Exponents and Powers

Exercise 2.1

Solution 1

(i) $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2}$ $\left(\because \left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^{m}\right)$

$= \frac{5 \times 5}{3 \times 3}$

$= \frac{25}{9}$

(ii) $(-3)^{-3} = \frac{1}{(-3)^{3}}$ $\left(\because (a)^{-m} = \frac{1}{(a)^{m}}\right)$

$= \frac{1}{(-3) \times (-3) \times (-3)}$

$= \frac{1}{-27} = -\frac{1}{27}$

(iii) $\left(\frac{2}{7}\right)^{-4} = \left(\frac{7}{2}\right)^{4}$

$= \frac{7 \times 7 \times 7 \times 7}{2 \times 2 \times 2 \times 2}$

$= \frac{2401}{16}$

Solution 2

(i) $\left[(2)^{-1}+(4)^{-1}+(3)^{-1}\right]^{-1} = \left[\frac{1}{2}+\frac{1}{4}+\frac{1}{3}\right]^{-1} \quad \left(\because (a)^{-m} = \frac{1}{a^m}\right)$

$= \left[\frac{6+3+4}{12}\right]^{-1}$

$= \left(\frac{13}{12}\right)^{-1}$

$= \frac{12}{13}$

(ii) $\left[(4)^{-1}-(5)^{-1}\right]^2 \times \left(\frac{5}{8}\right)^{-1} = \left(\frac{1}{4}-\frac{1}{5}\right)^2 \times \left(\frac{8}{5}\right)^1$

$= \left(\frac{5-4}{20}\right)^2 \times \frac{8}{5}$

$= \frac{1}{20\times 20} \times \frac{8}{5}$

$= \frac{1}{250}$

(iii) $\left[4^0+4^2-2^3\right] \times 3^{-2} = \left[1+16-8\right] \times \frac{1}{3^2} \quad \left[\because a^0 = 1\right]$

$= (9) \times \frac{1}{9}$

$= 1.$

(iv) $\left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2} = \left[(25) - (4)^2\right] \times \left(\frac{4}{3}\right)^2$

$= [25 - 16] \times \frac{16}{9}$

$= 9 \times \frac{16}{9}$

$= 16.$

Solution 3.

(i) As the multiplicative inverse of $a^m = a^m$,

$\therefore$ the multiplicative inverse of $\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{81}{16}\right)^{\frac{3}{4}}$

$\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2}\right)^{\frac{3}{4}}$

$= \left(\frac{3^4}{2^4}\right)^{\frac{3}{4}}$

$= \left(\frac{3}{2}\right)^{4 \times \frac{3}{4}}$

$= \left(\frac{3}{2}\right)^3 = \frac{3 \times 3 \times 3}{2 \times 2 \times 2}$

$= \frac{27}{8}.$

(ii) $\left\{\left(\frac{-3}{2}\right)^{-4}\right\}^{1/2} = \left\{\left(\frac{-3}{2}\right)^{-4\times\frac{1}{2}}\right\}$

$= \left(\frac{-3}{2}\right)^{-2}$

Multiplicative inverse of $\left(\frac{-3}{2}\right)^{-2} = \left(\frac{-3}{2}\right)^{2}$

$\left(\frac{-3}{2}\right)^{2} = \frac{-3\times-3}{2\times2} = \frac{9}{4}$

(iii) $\left(\frac{5}{7}\right)^{-2} \times \left(\frac{5}{7}\right)^{4} \div \left(\frac{5}{7}\right)^{3} = \left(\frac{5}{7}\right)^{-2+4-3}$ $\left(\because a^m \times a^n = a^{m+n},\ a^m \div a^n = a^{m-n}.\right)$

$= \left(\frac{5}{7}\right)^{-1}$

Multiplicative inverse of $\left(\frac{5}{7}\right)^{-1} = \left(\frac{5}{7}\right)$.

Solution 4.

(i) $(16)^{-2} = (2\times2\times2\times2)^{-2}$

$= (2^4)^{-2}$

$= 2^{4\times-2}$ $\left(\because (a^m)^n = a^{mn}\right)$

$= 2^{-8}$.

