Direct and Inverse Variation

EXERCISE : 9.1

1.

i) $\frac{5}{15} = \frac{1}{3}, \frac{8}{24} = \frac{1}{3}, \frac{12}{36} = \frac{1}{3}, \frac{15}{60} = \frac{1}{4}, \frac{18}{72} = \frac{1}{4}, \frac{20}{100} = \frac{1}{5}$

Here $\frac{x}{y} = \frac{5}{15} = \frac{8}{24} = \frac{12}{36} = \frac{1}{3} \neq \frac{15}{60} = \frac{18}{72} = \frac{1}{4} \neq \frac{20}{100} = \frac{1}{5}$

$\therefore$ x and y are not proportional.

ii) $\frac{3}{9} = \frac{1}{3}, \frac{5}{15} = \frac{1}{3}, \frac{7}{21} = \frac{1}{3}, \frac{9}{27} = \frac{1}{3}, \frac{10}{30} = \frac{1}{3}$

Here $\frac{x}{y} = \frac{3}{9} = \frac{5}{15} = \frac{7}{21} = \frac{9}{27} = \frac{10}{30} = \frac{1}{3}$ = Constant

$\therefore$ x and y are directly proportional.

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i) $\frac{x}{y} = \frac{3}{45} = \frac{5}{y_2} = \frac{x_3}{90} = \frac{x_4}{120} = \frac{10}{y_5} = \frac{1}{15}$ = Constant

$\frac{5}{y_2} = \frac{1}{15}$

$y_2 = 15 \times 5$

$y_2 = 75$

$\frac{x_3}{90} = \frac{1}{15}$

$x_3 = 90/15$

$x_3 = 6$

$\frac{x_4}{120} = \frac{1}{15}$

$x_4 = 120/15$

$x_4 = 8$

$\frac{10}{y_5} = \frac{1}{15}$

$y_5 = 15 \times 10 = 150$

$\therefore x_3 = 6, x_4 = 8, y_2 = 75, y_5 = 150$

ii) $\frac{x}{4} = \frac{4}{7} = \frac{8}{y_2} = \frac{x_3}{21} = \frac{20}{y_4} = \frac{28}{y_5}$ = Constant

$\frac{4}{7} = \frac{8}{y_2}$

$4y_2 = 7 \times 8$

$y_2 = \frac{7 \times 8^2}{4}$

$y_2 = 14$

$\frac{4}{7} = \frac{x_3}{21}$

$x_3 = \frac{4 \times 21^3}{7}$

$x_3 = 4 \times 3$

$x_3 = 12$

$\frac{4}{7} = \frac{20}{y_4}$

$y_4 = \frac{20^5 \times 7}{4}$

$y_4 = 5 \times 7$

$y_4 = 35$

$\frac{4}{7} = \frac{28}{y_5} \Rightarrow y_5 = \frac{7 \times 28^7}{4} \Rightarrow y_5 = 49$

$\therefore y_2 = 14, \; x_3 = 12, \; y_4 = 35, \; y_5 = 49$

3. Let the cost of 5.8 mts of cloth be ₹ x.

Cost of 8 mts of cloth cost ₹ 250

So it is a case of direct variation

i.e $\frac{8}{250} = \frac{5.8}{x}$

$x = \frac{250 \times 5.8}{8} = \frac{25 \times 58}{8}$

$x = ₹ 181.25$

∴ Cost of 5.8 mts of cloth ₹ 181.25

4. Labourer earns ₹ 672 per week ie 7 days

Labourer in 7 days he earns ₹ 672

in 1 day he earns ₹ $\frac{672}{7}$

$= ₹ 96$

Now in 18 days he earns ₹ 96×18

$= ₹$

5. Given 175 dollars Cost ₹ 7,350

how many dollars Cost ₹ 24,024

Let the no of dollars be "x"

$$\frac{x}{175} = \frac{24024}{7350}$$

$$x = \frac{24024 \times 175}{7350}$$

$$x = 572$$

∴ 572 dollars Cost ₹ 24,024

6. Let the number of kilometers travelled be "x".

Car travels 67.5 km in 4.5 litres of petrol.

x km in 26.4 litres of petrol

i.e $$\frac{x}{67.5} = \frac{26.4}{4.5}$$

$$x = \frac{26.4 \times 67.5}{4.5}$$

$$x = 396 \text{ km}$$

∴ Number of kilometers travelled is 396 km

7 Let the number of sheets of cardboard be "x"

Thickness of 12 cardboard sheets is 45 mm

Thickness of x cardboard sheets is 90 cm i.e 900 mm

i.e $\frac{x}{12} = \frac{900}{45}$

$$x = \frac{^{20}\cancel{900} \times 12}{\cancel{45}_{1}}$$

$$x = 240$$

∴ Number of sheets of cardboard are 240.

