $(\because \angle 1 = \angle 2)$

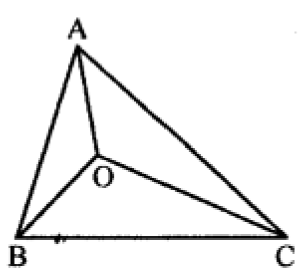
$\therefore AR > AQ$

Question 16.

If O is any point in the interior of a triangle ABC, show that

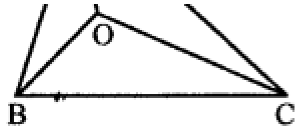
$OA + OB + OC > \frac{1}{2}$

(AB + BC + CA).



Solution:

Given : In the figure, O is any point in the interior of ΔABC.



To prove : $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

Construct : Join B and C.

Proof : In ΔOBC

$OB + OC > BC$ ...(*i*)

(Sum of two sides of a triangle is greater than its third side)

Similarly $OC + OA > CA$

and $OA + OB > AB$

Adding are get,

$(OB + OC + OC + OA + OA + OB) > BC + CA + AB$

$\Rightarrow 2(OA + OB + OC) > AB + BC + CA$

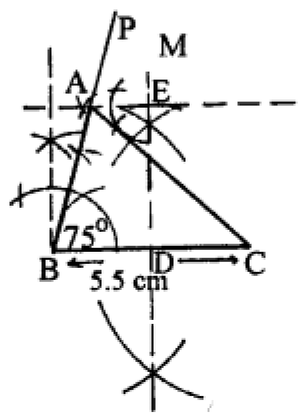
$\Rightarrow OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

Question P.Q.

Construct a triangle ABC given that base BC = 5.5 cm, ∠ B = 75° and height = 4.2 cm.

Solution:

Given. In a triangle ABC, Base BC = 5.5. cm, $\angle B = 750°$ and height = 4.2 cm.



Required. To construct a triangle ABC.

Steps of Construction :

(1) Draw a line BC = 5.5 cm.

(2) Draw $\angle PBC = 75°$.

(3) Draw the perpendicular bisector of BC and cut the BC at point D.

(4) Cut the DM at point E such that DE = 4.2 cm.

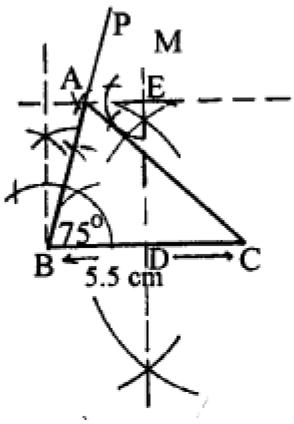
(5) Draw the line at point which is parallel to line BC.

(6) This parallel line cut the BP at point A.

(7) Join AC.

(8) ABC is the required triangle.

Given. In a triangle ABC, Base BC = 5.5. cm, $\angle B = 750°$ and height = 4.2 cm.



Required. To construct a triangle ABC.

Steps of Construction :

(1) Draw a line BC = 5.5 cm.

(2) Draw $\angle PBC = 75°$.

(3) Draw the perpendicular bisector of BC and cut the BC at point D.

(4) Cut the DM at point E such that DE = 4.2 cm.

(5) Draw the line at point which is parallel to line BC.

(6) This parallel line cut the BP at point A.

(7) Join AC.

(8) ABC is the required triangle.

Question P.Q.

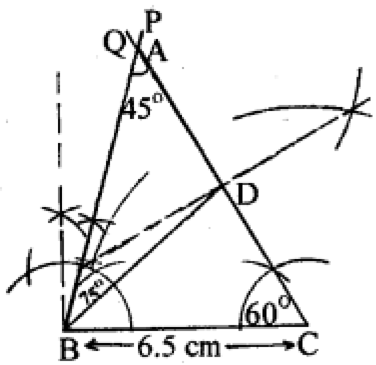
Construct a triangle ABC in which BC = 6.5 cm, ∠ B = 75° and ∠ A = 45°. Also construct median of A ABC passing through B.

Solution:

Given. In ΔABC, $BC = 6.5$ cm, $\angle B = 75°$ and $\angle A = 45°$.

Required. (*i*) To construct a triangle ABC.

(*ii*) Construct median of ΔABC passing through B.



Step of Construction.

(1) Draw a line BC = 6.5 cm.

(2) Make $\angle PBC = 75°$.

(3) Make $\angle BCQ = 60°$.

(4) BP and CQ cut at point A.

(5) ABC is the required triangle.

(6) Draw the bisector of AC.

(7) The bisector line cut the line AC at point D.

(8) Join BD.

(9) BD is the required median of ΔABC passing through B.

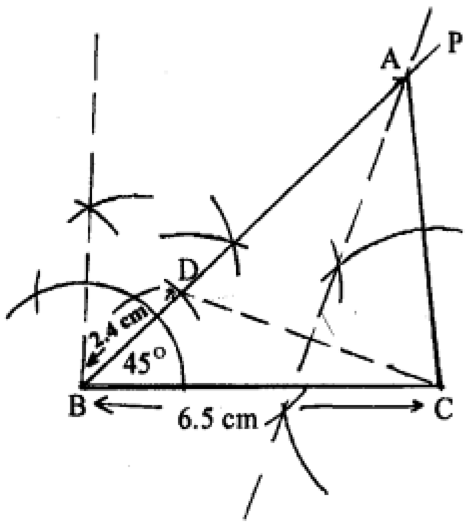
Question P.Q.

Construct triangle ABC given that AB – AC = 2.4 cm, BC = 6.5 cm. and $\angle$ B = 45°.

Solution:

Given. A triangle ABC in which AB − AC = 2.4 cm, BC = 6.5 cm, ∠B = 4.5°.

Required. To construct a triangle ABC.



Steps of Construction :

(1) Draw BC = 6.5 cm.

(2) Draw BP making angle 65° with BC.

(3) From BP, cut BD = 2.4 cm.

(4) Join D and C.

(5) Draw perpendicular bisector of DC which cuts BP at A.

(6) Join A and C.

(7) ABC is the required triangle.