(ii) $(125)^{-4} = (5\times5\times5)^{-4}$

$= \left((5)^3\right)^{-4}$

$= 5^{3\times(-4)}$

$= 5^{(-12)}$

Solution 5

(i) $2789.453 = 2\times1000 + 7\times100 + 8\times10 + 9\times1 + \frac{4}{10} + \frac{5}{100} + \frac{3}{1000}$

$= 2\times10^3 + 7\times10^2 + 8\times10^1 + 9\times10^0 + 4\times10^{-1} + 5\times10^{-2} + 3\times10^{-3}$

(ii) 3007.805

$= 3\times1000 + 0\times100 + 0\times10 + 7\times1 + \frac{8}{10} + \frac{0}{100} + \frac{5}{1000}$

$= 3\times10^3 + 0\times10^2 + 0\times10^1 + 7\times10^0 + 8\times10^{-1} + 0\times10^{-2} + 5\times10^{-3}$.

Solution 6

(i) $\left[\left\{\left(\frac{5}{7}\right)^2\right\}^{-1}\right]^{-3} = \left(\frac{5}{7}\right)^{2\times-1\times-3} \quad \left(\because (a^m)^n = a^{mxn}\right)$

$= \left(\frac{5}{7}\right)^6$

(ii) $\left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

$= \left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^3 \div \left(\frac{7}{5}\right)^{(-2)\times(-4)}$ $\left[\because \left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^m\right]$

$= \left(\frac{2}{7}\right)^5 \div \left(\frac{7}{5}\right)^8$

$= \left(\frac{2}{7}\right)^5 \times \left(\frac{5}{7}\right)^8$

$= \frac{2^5 \times 5^8}{7^5 \times 7^8} = \frac{2^5 \times 5^8}{7^{(5+8)}}$ $(\because a^m \times a^n = a^{m+n})$

$= \frac{2^5 \times 5^8}{7^{13}}$

(iii) $\left(\frac{4}{5}\right)^2 \times (5)^4 \times \left(\frac{2}{5}\right)^{-2} \div \left(\frac{5}{2}\right)^{-3}$

$= \left(\frac{4}{5}\right)^2 \times (5)^4 \times \left(\frac{5}{2}\right)^2 \div \left(\frac{2}{5}\right)^3$

$= \frac{4^2 \times 5^4}{5^2} \times \frac{5^2}{2^2} \times \frac{5^3}{2^3}$

$= \frac{5^{4+2+3-2} \times (2^2)^2}{2^{2+3}} = \frac{5^7}{2^{5-4}} = \frac{5^7}{2^1}$

(iv) $\dfrac{8^{-1} \times 5^3}{2^{-4}} = \dfrac{(2^3)^{-1} \times 5^3}{2^{-4}}$

$= 2^{-3+4} \times 5^3$

$= 5^3 \times 2^1$.

Solution 7

(i) $\left((-2)^3\right)^2 + (5)^{-3} \div (5)^{-5} - \left(-\frac{1}{2}\right)^0$

$= (-2)^{3\times 2} + (5)^{-3} \times (5)^{+5} - 1 \quad \left(a^0 = 1 \; ; (a^m)^n = a^{mn}\right)$

$= 2^6 + 5^2 - 1$

$= 64 + 25 - 1$

$= 88$

$= 8 \times 11$

$= 2^3 \times 11$.

(ii) $3^{-5} \times 3^2 \div 3^{-6} + \left(2^2 \times 3^1\right)^2 + \left(\frac{2}{3}\right)^{-1} + 2^{-1} + \left(\frac{1}{19}\right)^{-1}$

$= 3^{-5+2} \times 3^6 + 2^4 \times 3^2 + \left(\frac{3}{2}\right)^1 + \left(\frac{1}{2}\right)^1 + (19)^1$

$= 3^{-3+6} + 2^4 \times 3^2 + \frac{3+1}{2} + 19$

$= 3^3 + 2^4 \times 3^2 + \frac{4}{2} + 19$

$= 27 + 144 + 2 + 19$

$= 192 = 3 \times 64$

$= 2^6 \times 3.$

Solution 8

(i) $5^3 \times \left(\frac{4}{5}\right)^3 = 5^3 \times \frac{4^3}{5^3}$

$= 4^3 \times 5^{3-3}$

$= 4^3$

$= \frac{1}{(4)^{-3}}$

(ii) $\left[\left(\frac{3}{7}\right)^{-2}\right]^{-3} = \left(\frac{3}{7}\right)^{-2\times-3}$ $\left(\because (a^m)^n = a^{m\times n}\right)$