8 Mast of ship model is 6cm

Mast of actual ship is 9 m = 900cm

Length of actual ship is 33 m = 3300 cm

Let Length of model ship is x

i.e $\frac{l}{900} = \frac{x}{3300}$

$$x = \frac{^{2}\cancel{6} \times 33 \; 11}{\cancel{3}_{1}\cancel{0}_{1}}$$

$$x = 22 \text{ cm}$$

Model length of ship = 22 cm.

9. Mass of aluminium rod varies directly with length

$$M \propto l.$$

Mass of 16 cm long rod has 192 g
Mass of x cm long rod has 105 g

$$\text{i.e } \frac{x}{16} = \frac{105}{192}$$

$$x = \frac{105 \times 16^{1}}{\cancel{192}\ 12}$$

$$x = 8.75 \text{ cm}$$

105g mass has length of rod is 8.75cm

10. Given Map scale 1cm = 20 km.

Anita measures a distance from village A to village B is 3.5 cm

Actual distance between them is 3.5×20

$= 35 \times 2$

$= 70$ km.

11. i) Height of water tank is 23m 75cm ie 2375 cm
Length of shadow is 20m ie 2000 cm

if height of tree is 9m50cm i.e 950cm

Length of shadow is "x" = ?

i.e $\frac{x}{2000} = \frac{950}{2375}$

$x = \frac{950 \times 2000}{2375}$

$x = 800$ cm i.e 8m

∴ Length of shadow is 8m

ii) if Length of shadow is 12m, 1200 cm

Height of tree = ? Let = "x" cm

$\frac{x}{2375} = \frac{1200}{2000}$

$x = \frac{1200 \times 2375}{2000}$

$x = 1425$ cm i.e 14m 25cm

Height of tree is 14m 25cm high

12 Earning of 5 men = Earning of 7 women

Earning of 1 man = earning of $\frac{7}{5}$ woment

Earning of 10 men = earning of $\left(\frac{7}{5} \times 10^{2}\right)$ women

= earning of 14 women.

Earning of 10 men and 13 women = earning of (14+13) women
= earning of 27 women

Let 10 men and 13 women ie 27 women earn ₹ x in a day

Note that more women will earn more per day

Hence, it is a case of direct variation

$$\therefore \quad \frac{7}{525} = \frac{27}{x} \Rightarrow 7x = 525 \times 27$$

$$x = \frac{525 \times 27}{7} = 2025$$

Hence, 10 men and 13 women will earn ₹ 2025 in a day

EXERCISE : 9.2

1

ii) More speed, less time taken by train to cover a fixed distance

i.e Speed and Time are inversely proportional

iv) More people at work, less time taken to finish work

Less people at work, more time takes to finish work

i.e This is a inverse variation

2

i) $90 \times 10 = 900$; $15 \times 60 = 900$; $45 \times 20 = 900$; $30 \times 30 = 900$; $20 \times 45 = 900$.

i.e $90 \times 10 = 15 \times 60 = 20 \times 45 = 30 \times 30 = 45 \times 20 = 900 =$ Constant

i.e $xy =$ Constant

$\therefore$ x and y are in inverse variation

ii) $75 \times 10 = 750$; $45 \times 30 = 1350$; $30 \times 25 = 750$; $20 \times 35 = 750$; $10 \times 65 = 650$.

i.e $75 \times 10 \neq 45 \times 30 \neq 30 \times 25 = 20 \times 35 \neq 10 \times 65 \neq$ Constant

$\therefore$ x and y are not in inverse variation

3. Given Volume inversely proportional to pressure

$$V \propto \frac{1}{P}$$

i.e PV = Constant

Volume of a gas = 630 cm^3 = V_1

pressure of mercury = 360 mm = P_1

if Volume is 720 cm^3 = V_2, pressure = ? P_2 = ?

$$P_1V_1 = P_2V_2$$

$$360 \times 630 = 720 \times P_2$$

$$315 \qquad 2$$

$$P = 315 \text{ mm}$$

∴ pressure of mercury = 315 mm.

4. No of children are 20.

Each received 4 sweets

∴ Total no of sweets are 20 × 4 = 80 sweets

Given number of children reduced by 4 i.e 20 − 4 = 16.

Now 16 children were present

∴ It is a case of inverse variation

∴ Each children get $\frac{80}{16}$ sweets i.e 5 sweets

5. Pooja has money to buy 36 oranges at the rate of ₹4.5 per orange i.e

She has money ₹ 36×4.5

$= ₹ 162$

Now the price of orange is increased by 90 paise

i.e New price of orange is $4.50 + 0.90 = ₹ 5.40$.

Now she takes only $\frac{162}{5.4}$ oranges

i.e 30 oranges only

6. In 8 days, number of men required to construct a wall = 12

In 6 days, how many were required ?

i.e No. of men required = x.

i.e $\frac{x}{12} = \frac{8}{6}$

$x = \frac{8 \times 12}{6}$

$x = 16$

$\therefore$ 16 men required to construct a wall in 6 days

7. Total no. of taps = 8

These takes 27 minutes to fill a tank

Out of 8, 2 taps go out of order i.e

the remaining no of taps are 6

Let the 6 taps takes x minutes

Note that lesser the number of pipes, more will be the time required to fill the tank

So, it is a case of inverse variation

$$8 \times 27 = x \times 6$$

$$x = \frac{4\cancel{8} \times 27}{\cancel{6}}$$

$$x = 36 \text{ minutes}$$

∴ Time taken to fill the tank by 6 Taps is 36 minutes

8. No. of person Contract to Complete a part of Stadium in 9 months = 560 persons.

Let the no of persons to Complete a part of Stadium in 5 months = x.

More month, less persons are required

i.e It is a Case of inverse variation

$9 \times \overset{70}{560} = 8 \times x$

$x = 9 \times 70$

$x = 630.$

Now, no. of persons extra required is $630 - 560 = 70$ persons

9. A batch of bottles were packed in 30 boxes with 10 bottles in each box.

Now 12 bottles in each box can be filled or packed in how many boxes, Let it be x.

more boxes, less bottles were packed

i.e it is a case of inverse variation

$\therefore 30 \times 10 = 12 \times x$

$x = \frac{\overset{5}{\cancel{30}} \times 10^{5}}{\cancel{12}}$

$x = 25$

$\Rightarrow$ 25 boxes with 12 bottles in each box.

10. Vandana takes 24 minutes to reach school with a speed of 5 km/h

Now, how much speed is required to reach school with in 20 minutes

More time, less speed is required

i.e It is a case of inverse variation

∴ let the speed be x

$$5 \times 24 = x \times 20$$

$$x = \frac{5 \times 24}{20}^{6}$$

$$x = 6 \text{ km/h}$$

∴ Speed required = 6 km/h

11. After 15 days, the food for 80 soldiers for (60 − 15) days
i.e 45 days

So the number of soldiers in ford = 80 + 20 = 100.

Let the ford last for x days, when there are 100 soldiers in ford.

Note that more the number of soldiers in ford, the sooner the food exhaust

$80 \times 45 = 100 \times x$

$$x = \frac{80 \times 45}{100}$$

$x = 36$.

∴ The food will last for 36 days

12 1200 soldiers in a fort had enough food for 28 days.

After 4 days, some soldiers were sent be x.

∴ The food lasted for 32 more days

After 4 days, the food for 1200 soldiers for $(28-4)$ days is 24 days

∴ The number of soldiers in fort $= 1200 - x$

It is a case of inverse variation

$1200 \times 24 = (1200 + x) \times 32$

$$1200 - x = \frac{1200 \times 24}{32}$$

$1200 - x = 900$

$x = 1200 - 900 = 300$

∴ No of soldier were left $= 300$ Soldiers

EXERCISE : 9.3.