$= \left(\frac{3}{7}\right)^{6}$

$= \left(\frac{7}{3}\right)^{-6}$

(iii) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^{2} \div \left(\frac{1}{5}\right)^{-2} = \frac{\left(\frac{9}{5}\right)^{2} \times \left(\frac{5}{3}\right)^{2}}{\left(\frac{1}{5}\right)^{-2}}$

$= \frac{9^2}{5^2} \times \frac{5^2}{3^2} \times \frac{1^2}{5^2}$

$= 3^{4-2} \times 5^{2-4}$

$= 3^{+2} \times 5^{-2} = 5^{-2} \times \frac{1}{(3)^{-2}}$

$= \left(\frac{5}{3}\right)^{-2}$

(iv) $2^{-1}\left[\left(\frac{5}{3}\right)^4+\left(\frac{3}{5}\right)^{-2}\right] \div \frac{17}{9}$

$= \frac{1}{2}\left[\left(\frac{5}{3}\right)^4+\left(\frac{5}{3}\right)^2\right] \times \frac{9}{17}$

$= \frac{1}{2}\left[\left(\frac{5}{3}\right)^2\left[\left(\frac{5}{3}\right)^2+1\right]\right] \times \frac{9}{17}$

$= \frac{1}{2}\left[\frac{25}{9}\left(\frac{25}{9}+1\right)\right] \times \frac{9}{17}$

$= \frac{1}{2}\left[\frac{25}{9}\left(\frac{25+9}{9}\right)\right] \times \frac{9}{17}$

$= \frac{1}{2}\left[\frac{25}{9} \times \frac{34}{9}\right] \times \frac{9}{17}$

$= \frac{1}{2}\left[\frac{25}{9} \times \frac{17 \times 2}{9} \times \frac{9}{17}\right]$

$= \frac{5^2}{3^2} = \left(\frac{5}{3}\right)^2$

$= \left(\frac{3}{5}\right)^{-2}$

(v) $(-7)^3 \times \left(\frac{1}{-7}\right)^{-9} \div (-7)^{10}$

$= (-7)^3 \times (-7)^9 \div (-7)^{10}$

$= \frac{(-7)^3 \times (-7)^9}{(-7)^{10}}$

$= (-7)^{3+9-10}$

$= (-7)^2 = (7)^2$

$= \left(\frac{1}{(-7)}\right)^{-2} = \left(\frac{1}{7}\right)^{-2}$

$= \frac{1}{(+7)^2}$

Solution 9

(i) $\frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} = \frac{7^2 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}}$

$= \frac{7^{2+3} \times z^{-3+5}}{10}$ $\left(\because \frac{a^m}{a^n} = a^{m-n}\right)$

$= \frac{7^5 \times z^2}{10}$

(ii) $\frac{9^3 \times 27 \times t^4}{(3)^2 \times (3)^4 \times t^2} = \frac{(3^2)^3 \times 3^3 \times t^4}{(3)^2 \times (3)^4 \times t^2}$

$= 3^6 \times 3^3 \times t^4 \times 3^{-2} \times 3^{-4} \times t^{-2}$

$= 3^{6+3-2-4} \times t^{4-2}$ $\left(\because a^m \times a^n = a^{m+n}\right)$

$= 3^3 \times t^2$

$= 27 \times t^2$

(iii) $\frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3} = \frac{3^{-4} \times 5^{-6} \times t^{-6}}{3^{-10} \times 5^{-6} \times t^{-12}}$ $\left(\because (a^m)^n = a^{mxn}\right)$

$= 3^{-4+10} \times 5^{-6+6} \times t^{-6+12}$ $\left(\because \frac{a^m}{a^n} = a^{m-n}\right)$

$= 3^6 \times 5^0 \times t^6$

$= 3^6 \times t^6$ $\left(\because a^0 = 1\right)$

(iv) $\frac{2^{-5} \times 15^{-5} \times 50^2}{5^{-6} \times 6^{-5}} = \frac{2^{-5} \times (3 \times 5)^{-5} \times 5^3 \times 2^2}{5^{-6} \times (2 \times 3)^{-5}}$

$= \frac{2^{-5+2} \times 3^{-5} \times 5^{-5} \times 5^3}{5^{-6} \times 2^{-5} \times 3^{-5}}$

$= 2^{-5+2+5} \times 3^{-5+0+5} \times 5^{-5+6+3} = 2^2 \times 3^0 \times 5^4$

$= 2500$.