1. Farmer can reap a field in 10 days
His wife can reap it in 8 days

Farmer's one day work i.e reap a field is $\frac{1}{10}$.
His wife one day work is $\frac{1}{8}$

Farmer and wife can completed if they work together

i.e $\frac{1}{10} + \frac{1}{8}$

$= \frac{4+5}{40} = \frac{9}{40}$

$\therefore$ Both together can complete in $\frac{40}{9}$ days

2. Since A can complete $\frac{1}{5}$th of work in 2 days

$\therefore$ A's one day work $= \frac{1}{2}$ of $\frac{1}{5} = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

Since B can complete $\frac{2}{3}$rd of work in 8 days

$\therefore$ B's one day work $= \frac{2}{3}$ of $\frac{1}{8} = \frac{2}{3} \times \frac{1}{84} = \frac{1}{12}$

One day's work of A and B together $= \frac{1}{10} + \frac{1}{12}$

$= \frac{6+5}{60} = \frac{11}{60}$.

$\therefore$ A and B working together can complete the work in $\frac{60}{11}$ days

3. 'A' tap can fill a tank in 20 minutes

'A' tap in one minutes can fill $\frac{1}{20}$ th of tank

'B" Tap can fill a tank in 12 minutes

In one minute, Tap B can fill $\frac{1}{12}$ th of Tank

If both taps were opened then

In one minutes, Tap A and B can fill $= \frac{1}{20} + \frac{1}{12}$

$= \frac{3+5}{60}$

$= \frac{8}{60}$

∴ Both A and B will fill the tank in $\frac{60}{8}$ minutes

4. A can do a work in 6 days

B can do a work in 8 days.

A's one day work $= \frac{1}{6}$

B's one day work $= \frac{1}{8}$.

one day's work of A and B Together $= \frac{1}{6} + \frac{1}{8}$

$= \frac{4+3}{24} = \frac{7}{24}$

∴ 2 day's work of A and B Together $= 2 \times \frac{7}{24} = \frac{7}{12}$

$\therefore$ Remaining work $= 1 - \frac{7}{12} = \frac{5}{12}$

$\therefore$ The no. of days taken by A to finish the remaining work

$$= \frac{\text{work to be done}}{\text{A's one day work}} = \frac{5/12}{1/6} = \frac{5}{12} \times 6 = \frac{5}{2} \text{ days}$$

Hence, A will complete the remaining work in $\frac{5}{2}$ days

5. A can do a piece of work in 40 days

A's one day work $= \frac{1}{40}$.

He works for 8 days, he completes $8 \times \frac{1}{40} = \frac{1}{5}$ th work

Remaining work $= 1 - \frac{1}{5} = \frac{4}{5}$.

B finishes remaining work in 16 days

ie B finishes $\frac{4}{5}$th work in $16 \times \frac{5}{4}$ days

$= 20$ days

A and B can completed in $= \frac{1}{20} + \frac{1}{40}$

$= \frac{2+1}{40} = \frac{3}{40}$.

$\therefore$ It takes $\frac{40}{3}$ days if they do together.

6. A and B Separately do work in 10 and 15 days

A's one day work = $\frac{1}{10}$ days

B's one day work = $\frac{1}{15}$ days

A and B one's day work = $\frac{1}{10} + \frac{1}{15}$

$= \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$.

A Completed the remaining work in 5 days

i.e $\frac{1}{2}$ half of the work had Completed

Remaining half work had Completed by A and B together

i.e $\frac{1}{2} \times 6 = 3$ days

7 3 women or 5 girls take 17 days to Complete a piece of work

Since 3 women's work = 5 girl's work

$\therefore$ 1 woman's work = $\frac{5}{3}$ girl's work

$\therefore$ 7 women work = $\frac{5}{3} \times 7$ i.e $\frac{35}{3}$ girl's work.

$\therefore$ 7 women and 11 girls work = $\frac{35}{3} + 11$ i.e $\frac{68}{3}$ girl's work

Since 5 girls can do work in 17 days,

$\therefore$ 1 girl can do work in 5×17 i.e 85 days

$\frac{68}{3}$ girls can do the work in $\frac{85}{68/3}$ days. $\frac{85 \times 3}{68}$ days

$= \frac{15}{4}$ days

Hence 7 women and 11 girls working together will complete the work in $\frac{15}{4}$ days

8. A's one day work $= \frac{1}{10}$

B's one day work $= \frac{1}{15}$

So they divide money in the ratio $\frac{1}{10} : \frac{1}{15}$

i.e $\frac{1}{10} \times 30 : \frac{1}{15} \times 30$ i.e $3:2$, Sum $= 3+2 = 5$

A's share $= \frac{3}{5} \times 3500 = 3 \times 700 = 2100$

B's share $= \frac{2}{5} \times 3,500 = 2 \times 700 = 1400$

9. A's one day work $= \frac{1}{2}$

B's one day work $= \frac{1}{6}$

C's one day work $= \frac{1}{3}$

A, B, C one's day's work together $= \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$= \frac{3+2+1}{6} = \frac{6}{6} = 1$

A, B, C Completed their work by working together in 1 day

So they divide the money in the ratio $\frac{1}{2} : \frac{1}{6} : \frac{1}{3}$.

i.e $\frac{1}{2} \times 6 : \frac{1}{6} \times 6 : \frac{1}{3} \times 6$

$= 3 : 1 : 2$

Sum of these terms $= 3+1+2 = 6$.

A's share $= \frac{3}{6} \times 960 = 3 \times 160 = 480$

B's share $= \frac{1}{6} \times 960 = 1 \times 160 = 160$

C's share $= \frac{2}{6} \times 960 = 2 \times 160 = 320$

10. A and B and C together do a piece of work in 15 days

One's day's work of A, B and C together is $\frac{1}{15}$

B's one day work $= \frac{1}{30}$

C's one day work $= \frac{1}{40}$

A's one day work $= \frac{1}{15} - \left\{\frac{1}{30} + \frac{1}{40}\right\}$

$= \frac{1}{15} - \left\{\frac{4+3}{120}\right\}$

$= \frac{1}{15} - \frac{7}{120}$

$$= \frac{8-7}{120}$$

A's one day work $= \frac{1}{120}$

$\therefore$ A alone do the work in 120 days

11. A's one day + B's one day + C's one day $= \frac{5}{24}$

A's one day + C's one day $= \frac{1}{8}$

B's one day work $= \frac{5}{24} - \frac{1}{8}$

$$= \frac{5-3}{24}$$

$$= \frac{2}{24} = \frac{1}{12}$$

$\therefore$ B' alone can plough field in 12 days

12 A's one day work + B's one day work $= \frac{1}{10}$

B's one day work + C's one day work $= \frac{1}{15}$

C's one day work + A's one day work $= \frac{1}{12}$

2(A's one day work + B's one day work + C's one day work)

$$= \frac{1}{10} + \frac{1}{15} + \frac{1}{12}$$

$= \frac{6+4+5}{60} = \frac{15}{60} = \frac{1}{4}$

A's one day work + B's one day work + C's one day work $= \frac{1}{4} \times \frac{1}{2}$

$= \frac{1}{8}$

$\therefore$ A, B, C together can complete in 8 days

A's one day work $= \frac{1}{8} - \frac{1}{15}$

$= \frac{15-8}{120} = \frac{7}{120}$

$\therefore$ A alone takes $\frac{120}{7}$ days

B's one day work $= \frac{1}{8} - \frac{1}{12}$

$= \frac{3-2}{24} = \frac{1}{24}$

B alone takes 24 days

C's one day work $= \frac{1}{8} - \frac{1}{10}$

$= \frac{5-4}{40} = \frac{1}{40}$

C alone takes 40 days

13. Pipe fill a tank in 12 hours

In one hour, pipe fills $\frac{1}{12}$ of the tank

A waste pipe is left opened and filled in 16 hours

In one hour, it fills $\frac{1}{16}$ of tank

∴ portion of wastank emptied by the waste pipe in one hour $= \frac{1}{12} - \frac{1}{16}$

$$= \frac{4-3}{48}$$

$$= \frac{1}{48}$$

∴ Waste pipe takes 48 hours to empty the tank