Solution 10

Let the number which should divide $\left(\frac{3}{-2}\right)^{-3}$ to get $\left(\frac{2}{3}\right)^{2}$ be x.

$$\Rightarrow \frac{\left(\frac{3}{-2}\right)^{-3}}{x} = \left(\frac{2}{3}\right)^{2}$$

$$\Rightarrow \frac{3^{-3}}{(-2)^{-3}} = \frac{x \times 2^{2}}{3^{2}}$$

cross multiply

$$\Rightarrow 3^{-3} \times 3^{2} = x \times 2^{2} \times (-2)^{-3}$$

$$\Rightarrow 3^{-3+2} = x \times (2)^{2-3} \qquad (\because (-a)^{2} = (a)^{2})$$

$$\Rightarrow 3^{-1} = x \times (-2)^{-1}$$

$$\Rightarrow x = \frac{3^{-1}}{(-2)^{-1}}$$

$$\therefore x = -2/3$$

Solution 11:

$$9^m \div 3^{-2} = 9^4.$$

$$\Rightarrow \frac{9^m}{3^{-2}} = 9^4$$

$$\Rightarrow 9^m \times 9 = 9^4 \qquad \left(\because \frac{1}{a^m} = a^{-m}\right)$$

$$\Rightarrow 9^{m+1} = 9^4$$

If $a^m = a^n$ then $m = n$

$$\Rightarrow m+1 = 4$$

$$\therefore m = 3.$$

Solution 12

$$\left(\frac{-5}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{12} = \left[\left(\left(\frac{-5}{7}\right)^3\right)\right]^x \times \left(\frac{-5}{7}\right)^{-1}.$$

$$\Rightarrow \left(\frac{-5}{7}\right)^{-4+12} = \left(\frac{-5}{7}\right)^{3x-1}$$

$$\Rightarrow -4+12 = 3x-1 \qquad \left(\because a^m = a^n \Rightarrow m = n\right)$$

$$\Rightarrow 3x = 9$$

$$\therefore x = 3.$$

Solution 13

$$\left(\frac{-2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^{8} = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-13} \times \left(\frac{-2}{3}\right)^{-8} = \left(\frac{-2}{3}\right)^{-2x+1} \qquad \left(\because \left(\frac{p}{q}\right)^{m} = \left(\frac{q}{p}\right)^{-m}\right)$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-13-8} = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\therefore -21 = -2x+1 \qquad (\because a^m = a^n \text{ then } m = n)$$

$$\Rightarrow -2x = -22$$

$$\therefore x = 11.$$

Solution 14

(i) $5^{2x-1} = \dfrac{1}{(125)^{x-3}} \Rightarrow 5^{2x-1} = \dfrac{1}{(5^3)^{x-3}}$

$$\Rightarrow 5^{2x-1} \times 5^{3(x-3)} = 1$$

$$\Rightarrow 5^{(2x-1)+3(x-3)} = 5^{0} \qquad (\because a^0 = 1)$$

$$\therefore 2x - 1 + 3x - 9 = 0$$

$$\Rightarrow 5x = 10$$

$$\therefore x = 2.$$

(ii) $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$

$\Rightarrow \frac{(3^2)^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3$

$\Rightarrow 3^{(2n+5+9)(-1-16)} = 3^3$

$\Rightarrow 3^{2n-3} = 3^3$

$\therefore 2n-3 = 3$

$\Rightarrow n = 3.$

Exercise 2.2

Solution 1

(i) $0.0000000000085 = \frac{85}{(10)^{13}}$

$= 8.5 \times 10^{-12}.$

(ii) $0.00000000000942 = \frac{942}{(10)^{15}}$

$= 9.42 \times 10^{-13}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}.$

Solution - 2

i) 3.02×10^{-6}

0.00000302

ii) 1.007×10^{11}

100700000000

iii) 5.375×10^{14}

537500000000000

iv) $7.579 \times 10^{-14} = 7.579 \times 10^{-14}$

0.00000000000007579

Solution - 3

i) The mass of a proton is 1.673×10^{-24} gram

ii) Thickness of a piece of paper is 1.6×10^{-3} cm

iii) Diameter of a wire on a Computer chip is 3×10^{-6} m

iv) A helium atom has a diameter of 32×10^{-11} m

v) Mass of a molecule of hydrogen gas is about 3.34×10^{-21} tons

vi) Human body has 10^{+12} of cells which vary in shapes and sizes

vii) The distance from earth to the Sun is 149.6×10^{9} m

viii) The speed of light is 3×10^{8} m/sec

ix) Mass of the Earth is 5.97×10^{24} kg

x) 3 years = 3×365 days

$= 3 \times 365 \times 24$ Hours

$= 3 \times 365 \times 24 \times 60$ Minutes

$= 3 \times 365 \times 24 \times 60 \times 60$ Seconds

$= 94608000$ seconds

3 years = 9.4608×10^7 seconds

xi) 7 hectares = $7 \times 10{,}000\ m^2$

$= 7 \times 10000 \times 10000\ cm^2$

7 hectares = $7 \times 10^8\ cm^2$

xii) A sugar factory has annual sales of 3,720,000,000 kilograms of sugar.

A sugar factory has annual sales of 3.72×10^9 kg of sugar.

4. Solution 4

Given

Size of plant cell = $0.00001275\ m = 1.275 \times 10^{-5}\ m$

Thickness of piece of paper = $0.0016\ cm = 1.6 \times 10^{-5}\ m$

Diameter of a wire on a Computer chip

$= 0.000003\ m = 3 \times 10^{-6}\ m$

i) $\frac{\text{Size of plant cell}}{\text{Thickness of piece of paper}}$

$= \frac{1.275 \times 10^{-5}}{1.6 \times 10^{-5}} = 0.796 \approx 0.8$

∴ Size of plant cell 0.8 times the Thickness of piece of paper

ii) $$\frac{\text{Size of plant cell}}{\text{Diameter of wire on computer chip}}$$

$$\frac{1.275 \times 10^{-5}}{3 \times 10^{-6}}$$

$$4.25$$

∴ Size of plant cell is 4.25 times bigger than the diameter of wire on computer chip.

iii) $$\frac{\text{Thickness of piece of paper}}{\text{Diameter of wire on computer chip}}$$

$$\frac{1.6 \times 10^{-5}}{3 \times 10^{-6}}$$

$$5.33$$

∴ Thickness of piece of paper is 5.33 times bigger than the diameter of wire on computer chip

8. Solution 5

no. of red blood cells per cubic millimeter

$= 5.5 \times 10^6 / mm^3$

$= 5.5 \times 10^6 / m.m^3$

total no. of red blood cells in 5 liters of blood

$= 5.5 \times 10^6 / mm^3 \times 5 \text{ lit}$

$= 5.5 \times 10^6 / mm^3 \times 5 \times 10^5 \ mm^3$

$= 5.5 \times 5 \times 10^6 \times 10^5$

$= 27.5 \times 10^{11}$ //

Solution 6

Mass of Mars = 6.42×10^{29} Kg

Mass of Sun = 1.99×10^{30} Kg

Total Mass = Mass of Mars + Mass of Sun

$= 6.42 \times 10^{29} + 1.99 \times 10^{30}$

$= 0.642 \times 10^{30} + 1.99 \times 10^{30}$

Total Mass = 2.632×10^{30} Kg (∵ make powers equal in both terms)

∴ Total mass = 2.632×10^{30} Kg

Solution 7

distance between star and Earth = 8.1×10^{13} Km

$= 8.1 \times 10^{13} \times 10^{3}$ m

$= 8.1 \times 10^{16}$ m

Speed of light = 3×10^{8} m/sec

$$\text{Time} = \frac{\text{distance}}{\text{Speed}}$$

$$= \frac{8.1 \times 10^{16}}{3 \times 10^{8}}$$

Time = 2.7×10^{8} Sec